

^3He impurity in a Bose-Einstein condensate

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(Received 11 February 2000; published 13 July 2000)

We analyze a configuration consisting of a ^3He droplet immersed in a Bose-Einstein condensate at $T \approx 0$. We investigate the stationary density and oscillations of the condensate, as well as the dynamics of the droplet in the frame of a simple model. Our paper takes into account the actual interaction potential between the ^3He and condensate atoms and reveals the stability of the dynamics of a small ^3He droplet inside the condensate.

PACS number(s): 03.75.Fi, 05.30.Jp, 32.80.Pj

I. INTRODUCTION

Since the beginning of experimental production of Bose-Einstein condensates via magnetic and magneto-optic cooling and trapping, it has been generally stressed that condensates are fragile and unstable against a variety of disturbances such as stray heating, field noise in the trap, inelastic collisions with background atoms and, in general, intrusions of foreign agents [1]. The theoretical treatment of these and other perturbations is delicate and largely model dependent, so that one can at most approach one particular issue in a very specific frame. In the present paper, our purpose is to analyze the effects of doping a boson condensate with an impurity with which the atoms can interact through a potential that exhibits an attractive well, in addition to an exclusion core.

In such a case, not only the structural and energetical modifications undergone by the condensate are a subject of interest, but also the impurity dynamics, together with the adiabatic evolution of the condensate in the presence of their mutual coupling, are worth a detailed analysis. For this sake, in order to fix ideas, we first present a simple Thomas-Fermi model for a boson condensate perturbed by a static atomic or molecular cluster located near its center. Simple estimates indicate that except for impurities macroscopically large with respect to the size of the trap, the excluded particles represent a very small fraction of the total number. Moreover, we improve the model with a definite selection of the cluster—a small ^3He droplet trapped in the magnetic potential well—and a relatively more realistic description of the impurity-atom interaction. We base this choice on the fact that it has been recently shown [2] that ^3He clusters are bound for particle numbers larger than 29 and that valence particles in open shell drops couple their spins to the maximum allowed value. This finding relies on a shell model-like calculation based on nonlocal density-functional theory and has been recently corroborated by a variational computation of energies of ^3He droplets, using Aziz two-body interactions and including two- and three-body Jastrow-like correlations [3]. In this case, the smallest bound drop contains 35 atoms,

slightly above the prediction of Ref. [2], and the calculation confirms the magnetic properties of these systems. The total magnetic moment of the cluster may then permit its confinement in a magnetic trap. We show that if such an impurity becomes trapped together with the bosonic atoms, its only effect upon the condensate is suppression of a moderate number of bosons from its vicinity; neither the density profile of the condensed particles, nor their low-energy excitations, are significantly disturbed by the foreign agent.

We perform a detailed calculation of the motion of the impurity, subjected to the force provided by the condensed density. Assuming various initial conditions, and also some simplifying calculatory assumptions that retain the main physics of the problem, we show that the cluster can undergo oscillations around the trap center. Anyway, its center of mass remains localized, at least during times comparable with the condensate lifetime. This outcome differs from previous findings by Chin and Forber [4], who predict expulsion of the molecule from the trap. To the best of our knowledge, the study of Ref. [4] is the only prior estimate of the response of a condensate to doping by a large molecular cluster; the calculation stresses the geometrical aspects of exclusion of bosons in the trap, induced by the presence of an impenetrable sphere, and on those grounds it has been concluded that due to the reaction of the excluded cloud, the impurity would be most likely expelled from the condensate. The result of the present paper is due to the fact that we have explicitly taken into account the attractive interaction well between atoms in the impurity and those in the condensate. This attraction is the source of the effective restoring force.

This paper is organized as follows. In Sec. II we present the Thomas-Fermi description of a condensate doped with a small ^3He cluster and in Sec. III, we discuss the stationary solutions of the Gross-Pitaevsky equations and the spectrum of small amplitude oscillations of the trapped atoms interacting with the impurity. In Sec. IV, we examine the dynamics of the drop. The final summary and conclusions are the subject of Sec. V.

