Circuits and Systems Expositions

Analysis of Linear Networks with Inconsistent Initial Conditions

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Abstract—**This paper presents a new method to analyze time-invariant linear networks allowing the existence of inconsistent initial conditions. This method is based on the use of distributions and state equations. Any time-invariant linear network can be analyzed. The network can involve any kind of pure or controlled sources. Also, the transferences of energy that occur at t=0 are determined, and the concept of connection energy is introduced. The algorithms are easily implemented in a computer program.**

I. Introduction

INITIAL conditions may be inconsistent when there occurs a change in the network topology, as when two capacitors with different initial voltages are connected in parallel to form a new network.

The instant when a network forms a new topology will be $t = 0$. Initial conditions at $0-$ will be called initial conditions simply, while the values immediately after switching are the initial conditions at $0+$. This paper will show an efficient and simple method to analyze an electric network knowing the initial conditions at $0-$.

Voltage and current values at $0+$ and $0-$ are related by charge conservation in capacitive cutsets and flux conservation in inductive loops. Nevertheless, the application of these laws does not always suffice for obtaining the initial values at $0+$ from initial values at $0-$: in the network of the second example of Section VIII, there is only one inductor, and its flux at $0-$ is different from its flux at $0+$.

Consider the two capacitors again. If initial voltages are different, the total energy stored in capacitors at $0+$ is smaller than the energy at $0-$ because at $t = 0$, the capacitors have transformed part of their energy stored in their electric field into electric energy, which is not zero if initial conditions are inconsistent. This energy, which we call the *connection energy*, is analyzed in detail in this paper.

The problem of determining initial values at 0+ has been studied before. However, the analysis of the electric energy absorbed in the network connection is a little studied subject that this paper treats in detail.

Dervisoglu [3] developed an algorithm to calculate initial values at $0+$ using a state variable approach. His method does not involve distributions directly, but introduces the Dirac impulse δ and its derivatives in the analysis. Murakami [4] proposed another way of determining the response of a network with inconsistent initial conditions. However, his method does not allow the existence of dependent sources and pure current sources. His analysis is based on the state equation and the use of distributions, although the application of distributions is not as interesting as in our new formulation. Recently, Opal and Vlach [5] proposed a new method to calculate initial values at $0+$ without introducing state equations. They use a numerical Laplace transform inversion, exact for impulses and its derivatives. In their algorithm, it is necessary to integrate a system of equations in a time interval Δt tending to zero. This fact introduces numerical errors difficult to measure.

The problem of conservation of energy when initial conditions are inconsistent has been studied by Göknar [12]. He considers networks consisting of capacitors or inductors only, without sources. He demonstrates that in those networks, the difference of the energy stored in capacitors (or inductors) from $0-$ to $0+$ is always positive, and that this difference equals the energy consumed in the interval $[0, +\infty[$ by some resistors properly included into the network. However, he does not explain why the principle of conservation of energy seems to be violated.

Our approach is based on currents and voltages defined as distributions. The method is simple: first, equations of the network (Kirchhoff's laws and Ohm's law) are stated for currents and voltages defined as distributions. Then, we obtain a singular system of differential equations of distributions. This system is similar to classic singular systems of differential equations for functions, which can be written as

$$SY(t) + T\frac{d}{dt}Y(t) = E(t)$$

where the matrix T may be singular. Next, this equation is transformed into the following pair of equations:

$$\frac{d}{dt}X = \mathcal{A}X + \mathcal{B}\left(\frac{d}{dt}\right)E \tag{1}$$

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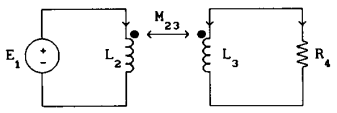

Fig. 1. Circuit with a perfect coupling.

$$Z = C\left(\frac{d}{dt}\right)E. \tag{2}$$

Then, state equation (1) is solved by the convolution of distributions.

In our analysis, the matrix $\mathcal{B}$ need not be constant, although there always exists a state equation where $\mathcal{B}$ is constant. This is derived immediately from the theory of polynomial matrices and the Kronecker form exposed by Gantmacher [11], which Verghese [10] uses in his analysis of singular systems. However, sometimes the computation of $\mathcal{B}$ in this formulation is very difficult.

To illustrate these different choices for the matrix $\mathcal{B}$, consider Fig. 1, with $L_2 = L_3 = M_{23} = L$ (the coupling is perfect and the order of the circuit is one). We derive

$$L\frac{d}{dt}I_2 + L\frac{d}{dt}I_3 = E_1$$
$$L\frac{d}{dt}I_2 + L\frac{d}{dt}I_3 = -R_4 I_3.$$

If we choose $I_2 + I_3$ as the state variable, the state equation is

$$\frac{d}{dt}(I_2 + I_3) = \frac{1}{L}E_1.$$

So $\mathcal{A}$ is zero and $\mathcal{B}$ is constant. On the other hand, if I_2 is the state variable, we obtain

$$\frac{d}{dt}I_2 = \frac{1}{L}E_1 + \frac{1}{R_4}\frac{d}{dt}E_1.$$

Then, $\mathcal{A}$ is zero and $\mathcal{B}$ depends on d/dt.

There are simple and efficient algorithms to determine a state equation when the matrix $\mathcal{B}$ is allowed to be not constant. Further, in this case, some physical magnitudes such as currents in inductors and voltages in capacitors can be chosen as state variables. In the Appendix, we explain an algorithm for determining a state equation with these characteristics, which is simpler and easier to program than the existing ones.

Generally, the matrix C of (2) also depends on d/dt, even if the state equation is derived from the Kronecker form. In fact, the degree of the polynomial matrix $C(\lambda)$ derived from the Kronecker form is equal to the nilpotency index of $S + \lambda T$ minus 1.

Our formulation allows the presence of any linear time-invariant element of first order: zero impedances and admittances, pure current and voltage sources, any controlled source, and perfect couplings are analyzed by this method. Also, the network can be not connected.

We assume that Ohm's law is valid for any instant of time, while Kirchhoff's laws are applicable only if $t \geq 0$. The instant $t = 0$ is included in the time interval where the network has a new topology. So voltage and current impulses are consistent in the new topology. These assumptions agree with the control theory approach to singular systems [10].

