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On the Industry Specificity of Human Capital and Business Cycles

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Abstract: We define specific (general) human capital as the set of occupations whose use is spread in a limited (wide) set of industries. Using the EU Labor Force Survey database, we identify these human capital types and analyze their employment and education. This exercise yields a persistent assignment of occupations into specific and general human capital types. The share of specific human capital varies across countries and has declined over time almost everywhere.
We consider a stylized two-sector model where one of the sectors uses both types of human capital and the other specializes on general human capital. We show that a mean preserving increase in the share of specific human capital reduces (increases) the contribution of shocks in non-specialized sector and increases (reduces) the contribution of shocks in specialized sector to the variance of final output, when sectoral outputs are gross complements (substitutes).

JEL Codes: E24,E30, I20, J23, J24, O41.

Keywords: Specific and General Human Capital Types, Propagation of Sectoral Shocks, Output Volatility.

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1 Introduction

At the individual level, the type of skills is an important determinant of employment opportunities across production plants and industries. For example, the services skills of managers and manual skills of cleaners are used almost everywhere. In contrast, the surgery skills of doctors and manual skills of craft workers are in demand in a small number of industries. At the aggregate level, the composition of such general and specific skills in a country can matter for the propagation of shocks and business cycles.

In this paper, we utilize a novel horizontal differentiation of skill types in order to analyze the impact of human capital portfolio composition on aggregate economic performance. We define two distinct types of human capital: "general" and "specific." As general human capital, we define a set of skills that enable individuals to perform generic tasks that are required for production in a wide range of industries. In contrast, specific human capital is defined as a set of skills that enable one to perform highly specialized tasks in a few industries.

We use harmonized individual level EU Labour Force Survey (ELFS) to identify specific and general human capital types and to analyze their employment and education fields and levels.[1] The empirical exercise yields remarkably stable assignment of occupations into specific and general human capital types. For example, according to the assignment, occupations such as Market-oriented Skilled Agricultural and Fishery Workers and Extraction and Building Trades Workers correspond to specific human capital in almost all countries and years in our sample. In turn, occupations such as General Managers and Sales and Services Elementary Occupations correspond to general human capital in almost all countries and years.

The share of the individuals employed in specific human capital occupations varies between and within countries. Moreover, it has declined over time almost everywhere. Both between and within industry shifts have contributed to this negative trend. Approximately 64 percent of variation in the share of specific human capital is because the share of almost all industries, which use specific human capital more intensively, has declined over time. The remaining variation stems from more intensive use of general human capital in almost all industries. Industries which use specific human capital very intensively are, for example, Agriculture, Hunting, and Fishing; Education; and Construction. In turn, industries which do not use it intensively are Financial Intermediation; Transport, Storage, and Communication; and Hotels and Restaurants. The ranking of industries according to their intensity of use of specific human capital is remarkably stable across countries and years.

Regarding the education of these human capital types, the graduates of education

[1] Jerbashian et al. (2015) use a similar assignment and offer a more limited empirical analysis for the Czech Republic.

fields such as Teacher Training and Education Science; Health and Welfare; and Agriculture and Veterinary are usually employed in specific human capital occupations. The graduates of education fields such as Social Sciences, Business, and Law; Services; and Science, Mathematics, and Computing usually have general human capital occupations. The level of education (skills) is very similar across the specific and general human capital types, which agrees with our horizontal differentiation of skills.

We build a stylized multi-sector model to illustrate how this horizontal differentiation of human capital types can matter for aggregate economic performance and to analyze the distortions which can arise if there are unanticipated shocks. We assume that general human capital is required for production in two sectors and is mobile across these sectors to capture the inherent flexibility of general human capital in the model. We call these sectors h and l and their outputs Y_h and Y_l. Specific human capital is required for production in sector h only. The outputs of these sectors are aggregated into consumption goods with a CES function. Sectors h and l are subject to i.i.d. shocks which happen in between hiring and compensating inputs. Our analysis focuses on the effect of human capital portfolio composition on prorogation of these shocks to final output fluctuations and on the effects of the variances of these shocks (uncertainty) on the demand for and supply of human capital types.

The elasticity of substitution between Y_h and Y_l turns to be important for our analysis. For brevity, we summarize our results for the case when Y_h and Y_l are gross complements.

We consider two countries which have different endowments of specific and general human capital types, but have the same expected output. In the country which has higher amount of specific human capital, the volatility of shocks in sector h (l) has a lower (higher) contribution to the volatility of final output. Therefore, the volatility of final output in this country is higher if, for example, there are no shocks in sector h.

Further, we turn off one of the sectoral shocks and consider a planner which has an option to marginally increase either the amount of general human capital or specific human capital at no cost/at the same cost. If the planner increases the amount of general human capital then the volatility of final output does not necessarily increase less than if it increases the amount of specific human capital. To make the trade-offs more comparable, we impose a condition that the marginal changes in either of human capital types should also deliver equal marginal changes in expected final output. In such a case, if the planner increases the amount of general human capital then the volatility of final output increases more (less) than increasing the amount of specific human capital if there are no shocks to sector l (h). Clearly, if the planner has a concave objective function then it would prefer investing in specific human capital if there are no shocks to sector l. In this respect, it would under-invest in specific human capital if it does not anticipate shocks to sector h.

Finally, we close the model in a trivial manner and assume that the economy is populated by one-period lived identical households. The representative household has concave utility from consumption of final goods. At the beginning of the period the household needs to decide how much specific and general human capital to acquire. The costs of acquiring both types skills are in terms of final goods.

The household makes its choices given the relative expected wage rates of human capital types. In the production side, the relative expected wage rate of general human capital declines with the volatility of shocks in sector l when Y_h and Y_l are gross complements. Moreover, the share of general human capital allocation to sector h increases with the volatility of shocks in sector l. Therefore, the equilibrium amount of general human capital declines and the allocation of general human capital to sector h increases with the volatility of shocks in sector l. If the shocks in sector l cannot be anticipated, the household will over-invest in general human capital, under-invest in specific human capital, and allocate less than optimal amount of general human capital to sector h.

There are several important assumptions in our analysis. We assume that occupations represent skills. A growing number of studies argue whether occupation, industry, firm, and the task content of the job determine skills (e.g., Helwege, 1992; Neal, 1995; Kambourov and Manovskii, 2009a,b; Longhi and Brynin, 2010; Ritter, 2014; Cortes and Gallipoli, 2015). These studies seem to have settled on that occupations are a major determinant of skills. We also assume that general human capital can be relatively easily transferred across industries. Given the data we have, we don't attempt to test this assumption for our particular assignment in this paper.

From the perspective of these studies, specific human capital can be called industry-specific human capital. One of the major differences of this paper from these studies is that we define the relative transferability of skills across industries using concentration measures instead of changes in earnings or switches across industries. In this sense, the identification of human capital specificity in this paper is based on the differences of industries in terms of technological demands for skills.

Our paper is also related to studies which horizontally differentiate among types of skills and examine the role of such differences for economic outcomes (e.g., Hanushek et al., 2011; Acemoglu and Autor, 2011; Jerbashian et al., 2015). It broadly relates to studies that examine the intra- and inter-temporal trade-offs between different types of human capital in environments with uncertainty, introduction of new technologies, or trade. Such mechanisms are analyzed in Goos et al. (2014), Hummels et al. (2014), Autor and Dorn (2013), Krueger and Kumar (2004a,b), and Gould et al. (2001), among others. We contribute to all these groups of studies by introducing a novel way for horizontally differentiating among types of skills and analyzing the effect of human capital portfolio composition in terms of these skills on propagation of sectoral shocks. We also contribute by identifying skills for which industry and occupational specificity can be rather hard to distinguish.

Gervais et al. (2008) is one of the closest studies to ours in terms of the theoretical analysis. Gervais et al. (2008) considers an economy where firms hire firm-specific and general human capital as in Becker (1962). Firms are subject to idiosyncratic productivity shocks and receive signals about next period values of the shocks. Firm-specific human capital is more productive but cannot be hired/fired after the shocks. General human capital can be hired and fired. Gervais et al. (2008) show that output is higher in economies with higher precision of signal because the amount of specific human capital is higher. However, output declines more in these economies after an unexpected decline in the precision of the signal. The latter effect arises because unexpected decline of the precision in their framework does not alter human capital portfolio composition and implies a higher misallocation of resources in these economies. Contrary to Gervais et al. (2008), we do not assume that specific human capital is necessarily more productive than general human capital. Moreover, we focus on distortions which arise if shocks are unanticipated and on assessing propagation of sectoral shocks in a stylized economy where factor inputs are not mobile after shocks. In additional results section, we also consider an economy where general human capital can be reallocated after the shocks and analyze the dependence of the elasticity of final output with respect to sectoral shocks on human capital portfolio composition. The results that we derive are similar to our results for the variance of final output.

A number of studies examine the determinants of aggregate volatility (e.g., see Acemoglu and Zilibotti, 1997; Acemoglu et al., 2012; Carvalho and Gabaix, 2013; Koren and Tenreyro, 2013). These studies have highlighted the importance of, for example, financial, sectoral, and product variety diversification. In turn, the accumulated evidence suggests that - overlooking the period of the global economic downturn - output volatility in many countries has declined in recent years (see, for example, Stock and Watson, 2005, and the references therein). The theoretical inference of this paper illustrates the importance of human capital portfolio composition and mobility of factor inputs for aggregate volatility.[2] According to our theoretical inference, the secular decline in the share of specific human capital can be one of the factors contributing to the documented trends in output volatility.

The paper is organized as follows. Section 2 discusses the composition of specific and general human capital. Section 3 presents the model and its results. Section 4 concludes.

2 Specific and General Human Capital

We treat each occupation as a set of skills which enable the performance of tasks that are necessary as a part of the production process. In this respect, occupations define the labor services input in the production in each industry. To the extent that industries

[2] As it will become apparent in the model section, our results are not specific to human capital portfolio composition. They apply to the portfolio of mobile assets/factors in general.

differ in their technological needs in terms of the types of labor services, their demand for occupations would also be different. We classify an occupation as "specific human capital" if it is used in a limited set of industries, i.e., its employment share exhibits a high degree of concentration across industries. Accordingly, we classify an occupation as "general human capital" if it is used in the production of a wide variety of products, i.e., its employment share has a high degree of dispersion.

Ideally, we would need data on industries' technological demand for occupations to identify the degree of an occupation's "industry specificity." In the data, however, we commonly observe the demand and supply together. Given that our classification is based on relative concentrations of occupations across industries, actually it is sufficient to have that industries' demands for different occupations and the supplies of these skills to any particular industry are not disproportionately affected by frictions and distortions in the economy, if any. Hereafter, we assume that this is the case.

We employ data from the harmonized individual level ELFS (yearly files of 2014 release) to identify specific and general human capital types and summarize how they are used and produced in our sample of European countries. We retrieve from this database information on the number of people in labor force in each country and year, their occupation in the main job (2-digit ISCO-88), the industry in which they are employed (1-digit NACE Rev. 1), and their education level and field of education (1-digit ISCED-97). Table 5 offers our sample of countries and years.

Using these data we compute the number of individuals employed in each occupation-industry cell for each country and year. From this matrix, we derive the distributions of *within-occupation employment share across industries, within-industry employment share across occupations*, and *total employment shares* by occupation.[3] Table 1 reports the results from an ANOVA exercise for within-occupation employment share. The variation of within-occupation employment share is mostly driven by industry and occupation differences, while time and country differences are much less important. Table 2 reports within-occupation employment share for each occupation and industry, where we take averages over countries and years. Figure 1 illustrates the trends in the share of employment in each occupation, where we take country-level and year- and country-level averages (see also Table 22 in Data Appendix).

For each country and year, we use the distribution of within-occupation employment shares to calculate five concentration statistics: coefficient of variation (CV), and Herfindahl (HI), Gini, Theil, and generalized entropy (GE) indices. According to simple ANOVA exercises, these concentration measures vary significantly across occupations but are remarkably stable across countries and years. Moreover, rank correlations among these concentration measures are almost perfect (see Tables 16-21 in Data Appendix).

We average each of these concentration measures over countries and years. For each

[3]Our results robust to using weekly hours worked instead of individuals employed.

of the averages, we create a dummy variable which equals 1 for higher than median values of the averaged concentration measures. An occupation is classified as specific human capital if the average of these five dummy variables is greater than 0.5 and as general otherwise. This ranking of the different occupations is offered in Table 3 together with the values of averaged concentration measures.

For example, our classification suggests that occupations such as Market-oriented Skilled Agricultural and Fishery Workers and Extraction and Building Trades Workers correspond to specific human capital. In turn, occupations such as General Managers and Sales and Services Elementary Occupations correspond to general human capital. This ranking is stable across countries and years, with very few exceptions, in line with ANOVA exercises and rank correlations. To illustrate this, we perform a similar assignment within countries using time averaged concentration measures and within years using country averages. Table 4 reports the number of times that an occupation is assigned into specific human capital type in the sample of countries and in the sample of years.[4]

We use this assignment to calculate the share of individuals employed in specific human capital occupations out of total employment in each country and year in our sample. We call this simply "the share of specific human capital" and present the results in Figure 2. Country-level basic statistics for the share of specific human capital are offered in Table 5 (see Table 23 for correlations).

The share of specific human capital displays considerable time and country variation (see also Table 24 in Data Appendix for an ANOVA exercise). It has a negative trend in almost all countries.[5] We average it over the countries and offer the results in Figure 3. On average, the share of specific human capital has declined in the countries in our sample by approximately 5 percentage points during the period of 1992–2010.[6] In the sample period, Southern European and former transition countries tend to be the most abundant of specific human capital. The UK and the Netherlands are at the other end of the spectrum.

