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Riskcenter

# On the practical implementation of retirement gains by using an upside and a downside terminal wealth constraint

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**Abstract**

We analyze an investment strategy for an investor with a savings plan for retirement consisting on constraining the terminal wealth accumulated after the savings period by setting an upper and lower bound. We carry out a simulation of the terminal wealth after a savings period of thirty years by using daily, monthly, weekly and yearly updates. We calculate the percentiles of the final wealth and the corresponding lifetime annuity that the pension saver will receive during the consumption period. We observe how that the simulated values converge to the theoretical values of the percentiles when the frequency of update increases. Finally, in the numerical example the effect of inflation is also considered.

## 1 Introduction

We carry out a practical implementation of a new investment strategy for retirement where the investor establishes and upper and a lower bound on the terminal wealth. An investment plan is usually characterized by a period of savings followed by a period of consumptions. We analyze the problem of setting a dynamic investment strategy where an initial wealth is invested in order to reach a target capital at the end of the savings period which is bounded by some guaranteed upper and lower bounds. The proposed mechanism results in a transparent and automatic investment product where the portfolio is rebalanced automatically so that the accumulated wealth at any moment is constraint by the lower and upper bounds.

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Donnelly et al. (2015) recently solved the portfolio selection problem of an investor with a deterministic savings plan who is constrained to have no more than a target wealth at retirement (an upper bound). Here we extend the results of Donnelly et al. (2015) by adding also a lower bound to the terminal wealth and implementing a numerical application. So, we assume that investors are willing to give up large gains if a minimum terminal wealth is guaranteed. Therefore, our contribution is about the reduction of the uncertainty of the terminal wealth being too large or too low, by providing an smoothing mechanism which includes an embedded guarantee on the terminal wealth.

The paper is organized as follows. In section 2 the background is presented. In section 3 we present the mathematical problem to solve. In section 4 we carry out the numerical illustration. Section 5 concludes.

# 2 Background

It is necessary to analyze the stochastic distribution of retirement wealth for proposing investment strategies (Greninger et al., 2000; Basu et al., 2011; Grossman and Zhou, 1996; Browne, 1999).

There are many authors introducing some constraint on the portfolio or the terminal wealth. Namely, Grossman and Zhou (1996) impose the constraint that the terminal wealth must be at least some fraction of the initial wealth. On the other hand, Korn and Trautmann (1995) set a constraint on the expected value of the final wealth.

Recently, Donnelly et al. (2015) found that by constraining the final wealth by using an upper bound, the investor increases their chance of attaining the desired target retirement wealth, and even if he fails to reach it, he still has a higher wealth than if he has no such upper bond. Note that Donnelly et al. (2015) proposed a different formulation compared to Dhaene et al. (2005), in which at least the target capital is attained with maximum probability. Donnelly et al. (2015) have also a different approach compared to Browne (1999), as Browne (1999) maximizes directly the probability of reaching the target retirement wealth. Here we consider the same approach as Donnelly et al. (2015) but adding also a lower bound for the final wealth. Note that our approach is also different from Gerrard et al. (2014) who analyzed the lowest part of the terminal wealth distribution after savings and consumption.

Here we concentrate on the savings phase (by choosing a saving period of thirty years) and we constrain the terminal wealth by using an upper and a lower bound. Additionally, we also provide the corresponding values of the lifetime annuities that the pension saver could receive during the consumption period. Other relevant contributions where some constrain on the terminal wealth is introduced can be found in Van Weert et al. (2010); Bouchard et al. (2010); Gaibh et al. (2009); Boyle and Tian (2007); Cuoco (1997); Zariphopoulou (1994), among others.

# 3 Presentation of mathematical problem to solve

We assume investment in a continuous-time market model over a finite time horizon $[0, T]$ for an integer $T > 0$. We also refer to $T$ as the *terminal time*.

The market consists of one risky stock and one risk-free bond. The price of the stock is driven by an 1-dimensional, standard Brownian motion $W = \{W(t); t \in [0,T]\}$. The Brownian motion is defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$. The risk-free bond has price process $\{S_0(t); t \in [0,T]\}$ and the risky stock has price process $\{S_1(t); t \in [0,T]\}$ with dynamics

$$\mathrm{d}S_0(t) = r_{\mathrm{N}} S_0(t)\,\mathrm{d}t, \qquad \mathrm{d}S_1(t) = S_1(t)\left(\mu dt + \sigma dW(t)\right), \tag{3.1}$$

with $\sigma > 0$, $S_0(0) = 1$ and $S_1(0)$ being a fixed, strictly positive constant. We assume that $\mu > r_{\mathrm{N}}$.

The information available to investors is represented by the filtration

$$\mathcal{F}_t := \sigma\{W(s), s \in [0,t]\} \vee \mathcal{N}(\mathbb{P}), \quad \forall t \in [0,T], \tag{3.2}$$

where $\mathcal{N}(\mathbb{P})$ denotes the collection of all $\mathbb{P}$-null events in the probability space $(\Omega, \mathcal{F}, \mathbb{P})$.

Define the usual $\mathbb{R}$-valued market price of risk

$$\theta := \frac{\mu - r_{\mathrm{N}}}{\sigma}.$$

## 3.1 Investor

An investor starts with a fixed non-random initial wealth $x_0 > 0$ and plans to make a sequence of known future savings $a > 0$. Define $C(t)$ to be the sum from time 0 to time $t$ of the investor's planned discrete savings, with

$$dC(t) = \begin{cases} a & \text{if } t = 1, 2, \ldots, T-1 \\ 0 & \text{otherwise.} \end{cases}$$

In other words, at the end of each unit time period, the investor pays an amount $a > 0$ into their fund.

