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Finite-temperature T matrix in a real-time formalism*

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A generalized off-shell unitarity relation for the two-body scattering T matrix in a many-body medium at finite temperature is derived, through a consistent real-time perturbation expansion by means of Feynman diagrams. We comment on perturbation schemes at finite temperature in connection with an erroneous formulation of the Dyson equation in a paper recently published.

I. INTRODUCTION

In a recent paper[1] scattering in a many-body medium at finite temperature in equilibrium was examined, and a generalized off-shell unitarity relation for the T matrix was derived in a real-time perturbative formalism. The starting point features a Dyson equation for the real-time Green's function at finite temperature in the context of non-time-ordered Feynman diagrams. To our knowledge the Dyson equation, Eq. (1) of Ref. 1, cannot be found in the current literature, and it is our belief that no suitable justification for it is given in Ref. 1 (see Sec. IIA). It is perhaps worthwhile to mention some of the significant developments related to the subject of perturbation schemes for real-time Green's functions at finite temperature.

The well-known zero-temperature Feynman diagram expansion relies upon the assumptions that at both times $-\infty$ and $+\infty$ the system is not interacting and that the states which develop from $-\infty$ forward in time and from $+\infty$ backward in time are the same.[2] For this reason it can be applied neither to systems at finite temperatures,[3] nor to systems in nonequilibrium[4] even when the system is described by its ground state.[5] In such cases no time-ordered product appears when transforming to the interaction representation, and Wick's theorem does not apply.

Nevertheless, for equilibrium situations, thermodynamic imaginary-time Green's functions can be defined (for finite temperature); from an ordering along the imaginary-time axis, a generalized Wick's theorem follows.[6] This theorem allows the use of a diagrammatic perturbation theory for the imaginary-time Green's function similar to the zero-temperature one. In the corresponding Lehmann representation an analytical continuation[7] to real frequencies can be performed, and it is then proved that the obtained weight function also determines the retarded and advanced real-time Green's functions.[8,2] It should be stressed that no diagrams are available in this equilibrium formalism for the real-time Green's functions.

For nonequilibrium situations, the iteration of the imaginary-time Green's function equation of motion results in a perturbative expansion that can be analytically continued to real time.[9]

It is interesting to note that these difficulties in the applicability of Wick's theorem do not arise in the recently derived[10] perturbative expansion in Feynman diagrams of the classical thermodynamic Green's functions.

In order to deal directly with real-time Green's functions in equilibrium and nonequilibrium at both zero and finite temperatures, two related formalisms have been developed. One of them is essentially due to Schwinger[11] and it is the functional counterpart of the diagrammatic perturbative scheme introduced by Keldysh and Craig independently,[12] which has been extensively used in the literature.[13,14] This diagram technique is based on the use of a closed contour C for time ordering (from $-\infty$ to $+\infty$ and then back to $-\infty$) in which the times on both branches of C are differentiated as a bookkeeping artifice. Contour C allows a consistent application of Wick's theorem and, therefore, formally the same diagrams are obtained as at zero temperature, only now the time labels of virtual states are integrated along C. When the two branches of the contour are disentangled, one is led to the real-time functions. It should be mentioned that suitable definitions of new dynamical pictures lead to Keldysh's results, considering time ordering from $-\infty$ to $+\infty$ only.[15]

It is the purpose of this paper to derive a generalized off-shell unitarity relation for the T matrix at finite temperature in the framework of Keldysh's consistent real-time perturbation theory, in which a Dyson equation and a self-energy can be well defined. In order to carry out

our derivation we find it necessary to review the formalism, giving the rules for practical calculations since they do not appear to be sufficiently explicit in the literature. At the end, we comment on an S-matrix approach to scattering in a many-body medium at finite temperature.

