Medieval Islamic Horary Quadrants for Specific Latitudes and their Influence on the European Tradition

M. Viladrich

I. Contents

1. Introduction

Determining the hour of day or night was one of the main problems facing the Muslim astronomers of the Middle Ages. Geometric or trigonometric solutions made it possible to tell the time, either using tables compiled according to the movements of the sun or of the stars, or by using a particular formula each time that the problem had to be solved[1]. They also developed a range of instruments in their attempts to solve it. The basic formulae of spherical astronomy had been known to Muslim astronomers from the ninth century onwards. Two different mathematical methods of finding time were available: one approximate, and one exact[2]. The exact formula gives time $t(h,\delta,\phi)$ in terms of the instantaneous solar altitude h, the celestial declination δ, and the local latitude ϕ, and the solar altitude h_i or the ith seasonal hour $h_i(\delta,\phi)$. In this study I shall deal with the graphical representation of the solar altitudes at the hours, as found on the instruments known as horary quadrants.

Both surviving instruments and Arabic textual sources will be considered. The different types of horary quadrants described here could also serve religious purposes, such as regulating the time of the daylight prayers (*ẓuhr* and *ʿaṣr*). Some of them, as we will seek to demonstrate, strongly influenced the European tradition of time-keeping instruments. We shall not, however, deal systematically with all the European materials on quadrants for specific latitudes: Latin, Byzantine, Hebrew and Romance

1 For a general overview of approaches to time-keeping problems used by Muslim legal scholars and scientists see KING, D. A. [1973, 1978, 1990b, 1993, 1996].
One general remark has to be made: as far as astronomical instruments are concerned, the Islamic Middle Ages cover the period from 750 to 1900.

2 Two approximate formulae, arithmetical and trigonometric, both independent of latitude provide a reasonable approximations for most practical purposes over the geographical latitudes of the central lands of Islam and their use has been discussed in KING, D. A. [1988], and KING, D. A. & GIRKE, D. [1988]. Time-keeping functions based on the approximate formulae were calculated and applied to draw different varieties of the approximate horary quadrant. In addition, many European instruments, even if conceived for a fixed latitude, bore additional approximate markings until the seventeenth century and even later. See NORTH, J. [1981].

texts are not beyond the scope of the present paper. Nevertheless, any history of the quadrant in Europe needs to pay careful attention to the Islamic material. Treatises on quadrants were not uncommon, and these instruments were described and constructed continuously in Europe until *ca.* 1700. The European examples that we will mention represent the end of a tradition which started in Abbasid Baghdad[3].

Several classical analyses of Islamic time-keeping methods and related instruments are available, and time-keeping tables have been studied[4]. However, most of these works are now in need of revision and, in the area that specifically concerns us, no survey of Islamic or European quadrants is available[5]. This paper represents a modest contribution to the colorful history of astronomical instrumentation in medieval Islamic civilisation.

3 The practice of writing treatises on horary quadrants and constructing instruments was remarkably well established in France, Italy, Germany and Netherland, throughout the Renaissance period. Exhaustive descriptions of such instruments are to be found, among others, in van CITTERT, P. H. [1954]; DRECKER, J. [1925a & 1925b]; GUNTHER, R. T. [1932]; GIBBS, S., HENDERSON, J. A. & de SOLLA PRICE, D. [1973]; GIBBS, S. & SALIBA, G. [1984]; HIGGINS, K. [1953]; SCHMALZL, P. [1929]; SCHOY, K. [1923a & 1923b]; SÉDILLOT, J. J. [1834-1835]; SÉDILLOT, L. A. [1844]; TURNER, A. J. [1985]; ZINNER, E. [1956].

4 As far as instruments are concerned, this is the case of the French translation of a text by the 13th century Egyptian astronomer al-Marrākushī due to SÉDILLOT, J. J., [1834-35] whose second part on astronomical instruments was summarized by SÉDILLOT, L. A. [1844]. See also the works by SCHOY, K. [1923a & 1923b] and SCHMALZL, P. [1929]. For the tables see KING. D. A. *SATMI* (forthcoming).

5 On the quadrant Islamic horay quadrant *rubᶜ al-sāᶜāt* see KING, D. A. [1994]. Also, large project preparing *A Catalogue of Medieval Astronomical Instruments: Astrolabes, Quadrants and Sundials* is currently in progress at the Institute of History of Science in Frankfurt am Main. The purpose of this work is to provide a useful research tool compiling critical descriptions of all historically-significant instruments, arranged chronologically according to origin or type. The compilation will include around 500 astrolabes of both Islamic and European origin, and 100 quadrants and sundials. This paper derives from it. The author acknowledges the generosity of Prof. David A. King in allowing her to work on these materials. The author also acknowledges Prof. King's helpful comments on and the revisions. The remaining errors are the sole responsibility of the author.

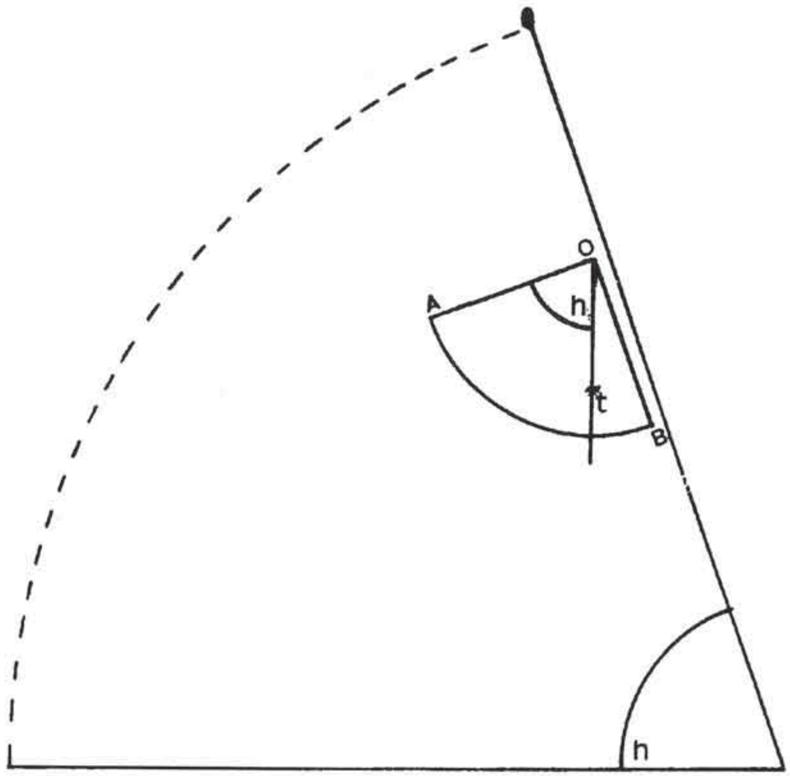


Figure i

1.1 The use of the standard horary quadrant for a specific latitude

The horary quadrants described in this paper are computational devices: vertical dials. They are observational instruments only in so far as one can measure the instantaneous solar altitude on the quadrant scale. The operation of finding the hour involves nothing more than an easy eye-interpolation on the grids displaying the solar altitudes at the hours.

On a particular date, six hour lines (rather than twelve) will suffice to tell the time throughout the day, because each one serves a pair of seasonal hours[6] which have the same solar altitude and which are symmetrical with respect to the local meridian: (I,XI), (II,X), (III,IX), (IV,VIII), (V,VII), (VI). Likeways, only six zodiacal signs (out of twelve) are needed, since the two halves of the ecliptic between solstices are symmetrically inclined to the celestial equator. From here on, "symmetrical degrees" will be understood as satisfying $\delta(\lambda_1)= \delta(\lambda_2)$ even if $\lambda_1 \neq \lambda_2$. Consequently, it suffices to tabulate and represent h_i for six signs, for example, from 90° to 270°. The quadrants in our figures have been constructed with tabular values for h_i at the beginning of every hour (for $i= 1$ to 6 hours), at $\phi=36°$.

This use is taken as understood by all the authors consulted. Nevertheless, they often fail to mention the thread with a movable weight which hangs from the corner of the quadrant O. The operations to be performed are described in *Figure i*. The weight indicates the solar longitude λ, measured on a radial scale (either linear or non-linear) on the axis OA and usually divided into "symmetrical" signs or degrees. These instruments carry a pair of sights on the axis for proper alignment. The rays of the sun pass through the upper sight to fall on the lowe one. At a particular time, the thread hanging from O will intersect an hour line at t. Angle h corresponds to the instantaneous solar altitude at that moment. The hour line passing through the weight indicates the time elapsed since sunrise, or the time remaining before sunset.

[6] Seasonal hours are one-twelfth divisions of the length of daylight and hence vary throughout the year and with terrestrial latitude.

1.2 Notation

Latin alphabet:

a azimuth (usually measured from the prime vertical)
d equation of half-daylight (= D-90°)
D half arc of daylight
h instantaneous solar or stellar altitude
h_a solar altitude at the beginning of the *ᶜaṣr* prayer
h_b solar altitude at the end of the *ᶜaṣr* prayer
h_n length of the gnomon (usually 12 units)
h_z solar altitude at the beginning of the *ẓuhr* prayer
h_q solar altitude when the sun is in the azimuth of Mecca
h_0 solar altitude in the prime vertical
H solar meridian altitude
i relates to seasonal hours, from 1 to 6
j relates to equinoctial hours
L_ϕ terrestial longitude
q *qibla*, azimuth of Mecca (measured from south)
R sexagesimal base (60)
r duration of morning twilight (as a subscript r relates to morning twilight)
s duration of evening twilight (as a subscript s relates to the evening twilight)
sdh seasonal dayhours
sdn seasonal nighthours
t hour-angle
t_a hour-angle at the beginning of the *ᶜaṣr* prayer
t_b hour-angle at the end of the *ᶜaṣr* prayer
T time since rising or time remaining until setting

Greek alphabet:

δ solar declination
ϵ obliquity of the ecliptic
σ independent variable (for angles or arcs)
λ solar longitude
λ longitude of the point of the ecliptic opposite the point with longitude

(= λ + 180°)
ϕ terrestial latitude
θ rising amplitude

Miscellaneous:
All entries in the tables are expressed sexagesimally with 60 as the base. The medieval functions Sin, Cos, Tan and Cot are based on R = 60, unless otherwise indicated[7].

Capitals used in the following descriptions and figures:
AOB is a given quadrant where
AO is the horizontal axis (or *khaṭṭ awwal al-rubᶜ*)
BO is the vertical axis (or *khaṭṭ akhir al-rubᶜ*)
O is the centre (*al-markaẓ*)
B is the "end of the quadrant" or last degree in the external graduation (*akhir al-rubᶜ*)
A is the "beginning of the quadrant" or first degree in the external graduation (*awwal qaws al-rubᶜ*)
H(S) is the meridian altitude of Cancer 0°, measured on the external graduation of the quadrant
H(W) is the meridian altitude of Capricorn 0°
EE' is the day-circle of the sun at the equinoxes
SS' is the day-circle of Cancer
WW' is the day-circle of Capricorn

1.3. Formulae used in the analysis

The formulae for time-keeping used in the tables to be analyzed here are (in medieval notation):[8]

I $t = \text{arcCos}\ (\text{Sin}\ h - \text{Sin}\ \delta\ \text{Sin}\ \phi / R) / \text{Cos}\ \delta\ \text{Cos}\ \phi / R^2$
$T(h,\lambda,\phi) = 90° + d(\delta,\phi) - t$

[7] On the medieval Islamic convention, see KENNEDY, E. S. [1956], 139-140.

[8] On modern formulation of functions for time-keeping see KING, D. A. [1973].

$= d + \text{arcSin} (R (R\text{Sin } h - \text{Sin } \delta \text{ Sin } \phi) / \text{Cos } \delta \text{ Cos } \phi)$
where d is the excess of half daylight over 90°, that is $d = \text{arcSin} (\text{Tan } \delta \text{ Tan } \phi / R)$

The equivalent formula for h_i
II $h_i = \text{arcSin} (R (\text{Cos } t_i \text{ Cos } \delta \text{ Cos } \phi + \text{Sin } \delta \text{ Sin } \phi)$

and for the solar altitude at the prayers
III $h_a(h_n) = \text{arcCot} (\text{Cot } h_n + R)$
$h_b(h_n) = \text{arcCot} (\text{Cot } h_n + 2R)$

2. References to the use of the exact formulae for time-keeping

Sources on Indian astronomy, such as the large astronomical compilations *Brāhmasphuṭasiddhānta*, *Paulisasiddhānta* or *Khandakhādyaka*, offered different solutions by using exact methods in plane and spherical trigonometry. Exact rules were developed in Varāhamihira's *Pañcasiddhāntikā* for determining the time elapsed after sunrise (before midday) from the length of the shadow of a vertical gnomon, and to solve the inverse problem[9]. The exact formula is also to be found in the *Surya-Siddhānta* (sixth c.), in a rule devised to find the sine of the solar altitude at any given hour of the day, when the sun's meridian distance, its declination, and the local latitude are known[10].

Determining the time from the solar altitude was a problem familiar to ancient and medieval astronomers, not least since it had to be solved in order to find the horoscopus, or the ascendant degree of the ecliptic, at a given time[11]. This exact formula originated in Indian astronomical sources, and was later introduced in Islamic astronomy. From the ninth century onwards, Islamic astronomers compiled tables of $T(h,H)$, $t(h,\lambda)$ for specific latitudes based on the exact formula for time-keeping.

[9] NEUGEBAUER, O. & PINGREE, D. [1970-1971/II: 43-45].

[10] *Sūrya-Siddhānta*, rule n. 36. See BURGESS, E. [1860: 132-134].

[11] See *KING, D. A. [1998, 176]*.

The exact formula was probably introduced in Islamic astronomy through the texts of al-Khwārizmī. Later, most Arab astronomers dealt with the problems of determining the time, either in their *zījes* or in independent treatises, in which they included both approximate and exact formulae.

Al-Khwārizmī, in the ninth century, was familiar with the exact formula, as Ibn al-Muthannā's commentary (tenth c.) on his astronomical tables reveals. The trigonometrical section of this work includes a rule for the exact determination of time in equatorial degrees from the solar altitude.[12]

Islamic astronomers interested in time-keeping applied either the Indian formula, probably introduced into medieval astronomy by al-Battānī, whose tables were widely known in Eastern and Western lands, even before any translation of the *Almagest* was available.[13] In addition, al-Battānī's work contains two spurious numerical tables, giving the solar altitude and the azimuth at each seasonal hour for the latitude of Raqqa. These tables are mentioned in relation to the construction of a horizontal sundial.[14] Many Islamic astronomers were involved in the time-keeping calculations: Ḥabash al-Ḥāsib, Abū-l-Wafā' al-Būzajānī, Ibn Yūnus and al-Bīrūnī, among others, whose works contributed to creating the corpus of the Islamic religious astronomy (*ʿilm al-mīqāt*). In the eleventh century, al-Bīrūnī described all methods of time-keeping known to him and, in his magnificent discussion of shadows, noted their association with Indian and Muslim astronomers. In his time, the exact procedures were widely known; they were applied to the construction of time-keeping devices, like the ones

[12] However, Ibn al-Muthannā claims that this formulation is equivalent to an earlier one, based on an approximate procedure for timekeeping. See the edition of the Hebrew text and the study by GOLDSTEIN, B. R. [1967: 82-83 and 207-209].

[13] The exact procedure in the Almagest permits the determination of the arc of half day-light from the solar declination and the local latitude. See NEUGEBAUER, O. [1975/1: 34-44 and 366-370].

[14] See NALLINO, C. A. [1899-1907, 2: 188 and 294-95].

described below.[15]

3. Sources

Instruments ans descriptions of instruments are listed together, chronologically as far as possible. Textual sources are referred to by a capital letter, and instruments by a capital letter and a subscript number (see also Appendix).

3.1 Instruments

Only a few examples of horary quadrants for a specific latitude have survived. Those we have attest to their use from the tenth century onwards. Complete horary quadrants for particular latitudes did not reach a wide public, since for practical reasons Muslim astronomers preferred universal instruments. Nevertheless, a few extant instruments for a single latitude bear witness to their use from the tenth century onwards.

Q_1. The thirteenth century Nishapur quadrant, preserved in the Metropolitan Museum of Art in New York. It was devised for $\phi = 36°$ since in it $H(90°) = 78°$ and $H(270°) = 30°$ and carries six sigmoid lines for the seasonal hours[16].

Q_2. A thirteenth century (?) horary quadrant for $\phi = 30°$ (*Plate I*) in which day-circles are equidistant and span the complete axis. It includes markings for $h_a(\lambda)$[17].

A_1. The astrolabe by Ḥāmid ibn ᶜAlī al-Wāsiṭī, a famous instrument maker

[15] A Persian attestation of reckoning time from solar altitude is analyzed in GOLDSTEIN, B. R. [1963].

[16] See KING, D. A. [1988b].

[17] Property of the late Alain Brieux in the seventies, now in a private collection in Kuwait. Published by KING, D. A. [1995:76-96].

from Baghdad. On the back, the instrument has an horary quadrant for the latitude of 33°, dated 348 H./959 A.D., preserved in the Museo Nazionale in Palermo[18].

A_2. The astrolabe by Ḥāmid b. al-Khīdr al-Khujandī, dated 374 H./984-5 A.D., for the latitude 33° (Baghdad), preserved in a private collection in Kuwait[19]

A_3. An astrolabe by an unidentified "ᶜAbdī" probably from Istanbul, made in the year 1125 H./1713-14 A.D. The instrument is preserved in the Museum of the History of Science in Oxford (*Plate II*)[20].

Q_3: A wooden Ottoman quadrant preserved in the Egyptian National Library (*Plate III*)[21]. It is dated 1178 H./1764-65 A.D. Our analysis reveals that it was made for the latitude $\phi = 41°$. Its plate is divided into four different sections, which will be referred to by subscript letters in our descriptions (Q_{3a}, Q_{3b}, Q_{3c}, Q_{3d}).

Q_4: Ottoman quadrant from *ca.* 1205 H./1790-91 B.C., preserved in the Egyptian National Library with drawings on both sides (referred to here by a subscript letter *a* and *b*)[22]. It was designed for $\phi = 41°$ and $\epsilon = 23;35°$.. According to the description in the Cairo *Catalogue*, one side,

[18] For the astronomer see SEZGIN, F. [1967on, *GAS*/VI: 207]; MAYER, L. A. [1956: 45]. For the instrument MONTILLARO, V. [1848], whose description is not to be trusted, and GUNTHER, R. T. [1932/I: 230, n. 100].

[19] For the astronomer see SEZGIN, F. [1967on, *GAS*/V: 307-8; *GAS*/VI: 220-22]; KING, D. A. [1986c: B50]. For the instrument KING, D. A. [1995: 76-96].

[20] Oxford University, Museum of the History of Science, 57 - 84/171 A, 171. See MAYER, L. A. [1956: 32]; BRIEUX, A. & MADDISON, F. [forthcoming, *sub* ABDI 1]; KING, D. A. [1985a: 375].

[21] KING, D. A [1981-1986, *Cairo Arabic Catalogue*, Instruments, n. 3].

[22] KING, D. A. [1981-1986, *Cairo Arabic Catalogue*, Instruments, n. 4].

Q_{4a}, contains the *sāᶜāt al-sharqīya* (*ab ortu*). The apex of both sides includes universal quadrants; side Q_{4b} contains the *sāᶜāt al-gharbīya* (*ab ocasu*) for $\phi=41°$. There is also the inscription *ᶜaṣr awwal*, referring to the hour line for the beginning of the *ᶜaṣr*.

3.2 Manuscripts and written sources

A. An anonymous Abbasid treatise on the construction of the horary quadrant preserved in Ms. Istanbul Aya Sofia 4830 (fols. 196*v*-197*v*) containing a table of h_i for the latitude of 33° (Baghdad)[23]. Research has demonstrated that the table contained in the text is based on the approximate formula[24]. The horary quadrant described has a graphic representation of the seasonal hour lines for a specific latitude.