A portfolio process $\pi = \{\pi(t); t \in [0,T]\}$ is a $\mathbb{R}$-valued, square-integrable, $\{\mathcal{F}_t\}$-progressively measurable process. The investor follows a self-financed strategy, investing at each instant $t \in [0,T]$ a monetary amount $\pi(t)$ in the stock such that the $\pi = \{\pi(t); t \in [0,T]\}$ is a portfolio process.

The wealth process $X^\pi = \{X^\pi(t); t \in [0,T]\}$ corresponding to a portfolio process is the $\{\mathcal{F}_t\}$-adapted, $\mathbb{R}$-valued process given by the *wealth equation*

$$dX^\pi(t) = \left(r_{\mathrm{N}} X^\pi(t) + \pi(t)\sigma\theta\right) dt + \pi(t)\sigma\, dW(t) + dC(t), \quad X^\pi(0) = x_0 \text{ a.s.} \tag{3.3}$$

Define the *savings plan* $g$ of the investor, i.e. the discounted sum of the future savings by the investor by

$$g(t) := \int_t^T e^{-r_{\mathrm{N}}(s-t)} dC(s), \quad \forall t \in [0,T]. \tag{3.4}$$

Then the set of *admissible portfolios* for the investor's initial wealth $x_0 > 0$ is defined to be

$$\mathcal{A} := \{\pi : \Omega \times [0,T] \to \mathbb{R} : X^\pi(0) = x_0, \text{ a.s. and } X^\pi(t) + g(t) \geq 0, t \in (0,T] \text{ a.s.}\}.$$

We say that a portfolio process $\pi$ is *admissible* if $\pi \in \mathcal{A}$.

Define the state price density process $H$ as $H(t) := \exp\left(-\left(r_{\mathrm{N}} + \frac{1}{2}\theta^2\right) t - \theta W(t)\right)$, for each $t \in [0, T]$. A portfolio $\pi$ must satisfy the *budget constraint* that

$$\mathbb{E}\left(H(T) X^{\pi}(T)\right) \leq x_0 + g(0). \tag{3.5}$$

The utility function of the investor is the power utility function

$$U(x) := \frac{1}{\gamma} x^{\gamma}, \qquad x > 0,$$

for a fixed constant $\gamma \in (-\infty, 1) \setminus \{0\}$. The investor seeks to maximise the expected utility of their terminal wealth, subject to constraints on the range of values of the terminal wealth.

Define the constant

$$A := \frac{\theta}{\sigma(1-\gamma)}$$

and the process

$$Z(t) = \exp\left(\left(r_{\mathrm{N}} + \theta\sigma A - \frac{1}{2}\sigma^2 A^2\right) t + \sigma A W(t)\right), \quad \forall t \in [0, T]. \tag{3.6}$$

## 3.2 Problem with an upper bound

Donnelly et al. (2015) introduced the constrained problem with an upper bound only, in which the investor seeks to maximize the expected utility of their terminal wealth, subject to the wealth being bounded above by the upper bound $K_U > 0$.

In order to avoid the uninteresting case that the investor can immediately be assured of maximizing the terminal utility, Donnelly et al. (2015) assume that $(x_0 + g(0))e^{r_{\mathrm{N}} T} < K_U$ and solve the following problem (Problem 4.1 in Donnelly et al. (2015)):

**Problem 3.1.** Find $\pi^{\star} \in \mathcal{A}$ such that

$$\mathbb{E}\left(U(X^{\pi^{\star}}(T))\right) = \sup_{\pi \in \mathcal{A}} \{\mathbb{E}\left(U(X^{\pi}(T))\right)\},$$

and $X^{\pi^{\star}}(T) \in [0, K_U]$, a.s.

**Proposition 3.2.** *Proposition 4.5 in Donnelly et al. (2015). An optimal investment strategy for Problem 3.1 is to invest the amount*

$$\pi^{\star}(t) := A\left[1 - \Phi(d_{+}(t, P(t); K_U))\right] P(t) \tag{3.7}$$

*in the risky stock and the amount* $X^{\pi^{\star}}(t) - \pi^{\star}(t)$ *in the risk-free bond, in which* $P(t) = (z_0 + g(0))\, Z(t)$ *and the function* $d_{+}$ *is defined by*

$$d_{\pm}(t, y; K_U) := \frac{1}{\sigma A\sqrt{T-t}}\left(\ln\left(\frac{y}{K_U}\right) + \left(r_N \pm \frac{1}{2}\sigma^2 A^2\right)(T-t)\right), \quad \forall y > 0. \tag{3.8}$$

*The wealth process corresponding to this optimal investment strategy is*

$$X^{\pi^\star}(t) = P(t) - g(t) - c(t, P(t); K_U), \tag{3.9}$$

*in which*

$$c(t, y; K_U) := y\Phi(d_+(t, y; K_U)) - K_U e^{-r_N(T-t)}\Phi(d_-(t, y; K_U)),$$

*and* $\Phi(z)$ *denotes the cumulative standard normal distribution function at* $z \in \mathbb{R}$.

*In particular, the relationship between the investor's initial wealth* $X^{\pi^\star}(0) = x_0$ *and the shadow initial wealth* $z_0$ *is*

$$x_0 = z_0 - c(0, z_0 + g(0); K_U). \tag{3.10}$$

The proof is found in Donnelly et al. (2015).

### 3.2.1 Problem with a lower and an upper bound

Here we extend of the problem to include a lower bound $K_L \in (0, K_U)$, below which the terminal wealth must not fall. Combined with the upper bound $K_U$, this means that the investor's terminal wealth lies in the range $[K_L, K_U]$.

