Turn-on-time statistics of modulated lasers subjected to resonant weak optical feedback

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We present analytical calculations of the turn-on-time probability distribution of intensity-modulated lasers under resonant weak optical feedback. Under resonant conditions, the external cavity round-trip time is taken to be equal to the modulation period. The probability distribution of the solitary laser results are modified to give reduced values of the mean turn-on-time and its variance. Numerical simulations have been carried out showing good agreement with the analytical results.

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I. INTRODUCTION

Lasers subjected to external feedback have been extensively studied in the last years. Light fed back into the cavity greatly modifies the steady-state operation of the solitary laser, giving frequency shifts [1], multistability [2] and low- [3] and high- [4] frequency fluctuations, depending on the amount of light reinjected. Moderate to strong feedback levels drive the laser dynamics through different routes to chaos [5] to a regime of coherence collapse [6]. Controlled inphase weak optical feedback provides a well-known method for linewidth reduction and output stabilization in lasers [7].

Q switching of gas, dye, or solid-state lasers is a common setup for narrow, high peak power pulse generation [8,9]. The losses are periodically modulated by means of tilted mirrors, saturable absorber, acousto-optics or electroabsorption modulators, and others. By a careful experimental arrangement, high repetition rates have been obtained [10].

Gain-switched semiconductor lasers provide power peaks of several mW at GHz modulation rates. The material gain is periodically driven lower to higher values by means of an increase of the current injection through the diode junction [11]. Synchronization of the different pulses is of special importance in practical applications. Desynchronized pulses cause a degradation of the temporal resolution and act as a limiting factor in high-bit rate optical communication systems, where semiconductor lasers are used as light sources [12].

Some works considered the transient dynamics of the laser. Analytical calculations of turn-on-time statistics are available for lasers in which the polarization (class-B lasers) [13], or polarization and population inversion (class-A lasers) [14] could be adiabatically eliminated. Transient dynamics of lasers subjected to feedback have also been analyzed . Frequency selection during the switch-on [15], mean turn-on-time and its variance were calculated for gain- and Q-switched conditions with short external cavities [16,17], where the theory is valid. Only numerical simulations of the equations describing time evolution have been done to study the transient behavior of the laser under periodic or pseudorandom word modulations [18,19].

In this paper we study the transient behavior of both class-A and -B lasers externally modulated, whether in losses or in gain, and subjected to weak optical feedback. We consider the special, but interesting situation, in which the period of the modulation coincides with the external cavity round-trip time, for any cavity length. Under this condition, we calculate the mean switch-on time and its variance. The most important assumption is that the field fed back into the laser cavity has the same statistical properties as the one that is generated at that time. This resonance condition is particularly interesting. The light pulse that is building up receives an incoming feedback from the previous light pulse, which we assume as statistically identical. The situation is very different to other feedback conditions analytically studied in previous works, where the light fed back was assumed to have their origin in the same light pulse [16,17,19]. This assumption allows us to perform analytical calculations which, as will be shown, are in good agreement with the numerical simulation of noise driven rate equations. The paper is organized as follows. In Secs. II and III, analytical results for class-A and -B lasers, respectively, are presented. Numerical solutions and discussion of the results are given in Sec. IV. A summary and conclusions are given in Sec. V.

II. ANALYTICAL RESULTS: CLASS-A LASERS

We describe the dynamics of a class-A laser by means of the time evolution of the slowly varying complex electric field $E(t)$. The material variables (polarization and population inversion) have been adiabatically eliminated. The evolution of the electric field of a class-A laser is described through the following stochastic differential equation, where the possibility for a cavity detuning θ has been included [20,21,15]:

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$$\frac{dE}{dt}=(1+i\theta)\left[\frac{g}{1+s|E|^2}-\gamma\right]E(t)+\kappa e^{-i\omega\tau}E(t-\tau)+\sqrt{\varepsilon}\xi(t), \tag{1}$$

where $\xi(t)$ is a white Gaussian process taking into account spontaneous emission with zero mean and a correlation $\langle\xi(t)\xi^*(t')\rangle=2\,\delta(t-t')$. Under Q-switching conditions, the cavity losses are modulated [8]. During the modulation period $T=T_{\rm on}+T_{\rm off}$, the losses parameter takes a value $\gamma=\gamma_1$ below threshold in $T_{\rm off}$, while during $T_{\rm on}$ the losses are reduced to $\gamma=\gamma_2$.