II. REAL-TIME FORMALISM T MATRIX

A. Real-time formalism at finite temperature

In Keldysh's formalism each physical magnitude $M = M(x_1, \ldots, x_n)$, where $x_i = (\vec{x}_i, t_i)$, is generalized to a corresponding $M(\vec{x}_1 t_{1\alpha}, \ldots, \vec{x}_n t_{n\beta})$ where the subindex $\alpha(\ldots\beta)$ is equal to + or − depending on whether $t_{1\alpha}(\ldots t_{n\beta})$ lies on the upper (from $-\infty$ to $+\infty$) or lower (back from $+\infty$ to $-\infty$) branch of circuit C. Each specification of the set $\{\alpha, \ldots, \beta\}$ defines a different functional dependence on the variables $(x_1, \ldots, x_n)$, that is, it defines one of the 2^n functions

$$M_{\alpha\ldots\beta}(x_1, \ldots, x_n) \equiv M(x_1 t_{1\alpha}, \ldots, x_n t_{n\beta}).$$

In particular when all the times lie on the upper branch, the $M_{+\ldots+}$ equals M. Internal degrees-of-freedom indices can also be made explicit, but in order not to encumber the notation they will be left implicit.

The generalized one-particle Green's function is defined in terms of fields in the Heisenberg picture ψ_H as

$$iG_{\alpha\beta}(x_1, x_2) \equiv \langle T_c[\psi_H(\vec{x}_1 t_{1\alpha})\psi_H^\dagger(\vec{x}_2 t_{2\beta})]\rangle , \qquad (2.1)$$

where $\langle\cdots\rangle$ is the expectation value $\mathrm{Tr}(\rho\ldots)/\mathrm{Tr}\rho$, where ρ is the density matrix. The time-ordering operator T_C orders times along contour C, that is,

$$\begin{aligned} T_C[A(t_\alpha)B(t'_\beta)] &= \delta_{\alpha\beta}\{\delta_{\alpha+}T[A_+(t)B_+(t')] + \delta_{\alpha-}\bar{T}[A_-(t)B_-(t')]\} \\ &\quad + \delta_{\alpha-}\delta_{\beta+}A_-(t)B_+(t') - \delta_{\alpha+}\delta_{\beta-}B_-(t')A_+(t) , \end{aligned} \qquad (2.2)$$

where T and $\bar{T}$ denote the usual time and anti-time-ordering operators.

The relevance of the generalized Green's function (2.1) is expressed by the following relations:

$$iG_{++}(x_1, x_2) = \langle T[\psi_H(x_1)\psi_H^\dagger(x_2)]\rangle \equiv iG(x_1, x_2) , \qquad (2.3a)$$

$$iG_{+-}(x_1, x_2) = -\langle \psi_H^\dagger(x_2)\psi_H(x_1)\rangle \equiv iG^<(x_1, x_2) , \qquad (2.3b)$$

$$iG_{-+}(x_1, x_2) = \langle \psi_H(x_1)\psi_H(x_2)\rangle \equiv iG^>(x_1, x_2) , \qquad (2.3c)$$

$$iG_{--}(x_1, x_2) = \langle \bar{T}[\psi_H(x_1)\psi_H^\dagger(x_2)]\rangle \equiv i\bar{G}(x_1, x_2) . \qquad (2.3d)$$

The study of $G_{\alpha\beta}$ will thus lead us directly to the knowledge of the four finite temperature real-time Green's functions.

It is easily verified that $G_{\alpha\beta}$ is T_c ordered when expressed in the interaction picture. The change of image of the fields is performed through an operator defined along the contour C. An adiabatic hypothesis is made at the physical time $t = -\infty$ only, so that a possible degeneracy of finite temperature states is irrelevant. (Nevertheless it can be proved[12] that for systems in equilibrium this procedure leads to the correct expectation values in terms of the true canonical distribution ρ.) A generalized Wick's theorem applies, so that $G_{\alpha\beta}$ can be expanded in a perturbation series which can be written as a Dyson equation in terms of the proper self-energy $\Sigma_{\gamma\delta}$ (see Fig. 1),

$$\begin{aligned} G_{\alpha\beta}(x_1, x_2) &= G_{0\alpha\beta}(x_1, x_2) \\ &\quad + \int_c \int_c d^4x_3\, d^4x_4\, G_{0\alpha\gamma}(x_1, x_3) \\ &\quad \times \Sigma_{\gamma\delta}(x_3, x_4) G_{\delta\beta}(x_4, x_2) . \end{aligned} \qquad (2.4)$$

It can be proved that disconnected diagrams do not appear in any order in this perturbation expansion.[16,17] Note that Eq. (2.4) for $G_{\alpha\beta}$ represents, in fact, four equations, one for each specification of $\{\alpha, \beta\}$.