Switches are not considered as elements of the network. Instead, we assume that the network topology for $t \geq 0$ is different from the topology for $t < 0$.

Our approach to the problem of energy is very different from [12]. Our analysis is valid for any linear time-invariant network, including controlled and uncontrolled sources. We will study the transferences of energy, and we will find out why the total energy stored in inductors and capacitors at $0-$ is different from that of $0+$.

II. Preliminaries

The concept of distribution [6] generalizes the concept of function. A distribution is defined as a continuous linear map from the space of all C^∞ real functions with compact support into $\mathbb{C}$. Locally integrable functions can be considered as a particular case of distributions. Any distribution is infinitely derivable. The set of distributions is a vector space which is denoted by $\mathcal{D}'$.

It is always possible to consider the restriction of a distribution to an open set (similar to the restriction of a function to an open set). Nevertheless, not all distributions defined in an open set can be extended to the whole of $\mathbb{R}$.

In an electrical network, distributions that define currents and voltages depend only on time. To define the values of distributions at $0-$, we suppose that there exists an interval $]-\epsilon, 0[$, $\epsilon > 0$ such that the restriction of those distributions to this interval is a continuous function such that its limit exists at $0-$. If f is such a distribution and f is defined in $]-\epsilon, +\infty[$, it is easy to demonstrate that f can be written in a unique way as follows:

$$f = f^- + f^+$$

where f^- is the restriction of f to the interval $]-\epsilon, 0[$ (extended by zero to the interval $]-\epsilon, +\infty[$ and f^+ is a distribution whose support is contained in $[0, +\infty[$. This fact follows immediately: given the distribution f and its restriction f^- extended by zero, we define $f^+ = f - f^-$ and verify that the support of $f - f^-$ is contained in $[0, +\infty[$. f^+ is unique because f^- is unique.

Therefore, the distributions U, I, and E corresponding to voltages, currents, and values of independent sources, respectively, will be written as follows:

$$U = U^- + U^+ \tag{3}$$

$$I = I^- + I^+ \tag{4}$$

$$E = E^- + E^+ \tag{5}$$

where U^-, I^-, and E^- are voltages, currents, and values of independent sources in the interval $]-\epsilon, 0[$ and U^+, I^+, and E^+ are voltages, currents, and independent sources in the interval $[0, +\infty[$. Values corresponding to $t = 0$ are included in U^+, I^+, and E^+. For example, if there are Dirac impulses in U, I, or E in $t = 0$, they are included in U^+, I^+, or E^+.

III. Network Equations

Let us consider a network in the time interval $]-\epsilon, +\infty[$. At $t = 0$, all branches are connected to form a new network topology. Currents and voltages are determined by Ohm's law and Kirchhoff's laws. Ohm's law is assumed to be valid in the whole interval $]-\epsilon, +\infty[$. The equation is expressed as

$$MU + NI + P\frac{d}{dt}U + Q\frac{d}{dt}I = E \tag{6}$$

where U and I are vectors of branch voltages and currents, respectively, and E is a vector whose elements correspond to independent sources in the network. The elements of vectors U, I, and E are distributions dependent on time. M, N, P, and Q are constant square matrices. Their dimension is equal to the number of network branches. Equation (6) is a very general expression: it allows the presence of any linear time-invariant element of first order in the network.

The restriction of distributions U and I to the interval $]-\epsilon, 0[$ is supposed to be defined by C^1 functions and the restriction of E by a continuous function. We also assume that the limits $U(0-)$, $(d/dt)U(0-)$, $(d/dt)I(0-)$, and $E(0-)$ exist and are finite. Therefore, U, I, and E can be written as in (3), (4), and (5).

Kirchhoff's laws can be applied to U^+ and I^+ only:

$$AI^+ = 0$$
$$BU^+ = 0$$

where A is the incidence matrix and B is the loop matrix. These equations are equivalent to

$$I^+ = B^t I_B^+ \tag{7}$$

$$U^+ = A^t V_N^+ \tag{8}$$

where I_B^+ is the vector whose components are link currents and V_N^+ is the vector of node voltages (it is not necessary to choose a normal tree: any tree is suitable in this formulation).

Equation (6) is also true if all distributions are restricted to $t < 0$:

$$MU_{|t<0} + NI_{|t<0} + P\frac{d}{dt}U_{|t<0} + Q\frac{d}{dt}I_{|t<0} = E_{|t<0}.$$

All distributions that appear in this equation are functions which can be extended by zero to $\mathbb{R}$ as distributions. If we denote their extension by zero by $\{U_{|t<0}\}$, $\{I_{|t<0}\}$, $\{(d/dt)U_{|t<0}\}$, $\{(d/dt)I_{|t<0}\}$, and $\{E_{|t<0}\}$, we have

$$M\{U_{|t<0}\} + N\{I_{|t<0}\} + P\left\{\frac{d}{dt}U_{|t<0}\right\} + Q\left\{\frac{d}{dt}I_{|t<0}\right\} = \{E_{|t<0}\}. \tag{9}$$

Then, $\{U_{|t<0}\} \equiv U^-$, $\{I_{|t<0}\} \equiv I^-$, $\{E_{|t<0}\} \equiv E^-$, and [7]

$$\frac{d}{dt}U^- = \left\{\frac{d}{dt}U_{|t<0}\right\} - \delta U(0-)$$
$$\frac{d}{dt}I^- = \left\{\frac{d}{dt}I_{|t<0}\right\} - \delta I(0-).$$

Replacing these expressions in (9), we obtain

$$MU^- + NI^- + P\left(\frac{d}{dt}U^- + \delta U(0-)\right) + Q\left(\frac{d}{dt}I^- + \delta I(0-)\right) = E^-.$$

Equation (6) is equivalent to

$$MU^+ + MU^- + NI^+ + NI^- + P\frac{d}{dt}U^+ + P\frac{d}{dt}U^- + Q\frac{d}{dt}I^+ + Q\frac{d}{dt}I^- = E^+ + E^-.$$

From the last two equations, we get the equation shown in (10) at the bottom of this page. Equations (7), (8), and (10) determine U^+ and I^+. So we have a system of algebraic and differential equations in the algebra of distributions with support contained in $\mathbb{R}^+$, that is to say, in the convolution algebra $\mathcal{D}'_+$ [6], [7].

