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High order hydrodynamical equations

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Summary. — Hydrodynamical equations act as a link between the local observed magnitudes of galactic motion and the general ones accounting for the behaviour of the Galaxy as a whole. Constraints are set usually in order to use them even in the lower order hierarchy. We present in this paper the complete expressions up to their fourth order. These equations will be used in the next future in their general form taking into account both the expected increase of kinematic data that the astrometric mission Hipparcos will provide, and some recent results indicating the possibility to obtain estimates for the momenta gradients.

Key words : galactic dynamics — statistical astronomy.

1. Introduction.

The hydrodynamical equations obtained by taking momenta of the collisionless Boltzmann equation establish relations among the stellar density derivatives, the velocity of the local standard of rest, their derivatives, the galactic force field, the momenta of the residual velocity distribution and their derivatives.

To our knowledge only Eelsalu (1979, 1982) has published incomplete fourth order equations for an axi-symmetric system, being the usual procedure to consider suitable hypothesis to reduce the number of variables (Turon, 1971 ; Vandervoort, 1975 ; Núñez, 1982). It is also possible to introduce new variables, e.g. King (1967) pseudomomenta, in which the low order equations are separable. Hydrodynamical equations have been also used when considering dynamical problems in external galaxies, e.g. Sargent *et al.* (1977), and also in the basis of the development of the Lin theory (Rohlfs, 1976).

On the other hand, Erickson (1975) has shown the possibility of determining with reasonable accuracy the central momenta up to the 4th order of the velocity distribution. Other determinations have been done by Núñez and Torra (1982). Results obtained by Torra (1984) using a rough estimation of the magnitudes involved from a compiled catalogue of kinematic data confirm the presence of local irregularities, also determined by other authors (Menge da Freitas, 1980, 1982 ; Oblak, 1983) and ratify that at least some of the terms involving momenta gradients cannot be considered null. As stated by Crézé (1978), one of the goals of the Hipparcos mission will be the determination of the momenta gradients.

Then, it will be possible to work with improved observational data in the next future using the hydrodynamical equations in their complete form, as well as they can be used in order to build up theoretical models either in their complete form or assuming hypotheses that make null some of the terms involved.

2. Derivation of the hydrodynamical equations.

As it is well known, stellar systems can be described by a distribution function $f(t, \mathbf{r}, \mathbf{V})$, or density in the phase space which gives the number of stars with positions in the range $(\mathbf{r}, \mathbf{r} + d\mathbf{r})$, and velocities in the range $(\mathbf{V}, \mathbf{V} + d\mathbf{V})$ at a certain time t. Applying Liouville's theorem to this stellar system we can write the so called fundamental equation of the stellar dynamics :

$$\frac{\mathrm{D}f}{\mathrm{D}t} = \frac{\partial f}{\partial t} + \mathbf{V} \cdot \nabla_{\mathbf{r}} f - \nabla_{\mathbf{r}} U \cdot \nabla_{\mathbf{V}} f = 0$$

where $U(t, \mathbf{r})$ is the potential that governs the system, and where we have neglected the effect of stellar encounters which can be included as a perturbational term in the right hand side of the fundamental equation.

If we use cartesian coordinates :

$$\mathbf{r} = (x, y, z) \quad \mathbf{V} = (X, Y, Z)$$

the expression of the fundamental equation is :

$$\frac{\partial f}{\partial t} + X \frac{\partial f}{\partial x} + Y \frac{\partial f}{\partial y} + Z \frac{\partial f}{\partial z} - \frac{\partial U}{\partial x} \frac{\partial f}{\partial X} - \frac{\partial U}{\partial y} \frac{\partial f}{\partial Y} - \frac{\partial U}{\partial z} \frac{\partial f}{\partial Z} = 0 . \quad (1)$$

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The hydrodynamical equations of stellar dynamics are obtained when performing the different momenta

$$\int_{\mathbf{V}} \mathbf{V}^n \frac{\mathrm{D}f}{\mathrm{D}t} \, \mathrm{d}\mathbf{V} = 0 \tag{2}$$

of the fundamental equation (1), assuming that f and its products by $\mathbf{V}^n$ vanish in the boundary of the integration domain.

Let us consider the velocity of the local standard of rest $\mathbf{V}_0 = (X_0, Y_0, Z_0)$ defined by

$$\mathbf{V}_0(t, \mathbf{r}) = \frac{1}{N(t, \mathbf{r})} \int_{\mathbf{V}} \mathbf{V} f(t, \mathbf{r}, \mathbf{V}) \, \mathrm{d}\mathbf{V}$$

being

$$N(t, \mathbf{r}) = \int_{\mathbf{V}} f(t, \mathbf{r}, \mathbf{V}) \, \mathrm{d}\mathbf{V}$$

the stellar density.

We introduce now

$$A_{pqr} = \int_{\mathbf{V}} X^p Y^q Z^r f \, \mathrm{d}\mathbf{V}$$

and express it and the distribution function in terms of the residual velocity :

$$\mathbf{v} = \mathbf{V} - \mathbf{V}_0 .$$

After some algebra we can write

$$A_{pqr} = \sum_{a=0}^{p} \sum_{b=0}^{q} \sum_{c=0}^{r} \binom{p}{a} \binom{q}{b} \binom{r}{c} \times X_0^{p-a} Y_0^{q-b} Z_0^{q-c} P_{abc} \tag{3}$$

where

$$P_{abc} = \int_{\mathbf{v}} (X - X_0)^a (Y - Y_0)^b (Z - Z_0)^c f \, \mathrm{d}\mathbf{v}$$

is the so called pressure, related to the corresponding central momentum μ_{abc} of the residual velocities distribution function by :

$$P_{abc} = N(t, \mathbf{r}) \, \mu_{abc} . \tag{4}$$

Substituting (1) in (2) and taking into account the expression for A_{pqr} we obtain for the $n = p + q + r$ order equation :

$$\frac{\partial A_{pqr}}{\partial t} + \frac{\partial A_{(p+1)qr}}{\partial x} + \frac{\partial A_{p(q+1)r}}{\partial y} + \frac{\partial A_{pq(r+1)}}{\partial z} + + p \frac{\partial U}{\partial x} A_{(p-1)qr} + q \frac{\partial U}{\partial y} A_{p(q-1)r} + r \frac{\partial U}{\partial z} A_{pq(r-1)} = 0 \tag{5}$$

If, instead of cartesian, cylindrical coordinates

$$\mathbf{r} = (\omega, \theta, z) \quad \mathbf{V} = (\Pi, \Theta, Z)$$

are used the fundamental equation is :

$$\frac{\partial f}{\partial t} + \Pi \frac{\partial f}{\partial \omega} + \frac{\Theta}{\omega} \frac{\partial f}{\partial \theta} + Z \frac{\partial f}{\partial z} - \left(\frac{\partial U}{\partial \omega} - \frac{\Theta^2}{\omega} \right) \frac{\partial f}{\partial \Pi} - \left(\frac{1}{\omega} \frac{\partial U}{\partial \theta} + \frac{\Pi \Theta}{\omega} \right) \frac{\partial f}{\partial \Theta} - \frac{\partial U}{\partial z} \frac{\partial f}{\partial Z} = 0 .$$

Then, the general expression for the n-order equation is :

$$\frac{\partial A_{pqr}}{\partial t} + \frac{\partial A_{(p+1)qr}}{\partial \omega} + \frac{1}{\omega} \frac{\partial A_{p(q+1)r}}{\partial \theta} + \frac{\partial A_{pq(r+1)}}{\partial z} + + p \frac{\partial U}{\partial \omega} A_{(p-1)qr} + \frac{q}{\omega} \frac{\partial U}{\partial \theta} A_{p(q-1)r} + r \frac{\partial U}{\partial z} A_{pq(r-1)} - \frac{p}{\omega} A_{(p-1)(q+2)r} + \frac{q+1}{\omega} A_{(p+1)qr} = 0 \tag{6}$$

being now :

$$A_{pqr} = \sum_{a=0}^{p} \sum_{b=0}^{q} \sum_{c=0}^{r} \binom{p}{a} \binom{q}{b} \binom{r}{c} \Pi_0^{p-a} \Theta_0^{q-b} Z_0^{q-c} P_{abc}$$

where

$$P_{abc} = \int_{\mathbf{v}} (\Pi - \Pi_0)^a (\Theta - \Theta_0)^b (Z - Z_0)^c f \, \mathrm{d}\mathbf{v}$$

being the central momentum μ_{abc} defined also by (4).

General expressions (5) and (6) allow us to extend the hierarchy of hydrodynamical equations to the n-order. Taking into account (4) we can express these equations in terms of central momenta of the residual velocities distribution. In Appendices A and B we present the extended form of these equations up to the fourth order in both cartesian and cylindrical coordinates respectively. These extended equations up to the fourth order are also available, on request, in terms of pressures.

3. Concluding remarks.

When deriving from (5) and (6) the equations of the Appendices A and B there is the possibility to express the resulting 3rd and 4th order equations including the partial derivatives of the potential, or substitute them taking into account the 2nd order equations.

The system formed by the hydrodynamical equations up to the n-th order is constituted by :

$$\sum_{i=0}^{n} \binom{i+2}{2} = \binom{n+3}{3}$$

equations, being the number of components of the velocity of the local standard of rest and momenta of order higher than 2

$$\sum_{i=0}^{n} \binom{i+3}{2} = \binom{n+4}{3}$$

These $\binom{n+4}{3}$ magnitudes in addition to density and potential appear also derived with respect to time and position.

So, the system is far away to be closed by increasing the number of equations. Difference between magnitudes and equations is

$$2 + \binom{n+4}{3} - \binom{n+3}{3} = 2 + \binom{n+3}{2}.$$

Any observational work that determines enough magnitudes can allow us to close the system and determine the rest of them by solving it. For instance, if estimates of high order momenta and their gradients are obtained, and assuming the values for the density gradient, the system becomes linear and can be treated by a least squares method to obtain the components of the galactocentric velocity of the local standard of rest and the components of the galactic force.

Another possibility is, as usual, to consider a set of simplifying hypotheses that enable to solve the differential system and to determine the form of the unknown functions involved. Both possibilities will be used in forthcoming papers.

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Appendix A.

$$\frac{\partial \ln N}{\partial t} + X_0\frac{\partial \ln N}{\partial x} + Y_0\frac{\partial \ln N}{\partial y} + Z_0\frac{\partial \ln N}{\partial z} + \frac{\partial X_0}{\partial x} + \frac{\partial Y_0}{\partial y} + \frac{\partial Z_0}{\partial z} = 0$$

$$\frac{\partial X_0}{\partial t} + X_0\frac{\partial X_0}{\partial x} + Y_0\frac{\partial X_0}{\partial y} + Z_0\frac{\partial X_0}{\partial z} + \mu_{200}\frac{\partial \ln N}{\partial x} + \mu_{110}\frac{\partial \ln N}{\partial y} +$$
$$+ \mu_{101}\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{200}}{\partial x} + \frac{\partial \mu_{110}}{\partial y} + \frac{\partial \mu_{101}}{\partial z} = -\frac{\partial U}{\partial x}$$

$$\frac{\partial Y_0}{\partial t} + X_0\frac{\partial Y_0}{\partial x} + Y_0\frac{\partial Y_0}{\partial y} + Z_0\frac{\partial Y_0}{\partial z} + \mu_{110}\frac{\partial \ln N}{\partial x} + \mu_{020}\frac{\partial \ln N}{\partial y} +$$
$$+ \mu_{011}\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{110}}{\partial x} + \frac{\partial \mu_{020}}{\partial y} + \frac{\partial \mu_{011}}{\partial z} = -\frac{\partial U}{\partial y}$$

$$\frac{\partial Z_0}{\partial t} + X_0\frac{\partial Z_0}{\partial x} + Y_0\frac{\partial Z_0}{\partial y} + Z_0\frac{\partial Z_0}{\partial z} + \mu_{101}\frac{\partial \ln N}{\partial x} + \mu_{011}\frac{\partial \ln N}{\partial y} + \mu_{002}\frac{\partial \ln N}{\partial z} +$$
$$+ \frac{\partial \mu_{101}}{\partial x} + \frac{\partial \mu_{011}}{\partial y} + \frac{\partial \mu_{002}}{\partial z} = -\frac{\partial U}{\partial z}$$

$$\frac{\partial \mu_{200}}{\partial t} + X_0\frac{\partial \mu_{200}}{\partial x} + Y_0\frac{\partial \mu_{200}}{\partial y} + Z_0\frac{\partial \mu_{200}}{\partial z} + 2\mu_{200}\frac{\partial X_0}{\partial x} + 2\mu_{110}\frac{\partial X_0}{\partial y} +$$
$$+ 2\mu_{101}\frac{\partial X_0}{\partial z} + \mu_{300}\frac{\partial \ln N}{\partial x} + \mu_{210}\frac{\partial \ln N}{\partial y} + \mu_{201}\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{300}}{\partial x} + \frac{\partial \mu_{210}}{\partial y} +$$
$$+ \frac{\partial \mu_{201}}{\partial z} = 0$$

$$\frac{\partial \mu_{110}}{\partial t} + X_0\frac{\partial \mu_{110}}{\partial x} + Y_0\frac{\partial \mu_{110}}{\partial y} + Z_0\frac{\partial \mu_{110}}{\partial z} + \mu_{200}\frac{\partial Y_0}{\partial x} + \mu_{110}\Big[\frac{\partial X_0}{\partial x} +$$
$$+ \frac{\partial Y_0}{\partial y}\Big] + \mu_{101}\frac{\partial Y_0}{\partial z} + \mu_{020}\frac{\partial X_0}{\partial y} + \mu_{011}\frac{\partial X_0}{\partial z} + \mu_{210}\frac{\partial \ln N}{\partial x} +$$
$$+ \mu_{120}\frac{\partial \ln N}{\partial y} + \mu_{111}\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{210}}{\partial x} + \frac{\partial \mu_{120}}{\partial y} + \frac{\partial \mu_{111}}{\partial z} = 0$$