An essential feature of this finite temperature real-time Dyson equation is the fact of the four functions G, $G^<$, $G^>$, and $\bar{G}$ being mixed up in the self-energy term. Hence it should be stressed that the euqation for $G_{\alpha\beta}$ is closed, but that each of the four equations to which it gives rise for its four components is not closed.

Before proceeding further we give the explicit rules used in this formalism:

(i)
$$\begin{aligned} \int_C d^4x\, M_{\ldots\alpha\ldots}(\cdots x\cdots) &= \int d^3\vec{x}\left(\int_{-\infty}^{+\infty} dt\, M_{\ldots+\ldots}(\cdots x\cdots) + \int_{+\infty}^{-\infty} dt\, M_{\ldots-\ldots}(\cdots x\cdots)\right) \\ &= \sum_{\alpha=+,-} \mathrm{sgn}\,\alpha \int d^4x\, M_{\ldots\alpha\ldots}(\cdots x\cdots) . \end{aligned} \qquad (2.5)$$

We write $\mathrm{sgn}\,\alpha(=\alpha)$ instead of α for a clearer understanding.

(ii) In the same way as in the usual zero-temperature formalism the static instantaneous interparticle potential $V(|\vec{x}_1 - \vec{x}_2|)$ is rewritten as $U(x_1, x_2) = V(|\vec{x}_1 - \vec{x}_2|)\delta(t_1 - t_2)$ in order to express the Dyson equation in a covariant way (see Ref. 2, p. 85), it is here necessary to consider the following generalized interparticle potential:

$$U_{\alpha\beta}(x_1, x_2) \equiv V(|\vec{x}_1 - \vec{x}_2|)\delta_{\alpha\beta}(t_1, t_2) , \qquad (2.6)$$

where

$$\delta_{\alpha\beta}(t_1, t_2) = \mathrm{sgn}\alpha\, \delta_{\alpha\beta}\delta(t_1 - t_2) \qquad (2.7)$$

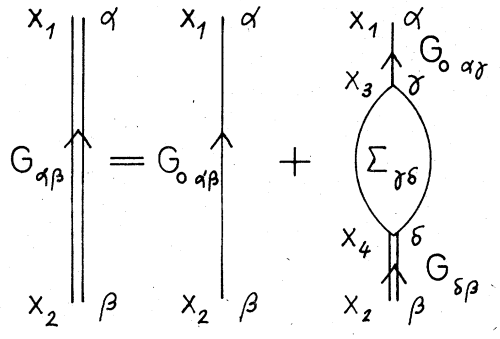

FIG. 1. Dyson's equation in Keldysh's formalism. Subindices are double valued. The sense of the Green's functions' arrows refer to the relative order of subindices and space-time coordinates.

verifies

$$\int_C dt_2\, \delta_{\alpha\beta}(t_1,t_2) M_{\ldots\beta\ldots}(\cdots t_2\cdots) = M_{\ldots\alpha\ldots}(\cdots t_1\cdots)\,, \quad (2.8)$$

and where $\delta_{\alpha\beta}$ and $\delta(t_1-t_2)$ are the usual Kronecker and Dirac deltas, respectively. From (2.5) to (2.7) it follows that

$$\int_C dt_2\, U_{\alpha\beta}(x_1,x_2) M_{\ldots\beta\ldots}(\cdots t_2\cdots) = V(|\vec{x}_1-\vec{x}_2|) M_{\ldots\alpha\ldots}(\cdots t_1\cdots)\,. \quad (2.9)$$

(iii) The same formal rules apply for constructing Feynman diagrams for $G_{\alpha\beta}$ as in the usual formalism for zero-temperature Green's function (see Ref. 2, p. 97); the essential difference lies in the fact that the time variables are defined along C, or in other words, that all magnitudes carry subindices.