II. THOMAS-FERMI DESCRIPTION OF A DOPED CONDENSATE

Let us assume that the condensed atoms interact with an impurity located at position \mathbf{R} in the trap through a Lennard-

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Jones (LJ) potential V_{LJ} with suppressed infinite core, so that the effective external field felt by the particles is

$$V(\mathbf{r}) = \frac{m\omega^2}{2}r^2 + 4\varepsilon \left[\left(\frac{\sigma}{|\mathbf{r}-\mathbf{R}|} \right)^{12} - \left(\frac{\sigma}{|\mathbf{r}-\mathbf{R}|} \right)^6 \right] \quad (2.1)$$

for $|\mathbf{r}-\mathbf{R}| \geq r_c$ and

$$V(\mathbf{r}) = \frac{m\omega^2}{2}r^2 + V_{\text{LJ}}(r_c) \quad (2.2)$$

for $|\mathbf{r}-\mathbf{R}| \leq r_c$, with r_c sufficiently small to guarantee that the chemical potential μ lies well below $V_{\text{LJ}}(r_c)$. For an atomic impurity, i.e., a molecule or a small atomic cluster, typical energy and length scales for the parameters of the LJ potential are a few K and a few Å (or fractions), respectively. The classical turning points $\mathbf{r}_{1,2}$ are thus defined by the relationship

$$\mu = V(\mathbf{r}_{1,2}). \quad (2.3)$$

For the sake of some estimates, it is convenient to consider that the impurity is motionless at the center of the trap, i.e., $\mathbf{R}=\mathbf{0}$. Since the size of the condensate amounts to several microns, and μ is, at most, around 1 μK , we can safely assume that r_1 and r_2 , respectively, coincide with the turning points of a pure LJ and pure harmonic trap potentials, respectively. Accordingly, we obtain

$$r_1 = \sigma \left(\frac{2\varepsilon}{\mu} \right)^{1/6} \left(\sqrt{1 + \frac{\mu}{\varepsilon}} - 1 \right)^{1/6}, \quad (2.4)$$

$$r_2 = \left(\frac{2\mu}{m\omega^2} \right)^{1/2}. \quad (2.5)$$

The Thomas-Fermi (TF) density thus reads [5,6]

$$\rho_{\text{TF}} = \frac{\mu - V_{\text{trap}} - V_{\text{LJ}}}{g} \Theta(\mu - V_{\text{trap}} - V_{\text{LJ}}) \quad (2.6)$$

with $V_{\text{trap}} = m\omega^2 r^2/2$ and $g = 4\pi\hbar^2 a/m$ the interaction parameter between the boson atoms, with a being the s -wave scattering length. It is worthwhile noticing that for a strongly attractive, highly localized LJ potential, the density (2.6) exhibits a large peak, whose presence might invalidate the TF approximation. However, as discussed further in Sec. III, explicit calculations of the density profile in the mean-field description support the quantitative analysis presented below.

As usual, the chemical potential μ is fixed by particle number conservation, namely,

$$N = \int_{r_1}^{r_2} d^3r \rho_{\text{TF}}(r) = N_{\text{trap}}(\mu, \omega) + N_{\text{LJ}}(\mu, \omega, \sigma, \varepsilon) \quad (2.7)$$

with

$$N_{\text{trap}} = \int_0^{r_2} d^3r \frac{\mu - V_{\text{trap}}}{g} = \frac{8\pi}{15g} \left(\frac{2}{m\omega^2} \right)^{3/2} \mu^{5/2}, \quad (2.8)$$

the number of atoms held by the bare harmonic trap and

$$N_{\text{LJ}} = - \int_0^{r_1} d^3r \frac{\mu - V_{\text{trap}}}{g} - \int_{r_1}^{r_2} d^3r \frac{V_{\text{LJ}}}{g} \quad (2.9)$$

the amount of atoms expelled by the LJ exclusion core, that become concentrated at the large density peak sitting at the LJ well.

Taking advantage of the fact that the LJ potential is negligible at r_2 and that μ/ε is much smaller than unity—which in turns permits us to approximate $r_1 \approx \sigma + o(\mu/\varepsilon)$ —we obtain

$$N_{\text{LJ}}(\mu, \omega, \sigma, \varepsilon) = \frac{4\pi}{g} \left(-\frac{\mu\sigma^3}{3} + \frac{m\omega^2}{10}\sigma^5 + \frac{8}{9}\varepsilon\sigma^3 \right). \quad (2.10)$$

