Relative space-time scaling of electromagnetic soundings arrays

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Abstract: There are many methods for the study of near-surface applied geophysics. This study will not only help understand how the main geoelectrical methods works (DC and MT) but also improve the technique used for analysis, combining the best of both methods and takes advantage of the best each. I test an empirical relation between the DC and MT data on synthetic models and I applied it on real data. A discussion of the limits and utility of this relationship is also presented.

I. INTRODUCTION

There are different electromagnetic methods in geophysics used to characterize the subsoil, each with a specific sensibility depending on penetration, but all with the same objective; to obtain the electrical properties of the subsurface for later geologic studies.

Due to technical limitations, no single electrical conductivity depth-sounding technique provides complete, consistent and sufficient data to characterize the subsurface. The integration of electrical and electromagnetic data can improve the robustness of model interpretation and the cumulative probability of detection of subsurface targets.

A. Electrical Resistivity method

The Direct Current (DC) resistivity method (also named electrical resistivity method) has a long history in applied geophysics and has been one of the most important methods for the subsurface studies. Surface electrical resistivity is based on the principle that the distribution of electrical potential in the ground around a current-carrying electrode depends on the electrical resistivities and distribution of the surrounding soils and rocks. The fundamental steps involved in this method may be outlined as follows. When an electrical direct current $I$ [A] is applied between two electrodes (Fig. 1 electrodes A and B) implanted in the ground and the difference of potential $V$ [V] is measured between two additional electrodes that do not carry current (Fig. 1 electrodes P and Q), the impedance of the ground $Z=V/I$ [V/A] is known. This impedance is then transformed into an apparent resistivity $\rho_a[\Omega m]$ which is an indicator of the electrical resistivity structure of the ground. Apparent resistivity is obtained under the false assumption that the Earth has a uniform resistivity $\rho$.

Apparent resistivity is interpreted to be the resistivity that would have been measured if the Earth was in fact homogeneous and it can be described like $\rho_a = \kappa \cdot Z$. Being $Z$ the impedance and $\kappa$ a geometric factor that depends only on the arrangement of the four electrodes used in the method.

Different arrangement of the electrodes allows the apparent resistivity being determined at different depths and lateral positions.

The choice of the best array for a field survey depends on the type of structure to be mapped, the sensitivity of the resistivity meter and the background noise level. In practice, the arrays that are most commonly used for 2-D imaging surveys are the Schlumberger, Wenner Array dipole-dipole and pole-dipole. Among the characteristics of an array that should be considered are (i) the sensitivity of the array to vertical and horizontal changes in the subsurface resistivity, (ii) the depth of investigation, (iii) the horizontal data coverage, (iv) the signal strength and (v) the easiest to deploy.

The Schlumberger array will be detailed below because is the one used for the obtaining of the experimental data in this study.

![Schlumberger array](image)

Fig. 1: Schlumberger array [1].

A Schlumberger sounding can achieve excellent depth penetration with sufficiently large $AB$ separations. The array has limited lateral resolution and since it is designed for vertical sounding, it is named VES (Vertical Electrical Sounding). The geometric factor [1] for the Schlumberger array is $\kappa = (n - 1)(n + 1)\pi a/2$.

B. Magnetotelluric method

The magnetotelluric method (MT) is an electromagnetic geophysical technique that determines ground electrical
resistivity distribution from the simultaneous measurements of the fluctuations of the natural electromagnetic field. The relationship between the electric, $E$, and magnetic, $H$, fields at a given frequency, $f$, are expressed as follow:

$$E(f) = Z(f) \cdot H(f)$$

Under the plane wave assumption, the relationship between the horizontal components are:

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \cdot \begin{pmatrix} H_x \\ H_y \end{pmatrix}$$

where $Z$ is the impedance tensor.

The impedance is a complex magnitude, from which it is customary to define the apparent resistivity and the phase for each component of the tensor as:

$$\rho_{aij} = \frac{1}{2\pi f \mu} |Z_{ij}(f)|^2$$

$$\varphi_{ij}(f) = \arctan \left( \frac{\text{Im}[Z_{ij}(f)]}{\text{Re}[Z_{ij}(f)]} \right)$$

where $\mu$ is the magnetic permeability, and $i, j$ denote any horizontal component [2].

II. THEORY

A. MT and VES data relationship

Correlation, comparison or integration of data from the various electrical and electromagnetic sounding techniques is a non-trivial task. For example, in the direct current (DC) resistivity method where depth sounding is achieved by varying the electrode separations, the experimental data is shown as apparent resistivity versus electrode separation. However, in the magnetotelluric (MT) method that employs natural EM field variations on the surface to probe the subsurface, the measured apparent-resistivity data is presented as a function of period (or its reciprocal, frequency).

There is no simple generalized scheme for comparing these depth-sounding arrays, and the non-specialist end-user sometimes views the experimental data obtained by these methods as disparate data sets constituting different data spaces.

A scaling relationship for MT and VES has been empirically determined [3] as,

$$T = 2\pi \mu \sigma L^2,$$  \hfill (1)

where $T$ is the MT period in seconds, $\mu$ is the magnetic permeability (taken to be equal to that of free-space: $\mu_0 = 4\pi \cdot 10^{-7} \Omega s/m$), $L$ is one-half the electrode-array length (AB) in meters, and $\rho \left( = \frac{1}{\sigma} \right)$ is the homogeneous subsurface resistivity in (\Omega m), which is only known after data inversion and is hence conveniently approximated here by apparent resistivity $\rho_a$.