C. An anonymous twelfth century Andalusian treatise on the different kinds of horary quadrants in MS Cairo Ṭalᶜat *mīqāt* 155,3 fols. 19*r*-21*v*[25]. It was copied *ca.* 1150 in *naskhī* script, probably in Egypt[26]. The treatise offers several possible methods of drawing universal horary quadrants, and three basic horary quadrants for specific latitudes. It shows: hour lines either as curves or straight lines, hour lines for seasonal or equinoctial hours, or for time elapsed since sunrise, all of which are described here. This source may be an example of a class of Arabic treatises on instruments, no longer extant, but from which the Cairo astronomer al-Marrākushī probably compiled the sections of his treatise on quadrants (*ca.* 1280). The different quadrants described in it will be referred to as C_1 to C_3 from here on.

23 The text has been translated by KING, D. A. [1988c].

24 KING, D. A. [1983a: 31].

25 KING, D. A. [1981-1986 *Cairo Catalogue Arabic*/1: 675, n. 4]; KING, D. A. [1986c: 135, F6].

26 The manuscript was purported to be copied *ca.* 1700. KING, D. A. [1988c] predated it by about five centuries, on the basis of internal evidence and inspection of contents.

M. The main work of the thirteenth century astronomer al-Marrākushī (fl. Cairo 1280)[27], *Kitāb al-mabādi' wa-l-ghāyāt fī ᶜilm al-mīqāt*, which deals with spherical astronomy and instruments of all kinds. His work is available in several manuscripts, many of which are in libraries in Istanbul. I consulted the facsimile of Ms. Topkapi Ahmet III (No. 3343), copied in 747 H. The part of this work concerning universal instruments - universal horary quadrants and sine quadrants - has recently been studied[28]. Al-Marrākushī's work seems to be characterized by eclecticism; the author shows his familiarity with the Andalusian and Maghribi tradition of astronomy, quoting al-Zarqālluh and Ibn al-Kammād[29]. His other sources were of Eastern origin and most of his tables were computed for the latitude of Cairo, apparently by himself. In his treatises on instruments he does not generally mention his sources. The different quadrants described in it will be referred to as M_1 to M_4 from here on.

S. A text by Pseudo Ibn al-Sarrāj, working in Aleppo *ca.* 1300[30]. The manuscript preserved in the Chester Beatty Library in Dublin (Ms. 102,2, fols. 88*v*-93*r*) was copied *ca.* 1350. The relevant quadrants mentioned in it will be designated by the abbreviations S_1 to S_9.

[27] SUTER, H. [1900, n. 363]; KING, D. A. [1986c, C17]. Concerning al-Marrākushī's horary quadrants for specific latitudes see AL-MARRĀKUSHĪ [fl. Cairo 1280, ed. *facsimilae*/1 SEZGIN, F. 1984, 366-371].

[28] KING, D. A. & GIRKE, D. [1988]

[29] Al-Zarqālluh: SUTER, H. [1900: n. 255]; Ibn al-Kammād: SUTER, H. [1900: n. 487]; KENNEDY, E. S. [1956: nn. 5, 24, 48, 66 and 72].

[30] On Ibn al-Sarrāj see SUTER, H. [1900: n. 508 (confused)]; KING, D. A [1986c: C26]. On Ibn al-Sarrāj's instruments see JANIN, L. & KING, D. A. [1977: 206 and 216]; KING, D. A. [1983b: 544-546] [1987a: 7] [1987b: 1-3]. For the past 15 years or so D. King has claimed that the author of this anonymous treatise was the astronomer and instrument-maker Ibn al-Sarrāj. François Charette (J-W. Goethe Universität, Frankfurt am Main), has shown that Ibn al-Sarrāj was not the author. Details are in his forthcoming doctoral thesis on the Dublin manuscript.

Z. An Ottoman text by (Ghāzī) Aḥmad Mukhtār (Pasha), *Riyāḏ-Mukhtār. Mirāt al-mīqāt wa-l-adwār maᶜa majmūᶜat al-ashkāl* published at Būlāq, where the author held an official post, in 1303 H/ 1885-6 A.D. A copy of the book is preserved in the Bodleian Library in Oxford[31].

4. Some tables based on the exact formula

The tables in the sources analyzed in this section were designed for the specific purpose of marking horary quadrants with a graph which saves the user the task of tabular interpolation. The earliest tables of time-keeping functions were calculated by the approximate formula. Later, the mathematically exact formula for reckoning time was introduced[32]; the use of the accurate formula in tabulating the functions h_i (λ), or T(λ) and H(λ) made it possible to reduce the size of the tables. This advantage encouraged Muslim astronomers in both the Indian and the Ptolemaic tradition to tabulate these functions and to include them in their treatises on the construction of astronomical instruments[33]. This is the case, for instance, of a treatise on the sine quadrant by al-Khwārizmī, which confirms the attribution of the first known description of the sine quadrant to this author. It contains a small table of a function, whose arguments are seasonal hours and solar longitude[34]. The backs of many later astrolabes

[31] This source was studied by WÜRSCHMIDT, von J. [1919] in his analysis of Islamic prayer times h_a and h_b and the lines drawn on the instruments serving this purpose. SCHMALZ, P. [1929: 39-40] also mentions it in his description of the almucantar quadrant.

[32] All kinds of Islamic tables for time-keeping based in both formulae are analysed in KING, D. A. [forthcoming: *Studies in Astronomical Timekeeping in Medieval Islam, I: A Survey of Tables for Reckoning Time by the Sun and Stars*].

[33] See, for instance, in KING, D. A. [1985b] the discussion on the astrolabe by al-Ashraf, the Rasulid prince who constructed his own astrolabes by using tables for spherical astronomy (for specific latitudes in Yemen and the Hijaz) rather than the more common geometrical construction.

[34] On fols. 96*v*-97*v* of MS Berlin Ahlwardt 5793. The table is described in KING, D. A. [1983a: 27-29], and has now been analyzed by HOGENDIJK.

also show quadrants on which a single horary mark H (*khaṭṭ niṣf al-nahār*) is drawn for two or three different latitudes, or lines for $h_a(\lambda)$ at a specific latitude, or for $h_q(\lambda)$ in order to find the *qibla* direction in several localities[35]. Some of these functions were intended for a particular latitude, based on the accurate formula and included in a few *zījes*[36]. Tables calculated with the exact formula are found in the sources M and S. The results of our analysis are presented below.

1. Source M (p. 243) presents a "Table of the solar altitude for the seasonal hours and the *ʿaṣr* at the beginning of signs for $\phi = 30°$", that is, $h_i(\lambda)$ and $h_a(\lambda)$. Parameters are $\phi = 30°$ (Cairo) and $\epsilon = 23;35°$.

2. This source also has a "Table of the solar altitude for the equinoctial hours for the beginning of the signs and in the prime vertical for the northern signs, for $\phi = 30°$, the altitude of the sun in the *qibla* for the region which longitude is 65° West from Arin" (p. 367, *Plate IVa*). In this case, for the same parameters and arguments, the table gives three functions: h_j (j being the number of equal hours from 1 to 6), h_0 and h_q.

35 For an overview of mathematical methods and tables used to find the *qibla* direction see KING, D. A. [1999].

36 For instance, al-Battānī tabulated h_i and a_i for $\phi = 36°$ and $\lambda = 90°$ and 270°, and the length of the seasonal day hours for Mecca, Baghdad and Harran respectively. See NALLINO, C. A. [1907/II: 67 and 188]. Other examples are found in some large handbooks for time-keeping of Ibn Yūnus for Cairo (*ca.* 975) and al-Khalīlī (*ca.* 1370) for Damascus, see KING, D. A. [1996: 147-148 and 155]. In addition, the tenth c. Baghdad astronomer ʿAlī ibn Amājūr tabulated in his *Zīj al-Ṭaylasan* (copied *ca.* 1215, ms. Paris B. N. Ar. 2486, fols. 239*r*-255*r*) the function T(h,H) in equatorial degrees and minutes for $\phi = 32;25°$ based on the exact formula. See KENNEDY, E. S. [1956: 125, n. 8]; SEZGIN, F. [1967on, *GAS*/VI: 177-78]. Two more examples of large sets of tables for time-keeping are due to the Yemeni astronomer Abū-l-ʿUqūl who worked for the Sultan al-Mu'ayyad and computed tables for the latitude of Taiz, Yemen, and to the architect Ṣāliḥ Efendī (*ca.* 1750), who produced a corpus of time-keeping for Istanbul. A summary of the activities of these astronomers in computing time-keeping tables can be found in KING, D. A. [1990b].

3. Source S includes a "Table for the altitude of the seasonal hours and their times, the solar altitude at the beginning and the end of *ᶜaṣr* and the rising amplitude at the beginning of the signs for $\phi = 36°$" (fol. 47 *v*). Consequently, parameters are: $\phi = 36°$ and $\epsilon = 23;35°$. Entries in the table correspond to:

$h_i(\lambda)$ by formula II
$h_a(\lambda)$ and $h_b(\lambda)$ by formula III

The tables also show the time elapsed since sunrise T_i, expressed in degrees. Tables 1 and 2 for source M, and 3 for source S, have been recomputed with the format they present in the manuscripts, and to the same degree of accuracy, by using the exact modern formula, and checked against mechanical results (see below Tables I, I and III). The comparison revealed al-Marrākushī's calculations to be highly accurate. Entries in Table 1 are arranged in seven columns, for h_i and h_a carried to minutes of arc. Most of al-Marrākushī's values are accurate or in error by ± 1 minute of arc. Larger mistakes are found randomly distributed. Values in Table I below give some idea of his precision. Errors in the second digit are shown in square brackets, and derived by the convention: error = text - recomputation.

Table I

λ	h_1	h_2	h_3	h_4	h_5	h_6
90	13;51	28;21	43;15 [−1]	58;22 [+1]	73;8 [−4]	83;35
120	13;55	28;21	43;4	57;47 [+1]	71;49 [+2]	80;16
150	13;46	27;43	41;35 [+1]	54;50 [−1]	66;13 [−1]	71;32
180	12;58 [−1]	25;40	37;49 [−3]	48;36 [−1]	56;46	60;0
210	11;29	22;23	32;10 [−8]	40;38 [+1]	46;23 [+1]	48;28
240	10	19;13	27;22 [+1]	33;54 [+1]	38;13 [+1]	39;43 [−1]
270	9;21	17;53 [−1]	25;20	31;15 [−1]	35;5 [+1]	36;25

λ	h_a	λ	h_a
90	41;58 [+1]	210	27;56
120	40;30 [+1]	240	2[4];25[37]
150	36;51 [+1]	270	23
180	32;22		

[37] Ms. 25;25

Entries in Table 2 are displayed in eight columns, for h_j, h_0 and h_q. In the columns corresponding to h_6 and h_0 three and four entries are missing and have been replaced with zeros. Most of the entries are in error by ± 0;1°. Table II shows a set of selected values and their recomputation difference.

Table II

λ	h_1	h_2	h_3	h_4	h_5	h_6
90	11;4	20;55 [−1]	28;13 [−1]	34;25	36;25	–
120	11;30 [−1]	21;56 [−1]	30;42	36;56 [+1]	39;41 [+1]	–
150	12;24	27;4 [+1]	34;31 [+1]	42;50 [+1]	47;43	–
180	12;18 [+1]	25;40[38]	37;45 [−1]	48;36 [+1]	56;47 [+1]	60
210	12;47 [−1]	25;44 [−2]	38;41	51;54	62;36 [−2]	70;29 [−1]
240	12;11 [−1]	24;50	37;44	50;44 [+1]	63;31	74;59
270	11;52	24;15	36;59	49;53 [−2]	62;51 [−3]	75;22

λ	h_0
210	23;35
240	43;[5]3 [+1]
270	53;9

Entries in source S are arranged in two parallel columns (*da'īr* and *irtifāᶜ*) of nine entries for each of the signs, from Cancer to Capricorn. The first six entries correspond to h_i. At the bottom of each column appear the beginning and the end of the *ᶜaṣr* (h_a, h_b) and the rising amplitude (θ). All entries are carried to minutes of arc. A few entries for h_i are accurate, and the error in the remainder is of minutes of arc in most cases. Some scribal or reading mistakes, attributable to copyists, have been detected. Table III shows a set of selected entries from Table 3.

[38] Ms. 25;47

Table III

λ	h_1	h_2	h_3	h_4	h_5	h_6
90	13;16 [−1]	27;24	41;55 [−2]	56;30 [+1]	70;11 [+8]	77;35
120	13;8 [−12]	27;14 [+13]	41;43 [−7]	55;11 [−51]	67;8	74;16
150	13;2 [−3]	26;0 [−21]	38;35 [−49]	50;30 [1;5]	60;31 [+8]	65;32
180	12;4 [−1]	23;51 [−1]	34;54	44;28 [−1]	51;24	54;0
210	10;16 [−9]	20;11 [+1]	28;54 [+1]	36;6	40;27 [−19]	42;28
240	8;23 [−23]	16;9 [−35]	23;54 [+17]	28;48 [−14]	32;0 [−23]	33;44
270	8;6 [+3]	15;16 [−3]	21;30	25;18 [+1;0]	29;6	30;25

Functions h_i and t_i show substantial errors, distributed irregularly throughout the argument. Function h_i has entries for the sixth hour for all the signs. Functions h_a and h_b were calculated with remarkable precision; the only random error greater than 0;1° is found in h_a for $\lambda = 270°$.

5. The varieties of horary quadrants with sigmoid lines

In this paper the reader will find a large variety of solutions to the problem of constructing an horary quadrant, most of them arising from the Eastern Islamic lands (Source A) and the Andalusian tradition represented by the Cairo Ṭalᶜat Manuscript (Source C). Their accuracy depends on the care with which the underlying table for h_i or h_j was calculated, and on the care with which they were designed. As for their practical purposes, their use always involves the operations mentioned above in Section 1. 1. The following sections describe several varieties of horary quadrants, some of them for the first time in the contemporary literature. We start with those bearing hour-lines as sigmoids.

5.1 Horary quadrants with sigmoid lines for the seasonal hours and uniform solar scale. (Types 1 & 2)

The main advantage of the horary quadrant with sigmoid hour-lines was that they were easy to draw. These instruments had the following basic elements:

- Sigmoid lines for the seasonal hours, serving a particular latitude
- Day-circles for zodiacal signs as concentric equidistant arcs drawn

according to a radial solar longitude scale divided uniformly.

The construction of the horary quadrant satisfying these two conditions is described in several Arabic sources, namely: A, C and M. Sources A and M present an apparently trivial difference: the external arc of the quadrant corresponds to Capricorn while the day-circle of Cancer is placed on the edge. This difference will generate two different quadrants with sigmoid hour-lines and uniform longitude scale whose construction is described below (**Types 1 & 2**). Even if based on the same table and drawn with the same kind of curves, the final shape will be different, as can be seen by comparing *Figures 1* and *2*[39]. The fact that the order of the day-circles is modified in both treatises allows us to establish text A as the main source for one of the quadrants described by al-Marrākushī.

5.1.1 Type 1 construction

A general description of the horary quadrant referred to as to **Type 1** and represented in *Figure 1* is given by sources A and M. The present description is based on the latter text. However, here we will consider its instructions, which are more eloquent than in source A, only in order to draw the basic lines of the quadrant, that is, to establish the day-circles and the solar longitude scale. No one textual source gives a description sufficiently clear enough for us to draw the **Type 1** quadrant.

Sources A and M establish that the external arc of the quadrant corresponds to the day-circle of Capricorn, this being its main characteristic. In addition, in both texts the circles of the signs are represented by concentric arcs spanning three-quarters of the axis of the

[39] A comparison of *Figures 1 & 2* indicates that the inversion in the order of zodiacal signs produces very different aesthetic effects. **Type 2** appears easier to manipulate and clearer than **Type 1** for a small latitude. MICHEL, H. [1947: 79-80] in his description of the graph of solar meridian altitudes indicated: *"Pour la clarté du dessin, on considérera les arcs des signes dans l'ordre marqué sur le rayon vertical, c'est-à-dire le Cancer à l'extérieur"*.

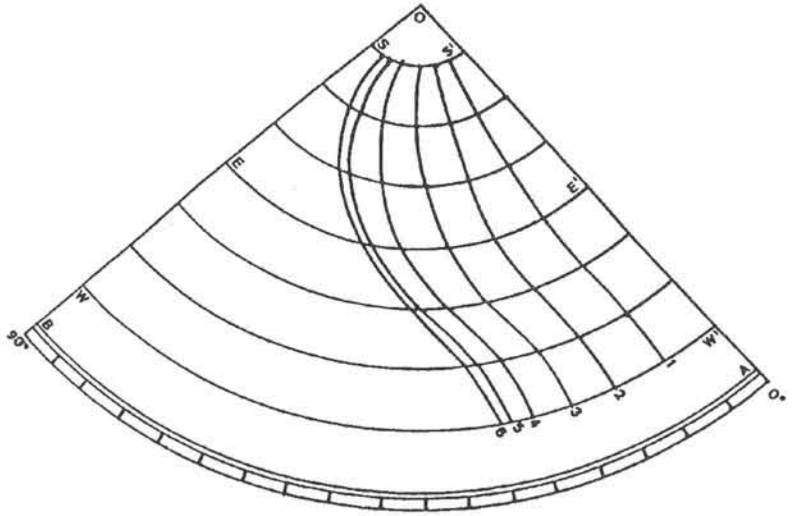


Figure 1

solar longitude[40]. Al-Marrākushī suggests the possibility of drawing the quadrant for either seasonal or equinoctial hours, but does not specify whether they should be sigmoids or straight-lines. In fact, his figure has

[40] According to Source A, the basic marks in the horary quadrant are thirteen concentric quarter of circles as day-circles for each half zodiacal sign of solar longitude, from Can 0° to Cap 0°, spanning some three-quarters of the two axes (the fraction is arbitrary). The hour curves are to be drawn across these using the table values of $h_i(\lambda)$ appended in the source.: "*If you want to divide the hours, divide the amount of three-quarters of the line standing on the side of the quadrant into thirteen divisions, then place the compass at the centre of the quadrant and open it (successively) to each of the thirteen marks, drawing with it circles which meet up with the two straight lines which have been divided. The largest arc will be for the first point of Capricorn and the smallest for the first point of Cancer.*" [Translation and study to be published by KING, D. A. See also KING, D. A. [1988b]. Similarly, Source M [ed. *facsimilae*, SEZGIN, F. 1984: vol. 1, 366-67] indicates: "*Divide line at the axis of solar longitude] by eight equal divisions...*", that is 8 x 3/4 = 6.

only one straight hour-line (quadrant M_1)[41]. He also suggests completing the plate by plotting h_a, h_q and h_0 (given in Table 2). Of course, different sets of hour lines would not combine well on the same quadrant.

Figure 1 represents a **Type 1** quadrant devised for 36° north. It has been drawn to scale according to a mechanical computation of h_i at the temporal hours for $\phi=36°$ and $\epsilon=23;35°$. Hour lines are drawn by taking the solar altitude from A to B for each one of the circles of the signs at the corresponding hours, from 6 to 1. These marks are connected from the centre through all the circles of the signs. Hour lines result in sigmoid curves.

Source C mentions that universal hour lines can be converted to serve a particular latitude by marking on the quadrant the day-circles of the signs as concentric arcs, whose centre is the centre of the quadrant (C_1)[42]. This modification will also be based on a radial solar longitude scale divided regularly and with the day-circle of Capricorn at the external rim. Nevertheless, there are no additional instructions for drawing the hour-lines, either as sigmoids or as straight lines. All the other sources refer to the practice of converting universal quadrants into quadrants for specific latitudes in their sections on the later instruments.

As regards complete horary quadrants for particular latitudes, only one Islamic instrument of **Type 1** has been preserved as far as the author

[41] See source M [ed. *facsimilae*, SEZGIN, F. 1984: vol. 1, 366-67]. In his review of the history of the quadrant among the Muslim astronomers SCHMALZ, P. [1929: 121-124] examines four kinds of horary quadrants for specific latitudes described by al-Marrākushī (M_1 to M_4). Quadrant M_1 is unfinished in the manuscript and the corresponding figure displays a uniform solar scale about three-quarters of the axis and a single straight hour-line.