Using Eq. (2.5), the Dyson equation (2.4) can be expressed in terms of physical time integration ($\int_{-\infty}^{+\infty}$). It is then straightforward to transform it to the Fourier-space representation, and, if a uniform system is assumed, an algebraic equation is obtained:

$$\begin{aligned} G_{\alpha\beta}(p) = {} & G_{0\alpha\beta}(p) + G_{0\alpha+}(p)\Sigma(p)G_{+\beta}(p) \\ & - G_{0\alpha+}(p)\Sigma^{<}(p)G_{-\beta}(p) \\ & - G_{0\alpha-}(p)\Sigma^{>}(p)G_{+\beta}(p) + G_{0\alpha-}(p)\bar{\Sigma}(p)G_{-\beta}(p)\,, \end{aligned} \quad (2.10)$$

where $p=(\vec{p},p_0)$ and $\hbar\vec{p}(\hbar p_0)$ is the momentum (energy). For $\Sigma_{\alpha\beta}$ a shorthand notation has been used $(\Sigma,\Sigma^{<},\Sigma^{>},\bar{\Sigma})$, in equivalence to the one for $G_{\alpha\beta}$ in Eq. (2.3). The $\{\alpha=+,\beta=+\}$ component of Eq. (2.10) is the Fourier-transformed equation for the real-time Green's function G, and it features a different and much more complicated self-energy term than Eq. (1) of Ref. 1, where a closed Dyson equation for G is written down in terms of Σ. It should also be noted that such an equation does not follow from the usual equilibrium imaginary-time formalism: a Dyson equation exists for the imaginary-time Green's function defined at a denumerable set of points on the imaginary frequency axis; and only after the relevant diagrams (i.e., in this paper, the ladder ones) have been calculated for these particular frequencies can the result be analytically continued to the whole complex-frequency plane and, in particular, to real frequencies. The weight function thus obtained for real frequencies determines also the Fourier-transformed real-time Green's functions[8,2]: this is the important result of the imaginary-time formalism. The imaginary-time Green's function analytically continued to real frequencies is not the same as the real-time Green's function (i.e., compare Eqs. (31.35), (31.22), and (31.24) of Ref. 2).

On the other hand, in the nonequilibrium imaginary-time formalism developed by Kadanoff and Baym,[9] the imaginary-time Dyson equation is analytically continued to real times. So a Dyson equation does exist but it turns out to be the same form as Eq. (2.10).[18] And it should be noted that their formalism is especially well suited for making contact with the theory of equilibrium imaginary-time Green's functions because it reduces to it in the absence of external fields.[4]

B. T-matrix unitarity relations

Our aim is to derive a real-time theory for two-body scattering in a many-body medium at finite temperature, in particular for the T matrix or generalized scattering amplitude. In the self-consistent ladder approximation all the diagrams in Fig. 2 are summed up; as explicitly seen below, this considers both particle-particle and hole-hole intermediate scattering states while the particle-hole ones are automatically excluded.

When the rules of Sec. II A are applied to these ladder diagrams, and after a somewhat lengthy but straightforward algebra, one arrives at the following compact expression:

$$\Sigma^{\rm lad}_{\alpha\beta}(p) = -\frac{i}{\hbar}\int \frac{d^4q}{(2\pi)^4}\, G_{\beta\alpha}(q) \times [\,\Gamma_{\alpha\alpha\beta\beta}(p,q;p,q) - \Gamma_{\alpha\alpha\beta\beta}(p,q;q,p)]\,, \quad (2.11)$$

where the effective two-particle interaction Γ is defined by the equation (see Figs. 3 and 4)