The relative magnitudes of the terms in this expression become clearer if we introduce some typical scales; $\tilde{\varepsilon} = \varepsilon/\hbar\omega$, $\tilde{\mu} = \mu/\hbar\omega$, $a_{\text{osc}} = \sqrt{\hbar/m\omega}$, we can write

$$N_{\text{LJ}} = \frac{\sigma^3}{3a_{\text{osc}}^2 a} \left(\frac{8\tilde{\varepsilon}}{3} - \tilde{\mu} \right) + \frac{\sigma^5}{10a_{\text{osc}}^4 a} \approx \frac{8}{9} \frac{\sigma^3}{a_{\text{osc}}^2 a} \tilde{\varepsilon}. \quad (2.11)$$

The last line holds in view of the fact that $\tilde{\varepsilon}$ is around six orders of magnitude larger than $\tilde{\mu}$, while σ is comparable with the scattering length a , thus three orders of magnitude below a_{osc} . Furthermore, it shows that the number of excluded particles is of order unity and independent of the total number of atoms in the condensate. Notice that the second term in Eq. (2.11) can be sizeable for an impurity core σ much larger than a_{osc} ; in this case, the first term becomes macroscopically large as well. However, if σ becomes comparable with the condensate diameter, the simple TF model becomes invalid. On the other hand, sensitivity of N_{LJ} to the total particle number could only be restored at the expense of drastically suppressing the attractive strength ε , which leaves one with the hard-core interaction discussed in Ref. [4]; in such a case, $N_{\text{LJ}} = -\sigma^3 \tilde{\mu}/3a_{\text{osc}}^2 a + \sigma^5/10a_{\text{osc}}^4 a$ would be the number of bosonic atoms excluded by the hard sphere.

Similar considerations allow us to derive a first correction to the chemical potential of the bosons in the deformed trap; Eq. (2.7) is an implicit one whose zeroth order approximation corresponds to μ_0 defined by Eq. (2.8). A first iteration then gives

$$\begin{aligned} N_{\text{LJ}}(\mu, \omega, \sigma, \varepsilon) &\approx N_{\text{LJ}}(\mu_0, \omega, \sigma, \varepsilon) \\ &= \frac{4\pi}{g} \left[-\left(\frac{15g}{8\pi} N \right)^{2/5} \left(\frac{m\omega^2}{2} \right)^{3/5} \frac{\sigma^3}{3} \right. \\ &\quad \left. + \frac{m\omega^2}{10} \sigma^5 + \frac{8}{9} \varepsilon \sigma^3 \right]. \end{aligned} \quad (2.12)$$

We now consider

$$N = N_{\text{trap}}(\mu) + N_{\text{LJ}}(\mu_0). \quad (2.13)$$

Using Eqs. (2.8) and (2.12), this relation can be explicitly solved, giving

$$\mu \approx \mu_0 \left\{ 1 + \frac{\sigma^3}{\lambda^3} \left[\frac{1}{(15N)^{3/5}} - \frac{\sigma^2}{25\lambda^2 N} - \frac{16}{45\pi N} \frac{\tilde{\epsilon}}{(\lambda/a_{\text{osc}})^2} \right] \right\}. \quad (2.14)$$

We have here defined a length scale λ as

$$\lambda^5 = a_{\text{osc}}^4 a \quad (2.15)$$

which, for $a \approx 10^{-3} a_{\text{osc}}$ takes the value $\lambda \approx 0.25 a_{\text{osc}}$. For the sizes and energies under consideration, this gives

$$\mu \approx \mu_0 \left(1 - \frac{0.004}{N} \right). \quad (2.16)$$

It is then clear that for currently large condensates, no modification in the chemical potential is to be expected in the presence of such perturbations. On the other hand, it should be kept in mind that if the impurity core σ becomes comparable with a_{osc} , the assumptions giving rise to Eqs. (2.11) and (2.14) are no longer valid.

III. THE ³He CLUSTER

In this section we develop a more specific model in which the impurity with phase-space coordinates (\mathbf{R}, \mathbf{P}) is a small droplet of liquid ³He, whose density profile $\rho_3(\mathbf{r}_{\text{He}})$ is computed in the density-functional frame of Ref. [7]. The trapped bosons are alkali atoms and the ³He-alkali interaction potential V_{3-a} is taken from Ref. [8]. The total Hamiltonian is then written as

$$\begin{aligned} H[\rho(\mathbf{r}), \mathbf{R}, \mathbf{P}] = & \int d\mathbf{r} \left[\frac{\hbar^2}{2m} \left| \nabla \sqrt{\rho(\mathbf{r})} \right|^2 \right. \\ & + V_{\text{trap}}(\mathbf{r}) \rho(\mathbf{r}) + \frac{g}{2} \rho^2(\mathbf{r}) \left. \right] + \frac{P^2}{2M} \\ & + \int \int d\mathbf{r} d\mathbf{r}_{\text{He}} \rho(\mathbf{r}) \rho_3(\mathbf{r}_{\text{He}}) V_{3-a}(\mathbf{r}, \mathbf{r}_{\text{He}}). \end{aligned} \quad (3.1)$$