[42] Source C establishes: "The hours are of two kinds, seasonal and equinoctial. The seasonal hours are universal if they are marked against a scale showing the maximum [altitudes] and are not universal if they are marked against a scale showing the zodiacal signs. The equinoctial hours can only be for a particular latitude when they are marked on the day-circles of the zodiacal signs. The shape [of the hour curves] differs according to the way they are marked, and, for this reason, the kinds of quadrants are numerous..." [Chapter on marking the hours on plane instruments (*al-musaṭṭaḥāt*). Translation and study to be published by KING, D. A. See also KING, D. A. [1988b].

knows, in which the arc of the quadrant represents the day-circle of Capricorn and the remaining day-circles span the complete axis. This instrument is the Nishapur quadrant Q_1.

5.1.2 Type 2 construction

Figure 2 shows the **Type 2** horary quadrant. Notice that the day-circles of the signs are equidistant and placed in order so that the arc of the quadrant corresponds to the summer solstice (SS'). Seasonal hour lines are drawn on it, following the procedure described for **Type 1** and result in sigmoids. Its construction improves the situation at the centre of the quadrant - too crowded in **Type 1** - and makes it easier to use.

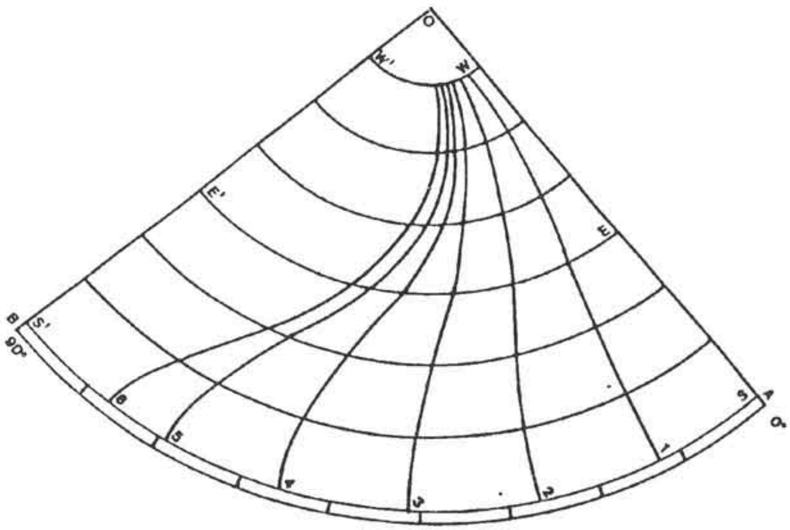


Figure 2

Three Islamic instruments present horary quadrants of **Type 2**, in which the external arc of the quadrant represents the day-circle for Cancer:
1. The astrolabe of al-Wāsiṭī A_1, with day-circles drawn every 10° and spanning the complete axis, has sigmoid hour-lines typical of **Type 2**.
2. The astrolabe of Ḥāmid b. al-Khīdr al-Khujandī A_2.

3. The anonymous quadrant Q_2, with an additional sigmoid line for h_a (in dots) (*Plate I*).

A few more astrolabes from the Islamic tradition have markings drawn by the same procedure as **Type 2** horary quadrants. For instance, Jaᶜfar ibn ᶜUmar ibn Daulatshāh al-Kirmānī[43], member of a reputed Persian school of astrolabe makers, drawn on the back of his astrolabe (dated 1354) a quadrant with hour-lines for the *ẓuhr* and the beginning of the *ᶜaṣr* for ϕ = 32° and six equidistant day-circles as arcs for the zodiacal signs. The solar longitude scale, divided regularly, fills 3/4 of the radial axis. Sigmoid prayer lines are drawn in the same style as seasonal hours in **Type 2**. The instrument is dated 755 (1354); in the fifties it was in the collection of M.G. Prin, and now in the Musée de l'Institut du Monde Arabe (Paris).[44]

5.2 Sigmoid equinoctial hour lines. Type 3

I know of no accurate description of this construction in an Arabic source; indeed, few diagrams of this kind, of any origin, have survived. However, the Andalusian source C mentioned the possibility of devising an horary quadrant with lines for equinoctial hours according to a solar longitude scale (see note 37 above). We will refer to this as **Type 3** (see *Figure 3*). Its use requires performing the same operations as **Types 1 & 2**.

6. Horary quadrants with curved hour lines

The replacement of sigmoid hour-lines with curved concave lines was historically successful. Two varieties of quadrants developed: those constructed to give time in seasonal hours **(Type 4 & Type 4')** and others giving equinoctial hours (**Type 5**). Both had a radial solar longitude scale which was not divided regularly.

43 MAYER, L. A. [1956, pp. 53-54, plate X].

44 See MOULIÉRAC, J. [1989].

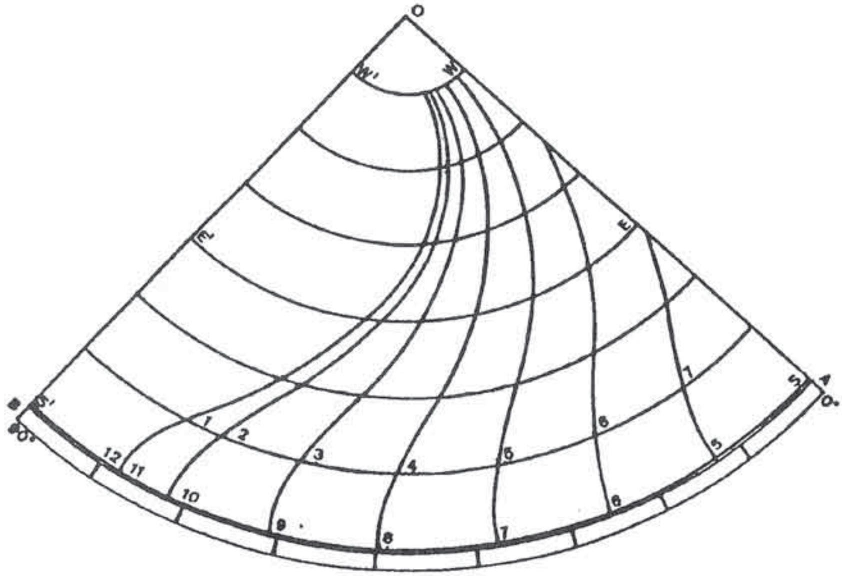


Figure 3

As we said, source C mentions that universal hour-lines can be converted to serve a particular latitude by being marked on the day-circles of the signs. Since the anonymous author refers to this conversion while describing a universal quadrant with hour-lines as curves, we suspect that he is referring to these curved lines for a specific latitude[45]. But, again, the Arabic text of this treatise is not at all clear; the first description known

[45] Source C establishes: "Thus it is necessary to derive [the hour-lines] by the above-mentioned method (universal horary quadrant) up to the sixth hour only, and they can be made particular to one latitude by marking on them the day-circles of the zodiacal signs, parallel [arcs] whose centre is the centre of the quadrant. The hours are marked on them by the altitudes corresponding to the hours for the day-circle of each sign." [Chapter on marking the hours on plane instruments (*al-musaṭṭaḥāt*)]... Translation and study to be published by KING, D. A. [1988b].

to us of curved hour-lines was given later, by Pseudo Ibn al-Sarrāj.

6.1 First variety with curved lines for seasonal hours and non-uniform solar scale (Type 4)

In one of Pseudo Ibn al-Sarrāj's quadrants - S_8, known as *dawā'īr al-taᶜdīl* (or the "proportional circles")[46], the resulting hour lines are no longer sigmoids, but curved lines (the text states "arcs of circles drawn with a compass"). In *Figure 4* it is drawn to scale.

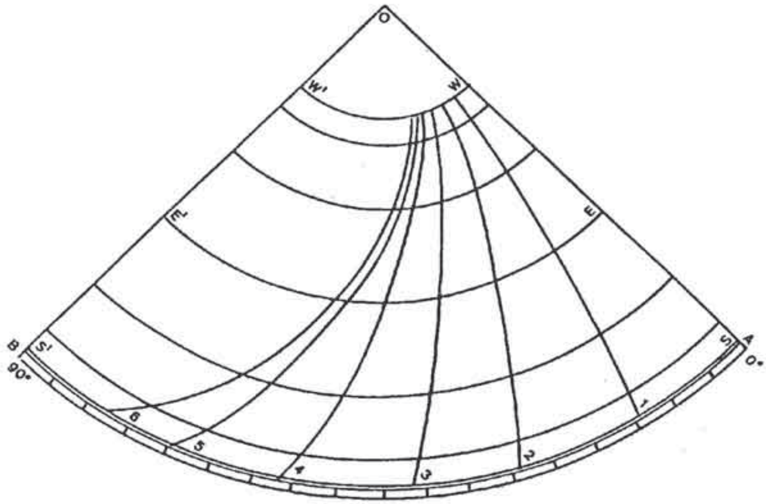


Figure 4

To construct it, first the day-circle of Capricorn WW' and the day-circle of Aries EE' are drawn. One has to take 45° on the external graduation and make a mark W with this aperture on the horizontal axis, taking on it a distance of 1 - crd 45°. This mark will be used for drawing the day-

[46] Source S, Chapter 73, fol. 92*r*.

circle of Capricorn WW' from the centre of the quadrant. Since the arc of the quadrant represents the day-circle of Cancer SS', the day-circle of Aries EE' is to be found exactly between them. The day-circles are concentric and stereographically projected. Since the hour lines are not sigmoids, the particular way of drawing the solar longitude scale backwards, from the values of H for the beginning of each one of the signs, has produced a non-linear scale for λ but a linear one for δ.

Significantly, the solar longitude scale on Pseudo Ibn al-Sarrāj's figure takes up approximately three-quarters of the vertical axis, a fraction which is also arbitrarily stated in sources A and M in connection with **Type 1**. Furthermore, Pseudo Ibn al-Sarrāj's drawing has thirteen concentric quarters of circles as day-circles for each third of the zodiacal signs, as source A suggests (see section 5. 1. 1.).

The three parallels of declination for the equinoxes and both solstices are stereographically projected on the quadrant, where they become day-circles of the signs. Distances WE and ES correspond to ϵ and distance OW to $(90°-\phi) - \epsilon$. This distance would be 30;25° considering Ibn al-Sarrāj's parameters, that is, for $\phi = 36°$ and $\epsilon = 23;35°$. The rest of the intersections with the other signs are determined backwards with the ruler from the altitude entries in the table. The dots S_6, E_6, W_6 represent H_6. S_6 E_6 W_6 is the curve of the solar meridian altitude, H. The remaining hour lines, which produce arcs of circles, instead of sigmoids, will be found in the same way.

An example of **Type 4**, of Ottoman origin, has been preserved: Q_{3a} (*Plate III*), whose inner part is engraved with two sets of seasonal hours as curved lines. These lines are based on h_i for the northern and the meridional signs. As far as one can tell from the approximate measurements over the plate, it seems that these sets were drawn to be used in or not far from Istanbul.

6.2 Second variety with curved lines for seasonal hours and non-uniform solar scale (Type 4')

There is a quadrant with arcs of circles that can be drawn only for a specific latitude. Constructed over the same scale as **Type 4**, it incorporates curved lines for seasonal hours by adjusting the radial solar longitude scale to them (see *Figure 4'*). The resulting solar longitude scale (WES) on axis OA is not divided regularly. Pseudo Ibn al-Sarrāj claimes to have invented this hour-dial (*sā^cāt al-makhṣūṣa*) himself with the purpose of drawing the 6 hour-lines of equal length and represented by the same kind of curve. It is referred to as S_1. Pseudo Ibn al-Sarrāj states that

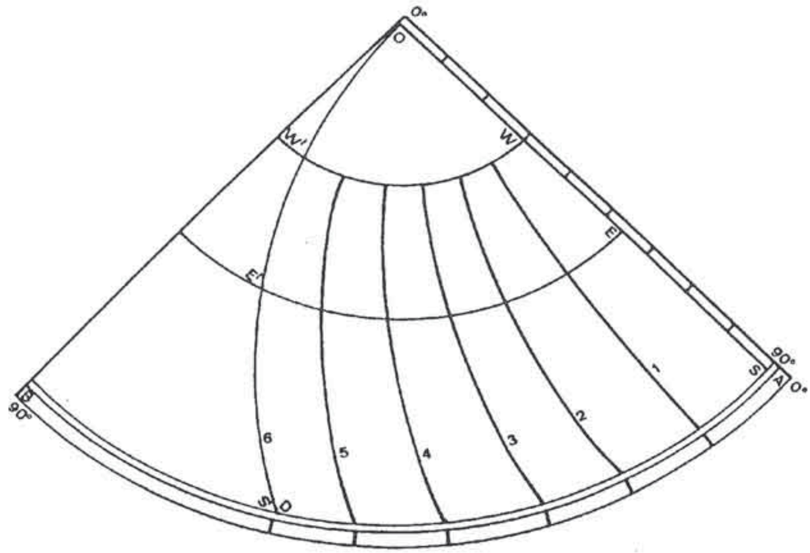


Figure 4'

this curve will be an arc of circle to be drawn by a compass[47]. The line OD (called *al-ufq*) or "horizon", is drawn from D using the compass with centre in A and radius AO. The divisions of the hours have been drawn on the *madār* of Cancer from the centre, every 10°, on the graduation of the arc AB; the day-circle of Capricorn WW'' and the day-circle of Aries EE'', taking OW = H for $\lambda = 270°$ and OE = H for $\lambda = 0°$ on the scale

[47] Source S, Chapter 66, fol. 88*v*. Pseudo Ibn al-Sarrāj seeks to draw the hour lines using the compass, which is not possible.

OA. The projection of the day-circles is stereographic. Each day-circle is divided into six equal parts. The quadrant is to be used with two threads: one in O, the centre, and the other in A. Pseudo Ibn al-Sarrāj's main objective seems to be to avoid the use of the weight on the plum line index. The figure in the manuscript appears to represent its construction for $\phi = 30°$ (serving Cairo, where he traveled to) instead of 36°, the common value in the rest of the cases serving Aleppo, but the external graduation of the quadrant was not accurate. Nonetheless, the distance OW taken on the horizontal axis OA seems to be approximately 36°. This distance corresponds to $(90°-\phi) - \epsilon = 36;25°$, for $\phi = 30°$, that is, it corresponds to H for $\lambda = 270°$. Likewise, the distance OE corresponds to H for $\lambda = 0°$. The fact that $\phi = 30°$ is used may reflect an Egyptian influence in time-keeping in early fourteenth century Syria, where Marrākushī's tradition survived, or that the author was an Egyptian.

6.3 Curved lines for equinoctial hours (Type 5)

The construction of an horary quadrant with curved lines serving equinoctial hours is not explicitly mentioned by our written sources. Only one of the Arabic texts can be said to refer to it: source C refers indirectly to equinoctial hours drawn against day-circles of the zodiacal signs[48].

Nevertheless, at least one Islamic example of a quadrant of this kind exists on an astrolabe: A_3, due to a certain "ᶜAbdī" from Istanbul (*Plate II* and *Figure 5*). It is on the back of his instrument which has, on the upper right quadrant, seven hour-lines shaped like arcs of circles and a curved

[48] However, the treatise on the astrolabes by the early 11th century astronomer al-Bīrūnī, *Kitāb fī istīᶜab al-wujūh al-mumkina fī ṣanaᶜat al-asṭurlāb*, extant in several manuscript copies (Ms. Carullah 1451 & Tunis National Library 5540) but still unpublished, describes the construction of an horary quadrant on the back of the astrolabe. A figure in the manuscript seems to represent an horary quadrant with curved hour lines for the equinoctial hours. Only a few sections of this work have been translated and studied, but Richard Lorch (Munich) is preparing a detailed analyses with edition and translation. See BOILOT, D. J. [1955-1956]; KENNEDY, E. S. [*DSB*, II: 147-58].

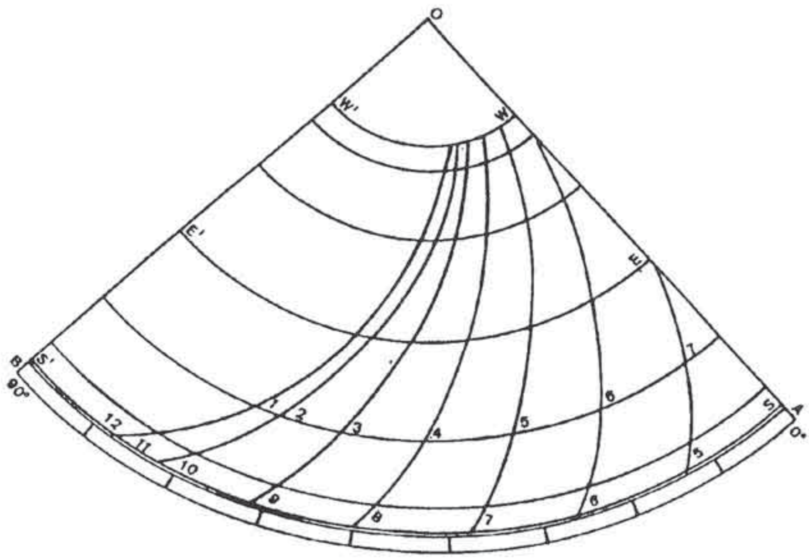


Figure 5

line for h_a. There is no indication of the latitude they are intended to serve. To deduce this parameter, values for h_j were measured on the figure throughout the day-circles of the solstices and the equinoxes. They were compared with recomputations by the exact formula for $\phi = 41°$ (Istanbul) and found to correspond. The radial solar longitude scale is divided irregularly, as is usual in this type of quadrant. If the unmarked alidade is original on the instrument, it could only be used to find the solar altitude four times a year, when $\lambda = 0°$ or $180°$, $90°$ or $270°$. The day-circle of Capricorn is divided only into five hours, as a consequence of the short length of daylight during the winter solstice. One more curved line on the plate indicates the beginning of the *ʿaṣr*[49].

49 It intersects day-circles at an altitude of approximately $h_a = 37°$, for $\lambda = 90°$; $h_a = 27°$ for $\lambda = 0°$ and $180°$; $h_a = 19°$, for $\lambda = 240°$. These values agree with results obtained by recomputation (37;17°, 28;9° and 19;30° respectively).

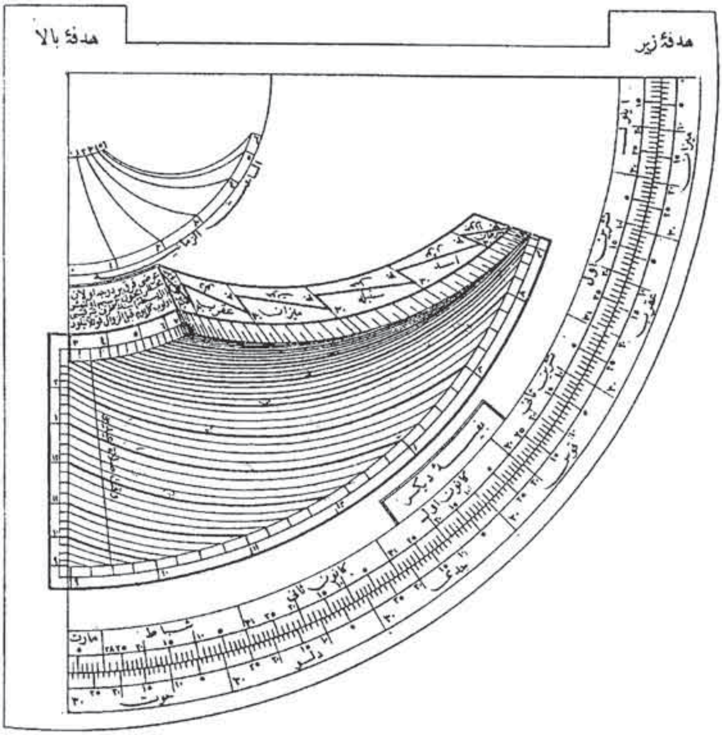


Figure 6

6.4 Curved lines in two sets for equinoctial hours measured from midday (Type 6 and 6')

The construction of these graphs was not widely discussed in the medieval sources but a description is found in the handbook on *mīqāt* by Pasha Ghazī Aḥmad Mukhtār dating from the end of the last century (source Z, from which we have taken *Figures 6 and 6'* for hours before and after midday at $\phi = 41°$ respectively). The Ottoman quadrants Q_3 and Q_4, both serving Istanbul, present two diagrams for equinoctial hours measured from midday, before (**Type 6**) and after it (**Type 6'**). In one case, they appear on the plate of the instrument (sections Q_{3c} & Q_{3d}, *Plate III*); in the other example on both sides Q_{4a} & Q_{4b}.