The delayed term $\kappa e^{-i\omega\tau}E(t-\tau)$ is added to the equation in order to take into account the amount of light fed back into the laser. The external reflector couples the two electric fields through the feedback strength κ. The external cavity round time τ and the emission frequency of the solitary laser ω combine to give the relative phase between the two fields. The external round time can be set arbitrarily to any value. In the present work, this value is adjusted to be $\tau=T$. Our model also applies to optical feedback coming from other emitted pulses giving $\tau=nT$, with n an integer.

The time is set to $t=0$ at one of the Q-switching times. Gain saturation can be neglected up to a given intensity reference I_r at which the switch-on time, defined as the time the optical intensity takes to reach the prefixed value I_r, is reached after the losses switch. As the time evolution of the electric field, or equivalently the optical intensity, is affected by spontaneous emission noise, the switch-on time is a stochastic quantity with a mean time and a variance. Both quantities will be analyzed. By neglecting gain saturation effects, instantaneous gain-switching turns-out to be formally equivalent to Q switching in class-A lasers.

In the case of the solitary laser ($\kappa=0$), the net losses $a_0=\gamma_1-g$ and the noise strength ε combine to give the mean intensity emitted in the off-state. Weak feedback is known to modify the expression to

$$I_0=\frac{\varepsilon}{a_0-\kappa\cos(\omega\tau)}, \tag{2}$$

so that light emission is enhanced when the external field is reinjected in phase with the internal one. The Q switch increases suddenly the net gain to $a=g-\gamma_2$. In the usual analytical approach, an amplification for the electrical field is assumed in the way $E(t)=h(t)e^{(1+i\theta)at}$. $h(t)$ results to be a stochastic quantity containing information about the seeding process, which gives rise to light amplification [14].

Statistical equivalence between this electrical field and the one corresponding to the previous light pulse allows us to apply the same kind of decomposition to $E(t-\tau)=g(t)e^{(1+i\theta)at}$. The key step of our calculations will be the assumption of statistical equivalence of $h(t)$ and $g(t)$. In particular, the relations $\langle h(t)\rangle=\langle g(t)\rangle=0$ and $\langle h(t)h^*(t')\rangle=\langle g(t)g^*(t')\rangle$ will be used. Using (1), the stochastic process $h(t)$ is given by

$$h(t)=h(0)+\int_0^t[\kappa e^{-i\omega\tau}g(t')+\sqrt{\varepsilon}\xi(t')e^{-(1+i\theta)at'}]dt'. \tag{3}$$

The variance of $h(t)$ is calculated as

$$\sigma_h^2(t)=\langle|h(t)|^2\rangle=\langle|h(0)|^2\rangle+\int_0^t dt'\int_0^t dt''[\kappa^2\langle g(t')g^*(t'')\rangle+\varepsilon\langle\xi(t')\xi^*(t'')\rangle e^{-(1+i\theta)at'}e^{-(1-i\theta)at''}], \tag{4}$$

due to the lack of correlation between $h(t)$, $g(t)$, and $\xi(t)$. More information about $g(t)$ has to be supplied to obtain $\sigma_h^2(t)$. In order to get a closed expression we make use of the relation, valid for the solitary laser,

$$\langle g(t)g^*(t')\rangle=\sigma_h^2(\min(t,t')). \tag{5}$$

This relation is totally independent of both laser parameters and the initial conditions for the electrical field. We will also assume that this equality holds in the case of the laser subjected to feedback. This *a priori* assumption gives us a new expression for $\sigma_h^2(t)$:

$$\sigma_h^2(t)=\sigma_0^2(t)+\kappa^2\int_0^t dt'\int_0^t dt''\sigma_h^2(\min(t',t'')), \tag{6}$$

where

$$\sigma_0^2(t)=\frac{\varepsilon}{a_0-\kappa\cos(\omega\tau)}+\frac{\varepsilon}{a}(1-e^{-2at}). \tag{7}$$

This integral equation for $\sigma_h^2(t)$ can be further reduced to a second-order ordinary differential equation in t as

$$\frac{d^2\sigma_h^2(t)}{dt^2}=\frac{d^2\sigma_0^2(t)}{dt^2}+2\kappa^2\sigma_h^2(t). \tag{8}$$