$$\frac{\partial \mu_{101}}{\partial t} + X_0\frac{\partial \mu_{101}}{\partial x} + Y_0\frac{\partial \mu_{101}}{\partial y} + Z_0\frac{\partial \mu_{101}}{\partial z} + \mu_{200}\frac{\partial Z_0}{\partial x} + \mu_{110}\frac{\partial Z_0}{\partial y} +$$
$$+ \mu_{101}\Big[\frac{\partial X_0}{\partial x} + \frac{\partial Z_0}{\partial z}\Big] + \mu_{011}\frac{\partial X_0}{\partial y} + \mu_{002}\frac{\partial X_0}{\partial z} + \mu_{201}\frac{\partial \ln N}{\partial x} +$$
$$+ \mu_{111}\frac{\partial \ln N}{\partial y} + \mu_{102}\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{201}}{\partial x} + \frac{\partial \mu_{111}}{\partial y} + \frac{\partial \mu_{102}}{\partial z} = 0$$

$$\frac{\partial \mu_{020}}{\partial t} + X_0\frac{\partial \mu_{020}}{\partial x} + Y_0\frac{\partial \mu_{020}}{\partial y} + Z_0\frac{\partial \mu_{020}}{\partial z} + 2\mu_{110}\frac{\partial Y_0}{\partial x} + 2\mu_{020}\frac{\partial Y_0}{\partial y} +$$
$$+ 2\mu_{011}\frac{\partial Y_0}{\partial z} + \mu_{120}\frac{\partial \ln N}{\partial x} + \mu_{030}\frac{\partial \ln N}{\partial y} + \mu_{021}\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{120}}{\partial x} +$$
$$+ \frac{\partial \mu_{030}}{\partial y} + \frac{\partial \mu_{021}}{\partial z} = 0$$

$$\frac{\partial \mu_{011}}{\partial t} + X_0\frac{\partial \mu_{011}}{\partial x} + Y_0\frac{\partial \mu_{011}}{\partial y} + Z_0\frac{\partial \mu_{011}}{\partial z} + \mu_{110}\frac{\partial Z_0}{\partial x} + \mu_{101}\frac{\partial Y_0}{\partial x} +$$
$$+ \mu_{020}\frac{\partial Z_0}{\partial y} + \mu_{011}\Big[\frac{\partial Y_0}{\partial y} + \frac{\partial Z_0}{\partial z}\Big] + \mu_{002}\frac{\partial Y_0}{\partial z} + \mu_{111}\frac{\partial \ln N}{\partial x} + \mu_{021}\frac{\partial \ln N}{\partial y} +$$
$$+ \mu_{012}\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{111}}{\partial x} + \frac{\partial \mu_{021}}{\partial y} + \frac{\partial \mu_{012}}{\partial z} = 0$$

APPENDIX A (*continued*).

$$\frac{\partial \mu_{002}}{\partial t} + X_0\frac{\partial \mu_{002}}{\partial x} + Y_0\frac{\partial \mu_{002}}{\partial y} + Z_0\frac{\partial \mu_{002}}{\partial z} + 2\mu_{101}\frac{\partial Z_0}{\partial x} + 2\mu_{011}\frac{\partial Z_0}{\partial y} + 2\mu_{002}\frac{\partial Z_0}{\partial z} +$$
$$+ \mu_{102}\frac{\partial \ln N}{\partial x} + \mu_{012}\frac{\partial \ln N}{\partial y} + \mu_{003}\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{102}}{\partial x} + \frac{\partial \mu_{012}}{\partial y} + \frac{\partial \mu_{003}}{\partial z} = 0$$

$$\frac{\partial \mu_{300}}{\partial t} + X_0\frac{\partial \mu_{300}}{\partial x} + Y_0\frac{\partial \mu_{300}}{\partial y} + Z_0\frac{\partial \mu_{300}}{\partial z} - 3\mu_{200}\left[\frac{\partial \mu_{200}}{\partial x} + \frac{\partial \mu_{110}}{\partial y} + \frac{\partial \mu_{101}}{\partial z}\right] +$$
$$+ 3\mu_{300}\frac{\partial X_0}{\partial x} + 3\mu_{210}\frac{\partial X_0}{\partial y} + 3\mu_{201}\frac{\partial X_0}{\partial z} +$$
$$+ [\mu_{400}-3\mu^2_{200}]\frac{\partial \ln N}{\partial x} + [\mu_{310}-3\mu_{200}\mu_{110}]\frac{\partial \ln N}{\partial y} + [\mu_{301}-3\mu_{200}\mu_{101}]\frac{\partial \ln N}{\partial z} +$$
$$+ \frac{\partial \mu_{400}}{\partial x} + \frac{\partial \mu_{310}}{\partial y} + \frac{\partial \mu_{301}}{\partial z} = 0$$

$$\frac{\partial \mu_{210}}{\partial t} + X_0\frac{\partial \mu_{210}}{\partial x} + Y_0\frac{\partial \mu_{210}}{\partial y} + Z_0\frac{\partial \mu_{210}}{\partial z} - \mu_{200}\left[\frac{\partial \mu_{110}}{\partial x} + \frac{\partial \mu_{020}}{\partial y} + \frac{\partial \mu_{011}}{\partial z}\right] -$$
$$- 2\mu_{110}\left[\frac{\partial \mu_{200}}{\partial x} + \frac{\partial \mu_{110}}{\partial y} + \frac{\partial \mu_{101}}{\partial z}\right] +$$
$$+ \mu_{300}\frac{\partial Y_0}{\partial x} + \mu_{210}\left[2\frac{\partial X_0}{\partial x} + \frac{\partial Y_0}{\partial y}\right] + \mu_{201}\frac{\partial Y_0}{\partial z} +$$
$$+ 2\mu_{120}\frac{\partial X_0}{\partial y} + 2\mu_{111}\frac{\partial X_0}{\partial z} + [\mu_{310}-3\mu_{200}\mu_{110}]\frac{\partial \ln N}{\partial x} +$$
$$+ [\mu_{220}-\mu_{200}\mu_{020}-2\mu^2_{110}]\frac{\partial \ln N}{\partial y} + [\mu_{211}-\mu_{200}\mu_{011}-2\mu_{110}\mu_{101}]\frac{\partial \ln N}{\partial z} +$$
$$+ \frac{\partial \mu_{310}}{\partial x} + \frac{\partial \mu_{220}}{\partial y} + \frac{\partial \mu_{211}}{\partial z} = 0$$

$$\frac{\partial \mu_{201}}{\partial t} + X_0\frac{\partial \mu_{201}}{\partial x} + Y_0\frac{\partial \mu_{201}}{\partial y} + Z_0\frac{\partial \mu_{201}}{\partial z} - \mu_{200}\left[\frac{\partial \mu_{101}}{\partial x} + \frac{\partial \mu_{011}}{\partial y} + \frac{\partial \mu_{002}}{\partial z}\right] -$$
$$- 2\mu_{101}\left[\frac{\partial \mu_{200}}{\partial x} + \frac{\partial \mu_{110}}{\partial y} + \frac{\partial \mu_{101}}{\partial z}\right] + \mu_{300}\frac{\partial Z_0}{\partial x} +$$
$$+ \mu_{210}\frac{\partial Z_0}{\partial y} + \mu_{201}\left[2\frac{\partial X_0}{\partial x} + \frac{\partial Z_0}{\partial z}\right] + 2\mu_{111}\frac{\partial X_0}{\partial y} + 2\mu_{102}\frac{\partial X_0}{\partial z} +$$
$$+ [\mu_{301}-3\mu_{200}\mu_{101}]\frac{\partial \ln N}{\partial x} + [\mu_{211}-\mu_{200}\mu_{011}-2\mu_{110}\mu_{101}]\frac{\partial \ln N}{\partial y} +$$
$$+ [\mu_{202}-\mu_{200}\mu_{002}-2\mu^2_{101}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{301}}{\partial x} + \frac{\partial \mu_{211}}{\partial y} + \frac{\partial \mu_{202}}{\partial z} = 0$$

$$\frac{\partial \mu_{120}}{\partial t} + X_0\frac{\partial \mu_{120}}{\partial x} + Y_0\frac{\partial \mu_{120}}{\partial y} + Z_0\frac{\partial \mu_{120}}{\partial z} - 2\mu_{110}\left[\frac{\partial \mu_{110}}{\partial x} + \frac{\partial \mu_{020}}{\partial y} + \frac{\partial \mu_{011}}{\partial z}\right] -$$
$$- \mu_{020}\left[\frac{\partial \mu_{200}}{\partial x} + \frac{\partial \mu_{110}}{\partial y} + \frac{\partial \mu_{101}}{\partial z}\right] + 2\mu_{210}\frac{\partial Y_0}{\partial x} +$$
$$+ \mu_{120}\left[\frac{\partial X_0}{\partial x} + 2\frac{\partial Y_0}{\partial y}\right] + 2\mu_{111}\frac{\partial Y_0}{\partial z} + \mu_{030}\frac{\partial X_0}{\partial y} + \mu_{021}\frac{\partial X_0}{\partial z} +$$
$$+ [\mu_{220}-\mu_{200}\mu_{020}-2\mu^2_{110}]\frac{\partial \ln N}{\partial x} + [\mu_{130}-3\mu_{110}\mu_{020}]\frac{\partial \ln N}{\partial y} +$$
$$+ [\mu_{121}-2\mu_{110}\mu_{011}-\mu_{101}\mu_{020}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{220}}{\partial x} + \frac{\partial \mu_{130}}{\partial y} + \frac{\partial \mu_{121}}{\partial z} = 0$$

APPENDIX A (*continued*).

$$\frac{\partial \mu_{111}}{\partial t} + X_0\frac{\partial \mu_{111}}{\partial x} + Y_0\frac{\partial \mu_{111}}{\partial y} + Z_0\frac{\partial \mu_{111}}{\partial z} - \mu_{110}\left[\frac{\partial \mu_{101}}{\partial x} + \frac{\partial \mu_{011}}{\partial y} + \frac{\partial \mu_{002}}{\partial z}\right] -$$

$$- \mu_{101}\left[\frac{\partial \mu_{110}}{\partial x} + \frac{\partial \mu_{020}}{\partial y} + \frac{\partial \mu_{011}}{\partial z}\right] - \mu_{011}\left[\frac{\partial \mu_{200}}{\partial x} + \frac{\partial \mu_{110}}{\partial y} +\right.$$

$$\left. + \frac{\partial \mu_{101}}{\partial z}\right] + \mu_{210}\frac{\partial Z_0}{\partial x} + \mu_{201}\frac{\partial Y_0}{\partial x} + \mu_{120}\frac{\partial Z_0}{\partial y} +$$

$$+ \mu_{111}\left[\frac{\partial X_0}{\partial x} + \frac{\partial Y_0}{\partial y} + \frac{\partial Z_0}{\partial z}\right] + \mu_{102}\frac{\partial Y_0}{\partial z} + \mu_{021}\frac{\partial X_0}{\partial y} + \mu_{012}\frac{\partial X_0}{\partial z} +$$

$$+ [\mu_{211} - \mu_{200}\mu_{011} - 2\mu_{110}\mu_{101}]\frac{\partial \ln N}{\partial x} + [\mu_{121} - 2\mu_{110}\mu_{011} - \mu_{101}\mu_{020}]\frac{\partial \ln N}{\partial y} +$$

$$+ [\mu_{112} - \mu_{110}\mu_{002} - 2\mu_{101}\mu_{011}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{211}}{\partial x} + \frac{\partial \mu_{121}}{\partial y} + \frac{\partial \mu_{112}}{\partial z} = 0$$

$$\frac{\partial \mu_{102}}{\partial t} + X_0\frac{\partial \mu_{102}}{\partial x} + Y_0\frac{\partial \mu_{102}}{\partial y} + Z_0\frac{\partial \mu_{102}}{\partial z} - 2\mu_{101}\left[\frac{\partial \mu_{101}}{\partial x} + \frac{\partial \mu_{011}}{\partial y} + \frac{\partial \mu_{002}}{\partial z}\right] -$$

$$- \mu_{002}\left[\frac{\partial \mu_{200}}{\partial x} + \frac{\partial \mu_{110}}{\partial y} + \frac{\partial \mu_{101}}{\partial z}\right] + 2\mu_{201}\frac{\partial Z_0}{\partial x} +$$

$$+ 2\mu_{111}\frac{\partial Z_0}{\partial y} + \mu_{102}\left[\frac{\partial X_0}{\partial x} + 2\frac{\partial Z_0}{\partial z}\right] + \mu_{012}\frac{\partial X_0}{\partial y} + \mu_{003}\frac{\partial X_0}{\partial z} +$$

$$+ [\mu_{202} - \mu_{200}\mu_{002} - 2\mu^2_{101}]\frac{\partial \ln N}{\partial x} + [\mu_{112} - \mu_{110}\mu_{002} - 2\mu_{101}\mu_{011}]\frac{\partial \ln N}{\partial y} +$$

$$+ [\mu_{103} - 3\mu_{101}\mu_{002}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{202}}{\partial x} + \frac{\partial \mu_{112}}{\partial y} + \frac{\partial \mu_{103}}{\partial z} = 0$$

$$\frac{\partial \mu_{030}}{\partial t} + X_0\frac{\partial \mu_{030}}{\partial x} + Y_0\frac{\partial \mu_{030}}{\partial y} + Z_0\frac{\partial \mu_{030}}{\partial z} - 3\mu_{020}\left[\frac{\partial \mu_{110}}{\partial x} + \frac{\partial \mu_{020}}{\partial y} + \frac{\partial \mu_{011}}{\partial z}\right] +$$

$$+ 3\mu_{120}\frac{\partial Y_0}{\partial x} + 3\mu_{030}\frac{\partial Y_0}{\partial y} + 3\mu_{021}\frac{\partial Y_0}{\partial z} +$$

$$+ [\mu_{130} - 3\mu_{110}\mu_{020}]\frac{\partial \ln N}{\partial x} + [\mu_{040} - 3\mu^2_{020}]\frac{\partial \ln N}{\partial y} + [\mu_{031} - 3\mu_{020}\mu_{011}]\frac{\partial \ln N}{\partial z} +$$

$$+ \frac{\partial \mu_{130}}{\partial x} + \frac{\partial \mu_{040}}{\partial y} + \frac{\partial \mu_{031}}{\partial z} = 0$$

$$\frac{\partial \mu_{021}}{\partial t} + X_0\frac{\partial \mu_{021}}{\partial x} + Y_0\frac{\partial \mu_{021}}{\partial y} + Z_0\frac{\partial \mu_{021}}{\partial z} - \mu_{020}\left[\frac{\partial \mu_{101}}{\partial x} + \frac{\partial \mu_{011}}{\partial y} + \frac{\partial \mu_{002}}{\partial z}\right] -$$

$$- 2\mu_{011}\left[\frac{\partial \mu_{110}}{\partial x} + \frac{\partial \mu_{020}}{\partial y} + \frac{\partial \mu_{011}}{\partial z}\right] + \mu_{120}\frac{\partial Z_0}{\partial x} +$$

$$+ 2\mu_{111}\frac{\partial Y_0}{\partial x} + \mu_{030}\frac{\partial Z_0}{\partial y} + \mu_{021}\left[2\frac{\partial Y_0}{\partial y} + \frac{\partial Z_0}{\partial z}\right] + 2\mu_{012}\frac{\partial Y_0}{\partial z} +$$

$$+ [\mu_{121} - 2\mu_{110}\mu_{011} - \mu_{101}\mu_{020}]\frac{\partial \ln N}{\partial x} + [\mu_{031} - 3\mu_{020}\mu_{011}]\frac{\partial \ln N}{\partial y} +$$

$$+ [\mu_{022} - \mu_{020}\mu_{002} - 2\mu^2_{011}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{121}}{\partial x} + \frac{\partial \mu_{031}}{\partial y} + \frac{\partial \mu_{022}}{\partial z} = 0$$