In Eq. (3.1), the impurity is treated as a structureless classical object with kinetic energy $P^2/2M$ and the first term is the usual Gross-Pitaevski (GP) density functional, which together with the coupling gives rise to an equation-of-motion for the condensate density of the form

$$\mu \sqrt{\rho(\mathbf{r})} = \left[-\frac{\hbar^2}{2m} \nabla^2 + V_{\text{eff}}(\mathbf{r}) + g\rho(\mathbf{r}) \right] \sqrt{\rho(\mathbf{r})} \quad (3.2)$$

with the effective external potential

$$V_{\text{eff}} = V_{\text{trap}} + \int d\mathbf{r}_{\text{He}} \rho_3(\mathbf{r}_{\text{He}}) V_{3-a}(\mathbf{r}, \mathbf{r}_{\text{He}}). \quad (3.3)$$

Furthermore, throughout this paper we disregard the restoring force exerted on the impurity by the magnetic trap,

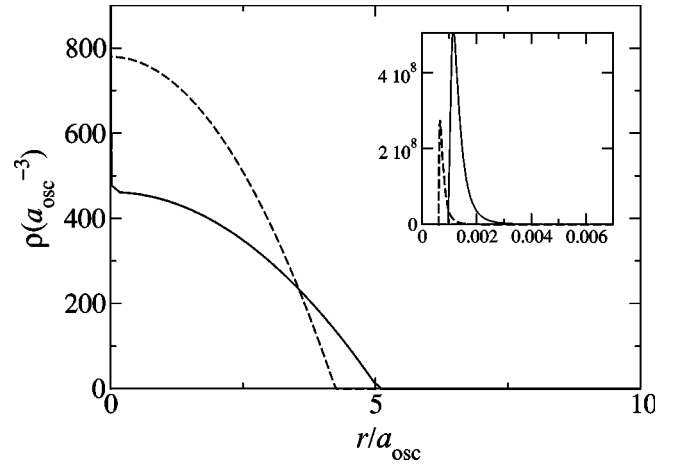


FIG. 1. Density profiles of condensates of Rb (full line) and Na (dashed line) for particle number $N=10^5$. The inset shows a zoom of the density around the potential minimum.

since our aim is to focus upon the mutual interaction effects between cluster and condensate, since such a restoring force would only enhance the condensate action.

Assuming that the impurity remains motionless at $\mathbf{R}=\mathbf{0}$, we have performed a series of calculations solving the above modified GP equation for alkali condensates of Rb, Na, and Li, as well as for polarized hydrogen; in spite of the relative differences in the description of the cluster-atom interaction, the overall effects remain identical. A typical droplet consistent with the simplifying assumptions of this paper contains 30 helium atoms; according to Ref. [2], this is the smallest, even atom number for which the droplet is bound and exhibits a large magnetic moment.

Density-functional theory permits a full calculation of the structural aspects of the helium densities interacting with the condensate-alkali density. A combined energy functional can be derived as the sum of the GP energy plus a density functional for the ³He system, which is presently available [9], and a condensate-alkali interaction energy term. This leads to a system of coupled GP-like equations for both the boson and the ³He density profiles, which provides the correct dynamics of the whole system. However, application of this procedure would imply a much higher computational cost with little extra insight in the main issues addressed in this paper; a self-consistent density profile for the helium cluster would only slightly modify the details of the effective potential (3.3) at the small scale level, preserving its general shape and the qualitative characteristics of the driving forces. It should be kept in mind that while the condensate is a dilute, the impurity is a highly incompressible liquid cluster [10,11].