IV. Determination of the State Equation

We denote the following expression by $K(0-)$:

$$K(0-) = PU(0-) + QI(0-). \tag{11}$$

From Section III, we derive this equation:

$$[MA^t \mid NB^t]\begin{bmatrix} V_N^+ \\ I_B^+ \end{bmatrix} + [PA^t \mid QB^t]\frac{d}{dt}\begin{bmatrix} V_N^+ \\ I_B^+ \end{bmatrix} = E^+ + \delta K(0-).$$

If we denote the vector whose components are V_N^+ and I_B^+ by Y^+ and the other matrices of the first member by S and T, we obtain

$$SY^+ + T\frac{d}{dt}Y^+ = E^+ + \delta K(0-). \tag{12}$$

If the network is solvable, the polynomial $\det(S+\lambda T) \not\equiv 0$. In this case, it is easy to find matrices $F(\lambda)$ and D such that

$$F(\lambda)(S+\lambda T)D = \left[\begin{array}{c|c} -\mathcal{A} + \lambda Id_a & 0 \\ \hline 0 & Id_b \end{array}\right] \tag{13}$$

where D is an invertible matrix, $F(\lambda)$ is a unimodular matrix, and Id_a, Id_b are identity matrices of orders a and b. The matrices $F(\lambda)$ and D can be calculated using the algorithm in the Appendix or any other method.

$$\boxed{MU^+ + NI^+ + P\tfrac{d}{dt}U^+ + Q\tfrac{d}{dt}I^+ = E^+ + \delta(PU(0-) + QI(0-)).} \tag{10}$$

Then, (12) is equivalent to

$$\left[\begin{array}{c|c} -\mathcal{A} + Id_a \frac{d}{dt} & 0 \\ \hline 0 & Id_b \end{array}\right] \left[\begin{array}{c} X^+ \\ \hline Z^+ \end{array}\right] = \left[\begin{array}{c} \mathcal{B}\left(\frac{d}{dt}\right) \\ \hline \mathcal{C}\left(\frac{d}{dt}\right) \end{array}\right] (E^+ + \delta K(0-))$$

with the following change of variables:

$$\left[\begin{array}{c} V_N^+ \\ \hline I_B^+ \end{array}\right] = Y^+ = D \left[\begin{array}{c} X^+ \\ \hline Z^+ \end{array}\right] \tag{14}$$

and the matrix $F(d/dt)$ has been written as

$$F\left(\frac{d}{dt}\right) = \left[\begin{array}{c} \mathcal{B}\left(\frac{d}{dt}\right) \\ \hline \mathcal{C}\left(\frac{d}{dt}\right) \end{array}\right].$$

So the system (12) is equivalent to

$$\frac{d}{dt} X^+ = \mathcal{A} X^+ + \mathcal{B}\left(\frac{d}{dt}\right)(E^+ + \delta K(0-)) \tag{15}$$

$$Z^+ = \mathcal{C}\left(\frac{d}{dt}\right)(E^+ + \delta K(0-)). \tag{16}$$

Equation (15) is the network state equation for distributions. The components of X^+ are the state variables. The derivatives which appear in (15) and (16) are in the sense of distributions. It is also interesting to observe that E^+ can be any distribution, not only a continuous function. For instance, E^+ can include any impulses $\delta^{(n)}$, $n \geq 0$.

V. Solution of the State Equation

The state equation is equivalent to the following system of convolution equations in the algebra $\mathcal{D}'_+$:

$$(\delta' Id - \delta \mathcal{A}) * X^+ = \mathcal{B}\left(\frac{d}{dt}\right)(E^+ + \delta K(0-))$$

(the symbol $*$ stands for the convolution of distributions). This result is due to the fact that for any distribution f,

$$\delta^{(n)} * f = \frac{d^n}{dt^n} f, \qquad n = 0, 1, 2 \cdots.$$

Also, we have

$$(\delta' Id - \delta \mathcal{A})^{*-1} = h(t) e^{t\mathcal{A}}$$

where $h(t)$ is the Heaviside function ($h(t) = 0$ if $t < 0$ and $h(t) = 1$ if $t \geq 0$). Then the solution of the state equation exists and is unique [6], [7], and it is equal to

$$\boxed{X^+ = (h(t) e^{t\mathcal{A}}) * \mathcal{B}\left(\frac{d}{dt}\right)(E^+ + \delta K(0-)).} \tag{17}$$

VI. Solution for Independent Sources Defined as Functions

Given a solvable network, let us suppose that E is defined by a function $E(t)$ continuous in the interval $]-\epsilon, +\infty[$.

1) If the matrix $\mathcal{B}$ does not depend on d/dt, from (17) we derive

$$X^+(t) = \int_{-\infty}^{+\infty} h(t-x) e^{(t-x)\mathcal{A}} \mathcal{B} E^+(x)\, dx + h(t) e^{t\mathcal{A}} \mathcal{B} K(0-).$$

That is to say,

$$\boxed{X^+(t) = \int_0^t e^{(t-X)\mathcal{A}} \mathcal{B} E^+(x)\, dx + h(t) e^{t\mathcal{A}} \mathcal{B} K(0-).}$$

Then, the initial values of state variables at $0+$ are

$$X(0+) = \mathcal{B}(PU(0-) + QI(0-)).$$

2) If the matrix $\mathcal{B}(d/dt)$ depends on d/dt and it is written as $\sum_{i=0}^n \mathcal{B}_i (d^i/dt^i)$,

a) if $n = 1$ and E is derivable, (17) is equivalent to

$$\begin{aligned} X^+(t) = \int_0^t e^{(t-x)\mathcal{A}} \left(\mathcal{B}_0 E^+(x) + \mathcal{B}_1 \left\{\frac{d}{dt} E^+(x)\right\}\right) dx \\ + h(t) e^{t\mathcal{A}} (\mathcal{B}_1 E(0) + \mathcal{B}_0 K(0-) \\ + \mathcal{A}\mathcal{B}_1 K(0-)) + \delta \mathcal{B}_1 K(0-) \end{aligned}$$