Appendix A (*continued*).

$$\begin{aligned}
&\frac{\partial \mu_{012}}{\partial t} + X_0\frac{\partial \mu_{012}}{\partial x} + Y_0\frac{\partial \mu_{012}}{\partial y} + Z_0\frac{\partial \mu_{012}}{\partial z} - 2\mu_{011}\left[\frac{\partial \mu_{101}}{\partial x} + \frac{\partial \mu_{011}}{\partial y} + \frac{\partial \mu_{002}}{\partial z}\right] - \\
&- \mu_{002}\left[\frac{\partial \mu_{110}}{\partial x} + \frac{\partial \mu_{020}}{\partial y} + \frac{\partial \mu_{011}}{\partial z}\right] + 2\mu_{111}\frac{\partial Z_0}{\partial x} + \\
&+ \mu_{102}\frac{\partial Y_0}{\partial x} + 2\mu_{021}\frac{\partial Z_0}{\partial y} + \mu_{012}\left[\frac{\partial Y_0}{\partial y} + 2\frac{\partial Z_0}{\partial z}\right] + \mu_{003}\frac{\partial Y_0}{\partial z} + \\
&+ [\mu_{112}-\mu_{110}\mu_{002}-2\mu_{101}\mu_{011}]\frac{\partial \ln N}{\partial x} + [\mu_{022}-\mu_{020}\mu_{002}-2\mu^2{}_{011}]\frac{\partial \ln N}{\partial y} + \\
&+ [\mu_{013}-3\mu_{011}\mu_{002}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{112}}{\partial x} + \frac{\partial \mu_{022}}{\partial y} + \frac{\partial \mu_{013}}{\partial z} = 0
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial \mu_{003}}{\partial t} + X_0\frac{\partial \mu_{003}}{\partial x} + Y_0\frac{\partial \mu_{003}}{\partial y} + Z_0\frac{\partial \mu_{003}}{\partial z} - 3\mu_{002}\left[\frac{\partial \mu_{101}}{\partial x} + \frac{\partial \mu_{011}}{\partial y} + \frac{\partial \mu_{002}}{\partial z}\right] + \\
&+ 3\mu_{102}\frac{\partial Z_0}{\partial x} + 3\mu_{012}\frac{\partial Z_0}{\partial y} + 3\mu_{003}\frac{\partial Z_0}{\partial z} + [\mu_{103}-3\mu_{101}\mu_{002}]\frac{\partial \ln N}{\partial x} + \\
&+ [\mu_{013}-3\mu_{011}\mu_{002}]\frac{\partial \ln N}{\partial y} + [\mu_{004}-3\mu^2{}_{002}]\frac{\partial \ln N}{\partial z} + \\
&+ \frac{\partial \mu_{103}}{\partial x} + \frac{\partial \mu_{013}}{\partial y} + \frac{\partial \mu_{004}}{\partial z} = 0
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial \mu_{400}}{\partial t} + X_0\frac{\partial \mu_{400}}{\partial x} + Y_0\frac{\partial \mu_{400}}{\partial y} + Z_0\frac{\partial \mu_{400}}{\partial z} - 4\mu_{300}\left[\frac{\partial \mu_{200}}{\partial x} + \frac{\partial \mu_{110}}{\partial y} + \frac{\partial \mu_{101}}{\partial z}\right] + \\
&+ 4\mu_{400}\frac{\partial X_0}{\partial x} + 4\mu_{310}\frac{\partial X_0}{\partial y} + 4\mu_{301}\frac{\partial X_0}{\partial z} + [\mu_{500}-4\mu_{300}\mu_{200}]\frac{\partial \ln N}{\partial x} + \\
&+ [\mu_{410}-4\mu_{300}\mu_{110}]\frac{\partial \ln N}{\partial y} + [\mu_{401}-4\mu_{300}\mu_{101}]\frac{\partial \ln N}{\partial z} + \\
&+ \frac{\partial \mu_{500}}{\partial x} + \frac{\partial \mu_{410}}{\partial y} + \frac{\partial \mu_{401}}{\partial z} = 0
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial \mu_{310}}{\partial t} + X_0\frac{\partial \mu_{310}}{\partial x} + Y_0\frac{\partial \mu_{310}}{\partial y} + Z_0\frac{\partial \mu_{310}}{\partial z} - \mu_{300}\left[\frac{\partial \mu_{110}}{\partial x} + \frac{\partial \mu_{020}}{\partial y} + \frac{\partial \mu_{011}}{\partial z}\right] - \\
&- 3\mu_{210}\left[\frac{\partial \mu_{200}}{\partial x} + \frac{\partial \mu_{110}}{\partial y} + \frac{\partial \mu_{101}}{\partial z}\right] + \mu_{400}\frac{\partial Y_0}{\partial x} + \mu_{310}\left[3\frac{\partial X_0}{\partial x} + \frac{\partial Y_0}{\partial y}\right] + \\
&+ \mu_{301}\frac{\partial Y_0}{\partial z} + 3\mu_{220}\frac{\partial X_0}{\partial y} + 3\mu_{211}\frac{\partial X_0}{\partial z} + [\mu_{410}-\mu_{300}\mu_{110}-3\mu_{210}\mu_{200}]\frac{\partial \ln N}{\partial x} + \\
&+ [\mu_{320}-\mu_{300}\mu_{020}-3\mu_{210}\mu_{110}]\frac{\partial \ln N}{\partial y} + \\
&+ [\mu_{311}-\mu_{300}\mu_{011}-3\mu_{210}\mu_{101}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{410}}{\partial x} + \frac{\partial \mu_{320}}{\partial y} + \frac{\partial \mu_{311}}{\partial z} = 0
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial \mu_{301}}{\partial t} + X_0\frac{\partial \mu_{301}}{\partial x} + Y_0\frac{\partial \mu_{301}}{\partial y} + Z_0\frac{\partial \mu_{301}}{\partial z} - \mu_{300}\left[\frac{\partial \mu_{101}}{\partial x} + \frac{\partial \mu_{011}}{\partial y} + \frac{\partial \mu_{002}}{\partial z}\right] - \\
&- 3\mu_{201}\left[\frac{\partial \mu_{200}}{\partial x} + \frac{\partial \mu_{110}}{\partial y} + \frac{\partial \mu_{101}}{\partial z}\right] + \mu_{400}\frac{\partial Z_0}{\partial x} + \\
&+ \mu_{310}\frac{\partial Z_0}{\partial y} + \mu_{301}\left[3\frac{\partial X_0}{\partial x} + \frac{\partial Z_0}{\partial z}\right] + 3\mu_{211}\frac{\partial X_0}{\partial y} + 3\mu_{202}\frac{\partial X_0}{\partial z} + \\
&+ [\mu_{401}-\mu_{300}\mu_{101}-3\mu_{201}\mu_{200}]\frac{\partial \ln N}{\partial x} + [\mu_{311}-\mu_{300}\mu_{011}-3\mu_{201}\mu_{110}]\frac{\partial \ln N}{\partial y} + \\
&+ [\mu_{302}-\mu_{300}\mu_{002}-3\mu_{201}\mu_{101}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{401}}{\partial x} + \frac{\partial \mu_{311}}{\partial y} + \frac{\partial \mu_{302}}{\partial z} = 0
\end{aligned}$$

APPENDIX A (*continued*).

$$
\begin{aligned}
&\frac{\partial \mu_{220}}{\partial t} + X_0\frac{\partial \mu_{220}}{\partial x} + Y_0\frac{\partial \mu_{220}}{\partial y} + Z_0\frac{\partial \mu_{220}}{\partial z} - 2\mu_{210}\left[\frac{\partial \mu_{110}}{\partial x} + \frac{\partial \mu_{020}}{\partial y} + \frac{\partial \mu_{011}}{\partial z}\right] - \\
&- 2\mu_{120}\left[\frac{\partial \mu_{200}}{\partial x} + \frac{\partial \mu_{110}}{\partial y} + \frac{\partial \mu_{101}}{\partial z}\right] + 2\mu_{310}\frac{\partial Y_0}{\partial x} + \\
&+ 2\mu_{220}\left[\frac{\partial X_0}{\partial x} + \frac{\partial Y_0}{\partial y}\right] + 3\mu_{211}\frac{\partial Y_0}{\partial z} + 2\mu_{130}\frac{\partial X_0}{\partial y} + \\
&+ 2\mu_{121}\frac{\partial X_0}{\partial z} + \left[\mu_{320} - 2\mu_{210}\mu_{110} - 2\mu_{120}\mu_{200}\right]\frac{\partial \ln N}{\partial x} + \\
&+ \left[\mu_{230} - 2\mu_{210}\mu_{020} - 2\mu_{120}\mu_{110}\right]\frac{\partial \ln N}{\partial y} + \left[\mu_{221} - 2\mu_{210}\mu_{011} - 2\mu_{120}\mu_{101}\right]\frac{\partial \ln N}{\partial z} + \\
&+ \frac{\partial \mu_{320}}{\partial x} + \frac{\partial \mu_{230}}{\partial y} + \frac{\partial \mu_{221}}{\partial z} = 0
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial \mu_{211}}{\partial t} + X_0\frac{\partial \mu_{211}}{\partial x} + Y_0\frac{\partial \mu_{211}}{\partial y} + Z_0\frac{\partial \mu_{211}}{\partial z} - \mu_{210}\left[\frac{\partial \mu_{101}}{\partial x} + \frac{\partial \mu_{011}}{\partial y} + \frac{\partial \mu_{002}}{\partial z}\right] - \\
&- \mu_{201}\left[\frac{\partial \mu_{110}}{\partial x} + \frac{\partial \mu_{020}}{\partial y} + \frac{\partial \mu_{011}}{\partial z}\right] - 2\mu_{111}\left[\frac{\partial \mu_{200}}{\partial x} + \frac{\partial \mu_{110}}{\partial y} + \right. \\
&\left. + \frac{\partial \mu_{101}}{\partial z}\right] + \mu_{310}\frac{\partial Z_0}{\partial x} + \mu_{301}\frac{\partial Y_0}{\partial x} + \mu_{220}\frac{\partial Z_0}{\partial y} + \\
&+ \mu_{211}\left[2\frac{\partial X_0}{\partial x} + \frac{\partial Y_0}{\partial y} + \frac{\partial Z_0}{\partial z}\right] + \mu_{202}\frac{\partial Y_0}{\partial z} + \\
&+ 2\mu_{121}\frac{\partial X_0}{\partial y} + 2\mu_{112}\frac{\partial X_0}{\partial z} + \left[\mu_{311} - \mu_{210}\mu_{101} - \mu_{201}\mu_{110} - 2\mu_{111}\mu_{200}\right]\frac{\partial \ln N}{\partial x} + \\
&+ \left[\mu_{221} - \mu_{210}\mu_{011} - \mu_{201}\mu_{020} - 2\mu_{111}\mu_{110}\right]\frac{\partial \ln N}{\partial y} + \\
&+ \left[\mu_{212} - \mu_{210}\mu_{002} - \mu_{201}\mu_{011} - 2\mu_{111}\mu_{101}\right]\frac{\partial \ln N}{\partial z} + \\
&+ \frac{\partial \mu_{311}}{\partial x} + \frac{\partial \mu_{221}}{\partial u} + \frac{\partial \mu_{212}}{\partial z} = 0
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial \mu_{202}}{\partial t} + X_0\frac{\partial \mu_{202}}{\partial x} + Y_0\frac{\partial \mu_{202}}{\partial y} + Z_0\frac{\partial \mu_{202}}{\partial z} - 2\mu_{201}\left[\frac{\partial \mu_{101}}{\partial x} + \frac{\partial \mu_{011}}{\partial y} + \frac{\partial \mu_{002}}{\partial z}\right] - \\
&- 2\mu_{102}\left[\frac{\partial \mu_{200}}{\partial x} + \frac{\partial \mu_{110}}{\partial y} + \frac{\partial \mu_{101}}{\partial z}\right] + 2\mu_{301}\frac{\partial Z_0}{\partial x} + \\
&+ 2\mu_{211}\frac{\partial Z_0}{\partial y} + 2\mu_{202}\left[\frac{\partial X_0}{\partial x} + \frac{\partial Z_0}{\partial z}\right] + 2\mu_{112}\frac{\partial X_0}{\partial y} + 2\mu_{103}\frac{\partial X_0}{\partial z} + \\
&+ \left[\mu_{302} - 2\mu_{201}\mu_{101} - 2\mu_{102}\mu_{200}\right]\frac{\partial \ln N}{\partial x} + \left[\mu_{212} - 2\mu_{201}\mu_{011} - 2\mu_{102}\mu_{110}\right]\frac{\partial \ln N}{\partial y} + \\
&+ \left[\mu_{203} - 2\mu_{201}\mu_{002} - 2\mu_{102}\mu_{101}\right]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{302}}{\partial x} + \frac{\partial \mu_{212}}{\partial y} + \frac{\partial \mu_{203}}{\partial z} = 0
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial \mu_{130}}{\partial t} + X_0\frac{\partial \mu_{130}}{\partial x} + Y_0\frac{\partial \mu_{130}}{\partial y} + Z_0\frac{\partial \mu_{130}}{\partial z} - 3\mu_{120}\left[\frac{\partial \mu_{110}}{\partial x} + \frac{\partial \mu_{020}}{\partial y} + \frac{\partial \mu_{011}}{\partial z}\right] - \\
&- \mu_{030}\left[\frac{\partial \mu_{200}}{\partial x} + \frac{\partial \mu_{110}}{\partial y} + \frac{\partial \mu_{101}}{\partial z}\right] + 3\mu_{220}\frac{\partial Y_0}{\partial x} + \\
&+ \mu_{130}\left[\frac{\partial X_0}{\partial x} + 3\frac{\partial Y_0}{\partial y}\right] + 3\mu_{121}\frac{\partial Y_0}{\partial z} + \mu_{040}\frac{\partial X_0}{\partial y} + \mu_{031}\frac{\partial X_0}{\partial z} + \\
&+ \left[\mu_{230} - 3\mu_{120}\mu_{110} - \mu_{030}\mu_{200}\right]\frac{\partial \ln N}{\partial x} + \left[\mu_{140} - 3\mu_{120}\mu_{020} - \mu_{030}\mu_{110}\right]\frac{\partial \ln N}{\partial y} + \\
&+ \left[\mu_{131} - 3\mu_{120}\mu_{011} - \mu_{030}\mu_{101}\right]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{230}}{\partial x} + \frac{\partial \mu_{140}}{\partial y} + \frac{\partial \mu_{131}}{\partial z} = 0
\end{aligned}
$$