The linear differential equation can be readily solved for $\sigma_h^2(t)$ to give

$$\sigma_h^2(t)=C_1e^{-\sqrt{2}\kappa t}+C_2e^{\sqrt{2}\kappa t}+C_3e^{-2at}, \tag{9}$$

being C_1, C_2, C_3 time-independent coefficients. In previous works where other situations were analyzed (solitary lasers [14], weak feedback [16], weak injected field [22], . . . ,) the variance $\sigma_h^2(t)$ was shown to relax to an asymptotic value $\sigma_h^2(\infty)$ before reaching the intensity reference. The stochastic process $h(t)$ was then treated as a stochastic variable, and the expressions for both the mean switch-on time and its variance were analytically solved to get simple expressions. In the present case, however, $\sigma_h^2(t)$ diverges exponentially at long times even for very low feedback levels. A method to overcome this limitation was used in the case of strong light injection [23]. A more general method [24] based on the exact probability distribution of switch-on times is needed here because of the lack of a deterministic drift.

Our analysis considers the transient dynamics of the laser, from the initial below threshold emission up to the switch-on time, always before reaching continuous-wave operation. Under this assumption [24], the electric field is a zero-mean complex circularly Gaussian distributed variable, so that the probability density distribution for the light intensity at a given time $P(I;t)$ reads

$$P(I;t)=\frac{1}{\langle I(t)\rangle}e^{-\frac{I}{\langle I(t)\rangle}}. \tag{10}$$

From this expression we can find the probability density distribution for the switch-on time at a given intensity reference I_r as

$$Q(t;I_r)=\frac{\langle \dot{I}(t)\rangle}{\langle I(t)\rangle^2}e^{-\frac{I_r}{\langle I(t)\rangle}}I_r. \tag{11}$$

The above distribution allows a simple way of evaluating all the moments of the switch-on time distribution

$$\langle t^n\rangle=\int_0^\infty t^n Q(t;I_r)dt. \tag{12}$$

The mean intensity functions $\langle I(t)\rangle$ needed in all these calculations can be readily evaluated in terms of $\sigma_h^2(t)$ as $\langle I(t)\rangle=\sigma_h^2(t)e^{2at}$ to give a final result.

It is important to note what is the role of the external round-trip time and the modulation period. Because of the resonance, the results become quite independent of these two times. In fact, only the relative phase between the internal and the reinjected field given by (2) appears in the calculations through the constants C_i. All modulation periods equally spaced in time to give the same value of $\cos(\omega\tau)$ would also give the same mean switch-on time and variance. This is quite in contrast with other feedback situations, where any independent variation of these parameters (external round time or frequency modulation) is linked to significant changes in the switch-on time distribution [19,25]. As in the solitary laser case, the detuning parameter θ only affects phase dynamics, and is irrelevant in the calculation of switch-on time statistics [26].

III. ANALYTICAL RESULTS: CLASS-B LASERS

In class-B lasers, the slow dynamics of the population inversion does not allow an adiabatic elimination of this material variable. The simpler mathematical description of these lasers involves a set of two coupled stochastic nonlinear differential equations for both the slowly varying complex electric field and the population inversion. We will use from now on the time evolution equations for a semiconductor laser subjected to feedback [1]:

$$\frac{dE}{dt}=\frac{1+i\alpha}{2}\left[\frac{g(N-N_0)}{1+s|E|^2}-\gamma\right]E(t)+\kappa e^{-i\omega\tau}E(t-\tau)+\sqrt{2\beta N}\xi(t), \tag{13}$$

$$\frac{dN}{dt}=C-\gamma_e N-\frac{g(N-N_0)}{1+s|E|^2}|E|^2, \tag{14}$$

where $\xi(t)$ is a white Gaussian process taking into account spontaneous emission, with zero mean and a correlation given by $\langle\xi(t)\xi^*(t')\rangle=2\,\delta(t-t')$. Feedback and noise additive terms have the same meaning as in the previous section. The linewidth enhancement factor α couples the intensity and phase dynamics, resulting in changes of the emission frequency with carrier variations [26].

It was recently shown in [27] the intrinsic difference between Q switching and gain switching in class-B lasers. Our attention concentrates on gain switching because this operating condition is the most usual one in semiconductor lasers. On the other hand, the switching dynamics of Q-switched class-B lasers can be described in terms of that of class-A lasers, due to the fact that the losses switch is fast and the population inversion is clamped to a fixed value.

The presence of weak optical feedback [7] modifies the mean light intensity emitted in the off-state before the current switch to

$$I_0=\frac{4\beta C_b}{\gamma\gamma_e-g(C_b-\gamma_e N_0)-2\gamma_e\kappa\cos(\omega\tau)}. \tag{15}$$

Under conditions of moderate or strong feedback, the above expression must be replaced to give the actual mean values for both light intensity and carrier number. The presence of a significant amount of photons in the cavity due to the optical feedback reduces the initial carrier number through the nonlinear coupling in (14).