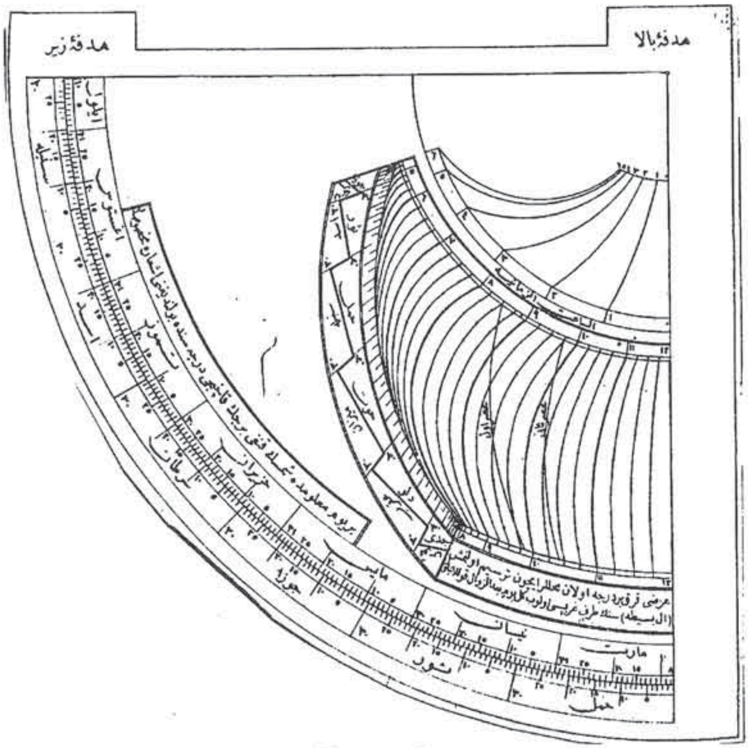


Figure 6′

The graph in Q_{3d} & Q_{4a} has curved lines designed to display equinoctial hours before midday for a particular latitude. Line OB represents the *khaṭṭ ṭulū*ᶜ or rising line. The hour-markings are found between the arc of the quadrant, representing the day-circle of Cancer, in which H = 70°, and the inner arc serving as a day-circle for Capricorn (four hours corresponding to the winter solstice). The day circles are not clearly drawn and it is practically impossible to recognize the names of the zodiacal signs. The solar longitude scale is divided backwards from tabular values of H along an inner arc which is not regularly graduated and which bears six pairs of symmetric signs. The arc $H_S H_E H_W$, marked with the meridian altitudes throughout the year, represents the ecliptic, and is graduated in both cases. Source Z shows the method of dividing the ecliptic for time since rising and remaining until setting.

The graph found in Q_{3c} and Q_{4b}, for the hours *baᶜda al-zawāl*, completes the quadrant described in source Z (*Figure 6*)[50]. The order of the signs has been inverted, Cancer now being in the inner part and Capricorn at the external arc of the quadrant. The solar longitude scale is not regular. Line OA represents the *khaṭṭ ghurūb* or setting line, which coincides with h_{12}.

An unsigned Italian quadrant for latitude 42° shows horary quadrants of **Type 6** and **Type 6'** combined on top of each other in the same plate. It is dated 1611 and preserved in Oxford (Lewis Evans Collection) (*Plate VI*)[51]. The Italian quadrant predates the Ottoman instruments Q_3 and Q_4 by more than a century. However, there seems to be at least an indirect Islamic precedent for **Type 6** and **Type 6'** quadrants in the descriptions of S_2 and S_7 by Pseudo Ibn al-Sarrāj to which we would draw the reader's attention, as they establish a relationship between Mamluk and Ottoman astronomy[52].

7. Horary quadrants with straight hour-lines

7.1. Equinoctial and seasonal hours as straight lines (Type 7 and

[50] In addition, the figure in Z has a solar calendar and a regular longitude scale along the arc of the quadrant; Max H of 72° and min H of 25° serve Istanbul. Its inner section contains an horary quadrant of **Type 4**.

[51] Displays graphs for *ante* and *post meridie* (by dots) equinoctial hours. It has a calendar on one of the axes. The inner arc represents the tropic of Cancer, and the external arc the tropic of Capricorn.

[52] It is interesting to compare *Plate III* (or drawings on Q_{3d} & $_{Q4a}$) with one of the quadrants devised by Pseudo Ibn al-Sarrāj described below (Section 9. 2, *Plate IXb*), the "hour-dial with the slivers" S_7 (*sh-ṭ-b* or *shaṭaba* lines). The drawing on Q_{3d} seems related to quadrant S_7. In quadrant S_7, intended for seasonal hours, the day-circles are drawn as straight lines while in Q_{3d} they are curved; h_i is measured along the horizontal axis, while in Q_{3d} it is measured along the external arc. The quadrants are obviously different, but the idea of having a solar scale on an inner-arc seems to be the same and is probably one of Pseudo Ibn al-Sarrāj's innovations. This idea is confirmed by the second diagram (Q_{3c} & Q_{4b}), whose shape recalls the product of Pseudo Ibn al-Sarrāj design S_2.

Type 7')

Equinoctial hours as straight lines occur in some European instruments. This is the case of Diego Ribero's quadrant, drawn in his Planisferio de Roma (1529)[53], a work dedicated to Emperor Charles V. Equinoctial and seasonal hours as straight lines were also described in a European treatise dating from 1535, written in Germany (Moguntiae) by Johann Stöffler, titled *De compositione aut fabrica astrolabii, eiusdemque usus*[54]. A magnificent example of an instrument constructed according to the German author's instructions has been preserved, and bears the quadrant *"dello stoflerino"*[55]. The instrument is signed "Auctore Tobia Volckmero Brunsvicensi": the leading south German instrument maker Tobias Volckmer[56], dated 1608 (*Plate VII*). It was probably taken from Germany by Prince Mattias and donated to the Galleria dei Uffizi. The instrument was constructed for $\phi = 48°$, serving either Freiburg, Munich or Salzburg, and also used for Viena and Nuremberg. Its back displays two diagrams: the first has equinoctial straight hour-lines (*horae communes ante et post meridiem*) in the external section. The inner circle serves the equator and the external line corresponds to both tropics. The solar longitude scale is stereographical. We will refer to this variety as **Type 7**.

However, the inner section of Volckmer's quadrant displays seasonal hours (*horae inequales*) as straight lines, a variety for which no Islamic or European instrument is known to the author, (with the sole exception of a

[53] See GARCÍA FRANCO, S. [1945: 377-382].

[54] ZINNER, E. [1956: 162] refers to manuscript works on horary quadrants by Johann Stöffler, Johann Volmar and Jakob Köbel. He attributes to the last a drawing of curved seasonal hour-lines.

[55] Johann Stöffler treatise is refered by MINIATI, M. as the source of the Tobias Volckmer's Quadrant. See MINIATI, M. [1986] and ZINNER, E. [1956: 163].

[56] ZINNER, E. [1956: 574-577]; GERKE, K. [1974]; WUNDERLICH, H. [1977: 104-116]; TURNER, A. [1985: 45].

figure in Source M related to the sail-shaped quadrant M_1[57]). We will refer to this variety as **Type 7'**. The solar longitude scale is stereographically projected, as is the other diagram on the same plate. Both diagrams comprise two different sets of hour lines, with a wedge-shaped effect. In both cases the external arc of the quadrant represents the day-circles of Cancer and Capricorn, while the inner arc represents the day-circle of Aries.

Two sets of equinoctial hours as straight lines for northern solar declination (six lines) and southern solar declination (eight lines) drawn to a non-regular longitude scale (**Type 7**) appear in Q_{3b} (*Plate III*), for $\phi \approx 41°$ and $\epsilon \approx 23;35°$. The external arc of one of the sections of this quadrant, graduated to each 5°, bears the inscription: [*sā*ᶜ*āt*] *mustawīya shamālīya* and *mustawīya janūbīya*. The use of the instrument is standard, but one has to consider which sets of lines to look at, depending on the time of year. The Islamic instrument maker of Q_{3b} drew two sets of hour-lines: for the first one, yielding *sā*ᶜ*āt mustawīya shamālīya*, the equator is in the inner part of the quadrant and Cancer is represented by its arc. For the second set of hour-lines, yielding *sā*ᶜ*āt mustawīya janūbīya*, the equinox is placed on the arc of the quadrant and Capricorn at the inner part. The arrangement of signs is of interest in this case since it attests to an [Islamic] tradition which may be independent of the one on which the European instrument makers based their devices.

8. Sail-shaped horary quadrants

Sail-shaped horary quadrants are those in which the day-circles of the signs, usually curved lines, have been changed to straight lines with hour-lines shaped like sails. They were specially designed for use in places where the latitude is more than 24°; otherwise drawing would have been difficult and the instrument would be too crowded for proper use. The earliest source known to us which mentions them is the Andalusian text C, which refers to *qil*ᶜ*ī/qulū*ᶜ*ī* quadrants, *i. e.* like "sails". It describes a

[57] See Section 5. 1. 1 and note 36. One has to remember that M_1 has a regularly dividied solar longitude scale, and consequently differs from **Type 7'**.

variety of two sail-shaped horary quadrants (C_2, C_3), one of which, C_2, can be made universal.

Al-Marrākushī was the earliest Mamluk astronomer to devote a substantional proportion of his work to the description of astronomical instruments; there is no doubt of their historical and scientifical interest. His *Kitāb al-mabādi' wa'-l-ghāyāt fī ᶜilm al-mīqāt* is a major survey of spherical astronomy and astronomical instruments, most of which are considered in the contemporary literature as genuine original inventions of the *muwaqqit*. Al-Marrākushī mentions Andalusian and Maghribi sources, among them al-Zarqālluh and Ibn al-Kammād, but in the section of his work dealing with the present subject he does not mention any source. This has led historians (mainly influenced by Sédillot's studies) to assume that al-Marrākushī was one of the first inventors of horary quadrants, among them those for specific latitudes[58]. This assumption is also to be questioned here by comparing sources C and M. Source M describes three sail-shaped horary quadrants for specific latitudes (numbered here M_2 to M_4). As far as sail-shaped horary quadrants are concerned, most of their descriptions coincide with source C. The analysis has shown their dependence on source C, and again, as in the case of **Type 1**, source C appears to be an obvious source for M. Later, Pseudo Ibn al-Sarrāj re-appraised the achievements of earlier astronomers and sought to improve on them.

8.1 Sail-shaped horary quadrant (Type 8)

Quadrants M_2 (*Plate IVa*, where it is incomplete) and S_5 *sāᶜāt al-shuᶜubiyya*, or "the branch" *(Plate IVb)*[59].

The straight vertical lines represent the day-circles; each one serves two "symmetrical" signs. Their position, and consequently the division of the solar longitude scale OA, has been determined working backwards from the altitudes measured on the graduation of the quadrant. The arc of the

58 This was the case of SCHMALZ, P. [1929: 121-124].

59 Source S, chapter 70, fol 90*v*.

quadrant coincides with measurements of H. Each temporal hour is traced as a curve line across the day-circles. It is to be used with a single thread.

8.2 Sail-shaped horary quadrants (Type 9 and Type 9')

First variety: quadrants C_2 & M_3 (*Plate VIIIa*, unfinished), the so-called "quadrant of the straight lines" (*khuṭūṭ al-mustaqīma*).

The radius of the so-called "small quadrant" A'B' is determined by intersecting $OH_{(S)}$ and an imaginary line AB. Line $OH_{(W)}$ is also drawn imaginarily. Dot *m* in *Plate VIIIa*— is marked on it "arbitrarily and near the apex" according to al-Marrākushī (notice below that Pseudo Ibn al-Sarrāj gives more details in his procedure). Connecting $H_{(S)}$ and *m*, one draws the sixth hour-line ("beginning of the seventh hour" in al-Marrākushī's text). The *madārāt* of the signs originate in A'. They are marked by measuring $H_{(\lambda)}$ on the quadrant and connecting them to the centre O by radii. These radii intersect SEW. Values for h_i at the beginning of each sign will be taken on the exterior graduation and joined to the centre O with a ruler. The ruler will intersect the *madārāt* at each hour. Hour-lines will finally be connected. Al-Marrākushī suggests completing the figure with lines for h_a, h_q and h_0. The Andalusian text has almost the same description (C_2) but does not mention the "small circle". It suggests drawing lines for the beginning and the end of the afternoon prayer, the *qibla*, "and so on". The same remark about its use is made, giving a value for $\epsilon = 24°$. It also points out that this kind of quadrant can be made universal, to be used in all latitudes.

Second variety (**Type 9'**)

Quadrant S_4[60] (*Figure 7*), called *al-junk* or quadrant of the "harp hours".

It is practically identical to **Type 9** but presents a few differences. It has no small quadrant. H_W is measured on the scale of the quadrant. We then imagine a line H_WW' parallel to the horizontal axis. AW_6 is the day-circle of Capricorn, the point W_6 being arbitrarily determined as the intersection

[60] Source S, chapter 69, fol. 90*r*.

of AW and AW_6. Point S_6 on the figure corresponds to the intersection of line AB with h_6 for Cancer measured in the divisions of the quadrant. The day-circle of Aries AE_6 is drawn by dividing W'B in two equal parts (point E' on the vertical axis). Points W_6, E_6 and S_6 are used to draw the so-called "arc of the sixth" for the zodiacal signs. This quadrant is to be used with one thread placed at the centre. Sights should be on axis BO.

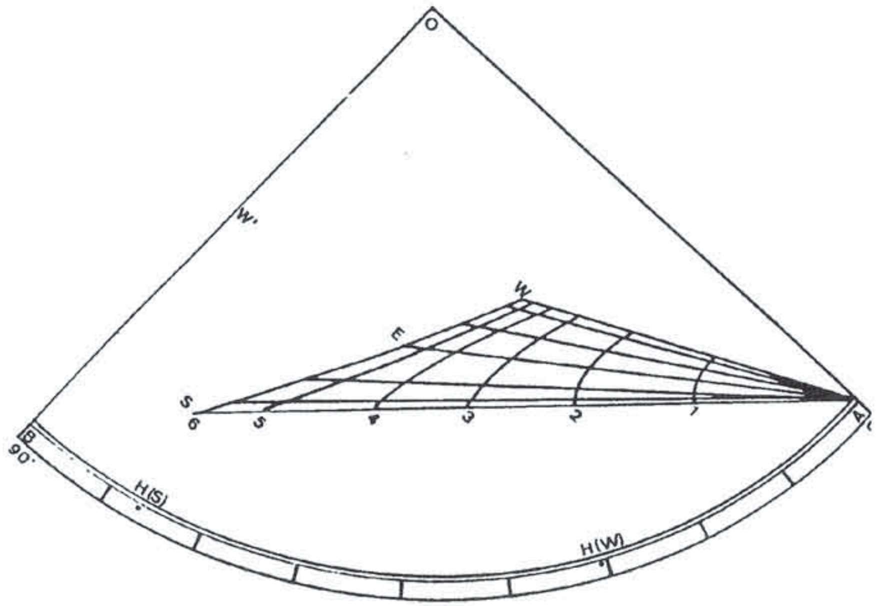


Figure 7

The final result is a crowded quadrant, difficult to draw, and of doubtful use.

8.3 Sail-shaped horary quadrant (Type 10)

Quadrants C_3 & M_4 (*Plate VIIIb*, unfinished)[61].

An arbitrary dot D is marked on axis AO, from which the day-circle are drawn, by marking the meridian altitude of the signs in the arc, and connecting it to dot D. Values for h_i at the beginning of each sign are taken on the exterior graduation and joined to the centre O with a ruler. The ruler intersects the day-circles at each hour. Hour-lines are connected. The text says the figure is intended for $\phi = 30°$. As in the preceding case, the same warnings are made, as well as the suggestion for h_a, h_q and h_0. No surviving instruments with sail-shaped hour-lines are known to the author.

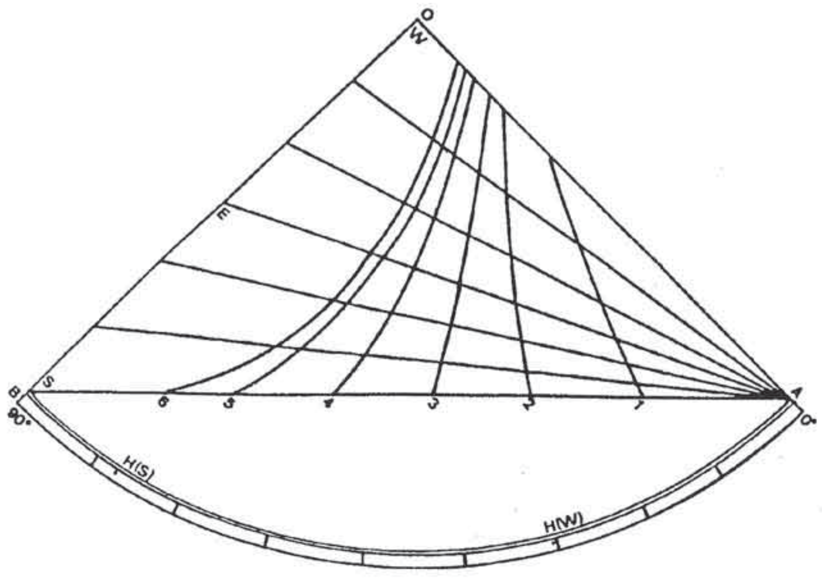


Figure 8

[61] Source M: 370 . See *Plate VIIIb*.

8.4 An additional sail-shaped horary quadrant by Pseudo Ibn al-Sarrāj (Type 11)

Quadrant S_3 *sāʿāt al-watar*, the "chord hours" (*Figure 8*)[62].

Axis OB has a linear scale for longitude. The straight lines from A to the axis OB represent the day-circles of the signs. They have been drawn by dividing the vertical axis OB into six equal parts and connecting each division to point A. It is to be used with two threads, one at O and the other at A. The thread hanging from A and moving along OB will represent the different day-circles of the longitudes along the year. It has to be kept fixed during the observation of the solar altitude. The sights should be on axis BO. The thread hanging from O will measure h_i in the external arc. The intersection of both threads determines the hour.

9. Some unusual hour-dials and horary quadrants by Pseudo Ibn al-Sarrāj

9.1 Curved lines for seasonal hours and Cancer at the inner.

Quadrant S_2 (*Plate IXa*)[63].

This graph, described by Pseudo Ibn al-Sarrāj, is based on the same inversion that causes the difference between the **types 1 & 2**: the external arc of the quadrant will represent the day-circle of Capricorn, while Cancer is in the inner part. The solar longitude scale is divided backwards from values of h_i. Values for $h_i(W)$, $h_i(S)$ and $h_i(E)$ are measured on the external graduation. Their intersections with the day-circles are determined with a ruler from the centre. The quadrant has only one thread at its centre. The sights should be on axis BO. The figure on the manuscript claims to be for $\phi = 36°$. On it, the angle FOB $\approx 31°$ and it may correspond to $H_W = (90°-\phi)-\epsilon = 30;25°$ for $\phi = 36°$, $\lambda = 270°$ and $\epsilon = 23;35°$. In addition, WOF' $= 77°$, which is h_m for $\lambda = 90°$.