b) the solution $\forall n$ when the function $E(t)$ is n-derivable is

$$\begin{aligned} X^+(t) = \int_0^t e^{(t-x)\mathcal{A}} \sum_{i=0}^n \mathcal{B}_i \left\{\frac{d^i}{dt^i} E^+(x)\right\} dx \\ + \sum_{i=1}^n \sum_{j=0}^{i-1} h(t) e^{t\mathcal{A}} \mathcal{A}^j \mathcal{B}_i \frac{d^{i-1-j}}{dt^{i-1-j}} E(0) \\ + \sum_{i=0}^n h(t) e^{t\mathcal{A}} \mathcal{A}^i \mathcal{B}_i K(0-) \\ + \sum_{i=2}^n \sum_{j=1}^{i-1} \sum_{k=0}^{j-1} \delta^{(k)} \mathcal{A}^{j-1-k} \mathcal{B}_i \frac{d^{i-1-j}}{dt^{i-1-j}} E(0) \\ + \sum_{i=1}^n \sum_{j=0}^{i-1} \delta^{(j)} \mathcal{A}^{i-1-j} \mathcal{B}_i K(0-). \end{aligned}$$

This expression is easy to introduce in a computer program. The initial values at 0^+ are obtained setting $t = 0$ in the last equation:

$$X^+(0+) = \sum_{i=1}^n \sum_{j=0}^{i-1} \mathcal{A}^j \mathcal{B}_i \frac{d^{i-1-j}}{dt^{i-1-j}} E(0) + \sum_{i=0}^n \mathcal{A}^i \mathcal{B}_i K(0-).$$

Voltages and currents are determined from X^+ and Z^+ using (14), (7), and (8).

If the matrix $\mathcal{C}(d/dt)$ is written as $\sum_{i=0}^{m} C_i(d^i/dt^i)$, we have

$$Z^+(t) = \sum_{i=0}^{m} C_i \left\{ \frac{d^i}{dt^i} E^+(t) \right\} + \sum_{i=1}^{m} \sum_{j=0}^{i-1} \delta^{(j)} C_i \frac{d^{i-1-j}}{dt^{i-1-j}} E(0) + \sum_{i=0}^{m} \delta^{(i)} C_i K(0-).$$

VII. Energy Analysis

A. Multiplication of Distributions

Electric power is equal to the product of current and voltage. If currents and voltages are defined by distributions, their product is not possible in general (only the product of a distribution and a C^∞ function is well defined). Due to this fact, power and energy cannot be analyzed in the space of distributions.

To multiply distributions, it is necessary to introduce another space where multiplication is possible. Such a space is the space $\mathcal{G}$ of generalized functions defined by Colombeau [8]. In this formulation, a distribution is a particular case of generalized function. So the space $\mathcal{D}'$ of distributions is a subspace of $\mathcal{G}$.

The product of two generalized functions always exists in $\mathcal{G}$, in particular if these generalized functions are distributions. For example, the square of the Dirac impulse δ is the generalized function δ^2, which is not a distribution.

Electric energy is defined as the definite integral of power, which is a generalized function that depends on time. The definite integral of a generalized function in an interval $[a, b]$ is introduced in [8] too. It is always defined and is equal to a generalized number (the set of generalized real numbers is an extension of $\mathbb{R}$). Obviously, if a generalized function is defined by a continuous function, its definite integral as a generalized function coincides with its usual definite integral as a function.

We assume that the space $\mathcal{D}'$ of distributions is included in the space $\mathcal{G}$ of generalized functions, where power and energy can be analyzed correctly.

B. Energy Analysis

The electric power absorbed by a network branch "i" is the generalized function defined by the product of branch current and branch voltage (we suppose that the current leaves the positive node and arrives at the negative node):

$$P_i = U_i I_i.$$

The electric energy W_i absorbed by a network branch "i" in the time interval $[a, b]$ is the following generalized number:

$$W_i = \int_a^b P_i \, dt.$$

The electric power absorbed by the network is

$$P = \sum_{i=1}^{n} P_i = \sum_{i=1}^{n} U_i I_i = U^t I.$$

Thus, the total electric energy consumed by the network in the time interval $[a, b]$ is

$$W = \sum_{i=1}^{n} W_i = \int_a^b P \, dt.$$

In the interval $[a, b]$, $-\epsilon < a < 0 < b$, W_i can be written as

$$W_i = \int_a^b U_i I_i \, dt = \int_a^b (U_i^+ + U_i^-)(I_i^+ + I_i^-) \, dt = \int_a^b U_i^+ I_i^+ \, dt + \int_a^b U_i^- I_i^- \, dt + \int_a^b U_i^+ I_i^- \, dt + \int_a^b U_i^- I_i^+ \, dt.$$

Similarly, we derive

$$W = \int_a^b U^{+t} I^+ \, dt + \int_a^b U^{-t} I^- \, dt + \int_a^b U^{+t} I^- \, dt + \int_a^b U^{-t} I^+ \, dt.$$

Let us define

$$W_i^+ = \int_a^b U_i^+ I_i^+ \, dt, \qquad W_i^- = \int_a^b U_i^- I_i^- \, dt$$

$$\boxed{W_i^c = \int_a^b U_i^+ I_i^- \, dt + \int_a^b U_i^- I_i^+ \, dt.} \tag{18}$$

W_i^- is the electric energy absorbed by the branch "i" in the interval $]a, 0[$. W_i^+ is the electric energy absorbed by the branch "i" in the interval $[0, b[$. W_i^c is an electric energy absorbed by the branch "i" due to the topology change. We define W_i^c as the connection energy of the branch "i." Then, we have

$$W_i = W_i^- + W_i^+ + W_i^c.$$

We also define

$$W^+ = \sum_{i=1}^{n} W_i^+ = \int_a^b U^{+t} I^+ \, dt,$$
$$W^- = \sum_{i=1}^{n} W_i^- = \int_a^b U^{-t} I^- \, dt \tag{19}$$

$$\boxed{W^c = \sum_{i=1}^{n} W_i^c = \int_a^c U^{+t} I^- \, dt + \int_a^b U^{-t} I^+ \, dt.} \tag{20}$$

The energy W^+ is equal to zero provided that

$$U^{+t} I^+ = V_N^{+t} (AB^t) I_B^+ = 0.$$

Replacing this expression in (19), we obtain

$$W^+ = 0. \tag{21}$$

Therefore, $W = W^- + W^c$.