During the modulation period $T=T_{\rm on}+T_{\rm off}$, the pump parameter (current injection) takes a value $C=C_b$ below threshold in $T_{\rm off}$, while during $T_{\rm on}$ the pumping is increased to a value $C=C_{\rm on}$ above threshold. $T_{\rm off}$ must be large enough in order to guarantee the return to the same initial conditions for all the gain switches. The carrier lifetime γ_e^{-1} is the intrinsic parameter controlling the relaxation time of the population inversion to the steady state. For typical values of γ_e^{-1} in semiconductor lasers, $T_{\rm off}$ has to be much longer than the time needed to trigger the switch-on [28]. This limitation was not present for class-A lasers, where population inversion follows almost immediately the pump control.

Similar to the case of class-A lasers, our modulation scheme assumes $\tau=T$. Gain compression effects are irrelevant ($s=0$) during the switch-on. When the nonlinear term in the carrier equation can be neglected because of low intensity, population inversion dynamics decouple from light evolution. Carrier evolution can then be readily solved as [26]

$$N(t)=\frac{C_t}{\gamma_e}e^{-\gamma_e t}+\frac{C_{\rm on}}{\gamma_e}(1-e^{-\gamma_e t}), \tag{16}$$

where the time is set to $t=0$ when the population inversion crosses threshold after one current switch. The current value $C_t=\gamma_e(N_0+\gamma/g)$ corresponds to the threshold value for laser emission. The time delay between the current switch and the threshold crossing is given by

$$t_{\rm th}=\frac{1}{\gamma_e}\ln\left[\frac{C_{\rm on}-C_b}{C_{\rm on}-C_t}\right]. \tag{17}$$

As usual, the switch-on time is defined as the time delay between the current switch and light emission, and is therefore given by $t_{\rm th}+t$. Light amplification is only possible for carrier populations above threshold, when $t>0$. We use the decomposition $E(t)=h(t)e^{A(t)/2}$ to solve (13). Statistical equivalence of different light pulses leads to $E(t-\tau)=g(t)e^{A(t)/2}$, where

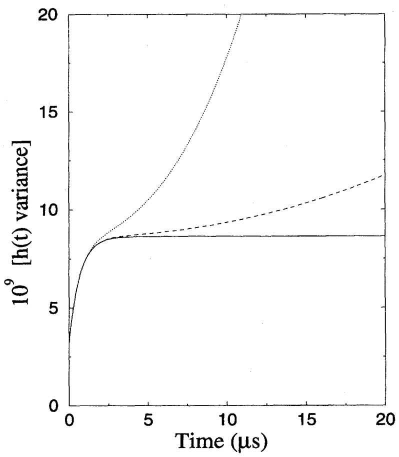

FIG. 1. Variance $\sigma_h^2(t)$ as a function of time for the class-A laser. The values correspond to the solitary laser (solid line) and to the feedback levels $\kappa=0.03$ μs^{-1} (dashed line) and $\kappa=0.1$ μs^{-1} (dotted line).

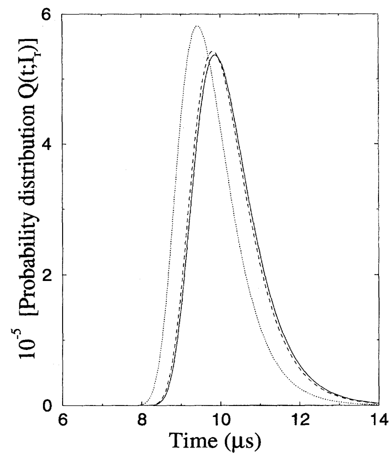

FIG. 2. Probability distribution of switch-on times $Q(t;I_r)$ for the class-A laser. The values correspond to the solitary laser (solid line) and to the feedback levels $\kappa=0.03$ μs^{-1} (dashed line) and $\kappa=0.1$ μs^{-1} (dotted line).