$$\Gamma_{\alpha\alpha\beta\beta}(p_1,p_2;p_3,p_4)$$

$$= \delta_{\alpha\beta}\mathrm{sgn}\alpha\, V(|\vec{p}_1-\vec{p}_3|)+\frac{i}{\hbar}\sum_{\epsilon}\mathrm{sgn}\epsilon\int\frac{d^4k}{(2\pi)^4}\; V(|\vec{k}|)G_{\alpha\epsilon}(p_1-k)G_{\alpha\epsilon}(p_2+k)\Gamma_{\epsilon\epsilon\beta\beta}(p_1-k,p_2+k;p_3,p_4) \quad (2.12a)$$

$$= \delta_{\alpha\beta}\mathrm{sgn}\alpha\, V(|\vec{p}_1-\vec{p}_3|)+\frac{i}{\hbar}\sum_{\epsilon}\mathrm{sgn}\epsilon\int\frac{d^4k}{(2\pi)^4}\; V(|\vec{k}|)G_{\epsilon\beta}(p_3-k)G_{\epsilon\beta}(p_4+k)\Gamma_{\alpha\alpha\epsilon\epsilon}(p_1,p_2;p_3-k,p_4+k)\,. \quad (2.12b)$$

From the equations for the four components of $\Gamma_{\alpha\alpha\beta\beta}$ (denoted as Γ, $\Gamma^<$, $\Gamma^>$ and $\overline{\Gamma}$ in an equivalent notation to that of $G_{\alpha\beta}$ and $\Sigma_{\alpha\beta}$) that follow from Eq. (2.12), and using the fact that under a frequency convolution integration $GG - G^>G^>$ is equal to $G^<G^< - \overline{G}\overline{G}$, it can be seen that $\Gamma - \Gamma^<(\Gamma^< - \overline{\Gamma})$ and $\Gamma^> - \overline{\Gamma}(\Gamma - \Gamma^>)$ satisfy the same equation. For this reason it is useful to define the retarded and advanced amplitudes, Γ^R and Γ^A:

$$\Gamma^R(p_1,p_2;p_3,p_4)\equiv\Gamma(p_1,p_2;p_3,p_4)-\Gamma^<(p_1,p_2;p_3,p_4)=\Gamma^>-\overline{\Gamma}\,, \quad (2.13a)$$

$$\Gamma^A(p_1,p_2;p_3,p_4)\equiv\Gamma^<(p_1,p_2;p_3,p_4)-\overline{\Gamma}(p_1,p_2;p_3,p_4)=\Gamma-\Gamma^>\,, \quad (2.13b)$$

which verify the following equations:

$$\overset{R}{\Gamma^A}(p_1,p_2;p_3,p_4)$$

$$= V(|\vec{p}_1-\vec{p}_3|)+\frac{i}{\hbar}\int\frac{d^4k}{(2\pi)^4}\; V(|\vec{k}|)[G(p_1-k)G(p_2+k)-\overset{<}{G^>}(p_1-k)\overset{<}{G^>}(p_2+k)]\overset{R}{\Gamma^A}(p_1-k,p_2+k;p_3,p_4) \quad (2.14a)$$

$$= V(|\vec{p}_1-\vec{p}_3|)+\frac{i}{\hbar}\int\frac{d^4k}{(2\pi)^4}\; V(|\vec{k}|)[G(p_3-k)G(p_4+k)-\overset{<}{G^>}(p_3-k)\overset{<}{G^>}(p_4+k)]\overset{R}{\Gamma^A}(p_1,p_2;p_3-k,p_4+k)\,, \quad (2.14b)$$