We compute the share of specific human capital in industries to assess its use. Table 6 offers the share of specific human capital in each industry, where we take country and time averages. For example, specific human capital is very intensively used in industries

[4] We also perform a similar assignment for each country and year. In line with the results in Table 4, this assignment has very little variation across countries and years. It is also highly correlated with our original assignment ($\rho = 0.913$).

[5] The ELFS database is based on stratified sampling. We use the available sample weights in all our calculations. However, these weights might not be very precise for the level of disaggregation we are interested in. This and sample imperfections can be responsible for some of the variation and spikes in the share of specific human capital in Figure 2. In Appendix - Treatment of Spikes in the Shares of Human Capital, we alleviate concerns with spikes and sample imperfections using polynomials to predict occupation shares in each industry-country pair.

[6] Kambourov and Manovskii (2008) find that industry mobility of workers has increased over time in the US. If the share of specific human capital displays similar negative trend in the US, then that negative trend could be a potential explanation for the higher industry mobility.

Agriculture, Hunting, and Fishing (1-digit NACE A-B) and Construction. It is not used intensively in industries Transport, Storage, and Communication, and Financial Intermediation. Figure 4 plots the share of specific human capital in each industry, where we take country and country-year averages. Tables 7 and 8 present time-averaged share of specific human capital in each country and industry and country-averaged share in each year and industry. The ranking of industries according to their use of specific human capital appears to be quite stable over time and across countries, which is further confirmed with country-level rank correlations as reported in Table 25 in Data Appendix.

The country-level trends in the share of specific human capital can be decomposed into changes in the employment shares of industries (between-industry) and industry-level changes in the shares of specific human capital (within-industry). Let $\omega^s_{c,t}$ and $\omega^s_{c,i,t}$ be the shares of specific human capital in country c at time t and in industry i in country c at time t. In turn, let $\omega_{c,i,t}$ be the share of employment in industry i in country c at time t. We have that

$$\Delta\omega^s_{c,t} = \sum_i \bar{\omega}^s_{c,i}\Delta\omega_{c,i,t} + \sum_i \bar{\omega}_{c,i}\Delta\omega^s_{c,i,t}, \tag{1}$$

where we use Δ to denote first difference operator and bar to denote the average over two consecutive periods.

We perform this decomposition for each country in the sample and average the changes in the share of specific human capital and its between- and within-industry components across countries and years. The average yearly change in the share of specific human capital is equal to -0.0035. In turn, the between- and within-industry components are equal to -0.0024 and -0.0012, correspondingly. This means that on average the share of industries which use specific human capital more intensively has declined over time (69 percent of variation) and industries have started using general human capital more intensively (31 percent of variation).[7]

These between- and within-industry components have non-trivial variation in sample countries. Table 9 offers the basics statistics of results from this decomposition for each country in the sample (see Table 26 Data Appendix for decomposition for each sample year). For example, the mean of within-industry component is not uniformly negative across countries. It is positive and relatively large for France, which indicates that industries in France have increased their specific human capital intensity over time.

Further, we retrieve from the ELFS database information on the workers' fields of studies for the highest degree (1-digit ISCED-97) and their levels of education. The levels of education are classified into three groups: pre-primary to lower-secondary (low-level; ISCED-97 0-2), secondary to post-secondary and non-tertiary (medium-level; ISCED-

[7] About half of the between-industry component is attributable to the decline of Agriculture, Hunting, and Fishing industry. Within-industry component is quite similar across industries.

97 3-4), and tertiary (high-level; ISCED-97 5-6). These data are used to identify how the workers' background in terms of education field maps onto occupations in the labor market and the education levels of specific and general human capital types.

We calculate the number of workers in each occupation-education field cell and the within-education field share of workers across different occupations. This share varies a lot with occupations and shows very little variation over time and countries according to Table 10 (see also Table 30 in Data Appendix). We sum this share across specific human capital occupations and across general human capital occupations and average these sums across countries and years. Table 11 offers for each education field the share of workers who have specific human capital occupation and their highest education degree in that field out of total number of employed individuals who have their highest degrees in that field. More than 70 percent of the graduates of education field Teacher Training and Education Science are employed in specific human capital occupations. Meanwhile, less than 20 percent of the graduates of education field Social Sciences, Business, and Law are employed in specific human capital occupations.[8] These disparities in the shares suggest that the differences of skills between general and specific human capital types are not solely associated with differences in occupations. They can be also associated with formal education and, especially, with the fields of education.

We list the levels of education/skills for all occupations in Table 12. This table offers the share of workers in each "level of education"-occupation cell out of total number of workers in each occupation, which we have averaged over countries and years.[9] Table 13 offers basic statistics for the levels of skills across specific and general human capital occupations. The distribution of skill-levels across the two types of human capital is such that no human capital type is singled out as exclusively low- or highly skilled. As an illustration, 75 percent of workers who have specific human capital have completed the pre-primary to lower-secondary education, while 25 percent are graduates of tertiary education. These percentages are almost exactly the same for the workers with general human capital.[10]

The ELFS database also provides us with information about individuals' professional status at the job in terms of being self-employed, employee, and family worker, if they have

[8] Tables 27, 28, and 29 in Data Appendix offer the share of workers in each education field for sample countries and sample years and the share of workers in each education-occupation cell out of total number in education fields, averaged across countries and time.

[9] Tables 31 and 32 in Data Appendix offer for each country and for each year the shares of employed individuals in each level of highest attained education out of total number of employed individuals which we have averaged across years and countries correspondingly.

[10] For the majority of sample countries we can use 3-digit disaggregation of occupations. We prefer 2-digit disaggregation since it allows us to focus on sets of skills which are neither very broad nor narrow so that they can be used in many industries and have relatively high scope of difference. We perform a similar assignment for 1-digit and 3-digit occupations and report the results in Tables 33 and 35 in Data Appendix. Similarly to 2-digit occupations, we observe negative trends. We also observe uniform levels of education for 3-digit occupations (see Figures 6 and 7 and Tables 33–36 in Data Appendix).

second jobs, and their age group and gender. For each of category of these variables, we compute the share of workers in specific human capital occupations out of total number of workers in specific human capital occupations. We do the same for general human capital occupations. Table 14 provides the basic statistics for these shares and tests the differences in means. The differences tend to be very small but statistically significant. For example, the share of self-employed is slightly higher and the share of employees is slightly lower among individuals who have specific human capital than among individuals who have general human capital. Moreover, the share of individuals who have more than 1 jobs and the share of females are slightly lower among individuals who have specific human capital than among individuals who have general human capital. We also compute the share of specific human capital within age groups and offer the results in Figure 5. The share of specific human capital has declined in all age groups. The largest reductions in the share of specific human capital can be observed among workers of age groups 17 to 22, 52 to 57, and 57 to 62. These groups are most likely to be comprised of individuals entering and exiting the labor market.[11]

Finally, we run simple OLS regressions where the dependent variables is the logarithm of real GDP per-capita and the main explanatory variable is the share of specific human capital. The data for GDP per capita we obtain from the WDI database. The results are presented in Table 15. The share of specific human capital and GDP per capita appear to be strongly negatively correlated.[12]

The Model

In this section, we offer a simplistic model where we write the production side so that to capture the inherent flexibility of general human capital and the inflexibility of specific human capital. There are two intermediate goods sectors: h and l. These sectors produce homogenous h-goods Y_h and l-goods Y_l. The producers of the h-goods have a nested-CES production function. Their inputs are specific and general types of human capital H_s and H_g, and K, which we call physical capital. In turn, the producers of l-goods have a single input of general human capital.[13] The final goods producers have a CES production function. Their inputs are Y_h and Y_l and they produce homogenous goods Y. Perfect competition prevails in all markets, and all producers maximize their

[11] The negative trend for age groups in between 22 and 52 is not very pronounced and it can be because of occupational change/mobility. Similarly, occupational change can be one of the reasons for the decline in the share of specific human capital between age groups 17 to 22 and 22 to 27.

[12] By definition and identification our classification of human capital types is different than the abstract, manual, and routine skills classification used by Acemoglu and Autor (2011) and Autor and Dorn (2013). Nevertheless, we check if the trends observed for the share of specific human capital repeat for the shares of employment in occupations requiring abstract, manual, and routine skills in Appendix - Abstract, Manual, and Routine Skills.

[13] According to Table 6, sector h can be thought to be comprised of 1-digit NACE Rev. 1 industries A-B, C, D, E, F, G, L, M, N, and Q and sector l of the reminder.

instantaneous profits. For the time being, we keep all factor inputs in a fixed supply.

The production function of the representative final goods producer is

$$Y = \lambda \left[\gamma_1 Y_h^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1-\gamma_1) Y_l^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}, \tag{2}$$

where $\varepsilon_1 > 0$ is the elasticity of substitution between h- and l-goods, $\gamma_1 \in (0,1)$, and λ is a random variable with log-normal distribution $\ln \mathcal{N}(\mu_z, \sigma_z^2)$. We set the price of final goods as the numeraire and denote the prices of the h- and l-goods by p_{Y_h} and p_{Y_l}. From the usual profit maximization problem it follows that the (inverse) demand functions for h- and l-goods are given by

$$p_{Y_h} = \omega_{Y_h}^Y \frac{Y}{Y_h}, \tag{3}$$

$$p_{Y_l} = \left(1 - \omega_{Y_h}^Y\right) \frac{Y}{Y_l}, \tag{4}$$

where $\omega_{Y_h}^Y$ is the share of Y_h compensation:

$$\omega_{Y_h}^Y = \frac{\gamma_1 Y_h^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\gamma_1 Y_h^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1-\gamma_1) Y_l^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}. \tag{5}$$

The production function of the representative h-goods producer is

$$Y_h = \lambda_h \left[\gamma_2 \left(u_h^g H_g\right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + (1-\gamma_2) Y_m^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} \right]^{\frac{\varepsilon_2}{\varepsilon_2 - 1}}, \tag{6}$$

where

$$Y_m = \left[\gamma_3 K^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + (1-\gamma_3) H_s^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} \right]^{\frac{\varepsilon_3}{\varepsilon_3 - 1}}, \tag{7}$$

u_h^g is the share of general human capital in h-sector, and γ_2, $\gamma_3 \in (0,1)$. In this nested-CES production function, $\varepsilon_2 > 0$ is the elasticity of substitution between Y_m and general human capital. It characterizes the elasticity of substitution between physical capital and general human capital and the elasticity of substitution between specific and general human capital. For brevity, we will say that ε_2 *is the elasticity of substitution between the pairs K and H_g and H_s and H_g*. In turn, $\varepsilon_3 > 0$ is the elasticity of substitution between physical capital and specific human capital. The shocks to h-goods production are given by λ_h, which has a log-normal distribution $\ln \mathcal{N}\left(\mu_{z_h}, \sigma_{z_h}^2\right)$.

We denote the wage rates of specific and general types of human capital by w_s and w_g and the return on K by r. The h-goods producer's (inverse) demand functions for physical capital and specific and general human capital are given by

$$r = \omega_{Y_m}^{Y_h} \omega_K^{Y_m} \frac{p_{Y_h} Y_h}{K}, \tag{8}$$

$$w_s = \omega_{Y_m}^{Y_h} \left(1 - \omega_K^{Y_m}\right) \frac{p_{Y_h} Y_h}{H_s}, \tag{9}$$

$$w_g = \left(1 - \omega_{Y_m}^{Y_h}\right) \frac{p_{Y_h} Y_h}{u_h^g H_g}, \tag{10}$$

where $\omega_{Y_m}^{Y_h}$ and $\omega_K^{Y_m}$ are shares and are given by:

$$\omega_{Y_m}^{Y_h} = \frac{(1-\gamma_2) Y_m^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}}{\gamma_2 \left(u_h^g H_g\right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + (1-\gamma_2) Y_m^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}}, \tag{11}$$

$$\omega_K^{Y_m} = \frac{\gamma_3 K^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}}{\gamma_3 K^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + (1-\gamma_3) H_s^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}}. \tag{12}$$

The representative l-goods producer has the following production technology

$$Y_l = \lambda_l \left(u_l^g H_g\right)^{\gamma_4}, \tag{13}$$

where u_l^g is the share of general human capital in l-sector and $\gamma_4 \in (0,1)$. The shocks to l-goods production are given by λ_l, which has a log-normal distribution $\ln \mathcal{N}\left(\mu_{z_l}, \sigma_{z_l}^2\right)$.

The usual profit maximization problem implies that l-goods producer's (inverse) demand function for general human capital is given by

$$w_g = \gamma_4 \frac{p_{Y_l} Y_l}{u_l^g H_g}. \tag{14}$$

Any profits are distributed to the households, which are discussed at the end of the section. Firms hire inputs before observing the values of the shocks. They compensate inputs after the realization of the shocks.