The density profiles and effective external fields felt by the bosonic atoms are, respectively, displayed in Figs. 1 and 2 for Rb and Na condensates carrying $N=10^5$ particles. In Fig. 1 we observe that for radii greater than the droplet core, the condensate density is similar to that in a standard harmonic trap; the effect of the impurity can only be seen on a much smaller scale. The cluster drills a hole in the density of the size of the V_{3-a} core, however the density develops a strong peak around the potential minimum. This can be understood if one compares the minimum interaction energy of -0.01 K (see Fig. 2) with the nanoKelvin energy scale im-

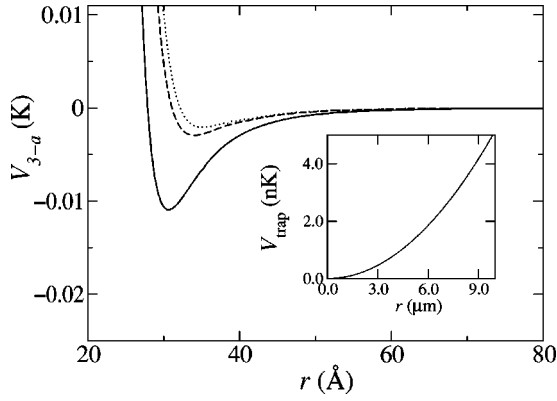


FIG. 2. External potentials seen by the atoms. Full, dashed and dotted lines correspond to Rb, Na, and Li atoms, respectively. The inset shows the standard harmonic trap potential (in K) for the Rb atoms and $\omega = 100 \text{ s}^{-1}$. Notice that the scale a_{osc} is roughly 10^4 \AA .

posed by the condensate, namely, the chemical potential μ . We must also point out that both the density peak, relative to the unperturbed value at $r=0$, and the hole width, decrease with increasing number of particles N . This can be visualized taking into account the growth of the chemical potential with total particle number as $N^{2/5}$.

In order to estimate the quality of the TF approach we have compared the TF and the GP density profiles, finding that for the number of particles addressed in this paper no visible difference appears. We have also verified the validity of Eq. (2.11) by integrating the peak density of the GP solution shown in Fig. 1.

Furthermore, we have computed the hydrodynamic, low-energy excitation spectrum of the perturbed condensates that correspond to the stationary solutions of the linearized equations-of-motion for density fluctuations $\delta\rho(\mathbf{r},t) = \exp(-i\Omega t)\delta\rho(\mathbf{r})$ [12],

$$m\Omega^2\delta\rho = -\nabla \cdot [(\mu - V_{\text{eff}})\nabla\delta\rho]. \quad (3.4)$$

The solutions of this equation have the form $\delta\rho(\mathbf{r}) = P_l^{(2n)}(r)r^l Y_{lm}(\theta, \phi)$ where the polynomials $P_l^{(2n)}(r)$ are defined inside the condensate and satisfy the orthogonality condition

$$\int_{r_1}^{r_2} P_l^{(2n)}(r)P_l^{(2n')}(r)r^{2l+2}dr = 0 \quad \text{if } n \neq n'.$$

The numerical solution of Eq. (3.4) is performed in the standard way, expanding $\delta\rho$ in a suitable basis and then diagonalizing the resulting matrix; it should be emphasized however that extra care has to be taken when dealing with the different scales of the condensate density.

We have made calculations for excitations with $l=0,1$ as a function of the chemical potential, for a condensate of Rb atoms subjected to the interaction with a cluster with 30 helium atoms. The relative frequency correction $\chi^2 = 100(\Omega^2 - \Omega_0^2)/\Omega_0^2$, with Ω_0^2 the excitation frequency in a pure harmonic trap, is plotted in Fig. 3 as a function of the chemical potential. We observe that χ^2 is completely negligible for moderate to large values of N . The corrections are

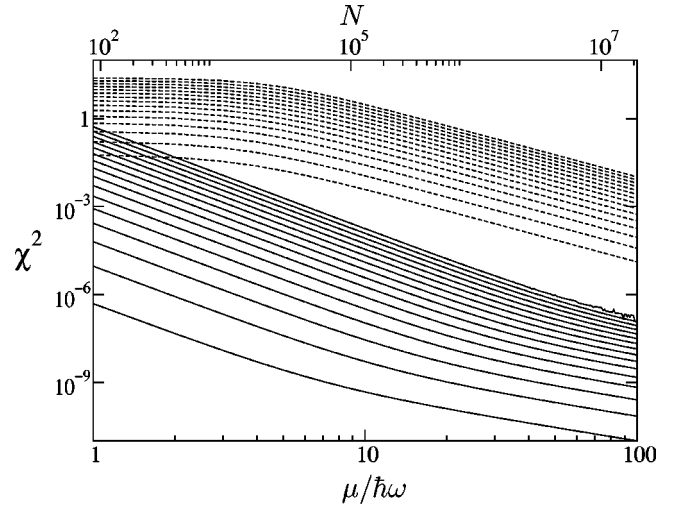


FIG. 3. Log-log plot of χ^2 , the relative correction to the eigenfrequencies of the condensate (see the text). Full (dashed) lines correspond to excitations with $l=0$ ($l=1$). The different lines with the same linestyle distinguish among the radial quantum numbers n , corresponding a lower n to a lower line.