in an obvious shorthand notation. From Eqs. (2.12) and (2.14) we get

$$\overset{<}{\Gamma^>}(p_1,p_2;p_3,p_4)=\frac{i}{\hbar}\int\frac{d^4k}{(2\pi)^4}\; V(|\vec{k}|)\{\overset{<}{G^>}(p_1-k)\overset{<}{G^>}(p_2+k)\Gamma^A(p_1-k,p_2+k;p_3,p_4)$$
$$+[G(p_1-k)G(p_2+k)-G^<(p_1-k)G^<(p_2+k)]\overset{<}{\Gamma^>}(p_1-k,p_2+k;p_3,p_4)\} \quad (2.15a)$$

$$=\frac{i}{\hbar}\int\frac{d^4k}{(2\pi)^4}\; V(|\vec{k}|)\{\overset{<}{G^>}(p_3-k)\overset{<}{G^>}(p_4+k)\Gamma^R(p_1,p_2;p_3-k,p_4+k)$$
$$+[G(p_3-k)G(p_4+k)-G^>(p_3-k)G^>(p_4+k)]\overset{<}{\Gamma^>}(p_1,p_2;p_3-k,p_4+k)\}\,, \quad (2.15b)$$

and iterating this equation we obtain

$$\overset{<}{\Gamma^>}(p_1,p_2;p_3,p_4)=\frac{i}{\hbar}\int\frac{d^4k}{(2\pi)^4}\; \Gamma^R(p_1,p_2;p_1-k,p_2+k)\overset{<}{G^>}(p_1-k)\overset{<}{G^>}(p_2+k)\Gamma^A(p_1-k,p_2+k;p_3,p_4) \quad (2.16a)$$

$$=\frac{i}{\hbar}\int\frac{d^4k}{(2\pi)^4}\; \Gamma^A(p_3-k,p_4+k;p_3,p_4)\overset{<}{G^>}(p_3-L)\overset{<}{G^>}(p_4+k)\Gamma^R(p_1,p_2;p_3-k,p_4+k)\,. \quad (2.16b)$$

As it can easily be proved from the previous equations, the following relations hold:

$$(\overset{<}{G^>}(p))^*=\overset{<}{G^>}(p)\,,\quad (G(p))^*=\overline{G}(p)\,, \quad (2.17)$$

$$\Gamma^{R\dagger}(p_1,p_2;p_3,p_4)\equiv(\Gamma^R(p_3,p_4;p_1,p_2))^*$$
$$=\Gamma^A(p_1,p_2;p_3,p_4)\,, \quad (2.18a)$$

$$\overset{<}{\Gamma^>}{}^{\dagger}(p_1,p_2;p_3,p_4)=\overset{<}{\Gamma^>}(p_1,p_2;p_3,p_4)\,. \quad (2.18b)$$

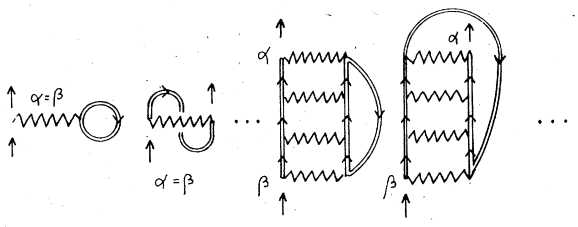

FIG. 2. Ladder diagrams in Keldysh's formalism.

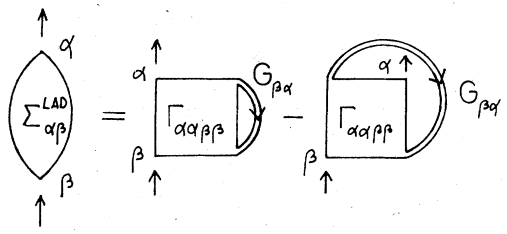

FIG. 3. Self-consistent ladder approximation to the proper self-energy in Keldysh's formalism.

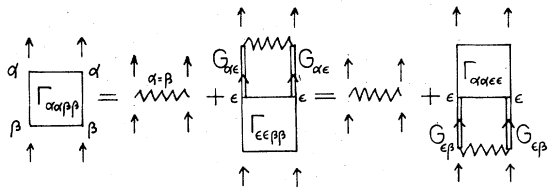

FIG. 4. Self-consistent T-matrix equation in Keldysh's formalism (ladder approximation).