Even being an empiric relation, the equation (1) works very well. It is indeed based in the equation (2), which is a semi analytic relation between MT and TEM (both are electromagnetic methods but MT works in the frequency domain (T period) and TEM is worked in the time domain (t)).

$$T \approx 3.9t,$$  \hfill (2)

and it’s later application in the empiric relation between TEM and VES valid for symmetric in-line 4-electrode arrays:

$$t = 0.5\pi \mu \sigma L^2$$  \hfill (3)

Because of this, the final relation obtained, represented in the equation (1), is a relation of great importance, especially in a practical case, where traditionally the VES and later the MT were used to study and now they are both combined to enlarge, corroborate and improve the results.

B. Methodology

In order to be able to evaluate the magnitude of this relation, a programme that generates VES and MT responses is needed for a specific layered ground model. To develop this report, two free-code commercial programmes have been used, the ZondIP2 [4] for the calculations in VES, and the ZondMT1d [5] for the MT ones. These programs solve forward and inverse problems for arbitrary arrays (in our case, data was obtained with a Schlumberger array) on the surface of horizontally-layered medium.

The method used for the getting of the different data is represented as follows, in order to be able to do the next comparative using the forward modelling.

FORWARD MODELLING

As it is shown in Figure 2, we start from a one-dimensional model, where MT and DC responses are calculated separately. In the latter, before being able to compare the results, the transformation represented in eq. (1) is needed to be applied in order to compare the results obtained by the two methods.
C. Implementation

For data evaluation, and following the diagram in figure 2, a synthetic data set to test the behaviour of the transformation and its effectivity has been generated.

Synthetic data has been generated through three different models of ground to be able to evaluate the behaviour and reliability of the transformation in different types of subsoil. All three models consist of a six layer structure with different resistivities ($\rho$) depending on the thickness of the layer ($h$) for a determined depth ($z$). The three created models are described below.

The first model (Fig. 3) is inspired by the model used by Meju, M.A. [3] to check the relation X. It is a soft model, without many changes in resistivity depth, in which we can find a first less resistant block, a central more resistive block, and finally a last block with the same resistance as the initial.

The second and the third models were created with large variation for the resistivity in order look for the limits of the transformation and its efficacy for different ranges in a more realistic approach.

In the second model (Fig. 4), it is intended to show the response of the transformation of a profile, with its variations in resistivity in depth in order to evaluate how the transformation is affected in the zones with resistance changes.

Finally, the third model (Fig. 5) is intended to study how the transformation is affected in a wide range of resistivities, including resistivities from three different magnitude orders.

After applying the equation (1) in each of the models, and comparing the results obtained with the result of applying the model directly in the program of the MT, it is obtained:

![Graph of the result comparison between VES and MT models.](image-url)
We can observe for the three models that there is in general a first and last part of the graphic, corresponding to the zones where the behaviour is asymptotic, that it is where the transformation is better adjusted. The adjustment is also suitable in the zones where the change in resistivity is very soft; however, it is worse in the zones showing more pronounced changes. Nevertheless, it can be observed that by moving a curve the transformation is well adjusted in the displaced parts.

III. APPLICATION

A. Dataset

The data used in this study were not obtained by the author, they were already obtained previously. I used a set of VES data from 1985 and a set of MT data from 2014 provided by the Institut Cartogràfic i Geològic de Catalunya (ICGC). All data was taken in an area of the Vallès, where, even being from different year, the locations of the different methods were very near and so could be grouped by station pairs (one of the VES and one of the MT).

B. Processing and inversion

Here it is shown the method used for the obtaining of the data that will be used for comparison using the inverse problem.

INVERT EXPERIMENTAL DATA

As we can see in Fig. 6, from the experimental data of the electrical sounding given, equation (1) is used for one way, and for the other the data is inverted using the ZondIP2 program, in order to obtain a model that represents them and in this way to be able to use this model in the MT ZondMT1d program so that the data may finally be compared.

C. Results

Four groups of VES data and four groups of MT data obtained in 2014 and named “MT3”, “MT2”, “MT45” and “MT4” have been analysed. In all groups a table was provided with apparent resistivities and distances AB/2 for VES or periods T for MT.

Data analysis has been completed, following the inverse problem depicted in Fig. 6. The results that have been obtained for each group of data, are shown in Figs. 7, 8, 9 and 10, where the MT data of 2014 campaign has been added for a better comparison.

Fig. 6: VES data inversion process

Fig. 7: VES, MT calculated and real MT data for the first station.

Fig. 8: VES, MT calculated and real MT data for the second station.
For all four groups (Fig. 7 to 10), it can be seen that real data adjusts much better than synthetic data previously studied. Even so, as it was also seen in synthetic data, it can also be observed that it is well adjusted for asymptotic zones but that the transformation fails in zones with more abrupt slope changes.

When using the VES data from 1985, the range evaluated in the 2014 campaign can be widened for all four stations, allowing in this way obtaining a more determined depth profile, especially for the first layers studied. This implies that the least profound zones is where VES has a better resolution.

IV. CONCLUSIONS

- The main conclusion of this study is the proved effectiveness of the proposed empiric relationship (eq. 1), to relate the Magnetotelluric Method and the Vertical Electrical Sounding.

- This relationship permits to obtaining with better resolution, giving access to a wider range of studied periods, and widening in this way the nowadays studies of MT conducted with already existing VES data.

- Finally, for the studied data it is clearly observed on the apparent resistivity curves, that in general there is an upper region (periods up to $10^{-2}$ s approx.) of low resistivities and a lower region where the resistivity is increasing. This fact represents clearly the location where the data was collected, the Vallès basin, filled with sediments (conductives layers showed on the first part of the curves up to the minimum) reaching the resistive rock basement, the bottom of the basin.

- In a posterior study, the misfits observed could be considered and probably it would be shown that they are within the expected range of error. Using a covariance matrix during the inversion, the adjustment could be improved having less misfit in the obtained results.

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