W^- is the electric energy absorbed by the network in the interval $]a, 0[$, while W^+ is the electric energy absorbed in the time interval $[0, b[$. W^+ is null, as we have derived. W^-

is also zero if we assume that there exists an initial network topology in the interval $]-\epsilon, 0[$ (in general, this topology is different from the final topology). However, this assumption is not necessary in our analysis since we only require the existence of a network topology for $t \geq 0$.

The energy W^c is not zero in general. We define W^c as the network connection energy. This electric energy is absorbed by the network in $t = 0$ as a consequence of the topology change.

Consider the energy W_i^c again. We can observe that the total energy absorbed by the branch "i" in $t = 0$ is not only W_i^c, but the addition of W_i^c and the part of W_i^+ corresponding to $t = 0$, which we will denote as W_i^0.

However, the part of the energy absorbed by the whole network in $t = 0$ corresponding to W^+, which we denote as W^0, is zero [this is derived immediately from (21)]. Therefore, the total energy absorbed by the network in $t = 0$ is equal to W^c.

Let us assume that there exists an interval $]0, \epsilon'[$ where the restrictions of distributions I_i^+ and U_i^+ are continuous functions with limit at $0+$. Then, operating as in Section II, we obtain

$$U_i^+ = U_i^0 + U_i^{+0}$$
$$I_i^+ = I_i^0 + I_i^{+0}$$

where U_i^{+0} and I_i^{+0} are the restriction of distributions U_i^+ and I_i^+ to the open interval $]0, +\infty[$ extended by zero and U_i^0, I_i^0 are distributions with support in $\{0\}$ (this is always possible if the vector E of independent sources is defined by a piecewise C^∞ function). From [8], we derive

$$\int_a^b U_i^{+0} I_i^- = 0, \qquad \int_a^b I_i^{+0} U_i^- = 0.$$

So we have

$$\boxed{W_i^c = \int_a^b U_i^0 I_i^- \, dt + \int_a^b U_i^- I_i^0 \, dt.} \tag{22}$$

$W_i^c = 0$ if initial conditions are consistent since there are no impulses in the network and U_i^0 and I_i^0 are equal to zero. Also, $W_i^c = 0$ if initial values at $0-$ are null.

With similar assumptions, we have

$$U^+ = U^0 + U^{+0}$$
$$I^+ = I^0 + I^{+0}$$

where U^{+0} and I^{+0} are the restriction of distributions U^+ and I^+ to the open interval $]0, +\infty[$ extended by zero and U^0, I^0 are distributions with support in $\{0\}$. We derive

$$\boxed{W^c = \int_a^b U^{0t} I^- \, dt + \int_a^b U^{-t} I^0 \, dt.} \tag{23}$$

In the analysis of the connection energy, the distributions I^- and U^- are as essential as I^+ and U^+. In other formulations of network analysis with inconsistent initial conditions [3]–[5], the existence of I^- and U^- is not analyzed. This is one of the reasons why the energy transferences cannot be understood in those formulations.

C. Determination of the Connection Energy

To calculate the integrals of (23), we must take into account that U^0, I^0, U^-, and I^- are generalized functions. The following results are derived from the formulation given by Colombeau [8].

If $a < 0 < b$ and if f is a continuous function in $]-\epsilon, 0[$ and $]0, \epsilon[$ and the limits $f(0-)$ and $f(0+)$ exist, then

$$\int_a^b \delta f \, dt = \frac{f(0-) + f(0+)}{2}. \tag{24}$$

On the other hand, the integral

$$\int_a^b \delta^{(n)} f \, dt, \qquad n \geq 1 \tag{25}$$

is a generalized number, but not a classical real number. If $n = 1$, it can be considered as the product $+\infty(f(0-) - f(0+))$.

To illustrate the use of these equations, let us consider a circuit where the voltage in the branch "i" is the impulse $k\delta$ and the current in $t = 0$ is finite. From (22), we obtain

$$W_i^c = \int_a^b (U_i^0 I_i^- + U_i^- I_i^0) \, dt = \int_a^b (k\delta I_i^- + 0) \, dt = k \int_a^b \delta I_i^-.$$

We can use (24) to calculate this integral, provided that I_i^- is a continuous function in $]-\epsilon, 0[$ and is zero in $[0, +\infty[$. Therefore,

$$W_i^c = k\frac{1}{2}(I_i^-(0-) + I_i^-(0+))$$
$$= k\frac{1}{2}(I_i(0-) + 0) = \frac{1}{2} k I_i(0-).$$

In an *RLCM* network, no derivatives of the Dirac distribution δ appear in voltages and currents if $E(t)$ is derivable enough. Therefore, the connection energy can be calculated using (24), and the generalized number that defines the connection energy is a classical real number.

In other circuits which include impulses $\delta^{(n)}$, $n \geq 1$, the connection energy is calculated using (25). This integral has a mathematical sense as a generalized number. In fact, following Colombeau's theory, the generalized number defined by (25) is "like an infinite real number." Similarly, the value of the generalized function δ at 0 is a generalized number with a mathematical sense which we can consider as an infinite real number. So in networks with impulses $\delta^{(n)}$, $n \geq 1$, transferences of energy are defined by generalized numbers which can be different from classical real numbers.

VIII. Examples

Example 1: In the network of Fig. 2, the switch S is closed for $t < 0$ and open for $t \geq 0$. In the new topology, the currents I_1 and $-I_2$ must be equal. Because of this sudden change of currents, voltage impulses appear in the network.