$$A(t)=(1+i\alpha)\int_0^t [g(N(t')-N_0)-\gamma]dt'. \tag{18}$$

Following the same procedure as in the previous section, we get $\langle h(t)\rangle=0$ and

$$\sigma_h^2(t)=\langle |h(t)|^2\rangle=\sigma_0^2(t)+\kappa^2\int_0^t dt'\int_0^t dt''\langle g(t')g^*(t'')\rangle, \tag{19}$$

where

$$\sigma_0^2(t)=I_0+4\beta\int_0^t N(t')e^{-\mathrm{Re}[A(t')]}dt'. \tag{20}$$

The relation (5) also holds for a solitary class-B laser. Using this relation in (19), Eqs. (6) and (8) can be reproduced and solved for $\sigma_h^2(t)$ to give

$$\begin{aligned}\sigma_h^2(t)=&I_0\cosh(\sqrt{2}\kappa t)+\frac{2\sqrt{2}\beta C_t}{\gamma_e\kappa}\sinh(\sqrt{2}\kappa t)\\&+\frac{\sqrt{2}\beta}{\kappa}\int_0^t[e^{-\sqrt{2}\kappa t'+\sqrt{2}\kappa t}-e^{\sqrt{2}\kappa t'-\sqrt{2}\kappa t}]e^{-\mathrm{Re}[A(t')]}\\&\times\{C_{\mathrm{on}}-\gamma_e N(t')-N(t')\mathrm{Re}[\dot{A}(t')]\}dt'.\end{aligned} \tag{21}$$

The switch-on time probability distribution can be calculated from $\sigma_h^2(t)$ as in the previous section (11) and (12) taking into account that $\langle I(t)\rangle=\sigma_h^2(t)e^{\mathrm{Re}[A(t)]}$. The switch-on time statistics is independent of the linewidth enhancement factor α. Conclusions about the role of the external round time and the frequency modulation are the same as those for class-A lasers.

IV. NUMERICAL RESULTS AND DISCUSSION

We show in Fig. 1 the variance $\sigma_h^2(t)$ for several feedback strengths (class-A dye laser). This function grows exponen-

TABLE I. Parameters corresponding to the class-A laser.

Parameter	Meaning	Value
g	Gain	1.323×10^7 s^{-1}
γ_1	Losses before switching	1.45×10^7 s^{-1}
γ_2	Losses after switching	1.25×10^7 s^{-1}
s	Gain saturation	0.0756
ε	Noise intensity	0.004 s^{-1}
ω	Angular frequency of light	2.48×10^{15} rad s^{-1}
θ	Cavity detuning	0.1
τ	External round-trip time	40×10^{-6} s
κ	Feedback strength	Variable
T_{on}	Time with net gain inside a period	20×10^{-6} s
T_{off}	Time with net losses inside a period	20×10^{-6} s
I_r	Intensity reference	0.015

TABLE II. Parameters corresponding to the class-B laser.

Parameter	Meaning	Value
g	Differential gain	5.6×10^4 s^{-1}
γ	Losses	4×10^{11} s^{-1}
γ_e	Inverse carrier lifetime	5×10^8 s^{-1}
N_0	Transparency value for the carrier number	6.8×10^7
s	Gain compression factor	10^{-6}
β	Spontaneous emission rate	1.1×10^4 s^{-1}
ω	Angular frequency of light	1.216×10^{15} rad s^{-1}
α	Linewidth enhancement factor	5
τ	External round-trip time	6500×10^{-12} s
κ	Feedback strength	variable
$C_{\rm on}$	Current above threshold	13.16×10^{16} s^{-1}
C_b	Current below threshold	3.68×10^{16} s^{-1}
$T_{\rm on}$	Time with the current on inside a period	90×10^{-12} s
$T_{\rm off}$	Time with the current off inside a period	6410×10^{-12} s
I_r	Intensity reference	23 500

tially for long times separating from the solitary laser behavior and giving an earlier switch-on. The probability distributions of switch-on time are plotted in Fig. 2. The introduction of feedback induces a shift to shorter times in the probability distribution and causes a narrower width. The usual asymmetric shape with long tails at long times [29] persists also when the laser is subjected to feedback. The same behavior can be shown for class-B lasers as well.

The stochastic differential equations (1), (13), and (14) have been numerically solved by means of a first-order Euler algorithm [30] taking into account the delayed additive terms. Statistics were performed after a significant number (100) of initial modulation periods. For the class-A laser, 10^4 turn-on events were considered in the averages to get the mean switch-on time and its variance. The integration time step was 0.01 μs. In the case of a class-B laser, these values were 10^3 and 0.01 ps, respectively. The laser parameters and the driving conditions of each laser are summarized in Tables I and II. The results of these numerical simulations are plotted in Figs. 3 and 4 for class-A and class-B lasers, respectively. The theoretical results of Secs. II and III are also plotted in these figures. Although the external reflector distance for the dye laser is unrealistically large (several kilometers) in order to meet resonance with the time constants involved, the analytical results are expected to apply to class-A lasers with arbitrary resonant modulation periods.