The addition of a lower bound has already been well studied in the literature (for example, see (Basak, 1995)).

In order to avoid both the uninteresting case that the investor can immediately be assured of maximizing the terminal utility and the breaching of the non-arbitrage condition, we assume that

**Assumption 3.3.** $K_L < (x_0 + g(0))\, e^{r_N T} < K_U$.

**Problem 3.4.** Find $\pi^\star \in \mathcal{A}$ such that

$$\mathbb{E}\left(U(X^{\pi^\star}(T))\right) = \sup_{\pi\in\mathcal{A}}\{\mathbb{E}\left(U(X^{\pi}(T))\right)\},$$

and $X^{\pi^\star}(T) \in [K_L, K_U]$, a.s.

The next proposition gives an expression for the optimal terminal wealth for Problem 3.4, when there is both a lower and upper bound constraint on the terminal wealth.

**Proposition 3.5.** *A solution to the constrained problem at the terminal time* $T$ *is*

$$\begin{aligned} X^\star(T) =&(z_0 + g(0))Z(T) - \max\{0, (z_0 + g(0))Z(T) - K_U\} \\ &+ \max\{0, K_L - (z_0 + g(0))Z(T)\}, \end{aligned} \tag{3.11}$$

*with the* shadow wealth $z_0 > 0$ *chosen so that the budget constraint (3.5) is satisfied with equality by* $X^\star$*, given the investor's initial wealth* $X^\star(0) = x_0$*, a.s. and savings plan* $g$.

The proof is found in Appendix A. Next we derive the value and replicating portfolio of the put option with maturity value $\max\{0, K_L - (z_0 + g(0))Z(T)\}$.

**Lemma 3.6.** *The price at time $t \in [0, T]$ of a European put option with maturity value* $\max\{0, K_L - (z_0 + g(0))Z(T)\}$ *is given by $p(t, P(t); K_L)$ with*

$$p(t, y; K_L) := K_L e^{-r_N(T-t)}\Phi(-d_-(t, y; K_L)) - y\Phi(-d_+(t, y; K_L)).$$

*with $d_\pm(t, y; K_L)$ defined by equation (3.8).*

*The replicating portfolio for the put option is to hold in the risky asset at time $t$ the amount $\pi_p(t, P(t); K_L)$, with*

$$\pi_p(t, y; K_L) := -Ay\,\Phi(-d_+(t, y; K_L)), \quad \forall t \in [0, T], \quad y > 0 \tag{3.12}$$

*and the remaining amount $p(t, P(t); K_L) - \pi_p(t, P(t); K_L)$ in the risk-free bond.*

The proof is found in Appendix A. The optimal strategy for Problem 3.4 is given next.

**Proposition 3.7.** *An optimal investment strategy for Problem 3.4 is to invest the amount*

$$\pi^\star(t) := A\left[1 - \Phi(d_+(t, P(t); K_U) - \Phi(-d_+(t, P(t); K_L))\right] P(t) \tag{3.13}$$

*in the risky stock and the amount $X^{\pi^\star}(t) - \pi^\star(t)$ in the risk-free bond, in which $P(t) = (z_0 + g(0))\,Z(t)$ and the function $d_+$ is defined by equation (3.8).*

*The wealth process corresponding to this optimal investment strategy is*

$$X^{\pi^\star}(t) = P(t) - g(t) - c(t, P(t); K_U) + p(t, P(t); K_L). \tag{3.14}$$

*In particular, the relationship between the investor's initial wealth $X^{\pi^\star}(0) = x_0$ and the shadow initial wealth $z_0$ is*

$$x_0 = z_0 - c(0, z_0 + g(0); K_U) + p(0, z_0 + g(0); K_L). \tag{3.15}$$

The proof follows trivially from the previous lemmas.

The relative value of the shadow initial wealth $z_0$ over the investor's actual initial wealth $x_0$ has a concrete interpretation. For the $p$-quantiles of the constrained terminal wealth that fall below the target wealth $K_U$, it gives their uplift over those for the unconstrained terminal wealth.

To see this, we calculate the $p$-quantiles under the constrained strategy. For the constrained strategy, there is a probability mass at the target wealth $K_U$. For this reason we use the following generalised definition of the $p$-quantile.

**Definition 3.8.** The $p$-quantile for a random variable $X$ is

$$\mathcal{Q}_p(X) = \inf\{y \in \mathbb{R} : \mathbb{P}[X \le y] \ge p\},$$

with the convention that $\inf\{\emptyset\} = \infty$.

**Proposition 3.9** ($p$-quantiles)**.** *Suppose an investor has initial wealth $x_0 > 0$ and follows the savings plan $g$. Define*

$$\beta_p := \sigma A\sqrt{T}\,\Phi^{-1}(p) + \left(r_N + \theta\sigma A - \frac{1}{2}\sigma^2 A^2\right)T. \tag{3.16}$$

*If the investor follows the optimal constrained strategy, i.e. the terminal wealth is constrained to lie in the range $[K_L, K_U]$, then the p-quantile of the investor's terminal wealth $X(T)$ is*

$$\mathcal{Q}_p(X(T);(K_L,K_U)) = \max\left\{K_L, \min\left\{K_U,(z_0+g(0))e^{\beta_p}\right\}\right\}. \tag{3.17}$$

The proof is found in Appendix A.

# 4 Numerical illustration

In this example, we fix the parameter values $r_{\mathrm{N}} = 0$, $\mu = 0.0343$, $\sigma = 0.1544$, $A = 1$ , $T = 30$, $g = 0$ and $x_0 = 300$. Note that the choice of the parameters implies that the investor's risk aversion constant is $\gamma = -0.44$.