APPENDIX A *(continued)*.

$$
\begin{aligned}
&\frac{\partial \mu_{121}}{\partial t} + X_0\frac{\partial \mu_{121}}{\partial x} + Y_0\frac{\partial \mu_{121}}{\partial y} + Z_0\frac{\partial \mu_{121}}{\partial z} - \mu_{120}\left[\frac{\partial \mu_{101}}{\partial x} + \frac{\partial \mu_{011}}{\partial y} + \frac{\partial \mu_{002}}{\partial z}\right] - \\
&- 2\mu_{111}\left[\frac{\partial \mu_{110}}{\partial x} + \frac{\partial \mu_{020}}{\partial y} + \frac{\partial \mu_{011}}{\partial z}\right] - \mu_{021}\left[\frac{\partial \mu_{200}}{\partial x} + \frac{\partial \mu_{110}}{\partial y} + \frac{\partial \mu_{101}}{\partial z}\right] + \\
&+ \mu_{220}\frac{\partial Z_0}{\partial x} + 2\mu_{211}\frac{\partial Y_0}{\partial x} + \mu_{130}\frac{\partial Z_0}{\partial y} + \mu_{121}\left[\frac{\partial X_0}{\partial x} + 2\frac{\partial Y_0}{\partial y} + \frac{\partial Z_0}{\partial z}\right] + \\
&+ 2\mu_{112}\frac{\partial Y_0}{\partial z} + \mu_{031}\frac{\partial X_0}{\partial y} + \mu_{022}\frac{\partial X_0}{\partial z} + [\mu_{221}-\mu_{120}\mu_{101}-2\mu_{111}\mu_{110}-\mu_{021}\mu_{200}]\frac{\partial \ln N}{\partial x} + \\
&+ [\mu_{131}-\mu_{120}\mu_{011}-2\mu_{111}\mu_{020}-\mu_{021}\mu_{110}]\frac{\partial \ln N}{\partial y} + \\
&+ [\mu_{122}-\mu_{120}\mu_{002}-2\mu_{111}\mu_{011}-\mu_{021}\mu_{101}]\frac{\partial \ln N}{\partial z} + \\
&+ \frac{\partial \mu_{221}}{\partial x} + \frac{\partial \mu_{131}}{\partial y} + \frac{\partial \mu_{122}}{\partial z} = 0
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial \mu_{112}}{\partial t} + X_0\frac{\partial \mu_{112}}{\partial x} + Y_0\frac{\partial \mu_{112}}{\partial y} + Z_0\frac{\partial \mu_{112}}{\partial z} - 2\mu_{111}\left[\frac{\partial \mu_{101}}{\partial x} + \frac{\partial \mu_{011}}{\partial y} + \frac{\partial \mu_{002}}{\partial z}\right] - \\
&- \mu_{102}\left[\frac{\partial \mu_{110}}{\partial x} + \frac{\partial \mu_{020}}{\partial y} + \frac{\partial \mu_{011}}{\partial z}\right] - \mu_{012}\left[\frac{\partial \mu_{200}}{\partial x} + \frac{\partial \mu_{110}}{\partial y} + \frac{\partial \mu_{101}}{\partial z}\right] + \\
&+ 2\mu_{211}\frac{\partial Z_0}{\partial x} + \mu_{202}\frac{\partial Y_0}{\partial x} + 2\mu_{121}\frac{\partial Z_0}{\partial y} + \mu_{112}\left[\frac{\partial X_0}{\partial x} + \frac{\partial Y_0}{\partial y} + 2\frac{\partial Z_0}{\partial z}\right] + \mu_{103}\frac{\partial Y_0}{\partial z} + \\
&+ \mu_{022}\frac{\partial X_0}{\partial y} + \mu_{013}\frac{\partial X_0}{\partial z} + [\mu_{212}-2\mu_{111}\mu_{101}-\mu_{102}\mu_{110}-\mu_{012}\mu_{200}]\frac{\partial \ln N}{\partial x} + \\
&+ [\mu_{122}-2\mu_{111}\mu_{011}-\mu_{102}\mu_{020}-\mu_{012}\mu_{110}]\frac{\partial \ln N}{\partial y} + \\
&+ [\mu_{113}-2\mu_{111}\mu_{002}-\mu_{102}\mu_{011}-\mu_{012}\mu_{101}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{212}}{\partial x} + \frac{\partial \mu_{122}}{\partial y} + \frac{\partial \mu_{113}}{\partial z} = 0
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial \mu_{103}}{\partial t} + X_0\frac{\partial \mu_{103}}{\partial x} + Y_0\frac{\partial \mu_{103}}{\partial y} + Z_0\frac{\partial \mu_{103}}{\partial z} - 3\mu_{102}\left[\frac{\partial \mu_{101}}{\partial x} + \frac{\partial \mu_{011}}{\partial y} + \frac{\partial \mu_{002}}{\partial z}\right] - \\
&- \mu_{003}\left[\frac{\partial \mu_{200}}{\partial x} + \frac{\partial \mu_{110}}{\partial y} + \frac{\partial \mu_{101}}{\partial z}\right] + 3\mu_{202}\frac{\partial Z_0}{\partial x} + \\
&+ 3\mu_{112}\frac{\partial Z_0}{\partial y} + \mu_{103}\left[\frac{\partial X_0}{\partial x} + 3\frac{\partial Z_0}{\partial z}\right] + \mu_{013}\frac{\partial X_0}{\partial y} + \mu_{004}\frac{\partial X_0}{\partial z} + \\
&+ [\mu_{203}-3\mu_{102}\mu_{101}-\mu_{003}\mu_{200}]\frac{\partial \ln N}{\partial x} + [\mu_{113}-3\mu_{102}\mu_{011}-\mu_{003}\mu_{110}]\frac{\partial \ln N}{\partial y} + \\
&+ [\mu_{104}-3\mu_{102}\mu_{002}-\mu_{003}\mu_{101}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{203}}{\partial x} + \frac{\partial \mu_{113}}{\partial y} + \frac{\partial \mu_{104}}{\partial z} = 0
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial \mu_{040}}{\partial t} + X_0\frac{\partial \mu_{040}}{\partial x} + Y_0\frac{\partial \mu_{040}}{\partial y} + Z_0\frac{\partial \mu_{040}}{\partial z} - 4\mu_{030}\left[\frac{\partial \mu_{110}}{\partial x} + \frac{\partial \mu_{020}}{\partial y} + \frac{\partial \mu_{011}}{\partial z}\right] + \\
&+ 4\mu_{130}\frac{\partial Y_0}{\partial x} + 4\mu_{040}\frac{\partial Y_0}{\partial y} + 4\mu_{031}\frac{\partial Y_0}{\partial z} + \\
&+ [\mu_{140}-4\mu_{030}\mu_{110}]\frac{\partial \ln N}{\partial x} + [\mu_{050}-4\mu_{030}\mu_{020}]\frac{\partial \ln N}{\partial y} + [\mu_{041}-4\mu_{030}\mu_{011}]\frac{\partial \ln N}{\partial z} + \\
&+ \frac{\partial \mu_{140}}{\partial x} + \frac{\partial \mu_{050}}{\partial y} + \frac{\partial \mu_{041}}{\partial z} = 0
\end{aligned}
$$

Appendix A (*continued*).

$$\frac{\partial \mu_{031}}{\partial t} + X_0\frac{\partial \mu_{031}}{\partial x} + Y_0\frac{\partial \mu_{031}}{\partial y} + Z_0\frac{\partial \mu_{031}}{\partial z} - \mu_{030}\left[\frac{\partial \mu_{101}}{\partial x} + \frac{\partial \mu_{011}}{\partial y} + \frac{\partial \mu_{002}}{\partial z}\right] -$$
$$- 3\mu_{021}\left[\frac{\partial \mu_{110}}{\partial x} + \frac{\partial \mu_{020}}{\partial y} + \frac{\partial \mu_{011}}{\partial z}\right] + \mu_{130}\frac{\partial Z_0}{\partial x} +$$
$$+ 3\mu_{121}\frac{\partial Y_0}{\partial x} + \mu_{040}\frac{\partial Z_0}{\partial y} + \mu_{031}\left[3\frac{\partial Y_0}{\partial y} + \frac{\partial Z_0}{\partial z}\right] + 3\mu_{022}\frac{\partial Y_0}{\partial z} +$$
$$+ [\mu_{131}-\mu_{030}\mu_{101}-3\mu_{021}\mu_{110}]\frac{\partial \ln N}{\partial x} + [\mu_{041}-\mu_{030}\mu_{011}-3\mu_{021}\mu_{020}]\frac{\partial \ln N}{\partial y} +$$
$$+ [\mu_{032}-\mu_{030}\mu_{002}-3\mu_{021}\mu_{011}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{131}}{\partial x} + \frac{\partial \mu_{041}}{\partial y} + \frac{\partial \mu_{032}}{\partial z} = 0$$

$$\frac{\partial \mu_{022}}{\partial t} + X_0\frac{\partial \mu_{022}}{\partial x} + Y_0\frac{\partial \mu_{022}}{\partial y} + Z_0\frac{\partial \mu_{022}}{\partial z} - 2\mu_{021}\left[\frac{\partial \mu_{101}}{\partial x} + \frac{\partial \mu_{011}}{\partial y} + \frac{\partial \mu_{002}}{\partial z}\right] -$$
$$- 2\mu_{012}\left[\frac{\partial \mu_{110}}{\partial x} + \frac{\partial \mu_{020}}{\partial y} + \frac{\partial \mu_{011}}{\partial z}\right] + 2\mu_{121}\frac{\partial Z_0}{\partial x} +$$
$$+ 2\mu_{112}\frac{\partial Y_0}{\partial x} + 2\mu_{031}\frac{\partial Z_0}{\partial y} + 2\mu_{022}\left[\frac{\partial Y_0}{\partial y} + \frac{\partial Z_0}{\partial z}\right] + 2\mu_{013}\frac{\partial Y_0}{\partial z} +$$
$$+ [\mu_{122}-2\mu_{021}\mu_{101}-2\mu_{012}\mu_{110}]\frac{\partial \ln N}{\partial x} + [\mu_{032}-2\mu_{021}\mu_{011}-2\mu_{012}\mu_{020}]\frac{\partial \ln N}{\partial y} +$$
$$+ [\mu_{023}-2\mu_{021}\mu_{002}-2\mu_{012}\mu_{011}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{122}}{\partial x} + \frac{\partial \mu_{032}}{\partial y} + \frac{\partial \mu_{023}}{\partial z} = 0$$

$$\frac{\partial \mu_{013}}{\partial t} + X_0\frac{\partial \mu_{013}}{\partial x} + Y_0\frac{\partial \mu_{013}}{\partial y} + Z_0\frac{\partial \mu_{013}}{\partial z} - 3\mu_{012}\left[\frac{\partial \mu_{101}}{\partial x} + \frac{\partial \mu_{011}}{\partial y} + \frac{\partial \mu_{002}}{\partial z}\right] -$$
$$- \mu_{003}\left[\frac{\partial \mu_{110}}{\partial x} + \frac{\partial \mu_{020}}{\partial y} + \frac{\partial \mu_{011}}{\partial z}\right] + 3\mu_{112}\frac{\partial Z_0}{\partial x} +$$
$$+ \mu_{103}\frac{\partial Y_0}{\partial x} + 3\mu_{022}\frac{\partial Z_0}{\partial y} + \mu_{013}\left[\frac{\partial Y_0}{\partial y} + 3\frac{\partial Z_0}{\partial z}\right] + \mu_{004}\frac{\partial Y_0}{\partial z} +$$
$$+ [\mu_{113}-3\mu_{012}\mu_{101}-\mu_{003}\mu_{110}]\frac{\partial \ln N}{\partial x} + [\mu_{023}-3\mu_{012}\mu_{011}-\mu_{003}\mu_{020}]\frac{\partial \ln N}{\partial y} +$$
$$+ [\mu_{014}-3\mu_{012}\mu_{002}-\mu_{003}\mu_{011}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{113}}{\partial x} + \frac{\partial \mu_{023}}{\partial y} + \frac{\partial \mu_{014}}{\partial z} = 0$$

$$\frac{\partial \mu_{004}}{\partial t} + X_0\frac{\partial \mu_{004}}{\partial x} + Y_0\frac{\partial \mu_{004}}{\partial y} + Z_0\frac{\partial \mu_{004}}{\partial z} - 4\mu_{003}\left[\frac{\partial \mu_{101}}{\partial x} + \frac{\partial \mu_{011}}{\partial y} + \frac{\partial \mu_{002}}{\partial z}\right] +$$
$$+ 4\mu_{103}\frac{\partial Z_0}{\partial x} + 4\mu_{013}\frac{\partial Z_0}{\partial y} + 4\mu_{004}\frac{\partial Z_0}{\partial z} + [\mu_{104}-4\mu_{003}\mu_{101}]\frac{\partial \ln N}{\partial x} +$$
$$+ [\mu_{014}-4\mu_{003}\mu_{011}]\frac{\partial \ln N}{\partial y} + [\mu_{005}-4\mu_{003}\mu_{002}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{104}}{\partial x} + \frac{\partial \mu_{014}}{\partial y} +$$
$$+ \frac{\partial \mu_{005}}{\partial z} = 0$$