In equilibrium, the shares of general human capital across the sectors sum to one

$$1 = u_h^g + u_l^g. \tag{15}$$

Moreover, the expected wage rates of general human capital should be equal across h- and l-sectors. Therefore, from (3), (4), (10), and (14) it follows that

$$\mathbb{E}\left[\frac{p_{Y_l} Y_l}{u_h^g H_g}\left[\gamma_4 \frac{u_h^g}{u_l^g} - \left(1 - \omega_{Y_m}^{Y_h}\right) \frac{\gamma_1}{1-\gamma_1} \left(\frac{Y_h}{Y_l}\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right]\right] = 0.$$

This equation can be rewritten in the following way

$$\gamma_4 \frac{u_h^g}{u_l^g} = \left(1 - \omega_{Y_m}^{Y_h}\right) \frac{\mathbb{E}\left[\frac{\gamma_1}{1-\gamma_1}\left(\frac{Y_h}{Y_l}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} \lambda_l \left[\frac{\gamma_1}{1-\gamma_1}\left(\frac{Y_h}{Y_l}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} + 1\right]^{\frac{1}{\varepsilon_1-1}}\right]}{\mathbb{E}\left[\lambda_l \left[\frac{\gamma_1}{1-\gamma_1}\left(\frac{Y_h}{Y_l}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} + 1\right]^{\frac{1}{\varepsilon_1-1}}\right]}. \tag{16}$$

To keep things simple, we assume that the last term in this equation is equal to

$$\mathbb{E}\left[\frac{\gamma_1}{1-\gamma_1}\left(\frac{Y_h}{Y_l}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right].$$ [14]

Further, we use (6) and (13) to rewrite this equation as

$$\left(u_h^g\right)^{1-\frac{\varepsilon_2-1}{\varepsilon_2}} \left(1-u_h^g\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4-1} = \mathbb{E}\left[\left(\frac{\lambda_h}{\lambda_l}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right] \frac{\gamma_2}{\gamma_4} \frac{\gamma_1}{1-\gamma_1} \left(H_g\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}-\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4} \tag{17}$$

$$\times \left[\gamma_2 \left(u_h^g H_g\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \left(1-\gamma_2\right) Y_m^{\frac{\varepsilon_2-1}{\varepsilon_2}}\right]^{\frac{\varepsilon_2}{\varepsilon_2-1}\frac{\varepsilon_1-1}{\varepsilon_1}-1}.$$

The expression in (17) characterizes the equilibrium share of general human capital in sector h (u_h^g). The following proposition describes the behavior of u_h^g in response to changes in K, H_s, and H_g.

Proposition 1. *1. The share of general human capital in sector h (u_h^g) declines with K and H_s when $\varepsilon_2 > \varepsilon_1$ and increases with them when $\varepsilon_1 > \varepsilon_2$.*

2. It increases with H_g when $\varepsilon_2 > \varepsilon_1 > 1$ or $\varepsilon_2 > \varepsilon_1$ and $\gamma_4 = 1$ and declines with it when $1 > \varepsilon_1 > \varepsilon_2$ or $\varepsilon_1 > \varepsilon_2$ and $\gamma_4 = 1$.

Proof. See Proofs Appendix. □

For example, when $\varepsilon_2 > \varepsilon_1$, h- and l-goods are less substitutable than the pairs K and H_g and H_s and H_g in the production of h-goods. The share of general human capital in sector h (u_h^g) declines with K and H_s because of this.

In (17) shocks λ_h and λ_l enter into the expected value operator with an exponent of $(\varepsilon_1 - 1)/\varepsilon_1$. This implies the following proposition.

Proposition 2. *1. The share of general human capital in h-sector declines with μ_{z_h} and increases with μ_{z_l} when h- and l-goods are gross complements ($1 > \varepsilon_1$). Moreover, it increases with $\sigma_{z_l}^2$.*

[14] Adimitedly, this is not a trivial assumption. We relax this assumption and use numerical methods to check our results.

2. *The share of general human capital in h-sector increases with μ_{z_h} and declines with μ_{z_l} when h- and l-goods are gross substitutes ($\varepsilon_1 > 1$). Moreover, it increases with $\sigma^2_{z_h}$.*

Proof. See Proofs Appendix. □

The substitutability between Y_h and Y_l is decisive for these results because shocks λ_h and λ_l are Hicks-neutral in Y_h and Y_l. For example, u^g_h declines with μ_{z_h} when Y_h and Y_l are gross complements because, *ceteris paribus*, higher μ_{z_h} implies higher μ_{Y_h}.

The *ex ante* relative inverse demand ($\mathbb{E}[w_g]/\mathbb{E}[w_s]$) for general human capital can be derived using (9), (10), and (11). We denote it by $\tilde{w}_g$, and it is given by

$$\tilde{w}_g = \frac{\gamma_2}{1-\gamma_2}\frac{1}{1-\omega_K^{Y_m}}\left(\frac{1}{u^g_h H_g}\right)^{\frac{1}{\varepsilon_2}}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} H_s. \tag{18}$$

This expression implies that the previous proposition also applies to the relative (inverse) demand for general human capital.

Corollary 1. 1. *The relative (inverse) demand for general human capital increases with μ_{z_h} and declines with μ_{z_l} when $1 > \varepsilon_1$. Moreover, it declines with $\sigma^2_{z_l}$.*

2. *The relative (inverse) demand for general human capital declines with μ_{z_h} and increases with μ_{z_l} when $\varepsilon_1 > 1$. Moreover, it declines with $\sigma^2_{z_h}$.*

The relative (inverse) demand for general human capital also depends on the amounts of factor inputs in the following way:

Proposition 3. *The relative (inverse) demand for general human capital increases with K when $\varepsilon_3 > 1 \geq \varepsilon_2 > \varepsilon_1$ and declines with K when $\varepsilon_1 > \varepsilon_2 \geq 1 > \varepsilon_3$. It declines with H_g and increases with H_s.*

Proof. See Proofs Appendix. □

The elasticity of substitution between H_g and K in the production of h-goods is governed by ε_2. The elasticity of substitution between H_s and K is ε_3. This proposition states that when h- and l-goods and general human capital and physical capital are gross complements ($1 \geq \varepsilon_2 > \varepsilon_1$) and physical capital and specific human capital are gross substitutes ($\varepsilon_3 > 1$) then the relative demand for general human capital increases with K. In turn, the relative demand for general human capital declines with K when h- and l-goods and general human capital and physical capital are gross substitutes ($\varepsilon_1 > \varepsilon_2 \geq 1$) and physical capital and specific human are gross complements ($1 > \varepsilon_3$).

In this framework, the composition of human capital portfolio matters for the contribution of the volatility of sectoral shocks to the volatility of final output.

Proposition 4. *Consider two countries where the endowments of H_s and H_g are different but the level of expected output is the same.*

1. *In the country where H_s is higher the variance of λ_h has a lower contribution to the variance of final output when $1 > \varepsilon_1$. It has a higher contribution when $\varepsilon_1 > 1$.*

2. *In the country where H_s is higher the variance of λ_l has a higher contribution to the variance of final output when $1 > \varepsilon_1$. It has lower a contribution when $\varepsilon_1 > 1$.*

Proof. See Proofs Appendix. □

This result holds because H_s is not flexible across h- and l-sectors and increasing H_s increases Y_h/λ_h. Therefore, $(Y_h/\lambda_h)^{\frac{\varepsilon-1}{\varepsilon}}$ declines with H_s when $1 > \varepsilon_1$ and increases with it when $\varepsilon_1 > 1$. To keep the expected level of output constant then $(Y_l/\lambda_l)^{\frac{\varepsilon-1}{\varepsilon}}$ has to increase when $1 > \varepsilon_1$ and it has to decline when $\varepsilon_1 > 1$. These two quantities multiply the volatilities of λ_h and λ_l, respectively, as they stand in front of these shocks in Y.

For example, this result implies that in the country where H_s is higher the volatility of final output is higher if either $1 > \varepsilon_1$ and $\sigma^2_{z_l} > \sigma^2_{z_h} = \sigma^2_z = 0$ or $\varepsilon_1 > 1$ and $\sigma^2_{z_h} > \sigma^2_{z_l} = \sigma^2_z = 0$. It is lower if either $1 > \varepsilon_1$ and $\sigma^2_{z_h} > \sigma^2_{z_l} = \sigma^2_z = 0$ or $\varepsilon_1 > 1$ and $\sigma^2_{z_l} > \sigma^2_{z_h} = \sigma^2_z = 0$. This result also implies the following corollary.

Corollary 2. *Consider two countries where H_s and H_g are different but the level of expected output is the same. Suppose that $\sigma^2_z = 0$, $\varepsilon_1 > 1$ and $\sigma^2_{z_h} > \sigma^2_{z_l}$ (i.e., the coefficient of variation of λ_h is higher than the coefficient of variation of λ_l). Further, suppose that the share of Y_h is higher than or equal to the share of Y_l:*

$$\omega^Y_{Y_h} \geq 0.5.$$

In such a case, the volatility of final output is higher in the country where H_s is higher. The volatility of final output is also higher in that country in case when $\sigma^2_z = 0$, $1 > \varepsilon_1$, $\sigma^2_{z_l} > \sigma^2_{z_h}$, and $0.5 \geq \omega^Y_{Y_h}$.

According to Table 6, mostly services industries are very intensive in general human capital (e.g., 1-digit NACE industries H, I, J, and K) and in this sense these industries could represent Y_l. Their output (and employment) share is usually lower than 0.5 and they tend to be less volatile than the other industries (e.g., see Koren and Tenreyro, 2007). This proposition then implies that, if $\varepsilon_1 > 1$, the secular decline in the share of specific human capital, as observed in our sample countries, can explain the negative trend in output volatility documented by, for example, Stock and Watson (2005).[15]

[15] We present an attempt to estimate ε_1 in Appendix - Elasticity of Substitution. We obtain both lower and higher than 1 values for ε_1. Lower than 1 estimated values deliver slightly superior results in terms of the Root Square Mean Error.

Further, we consider a planner (e.g., household, policy-maker) who has an option to marginally increase either H_g or H_s at no cost (or at the same cost). We establish the following results for the cases when $\sigma_z^2 = 0$ and either $\sigma_{z_h}^2 > \sigma_{z_l}^2 = 0$ or $\sigma_{z_l}^2 > \sigma_{z_h}^2 = 0$.

Proposition 5. *Suppose the planner has an option to marginally increase either H_g or H_s by the same amount. In such a case, increasing H_g does not necessarily increase the volatility of final output less than so does increasing H_s, and*

$$\frac{\partial \sigma_Y^2}{\partial H_g} \Big/ \frac{\partial \sigma_Y^2}{\partial H_s} < 1$$

depends on the values of model parameters.

Proof. See Proofs Appendix. □

Proposition 6. *Suppose the planner has an option to increase either H_g or H_s by amounts that deliver the same marginal increase in expected final output,*

$$\frac{\partial \mu_Y}{\partial H_g} = \frac{\partial \mu_Y}{\partial H_s}. \tag{19}$$

1. *When $\sigma_{z_h}^2 > \sigma_{z_l}^2 = 0$ and $1 > \varepsilon_1$ ($\varepsilon_1 > 1$) the volatility of the final output increases more (less) with H_g than with H_s.*

2. *When $\sigma_{z_l}^2 > \sigma_{z_h}^2 = 0$ and $1 > \varepsilon_1$ ($\varepsilon_1 > 1$) the volatility of the final output increases less (more) with H_g than with H_s.*

Proof. See Proofs Appendix. □

Suppose that the objective function of the planner (e.g., the utility function of the households) is increasing and concave in final output. In such a case, the planner would like to insure against the sectoral shocks. The results in this proposition imply that under condition (19) when $\sigma_{z_l}^2 > \sigma_{z_h}^2 = 0$ and $1 > \varepsilon_1$ the planner would prefer investing in H_g. It would over-invest in H_s if it does not anticipate the shocks and treats λ_l as a deterministic variable at its mean value. Clearly, it would overinvest in H_s also in case when $\sigma_{z_h}^2 > \sigma_{z_l}^2 = 0$ and $\varepsilon_1 > 1$ if it does not anticipate the shocks and treats λ_l as a deterministic variable at its mean value.

We close the model and endogenize the supply of human capital types in a trivial manner. The economy is populated by a mass one of one period lived and identical households. The households own all assets and have strictly increasing, concave, and twice continuously differentiable utility from consumption (C) of final goods. At the beginning of the period they are endowed with K amount of physical capital and no human capital. They need to decide how much specific and general human capital to acquire. The costs of acquiring both types skills (S) are in terms of final goods. The production happens after the households supply physical capital and both types of human capital.

The representative household then solves the following problem

$$\begin{aligned}
&\max_{C,S_s,S_g} \mathbb{E}\left[u\left(C\right)\right] \\
&s.t. \\
&C + S_s + S_g = rK + w_s H_s + w_g H_g, \\
&H_s = \lambda_H S_s, \\
&H_g = \lambda_H S_g,
\end{aligned}$$

where S_s and S_g are the expenses for acquiring the corresponding skills and $\lambda_H > 0$ is an exogenous productivity level.

Clearly, it is optimal to supply all K, and the household chooses the amounts of H_s and H_g so that

$$\begin{aligned}
&\mathbb{E}\left[w_s\right] = \mathbb{E}\left[w_g\right] = \lambda_H, \\
&\tilde{w}_g = 1.
\end{aligned} \tag{20}$$

The allocations of general human capital and the levels of output and human capital types can be solved from (17), (15), (2), (6), (13), (7), (5), (11), (12), (9), (10), and (20).

The supply of human capital fixes the ratio of expected wages. Therefore, for example, H_g/H_s declines and u_h^g increases with $\sigma_{\lambda_l}^2$ when $1 > \varepsilon_1$ according to (18), (20), Proposition 2 and Corollary 1. The household then will over-invest in general human capital, H_g, if it does not anticipate the shocks to λ_l and treats λ_l as constant at its mean it. Moreover, it will allocate more than optimal amount of general human capital to l-goods production.