larger for $l=1$ than for $l=0$ and for a given value of angular momentum, the frequency shift increases with increasing radial quantum number n . This can be interpreted in view of the fact that the density peak develops along the radius of the attractive well, thus its motion can only be triggered by the smallest wavelengths, i.e., $\lambda \leq \lambda_{\text{crit}} \approx \sigma$, in other words, by high-energy excitations, which are then the most sensitive to its presence.

IV. THE IMPURITY MOTION

We now analyze the effects of coupling between condensate and impurity motion. To simplify the description, we derive an effective droplet-atom interaction folding the interaction V_{3-a} from Refs. [8] and [13] with the density $\rho_3(\mathbf{r}_{He})$ computed as in Ref. [7] [cf. Eq. (3.1)] and fitting the result to a LJ potential, the corresponding parameters being $\sigma = 27 \text{ \AA}$ and $\varepsilon = 0.011 \text{ K}$ (see Fig. 4). The dynamics generated by the Hamiltonian (3.1) is contained in the time-dependent GP equation for the condensate density $\rho(\mathbf{r})$ in the presence of a total external field of the form (2.1), together with the classical equation of motion

$$M \frac{d^2 \mathbf{R}}{dt^2} = -\nabla_{\mathbf{R}} \int d\mathbf{r} V_{\text{LJ}}(|\mathbf{r} - \mathbf{R}|) \rho(\mathbf{r}). \quad (4.1)$$

At this point we incorporate a further simplification to the numerical resolution of this coupled dynamics as follows. Since the impurity is a macroscopic object with respect to the individual atoms, one can safely assume that its motion takes place in a much larger characteristic time that the equilibration scale of the condensate. The boson dynamics could then be regarded as adiabatic with respect to the impurity motion. This hypothesis can be also supported by the fact, already discussed at the end of the previous section, that a localized perturbation such as the droplet can only disturb the highest frequency modes of the condensate motion, however at the very small rate shown there.

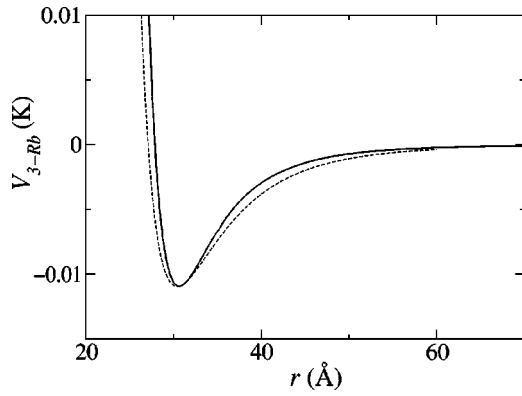


FIG. 4. Interaction potential between one Rb atom and the He cluster. The full line corresponds to the folded interaction in Eq. (3.3), the dashed line shows the fitted Lennard-Jones potential used in the calculations. Notice that the moderate spread of the approximate potential with respect to the folded one is nonrelevant within the condensate scales.

Moreover, we choose to describe the instantaneous condensate density by its TF approximation (2.6). As long as the dynamics of the condensate remains adiabatic this approximation is consistent with the TF approximation for the static density discussed in Secs. II and III. The coupled dynamics is then iteratively solved according to the sequence (i) compute the force exerted by an unperturbed condensate on a helium droplet initially located at $(\mathbf{R}_0, \mathbf{P}_0)$, (ii) allow the cluster to undergo a small displacement $\Delta \mathbf{R}$ during a short time interval Δt , (iii) compute the new condensate TF density due to the effective external field created by an impurity at $\mathbf{R}_0 + \Delta \mathbf{R}$ and back to (i) for the next displacement. We have verified that this numerical sequence is stable and energy conserving; in addition, we have found that a peculiarity of the present description is the fact that if one replaces the true condensate density $\rho(\mathbf{r}, t)$ in Eq. (4.1) by its TF approximation $\rho_{\text{TF}}[\mathbf{r}, \mathbf{R}(t)]$, the Jacobi integral