Here we describe the algorithm that calculates the real wealth accumulated by the investor after the investment period. The values of $K_L$ and $K_U$ are set to 225 and 450 respectively. For these vales of $K_L$ and $K_U$ we calculate the shadow initial wealth $z_0$ satisfying (3.15), which results $z_0 = 302.2626$.

We do 10000 simulations of a $T-$dimensional vector of standard normal random values $W(t)$, $t = 1, ...T$. Then, we simulate the process $S(t)$ according to $S(t) = S(t-1)exp((r_{\mathrm{N}} + \theta\sigma A - \frac{1}{2}\sigma^2\ A^2)t + \sigma AW(t))$ for $t = 2, ...T$ where $\theta = \frac{\mu - r_{\mathrm{N}}}{\sigma}$ and $S(1) = 1$. The real wealth process $X_t$ is initialized with the value of $x_0 = 300$. We calculate the amount invested in stocks in the first investment period, $\pi(1)$ by using expression (3.13) for $t = 1$ where $P(1) = (z_0 + g(0))S(1)$ and with the two following restrictions: 1) this amount cannot be higher than the initial level of real wealth ($x_0 = 300$), and 2) this amount must be positive. Then, the simulation loop starts and for each moment in time $j = 2, ..., 30$ we calculate which is the amount in stocks just before time $j$ (updating the amount in stock in the previous period $\pi(j-1)$ by using the factor $S(j)/S(j-1)$). The amount in bonds just before time $j$ is given by the difference between the real wealth and the amount in stocks at that time. Then, the amount invested in stocks for the next period is calculated by using expression (3.13) but with the following two restrictions: 1) this amount cannot be larger than the current real wealth, and 2) this amount must be positive. Then, the loop jumps to the next period. When the algorithm is finished, we calculate the quantiles of the real wealth at $T = 30$. The quantiles, obtained by simulating the final wealth, can be compared to the theoretical ones, those resulting from expression 3.17.

The algorithm is summarized as follows:

1. Initialize parameters ($r_{\mathrm{N}} = 0$, $\mu = 0.0343$, $\sigma = 0.1544$, $A = 1$ , $T = 30$, $g = 0$, $x_0 = 300$, $K_L = 250$ and $K_U = 415$.)

2. Compute $z_0$ from $x_0 = z_0 - c(0, z_0 + g(0); G) + p(0, z_0 + g(0); F)$.

3. Do 10000 replications.

4. Simulate $S(t)$ from $S(t) = S(t-1)exp((r_{\mathrm{N}} + \theta\sigma A - \frac{1}{2}\sigma^2\ A^2)t + \sigma AW(t))$ for $t = 2, ...T$ where $\theta = \frac{\mu - r_{\mathrm{N}}}{\sigma}$ and $S(1) = 1$.

5. Calculate $\pi(1) = max(min(A(1 - \Phi(d_+(1, P(1)); K_U) - \Phi(-d_+(1, P(1); K_L)))P(1), x_0)0)$, where $P(1) = z_0 S(1)$.

6. Compute real wealth as the initial wealth $x_0$.

7. For $t = 2, ..., 30$

   7.1 Calculate the value invested in bonds as current real wealth minus $\pi(t-1)$.

   7.2 Compute the value of $\pi(t-1)$ after period $t-1$.

   7.3 Update real wealth at $t$ as the sum of the two.

   7.4 Calculate $\pi(t)$ from $\pi(t) = max(min(A(1 - \Phi(d_+(t, P(t)); K_U) - \Phi(-d_+(t, P(t); K_L)))P(t), RealWealth(t), 0)$, where $P(t) = (z_0 + g(0))S(t)$.

8. End of loop.

9. End of loop.

10. Compute quantiles of the real wealth at $T = 30$.

Table 1 shows the distribution of the real wealth at $T = 30$ for $K_L = 250$ and $K_U = 415$. The first column shows the values of $p$ for the $p$-quantiles. The second column shows the theoretical quantiles obtained by using expression 3.17. The next four columns show the quantiles of the simulated final wealth at $T = 30$ for different updates: yearly, monthly, weekly and daily. Note how the values of the quantiles converge to the theoretical values as the frequency of the update increases.

We have also calculated the life-long annuity payable every year that a 65 year old investor will receive corresponding the final wealth of $X(T)$ accumulated after the investment period. The expression is:

$$Annuity(Age = 65, X(T)) = \frac{X(T)}{\sum_{t=66}^{111}(1+r)^{-(t-65)}{}_{t-65}P_{65}}$$

where $r = 0$ and the values for the survival probabilities ${}_{t-65}P_{65}$ have been calculated by using the Society of Actuaries Life Table available in the $lifecontingencies$ R package. The denominator in the previous expression is obtained by using the *axn* function of the $lifecontingencies$ R package.