[62] Source S, chapter 68, fol 89*v*.

[63] Source S, chapter 67, 89*r*.

9.2 Al-shaṭaba[64]: the "hour-dial with the slivers"

Quadrant S_7 (*Plate IXb*).

This hour-dial has three scales: AB on the arc; AO on the horizontal axis; OC as solar longitude scale inside the curve for $i=6$. Its construction is as follows: the day-circles of the signs are drawn from the radii for each 15° on the arc of the quadrant to the solar longitude scale. For (a) given sign(s) mark the h_i on the thread using the scale AO. Then move the thread to the day-circles. Join the position for each hour to produce hour curves. The quadrant can be used with a thread hanging from B. The sights should be placed on axis BO. A weight moving along the thread would measure the solar longitude λ on the inner scale; h_i would be measured on axis AO, a non-uniform altitude scale.

9.3 Sāᶜāt al-tisᶜīni, "based on two scales of 90°"

Quadrant S_6 (*Figure 9*)[65].

This quadrant has two graduations: AB on the arc; OA on the horizontal axis for the meridian altitudes. To construct it, H_S for each one of the signs is measured on the horizontal axis. These dimensions of H are moved to the axis BO to draw the day-circles with centre at B. This results in a solar longitude scale which is not regularly divided. The complementary of the altitudes h_1 to h_6 is measured on the OA scale to find the points where the hour lines intersect the day-circles of the signs. The hour lines have to be traced across these intersections by using a compass, if possible. Pseudo Ibn al-Sarrāj's text does not mention the latitude for which his figure is intended, but it is very similar to the one drawn to scale for $\phi = 36°$. There is no reference to any thread. It could be used with a thread hanging from B, and by observing the sun from O to B.

[64] Meaning unknown to the author. Source S, chapter 72, fol. 91*v*.

[65] Source S, chapter 71, fol. 91*r*.

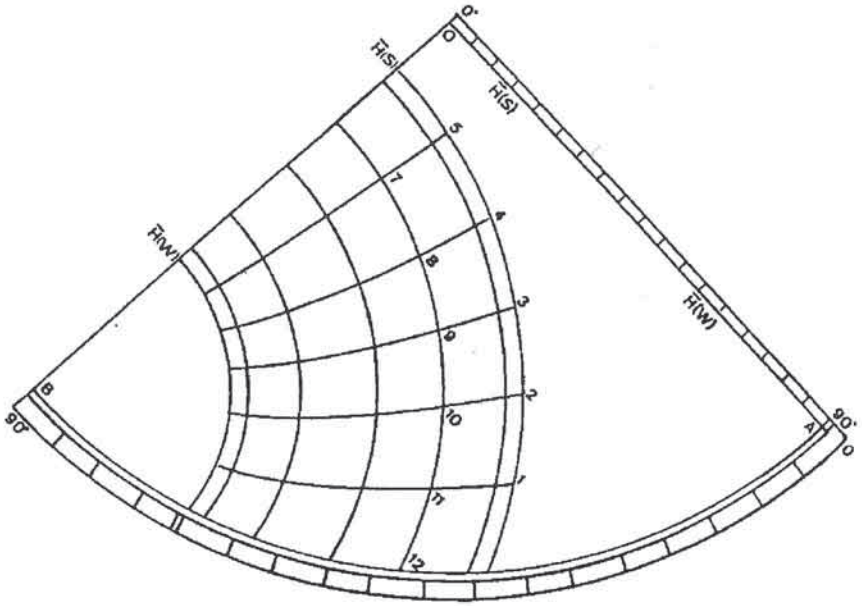


Figure 9

9.4 Sā^cāt al-zāwiya, or the "angular instrument"

Quadrant S_9 (*Figure 10*)[66].

This instrument is not a quadrant but a square. It comprises an OAXB square, divided diagonally. An auxiliary quadrant is drawn only as a reference to graduate the diagonal AB with a non-uniform altitude scale. This graduation is achieved by drawing radii to the uniform divisions of the altitude scale on the auxiliary quadrant. AO and BO are each divided into three equal parts. AE, EW, WO and BE', E'W'', W'O. The diagonal AB corresponds to the day-circle of Cancer. The distances AW and SW'' are

[66] Source S, chapter 75, fol 93*r*.

divided by six (equal) parts to construct the remaining day circles. The weight will move along the scale AW. The altitude of the hours for sign(s) is measured on the diagonal scale, one by one, to complete the hour curves. The sights should be on axis BO and h_i should be observed in that direction.

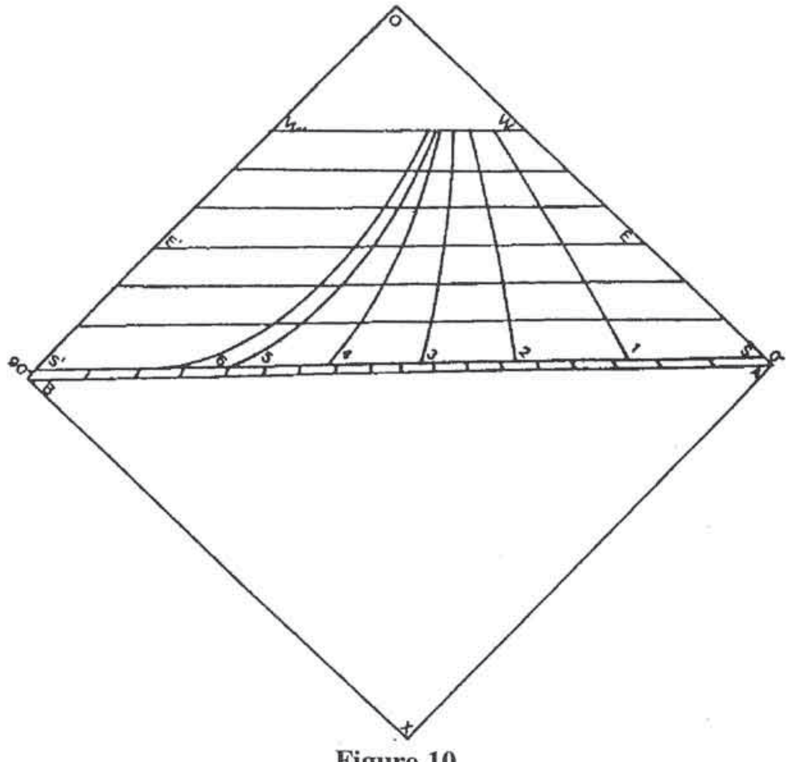


Figure 10

10. Other markings on horary quadrants for a particular latitude

10.1 Solar meridian altitudes (H) and prayer hour-lines (h_a, h_b, h_z)

From the tenth century onwards, mcst treatises on drawing quadrants for particular latitudes refer to a graphic representation of the hour-line for H, the solar meridian altitude for one or for a series of latitudes, and to hour-lines drawn specifically to perform the orthodox Muslim prayers, one of the main practical uses of the horary quadrants. Drawing them requires a table for H, h_a, h_b and h_z. Horary quadrants, almucantar quadrants and astrolabes display these lines engraved by the same methods as those used to draw the common hour-lines.

On earlier horary quadrants with linear solar longitude, scale H and prayer hour-lines (h_a,h_b,h_z) are represented as sigmoids; this is the case of Q_2, which has an additional sigmoid line for h_a by dots (see *Plate I* and section 5.1.2.). On later quadrants with stereographical solar scales the curves were closer to arcs of circles, as was the case of instrument A_3 (see *Plate II* and section 6.3). This is also the case of the Maghribi almucantar quadrant by ᶜAbd Allāh Aḥmad ibn ᶜAlī, for the latitude of Mequinez ϕ = 34°, dated 1219 H./1804 A. D. and preserved in the Museum for the History of Science in Oxford[67]. It is not our purpose to list all the instruments that displayed prayer hour-lines, either as sigmoids or curves, but in the Appendix we will give a few representative examples which are not mentioned in other sections of this paper. Most of them are Indo-Persian astrolabes, rather than horary quadrants — even though most of these instruments do not require the diagram, as they tend to have prayer hour-lines on the plates for specific latitudes.

10.2 Twilight markings as sigmoids

The astrolabe of Shams al-Dīn Muḥammad Ṣaffār[68] (*Plate X*) dated 882 H./1477-78 A. D. has an unusual sigmoid line to indicate when twilight

[67] Museum for the History of Science (Oxford): 56-127. See VERNET, J., SAMSÓ, J. *et alii* [1985: 100-101], where it is erroneously described as "*novus*".

[68] MAYER, L. A. [1956: 75-76, plate XIII]. Shams al-Dīn Muḥammad Ṣaffār, an astrolabist of the late 9th and early 10th centuries/ 15th-16th c., constructed two astrolabes in 882 H./1477-1478 A. D., one of which is preserved in the Museum of Islamic Art in Cairo, while the other is in the Fitzwilliam Museum in Cambridge (*Plate X*). In the back upper right quadrant of the latter one there is a diagram with the prayer lines for the *ẓuhr* and the *ᶜaṣr* at 32° of latitude.

begins in the morning in equinoctial hours for $\phi = 32°$, that is, *ma bayna al-ṣubḥ min al-mustawiya li ᶜarḍ 32*: few astrolabes include a line of this kind because morning and evening twilights can be easily measured on the front of the astrolabe. Ṣaffār's astrolabe has the drawing on the upper back right quadrant, in addition to the prayer lines h_a and h_z[69].

This line indicating twilight is drawn as a sigmoid by taking values of duration of the morning twilight $r \approx 22°$ for $\lambda = 0°$; $r \approx 26;30°$ for $\lambda = 90°$; $r \approx 23°$ for $\lambda = 270°$, according to which it can be measured on the figure taking the arc of the quadrant as the winter solstice. These values seem to correspond to an angle of depression $h_s \approx 18°$, a common parameter in medieval sources.[70] As far as we know, the drawing occurs only on one other Islamic astrolabe by Muḥammad ibn Abī l-Qasim, an instrument maker from Isfahan, who in 496 H. /1102-3 A. D. made an astrolabe now preserved in the Museo di Storia della Scienza in Florence (*Plate XI*)[71]. On the back of his astrolabe he engraved a quadrant with six equidistant day-circles, spanning 3/4 of the radial axis, and crossed by a number of lines: two sigmoid hour-lines for the beginning of the *ẓuhr* and the *ᶜaṣr* at latitude 32° (Isfahan), the direction of the *qibla* for the Persian city, and the uncommon sigmoid diagram *ma bayna al-ṣubḥ min al-mustawīya li ᶜarḍ 32*. Values for r used on it are approximately the same as in the preceding example.

[69] Together with a diagram probably related to the calculation of the sine and cosine. For the twilight markings, we assume that the radial axis of this quadrant is divided into six equal parts representing the equinoctial hours, numbered 0, 1, 2... from sunrise. From each one, six equidistant quadrants have been drawn, representing the arc of daylight for the day in which the sun rises at this hour. Each diurnal arc has been divided into three equal parts. Three sigmoids (two for the diagram *ma bayna al-ṣubḥ min al-mustawiya li ᶜarḍ 32*, and one more for the beginning of the *ᶜaṣr* prayer) are drawn against the scale of daylight arcs, not solar longitudes. A solar longitude scale could be incorporated in an alidade.

[70] See WIEDEMANN, E.[-KING, D. A.] [1996, 185-186].

[71] See MAYER [1956: 59, plate IV]. On the instrument see also GUNTHER, R. T. [1932, I: 263].

10.3 Qibla markings

Another line is shown in late Persian astrolabes (though not on horary quadrants) which is not related to time-keeping but has a direct religious application - the determination of the *qibla* direction for prayer. It is drawn in the same way as hour-lines. The line corresponds to the solar altitude when the sun is in the azimuth of Mecca (h_q, *inḥirāf al-qibla*). *Qibla* markings are usually drawn for a series of latitudes or cities, and combined with the diagram of H for the same cities, they produce a pleasing effect, especially if each series of lines are drawn on a different longitude scale: one with the summer solstice in the external arc of the quadrant and the other reversed[72].

11. Some European examples of tables and instruments

As stated in the introduction, this study does not intend to deal systematically with all the European materials on quadrants for specific latitudes. Nevertheless, a few examples will illustrate how the bases of the construction of the horary quadrant were applied in Europe on instruments of considerable geometrical complexity for particular latitudes.

As far as the availability of tables for $h_i(t,\delta,\phi)$ is concerned, in the European context the situation confirms our comments regarding the Islamic world: determining the solar altitude for the day hours at a particular latitude and tabulating it belongs to the domain of constructing scientific instruments rather than to the large astronomical compilations. In fact, the first European example of hour-lines for a specific latitude known to the author is a table of solar altitude contained on the treatise "*De utilitatibus astrolabii*" by the German monk Hermann Contractus (1013-1054). This table would have made it possible to draw an horary

[72] Instruments with drawings of this nature are studied by KING, D. A. [1999].

quadrant to be used "*in climate nostro*"[73].

The situation is practically the same in Medieval Spain. The solar altitude function, so useful for time-keeping, is not tabulated in the *Toledan Tables* or in the *Alphonsine Tables*. Nevertheless, a table for h_i appears in the Iberian Peninsula, in this case as an exception, in an astrological context: Enrique de Villena introduced it in his "*Tratado de Astrologia*"[74]. It is calculated for $\phi = 38°$ degrees, serving Baez (al-Andalus), and it is so inaccurate that it makes one suspect it may even have been compiled backwards from an instrument[75].

Many preserved European instruments also testify to an evident interest in drawings for time-keeping. Several examples of horary quadrants in Europe from the fourteenth to the eighteenth century demonstrate their widespread use in the western world, all types being represented. Other show more than one diagram on the same plate. A good example is J. Vooghd's quadrant, with two different horary quadrants serving specific latitudes, or the sixteenth century horary quadrant created by Edmund Gunter, discussed in the next section. Also, in 1681, in Amsterdam, Claas Jansz Vooghd published a small book describing a few quadrants printed on paper to be pasted on wood, but which could be engraved on copper. One of Vooghd's quadrants, dating from the beginning of the eighteenth century, has been preserved (*Plate V*). It was published in Amsterdam by

[73] The text (*Liber Secundum*) was published by GUNTHER, R. T. [1932, II: 418-422]. The maximum meridian altitude of the sun according to Hermann Contractus is 66° which coincides with that expected for the latitude of Reichenau, where he lived (approx. 47;30°). See also KREN, C. [1977].

[74] See CÁTEDRA, P. & SAMSÓ, J. [1980: 56-57 and 171-176].

[75] KING, D. A. and GIRKE, D. [1988] analyzed two different types of universal horary quadrants (*type a*, with circular arcs, and *type b* with parallels) in the treatise by Enrique de Villena. According to him the table is badly computed: even the values at the equinoxes are in error by 2°. A universal quadrant of *type b* is constructed on the basis of the so-called *jayb al-saʿat li-rtifaʿ nisf al-nahar*, a function already known to al-Khwārizmī; see KING, D. A. [1983a: 10] and HOGENDIJK, J.P. [1991:1-12] for explanation of the *jayb al-saʿat*.

Han a Loon and is preserved today in the Utrecht University Museum[76]. The progress made in early modern Europe in devising fine astronomical instruments led to a demand for increased accuracy and a wider variety of instruments. Vooghd's quadrant, which displayed sigmoid lines for the equinoctial hours, seems to be the result of the development of aesthetic instruments that agreed with observations. It is one of the finest examples of an horary quadrant for a specific latitude in the European tradition. The external diagram on Vooghd's quadrant presents nine sigmoid lines according to a regularly divided solar longitude scale. These hour lines correspond to h_j at the symmetric equinoctial hours and have been proved to serve $\phi=51°$, which is appropriate for the Netherlands. Also, in 1972 Louis Janin reproduced a design of an horary quadrant of **Type 3** taken from a French classical book on gnomonics by Dom François Bedos de Celles, published in Paris in 1760[77].

Types 4, 5 and **6** were very succesful too. For instance, J. Vooghd's quadrant (*Plate V*) displays, close to the edge, curved lines for seasonal hours, probably intended for $\phi= 51°$ in **Type 4**. Also, an horary quadrant made in London for King Edward VI and dated 1551 contains, in its inner part, an horary quadrant of the same type. It presents an unusual solar longitude scale along a straight line not divided regularly and a calendar scale on one of the sides[78]. It was made for $\phi= 51;34°$.

The drawing of curved equinoctial hour-lines (**Type 5**) seems to have been eve more successful in early modern Europe, and became widely known between the fourteenth and the eighteenth centuries. A few examples corroborate this asertion. Late European additions to an astrolabe

[76] See van CITTERT, P. H. [1954: 23 and 35 (Obs. 129, plates XVIII & XIX)]. This instrument by Han a Loon, published by Joannes Loots in Amsterdam, is considered a new edition of a quadrant by Vooghd, from 1681, edited by Joannes van Keulen in Amsterdam too, referred to as A.9 in van CITTERT, P. H [1954: 33-35]. On one of its sides there are two different horary quadrants serving specific latitudes. (**Type 3 & 4**, inner).

[77] JANIN, L. [1972-1973: fig. 1]

[78] WARD, F. A. B., [1981, pl. XIX]. Now published Silke Ackermann

by Abū Bakr ibn Yūsuf in Marrākush *ca*. 1200 (and preserved in the Instituto de Valencia de Don Juan in Madrid) include curved-hour lines for equinoctial hours together with a universal horary quadrant[79].

Also, a small, unsigned ivory quadrant dated 1610 and intended for $\phi = 49°$ [80] (preserved in the Utrecht University Museum) has not uniform solar scale and the hour lines are parts of circles. Values of h_i at the equinoctial hours were mechanically recomputed for $\phi = 49°$ and $\epsilon = 23;35°$. Results were compared with approximate measurements on the plate and confirm these parameters.

Finally, a Dutch copper horary quadrant by Anthony Sneewins (Utrecht University Museum), dated 1645 [81] was made after the manner of Philip Lansberg, a distinguished instrument maker, author of several works on astronomical and geometrical quadrants. The inner part presents a universal horary quadrant for seasonal hours (a diagram probably included to provide the instrument - restricted to a particular latitude - with a universal aspect). The external diagram corresponds to **Type 5**. A non-uniform solar scale is engraved on one of the axes. An alidade with an index turns around the centre. There is no indication of the local latitude for which it was constructed. Nevertheless, it can be approximately deduced from the figure $\phi = 52;30°$, which serves Amsterdam.

A far as **Type 6** and **Type 6'** are concerned, the European tradition is represented by the horary quadrant in the National Maritime Museum of Haifa, dated 1709, made in wood and paper. It is signed only with the capital letters I.F.S.F., and, according to the published description, not highly accurate. It bears a calendar along the axis and a solar longitud scale which recalls S_2. Post meridian hour lines are represented by dots. It could be used for any locality between Barcelona and Constantinople, approximately[82].

[79] See VERNET, J., SAMSÓ, J. *et alii* [1992: 56].

[80] Van CITTERT [1954: 35-36, plate XX].

[81] Van CITTERT [1954: 26-32, plate XVI].

[82] JANIN, J. [1972-1973: 5, pl. VI].

A brass horary quadrant by Gerolamo Volpaia (1540-1614), dated 1568, serving Florence $\phi = 43;30°$, and preserved in that city in the Museum of History of Science[83], bears the same kind of drawings.

And a fascicle by Philip Apian, entitled *De utilitate trientis, instrumenti astronomici novi libellus*, published in Tübingen in 1586, describes an instrument which he calls "*triens*" and attributes to his father, the reputed astronomer Peter Apian[84]. In fact, the instrument corresponds to a section — one third approximately — of a plate for a particular latitude in the stereographic astrolabe, in which several hour systems can be drawn. Among them, the hour lines **Type 6** and **Type 6'**[85].