$$E = \frac{1}{2} M \left(\frac{d\mathbf{R}}{dt} \right)^2 - \int \mathbf{F}(\mathbf{R}) \cdot d\mathbf{R} \quad (4.2)$$

with

$$\mathbf{F}(\mathbf{R}) = - \int d\mathbf{r} [\nabla_{\mathbf{R}} V_{LJ}(|\mathbf{r} - \mathbf{R}|)] \rho_{\text{TF}}(\mathbf{r}, \mathbf{R}) \quad (4.3)$$

is a first integral of the motion. Numerical conservation of this quantity is then a fingerprint of desired stability and we have verified that this quantity is conserved within 1% during, at least 0.1 s.

As an illustration of this kind of motion, we show in Fig. 5(a) the potential energy $\mathcal{W}_F = - \int F(R) dR$ appearing by Eq. (4.2); (b) the associated phase-space dynamics and (c) the z -coordinate as a function of time. The impurity was released in a Rb condensate with 10^5 particles at a position $\mathbf{R}_0 = a_{\text{osc}} = 2.8 \mu\text{m} \hat{z}$ with zero velocity. Regardless the initial condition, the resulting motion is oscillatory around the center of the trap with a period $\tau \approx 21$ ms. In part (c) of Fig. 5 we observe that the evolution does not take place at constant

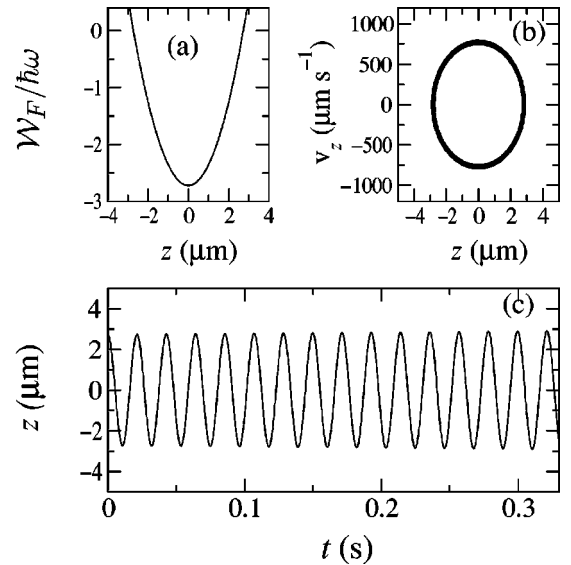


FIG. 5. Dynamics of the He droplet in the Rb condensate. (a) Potential energy \mathcal{W}_F felt by the droplet at a position z , (b) phase-space dynamics, and (c) motion along the z axis.

amplitude; in fact, we have verified that the amplitude increases exponentially with a time constant of 12 s. This outcome, however, is only a byproduct of the numerical scheme utilized and is in accordance with the accuracy mentioned in the preceding paragraph.

Due to the simplifying assumptions made on the interaction potential and condensate dynamics it is possible to extract an approximate analytic expression for the restoring coefficient k of the interaction force (4.3) in the z direction. We proceed to integrate Eq. (4.3) in the domain fixed by the TF density, compute $\partial F_z(z)/\partial z|_{z=0}$, and expand the result for large ϵ and small σ ; we thus find

$$k = m \omega^2 \frac{\sigma^3}{a_{\text{osc}}^2 a} \left[\frac{8\tilde{\epsilon} + 3\tilde{\mu}}{9} - \frac{8}{3} \tilde{\epsilon} \left(\frac{\sigma}{r_2} \right)^3 + \frac{8\tilde{\epsilon} - \tilde{\mu}}{4} \left(\frac{\sigma}{r_2} \right)^6 + \dots \right] \quad (4.4)$$

and then to leading order, we can calculate an approximate period

$$\tau_{\text{app}} = 2\pi \sqrt{\frac{M}{k}} = \frac{1}{\omega} \left(\frac{M}{m} \frac{a_{\text{osc}}^2 a}{\sigma^3} \right)^{1/2} \frac{6\pi}{(8\tilde{\epsilon} + 3\tilde{\mu})^{1/2}}. \quad (4.5)$$

From this expression we see that the oscillation time is indeed weakly dependent on the chemical potential, or number of particles, of the condensate. To verify this fact we have performed a series of calculations with the same initial conditions and lower numbers of particles. We have found that for particles ranging from 10^3 to 10^5 the oscillation period was constant within our numerical accuracy, being $\tau = (21.4 \pm 0.1)$ ms, while according to Eq. (4.5) the approximate period is $\tau_{\text{app}} = 21.6$ ms with a spread smaller than 1 ppm in the whole range.