R code for $X(T) = 415$:

```
library(lifecontingencies)
data(soaLt)
soa08Act=with(soaLt, new("actuarialtable",interest=0,x=x,lx=Ix,name="SOA2008"))
```

Table 1: Table showing the distribution of the real wealth at $T = 30$ for $K_L = 250$ and $K_U = 415$.

| **Update:** | Theoretical | Yearly | Monthly | Weekly | Daily |
|---|---|---|---|---|---|
| $p$ | $Q_p(X(T))$ | | | | |
| 1% | 250.0000 | 241.7170 | 246.1227 | 248.0315 | 249.0233 |
| 2.5% | 250.0000 | 246.4381 | 248.1896 | 248.9851 | 249.5117 |
| 5% | 250.0000 | 249.7004 | 249.3909 | 249.6616 | 249.8114 |
| 10% | 250.0000 | 254.9273 | 250.8434 | 250.3659 | 250.1304 |
| 15% | 250.0000 | 267.4020 | 254.2965 | 252.4664 | 251.3356 |
| 20% | 290.3133 | 292.8727 | 283.4717 | 284.7746 | 285.8970 |
| 25% | 334.3877 | 324.8488 | 328.2672 | 328.3217 | 328.3625 |
| 30% | 379.6421 | 354.7174 | 371.8317 | 373.9506 | 376.4171 |
| 35% | 415.0000 | 377.6660 | 402.7025 | 409.3999 | 412.5361 |
| 40% | 415.0000 | 392.5784 | 409.1703 | 412.2969 | 413.8254 |
| 45% | 415.0000 | 400.6617 | 411.3917 | 413.2157 | 414.2071 |
| 50% | 415.0000 | 406.0062 | 412.6297 | 413.7881 | 414.4618 |
| 55% | 415.0000 | 409.7468 | 413.5108 | 414.2082 | 414.6521 |
| 60% | 415.0000 | 412.5056 | 414.2754 | 414.5923 | 414.8100 |
| 65% | 415.0000 | 415.1566 | 414.9827 | 414.9076 | 414.9505 |
| 70% | 415.0000 | 417.7468 | 415.6342 | 415.2297 | 415.0977 |
| 75% | 415.0000 | 420.3110 | 416.3125 | 415.5666 | 415.2401 |
| 80% | 415.0000 | 423.2772 | 417.0502 | 415.9280 | 415.4017 |
| 85% | 415.0000 | 426.4701 | 417.9061 | 416.3479 | 415.5859 |
| 90% | 415.0000 | 430.3906 | 419.1283 | 416.9107 | 415.8299 |
| 95% | 415.0000 | 437.4809 | 421.1314 | 417.8511 | 416.2652 |
| 97.5% | 415.0000 | 444.0110 | 423.4497 | 418.8456 | 416.6837 |
| 99% | 415.0000 | 453.9527 | 426.0071 | 420.1559 | 417.3608 |

axn(soa08Act, x=65, payment="arrears")
15.02172
415/axn(soa08Act, x=65, payment="arrears")
27.62666

The results are shown in Table 3. in Appendix B.

## 4.1 Inflation

Critically, when income streams are presented in real terms, retirement investors choose flat or increasing *real* income streams over decreasing ones (Beshears et al., 2014). However, in practice, income streams are mostly presented in nominal terms rather than in real terms. It is difficult for the average person to understand the potential heavy toll of inflation on their retirement income, even though expected lifetimes are increasing.

We believe that the standard investment strategies followed by retirement investors are sub-optimal. The investment strategy followed by individual who wants a real income stream in retirement can have a considerable weighting of inflation-indexed assets. Today, the only inflation-indexed assets that are widely traded in the market are government-issued index-linked bonds (e.g. TIPS, OATis, ILGs). However, index-linked bonds give a negative real return to investors. They are highly demanded and in very short supply.

We believe that if inflation funds were introduced to the market, retirement investors would invest heavily in them. These funds could consist of real assets, such as infrastructure, commodities, equities, property. They would aim to broadly track price inflation. They would aim to give a positive real return over price inflation, although with some volatility around this.

The potential amount invested in inflation funds could be enormous, e.g. the total value of pension assets in the US was around 108% in 2012 (around \$16 851 billion in 2012) according to Towers Watson (2013). The US index-linked bond issuance is about 5% of this amount, e.g. in April 2012 it was \$866 billion (Krämer, 2013). Quite simply there is a massive potential demand for inflation-indexed assets that cannot be satisfied by the current volume of inflation-linked bonds.

Here we modify our algorithm so that some part of the wealth could be investing in an inflation fund. The algorithm is essentially the same, but now we assume that the amount not invested in stocks is giving some return. To do so, we first do 10000 simulations of a bivariate normal sample of size $T$, $(W_1(t), W_2(t))$, $t = 1..., T$ with mean values equal to 0, standard deviations equal to 1 and correlation $\rho$ equal to 0.5. Then, with these values we simulate the stock process $S_1(t)$ according to $S_1(t) = S_1(t-1)exp((r+\theta_1\sigma_1 A-\frac{1}{2}\sigma_1^2\ A^2)t+\sigma_1 AW_1(t))$ for $t = 2, ...T$ where $\mu_1 = 0.0343$, $\sigma_1 = 0.1544$, $\theta_1 = \frac{\mu_1 - r_{\mathrm{N}}}{\sigma_1}$ and $S_1(1) = 1$. Then, we also simulate the values of the inflation fund process $S_2(t)$ according to $S_2(t) = S_2(t-1)exp((r + \theta_2\sigma_2 A - \frac{1}{2}\sigma_2^2\ A^2)t + \sigma_2 AW_2(t))$ for $t = 2, ...T$ where $\mu_2 = 0.008$, $\sigma_2 = 0.02$, $\theta_2 = \frac{\mu_2 - r_{\mathrm{N}}}{\sigma_2}$ and $S_2(1) = 1$. Then, we use the values of $S_1(t)$, $t = 1, ..., T$ to update the amount invested in stocks and $S_2(t)$, $t = 1, ..., T$ to update the rest of money every year.

Namely, in the algorithm previously described, we introduce the following changes:

1. Initialize parameters ($r_{\mathrm{N}} = 0$, $\mu_1 = 0.0343$, $\sigma_1 = 0.1544$, $\mu_2 = 0.008$, $\sigma_2 = 0.02$, $A = 1$ , $T = 30$, $g = 0$, $x_0 = 300$, $K_L = 250$ and $K_U = 415$.)