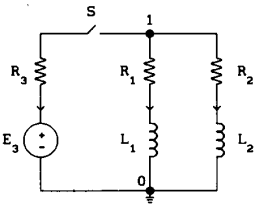

Fig. 2. Circuit of first example.

a) Time-Domain Analysis with the State Equation: From (7) and (8) and operating as in Section III, we obtain

$$\begin{bmatrix} 1 & 0 & R_1 \\ 1 & 0 & -R_2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ I_2^+ \end{bmatrix} + \begin{bmatrix} 0 & 0 & L_1 \\ 0 & 0 & -L_2 \\ 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} V_1^+ \\ V_2^+ \\ I_2^+ \end{bmatrix} = \begin{bmatrix} -\delta L_1 I_1(0-) \\ -\delta L_2 I_2(0-) \\ E_3^+ \end{bmatrix}.$$

After some elementary row and column transformations, we obtain the following state equation:

$$\begin{bmatrix} \frac{d}{dt} + \frac{R_1+R_2}{L_1+L_2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X^+ \\ \hline Z^+ \end{bmatrix} = \begin{bmatrix} \delta \frac{1}{L_1+L_2} I_2(0-) - L_1 I_1(0-)) \\ -\delta \frac{d}{dt}(I_1(0-) + I_2(0-)) \\ E_3^+ \end{bmatrix} \quad (26)$$

where

$$X^+ = I_2^+, \qquad Z^+ = \begin{bmatrix} V_1^+ - I_2^+ \frac{R_2 L_1 - R_1 L_2}{L_1+L_2} \\ V_2^+ \end{bmatrix}.$$

From (26), we derive

$$\begin{aligned} I_2^+ &= h(t) \exp\left(-\frac{R_1+R_2}{L_1+L_2}t\right) \\ &\quad * \delta \frac{1}{L_1+L_2}(L_2 I_2(0-) - L_1 I_1(0-)) \\ &= h(t) \exp\left(-\frac{R_1+R_2}{L_1+L_2}t\right) \frac{1}{L_1+L_2} \\ &\quad \cdot (L_2 I_2(0-) - L_1 I_1(0-)). \end{aligned}$$

Therefore,

$$I_2(0+) = \frac{1}{L_1+L_2}(L_2 I_2(0-) - L_1 I_1(0-)). \quad (27)$$

Also, from (26),

$$U_1^+ = U_2^+ = V_1^+ = I_2^+ \frac{R_2 L_1 - R_1 L_2}{L_1+L_2} - \delta \frac{L_1 L_2}{L_1+L_2}(I_1(0-) + I_2(0-)). \quad (28)$$

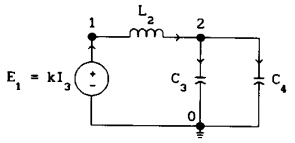

Fig. 3. Circuit of second example.

b) Analysis of the Network Connection Energy: The connection energy in branch 1 is

$$\begin{aligned} W_1^c &= \int_a^b I_1^- U_1^0 \, dt \\ &= \int_a^b I_1^- \left(-\delta \frac{L_1 L_2}{L_1+L_2}(I_1(0-) + I_2(0-))\right) dt \\ &= -\frac{1}{2}\frac{L_1 L_2}{L_1+L_2}(I_1(0-) + I_2(0-)) I_1(0-). \end{aligned}$$

In branch 2,

$$W_2^c = \int_a^b I_2^- U_2^0 \, dt = -\frac{1}{2}\frac{L_1 L_2}{L_1+L_2} \cdot (I_1(0-) + I_2(0-)) I_2(0-).$$

In branch 3, current and voltage are finite in $t = 0$. Therefore, $W_3^c = 0$.

The network connection energy is

$$W^c = \sum_{i=1}^{3} W_i^c = -\frac{1}{2}\frac{L_1 L_2}{L_1+L_2}(I_1(0-) + I_2(0-))^2. \quad (29)$$

From (27) and (28), we obtain

$$\begin{aligned} U_1^0 &= \delta L_1 (I_1(0+) - I_1(0-)) = \delta \Delta\phi_1 \\ U_2^0 &= \delta L_2 (I_2(0+) - I_2(0-)) = \delta \Delta\Phi_2 = U_1^0. \end{aligned}$$

That is to say, flux is conserved in the loop formed by both inductors: $\Delta\phi_1 = \Delta\phi_2 = \Delta\phi$. Then,

$$\begin{aligned} W^c = W_1^c + W_2^c &= \frac{1}{2}\Delta\phi I_1(0-) + \frac{1}{2}\Delta\phi I_2(0-) \\ &= \frac{1}{2}\Delta\phi(I_1(0-) + I_1(0+)) \\ &\quad + \frac{1}{2}\Delta\phi(I_2(0+) + I_2(0-)) \\ &= \frac{1}{2}L_1 I_1(0+)^2 - \frac{1}{2}L_1 I_1(0-)^2 \\ &\quad + \frac{1}{2}L_2 I_2(0+)^2 - \frac{1}{2}L_2 I_2(0-)^2. \end{aligned}$$

Thus, W^c is equal to the increment of the energy stored in the magnetic field created by both inductors. From (26), we derive that this increment is always ≤ 0.

Example 2: Given the network of Fig. 3, let us suppose that we interconnect its branches in $t = 0$. If capacitors C_3 and C_4 have different initial voltages, a current impulse is produced in the loop formed by both capacitors. This current impulse is converted into a voltage impulse by the controlled voltage source E_1. Due to this voltage impulse, the flux of inductor L_2 is not conserved and $I_2(0+) \neq I_2(0-)$.

a) Time-Domain Analysis with the State Equation: From Ohm's law and Kirchhoff's laws, we derive

$$\begin{bmatrix} -1 & 0 & k & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ I_3^+ \\ I_4^+ \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -L_2 & -L_2 \\ 0 & -C_3 & 0 & 0 \\ 0 & -C_4 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} V_1^+ \\ V_2^+ \\ I_3^+ \\ I_4^+ \end{bmatrix} = \delta \begin{bmatrix} 0 \\ -L_2 I_2(0-) \\ -C_3 U_3(0-) \\ -C_4 U_4(0-) \end{bmatrix}.$$

After some row and column elementary transformations, we obtain the following state equation:

$$\begin{bmatrix} \frac{d}{dt} & \frac{-1}{C_3+C_4} & 0 & 0 \\ \frac{1}{L_2} & \frac{d}{dt} + \frac{-C_3}{C_3+C_4}\left(\frac{1}{C_3} - \frac{k}{L_2}\right) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X^+ \\ \hline Z^+ \end{bmatrix} = \delta \begin{bmatrix} \frac{C_3 U_3(0-) + C_4 U_4(0-)}{C_3+C_4} \\ I_2(0-) + \left(\frac{1}{C_3} - \frac{k}{L_2}\right) \Delta Q_3 \\ \Delta Q_3 \\ 0 \end{bmatrix} \quad (30)$$

where the new variables are

$$X^+ = \begin{bmatrix} V_2^+ \\ I_2^+ \end{bmatrix}, \qquad Z^+ \begin{bmatrix} I_3^+ - I_2^+ \frac{C_3}{C_3+C_4} \\ V_1^+ - k I_3^+ \end{bmatrix}$$

and $\Delta Q_3 = (C_3 C_4 / C_3 + C_4)(U_4(0-) - U_3(0-))$. Therefore,

$$\begin{bmatrix} V_2^+ \\ I_2^+ \end{bmatrix} = h(t) e^{t\mathcal{A}} \begin{bmatrix} \frac{C_3 U_3(0-) + C_4 U_4(0-)}{C_3+C_4} \\ I_2(0-) + \left(\frac{1}{C_3} - \frac{k}{L_2}\right) \Delta Q_3 \end{bmatrix}.$$