A fourth example can be found in the astrolabe by L. Damery, *ca.* 1600, preserved in the Musée de la Vie Wallone in Liège. On the back side (upper-left section) it shows a **Type 6** horary quadrant, drawn together with a universal horary quadrant for seasonal hours[86]. And also, the back of the astrolabe by "Thornoe" (upper-right section) seems to display hour lines of **Type 6**. The instrument is preserved in the National Maritime Museum in Greenwich. It was included in Gunther's catalogue, but no picture was published[87]. A discussion on the history and the authorship of the instrument was given by E. Poulle[88].

There is an additional European example of **Type 7** in a brass quadrant from the end of eighteenth century by C. and D. Metz[89]. This quadrant, which was the property of J.H van Swinden (1746-1823), a member of the

[83] Museum of History of Science Firenze, n. 2503; for description and plate see JANIN, L. [1977].

[84] On the works by Peter Apian concerning horary quadrants see ZINNER, E. [1956: 160-162]; on the "triens" by Philipp Apian see ZINNER, E. [1956: 163-164].

[85] JANIN, L. [1979].

[86] MICHEL, H. [1976, pl. XII].

[87] GUNTHER, R. T. [1932, II *addendum*: 599].

[88] POULLE, E. [1963: 24, plate III].

[89] Van CITTERT, P. H. [1954: 36-38, plate XXII)].

Commission for "poids et mesures" who worked on the invention of the microscope[90], displays **Type 7** hour-lines. Above them, in the inner part of the instrument, can be found a system of azimuth lines which are straight and wedge-shaped too. The quadrant is intended for use with one thread and two beads in relation to a single non-linear solar longitude scale. In the European quadrants by Volckmer (**Type 7 and 7'**) the exterior arc corresponds to both the summer and winter solstices, producing the final effect of wedge-shaped hour lines, which has been reported to be characteristic of the Gunter type instrument. The instrument devised by Edmund Gunter (1581-1626) in his treatise *The Description and Use of the Sector, Cross-Staff and other Instruments* (London, 1623)[91] consists of a stereographic projection of part of the equator, tropics, ecliptic and horizon onto the plane of the equator (*Plate XII*). It is not our purpose to analyze its construction in detail, as full descriptions can be found in the modern literature; we will just highlight a number of points. In the particular case of the Gunter quadrant, the hour-lines were drawn in two different sets of hour-lines, one turning to the right to show the time in summer, and the other to the left for winter time. In addition, each set of hour-lines spans between the equator and the external arc of the quadrant, which represents either Cancer or Capricorn, while the equator is represented by the inner-arc, producing Gunter's characteristic wedge-shaped hour lines. The character of genuine innovation attributed to Gunter' quadrant has never been questioned. Nevertheless, seasonal hour-lines as curves drawn over a stereographic projection of the equator and the solstices have precedents in the Islamic tradition (Q_{3b}), which we have already mentioned. The wedge-shaped lines, though purported to be original, can also be found in a number of earlier instruments, both Islamic and European.

About 1325 double sets of lines were a common feature of the so-called *rub^c al-muqanṭarāt*. Pseudo Ibn al-Sarrāj, who described several different kinds of almucantar quadrants, wrote about an instrument which he called

[90] HACKMANN, W. D. [*DSB*, XIII: 183-184]; TURNER, A. J. [1985: 94].

[91] PEEPER, J. V. [*DSB*, V: 593-594]. A good description of Gunter's quadrant can be found in TURNER, A. J. [1985: 206-210]. See also GUNTER, E. [1623].

al-musattar or "covered quadrant", in that there were two sets of almucantars one on top of the other, one above the "horizon" and the other below, to serve northern and southern declinations. Markings on the inner quadrant serve the equinoxes and those on the external serve either solstice; the ecliptic is represented by only three zodiacal signs. To our knowledge, this instrument has not yet been described in the literature.

Another *muwaqqit* at the Umayyad Mosque in Damascus called al-Mizzī (died *ca.* 1350)[92], wrote about the almucantar quadrant, an instrument from which a few examples from the Islamic tradition are preserved. There are at least five Ottoman-Turkish or late Maghribian almucantar quadrants of the most straightforward variety.[93] One of them, in the Benaki Museum in Athens, is an ivory quadrant with markings for both Damascus and Cairo, based on Ibn al-Sarrāj's design[94]. In addition, the brass quadrant by Aḥmad ibn ᶜAbd al-Raḥmān al-Duhmānī (Morocco, 854 H./1450 A.D.), preserved in the Museo Arqueológico Nacional (Madrid)[95], presents a double set of almucantars and azimuth lines for $\phi = 36;40°$ (*Plate XIII*), as does the Maghribian almucantar quadrant by ᶜAbd Allāh Aḥmad ibn ᶜAlī (see section 10.1).

Wedge-shaped seasonal hour-lines as curves appear in Q_{3a}, while Q_{3b} displays straight hour-lines in two sets. In both cases the equinox remains in the middle of the day-circles of the solstices, a detail which may suggest that the Ottoman quadrants Q_3 and Q_4 are independent of Gunter's work, and most probably based on an old Islamic tradition which can be traced back to the ninth century and which reached its maximum splendour with the Syrian astronomer Ibn al-Sarrāj. Nevertheless, even in this case, there

92 SUTER, H. [1900, n. 406]; KING, D. A. [1986c, C34].

93 The best description of an instrument of this kind was given by MORLEY, W. H [1860]. See also WÜRSCHMIDT [1918]; FEHÉRVÁRI [1973]; JANIN & ROHR [1975]; KING, [1985a]; for the use of the instrument SCHMALZL [1929: 33-62]; and KING, D. A. & GIRKE, D. [1988].

94 See KING, D. A. [1984: 73-84].

95 Museo Arqueológico Nacional (Madrid), n. 50856. See VERNET, J., SAMSÓ, J. *et alii* [1985: 102-103].

is an European example of wedge-shaped hour-lines in the quadrant "dello stoflerino" by Tobias Volckmer, which predates Gunter's instrument (see section 7. 1 and *Plate VII*).

12. Conclusions

We conclude that Islamic horary quadrants for a particular latitude based on the exact formula for time-keeping originated in the ninth century and were widespread throughout the Islamic world, reaching Iran and Andalusia. The Andalusian tradition is represented by the Cairo Ṭalʿat manuscript (Source C). Al-Marrākushī was, most probably, the source that re-introduced and popularized the horary quadrant, in a number of varieties, among eastern and western *muwaqqits* in the Islamic cultural context.

Up to that moment, quadrants had three main shapes of hour-lines: sigmoid hour-lines or curve-lines, depending on the solar longitude scale (uniform, or stereographically projected), or sail-shaped quadrants in which day circles are represented as straight lines. During the fourteenth century the theory on the instrument reached its higher development. Pseudo Ibn al-Sarrāj investigated different ways to represent a trigonometric function graphically. In all probability, his instruments never became very popular, since no example has survived. Nevertheless, the tradition of the horary quadrant was to remain alive in the Islamic world for centuries (the latest source, Z, was published in Cairo in 1885). As far as the European tradition is concerned, there are no drawings which could be called genuinely original. A few horary quadrants from the European tradition present drawings which are uncommon in the Islamic tradition. However, we have questioned the high creativeness attributed to some of these designs, which are sometimes misleadingly purported as new ideas. We hope to have demonstrated the surviving influence of the Islamic Middle Ages traditions on time-keeping using the exact formula on the European instruments.

II. Appendix. Additional Extant Instruments

Function H

Several Persian astrolabes contain the graphic representation of the functions h_a, h_b, h_z and H, which were drawn according to the same tradition as hour lines for a particular latitude on the horary quadrants — though a diagram of this kind would not be needed on an astrolabe, since most of these instruments have hour lines on the plates for specific latitudes.

The representation of H seems to have been characteristic among the well-known group of astrolabists from Lahore working at the end of the seventeenth century.

1. * The Mughal astrolabe signed by the "two sons" (Muḥammad Muqīm and Ibn Qāsim Muḥammad) of ᶜĪsā b. Allāh-dād from Lahore, made in 1018 H. / 1609 A. D. In 1914 it was in possession of the Kestner-Museum in Hanover. The back of the astrolabe has a wavy representation of H serving 32° towards a regularly divided solar longitud scale[96].

2. * The astrolabe by Muḥammad Muqīm b. ᶜĪsā b. Allāh-dād asṭurlābī Humāyūnī Lahūrī, member of a reputed Lahore family of instrument-makers. The instrument (*ca.* 1650) is not dated, and characteristic in its style of the school of Lahore astrolabists[97].

3. * There also seems to be an attempt to draw this hour-line as a curve for $\phi = 20°$, on a stereographic solar longitude scale. On the back of a Lahore astrolabe by Muḥammad Muqīm, dated Ramadan 24th, 1102 / June 21st, 1691[98].

4. * An interesting example of H may be the one in a Sanskrit astrolabe (*ca.* 1800) (*Plate XIV*), unsigned, and with a drawing on its back upper right quadrant for $\phi = 25°$[99].

[96] FRANK, J., & MEYERHOF, M. [1925: Tafel 2].

[97] LINTON, L. [1980: 96-97].

[98] LINTON, L. [1980: 100-101].

[99] LINTON, L. [1980: 182].

A few representations of functions h_a, h_b, h_z

1. * The astrolabe of Ḥāmid ibn Maḥmūd al-Iṣfahānī[100], dated 547 H. / 1152 A. D., preserved in the National Museum of American History, presents two quadrants with linear solar longitude scale and respective sigmoid prayer lines h_a and h_z for latitudes 36° and 32;25° serving Rayy and Iṣfahān, drawn by the same method of **Type 1**[101].

2. * Jaᶜfar ibn ᶜUmar ibn Dawlatshāh al-Kirmānī[102], a member of a reputed Persian school of astrolabe makers, drew a quadrant with hour-lines h_a, h_b and h_z for the *ẓuhr* and the beginning of the *ᶜaṣr* for $\phi = 32°$ and six equidistant day-circles as arcs for the zodiacal signs on the back of his astrolabe dated 755 H./1354 A.D. The solar longitude scale, divided regularly, fills 3/4 of the radial axis. Sigmoid prayer lines are drawn in the same style as seasonal hours in **Type 2**. In the fifties it was M.G. Prin's collection, and is now preserved in the Musée de l'Institut du Monde Arabe (Paris).[103]

3. * The almucantar quadrant by Muḥammad b. Aḥmad al-Mizzī, dated 734 H. / 1333-1334 A. D., made in Damascus as a present for Nāṣīr al-Dīn Muḥammad b. ᶜAbd Allāh b. ᶜAbd al-Raḥīm, displays a curve for h_a [104].

4. * The almucantar quadrant by ᶜAbd Allāh Aḥmad ibn ᶜAlī[105], dated 1219 H./ 1804 A. D., made in Rabat under the supervision of the *muᶜaddil* Sayyid al-Muᶜṭī ibn al-Ṭayyib and to be given to Sayyid Muḥammad ibn al-Ḥājj ᶜAbd al-Salām al-Salāwī. It is preserved in the Museum of the History

[100] MAYER, L. A. [1956: 46].

[101] GUNTHER R. T. [1932, I: 117, no. 4, plate xxiv]; GIBBS and SALIBA [1984: 62-64, n. 4, fig. 38] for a proper description of the instrument.

[102] MAYER, L. A. [1956: 53-54, plate X].

[103] Mention Catalogue.

[104] It was described and printed by SCHMALZ, P. [1929: 36-37, fig. 5] where he refers to DORN, B. [1865] as his source for this information.

[105] See MICHEL, H. [1947: 123-128]; VERNET, J., SAMSÓ, J. *et alii* [1992: 250].

of Science in Oxford. Prayer hour-lines are constructed over a stereographic longitude scale as curves.

Qibla

1. One good example of *qibla* lines is the magnificent astrolabe made for Shāh ᶜAbbās II of Persia in 1057/1647-1648, preserved in the Museum of the History of Science in Oxford. It is signed Muḥammad Mahdī al-Yazdī. The instrument is a masterpiece of accurate work, decorated with coloured letters, in gold and turquoise. On the back it has the arcs of the solar altitudes in the azimuth of the *qibla* for Baghdad, Isfahan and Tus[106].

2. Another fine instrument by the same author, dated 1059/1649-1650, displays on its back the arcs of the solar altitudes in the azimuth of the *qibla* for a series of latitudes[107].

3. From the last century, dated 1234/1818-1819, the Persian astrolabe made by Muḥammad Akbar Afshār. The author included lines for h_q at Shiraz, Baghdad, Isfahan and Tus and the midday arcs for eight latitudes, every two degrees from 28° to 42°[108].

[106] TURNER, A. J. [1985: 86-91, fig. 67].

[107] BRIEUX, A. [1974: fig. 4]. Morley, reprinted in Gunther, I.

[108] TURNER, A. J. [1985: 104-107].

III. Plates

I. Q_2. An horary quadrant for $\phi = 30°$.
II. A_3. An astrolabe by "ᶜAbdī".
III. Q_3. A wooden Ottoman quadrant dated 1178 H./1764-65 A.D.
IV. *a*) Source M: 367 displaying Table II and M_2; *b*) Source S: S_5 *sāᶜāt al-shuᶜubiyya*.
V. Vooghd's quadrant (beginning 18th century).
VI. Italian horary quadrant dated 1611 (Oxford).
VII. Tobias Volckmer's quadrant dated 1608.
VIII. *a*) Source M: 369, M_3; *b*) Source M: 370, M_4.
IX. *a*) Source S: S_2; *b*) Source S: S_7.
X. The astrolabe of Shams al-Dīn Muḥammad Ṣaffār (882 H./1477-78 A. D.).
XI. The astrolabe by Muḥammad ibn Abī-l-Qasim (496 H. /1102-3 A. D.).
XII. Gunter's quadrant (1623).
XIII. The brass quadrant by Aḥmad ibn ᶜAbd al-Raḥmān al-Duhmānī (854 H./1450 A.D.).
XIV. A Sanskrit unsigned astrolabe (*ca.* 1800).

IV. References

BOILOT, D. J.
(1955-1956) "L'oeuvre d'al-Beruni. Essai bibliographique". *Mélanges de l'Institut Dominicain d'Etudes Orientales*, 2. Cairo, 1955, 161-256; "Corrigenda et addenda", *ibid.*, 3, Cairo, 1956, 391-396.

BRIEUX, A.
1974 *Les astrolabes. Tests d'authenticité. Art et Curiosité*. Paris.

BRIEUX, A. & MADDISON, F.
forthcoming *Repertoire des facteurs d'astrolabes et de leurs oeuvres/* I: Islam. Paris.

BURGESS, E. *Sūryasiddhānta* (Anonymous)
1860 *Translation of the Sūrya-Siddhānta, a text-book of Hindu astronomy, with notes, and an appendix, by Rev. Ebenezer Burgess, ...assisted by the Committe of publication* (trans. and notes by WHITNEY, W. D.) (*Journal of the American Oriental Society*, 6.) New Haven: The American Oriental Society, 1860, 141-498.

CÁTEDRA, P. & SAMSÓ, J.
1980 *Tratado de Astrología atribuído a Enrique de Villena*. Madrid.

Van CITTERT, P. H.
1954 *Astrolabes - A critical description of the astrolabes, noctilabes and quadrants in the care of the Utrecht University Museum*. Leiden.

DRECKER, J.
1925a *Die Theorie der Sonnenuhren*. Von BASSERMANN-JORDAN, E. (ed.) *Die Geschichte der Zeitmessung und der Uhren*, I. Berlin & Leipzig.

1925b *Zeitmessung und Sterndeutung in geschichtlicher Darstellung*. Berlin: Von Gebrüder Borntraeger.

DORN, B.
1865 *Drei in der Kaiserlichen öffentlichen Bibliothek befindliche astronomische Instrumente mit arabischen Inschriften.* (Mémoires de l'Académie Impériale des Sciences de St. Petersbourg, 9:1.) St. Petersbourg.

DSB

FEHÉRVÁRI, G.
1973 "An eighth/fourteenth-century quadrant of the astrolabist al-Mizzī". *Butlletin of the School of Oriental and African Studies* (London), 36, 1973, 115-117, 2 plates.

FRANK J. & MEYERHOF, M.
1925 *Ein Astrolab aus dem indischen Mogulreiche.* (Heidelberg Akten der von-Portheim-Stiftung, 13.) Heidelberg.

GARCÍA FRANCO, S.
1945 *Catálogo crítico de astrolabios existentes en España.* Madrid.

GERKE, K.
1974 *Tobias Volckmer aus Braunschweig.* (Mitteilungen der Technischen Universität Carolo-Wilhemina zu Braunschweig, IX, iii, iv.) Braunschweig.

GIBBS, S., HENDERSON, J. A. & de SOLLA PRICE, D.
1973 *A computerized checklist of astrolabes.* New Haven, Ct.: Yale University, Department of History of Science and Medicine.

GIBBS, S. & SALIBA, G.
1984 *Planispheric astrolabes from the National Museum of American History.* (Smithsonian Studies in History and Technology, 45.) Washington D. C.

GOLDSTEIN, B. R.
1963 "A medieval table for reckoning time from solar altitude". *Scripta Mathematica*, 27, 1963, 61-66. [Repr. KENNEDY, E. S. *et alii*, 1983,

293-298].

1967 *Ibn al-Muthannā's commentary on the astronomical tables of al-Khwārizmī*. New Haven, Conn.

GUNTER, E. (1581-1626)
1623 *The description and use of the sector, cross-staff and other instruments*. London.

GUNTHER, R. T.
1932 *The astrolabes of the world*, II. Oxford. [Repr. London: The Holland Press, 1976].

HACKMANN, W. D.
1976 "Swinden, J. H van". *Dictionary of scientific biography*, XIII (*DSB*, XIII). New York, 1976, 183-184.

HIGGINS, K.
1953 "The classification of sundials". *Annals of Science*, 9, 1953, 342-358.

JANIN, L.
1971 "Le cadran solaire de la Mosquée Umayyade à Damas". *Centaurus*, 16, 1971, 285-298.

1972-1973 "Un cadran solaire de hauteur". *Sefunim, Butlletin du Musée National Maritime de Haifa*, IV, 1972-1973, 60-63, 1 figure, 1 plate.

1977 "Un cadran de hauteur". *Annali dell' Istituto di Storia della Scienza di Firenze*, II, 1977, 21-23, 1 plate.

1979 "Astrolabe et cadran solaire en projection stéréographique horizontale". *Centaurus*, 2, 1979, 298-314.

JANIN, L. & ROHR, R. R. J.

1975 "Deux astrolabes-quadrants turcs". *Centaurus*, 19, 1975, 108-124.

JANIN, L: & KING, D. A.
1977 "Ibn al-Shāṭir's Ṣandūq al-Yawāqīt: an astronomical 'compendium'". *Journal for the History of Arabic Science*, 1, 1977, 187-256. [Repr. see KING, D. A. *IAI, Islamic Astronomical Instruments*. London: Variorum Reprints, 1987].

KAZEMI, F. & McCHESNEY, R. B. eds.
1988 *Islam and society: Arabic and Islamic studies in honor of B. Winder*, New York: New York University Press.

KENNEDY, E. S.
1956 "A survey of Islamic astronomical tables". *Transactions of the American Philosophical Society*, N.S., 46:2, 1956, 123-177.

KENNEDY, E. S.
1970 "Al-Bīrūnī". *Dictionary of scientific biography*, II (*DSB*, II), New York, 1970, 147-158. [Repr. *Studies in the Islamic exact sciences*, Beirut, 1983].

KENNEDY, E. S. *et alii*
1983 *Studies in the Islamic exact sciences*, Beirut: American University of Beirut.

KING, D. A.

1973 "Ibn Yunus' *Very Useful Tables* for reckoning time by the Sun". *Archive for History of Arabic Science,* 6, 1973, 342-394. [Repr. in *Islamic Mathematical Astronomy* (*IMA*). London: Variorum Reprints, 1986].

1978 "Astronomical timekeeping in fourteenth century Syria". *Proceedings of the First International Symposium for the History of Arabic Science (Alepp 1976)*/II. Alepp: 1978, 75-84. [Repr. in *Islamic Mathematical Astronomy* (*IMA*). London: Variorum Reprints, 1986].