4. Simulate $S_1(t) = S_1(t-1)exp((r_{\mathrm{N}} + \theta_1\sigma_1 A - \frac{1}{2}\sigma_1^2\ A^2)t + \sigma_1 AW_1(t))$ for $t = 2, ...T$ and $S_2(t) = S_2(t-1)exp((r_{\mathrm{N}} + \theta_2\sigma_2 A - \frac{1}{2}\sigma_2^2\ A^2)t + \sigma_2 AW_2(t))$ for $t = 2, ...T$ where $\theta_1 = \frac{\alpha_1 - r_{\mathrm{N}}}{\sigma_1}$ and $\theta_2 = \frac{\alpha_2 r_{\mathrm{N}}}{\sigma_2}$.

5. Calculate $\pi(1) = max(min(A(1-\Phi(d_+(1, P(1)); K_U)-\Phi(-d_+(1, P(1); K_L)))P(1), x_0)0)$, where $P(1) = z_0 S_1(1)$.

7.1. Subtract from the real wealth the amount $\pi(t-1)$ and update this difference by multiplying it by $S_2(t)/S_2(t-1)$.

7.4. Calculate $\pi(t)$ from $\pi(t) = max(min(A(1-\Phi(d_+(t, P(t)); K_U)-\Phi(-d_+(t, P(t); K_L)))P(t), RealWealth(t), 0)$, where $P(t) = (z_0 + g(0))S_1(t)$.

Table 2 shows the quantiles obtained by simulating the final wealth at $T = 30$, with $K_L = 250$ and $K_U = 415$ assuming that the amount not invested in stocks is invested in an inflation fund with 0.8% return and 2% volatility. The correlation used to simulate the bivariate normal distribution (as explained previously in the algorithm) is 0.5.

We have also calculated the life-long annuity payable every year that a 65 year old investor will receive for the final wealth of $X(T)$ accumulated after the investment period. The results are shown in Table 4 in Appendix B where the value of $r$ has been changed to 0.8% (the return assumed for the inflation fund).

# 5 Conclusion

We have shown the practical implementation of a new investment strategy that has the advantage of constraining the final wealth accumulated after the investment period between a lower and an upper bound. In this way, the saver is protected against extreme values, by providing an smoothing mechanism which includes an embedded guarantee on the terminal wealth.

Another advantage of the proposed strategy is that the portfolio is rebalanced automatically so that the accumulated wealth at any moment is constraint by the lower and upper bounds. We have also illustrated how the accumulated wealth can be translated into a life-long annuity that the investor will receive, which is easy to understand and communicate, increasing the transparency of the investment mechanism.

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Table 2: Table showing the distribution of the real wealth at $T = 30$ for $K_L = 250$ and $K_U = 415$. Yearly: with inflation 0.8% return, 2% volatility, $\rho = 0.5$

| **Update:** | Yearly |
|---|---|
| $p$ | $Q_p(X(T))$ |
| 1% | 251.8884 |
| 2.5% | 263.6504 |
| 5% | 276.6988 |
| 10% | 295.6553 |
| 15% | 315.1019 |
| 20% | 339.5316 |
| 25% | 368.7824 |
| 30% | 398.2215 |
| 35% | 421.3348 |
| 40% | 438.5349 |
| 45% | 451.9557 |
| 50% | 462.9167 |
| 55% | 472.8325 |
| 60% | 481.4466 |
| 65% | 490.6310 |
| 70% | 499.7588 |
| 75% | 509.7144 |
| 80% | 519.8703 |
| 85% | 532.6464 |
| 90% | 547.7482 |
| 95% | 570.9136 |
| 97.5% | 589.3045 |
| 99% | 612.7947 |

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# A Proof of optimal investment strategy in simple financial market model

*Proof.* of Proposition 3.5.

The proof is an adaption of the proof of [cite relevant proposition in the first paper].

Assume that we have chosen $z_0 > 0$ chosen so that the budget constraint (3.5) is satisfied with equality by $X^\star$.

For the investor's utility function, the first derivative $U'(x) = x^{\gamma-1}$, which is a strictly decreasing function, has a strictly decreasing inverse

$$I(y) := y^{\frac{1}{\gamma-1}}, \quad y > 0.$$

We can show that for the constant

$$y := (z_0 + g(0))^{\gamma-1} e^{\left(\gamma r_{\mathrm{N}} + \frac{1}{2}\frac{\gamma}{1-\gamma}\theta^2\right)T},$$

we have

$$(z_0 + g(0))Z(T) = I(yH(T)).$$

We work with $I(y(z_0)H(T))$ in the proof, rather than with $(z_0 + g(0))Z(T)$ due to the properties of $I(x)$ and $U'(x)$: they are both strictly decreasing functions of $x$.