We observe that the share of specific human capital has declined over time in almost all countries in the sample. The model offered above can match this observation in a straightforward manner. Suppose that physical capital (K) grows over time. Further, h- and l-goods and general human capital and physical capital are gross complements ($1 \geq \varepsilon_2 > \varepsilon_1$) while physical capital and specific human capital are gross substitutes ($\varepsilon_3 > 1$). In such a case, the relative demand for general human capital grows over time. Therefore, the households will acquire increasingly more general skills relative to specific skills, and the share of specific human capital will decline. Clearly, changes in the means of λ_h and λ_l can also drive this pattern. For example, over time the households will acquire increasingly more general skills relative to specific skills if $1 > \varepsilon_1$ and μ_{λ_h} grows relative to μ_{λ_l}.[16] In this reduced form model, the changes in K and $\mu_{\lambda_h}/\mu_{\lambda_l}$ can

[16]We assume that the production of Y_l does not require K. Given the way we model K, the mean value of λ_l can be thought to represent the amount of physical capital in the production of Y_l.

be thought to represent, for example, changes in the production of h-goods, because of factor accumulation or biased technical change and obsolescence, and changes of industrial composition in terms of sectors h and l (Appendix A.1 shows that increasing K increases Y_h/Y_l).[17]

Supply side factors can also be responsible for the negative trend in the share of specific human capital. In this model, H_g/H_s will increase if the productivity of schooling general human capital grows over time relative to the productivity of schooling specific human capital. According to Table 11 more than 70% of the graduates from the education field of Science, Mathematics, and Computing attain general human capital. It could be then reasonable to expect that the relative productivity in schooling of general human capital has increased if technical change implied by the introduction of ICT increases the efficiency in the education process in this education field, relative to other fields.[18]

According to our data, the share of specific human capital has declined over time within and between industries. Our model can match this observation too. In the model, the share of general human capital can be decomposed as

$$\begin{aligned}\frac{H_g}{H_g+H_s} &= \frac{u_l^g H_g}{u_l^g H_g}\frac{u_l^g H_g}{H_g+H_s} + \frac{u_h^g H_g}{u_h^g H_g + H_s}\frac{u_h^g H_g + H_s}{H_g+H_s} \\ &\equiv \omega_l^g \omega_l + \omega_h^g \omega_h.\end{aligned}$$

In this expression ω_l^g and ω_h^g are the shares of general human capital in l- and h-sector and ω_l and ω_h are the shares of industries in terms of employment out of total employment. The data suggests that ω_h^g and ω_l have grown. By construction this implies that ω_h has declined. Moreover, by construction ω_l^g is equal to 1 and does not vary.

Proposition 7. *For brevity, we define the following function*

$$\mathbb{I}\left(x,y\right) = \begin{cases} x & if \quad x>0, \\ y & otherwise. \end{cases}$$

In order for ω_h^g and ω_l to grow with K it is sufficient to have ε_1 higher than

$$\mathbb{I}\left(\frac{(1-\gamma_2)\left[1+\omega_{Y_h}^Y(1-\gamma_2)\right]-1}{(1-\gamma_2)\left[1+\omega_{Y_h}^Y(1-\gamma_2)\right]},0\right)$$

[17] As discussed above, changes in variances of sectoral shocks can also affect the relative demand for general human capital and drive this pattern.

[18] We take no stance on which of these channels is more likely to be behind the trend in Figure 3. We also do not take a firm stance on parameter values, although we present estimation results for ε_1 and γ_1 in Appendix - Elasticity of Substitution.

and lower than

$$\mathbb{I}\left(\frac{\frac{H_s}{H_g+H_s}\frac{1}{1-u_h^g}+(1-\gamma_2)\left[\frac{u_h^g}{1-u_h^g}-\frac{H_s}{H_g+H_s}(1-\gamma_2)\,\omega_{Y_h}^Y\right]}{(1-\gamma_2)\left[\frac{u_h^g}{1-u_h^g}-\frac{H_s}{H_g+H_s}(1-\gamma_2)\,\omega_{Y_h}^Y\right]},+\infty\right).$$

Proof. See Proofs Appendix. □

It is easy to check that the interval for ε_1 where ω_h^g and ω_l grow with K includes both lower than 1 and greater than 1 values of ε_1.

Additional Results

We further consider a version of the model where firms hire inputs after observing the values of shocks and $\varepsilon_2 = \gamma_4 = 1$ keeping the reminder of the model intact. The following propositions are true for such an economy.

Proposition 8. *1. The elasticity of final output Y with respect to λ_h (λ_l) increases less (more) with a marginal (percentage) increase in H_s than with a marginal (percentage) increase in H_g if $1 > \varepsilon_1$.*

2. The elasticity of final output Y with respect to λ_h (λ_l) increases more (less) with a marginal (percentage) increase in H_s than with a marginal (percentage) increase in H_g if $\varepsilon_1 > 1$.

Proof. See Proofs Appendix. □

Proposition 9. *Consider two countries where H_s and H_g are different but the level of (expected) output is the same.*

1. *In the country where H_s is higher the elasticity of final output with respect to λ_h (λ_l) is lower (higher) if $1 > \varepsilon_1$.*
2. *In the country where H_s is higher the elasticity of final output with respect to λ_h (λ_l) is higher (lower) if $\varepsilon_1 > 1$.*

Proof. See Proofs Appendix. □

Proposition 10. *The elasticity of w_s with respect to λ_h (λ_l) is higher (lower) than the elasticity of w_g with respect to λ_h (λ_l) if and only if $1 > \varepsilon_1$.*

Proof. See Proofs Appendix. □

3 Conclusions

In this paper, we consider industry-specificity as a distinct source of human capital heterogeneity that is defined irrespective of the skill-level accumulated through education. Accordingly, we define specific and general human capital types treating occupations as types of skills. Specific human capital is the set of skills/occupations whose use is spread in a limited set of industries. In turn, general human capital is the set of skills which are used in a wide range of industries.

We use harmonized individual level EU Labour Force Survey to identify these human capital types and analyze their employment and education. Our empirical exercise yields remarkably persistent assignment of occupations into specific and general human capital types. We find that the share of employment in specific human capital occupations varies significantly across countries and has declined over time almost everywhere.

This negative trend is attributable to both within and between industry shifts. Industries have started using general human capital more intensively and the share of industries which use specific human capital more intensively has declined. Importantly, the ranking of industries according to their intensity of use of specific human capital is also remarkably stable.

The fields of education of general and specific human capital types are quite different and this difference is very persistent over time and across countries. This suggests that in addition to the occupational skill differences these human capital types are different in terms of the formal education. We also find that education levels of specific and general types of human capital are very uniform, which agrees with our horizontal differentiation.

Finally, in a multi-sector model we assess the effect of human capital portfolio composition on the propagation of sectoral shocks and the effect of uncertainty on the composition. In the model, we split industries into two sectors, h and l, according to the intensity of use of specific human capital. General human capital is required for production in both sectors. Specific human capital is required for production in sector h only. The outputs of these sectors, Y_h and Y_l, are aggregated into consumption goods with a CES function.

Suppose Y_h and Y_l are gross complements. Among countries where expected output is the same, in the country where the amount of specific human capital is higher the volatility of shocks to sector h (l) has lower (higher) contribution to the volatility of final output. Therefore, the volatility of final output in this country is lower if, for example, there are no shocks to l.

Further, we turn off shocks either to sector h or l and consider a planner who has an option to marginally increase either the amount of general human capital or specific human capital at no cost (or the same cost) and at the same benefit in terms of expected final output. We show that increasing the amount of general human capital increases the volatility of final output more (less) than increasing the amount of specific human capital if there are no shocks to sector l (h). Clearly, if the planner has a concave objective function then it would prefer investing in specific human capital if there are no shocks to sector l. It would under-invest in general human capital if it does not anticipate shocks to sector h. The opposites of these results hold when Y_h and Y_l are gross substitutes.

Our theoretical framework can be also used to gain an insight into what can drive the decline in the share of specific human capital. Such a negative trend can stem from biased technical change in the production of Y_h and Y_l goods and/or in schooling of human capital types. It can also stem from factor accumulation.

Tables

Table 1: *ANOVA for Within-Occupation Share Across Industries*

Source	Partial SS	df	MS	F	P-stat
Model	517.522	86	6.018	219.470	0.000
Occupation	50.379	25	2.015	73.490	0.000
Industry	427.082	15	28.472	1038.390	0.000
Country	13.210	28	0.472	17.210	0.000
Year	1.083	18	0.060	2.190	0.003
Residual	3742.937	136507	0.027		
Total	4260.460	136593	0.031		

Note: This table reports the results from an ANOVA exercise for the share of workers in each occupation-industry cell out of total employment in each occupation in all industries. The variation in the data is at occupation-industry-country-year level, and we perform the ANOVA exercise along each of these dimensions. Number of Obs. = 136594; Adj. R-squared = 0.121.

Table 2: *The Employment Shares of Occupations in Industries*

	Industry (NACE)															
	A-B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
Occupation (ISCO-88)																
11	0.019	0.007	0.025	0.009	0.015	0.024	0.007	0.025	0.015	0.036	0.769	0.032	0.049	0.107	0.005	0.041
12	0.018	0.007	0.228	0.017	0.068	0.188	0.036	0.066	0.089	0.106	0.055	0.050	0.042	0.038	0.001	0.012
13	0.060	0.002	0.107	0.004	0.080	0.390	0.135	0.047	0.018	0.087	0.010	0.016	0.020	0.041	0.005	0.002
21	0.006	0.012	0.233	0.032	0.082	0.049	0.003	0.052	0.039	0.368	0.076	0.024	0.014	0.016	0.001	0.013
22	0.027	0.002	0.024	0.004	0.002	0.076	0.003	0.003	0.004	0.043	0.039	0.023	0.760	0.011	0.003	0.004
23	0.003	0.001	0.005	0.001	0.002	0.005	0.004	0.003	0.002	0.011	0.016	0.931	0.022	0.012	0.004	0.003
24	0.006	0.003	0.095	0.008	0.013	0.060	0.006	0.031	0.067	0.240	0.194	0.039	0.071	0.160	0.001	0.020
31	0.015	0.012	0.298	0.039	0.081	0.063	0.004	0.123	0.018	0.178	0.073	0.024	0.038	0.042	0.001	0.007
32	0.028	0.002	0.034	0.005	0.003	0.067	0.005	0.003	0.003	0.024	0.029	0.024	0.776	0.023	0.003	0.004
33	0.005	0.008	0.016	0.004	0.006	0.035	0.008	0.013	0.007	0.016	0.041	0.683	0.227	0.046	0.012	0.005
34	0.008	0.002	0.121	0.008	0.021	0.194	0.012	0.052	0.133	0.137	0.162	0.023	0.052	0.071	0.001	0.008
41	0.011	0.004	0.160	0.016	0.037	0.160	0.010	0.147	0.076	0.106	0.136	0.038	0.055	0.044	0.001	0.006
42	0.004	0.003	0.032	0.016	0.008	0.227	0.074	0.175	0.217	0.060	0.044	0.012	0.064	0.080	0.002	0.003
51	0.006	0.001	0.016	0.002	0.003	0.025	0.282	0.031	0.003	0.044	0.117	0.058	0.275	0.119	0.023	0.003
52	0.004	0.001	0.053	0.001	0.006	0.882	0.019	0.008	0.005	0.017	0.002	0.002	0.003	0.009	0.003	0.001
61	0.872	0.002	0.017	0.002	0.009	0.019	0.005	0.002	0.001	0.018	0.026	0.006	0.009	0.033	0.015	0.002
71	0.006	0.019	0.110	0.019	0.736	0.029	0.004	0.010	0.001	0.025	0.018	0.008	0.009	0.011	0.003	0.002
72	0.013	0.012	0.486	0.039	0.093	0.218	0.004	0.069	0.002	0.024	0.020	0.005	0.007	0.013	0.002	0.005
73	0.010	0.007	0.761	0.010	0.030	0.086	0.007	0.013	0.006	0.057	0.019	0.012	0.030	0.032	0.012	0.020
74	0.012	0.002	0.743	0.003	0.032	0.150	0.025	0.005	0.002	0.010	0.006	0.005	0.012	0.018	0.006	0.003
81	0.017	0.049	0.744	0.079	0.032	0.027	0.005	0.015	0.007	0.026	0.015	0.020	0.015	0.022	0.007	0.010
82	0.011	0.004	0.862	0.006	0.019	0.041	0.005	0.010	0.002	0.016	0.005	0.003	0.014	0.025	0.002	0.004
83	0.049	0.015	0.130	0.011	0.116	0.097	0.007	0.489	0.003	0.017	0.028	0.004	0.015	0.024	0.002	0.004
91	0.012	0.003	0.071	0.012	0.014	0.077	0.099	0.043	0.012	0.167	0.075	0.121	0.119	0.094	0.087	0.003
92	0.835	0.009	0.036	0.015	0.019	0.036	0.010	0.009	0.009	0.030	0.053	0.013	0.033	0.053	0.025	0.009
93	0.016	0.011	0.375	0.011	0.236	0.145	0.008	0.089	0.003	0.031	0.038	0.006	0.029	0.021	0.003	0.008

Note: We compute the share of workers in each occupation (2-digit ISCO-88) and industry (1-digit NACE Rev. 1) out of total employment in the occupation in all industries for each country and year. This table reports country and year average of these shares. See Table 3 and Table 6 for the definitions of occupations and industries.

