Reply to "Comment on 'Solutions of the telegrapher's equation in the presence of traps'"

Jaume Masoliver and Josep Porrà
Departmento de Física Fonamental, Universitat de Barcelona, 08027 Barcelona, Spain

George H. Weiss
National Institutes of Health, Bethesda, Maryland 20892
(Received 13 April 1992)

This reply adds a number of details to remarks by Foong and Kanno [preceding Comment, Phys. Rev. A **46**, 5296 (1992)] on our paper [Phys. Rev. A **45**, 2222 (1992)] regarding the discontinuities observed in the curves generated in that paper.

PACS number(s): 05.40.+j, 42.25.Bs

This is written to add a number of details to the comment by Foong and Kanno [1] on our paper [2]. We are in fundamental agreement with the Foong and Kanno that the observed discontinuity in the curves of the pdf plotted as a function of y that we have derived is due to the trapping of random walkers which would have undergone one reversal if there were no trap. It is also possible to derive a formula for the magnitude of that discontinuity, Δp, using a probabilistic argument rather than from the solution of the telegrapher's equation. Let the position of the turning point be denoted by τ_c, where $\tau_c = \tau - y_0$ in which τ is the dimensionless time and y_0 the dimensionless initial point. The discontinuity is then given by

$$\Delta p = \Pr\{\text{particle is moving to the left at time } \tau\} \times \Pr\{\text{exactly one switching during } (0,\tau)\} \times \Pr\{\text{particle is in the interval } (\tau_c \pm \tau) | \text{one switch}\} .$$

We may regard the occurrence of switching times as being generated by a Poisson process in which the rate (in dimensionless units) is equal to $\frac{1}{2}$. The three contributions appearing in our expression for Δp are, consecutively, $\frac{1}{2}$, $(\tau/2)\exp(-\tau/2)$, and $1/(2\tau)$ with the result that

$$\Delta p = \tfrac{1}{8} e^{-\tau/2} , \tag{1}$$

which can be verified from our Eq. (19).

The result in Eq. (1) can also be calculated by taking into account those particles, which, after a single switching event, are reflected in a neighborhood of the trap. For this purpose let us calculate the pdf for a particle to be at y at time τ, having switched directions at $\tau_1 < \tau$. Let $Y(\tau|\tau_1)$ be the position of the particle at time τ conditional on a switch having been made at τ_1. The velocity of the particle is, in our dimensionless units, equal to ± 1. Since the particle is initially at y_0 and initially moves to the left before switching at τ_1, $Y(\tau|\tau_1)$ can be decomposed as

$$Y(\tau|\tau_1) = y_0 - \tau_1 + \tau - \tau_1 = y_0 + \tau - 2\tau_1 . \tag{2}$$

The probability that a particle switches exactly once during the time interval $(0,\tau)$, the switching time occurring during the infinitesimal time interval $(\tau_1, \tau_1 + d\tau_1)$ is equal to $\frac{1}{2} e^{-\tau/2} d\tau_1$, which must be multiplied by the probability that the initial motion is in the direction of decreasing y. Since it follows that Eq. (2) that $dy = -2d\tau_1$, we verify the correctness of Eq. (1). Finally, if $\tau_1 > y_0$ the particle is absorbed by the trap, which implies a discontinuity in the density profile.

We have also corrected [3] a number of misprints in an Erratum to the original paper.

[1] S. K. Foong and S. Kanno, preceding Comment, Phys. Rev. A **46**, 5296 (1992).
[2] J. Masoliver, J. Porrà, and G. H. Weiss, Phys. Rev. A **45**, 2222 (1992).
[3] J. Masoliver, J. Porrà, and G. H. Weiss, Phys. Rev. A **46**, 3574(E) (1992).