Let $X(T) \in [K_L, K_U]$, a.s. be any attainable final wealth so that $\mathbb{E}(H(T)X(T)) \leq x_0$. We show that

$$\mathbb{E}(U(X(T))) \leq \mathbb{E}(U(X^\star(T))),$$

in which

$$X^\star(T) = \begin{cases} K_L & I(yH(T)) \leq K_L \\ I(yH(T)) & \text{if } I(yH(T)) \in (K_L, K_U) \\ K_U & I(yH(T)) \geq K_U. \end{cases}$$

As $I$ and $U'$ are strictly decreasing functions we can write:

$$X^\star(T) = \begin{cases} K_L & yH(T) \geq U'(K_L) \\ I(yH(T)) & \text{if } yH(T) \in (U'(K_L), U'(K_U)) \\ K_U & \text{if } yH(T) \leq U'(K_U) \end{cases}$$

As $U$ is a concave function then for any $a, b \in \mathbb{R}$, $U(a) - U(b) \leq U'(b) \cdot (a - b)$. In particular,

$$U(X(T)) - U(X^\star(T)) \leq U'(X^\star(T)) \cdot (X(T) - X^\star(T)), \quad \text{a.s.}$$

Take expectations:

$$\begin{aligned}
&\mathbb{E}(U(X(T)) - U(X^\star(T))) \\
\leq &\mathbb{E}(U'(X^\star(T)) \cdot (X(T) - X^\star(T))) \\
\leq &\mathbb{E}(U'(X^\star(T)) \cdot (X(T) - X^\star(T)) \,|\, yH(T) \geq U'(K_L)) \cdot \mathbb{P}[yH(T) \geq U'(K_L)] \\
&+ \mathbb{E}(U'(X^\star(T)) \cdot (X(T) - X^\star(T)) \,|\, yH(T) \in (U'(K_L), U'(K_U))) \cdot \mathbb{P}[yH(T) \in (U'(K_L), U'(K_U))] \\
&+ \mathbb{E}(U'(X^\star(T)) \cdot (X(T) - X^\star(T)) \,|\, yH(T) \leq U'(K_U)) \cdot \mathbb{P}[yH(T) \leq U'(K_U)].
\end{aligned}$$

Observe that on the event $[yH(T) \in (U'(K_L), U'(K_U))]$,

$$U'(X^\star(T)) = U'(I(yH(T))) = yH(T)$$

so that

$$\begin{aligned}&\mathbb{E}\left(U'(X^\star(T))\cdot(X(T)-X^\star(T))\,|\,yH(T)>U'(K_U)\right)\\=&\mathbb{E}\left(yH(T)\cdot(X(T)-X^\star(T))\,|\,yH(T)>U'(K_U)\right).\end{aligned}$$

Next observe that on the event $[yH(T)\le U'(K_U)]$, as $X(T)\in[K_L,K_U]$ a.s, then

$$X(T)-X^\star(T)=X(T)-K_U\le 0$$

and

$$U'(X^\star(T))=U'(K_U)\ge yH(T).$$

The negative sign of $X(T)-X^\star(T)$ reverses the inequality $U'(X^\star(T))\ge yH(T)$, giving that on the event $[yH(T)\le U'(K_U)]$,

$$U'(X^\star(T))\cdot(X(T)-X^\star(T))\le yH(T)\cdot(X(T)-X^\star(T)).$$

On the event $[yH(T)\ge U'(K_L)]$, as $X(T)\in[K_L,K_U]$ a.s, then

$$X(T)-X^\star(T)=X(T)-K_L\ge 0$$

and

$$U'(X^\star(T))=U'(K_L)\le yH(T).$$

Due to the positive sign of $X(T)-X^\star(T)$, the inequality $U'(X^\star(T))\le yH(T)$ is maintained, giving

$$U'(X^\star(T))\cdot(X(T)-X^\star(T))\le yH(T)\cdot(X(T)-X^\star(T)).$$

In summary, we find that

$$\mathbb{E}\left(U(X(T))-U(X^\star(T))\right)\le\mathbb{E}\left(yH(T)\cdot(X(T)-X^\star(T))\right).$$

As both solutions satisfy the budget constraint (3.5), the last line in the above inequality can be evaluated as

$$\mathbb{E}\left(yH(T)\cdot(X(T)-X^\star(T))\right)\le y\cdot((x_0+g(0))-(x_0+g(0)))=0,$$

which means

$$\mathbb{E}\left(U(X(T))-U(X^\star(T))\right)\le 0.$$

Hence

$$\mathbb{E}\left(U(X^{\pi^\star}(T))\right)=\sup_{\pi\in\mathcal{A}}\mathbb{E}\left(U(X^\pi(T))\right)\le\mathbb{E}\left(U(X^\star(T))\right)\le\mathbb{E}\left(U(X^{\pi^\star}(T))\right),$$

i.e. $X^{\pi^\star}(T)=X^\star(T)$, a.s.

□

*Proof.* of Lemma 3.6.

From [cite relevant lemma in our first paper], a European call option with maturity value $\max\{0,(z_0+g(0))Z(T)-K_L\}$ is given by $c(t,P(t);K_L)$ with

$$P(t):=(z_0+g(0))\,Z(t), \tag{A.1}$$

and

$$c(t,y;K_L):=y\Phi(d_+(t,y;K_L))-K_Le^{-r_{\mathrm{N}}(T-t)}\Phi(d_-(t,y;K_L)),$$

in which the functions $d_{\pm}(t, y; K_L)$ are defined by equation (3.8).

Thus by put-call parity, the value of the put option with the same strike price $K_L$ satisfies

$$p(t, y; K_L) = c(t, y; K_L) + K_L e^{-r_{\mathrm{N}}(T-t)} - y.$$

To find the replicating portfolio, we differentiate the put pricing function $p$ to get

$$p_t(t, y; K_L) = -\frac{y\phi(d_+(t, y; K_L))\sigma A}{2\sqrt{T-t}} + r_{\mathrm{N}} K_L e^{-r_{\mathrm{N}}(T-t)} \Phi(-d_-(t, y; K_L))$$

$$p_y(t, y; K_L) = \Phi(d_+(t, y; K_L)) - 1 = -\Phi(-d_+(t, y; K_L)), \qquad p_{yy}(t, y) = \frac{\phi(d_+(t, y; K_L))}{y\sigma A\sqrt{T-t}}.$$

By Ito's formula,

$$dp(t, P(t)) = p_t(t, P(t))dt + p_y(t, P(t))dP(t) + \frac{1}{2} p_{yy}(t, P(t))d\,[P]\,(t),$$

in which

$$dP(t) = (r_{\mathrm{N}} + \theta\sigma A)\, P(t)dt + \sigma A P(t) dW(t).$$

Substituting for: the derivatives of the pricing function $p$, the dynamics of $P$ and the candidate replicating portfolio $\pi_p\,(t, P(t)) := -AP(t)\,\Phi(-d_+(t, P(t)))$, we find that the dynamics of the pricing function $c$ satisfy the wealth equation (3.3). Hence $\pi_p\,(t, P(t))$ is the amount to be invested in the risky stock at time $t$ in order to replicate the payoff of the European put option.