Figure 3 shows a good agreement between theory and simulation. The shift to short times in the probability distri-

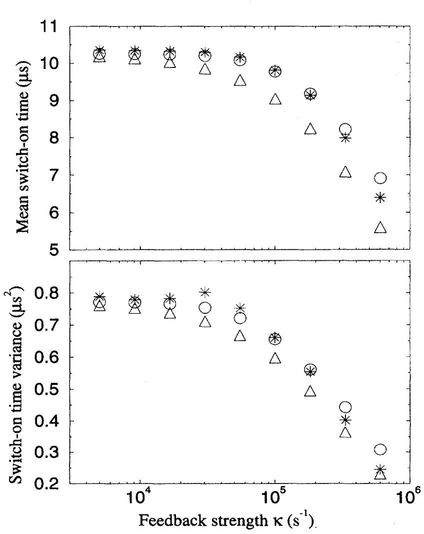

FIG. 3. Mean switch-on time and variance for the class-A laser at different feedback levels. The points correspond to the numerical simulation (asterisks) and to the theory developed (circles). The triangles show the result expected when the feedback comes from the same light pulse.

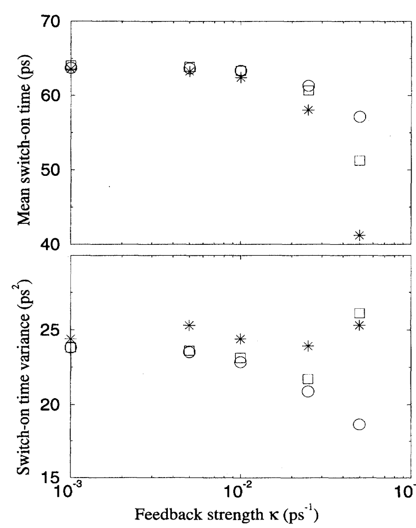

FIG. 4. Mean switch-on time and variance for the class-B laser at different feedback levels. The points correspond to the numerical simulation (asterisks) and to the theory developed (circles). The squares show the improvement in the theory when using the actual initial conditions for each light pulse.

bution is now evident. The mean switch-on time and its variance are significantly reduced when feedback is present. In our theory, feedback coming from previous light pulses has been considered. For completeness, we compare the results with those corresponding to feedback coming from the same light pulse $[E(t-\tau)=E(t)$ instead of $E(t-\tau)=E(t-T)]$. The reduction of the mean switch-on time and its variance starts at lower κ values, and is always larger than the one present with feedback from previous pulses. The explanation of this behavior lies in the reduction of the effective laser threshold (lower cavity losses) and the elimination of extra noise (intrinsically identical light pulses). Our theoretical model, developed in the previous sections, is able to distinguish the fine differences between these two feedback conditions.

The theoretical results for the class-B laser seem to be correct only at small feedback levels. In the calculations, the expression (15) was used to give account of the initial conditions for each switch-on. In fact, that expression is only valid for low κ values. We have checked in the numerical simulations that the mean value for the initial carrier number is significantly lower than C_b/γ_e and the mean intensity is much higher than the one given by (15). A lower carrier number increases the mean switch-on through $t_{\rm th}$, while a high initial intensity improves the seeding process, accelerating the switch-on. We include in Fig. 4 the results of the theory developed in Sec. III using the actual initial conditions for both carrier number and light intensity, as given by numerical simulations. In class-B lasers, the switch-on time variance turns out to be much less sensitive to feedback than in class-A lasers, even at strong feedback levels.

V. SUMMARY AND CONCLUSIONS

We have analytically calculated the mean switch on time and its variance for both class-A and -B lasers under periodic modulation, whether in losses or in gain, and subjected to optical feedback. We considered the special situation in which the period of the modulation T is equal to the external cavity round-trip time τ. For this situation our theory is valid at any cavity length. We observed that for weak feedback both the mean switch on time and its variance decrease with feedback strength. Our analytical results fit well with numerical simulation of noise driven rate equations. For class-A lasers the fit is especially good, while for class-B lasers the theory fails at feedback strengths for which the weak feedback condition is lost. From a practical point of view, we demonstrate an important jitter reduction when a reflector element is placed in front of a laser and the resonance with the modulation period is met.

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