Then, from (2.16) and (2.18) we obtain a generalized off-shell unitarity relation for the real-time scattering amplitude at finite temperature:

$$\Gamma^R(p_1,p_2;p_3,p_4)-\Gamma^A(p_1,p_2;p_3,p_4)$$
$$=\Gamma^{>}-\Gamma^{<}$$
$$=\frac{i}{\hbar}\int\frac{d^4k}{(2\pi)^4}\,\Gamma^R(p_1,p_2;p_1-k,p_2+k)$$
$$\times[G^{>}(p_1-k)G^{>}(p_2+k)-G^{<}(p_1-k)G^{<}(p_2+k)]$$
$$\Gamma^A(p_1-k,p_2+k;p_3,p_4)\,. \tag{2.19}$$

Equations (2.16) and (2.19) are a generalization of the ordinary scattering theory optical theorem. A first-order approximation to these self-consistent expressions is the replacement of the true Green's functions $G^{\gtrless}$ by the free ones $G_0^{\gtrless}$. Equation (2.19) in this case reduces to

$$[\Gamma^A(p_1,p_2;p_3,p_4)-\Gamma^R(p_1,p_2;p_3,p_4)]$$
$$\times\{1-\exp[\beta\hbar(p_{10}+p_{20})]\}^{-1}$$
$$=\frac{i}{\hbar}\int\frac{d^3\vec{k}}{(2\pi)^2}\,\Gamma^R(p_1,p_2;p_1-k,p_2+k)n^0_{p_1-k}n^0_{p_2+k}$$
$$\times\Gamma^A(p_1-k,p_2+k;p_3,p_4)$$
$$\times\delta[p_{10}+p_{20}-\hbar^{-1}(\epsilon^0_{p_1-k}+\epsilon^0_{p_2+k}-2\mu)]\,, \tag{2.20}$$

where the fact that Γ^R, Γ^A in the integrand do not depend upon k_0 has been used, and

$$n^0_p\equiv\{1+\exp[\beta(\epsilon^0_p-\mu)]\}^{-1}\,, \tag{2.21a}$$
$$\epsilon^0_p\equiv\hbar^2\vec{p}^2/2m\,. \tag{2.21b}$$

These relations generalize the on-shell optical theorems which have been proved in the imaginary-time formalisms.[19] The point of our discussion is that a correct optical theorem for the physical T matrix has been obtained through a consistent real-time perturbation scheme at finite temperature.

It should be stressed that as our starting equations are formally similar but different in nature to those in Ref. 1, essentially due to the matricial form[20] of these equations in Keldysh's formalism, our final results differ.

In the nonequilibrium situations of kinetic theory, a Markovian approximation is usually made which allows the development of a T-matrix theory along the same lines as in this paper. If in addition the approximation[21] of vanishing off-diagonal parts (in the matrix notation[20]) is taken at the beginning of the calculations, a simplified nonequilibrium T-matrix theory follows. These results agree exactly with Hall's,[21] who uses Keldysh's formalism in these approximations, and who extends previous results of Wyld and Fried to inhomogeneous systems. Alternatively, if we consider the approximation of vanishing off-diagonal parts in this paper [e.g., in Eq. (2.12)], we obtain closer results to those in Ref. 1; even so, the crucial difference of the G_0G_0 term under the integral signs being replaced by $G_0^{>}G_0^{>}-G_0^{<}G_0^{<}$ persists in this approximation.

Finally we would like to remark on the high desirability of an S-matrix complete description of two-body scattering processes in a many-body medium. The above defined T matrix Γ would then be the ladder approximation to the complete T matrix, that is, the approximation in which the kernel of the Bethe-Salpeter equation is substituted by the potential. Also, the interpretation of scattering wave functions would then become clear. This program has already been carried out at zero temperature in connection with scattering processes in atomic physics.[22] We are working at present on the nontrivial extension to finite temperatures in a real-time formalism, and hope to report our results elsewhere.

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