Appendix B.

$$\frac{\partial \ln N}{\partial t} + \Pi_0 \frac{\partial \ln N}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \ln N}{\partial \theta} + Z_0 \frac{\partial \ln N}{\partial z} + \frac{\partial \Pi_0}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \Theta_0}{\partial \theta} + \frac{\partial Z_0}{\partial z} + \frac{\Pi_0}{\varpi} = 0$$

$$\begin{aligned} &\frac{\partial \Pi_0}{\partial t} + \Pi_0 \frac{\partial \Pi_0}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \Pi_0}{\partial \theta} + Z_0 \frac{\partial \Pi_0}{\partial z} - \frac{\Theta^2{}_0}{\varpi} + \mu_{200}\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{110}}{\varpi}\frac{\partial \ln N}{\partial \theta} + \\ &+ \mu_{101}\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{200}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{110}}{\partial \theta} + \frac{\partial \mu_{101}}{\partial z} + \frac{\mu_{200}-\mu_{020}}{\varpi} = -\frac{\partial U}{\partial \varpi} \end{aligned}$$

$$\begin{aligned} &\frac{\partial \Theta_0}{\partial t} + \Pi_0 \frac{\partial \Theta_0}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \Theta_0}{\partial \theta} + Z_0 \frac{\partial \Theta_0}{\partial z} + \frac{\Pi_0\Theta_0}{\varpi} + \mu_{110}\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{020}}{\varpi}\frac{\partial \ln N}{\partial \theta} + \\ &+ \mu_{011}\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{110}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{020}}{\partial \theta} + \frac{\partial \mu_{011}}{\partial z} + \frac{2\mu_{110}}{\varpi} = -\frac{1}{\varpi}\frac{\partial U}{\partial \theta} \end{aligned}$$

$$\begin{aligned} &\frac{\partial Z_0}{\partial t} + \Pi_0 \frac{\partial Z_0}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial Z_0}{\partial \theta} + Z_0 \frac{\partial Z_0}{\partial z} + \mu_{101}\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{011}}{\varpi}\frac{\partial \ln N}{\partial \theta} + \mu_{002}\frac{\partial \ln N}{\partial z} + \\ &+ \frac{\partial \mu_{101}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{011}}{\partial \theta} + \frac{\partial \mu_{002}}{\partial z} + \frac{\mu_{101}}{\varpi} = -\frac{\partial U}{\partial z} \end{aligned}$$

$$\begin{aligned} &\frac{\partial \mu_{200}}{\partial t} + \Pi_0 \frac{\partial \mu_{200}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{200}}{\partial \theta} + Z_0 \frac{\partial \mu_{200}}{\partial z} + 2\mu_{200}\frac{\partial \Pi_0}{\partial \varpi} + 2\mu_{110}\left[\frac{1}{\varpi}\frac{\partial \Pi_0}{\partial \theta} - \frac{2\Theta_0}{\varpi}\right] + \\ &+ 2\mu_{101}\frac{\partial \Pi_0}{\partial z} + \mu_{300}\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{210}}{\varpi}\frac{\partial \ln N}{\partial \theta} + \mu_{201}\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{300}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{210}}{\partial \theta} + \\ &+ \frac{\partial \mu_{201}}{\partial z} + \frac{\mu_{300}-2\mu_{120}}{\varpi} = 0 \end{aligned}$$

$$\begin{aligned} &\frac{\partial \mu_{110}}{\partial t} + \Pi_0 \frac{\partial \mu_{110}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{110}}{\partial \theta} + Z_0 \frac{\partial \mu_{110}}{\partial z} + \mu_{200}\left[\frac{\partial \Theta_0}{\partial \varpi} + \frac{\Theta_0}{\varpi}\right] + \mu_{110}\left[\frac{\partial \Pi_0}{\partial \varpi} + \right. \\ &\left. + \frac{1}{\varpi}\frac{\partial \Theta_0}{\partial \theta} + \frac{\Pi_0}{\varpi}\right] + \mu_{101}\frac{\partial \Theta_0}{\partial z} + \mu_{020}\left[\frac{1}{\varpi}\frac{\partial \Pi_0}{\partial \theta} - \frac{2\Theta_0}{\varpi}\right] + \mu_{011}\frac{\partial \Pi_0}{\partial z} + \mu_{210}\frac{\partial \ln N}{\partial \varpi} + \\ &+ \frac{\mu_{120}}{\varpi}\frac{\partial \ln N}{\partial \theta} + \mu_{111}\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{210}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{120}}{\partial \theta} + \frac{\partial \mu_{111}}{\partial z} + \frac{2\mu_{210}-\mu_{030}}{\varpi} = 0 \end{aligned}$$

$$\begin{aligned} &\frac{\partial \mu_{101}}{\partial t} + \Pi_0 \frac{\partial \mu_{101}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{101}}{\partial \theta} + Z_0 \frac{\partial \mu_{101}}{\partial z} + \mu_{200}\frac{\partial Z_0}{\partial \varpi} + \frac{\mu_{110}}{\varpi}\frac{\partial Z_0}{\partial \theta} + \\ &+ \mu_{101}\left[\frac{\partial \Pi_0}{\partial \varpi} + \frac{\partial Z_0}{\partial z}\right] + \mu_{011}\left[\frac{1}{\varpi}\frac{\partial \Pi_0}{\partial \theta} - \frac{2\Theta_0}{\varpi}\right] + \mu_{002}\frac{\partial \Pi_0}{\partial z} + \mu_{201}\frac{\partial \ln N}{\partial \varpi} + \\ &+ \frac{\mu_{111}}{\varpi}\frac{\partial \ln N}{\partial \theta} + \mu_{102}\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{201}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{111}}{\partial \theta} + \frac{\partial \mu_{102}}{\partial z} + \frac{\mu_{201}-\mu_{021}}{\varpi} = 0 \end{aligned}$$

$$\begin{aligned} &\frac{\partial \mu_{020}}{\partial t} + \Pi_0 \frac{\partial \mu_{020}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{020}}{\partial \theta} + Z_0 \frac{\partial \mu_{020}}{\partial z} + 2\mu_{110}\left[\frac{\partial \Theta_0}{\partial \varpi} + \frac{\Theta_0}{\varpi}\right] + 2\mu_{020}\left[\frac{1}{\varpi}\frac{\partial \Theta_0}{\partial \theta} + \right. \\ &\left. + \frac{\Pi_0}{\varpi}\right] + 2\mu_{011}\frac{\partial \Theta_0}{\partial z} + \mu_{120}\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{030}}{\varpi}\frac{\partial \ln N}{\partial \theta} + \mu_{021}\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{120}}{\partial \varpi} + \\ &+ \frac{1}{\varpi}\frac{\partial \mu_{030}}{\partial \theta} + \frac{\partial \mu_{021}}{\partial z} + \frac{3\mu_{120}}{\varpi} = 0 \end{aligned}$$

$$\begin{aligned} &\frac{\partial \mu_{011}}{\partial t} + \Pi_0 \frac{\partial \mu_{011}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{011}}{\partial \theta} + Z_0 \frac{\partial \mu_{011}}{\partial z} + \mu_{110}\frac{\partial Z_0}{\partial \varpi} + \mu_{101}\left[\frac{\partial \Theta_0}{\partial \varpi} + \frac{\Theta_0}{\varpi}\right] + \\ &+ \frac{\mu_{020}}{\varpi}\frac{\partial Z_0}{\partial \theta} + \mu_{011}\left[\frac{1}{\varpi}\frac{\partial \Theta_0}{\partial \theta} + \frac{\partial Z_0}{\partial z} + \frac{\Pi_0}{\varpi}\right] + \mu_{002}\frac{\partial \Theta_0}{\partial z} + \mu_{111}\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{021}}{\varpi}\frac{\partial \ln N}{\partial \theta} + \\ &+ \mu_{012}\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{111}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{021}}{\partial \theta} + \frac{\partial \mu_{012}}{\partial z} + \frac{2\mu_{111}}{\varpi} = 0 \end{aligned}$$

APPENDIX B (*continued*).

$$\frac{\partial \mu_{002}}{\partial t} + \Pi_0\frac{\partial \mu_{002}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{002}}{\partial \theta} + Z_0\frac{\partial \mu_{002}}{\partial z} + 2\mu_{101}\frac{\partial Z_0}{\partial \varpi} + \frac{2\mu_{011}}{\varpi}\frac{\partial Z_0}{\partial \theta} + 2\mu_{002}\frac{\partial Z_0}{\partial z} +$$

$$+ \mu_{102}\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{012}}{\varpi}\frac{\partial \ln N}{\partial \theta} + \mu_{003}\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{102}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{012}}{\partial \theta} + \frac{\partial \mu_{003}}{\partial z} + \frac{\mu_{102}}{\varpi} = 0$$

$$\frac{\partial \mu_{300}}{\partial t} + \Pi_0\frac{\partial \mu_{300}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{300}}{\partial \theta} + Z_0\frac{\partial \mu_{300}}{\partial z} - 3\mu_{200}\left[\frac{\partial \mu_{200}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{110}}{\partial \theta} + \frac{\partial \mu_{101}}{\partial z} +\right.$$

$$\left.+ \frac{\mu_{200}-\mu_{020}}{\varpi}\right] + 3\mu_{300}\frac{\partial \Pi_0}{\partial \varpi} + 3\mu_{210}\left[\frac{1}{\varpi}\frac{\partial \Pi_0}{\partial \theta} - \frac{2\Theta_0}{\varpi}\right] + 3\mu_{201}\frac{\partial \Pi_0}{\partial z} +$$

$$+ [\mu_{400}-3\mu^2_{200}]\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{310}-3\mu_{200}\mu_{110}}{\varpi}\frac{\partial \ln N}{\partial \theta} + [\mu_{301}-3\mu_{200}\mu_{101}]\frac{\partial \ln N}{\partial z} +$$

$$+ \frac{\partial \mu_{400}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{310}}{\partial \theta} + \frac{\partial \mu_{301}}{\partial z} + \frac{\mu_{400}-3\mu_{220}}{\varpi} = 0$$

$$\frac{\partial \mu_{210}}{\partial t} + \Pi_0\frac{\partial \mu_{210}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{210}}{\partial \theta} + Z_0\frac{\partial \mu_{210}}{\partial z} - \mu_{200}\left[\frac{\partial \mu_{110}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{020}}{\partial \theta} + \frac{\partial \mu_{011}}{\partial z} +\right.$$

$$\left.+ \frac{2\mu_{110}}{\varpi}\right] - 2\mu_{110}\left[\frac{\partial \mu_{200}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{110}}{\partial \theta} + \frac{\partial \mu_{101}}{\partial z} + \frac{\mu_{200}-\mu_{020}}{\varpi}\right] +$$

$$+ \mu_{300}\left[\frac{\partial \Theta_0}{\partial \varpi} + \frac{\Theta_0}{\varpi}\right] + \mu_{210}\left[2\frac{\partial \Pi_0}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \Theta_0}{\partial \theta} + \frac{\Pi_0}{\varpi}\right] + \mu_{201}\frac{\partial \Theta_0}{\partial z} +$$

$$+ 2\mu_{120}\left[\frac{1}{\varpi}\frac{\partial \Pi_0}{\partial \theta} - \frac{2\Theta_0}{\varpi}\right] + 2\mu_{111}\frac{\partial \Pi_0}{\partial z} + [\mu_{310}-3\mu_{200}\mu_{110}]\frac{\partial \ln N}{\partial \varpi} +$$

$$+ \frac{\mu_{220}-\mu_{200}\mu_{020}-2\mu^2_{110}}{\varpi}\frac{\partial \ln N}{\partial \theta} + [\mu_{211}-\mu_{200}\mu_{011}-2\mu_{110}\mu_{101}]\frac{\partial \ln N}{\partial z} +$$

$$+ \frac{\partial \mu_{310}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{220}}{\partial \theta} + \frac{\partial \mu_{211}}{\partial z} + \frac{2(\mu_{310}-\mu_{130})}{\varpi} = 0$$

$$\frac{\partial \mu_{201}}{\partial t} + \Pi_0\frac{\partial \mu_{201}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{201}}{\partial \theta} + Z_0\frac{\partial \mu_{201}}{\partial z} - \mu_{200}\left[\frac{\partial \mu_{101}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{011}}{\partial \theta} + \frac{\partial \mu_{002}}{\partial z} +\right.$$

$$\left.+ \frac{\mu_{101}}{\varpi}\right] - 2\mu_{101}\left[\frac{\partial \mu_{200}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{110}}{\partial \theta} + \frac{\partial \mu_{101}}{\partial z} + \frac{\mu_{200}-\mu_{020}}{\varpi}\right] + \mu_{300}\frac{\partial Z_0}{\partial \varpi} +$$

$$+ \frac{\mu_{210}}{\varpi}\frac{\partial Z_0}{\partial \theta} + \mu_{201}\left[2\frac{\partial \Pi_0}{\partial \varpi} + \frac{\partial Z_0}{\partial z}\right] + 2\mu_{111}\left[\frac{1}{\varpi}\frac{\partial \Pi_0}{\partial \theta} - \frac{2\Theta_0}{\varpi}\right] + 2\mu_{102}\frac{\partial \Pi_0}{\partial z} +$$