1979 "On the early history of the universal astrolabe in Islamic astronomy and the origin of the term "shakkāzīya" in medieval scientific Arabic". *Journal for the History of Arabic Science*, 3, 1979, 244-257. [Repr. *IAI*].

1981-1986 *A catalogue of the scientific manuscripts in the Egyptian National Library* (in Arabic)/ 2. Cairo: General Egyptian Book Organization.

1983a *Al-Khwārizmī and new trends in mathematical astronomy in the ninth century* (Occasional papers on the Near East, Hagop Kevorkian Center for Near Eastern Studies, 2.) New York: New York University.

1983b "The astronomy of the Mamluks". *Isis,* 74, 1983, 531-555. [Repr. *IMA*].

1984 "The astronomy of the Mamluks. A brief overview". *Muqarnas*, 2, 1984, 73-84.

1985a "Osmanische astronomische Handschriften und Instrumente". *Türkische Kunst und Kultur der osmanischer Zeit. (Ausstellungskatalog, Museum für Kunsthandwerk, Frankfurt am Main, and Villa Hügel e V., Essen-Bredeney*/2.) Recklinghausen: Verlag Aurel Bongers, 1985, 373-378. [Repr. in *IAI*].

1985b "The medieval Yemeni astrolabe in the Metropolitan Museum of Modern Art in New York City". *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften*, 2, 1985, 99-122. [Repr. *IAI*]

1986a *Islamic mathematical astronomy (IMA)*. London: Variorum Reprints.

1986b "Astronomical timekeeping in Ottoman Turkey". *Proceedings of the first international symposium on the observatories in Islam*. Istanbul, 1977, 245-269. [Repr. *IMA*].

1986c *A survey of the scientific manuscripts in the Egyptian National Library*. Malibu, Ca.

1987a *Islamic astronomical instruments (IAI)*. London: Variorum Reprints.

1987b "The astronomical instruments of Ibn al-Sarrāj: a brief survey". *Islamic Astronomical Instruments*, London: Variorum Reprints, 1-3.

KING, D. A. & GIRKE, D.
1988 *An approximate trigonometric formula for astronomical timekeeping and related tables, sundials and quadrants from 800 to 1800*. Frankfurt am Main: Johann Wolfgang Goethe-Universität, Institut für Geschichte der Naturwissenschaften. *Preprint Series n. 1*.

1988b *The early history of the quadrant in Islamic astronomy*. Frankfurt am Main: Johann Wolfgang Goethe-Universität, Institut für Geschichte der Naturwissenschaften. *Preprint Series n. 1*.

1990a "Science in the service of religion: the case of Islam". *Impact of Science on Society*, 159, Paris: U.N.E.S.C.O., 1990, 245-262. [Repr. *Astronomy in the Service of Islam* (*ASI*). London: Variorum Reprints, 1993].

1990b "Mīkāt: astronomical timekeeping". *The Encyclopaedia of Islam*^{2}/VII, fac. 115-116, 27-32. [Repr. *ASI*].

1990c "A survey of medieval Islamic shadow schemes for simple time-reckoning. *Oriens*, 32, 1990, 191-249.

1993 *Astronomy in the Service of Islam* (*ASI*). London: Variorum Reprints.

1994 "Rub^c". *The Encyclopaedia of Islam*^{2}/VIII, fac. 139-140, 592-594.

1995 "Early islamic astronomical instruments in Kuwait collections".

Féhervari, G. & Fullerton, A. (eds.) *Kuwait: Arts and Architecture - A Collection of Essays*. Kuwait: 1995, 76-96.

(forthcoming) *Studies in astronomical timekeeping in medieval Islam, I: A survey of tables for reckoning time by the Sun and stars; II: A survey of tables for regulating the times of prayer. SATMI*. Barcelona.

KISH, G.
1970 "Apian, Peter". *Dictionary of scientific biography*, I (*DSB*, I), New York, 1970, 178-179.

LINTON, L.
1980 *Collection Leonard Linton et de divers amateurs: Instruments scientifiques, livres anciens. A catalogue of the Leonard Linton collection offered for sale by E. Libert, A. Castor and A. Brieux in 1980*. Paris: Alain Brieux.

LIVINGSTON, J.
1972 "The *mukḥula*, an Islamic conical sundial". *Centaurus*, 16, 1972, 115-120.

LORCH, R.
1981 "A note on the horary quadrant". *Journal for the History of Arabic Science*, 5, 1981, 115-120.

Al-MARRĀKUSHĪ
(fl. Cairo 1280) *Jāmiᶜ al-mabādi' wa'l-ghāyāt fī ᶜilm al-mīqāt*. SEZGIN, F. (ed.) *facsimilae*, 2. Repr. from Ms. 3343 Ahmet III Collection, Topkapi Sarayi Library Istanbul. Frankfurt-am-Main: Publications of the Institute of Arabic-Islamic Science, 1984.

MAYER, L. A.
1956 *Islamic astrolabists and their works*. Geneve, 1956. Supplement ETTINGHAUSEN, R. (ed.) *Aus der Welt der Islamischen Kunst*. Berlin, 1959, 293-296.

MICHEL, H.
1947 *Traité de l'astrolabe*. Paris.

MINIATI, M.
1986 "Le cadran universel de Tobias Volckmer conservé au Musée d'Histoire des Sciences de Florence". *Astrolabica,* 4, 1986, 21-28. [Repr. from *Bulletin de l'Association Nationale de Collectionneurs et Amateurs d'Horologerie Ancienne,* 46, 1986].

MORLEY, W. H.
1860 "Description of an Arabic quadrant". *Journal of the Royal Asiatic Society* (London), 17, 1860, 322-330, 2 figures.

MORTILLARO, V.
1848 "Illustrazione di un astrolabio arabo del nono secolo". *Opere del Mortillaro*/4. Palermo: 1848, 110-135, 2 plates.

NALLINO, C. A.
1899-1907 *Al-Battānī: Opus astronomicum*, 3vols., Milano.

NEUGEBAUER, O.
1975 *A history of ancient mathematical astronomy*, 3 vols., Berlin, Heidelberg, New York.

NEUGEBAUER, O. & PINGREE, D.
1970-1971 *The Pañchasiddhāntikā of Varāhamihira*, II. NEUGEBAUER, O, & PINGREE, D. (ed. & trans.) (Det Kongelige Danske Videnskabernes Selskaber Historisk-Filosofiske Skrifter, 6.) Copenhagen: Munksgaard.

NORTH, J. D.
1966 "Opus quorundam rotarum mirabilium". *Physis,* 8, 1966, 337-372.

1981 "Astrolabes and the hour-line ritual". *Journal for the History of Arabic Science*, 5, 1981, 113-114.

PEPPER, J. V.
1972 "Gunter, Edmund". *Dictionary of scientific biography*, V (*DSB*, V). New York: Ch. Scribner's Sons, 1972, 593-594.

POULLE, E.
1963 *Un constructeur d'instruments astronomiques au XV^{e} siècle: Jean Fusoris*. Paris: Librairie Honoré Champion.

PRICE, D.J. de Solla
1955 "An international checklist of astrolabes". *Archives Internationales d'Histoire des Sciences*, 8, 1955, 243-263 (eastern astrolabes); 363-381 (western astrolabes).

1957 "Precision instruments: to 1500". SINGER, Ch., HOLMYARD, E. S., HALL, A. R. & TREVOR, I. W. (eds.) *A History of Technology. From the Rennaisance to the Industrial Revolution, c. 1500-1750*, III. Oxford: 1957, 582-619.

1960 "The Little-ship of Venice". *Journal of the History of Medicine and Allied Sciences,* XV, 4, 1960, 399-593.

1969 "Portable sundials in Antiquity, including an account of a new example from Aphrodisias". *Centaurus*, 14, 1969, 242-266.

SCHMALZL, P.
1929 *Zur Geschichte des Quadranten bei den Arabern*. Munich.

SCHOY, K.
1923a *Gnomonik der Araber*. Von BASSERMAN-JORDAN, E. (ed.) *Die Geschichte der Zeitmessung und der Uhren*, I: F. Berlin & Leipzig: Vereinigung Wissenschaftlicher Verleger. [Repr. *Beiträge zur arabisch-islamischen Mathematik und Astronomie, I [reprints of collected works]* Frankfurt, 1988, 351-447].

1923b *Über den Gnomonschatten und die Schattentafeln der arabischen*

Astronomie. Hannover: Heinz Lafaire, 1923. [Repr. *Beiträge zur arabisch-islamischen Mathematik und Astronomie*, 1. Frankfurt, 1988, 324-349].

SÉDILLOT, J. J.
1834-1835 *Traité des instruments astronomiques des arabes composé au treizième siécle par Aboul Hhassan Ali de Maroc...*, 2. Paris. [Repr. Frankfurt, 1985].

SÉDILLOT, L. A.
1844 "Mémoire sur les instruments astronomiques des arabes". *Mémoires de l'Academie Royale des Inscriptions et Belles-lettres de l'Institut de France*, 1. Paris, 1-229. [Repr. Frankfurt, 19..].

SEZGIN, F.
1967 onwards *Geschichte des arabischen Schrifttums, X (GAS)*. Leiden: E. J. Brill.

SUTER, H.
1900 "Die Mathematiker und Astronomen der Araber und ihre Werke". (*Abhandlungen zur Geschichte der mathematischen Wissenschaften*). "Nachträge und Berichtigungen", *ibid.*, 14, 1902, 157-185. [Repr. *Beiträge in der Geschichte der Mathematik und Astronomie in Islam*, 2. Amsterdam: The Oriental Press, 1982. Frankfurt-am-Main: Institut für Geschichte der Arabisch-Islamischen Wissenschaften, 1986].

TURNER A. J.
1985 *Astrolabes, astrolabe related instruments*. (*The Time Museum. Catalogue of the Collection*, 1) Rockford, Ill.

VARĀHAMIHIRA
The Pañchasiddhāntikā, the astronomical work of Varāha Mihira, the text, edited with an original commentary in Sanskrit and an English translation and introduction by G. Thibaut ... and Mahāmahopādhāyya Sudhākara Dvivedi. Benares: E. J. Lazarus and Co. [Repr. Lahore, 1930].

VERNET, J., SAMSÓ, J. *et alii*

1985 *Instrumentos astronómicos en la España medieval. Su influencia en Europa*. Santa Cruz de la Palma.

VERNET, J., SAMSÓ, J. *et alii*
1992 *El legado científico andalusí*. Madrid.

WARD, F. A. B.
1981 *A catalogue of european scientific instruments in the Department of Medieval and Later Antiquities of the British Museum*. London.

WIEDEMANN, E. & WÜRSCHMIDT, J.
1916 "Über eine arabische kegelförmige Sonnenuhr". *Archiv für die Geschichte der Naturwissenschaften un der Technik,* 7, 1916, 359-376.

WUNDERLICH, H.
1977 *Kursächsische Feldmesskunst*, Berlin.

WÜRSCHMIDT, J.
1918 "Ein türkisch-arabisches Quadrant-Astrolab". *Archiv für die Geschichte der Naturwissenschaften und der Technik*, 8, 1918, 167-18.

WÜRSCHMIDT, J.
1919 "Die Bestimmung der krummen Stunden, der Deklination und der Gebetszeiten mittels des Astrolabs". *Mitteilungen zur Geschichte der Medizin und Naturwissenschaften*, 18, 1919, 183-190.

ZINNER, E.
1956 *Deutsche und niederländische astronomische Instrumente des 11.-18. Jahrhunderts*. München.

Plates

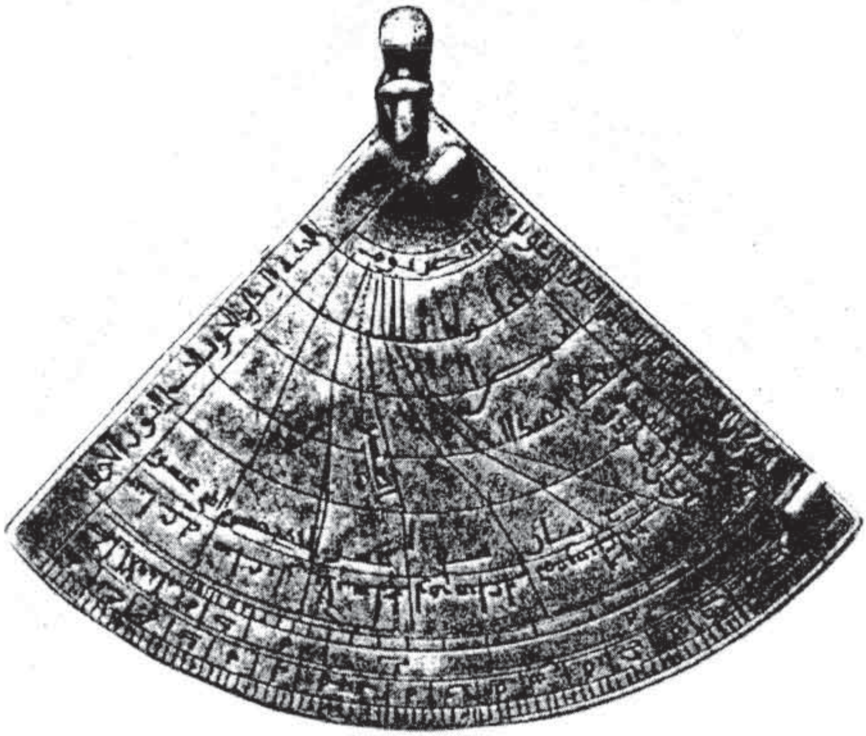


Plate I

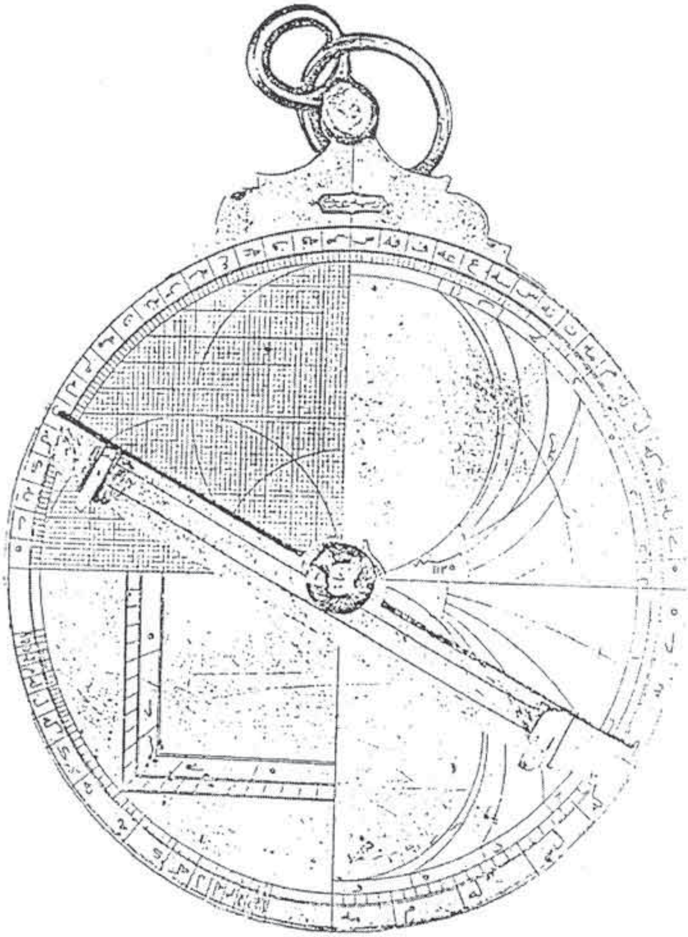
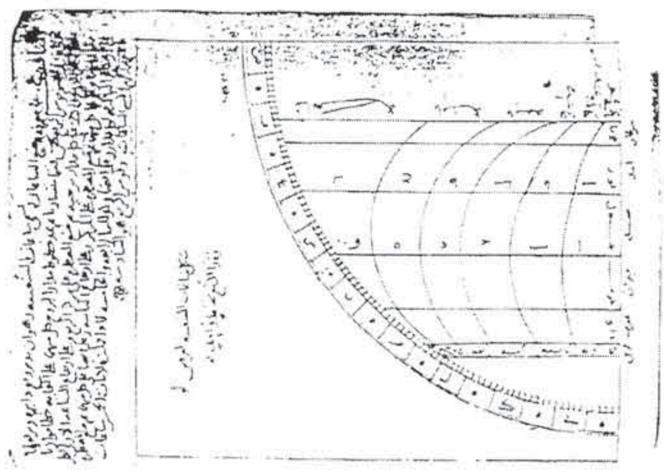