Table 3: *Concentrations of Occupations in Industries and the Assignment of Occupations into Specific and General Human Capital Types*

2-digit ISCO-88: Occupation Name	CV	GE	Gini	HI	Theil	Specific (= 1; General = 0)
11: Legislators and Senior Officials	3.114	4.761	0.870	0.665	2.026	1
12: Corporate Managers	1.195	0.719	0.559	0.154	0.587	0
13: General Managers	1.731	1.474	0.702	0.249	0.998	0
21: Physical, Mathematical, and Engineering Science Professionals	1.726	1.441	0.694	0.245	0.956	0
22: Life Science and Health Professionals	3.002	4.272	0.859	0.602	1.842	1
23: Teaching Professionals	3.697	6.423	0.916	0.874	2.430	1
24: Other Professionals	1.377	0.910	0.639	0.178	0.761	0
31: Physical and Engineering Science Associate Professionals	1.426	0.978	0.630	0.187	0.748	0
32: Life Science and Health Associate Professionals	3.052	4.452	0.859	0.625	1.874	1
33: Teaching Associate Professionals	3.011	4.388	0.876	0.618	1.982	1
34: Other Associate Professionals	1.214	0.702	0.596	0.152	0.649	0
41: Office Clerks	1.093	0.568	0.554	0.135	0.553	0
42: Customer Services Clerks	1.532	1.153	0.669	0.209	0.873	0
51: Personal and Protective Services Workers	1.718	1.458	0.706	0.247	0.996	0
52: Models, Salespersons, and Demonstrators	3.498	5.769	0.901	0.792	2.247	1
61: Market-oriented Skilled Agricultural and Fishery Workers	3.482	5.782	0.897	0.794	2.256	1
71: Extraction and Building Trades Workers	2.923	4.052	0.841	0.574	1.744	1
72: Metal, Machinery, and Related Trades Workers	2.064	2.029	0.767	0.319	1.223	0
73: Precision, Handicraft, Printing, and Related Trades Workers	3.041	4.426	0.867	0.622	1.912	1
74: Other Craft and Related Trades Workers	3.041	4.450	0.875	0.626	1.964	1
81: Stationary-plant and Related Operators	2.964	4.231	0.851	0.597	1.830	1
82: Machine Operators and Assemblers	3.426	5.552	0.888	0.764	2.164	1
83: Drivers and Mobile-plant Operators	1.995	1.902	0.742	0.303	1.128	0
91: Sales and Services Elementary Occupations	1.117	0.603	0.553	0.139	0.560	0
92: Agricultural, Fishery, and Related Laborers	3.329	5.315	0.892	0.735	2.179	1
93: Labourers in Mining, Construction, Manufacturing, and Transport	1.884	1.697	0.749	0.278	1.146	0

Note: This table offers the values of country- and year-averaged concentration measures for each occupation (2-digit ISCO-88) and the assignment of occupations into specific and general human capital types. An occupation corresponds to specific human capital if Specific dummy variable, which is offered in the last column of the table, is equal to 1. This dummy variable is constructed in the following manner. For each of the country- and year-averaged concentration measures (columns 2–6) we define a dummy variable which equals 1 for the values of the concentration measure which are higher than its median. We average these dummy variables over the concentration measures and set Specific dummy variable to 1 if the average is greater than 0.5, and to 0 otherwise.

Table 4: *Specific Human Capital Identified Separately for Each Country and Year*

2-digit ISCO-88: Occupation Name	Count in Countries	Count in Years
11: Legislators and Senior Officials	26	19
22: Life Science and Health Professionals	29	19
23: Teaching Professionals	29	19
32: Life Science and Health Associate Professionals	28	19
33: Teaching Associate Professionals	29	18
51: Personal and Protective Services Workers	2	0
52: Models, Salespersons, and Demonstrators	29	19
61: Market-oriented Skilled Agricultural and Fishery Workers	29	19
71: Extraction and Building Trades Workers	28	16
72: Metal, Machinery, and Related Trades Workers	2	0
73: Precision, Handicraft, Printing and Related Trades Workers	29	19
74: Other Craft and Related Trades Workers	27	19
81: Stationary-plant and Related Operators	28	19
82: Machine Operators and Assemblers	29	19
83: Drivers and Mobile-plant Operators	1	0
92: Agricultural, Fishery, and Related Laborers	28	19

Note: In this table, we offer the results from an exercise where we use our methodology to identify specific and general human capital types separately for each country and for each year. The second column offers the number of times that an occupation is assigned into specific human capital type in 29 sample countries. The third column offers the number of times that an occupation is assigned into specific human capital type in 19 sample years. For the second column, we average the concentration measures over years in each country and define a dummy variable for each of the averaged concentration measure which equals 1 for the values of the concentration measure which are higher than its median. Finally, we average these dummy variables over the concentration measures and call an occupation specific human capital if this average is greater than 0.5, and general otherwise. ISCO 11 occupation is classified as general human capital in Switzerland, Italy, and the UK. ISCO 32 is classified as general human capital in Spain. ISCO 51 is classified as specific human capital in Denmark and Sweden. ISCO 71 is classified as general human capital in Germany. ISCO 72 is classified as specific human capital in Germany and Spain. ISCO 74 is classified as general human capital in Luxembourg and Sweden. ISCO 81 is classified as general human capital in Denmark. ISCO 83 is classified as specific human capital in the UK. Our data do not contain ISCO 92 for France. For the third column, we repeat the exercise taking averages over years instead of countries. ISCO 33 is classified as specific human capital every year except 1992. ISCO 71 is classified as general human capital in 1997, 2001, and 2002. See Table 3 for the original assignment of occupations into specific and general human capital types.

Table 5: *Sample Countries and Years, and Basic Statistics for the Share of Specific Human Capital*

Country	Sample Period	Obs.	Mean	SD	Min	Max
Austria	1995–2010	16	0.343	0.036	0.294	0.379
Belgium	1993–2010	18	0.316	0.017	0.288	0.343
Bulgaria	2000–2010	11	0.378	0.013	0.352	0.392
Croatia	2002–2010	9	0.413	0.014	0.399	0.437
Cyprus	1999–2010	12	0.323	0.018	0.283	0.350
Czech Republic	1997–2010	14	0.354	0.013	0.335	0.376
Denmark	1992–2010	19	0.313	0.010	0.294	0.329
Estonia	1997–2010	14	0.343	0.012	0.319	0.365
Finland	1997–2010	14	0.322	0.027	0.290	0.365
France	1992–2010	19	0.315	0.020	0.281	0.364
Germany	1992–2010	19	0.310	0.015	0.286	0.330
Greece	1992–2010	19	0.442	0.039	0.389	0.511
Hungary	1996–2010	15	0.375	0.032	0.268	0.398
Iceland	1995–2010	16	0.405	0.022	0.372	0.443
Ireland	1992–2010	19	0.352	0.081	0.289	0.488
Italy	1992–2010	19	0.387	0.049	0.316	0.447
Latvia	1998–2010	13	0.350	0.040	0.274	0.389
Lithuania	1998–2010	13	0.396	0.036	0.354	0.455
Luxembourg	1992–2010	19	0.280	0.079	0.214	0.586
Netherlands	1992–2010	19	0.268	0.016	0.242	0.296
Norway	1996–2010	15	0.341	0.008	0.324	0.354
Poland	2000–2010	11	0.453	0.012	0.436	0.472
Portugal	1992–2010	19	0.389	0.041	0.340	0.460
Slovakia	1998–2010	13	0.376	0.011	0.363	0.400
Slovenia	1996–2010	15	0.425	0.030	0.373	0.471
Spain	1992–2010	19	0.358	0.026	0.308	0.399
Sweden	1997–2010	14	0.327	0.010	0.313	0.345
Switzerland	1996–2010	15	0.350	0.010	0.328	0.361
UK	1992–2010	19	0.270	0.013	0.254	0.291

Note: This table offers our sample of countries and years, and basic statistics for the share of workers in specific human capital occupations out of total employment (the share of specific human capital) in each country in the sample. See Table 3 for the assignment of occupations into specific and general human capital types.

Table 6: *The Share of Specific Human Capital in Industries*

1-digit NACE: Industry Name	Share
A-B: Agriculture, Hunting, and Fishing	0.795
C: Mining and Quarrying	0.327
D: Manufacturing	0.418
E: Electricity, Gas and Water Supply	0.193
F: Construction	0.569
G: Wholesale and Retail Trade; Repair of Goods	0.389
H: Hotels and Restaurants	0.053
I: Transport, Storage, and Communication	0.027
J: Financial Intermediation	0.013
K: Real Estate, Renting, and Business Activities	0.073
L: Public Administration; Social Security	0.132
M: Education	0.689
N: Health and Social Work	0.492
O: Other Community and Personal Service Activities	0.116
P: Households with Employed Persons	0.119
Q: Extra-territorial Organizations and Bodies	0.170

Note: This table offers the share of workers in specific human capital occupations out of total employment in industries (1-digit NACE). The data are averaged across countries and years. See Table 3 for the assignment of occupations into specific and general human capital types.

Table 7: *The Employment Share of Specific Human Capital in Countries and Industries*

Countries	Industries A-B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
Austria	0.945	0.306	0.340	0.166	0.552	0.373	0.027	0.021	0.009	0.051	0.120	0.759	0.498	0.108	0.029	0.078
Belgium	0.887	0.328	0.353	0.150	0.581	0.346	0.026	0.031	0.008	0.047	0.168	0.747	0.501	0.073	0.085	0.097
Bulgaria	0.714	0.272	0.507	0.214	0.416	0.469	0.036	0.020	0.008	0.068	0.103	0.664	0.596	0.081	0.121	0.235
Croatia	0.930	0.199	0.410	0.199	0.415	0.483	0.049	0.031	0.008	0.061	0.183	0.704	0.642	0.083	0.126	0.295
Cyprus	0.925	0.243	0.410	0.106	0.590	0.437	0.058	0.005	0.004	0.036	0.086	0.787	0.565	0.088	0.005	0.094
Czech Republic	0.586	0.376	0.421	0.245	0.563	0.407	0.031	0.020	0.007	0.059	0.141	0.657	0.621	0.106	0.132	0.158
Denmark	0.814	0.167	0.318	0.222	0.601	0.297	0.063	0.017	0.007	0.072	0.154	0.694	0.406	0.116	0.033	0.437
Estonia	0.642	0.237	0.440	0.222	0.547	0.397	0.083	0.023	0.024	0.079	0.113	0.575	0.509	0.086	0.269	0.384
Finland	0.905	0.247	0.332	0.202	0.494	0.344	0.021	0.012	0.008	0.105	0.092	0.616	0.413	0.159	0.333	0.162
France	0.901	0.266	0.418	0.141	0.628	0.297	0.062	0.032	0.012	0.077	0.128	0.659	0.400	0.168	0.088	0.144
Germany	0.868	0.420	0.313	0.253	0.617	0.294	0.075	0.023	0.006	0.095	0.152	0.741	0.506	0.141	0.054	0.156
Greece	0.989	0.423	0.537	0.112	0.752	0.343	0.023	0.006	0.006	0.028	0.157	0.834	0.595	0.080	0.036	0.124
Hungary	0.586	0.397	0.467	0.326	0.566	0.523	0.081	0.053	0.010	0.073	0.142	0.626	0.550	0.107	0.346	0.314
Iceland	0.802	0.593	0.493	0.183	0.654	0.476	0.140	0.039	0.010	0.109	0.129	0.715	0.457	0.156		0.169
Ireland	0.478	0.270	0.395	0.125	0.518	0.444	0.033	0.043	0.035	0.106	0.140	0.694	0.500	0.134	0.065	0.194
Italy	0.881	0.322	0.472	0.210	0.606	0.454	0.035	0.021	0.010	0.068	0.147	0.691	0.608	0.085	0.026	0.157
Latvia	0.671	0.148	0.423	0.203	0.452	0.403	0.068	0.049	0.032	0.103	0.144	0.560	0.551	0.138	0.220	0.559
Lithuania	0.775	0.246	0.481	0.213	0.481	0.392	0.074	0.036	0.017	0.108	0.120	0.555	0.587	0.098	0.225	0.098
Luxembourg	0.855	0.382	0.404	0.190	0.571	0.354	0.049	0.045	0.056	0.050	0.145	0.786	0.427	0.109	0.014	0.082
Netherlands	0.307	0.159	0.320	0.157	0.586	0.306	0.029	0.022	0.008	0.068	0.164	0.710	0.456	0.130	0.128	0.117
Norway	0.932	0.237	0.419	0.142	0.532	0.428	0.040	0.013	0.007	0.070	0.093	0.723	0.362	0.105	0.075	1.000
Poland	0.962	0.444	0.442	0.236	0.541	0.516	0.066	0.017	0.005	0.064	0.066	0.569	0.562	0.121	0.065	0.291
Portugal	0.912	0.531	0.547	0.131	0.608	0.367	0.023	0.025	0.010	0.054	0.175	0.582	0.355	0.087	0.007	0.197
Slovakia	0.521	0.429	0.478	0.289	0.541	0.491	0.042	0.026	0.007	0.069	0.100	0.632	0.571	0.107	0.071	0.354
Slovenia	0.949	0.432	0.500	0.201	0.468	0.445	0.057	0.027	0.005	0.053	0.099	0.694	0.581	0.087	0.060	0.196
Spain	0.896	0.446	0.462	0.253	0.602	0.346	0.024	0.017	0.011	0.056	0.120	0.773	0.433	0.082	0.028	0.171
Sweden	0.803	0.338	0.422	0.225	0.604	0.366	0.132	0.019	0.013	0.129	0.129	0.656	0.313	0.157	0.115	
Switzerland	0.954	0.328	0.365	0.183	0.628	0.435	0.094	0.057	0.031	0.113	0.144	0.770	0.563	0.227	0.207	0.156
UK	0.709	0.268	0.317	0.173	0.536	0.304	0.043	0.024	0.013	0.061	0.114	0.636	0.379	0.118	0.353	0.234

Note: For each industry (1-digit NACE), this table offers the yearly average share of workers in specific human capital occupations out of total employment in the industry in each country. We have no observations for 1-digit NACE industry P in Iceland and industry Q in Sweden. See Table 3 for the assignment of occupations into specific and general human capital types and Table 6 for the definitions of industries.