$$+ [\mu_{301}-3\mu_{200}\mu_{101}]\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{211}-\mu_{200}\mu_{011}-2\mu_{110}\mu_{101}}{\varpi}\frac{\partial \ln N}{\partial \theta} +$$

$$+ [\mu_{202}-\mu_{200}\mu_{002}-2\mu^2_{101}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{301}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{211}}{\partial \theta} + \frac{\partial \mu_{202}}{\partial z} +$$

$$+ \frac{\mu_{301}-2\mu_{121}}{\varpi} = 0$$

$$\frac{\partial \mu_{120}}{\partial t} + \Pi_0\frac{\partial \mu_{120}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{120}}{\partial \theta} + Z_0\frac{\partial \mu_{120}}{\partial z} - 2\mu_{110}\left[\frac{\partial \mu_{110}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{020}}{\partial \theta} + \frac{\partial \mu_{011}}{\partial z} +\right.$$

$$\left.+ \frac{2\mu_{110}}{\varpi}\right] - \mu_{020}\left[\frac{\partial \mu_{200}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{110}}{\partial \theta} + \frac{\partial \mu_{101}}{\partial z} + \frac{\mu_{200}-\mu_{020}}{\varpi}\right] + 2\mu_{210}\left[\frac{\partial \Theta_0}{\partial \varpi} + \frac{\Theta_0}{\varpi}\right] +$$

$$+ \mu_{120}\left[\frac{\partial \Pi_0}{\partial \varpi} + \frac{2}{\varpi}\frac{\partial \Theta_0}{\partial \theta} + \frac{2\Pi_0}{\varpi}\right] + 2\mu_{111}\frac{\partial \Theta_0}{\partial z} + \mu_{030}\left[\frac{1}{\varpi}\frac{\partial \Pi_0}{\partial \theta} - \frac{2\Theta_0}{\varpi}\right] + \mu_{021}\frac{\partial \Pi_0}{\partial z} +$$

$$+ [\mu_{220}-\mu_{200}\mu_{020}-2\mu^2_{110}]\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{130}-3\mu_{110}\mu_{020}}{\varpi}\frac{\partial \ln N}{\partial \theta} +$$

$$+ [\mu_{121}-2\mu_{110}\mu_{011}-\mu_{101}\mu_{020}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{220}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{130}}{\partial \theta} + \frac{\partial \mu_{121}}{\partial z} +$$

$$+ \frac{3\mu_{220}-\mu_{040}}{\varpi} = 0$$

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APPENDIX B (*continued*).

$$
\begin{aligned}
&\frac{\partial \mu_{111}}{\partial t} + \Pi_0 \frac{\partial \mu_{111}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{111}}{\partial \theta} + Z_0 \frac{\partial \mu_{111}}{\partial z} - \mu_{110}\Big[\frac{\partial \mu_{101}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{011}}{\partial \theta} + \frac{\partial \mu_{002}}{\partial z} + \\
&+ \frac{\mu_{101}}{\varpi}\Big] - \mu_{101}\Big[\frac{\partial \mu_{110}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{020}}{\partial \theta} + \frac{\partial \mu_{011}}{\partial z} + \frac{2\mu_{110}}{\varpi}\Big] - \mu_{011}\Big[\frac{\partial \mu_{200}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{110}}{\partial \theta} + \\
&+ \frac{\partial \mu_{101}}{\partial z} + \frac{\mu_{200}-\mu_{020}}{\varpi}\Big] + \mu_{210}\frac{\partial Z_0}{\partial \varpi} + \mu_{201}\Big[\frac{\partial \Theta_0}{\partial \varpi} + \frac{\Theta_0}{\varpi}\Big] + \frac{\mu_{120}}{\varpi}\frac{\partial Z_0}{\partial \theta} + \\
&+ \mu_{111}\Big[\frac{\partial \Pi_0}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \Theta_0}{\partial \theta} + \frac{\partial Z_0}{\partial z} + \frac{\Pi_0}{\varpi}\Big] + \mu_{102}\frac{\partial \Theta_0}{\partial z} + \mu_{021}\Big[\frac{1}{\varpi}\frac{\partial \Pi_0}{\partial \theta} - \frac{2\Theta_0}{\varpi}\Big] + \mu_{012}\frac{\partial \Pi_0}{\partial z} + \\
&+ [\mu_{211}-\mu_{200}\mu_{011}-2\mu_{110}\mu_{101}]\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{121}-2\mu_{110}\mu_{011}-\mu_{101}\mu_{020}}{\varpi}\frac{\partial \ln N}{\partial \theta} + \\
&+ [\mu_{112}-\mu_{110}\mu_{002}-2\mu_{101}\mu_{011}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{211}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{121}}{\partial \theta} + \frac{\partial \mu_{112}}{\partial z} + \frac{2\mu_{211}-\mu_{031}}{\varpi} = 0
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial \mu_{102}}{\partial t} + \Pi_0 \frac{\partial \mu_{102}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{102}}{\partial \theta} + Z_0 \frac{\partial \mu_{102}}{\partial z} - 2\mu_{101}\Big[\frac{\partial \mu_{101}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{011}}{\partial \theta} + \frac{\partial \mu_{002}}{\partial z} + \\
&+ \frac{\mu_{101}}{\varpi}\Big] - \mu_{002}\Big[\frac{\partial \mu_{200}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{110}}{\partial \theta} + \frac{\partial \mu_{101}}{\partial z} + \frac{\mu_{200}-\mu_{020}}{\varpi}\Big] + 2\mu_{201}\frac{\partial Z_0}{\partial \varpi} + \\
&+ \frac{2\mu_{111}}{\varpi}\frac{\partial Z_0}{\partial \theta} + \mu_{102}\Big[\frac{\partial \Pi_0}{\partial \varpi} + 2\frac{\partial Z_0}{\partial z}\Big] + \mu_{012}\Big[\frac{1}{\varpi}\frac{\partial \Pi_0}{\partial \theta} - \frac{2\Theta_0}{\varpi}\Big] + \mu_{003}\frac{\partial \Pi_0}{\partial z} + \\
&+ [\mu_{202}-\mu_{200}\mu_{002}-2\mu^2_{101}]\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{112}-\mu_{110}\mu_{002}-2\mu_{101}\mu_{011}}{\varpi}\frac{\partial \ln N}{\partial \theta} + \\
&+ [\mu_{103}-3\mu_{101}\mu_{002}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{202}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{112}}{\partial \theta} + \frac{\partial \mu_{103}}{\partial z} + \\
&+ \frac{\mu_{202}-\mu_{022}}{\varpi} = 0
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial \mu_{030}}{\partial t} + \Pi_0 \frac{\partial \mu_{030}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{030}}{\partial \theta} + Z_0 \frac{\partial \mu_{030}}{\partial z} - 3\mu_{020}\Big[\frac{\partial \mu_{110}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{020}}{\partial \theta} + \frac{\partial \mu_{011}}{\partial z} + \\
&+ \frac{2\mu_{110}}{\varpi}\Big] + 3\mu_{120}\Big[\frac{\partial \Theta_0}{\partial \varpi} + \frac{\Theta_0}{\varpi}\Big] + 3\mu_{030}\Big[\frac{1}{\varpi}\frac{\partial \Theta_0}{\partial \theta} + \frac{\Pi_0}{\varpi}\Big] + 3\mu_{021}\frac{\partial \Theta_0}{\partial z} + \\
&+ [\mu_{130}-3\mu_{110}\mu_{020}]\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{040}-3\mu^2_{020}}{\varpi}\frac{\partial \ln N}{\partial \theta} + [\mu_{031}-3\mu_{020}\mu_{011}]\frac{\partial \ln N}{\partial z} + \\
&+ \frac{\partial \mu_{130}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{040}}{\partial \theta} + \frac{\partial \mu_{031}}{\partial z} + \frac{4\mu_{130}}{\varpi} = 0
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial \mu_{021}}{\partial t} + \Pi_0 \frac{\partial \mu_{021}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{021}}{\partial \theta} + Z_0 \frac{\partial \mu_{021}}{\partial z} - \mu_{020}\Big[\frac{\partial \mu_{101}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{011}}{\partial \theta} + \frac{\partial \mu_{002}}{\partial z} + \\
&+ \frac{\mu_{101}}{\varpi}\Big] - 2\mu_{011}\Big[\frac{\partial \mu_{110}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{020}}{\partial \theta} + \frac{\partial \mu_{011}}{\partial z} + \frac{2\mu_{110}}{\varpi}\Big] + \mu_{120}\frac{\partial Z_0}{\partial \varpi} + \\
&+ 2\mu_{111}\Big[\frac{\partial \Theta_0}{\partial \varpi} + \frac{\Theta_0}{\varpi}\Big] + \frac{\mu_{030}}{\varpi}\frac{\partial Z_0}{\partial \theta} + \mu_{021}\Big[\frac{2}{\varpi}\frac{\partial \Theta_0}{\partial \theta} + \frac{\partial Z_0}{\partial z} + \frac{2\Pi_0}{\varpi}\Big] + 2\mu_{012}\frac{\partial \Theta_0}{\partial z} + \\
&+ [\mu_{121}-2\mu_{110}\mu_{011}-\mu_{101}\mu_{020}]\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{031}-3\mu_{020}\mu_{011}}{\varpi}\frac{\partial \ln N}{\partial \theta} + \\
&+ [\mu_{022}-\mu_{020}\mu_{002}-2\mu^2_{011}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{121}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{031}}{\partial \theta} + \frac{\partial \mu_{022}}{\partial z} + \frac{3\mu_{121}}{\varpi} = 0
\end{aligned}
$$

APPENDIX B (*continued*).

$$\begin{aligned}
&\frac{\partial \mu_{012}}{\partial t} + \Pi_0\frac{\partial \mu_{012}}{\partial \varpi} + \frac{\theta_0}{\varpi}\frac{\partial \mu_{012}}{\partial \theta} + Z_0\frac{\partial \mu_{012}}{\partial z} - 2\mu_{011}[\frac{\partial \mu_{101}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{011}}{\partial \theta} + \frac{\partial \mu_{002}}{\partial z} +\\
&+ \frac{\mu_{101}}{\varpi}] - \mu_{002}[\frac{\partial \mu_{110}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{020}}{\partial \theta} + \frac{\partial \mu_{011}}{\partial z} + \frac{2\mu_{110}}{\varpi}] + 2\mu_{111}\frac{\partial Z_0}{\partial \varpi} +\\
&+ \mu_{102}[\frac{\partial \theta_0}{\partial \varpi} + \frac{\theta_0}{\varpi}] + \frac{2\mu_{021}}{\varpi}\frac{\partial Z_0}{\partial \theta} + \mu_{012}[\frac{1}{\varpi}\frac{\partial \theta_0}{\partial \theta} + 2\frac{\partial Z_0}{\partial z} + \frac{\Pi_0}{\varpi}] + \mu_{003}\frac{\partial \theta_0}{\partial z} +\\
&+ [\mu_{112}-\mu_{110}\mu_{002}-2\mu_{101}\mu_{011}]\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{022}-\mu_{020}\mu_{002}-2\mu^2_{011}}{\varpi}\frac{\partial \ln N}{\partial \theta} +\\
&+ [\mu_{013}-3\mu_{011}\mu_{002}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{112}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{022}}{\partial \theta} + \frac{\partial \mu_{013}}{\partial z} + \frac{2\mu_{112}}{\varpi} = 0
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial \mu_{003}}{\partial t} + \Pi_0\frac{\partial \mu_{003}}{\partial \varpi} + \frac{\theta_0}{\varpi}\frac{\partial \mu_{003}}{\partial \theta} + Z_0\frac{\partial \mu_{003}}{\partial z} - 3\mu_{002}[\frac{\partial \mu_{101}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{011}}{\partial \theta} + \frac{\partial \mu_{002}}{\partial z} +\\
&+ \frac{\mu_{101}}{\varpi}] + 3\mu_{102}\frac{\partial Z_0}{\partial \varpi} + \frac{3\mu_{012}}{\varpi}\frac{\partial Z_0}{\partial \theta} + 3\mu_{003}\frac{\partial Z_0}{\partial z} + [\mu_{103}-3\mu_{101}\mu_{002}]\frac{\partial \ln N}{\partial \varpi} +\\
&+ \frac{\mu_{013}-3\mu_{011}\mu_{002}}{\varpi}\frac{\partial \ln N}{\partial \theta} + [\mu_{004}-3\mu^2_{002}]\frac{\partial \ln N}{\partial z} +\\
&+ \frac{\partial \mu_{103}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{013}}{\partial \theta} + \frac{\partial \mu_{004}}{\partial z} + \frac{\mu_{103}}{\varpi} = 0
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial \mu_{400}}{\partial t} + \Pi_0\frac{\partial \mu_{400}}{\partial \varpi} + \frac{\theta_0}{\varpi}\frac{\partial \mu_{400}}{\partial \theta} + Z_0\frac{\partial \mu_{400}}{\partial z} - 4\mu_{300}[\frac{\partial \mu_{200}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{110}}{\partial \theta} + \frac{\partial \mu_{101}}{\partial z} +\\
&+ \frac{\mu_{200}-\mu_{020}}{\varpi}] + 4\mu_{400}\frac{\partial \Pi_0}{\partial \varpi} + 4\mu_{310}[\frac{1}{\varpi}\frac{\partial \Pi_0}{\partial \theta} - \frac{2\theta_0}{\varpi}] + 4\mu_{301}\frac{\partial \Pi_0}{\partial z} +\\
&+ [\mu_{500}-4\mu_{300}\mu_{200}]\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{410}-4\mu_{300}\mu_{110}}{\varpi}\frac{\partial \ln N}{\partial \theta} + [\mu_{401}-4\mu_{300}\mu_{101}]\frac{\partial \ln N}{\partial z} +\\
&+ \frac{\partial \mu_{500}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{410}}{\partial \theta} + \frac{\partial \mu_{401}}{\partial z} + \frac{\mu_{500}-4\mu_{320}}{\varpi} = 0
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial \mu_{310}}{\partial t} + \Pi_0\frac{\partial \mu_{310}}{\partial \varpi} + \frac{\theta_0}{\varpi}\frac{\partial \mu_{310}}{\partial \theta} + Z_0\frac{\partial \mu_{310}}{\partial z} - \mu_{300}[\frac{\partial \mu_{110}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{020}}{\partial \theta} + \frac{\partial \mu_{011}}{\partial z} +\\
&+ \frac{2\mu_{110}}{\varpi}] - 3\mu_{210}[\frac{\partial \mu_{200}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{110}}{\partial \theta} + \frac{\partial \mu_{101}}{\partial z} + \frac{\mu_{200}-\mu_{020}}{\varpi}] + \mu_{400}[\frac{\partial \theta_0}{\partial \varpi} + \frac{\theta_0}{\varpi}] +\\
&+ \mu_{310}[3\frac{\partial \Pi_0}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \theta_0}{\partial \theta} + \frac{\Pi_0}{\varpi}] + \mu_{301}\frac{\partial \theta_0}{\partial z} + 3\mu_{220}[\frac{1}{\varpi}\frac{\partial \Pi_0}{\partial \theta} - \frac{2\theta_0}{\varpi}] + 3\mu_{211}\frac{\partial \Pi_0}{\partial z} +\\
&+ [\mu_{410}-\mu_{300}\mu_{110}-3\mu_{210}\mu_{200}]\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{320}-\mu_{300}\mu_{020}-3\mu_{210}\mu_{110}}{\varpi}\frac{\partial \ln N}{\partial \theta} +\\
&+ [\mu_{311}-\mu_{300}\mu_{011}-3\mu_{210}\mu_{101}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{410}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{320}}{\partial \theta} + \frac{\partial \mu_{311}}{\partial z} +\\
&+ \frac{2\mu_{410}-3\mu_{230}}{\varpi} = 0
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial \mu_{301}}{\partial t} + \Pi_0\frac{\partial \mu_{301}}{\partial \varpi} + \frac{\theta_0}{\varpi}\frac{\partial \mu_{301}}{\partial \theta} + Z_0\frac{\partial \mu_{301}}{\partial z} - \mu_{300}[\frac{\partial \mu_{101}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{011}}{\partial \theta} + \frac{\partial \mu_{002}}{\partial z} +\\
&+ \frac{\mu_{101}}{\varpi}] - 3\mu_{201}[\frac{\partial \mu_{200}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{110}}{\partial \theta} + \frac{\partial \mu_{101}}{\partial z} + \frac{\mu_{200}-\mu_{020}}{\varpi}] + \mu_{400}\frac{\partial Z_0}{\partial \varpi} +\\
&+ \frac{\mu_{310}}{\varpi}\frac{\partial Z_0}{\partial \theta} + \mu_{301}[3\frac{\partial \Pi_0}{\partial \varpi} + \frac{\partial Z_0}{\partial z}] + 3\mu_{211}[\frac{1}{\varpi}\frac{\partial \Pi_0}{\partial \theta} - \frac{2\theta_0}{\varpi}] + 3\mu_{202}\frac{\partial \Pi_0}{\partial z} +\\
&+ [\mu_{401}-\mu_{300}\mu_{101}-3\mu_{201}\mu_{200}]\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{311}-\mu_{300}\mu_{011}-3\mu_{201}\mu_{110}}{\varpi}\frac{\partial \ln N}{\partial \theta} +\\
&+ [\mu_{302}-\mu_{300}\mu_{002}-3\mu_{201}\mu_{101}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{401}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{311}}{\partial \theta} + \frac{\partial \mu_{302}}{\partial z} +\\
&+ \frac{\mu_{401}-3\mu_{221}}{\varpi} = 0
\end{aligned}$$