Plate II

Plate III

a



b

Plate IV

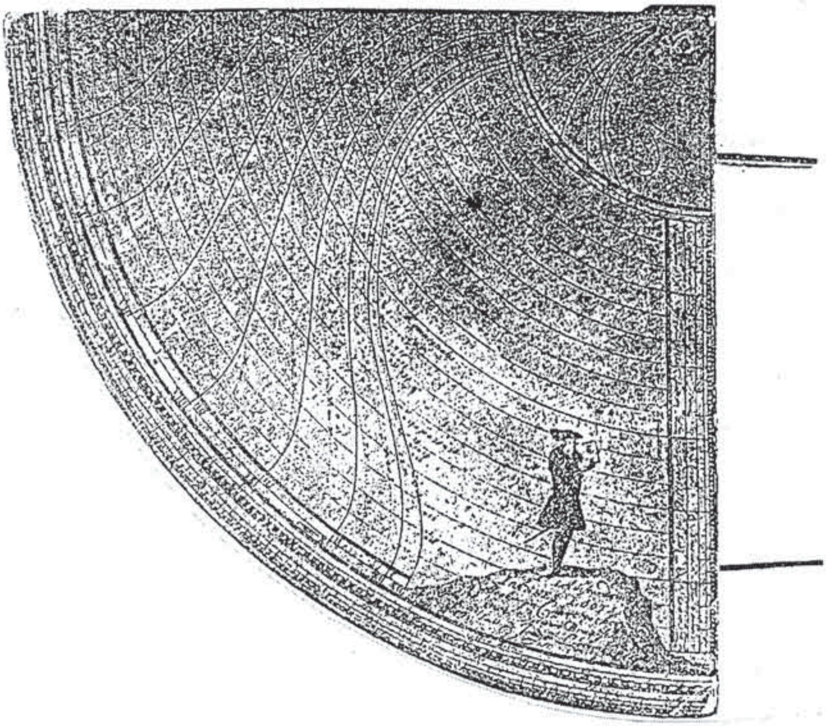


Plate V

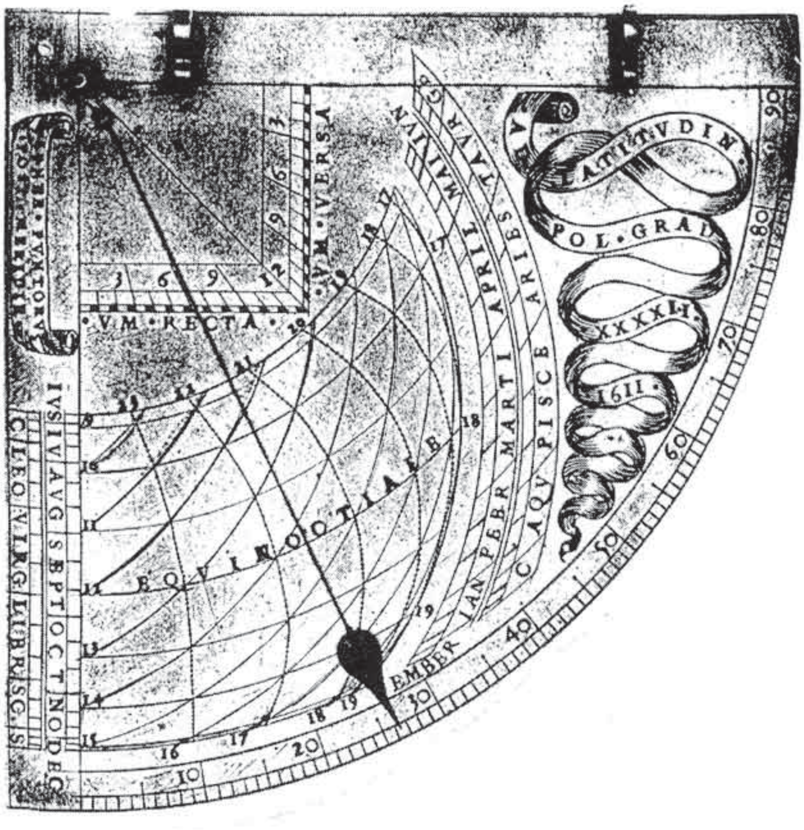


Plate VI

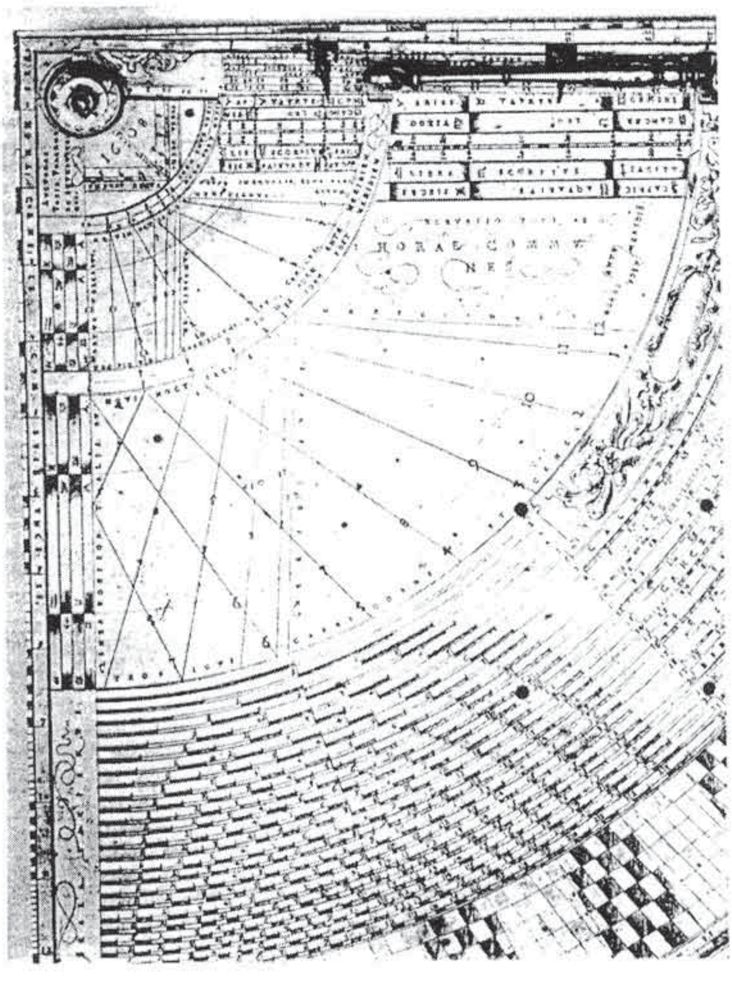
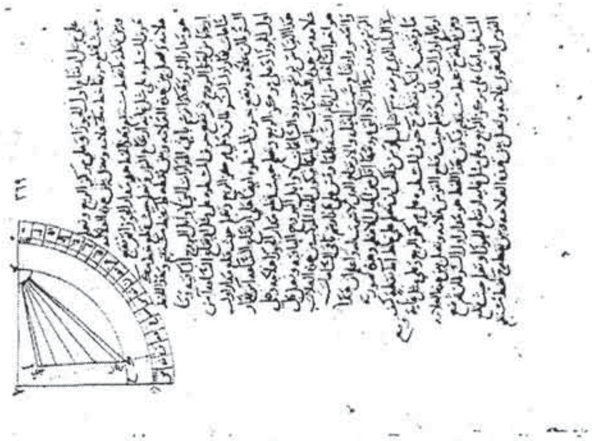
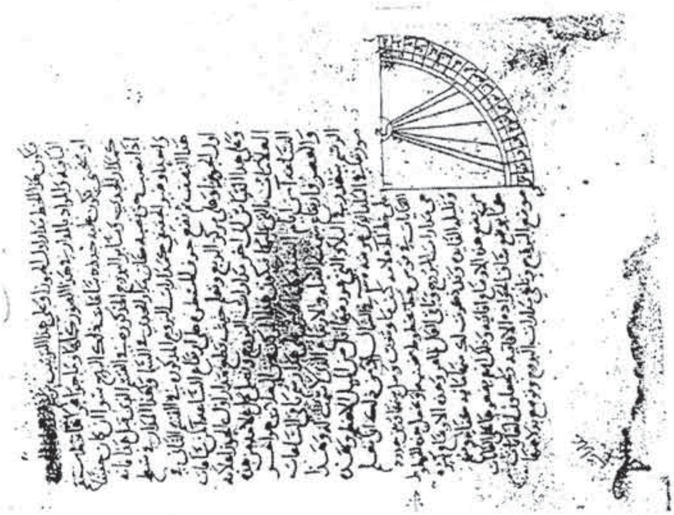


Plate VII

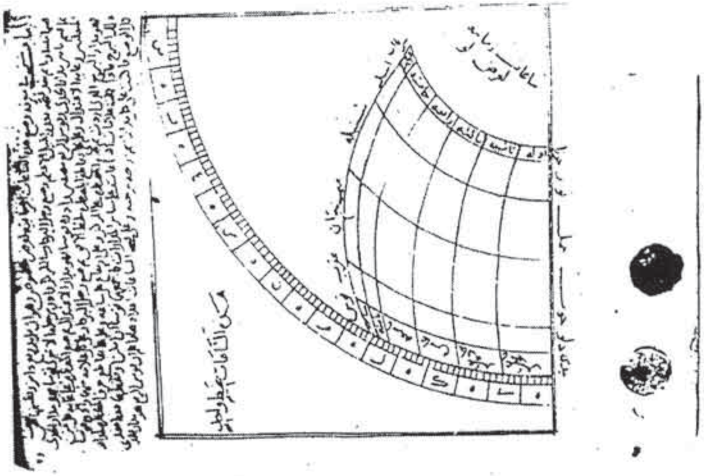


a



b

Plate VIII



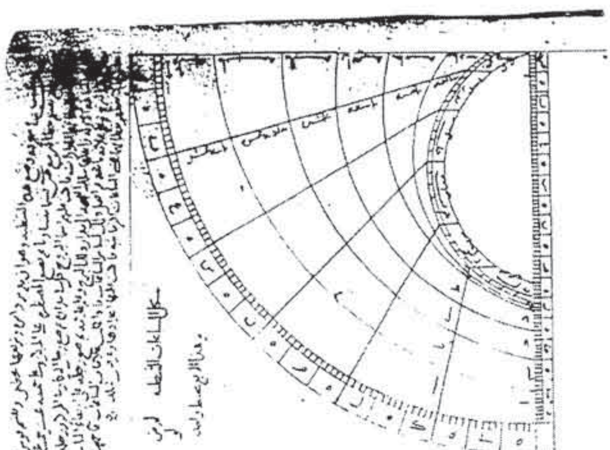


Plate IX

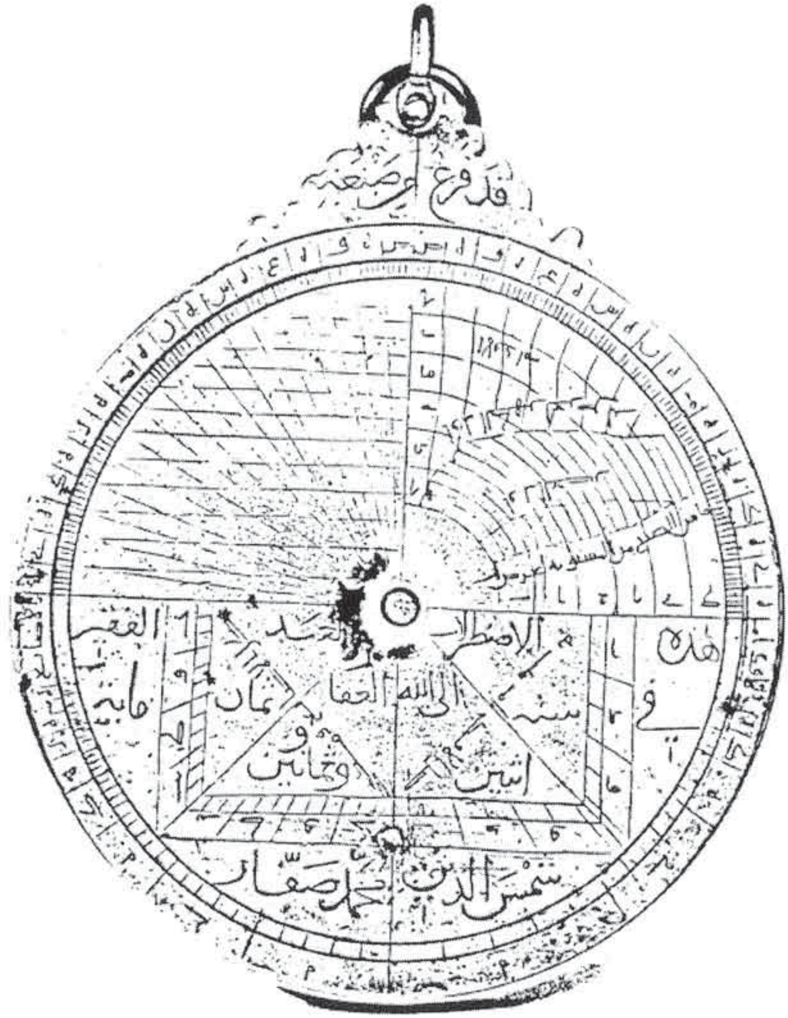


Plate X

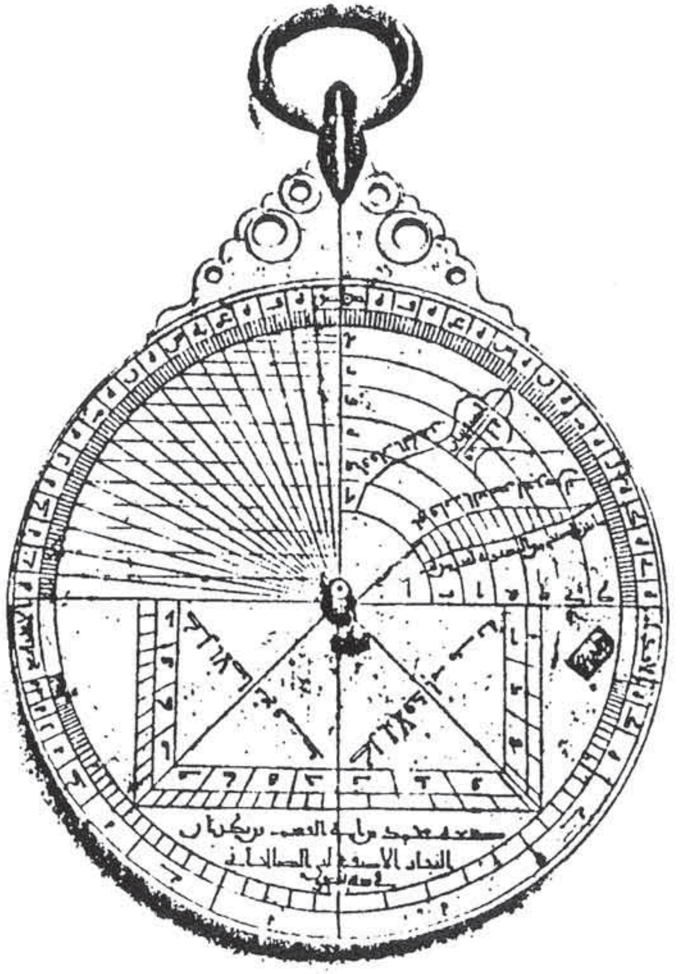


Plate XI

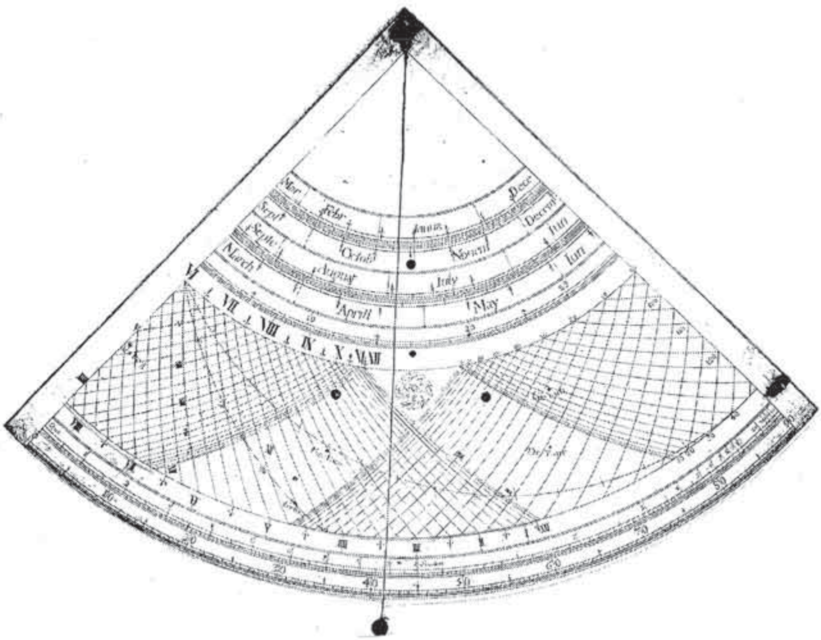


Plate XII

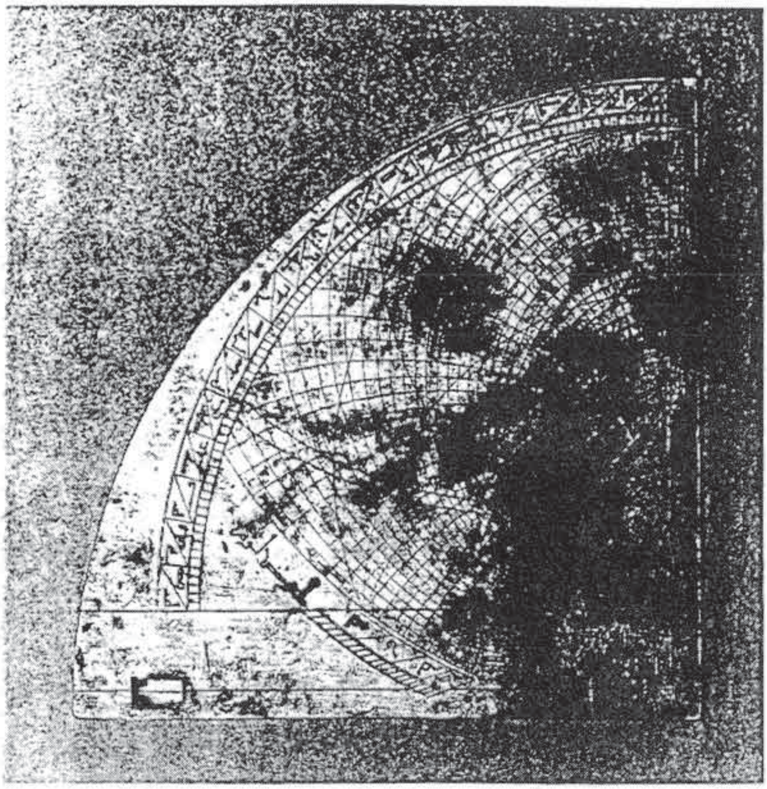


Plate XIII

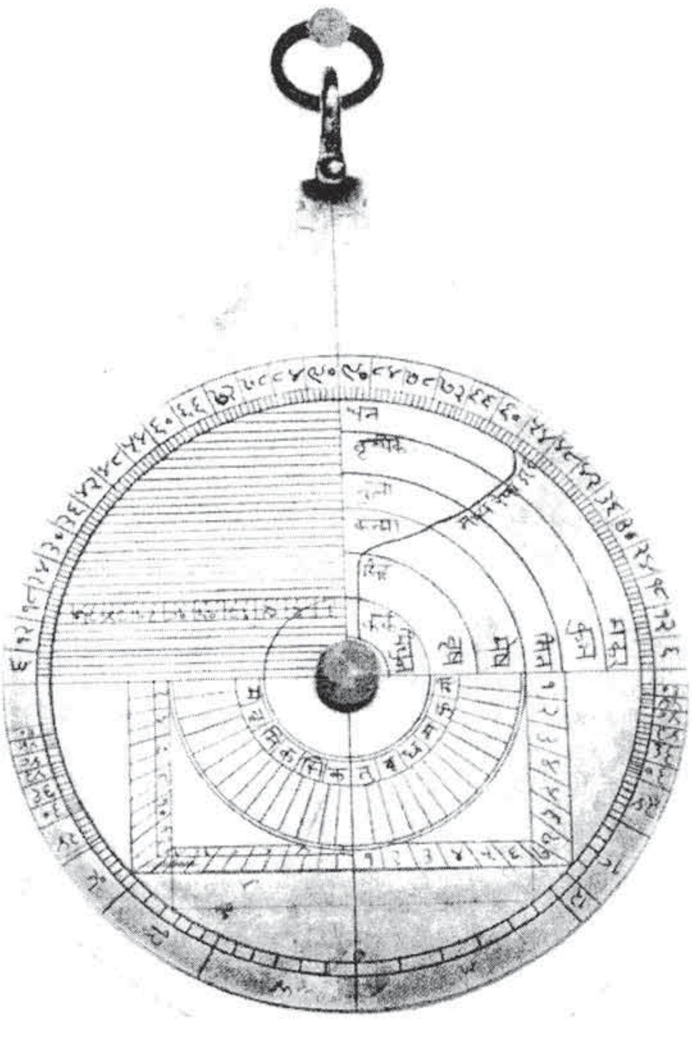


Plate XIV

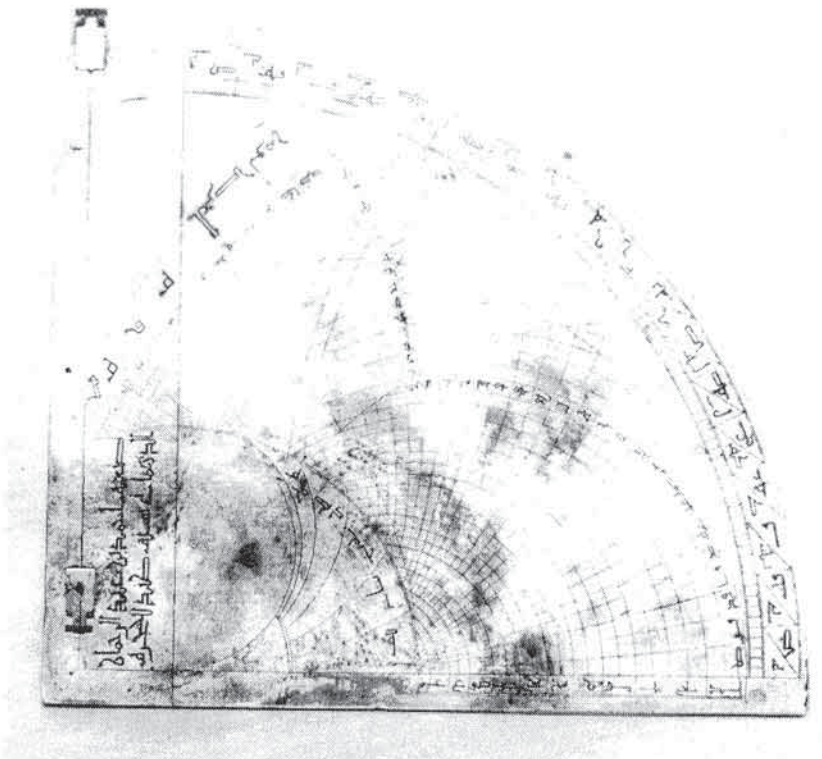


Plate XV