Table 8: *The Employment Share of Specific Human Capital in Years and Industries*

Year	A-B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1992	0.742	0.374	0.448	0.239	0.605	0.306	0.084	0.074	0.073	0.198	0.239	0.647	0.546	0.173	0.047	0.275
1993	0.789	0.347	0.433	0.208	0.594	0.366	0.058	0.031	0.013	0.076	0.183	0.727	0.505	0.105	0.045	0.233
1994	0.845	0.371	0.432	0.185	0.596	0.365	0.057	0.030	0.016	0.075	0.176	0.731	0.501	0.107	0.092	0.196
1995	0.844	0.391	0.435	0.169	0.599	0.375	0.042	0.028	0.013	0.065	0.165	0.729	0.488	0.119	0.075	0.180
1996	0.849	0.335	0.430	0.199	0.587	0.384	0.046	0.032	0.012	0.065	0.158	0.714	0.493	0.112	0.067	0.171
1997	0.829	0.377	0.421	0.198	0.585	0.382	0.043	0.026	0.009	0.069	0.156	0.698	0.496	0.126	0.103	0.158
1998	0.783	0.358	0.423	0.200	0.574	0.382	0.054	0.026	0.013	0.076	0.137	0.690	0.501	0.123	0.140	0.213
1999	0.786	0.336	0.427	0.205	0.573	0.387	0.060	0.028	0.014	0.070	0.128	0.696	0.507	0.122	0.104	0.134
2000	0.785	0.313	0.426	0.190	0.570	0.399	0.051	0.028	0.012	0.065	0.129	0.693	0.506	0.114	0.097	0.121
2001	0.787	0.324	0.431	0.210	0.570	0.404	0.055	0.028	0.012	0.065	0.126	0.685	0.498	0.109	0.112	0.162
2002	0.795	0.326	0.429	0.204	0.569	0.410	0.051	0.026	0.010	0.068	0.129	0.683	0.501	0.109	0.102	0.131
2003	0.796	0.337	0.423	0.206	0.560	0.411	0.050	0.022	0.013	0.059	0.127	0.680	0.493	0.113	0.109	0.243
2004	0.793	0.356	0.415	0.193	0.559	0.389	0.053	0.024	0.012	0.068	0.120	0.683	0.490	0.112	0.200	0.167
2005	0.791	0.306	0.412	0.204	0.562	0.391	0.052	0.026	0.011	0.066	0.116	0.687	0.493	0.113	0.157	0.178
2006	0.786	0.282	0.412	0.187	0.566	0.391	0.051	0.025	0.011	0.063	0.116	0.681	0.483	0.110	0.162	0.166
2007	0.776	0.299	0.407	0.183	0.564	0.392	0.054	0.024	0.010	0.062	0.116	0.682	0.479	0.107	0.138	0.145
2008	0.769	0.301	0.400	0.176	0.561	0.388	0.052	0.025	0.011	0.066	0.115	0.683	0.472	0.108	0.121	0.163
2009	0.799	0.294	0.395	0.168	0.546	0.387	0.053	0.021	0.009	0.091	0.116	0.675	0.473	0.118	0.111	0.115
2010	0.802	0.305	0.397	0.165	0.547	0.393	0.059	0.022	0.010	0.089	0.114	0.676	0.472	0.125	0.118	0.129

Note: For each industry (1-digit NACE), this table offers country-level average share of workers in specific human capital occupations out of total employment in the industry in each year. See Table 3 for the assignment of occupations into specific and general human capital types and Table 6 for the definitions of industries.

Table 9: *Industry-Level Decomposition of the Trends in the Share of Specific Human Capital for Each Country*

		Between Industries				Within Industries				Total			
Country	Obs.	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max
Austria	15	-0.002	0.005	-0.016	0.010	-0.003	0.015	-0.054	0.009	-0.005	0.019	-0.070	0.019
Belgium	17	-0.001	0.004	-0.007	0.009	-0.001	0.011	-0.030	0.027	-0.002	0.013	-0.038	0.032
Bulgaria	10	-0.003	0.005	-0.012	0.003	0.000	0.006	-0.012	0.011	-0.003	0.009	-0.020	0.014
Croatia	7	-0.003	0.009	-0.018	0.010	0.000	0.007	-0.012	0.007	-0.003	0.009	-0.014	0.016
Cyprus	11	-0.001	0.005	-0.008	0.010	-0.005	0.010	-0.019	0.009	-0.006	0.012	-0.027	0.014
Czech Republic	13	-0.002	0.002	-0.006	0.001	0.001	0.008	-0.016	0.013	-0.001	0.008	-0.018	0.014
Denmark	18	-0.001	0.003	-0.009	0.004	0.001	0.011	-0.017	0.032	0.000	0.012	-0.019	0.033
Estonia	13	-0.003	0.006	-0.013	0.007	0.001	0.014	-0.021	0.029	-0.002	0.013	-0.023	0.022
Finland	13	-0.003	0.002	-0.005	0.000	-0.003	0.011	-0.037	0.006	-0.006	0.011	-0.041	0.004
France	18	-0.003	0.005	-0.016	0.012	0.003	0.025	-0.030	0.099	0.001	0.023	-0.037	0.083
Germany	18	-0.001	0.003	-0.007	0.006	-0.001	0.003	-0.006	0.007	-0.002	0.004	-0.007	0.010
Greece	18	-0.006	0.005	-0.017	0.001	-0.001	0.004	-0.014	0.005	-0.007	0.008	-0.031	0.004
Hungary	14	0.008	0.036	-0.008	0.132	-0.001	0.005	-0.007	0.006	0.007	0.035	-0.014	0.125
Iceland	15	-0.003	0.009	-0.018	0.021	0.000	0.010	-0.019	0.017	-0.003	0.014	-0.022	0.028
Ireland	18	-0.004	0.009	-0.028	0.004	-0.006	0.031	-0.119	0.032	-0.010	0.033	-0.138	0.009
Italy	18	-0.003	0.004	-0.014	0.008	-0.004	0.013	-0.055	0.004	-0.007	0.015	-0.064	0.001
Latvia	12	-0.003	0.006	-0.010	0.010	-0.004	0.016	-0.040	0.018	-0.007	0.018	-0.044	0.023
Lithuania	12	-0.007	0.021	-0.059	0.035	-0.001	0.009	-0.017	0.012	-0.008	0.022	-0.047	0.043
Luxembourg	18	-0.005	0.008	-0.019	0.007	-0.016	0.062	-0.262	0.021	-0.021	0.066	-0.281	0.016
Netherlands	18	-0.001	0.007	-0.012	0.015	-0.002	0.005	-0.013	0.013	-0.003	0.008	-0.017	0.012
Norway	13	-0.003	0.005	-0.015	0.003	0.001	0.009	-0.014	0.016	-0.001	0.010	-0.019	0.013
Poland	9	-0.005	0.007	-0.018	0.010	0.001	0.004	-0.004	0.008	-0.004	0.009	-0.021	0.014
Portugal	18	-0.005	0.013	-0.051	0.020	0.002	0.009	-0.011	0.026	-0.003	0.018	-0.055	0.035
Slovakia	12	-0.002	0.004	-0.008	0.006	0.002	0.010	-0.010	0.027	0.000	0.013	-0.013	0.033
Slovenia	14	-0.004	0.007	-0.017	0.005	-0.001	0.009	-0.010	0.021	-0.005	0.014	-0.027	0.024
Spain	18	-0.005	0.003	-0.013	0.001	0.000	0.004	-0.008	0.008	-0.005	0.003	-0.011	0.001
Sweden	13	-0.001	0.004	-0.009	0.011	-0.001	0.007	-0.016	0.014	-0.002	0.007	-0.020	0.008
Switzerland	14	-0.003	0.009	-0.026	0.010	0.001	0.009	-0.020	0.019	-0.002	0.011	-0.027	0.018
UK	18	0.000	0.002	-0.003	0.003	-0.001	0.004	-0.011	0.005	-0.002	0.004	-0.013	0.005

Note: This table offers the basic statistics for between- and within-industry decomposition of changes in the share of specific human capital (1) for each country. Columns 1-4 and 5-8 offer the basic statistics for between- and within-industry components. Columns 9-12 offer the basic statistics for total change in the share of specific human capital. Figures 4 and 2 show that there are spikes in the share of specific human capital. These spikes can stem from sample imperfections and imperfections in sampling weights at this level of disaggregation. For example, in our data there are large persistent changes in the number of employees in occupations 82 and 93 in between 2000 and 2001 in the Czech Republic. Such imperfections can bias these decompositions in ambiguous directions. The effects of such biases are likely to be alleviated when we take country and/or year averages. In Appendix - Treatment of Spikes in the Shares of Human Capital we attempt to mitigate these imperfections.

Table 10: *ANOVA for the Within-Education Field Share Across Occupations*

Source	Partial SS	df	MS	F	P-stat
Model	192.222	294	0.654	403.920	0.000
Occupation	36.100	25	1.444	892.080	0.000
Occupation x Education Field	155.391	225	0.691	426.660	0.000
Education Field	0.000	9	0.000	0.000	1.000
Country	0.004	28	0.000	0.090	1.000
Year	0.000	7	0.000	0.000	1.000
Residual	88.164	54466	0.002		
Total	280.387	54760	0.005		

Note: This table reports the results from an ANOVA exercise for the share of workers in each highest-degree education field-occupation cell out of total number of workers who have their highest degree in that education field. The variation in the data are at occupation-education field-country-year level, and we perform the ANOVA exercise along each of these dimensions and the interaction of education fields and occupations. The data for education fields are available for the period of 2003–2010. Number of obs = 54761; Adj. R-squared = 0.684. See Table 3 for the list of occupations and Table 11 for education fields.

Table 11: *The Share of Specific Human Capital in Education Fields*

1-digit ISCED-97: Education Field Name	Share of Specific Human Capital
0: General Programs	0.262
1: Teacher Training and Education Science	0.716
2: Humanities, Languages, and Arts	0.351
3: Social Sciences, Business, and Law	0.165
4: Science, Mathematics, and Computing	0.266
5: Engineering, Manufacturing, and Construction	0.314
6: Agriculture and Veterinary	0.545
7: Health and Welfare	0.668
8: Services	0.210
9: Unknown	0.297

Note: This table offers for each education field (1-digit ISCED-97) the country-year averaged share of workers who have specific human capital occupation and their highest degree in that education field out of total number of workers who have their highest degrees in that field. The data for education fields are available for the period of 2003–2010. See Table 3 for the assignment of occupations into specific and general human capital types.

Table 12: *Skill-Levels Across Occupations*

Occupation (ISCO-88)	Skill-level (ISCED-97 0-2; 3-4; 5-6) Low-skilled	Medium-skilled	Highly-skilled
11	0.098	0.339	0.570
12	0.092	0.356	0.548
13	0.237	0.495	0.262
21	0.016	0.147	0.843
22	0.007	0.076	0.923
23	0.010	0.102	0.884
24	0.029	0.198	0.768
31	0.093	0.566	0.337
32	0.070	0.470	0.455
33	0.087	0.473	0.446
34	0.107	0.552	0.337
41	0.181	0.646	0.169
42	0.203	0.637	0.156
51	0.299	0.605	0.090
52	0.299	0.607	0.089
61	0.493	0.434	0.067
71	0.373	0.579	0.044
72	0.283	0.649	0.064
73	0.281	0.626	0.097
74	0.378	0.565	0.052
81	0.397	0.541	0.063
82	0.439	0.508	0.046
83	0.434	0.527	0.032
91	0.536	0.411	0.045
92	0.604	0.365	0.052
93	0.536	0.422	0.036

Note: This table offers for each occupation the share of workers in each level of highest attained education out of total number of workers in that occupation, which we have averaged over countries and years. There are three levels of highest attained education: pre-primary to lower-secondary (low-skilled; ISCED-97 0-2), secondary to post-secondary non-tertiary (medium-skilled; ISCED-97 3-4), and tertiary (highly-skilled; ISCED-97 5-6). See Table 3 for the list of occupations.

Table 13: *Skill-Levels of Specific and General Human Capital Types*

		Specific Human Capital				General Human Capital					
Level of Education	Obs.	Mean	SD	Min	Max	Mean	SD	Min	Max	Diff. in Means	SE
Low (ISCED-97 0-2)	441	0.301	0.193	0.031	0.867	0.237	0.159	0.03	0.778	0.063***	(0.012)
Medium (ISCED-97 3-4)	441	0.443	0.179	0.044	0.819	0.506	0.15	0.142	0.807	-0.063***	(0.011)
High (ISCED-97 5-6)	441	0.251	0.093	0.058	0.457	0.251	0.082	0.071	0.517	0.001	(0.006)

Note: This table offers basic statistics for the share of workers with low-, medium-, and high-level skills/education who have specific human capital occupation out of total employment in specific human capital occupations and the share of workers with low-, medium-, and high-level skills/education who have general human capital occupation out of total employment in general human capital occupations. The data are for all countries and years. There are three levels of highest attained education: pre-primary to lower-secondary (low-skilled; ISCED-97 0-2), secondary to post-secondary non-tertiary (medium-skilled; ISCED-97 3-4), and tertiary (highly-skilled; ISCED-97 5-6). The last two columns of the table use two-sided t-test to test the significance of differences in means. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. See Table 3 for the assignment of occupations into specific and general human capital types.

Table 14: *Professional Status, Number of Jobs, Gender, and Age of Specific and General Human Capital Types*

			Specific Human Capital				General Human Capital					
Variables	Categories	Obs.	Mean	SD	Min	Max	Mean	SD	Min	Max	Diff. in Means	SE
Professional Job Status	Self-employed	457	0.171	0.075	0.046	0.379	0.126	0.057	0.041	0.327	0.045***	(0.004)
	Employee	457	0.795	0.103	0.411	0.936	0.865	0.063	0.640	0.955	-0.071***	(0.006)
	Family Worker	457	0.034	0.039	0.000	0.214	0.008	0.008	0.000	0.038	0.025***	(0.002)
Number of Jobs	1 Job	457	0.951	0.033	0.806	0.996	0.955	0.031	0.725	0.995	-0.005***	(0.002)
	More than 1 Jobs	457	0.048	0.032	0.004	0.194	0.043	0.029	0.005	0.176	0.005***	(0.002)
Gender	Male	457	0.539	0.060	0.371	0.705	0.566	0.041	0.379	0.699	-0.026***	(0.003)
	Female	457	0.461	0.060	0.295	0.629	0.434	0.041	0.301	0.621	0.026***	(0.003)
Age Group	17 to 22	457	0.038	0.026	0.003	0.106	0.025	0.018	0.003	0.073	0.012***	(0.001)
	22 to 27	457	0.095	0.020	0.053	0.171	0.086	0.017	0.047	0.162	0.009***	(0.001)
	27 to 32	457	0.128	0.017	0.084	0.180	0.129	0.019	0.088	0.171	-0.001	(0.001)
	32 to 37	457	0.137	0.017	0.093	0.184	0.142	0.017	0.098	0.182	-0.005***	(0.001)
	37 to 42	457	0.140	0.014	0.099	0.175	0.145	0.014	0.107	0.181	-0.006***	(0.001)
	42 to 47	457	0.139	0.013	0.100	0.182	0.144	0.014	0.113	0.186	-0.005***	(0.001)
	47 to 52	457	0.132	0.015	0.093	0.183	0.136	0.016	0.096	0.178	-0.004***	(0.001)
	52 to 57	457	0.112	0.017	0.069	0.164	0.115	0.018	0.069	0.182	-0.003***	(0.001)
	57 to 62	457	0.079	0.021	0.037	0.127	0.078	0.021	0.027	0.132	0.000	(0.001)

Note: This table offers basic statistics for the share of workers in each category of variables Professional Job Status, Number of Jobs, Gender, Age Group who have specific human capital occupation out of total employment in specific human capital occupations and the share of workers who have general human capital occupation out of total employment in general human capital occupations. The data are for all countries and years. The last two columns of the table use two-sided t-test to test the significance of differences in means. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. See Table 3 for the assignment of occupations into specific and general human capital types.