APPENDIX B (*continued*).

$$\begin{aligned}
&\frac{\partial \mu_{220}}{\partial t} + \Pi_0\frac{\partial \mu_{220}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{220}}{\partial \theta} + Z_0\frac{\partial \mu_{220}}{\partial z} - 2\mu_{210}\Big[\frac{\partial \mu_{110}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{020}}{\partial \theta} + \frac{\partial \mu_{011}}{\partial z} +\\
&+ \frac{2\mu_{110}}{\varpi}\Big] - 2\mu_{120}\Big[\frac{\partial \mu_{200}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{110}}{\partial \theta} + \frac{\partial \mu_{101}}{\partial z} + \frac{\mu_{200}-\mu_{020}}{\varpi}\Big] + 2\mu_{310}\Big[\frac{\partial \Theta_0}{\partial \varpi} + \frac{\Theta_0}{\varpi}\Big] +\\
&+ 2\mu_{220}\Big[\frac{\partial \Pi_0}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \Theta_0}{\partial \theta} + \frac{\Pi_0}{\varpi}\Big] + 3\mu_{211}\frac{\partial \Theta_0}{\partial z} + 2\mu_{130}\Big[-\frac{1}{\varpi}\frac{\partial \Pi_0}{\partial \theta} - \frac{2\Theta_0}{\varpi}\Big] +\\
&+ 2\mu_{121}\frac{\partial \Pi_0}{\partial z} + [\mu_{320}-2\mu_{210}\mu_{110}-2\mu_{120}\mu_{200}]\frac{\partial \ln N}{\partial \varpi} +\\
&+ \frac{\mu_{230}-2\mu_{210}\mu_{020}-2\mu_{120}\mu_{110}}{\varpi}\frac{\partial \ln N}{\partial \theta} + [\mu_{221}-2\mu_{210}\mu_{011}-2\mu_{120}\mu_{101}]\frac{\partial \ln N}{\partial z} +\\
&+ \frac{\partial \mu_{320}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{230}}{\partial \theta} + \frac{\partial \mu_{221}}{\partial z} + \frac{3\mu_{320}-2\mu_{140}}{\varpi} = 0
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial \mu_{211}}{\partial t} + \Pi_0\frac{\partial \mu_{211}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{211}}{\partial \theta} + Z_0\frac{\partial \mu_{211}}{\partial z} - \mu_{210}\Big[\frac{\partial \mu_{101}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{011}}{\partial \theta} + \frac{\partial \mu_{002}}{\partial z} +\\
&+ \frac{\mu_{101}}{\varpi}\Big] - \mu_{201}\Big[\frac{\partial \mu_{110}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{020}}{\partial \theta} + \frac{\partial \mu_{011}}{\partial z} + \frac{2\mu_{110}}{\varpi}\Big] - 2\mu_{111}\Big[\frac{\partial \mu_{200}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{110}}{\partial \theta} +\\
&+ \frac{\partial \mu_{101}}{\partial z} + \frac{\mu_{200}-\mu_{020}}{\varpi}\Big] + \mu_{310}\frac{\partial Z_0}{\partial \varpi} + \mu_{301}\Big[\frac{\partial \Theta_0}{\partial \varpi} + \frac{\Theta_0}{\varpi}\Big] + \frac{\mu_{220}}{\varpi}\frac{\partial Z_0}{\partial \theta} +\\
&+ \mu_{211}\Big[2\frac{\partial \Pi_0}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \Theta_0}{\partial \theta} + \frac{\partial Z_0}{\partial z} + \frac{\Pi_0}{\varpi}\Big] + \mu_{202}\frac{\partial \Theta_0}{\partial z} +\\
&+ 2\mu_{121}\Big[-\frac{1}{\varpi}\frac{\partial \Pi_0}{\partial \theta} - \frac{2\Theta_0}{\varpi}\Big] + 2\mu_{112}\frac{\partial \Pi_0}{\partial z} + [\mu_{311}-\mu_{210}\mu_{101}-\mu_{201}\mu_{110}-2\mu_{111}\mu_{200}]\frac{\partial \ln N}{\partial \varpi} +\\
&+ \frac{\mu_{221}-\mu_{210}\mu_{011}-\mu_{201}\mu_{020}-2\mu_{111}\mu_{110}}{\varpi}\frac{\partial \ln N}{\partial \theta} +\\
&+ [\mu_{212}-\mu_{210}\mu_{002}-\mu_{201}\mu_{011}-2\mu_{111}\mu_{101}]\frac{\partial \ln N}{\partial z} +\\
&+ \frac{\partial \mu_{311}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{221}}{\partial \theta} + \frac{\partial \mu_{212}}{\partial z} + \frac{2(\mu_{311}-\mu_{131})}{\varpi} = 0
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial \mu_{202}}{\partial t} + \Pi_0\frac{\partial \mu_{202}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{202}}{\partial \theta} + Z_0\frac{\partial \mu_{202}}{\partial z} - 2\mu_{201}\Big[\frac{\partial \mu_{101}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{011}}{\partial \theta} + \frac{\partial \mu_{002}}{\partial z} +\\
&+ \frac{\mu_{101}}{\varpi}\Big] - 2\mu_{102}\Big[\frac{\partial \mu_{200}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{110}}{\partial \theta} + \frac{\partial \mu_{101}}{\partial z} + \frac{\mu_{200}-\mu_{020}}{\varpi}\Big] + 2\mu_{301}\frac{\partial Z_0}{\partial \varpi} +\\
&+ \frac{2\mu_{211}}{\varpi}\frac{\partial Z_0}{\partial \theta} + 2\mu_{202}\Big[\frac{\partial \Pi_0}{\partial \varpi} + \frac{\partial Z_0}{\partial z}\Big] + 2\mu_{112}\Big[-\frac{1}{\varpi}\frac{\partial \Pi_0}{\partial \theta} - \frac{2\Theta_0}{\varpi}\Big] + 2\mu_{103}\frac{\partial \Pi_0}{\partial z} +\\
&+ [\mu_{302}-2\mu_{201}\mu_{101}-2\mu_{102}\mu_{200}]\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{212}-2\mu_{201}\mu_{011}-2\mu_{102}\mu_{110}}{\varpi}\frac{\partial \ln N}{\partial \theta} +\\
&+ [\mu_{203}-2\mu_{201}\mu_{002}-2\mu_{102}\mu_{101}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{302}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{212}}{\partial \theta} + \frac{\partial \mu_{203}}{\partial z} +\\
&+ \frac{\mu_{302}-2\mu_{122}}{\varpi} = 0
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial \mu_{130}}{\partial t} + \Pi_0\frac{\partial \mu_{130}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{130}}{\partial \theta} + Z_0\frac{\partial \mu_{130}}{\partial z} - 3\mu_{120}\Big[\frac{\partial \mu_{110}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{020}}{\partial \theta} + \frac{\partial \mu_{011}}{\partial z} +\\
&+ \frac{2\mu_{110}}{\varpi}\Big] - \mu_{030}\Big[\frac{\partial \mu_{200}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{110}}{\partial \theta} + \frac{\partial \mu_{101}}{\partial z} + \frac{\mu_{200}-\mu_{020}}{\varpi}\Big] + 3\mu_{220}\Big[\frac{\partial \Theta_0}{\partial \varpi} + \frac{\Theta_0}{\varpi}\Big] +\\
&+ \mu_{130}\Big[\frac{\partial \Pi_0}{\partial \varpi} + \frac{3}{\varpi}\frac{\partial \Theta_0}{\partial \theta} + \frac{3\Pi_0}{\varpi}\Big] + 3\mu_{121}\frac{\partial \Theta_0}{\partial z} + \mu_{040}\Big[-\frac{1}{\varpi}\frac{\partial \Pi_0}{\partial \theta} - \frac{2\Theta_0}{\varpi}\Big] + \mu_{031}\frac{\partial \Pi_0}{\partial z} +\\
&+ [\mu_{230}-3\mu_{120}\mu_{110}-\mu_{030}\mu_{200}]\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{140}-3\mu_{120}\mu_{020}-\mu_{030}\mu_{110}}{\varpi}\frac{\partial \ln N}{\partial \theta} +\\
&+ [\mu_{131}-3\mu_{120}\mu_{011}-\mu_{030}\mu_{101}]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{230}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{140}}{\partial \theta} + \frac{\partial \mu_{131}}{\partial z} +\\
&+ \frac{4\mu_{230}-\mu_{050}}{\varpi} = 0
\end{aligned}$$

APPENDIX B (*continued*).