Throughout this section, we have only considered the one-dimensional motion of the droplet. Indeed, since the total force on the cluster is a central one, its angular momentum is conserved and the three-dimensional dynamics becomes a trivial issue. A feature to remark about is that the

TABLE I. Exact force F_z according to Eq. (4.3) vs approximate expression $-kz$ felt by the droplet at z . Data correspond to a Rb condensate carrying 10^5 particles.

$z(a_{osc})$	$F_z(\hbar\omega/a_{osc})$	$-kz(\hbar\omega/a_{osc})$
1.0	-6.131 2026	-6.131 1953
2.0	-12.262 3909	-12.262 3906
3.0	-18.393 5852	-18.393 5859
4.0	-24.524 7802	-24.524 7812
5.0	-30.655 9371	-30.655 9766

restoring force is not strictly linear in coordinate space; however, we have verified that within this model, due to the substantial difference between condensate and impurity scales, linearity is an excellent approximation. This is illustrated in Table I. In fact, we have observed that the three-dimensional (3D) orbits appears as closed trajectories within the lifetime of the condensate, indicating that the rotation induced by the nonlinear terms in the restoring force are unimportant. Furthermore, the quasiperiod of these orbits is a characteristic of the system and is unaffected by the diverse initial conditions.

V. DISCUSSION AND SUMMARY

In this paper we have examined the structural and energetical modifications undergone by a boson condensate in the presence of a static doping agent that provides an extra external field, and also the dynamics of the impurity subjected to the average force exerted by the atomic cloud. As specific input, we have considered a small ^3He cluster, since we can provide a reasonable description of both the helium atom-alkali atom interaction and of the droplet density. In view of its nonvanishing magnetic moment, the droplet can be trapped by the external field, however throughout this paper we have disregarded the magnetic force on the droplet in order to stress the role of the coupling between the impurity and the condensate, which is the most important contribution to the total restoring force.

The main results of this paper are, on the one hand, the fact that for the scales considered, i.e., impurity and condensate sizes, the effects of the doping agent upon the cloud are essentially negligible, meaning that the ground state and low

excitations of such condensates are robust against perturbations induced by such probes. Stability of the condensate prevails, in spite of the fact that the repulsive core of the cluster-atom interaction excludes some small fraction of the trap volume, which is however, of limited relevance regarding the overall size of the boson system. On the other hand, the impurity itself is sensitive to the surrounding cloud, which impresses an oscillatory motion around the center of the trap. This is due to the fact that the helium cluster feels an average effective restoring force, created by the boson atoms.

It is important to remark that in spite of the positiveness of the s -wave scattering length for such an interaction, the attraction is responsible for the existence of an effective, macroscopic restoring force exerted by the atomic cloud on the cluster. We have in fact verified that the s and p phase shifts correspond to predominant repulsion, but it should be kept in mind that insofar as the impurity is regarded as a macroscopic scattering center, phase shifts and scattering lengths are parameters characterizing dispersion of the individual bosonic atoms. The motion of this scattering center is due to the average, combined action of the cloud, which in the present case, strongly enhances the attractive part of the interaction.

We also stress that all the considerations in this section exclusively concern the present results, which are, in this sense, rather model-dependent. A different type of coupling would not necessarily give rise to a similar outcome, as evidenced by the conclusions of Ref. [4], where a purely repulsive interaction gives rise to expulsion of the impurity. However, these distinct results indicate that impurity dynamics in the bulk of a boson condensate is a nontrivial issue that merits deeper investigation, which in any specific situation requires an appropriate description of the interaction.

ACKNOWLEDGMENTS

This paper was supported by Grant No. PICT 1706 from the Agencia Nacional de Promoción Científica y Tecnológica of Argentina, Grant No. TW81 from the Universidad de Buenos Aires, Grant No. PB98-1247 from DGESIC, Spain, and Grant No. 1998SGR-00011 from the Generalitat of Catalonia. One of us (P.C.) is grateful to the Universidad de Buenos Aires for financial support.

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