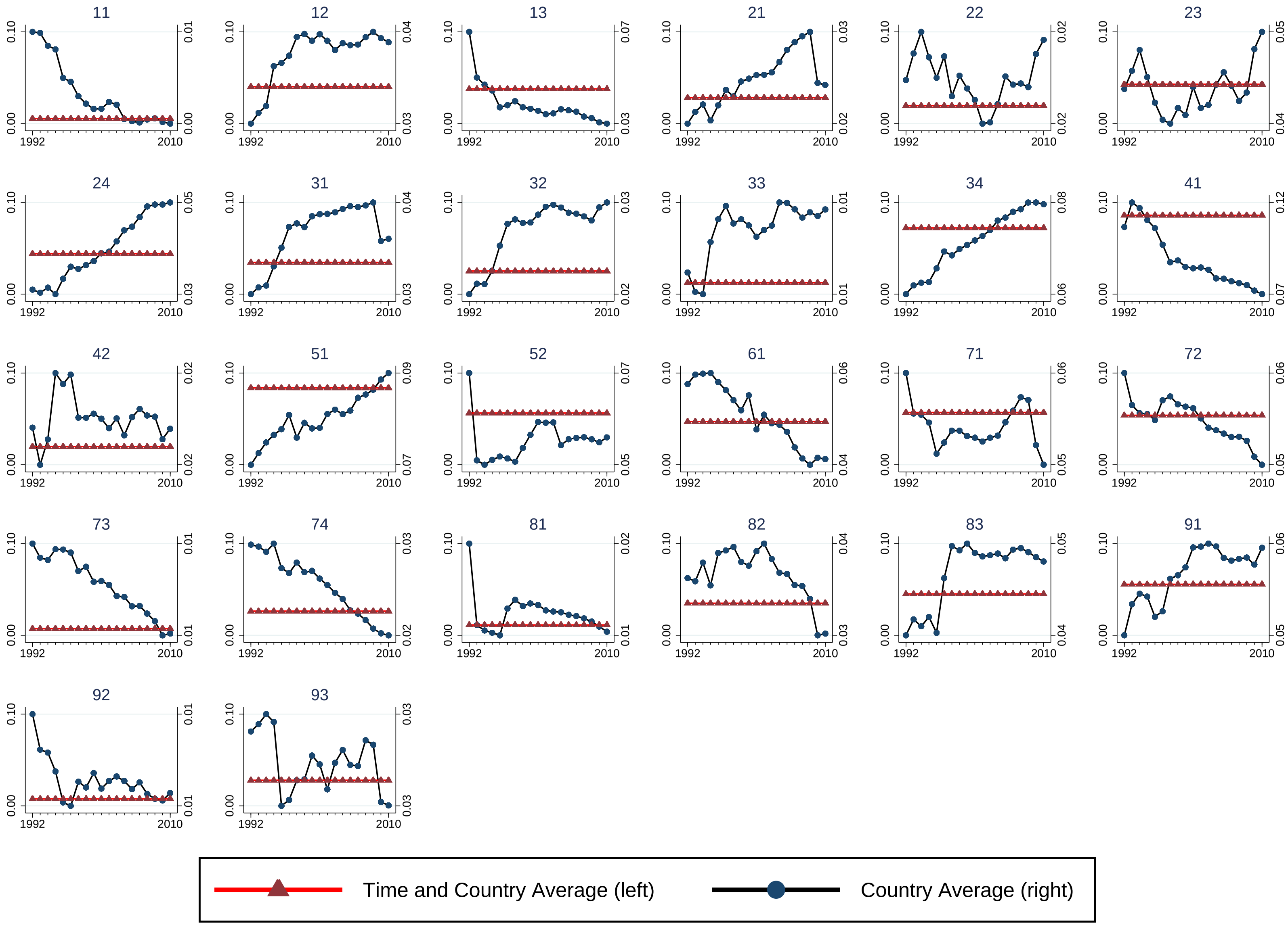
Table 15: *Regression Results for GDP per capita and the Share of Specific Human Capital*

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Share of Specific	-6.234***	-6.490***	-2.652***	-1.177**	-6.655***	-6.576***	-0.509**	-0.581**
Human Capital	(0.532)	(0.562)	(0.458)	(0.475)	(0.490)	(0.533)	(0.209)	(0.244)
Share of Medium- and Highly Skilled Employees	N	Y	N	Y	N	Y	N	Y
Country Fixed Effects	N	N	Y	Y	N	N	Y	Y
Year Fixed Effects	N	N	N	N	Y	Y	Y	Y
Number of Obs.	450	435	450	435	450	435	450	435
R2	0.250	0.387	0.977	0.984	0.278	0.397	0.991	0.991

Note: In regressions reported in this table, the dependent variable is the logarithm of real (PPP-adjusted) GDP per capita, which we obtain from the WDI database. The main explanatory variable is the share of specific human capital. In columns (2), (4), (6), and (8) we include as additional explanatory variables the shares of employed individuals who have secondary to post-secondary non-tertiary education (medium-skilled; ISCED-97 3-4) and tertiary education (highly-skilled; ISCED-97 5-6) out of total number of employed individuals (who report their education level). The variation in the data is at country-year level. Regressions are estimated using the OLS method. Robust standard errors are in parentheses. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. We obtain similar results if we use a third degree polynomial approximation for the share of specific human capital for each country.

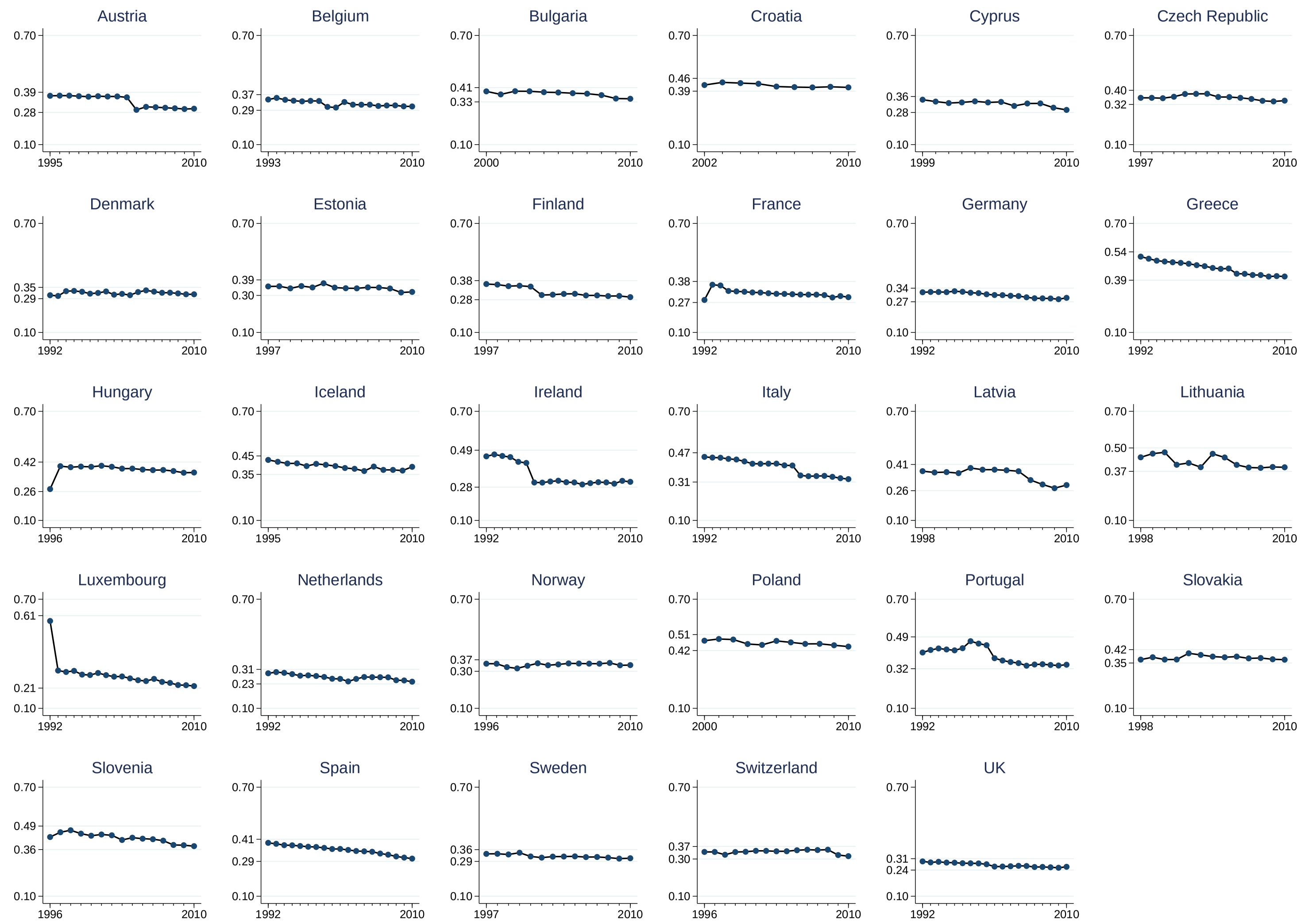
Figures

Figure 1: *The Employment Shares of Occupations*



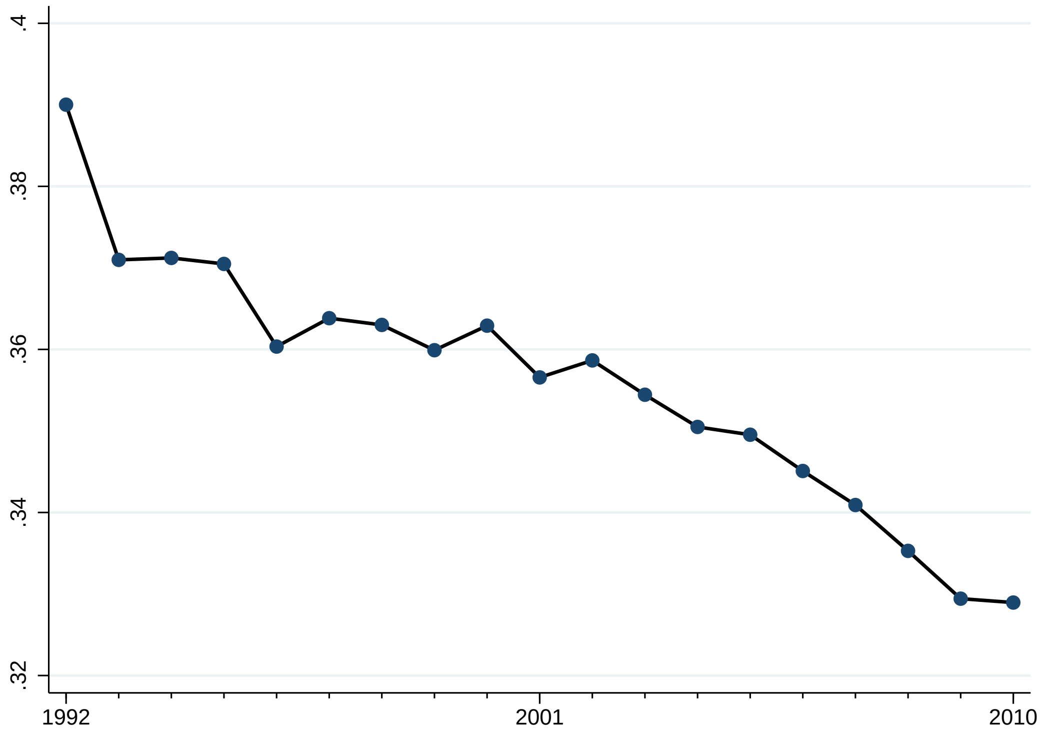
Note: This figure illustrates country- and year-level and country-level averages of the share of workers in 2-digit ISCO-88 occupations out of total employment. See Table 3 for the definitions of occupations.