□

*Proof.* of Lemma 3.9. Fix $p \in (0, 1)$. From [cite relevant lemma in our first paper], with no lower bound on the terminal wealth,

$$\mathcal{Q}_p(X(T); (0, K_U)) = \min\left\{K_U, (z_0 + g(0))e^{\beta_p}\right\}.$$

It is useful to consider another investor who has the same savings plan $g$ and the same upper bound $K_U$ as the first investor. However, this second investor has no lower bound on the terminal wealth, i.e. $K_L = 0$, and starts with an initial wealth $\tilde{x}_0$ that satisfies

$$\tilde{x}_0 = z_0 - c(0, z_0 + g(0); K_U).$$

. This second investor follows the optimal constrained strategy. Then, as $g(T) = 0$, the wealth at time $T$ of the second investor is

$$\tilde{X}(T) = (z_0 + g(0))Z(T) - g(T) - c(T, P(T); K_U) = \min\{K_U, P(T)\}.$$

Thus the terminal wealth of the constrained investor, who has a lower bound $K_L$ on their terminal wealth, is related to that of the second unconstrained investor by

$$X(T) = \begin{cases} \tilde{X}(T) & \text{if } \tilde{X}(T) \geq K_L \\ K_L & \text{if } \tilde{X}(T) < K_L. \end{cases}$$

The desired expression (3.17) follows by consideration of the last expression.

□

# B Lifetime annuities

Table 3: Distribution of the life-long annuity at 65 years old for $K_L = 250$ and $K_U = 415$.

| **Update:** | Theoretical | Yearly | Monthly | Weekly | Daily |
|---|---|---|---|---|---|
| $p$ | $Q_p(Annuity(65))$ | | | | |
| 1% | 16.64 | 16.09 | 16.38 | 16.51 | 16.58 |
| 2.5% | 16.64 | 16.41 | 16.52 | 16.58 | 16.61 |
| 5% | 16.64 | 16.62 | 16.60 | 16.62 | 16.63 |
| 10% | 16.64 | 16.97 | 16.70 | 16.67 | 16.65 |
| 15% | 16.64 | 17.80 | 16.93 | 16.81 | 16.73 |
| 20% | 19.33 | 19.50 | 18.87 | 18.96 | 19.03 |
| 25% | 22.26 | 21.63 | 21.85 | 21.86 | 21.86 |
| 30% | 25.27 | 23.61 | 24.75 | 24.89 | 25.06 |
| 35% | 27.63 | 25.14 | 26.81 | 27.25 | 27.46 |
| 40% | 27.63 | 26.13 | 27.24 | 27.45 | 27.55 |
| 45% | 27.63 | 26.67 | 27.39 | 27.51 | 27.57 |
| 50% | 27.63 | 27.03 | 27.47 | 27.55 | 27.59 |
| 55% | 27.63 | 27.28 | 27.53 | 27.57 | 27.60 |
| 60% | 27.63 | 27.46 | 27.58 | 27.60 | 27.61 |
| 65% | 27.63 | 27.64 | 27.63 | 27.62 | 27.62 |
| 70% | 27.63 | 27.81 | 27.67 | 27.64 | 27.63 |
| 75% | 27.63 | 27.98 | 27.71 | 27.66 | 27.64 |
| 80% | 27.63 | 28.18 | 27.76 | 27.69 | 27.65 |
| 85% | 27.63 | 28.39 | 27.82 | 27.72 | 27.67 |
| 90% | 27.63 | 28.65 | 27.90 | 27.75 | 27.68 |
| 95% | 27.63 | 29.12 | 28.03 | 27.82 | 27.71 |
| 97.5% | 27.63 | 29.56 | 28.19 | 27.88 | 27.74 |
| 99% | 27.63 | 30.22 | 28.36 | 27.97 | 27.78 |

Table 4: Distribution of the life-long annuity at 65 years old for $K_L = 250$ and $K_U = 415$. Yearly: final wealth simulated with inflation 0.8% return, 2% volatility, $\rho = 0.5$. Life-long annuity calculated with yearly interest rate of 0.8% .

| **Update:** | Yearly |
|---|---|
| $p$ | $Q_p(Annuity(65))$ |
| 1% | 18.17 |
| 2.5% | 19.02 |
| 5% | 19.96 |
| 10% | 21.33 |
| 15% | 22.73 |
| 20% | 24.5 |
| 25% | 26.61 |
| 30% | 28.73 |
| 35% | 30.4 |
| 40% | 31.64 |
| 45% | 32.61 |
| 50% | 33.4 |
| 55% | 34.11 |
| 60% | 34.73 |
| 65% | 35.4 |
| 70% | 36.05 |
| 75% | 36.77 |
| 80% | 37.51 |
| 85% | 38.43 |
| 90% | 39.52 |
| 95% | 41.19 |
| 97.5% | 42.52 |
| 99% | 44.21 |

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