The matrix $\mathcal{A}$ is obtained immediately from (30). From this result, it is easily derived that ΔQ_3 is equal to the charge increment in the capacitor C_3 between the instants $0-$ and $0+$.

b) Analysis of the Network Connection Energy: Applying (22), we can derive that the network connection energy is

$$W^c = \sum_{i=1}^{4} W_i^c = \frac{1}{2} \Delta Q_3 [k(I_2(0-) - I_1(0-)) + (U_3(0-) - U_4(0-))].$$

The total connection energy of both capacitors is

$$\begin{aligned} W_3^c + W_4^c &= \frac{1}{2} \Delta Q_3 (U_3(0-) - U_4(0-)) \\ &= \frac{1}{2} \Delta Q_3 [(U_3(0-) + U_3(0+)) - (U_4(0+) - U_4(0-))] \\ &= \frac{1}{2} C_3 U_3(0+)^2 - \frac{1}{2} C_3 U_3(0-)^2 + \frac{1}{2} C_4 U_4(0+)^2 - \frac{1}{2} C_4 U_4(0-)^2 \end{aligned}$$

which is equal to the increment of the energy stored in the electric field created by both capacitors.

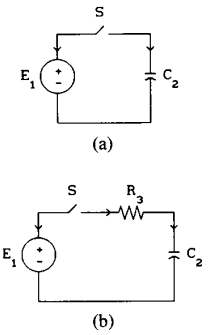

Fig. 4. Networks of third example. (a) Circuit with a CE loop. (b) The same circuit with an additional resistor.

Example 3: Let us consider the network of Fig. 4(a). The switch S is open for $t < 0$ and closed for $t \geq 0$. E_1 is a continuous function in $]-\epsilon, +\infty[$. We are going to analyze the connection energy of this circuit.

The capacitor current is $I_2^+ = \Delta Q\, \delta$. Then,

$$I_2^0 = \Delta Q\, \delta = -I_1^0$$

where ΔQ is the charge increment of the capacitor:

$$\Delta Q = C_2(E_1(0) - U_2(0-)).$$

The connection energy for each branch is

$$W_1^c = -\frac{1}{2} \Delta Q\, E_1(0) \qquad W_2^c = \frac{1}{2} \Delta Q\, U_2(0-).$$

So the network connection energy is

$$\begin{aligned} W^c &= \frac{1}{2} \Delta Q (U_2(0-) - E_1(0)) \\ &= -\frac{1}{2} C_2 (E_1(0) - U_2(0-))^2. \end{aligned}$$

Now, we are going to give a physical interpretation for the energy W^c in this circuit. To make the example simpler, we suppose that $E_1(t)$ is a constant function. Then, in this circuit, $-W^c$ is equal to the limit of the energy consumed in the resistor of the network of Fig. 4(b) when $R_3 \to 0$ in the interval $[0, t]$ for any t such that $0 < t < +\infty$: It is easy to verify that the resistor current for $t \geq 0$ is

$$I_3(t) = \frac{1}{R_3} (E_1 - U_2(0-)) \exp\left(\frac{-t}{R_3 C_2}\right).$$

Therefore,

$$\begin{aligned} \int_0^t R_3 I_3(t)^2\, dt &= (E_1 - U_2(0-))^2 \frac{1}{R_3} \int_0^t \exp\left(\frac{-2t}{R_3 C_2}\right) dt \\ &= (E_1 - U_2(0-))^2 \frac{1}{2} C_2 \left(1 - \exp\left(\frac{-2t}{R_3 C_2}\right)\right). \end{aligned}$$

It is clear that the limit of this expression when $R_3 \to 0$ is $-W^c$.

Given any value of R_3, the above expression shows that the energy absorbed by the resistor R_3 in $[0, +\infty[$ is $-W^c$ too. This result is similar to the interpretation given by Göknar [12], although Göknar's interpretation was stated only for L or

C circuits without sources. However, if $E_1(t)$ is not a constant function, the energy consumed by R_3 in $[0, +\infty[$ is generally different from $-W^c$ (for example, if $E_1(t)$ is sinusoidal, the energy consumed by R_3 is $+\infty$). On the other hand, it can be checked that our first interpretation of $-W^c$ as the limit of the energy consumed by R_3 when $R_3 \to 0$ in the finite interval $[0, t]$ can be extended to not constant functions $E_1(t)$ if $E_1(t)$ is derivable enough.

Therefore, we think that our interpretation of $-W^c$ is better than Göknar's. It should be investigated if our hypothesis is true for any *RLCM* circuit. However, this is a difficult problem since it involves singular perturbations in singular systems.

APPENDIX

Given a pencil of square matrices $S + \lambda T$, we are going to explain an algorithm to calculate a unimodular matrix $F(\lambda)$ and an invertible matrix D such that (13) holds. The matrices $F(\lambda)$ and D will be calculated using elementary row and column transformations. Our algorithm is purely algebraic, such as the algorithm given by Fettweis [9].

The concept of elementary row and column transformations of polynomial matrices can be found in [11]. If these transformations do not depend on λ, they are said to be strict. If one matrix is obtained from another by elementary transformations, these matrices are equivalent. If all the transformations are strict, then they are strictly equivalent.