$$
\begin{aligned}
&\frac{\partial \mu_{121}}{\partial t} + \Pi_0 \frac{\partial \mu_{121}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{121}}{\partial \theta} + Z_0 \frac{\partial \mu_{121}}{\partial z} - \mu_{120}\Big[\frac{\partial \mu_{101}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{011}}{\partial \theta} + \frac{\partial \mu_{002}}{\partial z} + \\
&+ \frac{\mu_{101}}{\varpi}\Big] - 2\mu_{111}\Big[\frac{\partial \mu_{110}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{020}}{\partial \theta} + \frac{\partial \mu_{011}}{\partial z} + \frac{2\mu_{110}}{\varpi}\Big] - \mu_{021}\Big[\frac{\partial \mu_{200}}{\partial \varpi} + \\
&+ \frac{1}{\varpi}\frac{\partial \mu_{110}}{\partial \theta} + \frac{\partial \mu_{101}}{\partial z} + \frac{\mu_{200}-\mu_{020}}{\varpi}\Big] + \mu_{220}\frac{\partial Z_0}{\partial \varpi} + 2\mu_{211}\Big[\frac{\partial \Theta_0}{\partial \varpi} + \frac{\Theta_0}{\varpi}\Big] + \\
&+ \frac{\mu_{130}}{\varpi}\frac{\partial Z_0}{\partial \theta} + \mu_{121}\Big[\frac{\partial \Pi_0}{\partial \varpi} + 2\frac{\partial \Theta_0}{\partial \theta} + \frac{\partial Z_0}{\partial z} + \frac{2\Pi_0}{\varpi}\Big] + 2\mu_{112}\frac{\partial \Theta_0}{\partial z} + \\
&+ \mu_{031}\Big[\frac{1}{\varpi}\frac{\partial \Pi_0}{\partial \theta} - \frac{2\Theta_0}{\varpi}\Big] + \mu_{022}\frac{\partial \Pi_0}{\partial z} + \\
&+ [\mu_{221}-\mu_{120}\mu_{101}-2\mu_{111}\mu_{110}-\mu_{021}\mu_{200}]\frac{\partial \ln N}{\partial \varpi} + \\
&+ \frac{\mu_{131}-\mu_{120}\mu_{011}-2\mu_{111}\mu_{020}-\mu_{021}\mu_{110}}{\varpi}\frac{\partial \ln N}{\partial \theta} + \\
&+ [\mu_{122}-\mu_{120}\mu_{002}-2\mu_{111}\mu_{011}-\mu_{021}\mu_{101}]\frac{\partial \ln N}{\partial z} + \\
&+ \frac{\partial \mu_{221}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{131}}{\partial \theta} + \frac{\partial \mu_{122}}{\partial z} + \frac{3\mu_{221}-\mu_{041}}{\varpi} = 0
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial \mu_{112}}{\partial t} + \Pi_0 \frac{\partial \mu_{112}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{112}}{\partial \theta} + Z_0 \frac{\partial \mu_{112}}{\partial z} - 2\mu_{111}\Big[\frac{\partial \mu_{101}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{011}}{\partial \theta} + \frac{\partial \mu_{002}}{\partial z} + \\
&+ \frac{\mu_{101}}{\varpi}\Big] - \mu_{102}\Big[\frac{\partial \mu_{110}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{020}}{\partial \theta} + \frac{\partial \mu_{011}}{\partial z} + \frac{2\mu_{110}}{\varpi}\Big] - \mu_{012}\Big[\frac{\partial \mu_{200}}{\partial \varpi} + \\
&+ \frac{1}{\varpi}\frac{\partial \mu_{110}}{\partial \theta} + \frac{\partial \mu_{101}}{\partial z} + \frac{\mu_{200}-\mu_{020}}{\varpi}\Big] + 2\mu_{211}\frac{\partial Z_0}{\partial \varpi} + \mu_{202}\Big[\frac{\partial \Theta_0}{\partial \varpi} + \frac{\Theta_0}{\varpi}\Big] + \\
&+ \frac{2\mu_{121}}{\varpi}\frac{\partial Z_0}{\partial \theta} + \mu_{112}\Big[\frac{\partial \Pi_0}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \Theta_0}{\partial \theta} + 2\frac{\partial Z_0}{\partial z} + \frac{\Pi_0}{\varpi}\Big] + \mu_{103}\frac{\partial \Theta_0}{\partial z} + \\
&+ \mu_{022}\Big[\frac{1}{\varpi}\frac{\partial \Pi_0}{\partial \theta} - \frac{2\Theta_0}{\varpi}\Big] + \mu_{013}\frac{\partial \Pi_0}{\partial z} + \\
&+ [\mu_{212}-2\mu_{111}\mu_{101}-\mu_{102}\mu_{110}-\mu_{012}\mu_{200}]\frac{\partial \ln N}{\partial \varpi} + \\
&+ \frac{\mu_{122}-2\mu_{111}\mu_{011}-\mu_{102}\mu_{020}-\mu_{012}\mu_{110}}{\varpi}\frac{\partial \ln N}{\partial \theta} + \\
&+ [\mu_{113}-2\mu_{111}\mu_{002}-\mu_{102}\mu_{011}-\mu_{012}\mu_{101}]\frac{\partial \ln N}{\partial z} + \\
&+ \frac{\partial \mu_{212}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{122}}{\partial \theta} + \frac{\partial \mu_{113}}{\partial z} + \frac{2\mu_{212}-\mu_{032}}{\varpi} = 0
\end{aligned}
$$

APPENDIX B (*continued*).

$$\frac{\partial \mu_{103}}{\partial t} + \Pi_0\frac{\partial \mu_{103}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{103}}{\partial \theta} + Z_0\frac{\partial \mu_{103}}{\partial z} - 3\mu_{102}\left[\frac{\partial \mu_{101}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{011}}{\partial \theta} + \frac{\partial \mu_{002}}{\partial z} +\right.$$

$$\left. + \frac{\mu_{101}}{\varpi}\right] - \mu_{003}\left[\frac{\partial \mu_{200}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{110}}{\partial \theta} + \frac{\partial \mu_{101}}{\partial z} + \frac{\mu_{200}-\mu_{020}}{\varpi}\right] + 3\mu_{202}\frac{\partial Z_0}{\partial \varpi} +$$

$$+ \frac{3\mu_{112}}{\varpi}\frac{\partial Z_0}{\partial \theta} + \mu_{103}\left[\frac{\partial \Pi_0}{\partial \varpi} + 3\frac{\partial Z_0}{\partial z}\right] + \mu_{013}\left[\frac{1}{\varpi}\frac{\partial \Pi_0}{\partial \theta} - \frac{2\Theta_0}{\varpi}\right] + \mu_{004}\frac{\partial \Pi_0}{\partial z} +$$

$$+ \left[\mu_{203}-3\mu_{102}\mu_{101}-\mu_{003}\mu_{200}\right]\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{113}-3\mu_{102}\mu_{011}-\mu_{003}\mu_{110}}{\varpi}\frac{\partial \ln N}{\partial \theta} +$$

$$+ \left[\mu_{104}-3\mu_{102}\mu_{002}-\mu_{003}\mu_{101}\right]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{203}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{113}}{\partial \theta} + \frac{\partial \mu_{104}}{\partial z} +$$

$$+ \frac{\mu_{203}-\mu_{023}}{\varpi} = 0$$

$$\frac{\partial \mu_{040}}{\partial t} + \Pi_0\frac{\partial \mu_{040}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{040}}{\partial \theta} + Z_0\frac{\partial \mu_{040}}{\partial z} - 4\mu_{030}\left[\frac{\partial \mu_{110}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{020}}{\partial \theta} + \frac{\partial \mu_{011}}{\partial z} +\right.$$

$$\left. + \frac{2\mu_{110}}{\varpi}\right] + 4\mu_{130}\left[\frac{\partial \Theta_0}{\partial \varpi} + \frac{\Theta_0}{\varpi}\right] + 4\mu_{040}\left[\frac{1}{\varpi}\frac{\partial \Theta_0}{\partial \theta} + \frac{\Pi_0}{\varpi}\right] + 4\mu_{031}\frac{\partial \Theta_0}{\partial z} +$$

$$+ \left[\mu_{140}-4\mu_{030}\mu_{110}\right]\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{050}-4\mu_{030}\mu_{020}}{\varpi}\frac{\partial \ln N}{\partial \theta} + \left[\mu_{041}-4\mu_{030}\mu_{011}\right]\frac{\partial \ln N}{\partial z} +$$

$$+ \frac{\partial \mu_{140}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{050}}{\partial \theta} + \frac{\partial \mu_{041}}{\partial z} + \frac{5\mu_{140}}{\varpi} = 0$$

$$\frac{\partial \mu_{031}}{\partial t} + \Pi_0\frac{\partial \mu_{031}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{031}}{\partial \theta} + Z_0\frac{\partial \mu_{031}}{\partial z} - \mu_{030}\left[\frac{\partial \mu_{101}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{011}}{\partial \theta} + \frac{\partial \mu_{002}}{\partial z} +\right.$$

$$\left. + \frac{\mu_{101}}{\varpi}\right] - 3\mu_{021}\left[\frac{\partial \mu_{110}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{020}}{\partial \theta} + \frac{\partial \mu_{011}}{\partial z} + \frac{2\mu_{110}}{\varpi}\right] + \mu_{130}\frac{\partial Z_0}{\partial \varpi} +$$

$$+ 3\mu_{121}\left[\frac{\partial \Theta_0}{\partial \varpi} + \frac{\Theta_0}{\varpi}\right] + \frac{\mu_{040}}{\varpi}\frac{\partial Z_0}{\partial \theta} + \mu_{031}\left[\frac{3}{\varpi}\frac{\partial \Theta_0}{\partial \theta} + \frac{\partial Z_0}{\partial z} + \frac{3\Pi_0}{\varpi}\right] + 3\mu_{022}\frac{\partial \Theta_0}{\partial z} +$$

$$+ \left[\mu_{131}-\mu_{030}\mu_{101}-3\mu_{021}\mu_{110}\right]\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{041}-\mu_{030}\mu_{011}-3\mu_{021}\mu_{020}}{\varpi}\frac{\partial \ln N}{\partial \theta} +$$

$$+ \left[\mu_{032}-\mu_{030}\mu_{002}-3\mu_{021}\mu_{011}\right]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{131}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{041}}{\partial \theta} + \frac{\partial \mu_{032}}{\partial z} + \frac{4\mu_{131}}{\varpi} = 0$$

$$\frac{\partial \mu_{022}}{\partial t} + \Pi_0\frac{\partial \mu_{022}}{\partial \varpi} + \frac{\Theta_0}{\varpi}\frac{\partial \mu_{022}}{\partial \theta} + Z_0\frac{\partial \mu_{022}}{\partial z} - 2\mu_{021}\left[\frac{\partial \mu_{101}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{011}}{\partial \theta} + \frac{\partial \mu_{002}}{\partial z} +\right.$$

$$\left. + \frac{\mu_{101}}{\varpi}\right] - 2\mu_{012}\left[\frac{\partial \mu_{110}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{020}}{\partial \theta} + \frac{\partial \mu_{011}}{\partial z} + \frac{2\mu_{110}}{\varpi}\right] + 2\mu_{121}\frac{\partial Z_0}{\partial \varpi} +$$

$$+ 2\mu_{112}\left[\frac{\partial \Theta_0}{\partial \varpi} + \frac{\Theta_0}{\varpi}\right] + \frac{2\mu_{031}}{\varpi}\frac{\partial Z_0}{\partial \theta} + 2\mu_{022}\left[\frac{1}{\varpi}\frac{\partial \Theta_0}{\partial \theta} + \frac{\partial Z_0}{\partial z} + \frac{\Pi_0}{\varpi}\right] + 2\mu_{013}\frac{\partial \Theta_0}{\partial z} +$$

$$+ \left[\mu_{122}-2\mu_{021}\mu_{101}-2\mu_{012}\mu_{110}\right]\frac{\partial \ln N}{\partial \varpi} + \frac{\mu_{032}-2\mu_{021}\mu_{011}-2\mu_{012}\mu_{020}}{\varpi}\frac{\partial \ln N}{\partial \theta} +$$

$$+ \left[\mu_{023}-2\mu_{021}\mu_{002}-2\mu_{012}\mu_{011}\right]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{122}}{\partial \varpi} + \frac{1}{\varpi}\frac{\partial \mu_{032}}{\partial \theta} + \frac{\partial \mu_{023}}{\partial z} + \frac{3\mu_{122}}{\varpi} = 0$$

APPENDIX B (continued).

$$\frac{\partial \mu_{013}}{\partial t} + \Pi_0 \frac{\partial \mu_{013}}{\partial \bar{\omega}} + \frac{\Theta_0}{\bar{\omega}} \frac{\partial \mu_{013}}{\partial \theta} + Z_0 \frac{\partial \mu_{013}}{\partial z} - 3\mu_{012}\left[\frac{\partial \mu_{101}}{\partial \bar{\omega}} + \frac{1}{\bar{\omega}}\frac{\partial \mu_{011}}{\partial \theta} + \frac{\partial \mu_{002}}{\partial z} + \right.$$

$$\left. + \frac{\mu_{101}}{\bar{\omega}}\right] - \mu_{003}\left[\frac{\partial \mu_{110}}{\partial \bar{\omega}} + \frac{1}{\bar{\omega}}\frac{\partial \mu_{020}}{\partial \theta} + \frac{\partial \mu_{011}}{\partial z} + \frac{2\mu_{110}}{\bar{\omega}}\right] + 3\mu_{112}\frac{\partial Z_0}{\partial \bar{\omega}} +$$

$$+ \mu_{103}\left[\frac{\partial \Theta_0}{\partial \bar{\omega}} + \frac{\Theta_0}{\bar{\omega}}\right] + \frac{3\mu_{022}}{\bar{\omega}}\frac{\partial Z_0}{\partial \theta} + \mu_{013}\left[\frac{1}{\bar{\omega}}\frac{\partial \Theta_0}{\partial \theta} + 3\frac{\partial Z_0}{\partial z} + \frac{\Pi_0}{\bar{\omega}}\right] + \mu_{004}\frac{\partial \Theta_0}{\partial z} +$$

$$+ \left[\mu_{113} - 3\mu_{012}\mu_{101} - \mu_{003}\mu_{110}\right]\frac{\partial \ln N}{\partial \bar{\omega}} + \frac{\mu_{023} - 3\mu_{012}\mu_{011} - \mu_{003}\mu_{020}}{\bar{\omega}}\frac{\partial \ln N}{\partial \theta} +$$

$$+ \left[\mu_{014} - 3\mu_{012}\mu_{002} - \mu_{003}\mu_{011}\right]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{113}}{\partial \bar{\omega}} + \frac{1}{\bar{\omega}}\frac{\partial \mu_{023}}{\partial \theta} + \frac{\partial \mu_{014}}{\partial z} + \frac{2\mu_{113}}{\bar{\omega}} = 0$$

$$\frac{\partial \mu_{004}}{\partial t} + \Pi_0 \frac{\partial \mu_{004}}{\partial \bar{\omega}} + \frac{\Theta_0}{\bar{\omega}} \frac{\partial \mu_{004}}{\partial \theta} + Z_0 \frac{\partial \mu_{004}}{\partial z} - 4\mu_{003}\left[\frac{\partial \mu_{101}}{\partial \bar{\omega}} + \frac{1}{\bar{\omega}}\frac{\partial \mu_{011}}{\partial \theta} + \frac{\partial \mu_{002}}{\partial z} + \right.$$

$$\left. + \frac{\mu_{101}}{\bar{\omega}}\right] + 4\mu_{103}\frac{\partial Z_0}{\partial \bar{\omega}} + \frac{4\mu_{013}}{\bar{\omega}}\frac{\partial Z_0}{\partial \theta} + 4\mu_{004}\frac{\partial Z_0}{\partial z} + \left[\mu_{104} - 4\mu_{003}\mu_{101}\right]\frac{\partial \ln N}{\partial \bar{\omega}} +$$

$$+ \frac{\mu_{014} - 4\mu_{003}\mu_{011}}{\bar{\omega}}\frac{\partial \ln N}{\partial \theta} + \left[\mu_{005} - 4\mu_{003}\mu_{002}\right]\frac{\partial \ln N}{\partial z} + \frac{\partial \mu_{104}}{\partial \bar{\omega}} + \frac{1}{\bar{\omega}}\frac{\partial \mu_{014}}{\partial \theta} +$$

$$+ \frac{\partial \mu_{005}}{\partial z} + \frac{\mu_{104}}{\bar{\omega}} = 0$$