The concept of the row echelon of a matrix is introduced by Campbell [13]. He gives this definition: a rectangular $m \times n$ matrix A which has rank r is said to be in row echelon form if A is of the form

$$\left[\frac{C_{r\times n}}{O_{(m-r)\times n}}\right]$$

where the elements c_{ij} of C $(= C_{r\times n})$ satisfy the following conditions:

1) $c_{ij} = 0$ if $i > j$.
2) The first nonzero entry in each row of C is 1.
3) If $c_{ij} = 1$ is the first nonzero entry of the ith column, then the jth column of C is the unit vector e_i whose only nonzero entry is in the ith position. This column is said to be a "distinguished" column.

For example, the following matrix is in row echelon form:

$$\begin{bmatrix} 1 & 3 & 0 & -2 & 0 & 4 & 0 \\ 0 & 0 & 1 & 3 & 4 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

It is easy to program an algorithm to obtain the row echelon matrix of any matrix by elementary row transformations. We have the following properties:

- Any rectangular matrix B can always be row reduced to row echelon form by elementary row operations. That is to say, there always exists an invertible matrix G such that $GB = A$, where A is in row echelon form.
- The rank of the matrix B equals the rank of its row echelon form A and is equal to the number of distinguished columns of A.

The algorithm to obtain (13) is shown in the following example. Let us consider the nonsingular polynomial matrix $S + \lambda T^{(1)}$:

1) Through strict elementary row transformations in the polynomial matrix $S + \lambda T$, the matrix $\mathbf{T}$ is transformed in its row echelon form. The resulting matrix is

$$S^{(1)} + \lambda T^{(1)} = \begin{bmatrix} S_{11}^{(1)} + \lambda T_{11}^{(1)} & \vdots & S_{12}^{(1)} + \lambda T_{12}^{(1)} \\ \cdots\cdots\cdots & \vdots & \cdots\cdots\cdots \\ S_{21}^{(1)} & \vdots & S_{22}^{(1)} \end{bmatrix}.$$

If the rank of $\mathbf{T}$ equals the dimensions of $S+\lambda T^{(1)}$, then we have finished, since the submatrix $T_{11}^{(1)}$ must be equal to the identity matrix and the submatrices $S_{21}^{(1)} y S_{22}^{(1)}$ cannot exist.

2) By means of strict elementary row transformations in $S^{(1)} + \lambda T^{(1)}$ we derive the row echelon form of the submatrix

$$\left[S_{21}^{(1)} \vdots S_{22}^{(1)}\right]$$

Therefore, the pencil $S^{(1)} + \lambda T^{(1)}$is transformed into

$$S^{(2)} + \lambda T^{(2)} = \begin{bmatrix} S_{11}^{(2)} + \lambda T_{11}^{(2)} & \vdots & S_{12}^{(2)} + \lambda T_{12}^{(2)} \\ \cdots\cdots\cdots & \vdots & \cdots\cdots\cdots \\ S_{21}^{(2)} & \vdots & S_{22}^{(2)} \end{bmatrix}.$$

3) The rank of the submatrix

$$\left[S_{21}^{(2)} \vdots S_{22}^{(2)}\right]$$

equals its number of rows, since $\det(S + \lambda T) \neq 0$. Thus, exchanging some columns in $S^2 + \lambda T^2$we obtain

$$S^{(3)} + \lambda T^{(3)} = \begin{bmatrix} S_{11}^{(3)} + \lambda T_{11}^{(3)} & \vdots & S_{12}^{(3)} + \lambda T_{12}^{(3)} \\ \cdots\cdots\cdots & \vdots & \cdots\cdots\cdots \\ S_{21}^{(3)} & \vdots & Id_{22} \end{bmatrix}$$

(the columns of the submatrix $Id_{22}$are the distinguished columns of $\left[S^{(2)}21 \vdots S_{22}^{(2)}\right]$). If the submatrices of $S^{(3)} + \lambda T^{(3)}$

$$S_{11}^{(3)} + \lambda T_{11}^{(3)}, \qquad S_{12}^{(3)} + \lambda T_{12}^{(3)}$$

do not exist (due to the fact that $S + \lambda T$is a unimodular matrix), then we have finished too.

4) Though (not strict) elementary row transformations, the pencil $S^{(3)} + \lambda T^{(3)}$is transformed into

$$S^{(4)} + \lambda T^{(4)} = \begin{bmatrix} S_{11}^{(4)} + \lambda T_{11}^{(4)} & \vdots & 0 \\ \cdots\cdots\cdots & \vdots & \cdots\cdots \\ S_{21}^{(4)} & \vdots & Id_{22} \end{bmatrix}.$$

5) Due to the fact that $\det(S + \lambda T) \neq 0$,

$$\det\left(S_{21}^{(4)} + \lambda T^{(4)}11\right) \neq 0.$$

Now we return to point 1 of this algorithm, operating in the submatrix $S_{11}^{(4)} + \lambda T_{11}^{(4)}$, instead of the matrix $S + \lambda T$.However, all the elementary transformations of points 1, 2, 3, and 4 must be done in the whole matrix $S^{(4)} + \lambda T^{(4)}$, not only in $S_{11}^{(4)} + \lambda T_{11}^{(4)}$. These operations will finish in point 1 or 3 after a finite number of loops, since in each loop the dimensions of submatrix $S_{11}^{(4)} + \lambda T_{11}^{(4)}$ is strictly smaller than the dimension of $S + \lambda T$. In fact, the number of loops must be $\leq$ rank (T)+1. At the end of this process we will obtain the matrix

$$\begin{bmatrix} -\mathcal{A} + \lambda\, Id_a & \vdots & 0 \\ \cdots\cdots\cdots & \vdots & \cdots\cdots \\ M & \vdots & Id_b \end{bmatrix}.$$

If all of the same elementary row transformations of points 1–5 are made in the identity matrix, the resulting matrix is a unimodular matrix $\mathbf{F}(\lambda)$. The only column transformations in points 1–5 are the column exchanges of point 3. Exchanging the same columns in the identity matrix, we derive an invertible matrix $\mathbf{D}'$. If we define

$$D = D\prime \begin{bmatrix} Id_a & \vdots & 0 \\ \cdots\cdots\cdots & \vdots & \cdots\cdots \\ M & \vdots & Id_b \end{bmatrix}$$

(13) holds. Observe that $\mathbf{D}'$ has been obtained through exchanges of columns in the identity matrix. Then, it is easily seen that the state variables X^+ must be a subset of the original variables Y^+in (12).

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