Figure 2: *The Share of Specific Human Capital in Sample Countries*



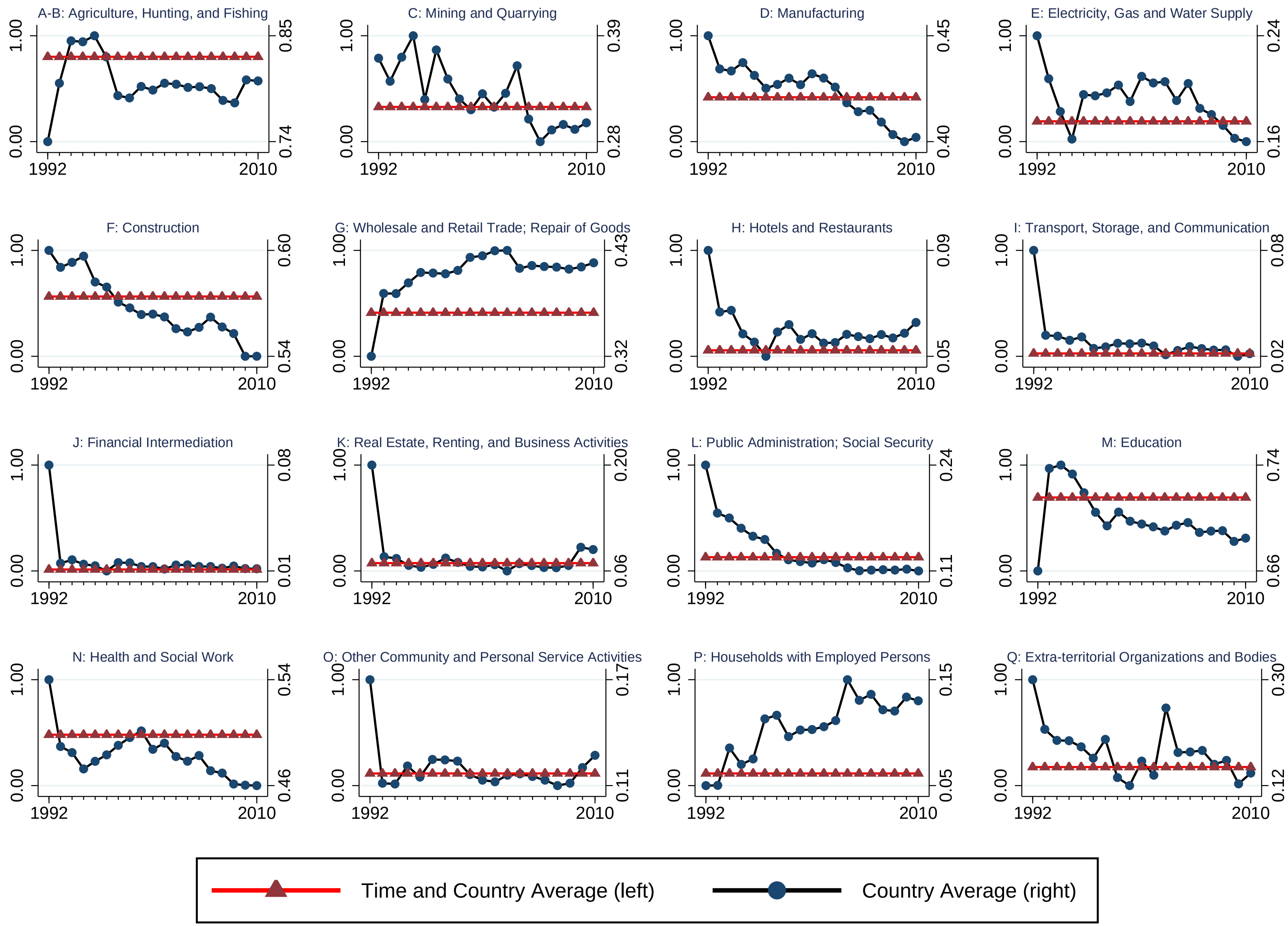
Note: This figure offers the share of workers in specific human capital occupations out of total employment in each country in our sample. See Table 3 for the assignment of occupations into specific and general human capital types. We obtain virtually the same figure when we weight observations using the number of hours worked in the reference week. Jerbashian et al. (2015) use data from Czech Labor Force Survey (2007Q2) and from Jeong et al. (2008) and find that the share of specific human capital has steadily declined in the Czech Republic in the period of 1994–2007, which somewhat contrasts with this figure. Such a difference can stem from imperfections in sampling weights at this level of disaggregation and from sample breaks/data imperfections. For example, in our data there are very large persistent changes in the number of employees in occupations 82 and 93 in between 2000 and 2001 in the Czech Republic. Our results for the Czech Republic are very similar to Jerbashian et al. (2015) when we predict employment in these occupations before 2001 using their values from 2001 onwards and polynomials of time. The persistence of the assignment indicates that these irregularities are not important. In the Czech Republic, the assignment is time invariant. In Appendix - Treatment of Spikes in the Shares of Human Capital we alleviate this problem using polynomials to predict occupation shares in each industry-country pair.

Figure 3: *The Average Employment Share of Specific Human Capital in Sample Countries*



Note: This figure offers country-averaged value of the share of specific human capital. Employment weighted average value displays similar negative trend and percentage change over time. We pull countries c and years t and run a regression of the following form: $\textit{Share of Specific Human Capital}_{c,t} = \alpha + \beta t + \eta_{c,t}$. The coefficient in front of time trend t is highly significant and negative. See Table 3 for the assignment of occupations into specific and general human capital types.

Figure 4: *The Average Employment Share of Specific Human Capital in Industries*



Note: For each industry, this figure offers the country- and year-average and country-average share of workers in specific human capital occupations out of total employment in the industry. See Table 3 for the assignment of occupations into specific and general human capital types.

Figure 5: *The Share of Specific Human Capital in Age Groups*

Note: This figure offers country-averaged value of the share of specific human capital in each age group. See Table 3 for the assignment of occupations into specific and general human capital types.

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Appendix Further Results

Appendix - Treatment of Spikes in the Shares of Human Capital

There are spikes/jumps in the shares of employment in occupations which lead to spikes in the data for the share of specific human capital. These spikes can be noticed in Figures 1, 2, and 4. They can be because of sample imperfections and because sample weights might not be very precise for the level of disaggregation we are interested in.

We alleviate the potential influence of such spikes on our analysis we fit 3rd degree polynomials on employment shares of occupations within each industry-country pair using year level variation. In each country, we drop those occupation-industry pairs for which we have less than 3 (year) observations. We also drop year 1992 and 1996 for Hungary. This exercise gives smoothed (trends in) employment shares in each industry-country pair. These smoothed shares are then used to compute concentration measures and to assign 2-digit ISCO-88 occupations into specific and general human capital types. For each country, year, and occupation, we compute 5 concentration measures (CV, GE, Gini, HI, Theil) for the distribution of smoothed within-occupation share across industries. We average the values of concentration measures across countries and years. The country- and year-averaged values of the concentration measures are offered in columns 2-6 of this table. We define dummy variables for each of the concentration measures which are equal to 1 for the values of the concentration measures that are higher than or equal to their medians. We take the average of these dummy variables and define Specific dummy variable which is equal to 1 if the average is greater than 0.5, and to 0 otherwise.

Table 37 offers the results from the assignment, which is exactly the same as our main assignment in Table 3. Figures 8 and 9 offer country-level and country averaged shares of specific human capital.

Similarly to our main analysis, the concentration measures have very low variation across countries and years. Moreover, if we perform the assignment using smoothed shares in each country and year then that assignment is highly correlated with the one in 37 ($\rho = 0.764$).

We also perform between and within industry decomposition for the smoothed share of specific human capital using (1). Between industry variation explains 76 percent of the variation in the data. The remainder is attributable to the within industry variation. These results are very close to the results offered in the main text. Tables 38 and 39 offer between and within industry decompositions for each sample country and year.

Appendix - Abstract, Manual, and Routine Skills

By definition and identification our classification of human capital types is different than the abstract, manual, and routine skills classification used by Acemoglu and Autor (2011) and Autor and Dorn (2013). Nevertheless, we check if the trends observed for the share of specific human capital repeat for the shares of employment in occupations requiring abstract, manual, and routine skills. We match our 2-digit ISCO-88 occupations with 5 groups of occupations in Table 2 of Autor and Dorn (2013) and assign occupations into abstract, manual, and routine types. Table 42 in Data Appendix presents the assignment.[19] According to it, 11 occupations require abstract skills, 8 require manual skills,

[19] This assignment can be noisy because of differences in occupation coding and matching between ISCO-88 and 5 groups of occupations in Table 2 of Autor and Dorn (2013).

and 7 require routine skills. Out of the first set 5 are specific human capital occupations. Out of each of the second and third sets 4 are specific human capital occupations. This uniformity should not be very surprising. In our classification, skill levels are quite uniform according to Table 13. In contrast, according to Figure 2 of Autor and Dorn (2013), skill levels correlate negatively with the order: abstract, routine, and manual skills.[20]

We compute employment shares in occupations requiring abstract, manual, and routine skills for each country and year. Table 43 in Data Appendix offers the basic statistics for these shares. Table 44 in Data Appendix offers correlations among these shares and the share of specific human capital. Further, we take the averages of these shares across sample countries and illustrate their behavior over time, together with the share of specific human capital, in Figure 10 of Data Appendix. The share of specific human capital is firmly positively correlated with the share of employment in occupations requiring routine skills and negatively correlated with the share of employment in occupations requiring abstract skills. It is also positively correlated with the share of employment in occupations requiring manual skills. We obtain very similar correlations if we exclude occupations requiring either routine or manual skills from occupations corresponding to specific human capital.

In the spirit of Acemoglu and Autor (2011) and Autor and Dorn (2013), Goos et al. (2014) perform an assignment of 2-digit ISCO-88 occupations into high-paying, middling, and low-paying, which map well to abstract, routine, and manual occupations (see Table 1 in Goos et al., 2014). Table 45 in Data Appendix presents the correspondence between this assignment and specific and general human capital occupations. Some of the specific and general human capital occupations are not matched with high-paying, middling, and low-paying occupations because Goos et al. (2014) drop these occupations from the analysis. Out of matched 8 specific human capital occupations, 2 are high-paying, 5 are middling, and 1 is low-paying.

We compute the shares in high-paying, middling, and low-paying occupations out of total employment for each country and year. Table 46 in Data Appendix offers the basic statistics for these shares.[21] Table 47 in Data Appendix offers correlations among these shares and the share of specific human capital. Further, we take the averages of these shares across sample countries and illustrate their behavior over time, together with the share of specific human capital, in Figure 11 of Data Appendix. The share of specific human capital is firmly positively correlated with the share of employment in middling occupations and negatively correlated with the share of employment in high-paying and middling occupations.

Appendix - Elasticity of Substitution

The value of the elasticity of substitution between h- and l-goods ε_1 is important for our results. In this section, we present an attempt to estimate ε_1.

Following current practice (e.g., see Herrendorf et al., 2015), we use equations (2), (3), and (4) to estimate ε_1. In these equations, Y_l is the real output in sectors which are (very) intensive in general human capital input and Y_h is the real output in the remaining sectors. Variable λ is productivity level, and p_h and p_l are the relative prices of Y_h and Y_l. We assume that $\ln \lambda(t)$ is a smooth function of time so that it can be represented by

[20] Autor and Dorn (2013) use wage data to measure skill levels.

[21] Similarly to Goos et al. (2014), we drop 1-digit NACE industries A-B, C, L, M, and Q.

a polynomial of the following form

$$g(t) = \delta_1 t + \delta_2 t^2 + \delta_3 t^3,$$

where $\{\delta_i\}$ are real numbers.

We normalize output Y and price levels dividing them to their geometric averages. Further, we take the logarithms of these equations and take the first differences of price equations so that we get

$$\ln\left(\frac{Y}{\bar{Y}}\right) = \frac{\varepsilon_1}{\varepsilon_1 - 1}\ln\left[\overline{\omega_{Y_h}^{Y}}\left(\frac{Y_h}{\bar{Y}_h}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \overline{\left(1-\omega_{Y_h}^{Y}\right)}\left(\frac{Y_l}{\bar{Y}_l}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right] + \tilde{g}(t), \quad (21)$$

$$\Delta\ln\left(\frac{Y/\bar{Y}}{Y_h/\bar{Y}_h}\right) = \varepsilon_1\Delta\ln\left(\frac{p_h}{\bar{p}_h}\right) + (1-\varepsilon_1)\Delta\tilde{g}(t), \quad (22)$$

$$\Delta\ln\left(\frac{Y/\bar{Y}}{Y_l/\bar{Y}_l}\right) = \varepsilon_1\Delta\ln\left(\frac{p_l}{\bar{p}_l}\right) + (1-\varepsilon_1)\Delta\tilde{g}(t), \quad (23)$$

where we use bars to denote geometric averages and

$$\tilde{g}(t) = g(t) - \frac{1}{T}\sum_{t=1}^{T}\tilde{g}(t),$$

$$\overline{\omega_{Y_h}^{Y}} = \gamma_1\left(\frac{\exp(\tilde{g}(t))\bar{Y}_h}{\bar{Y}}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}},$$

$$\overline{\left(1-\omega_{Y_h}^{Y}\right)} = (1-\gamma_1)\left(\frac{\exp(\tilde{g}(t))\bar{Y}_l}{\bar{Y}}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}.$$

We use data from the EU KLEMS database and estimate this system of equations jointly for each sample country. The identification of parameters is based on within country variation. In this database, there are no data for Iceland, Norway, and Switzerland. There are data for Australia, Korea, Japan, and the US, however. We carry our estimations also for these countries.

We use the total industrial real output for Y and real output in h- and l-sectors for Y_h and Y_l. In line with Table 6, h-sectors are NACE industries A-B, C, D, E, F, G, L, M, and N, and l-sectors are industries H, I, J-K, O, and P.[22] The data for prices and nominal output in the EU KLEMS database are at 1-digit level of aggregation. We use (current) nominal value-added weights to aggregate prices and obtain prices in h- and l-sectors:

$$p_h = \sum_{i\in h}\left(\frac{p_i Y_i}{\sum_{i\in h} p_i Y_i}\right)p_i,$$

$$p_l = \sum_{i\in l}\left(\frac{p_i Y_i}{\sum_{i\in l} p_i Y_i}\right)p_i.$$

The base year for prices is 1995 in the EU KLEMS database. For each country, we transform price series so that the base year is the first year in the sample. [23]

[22] As an auxiliary exercise, we obtain from the EU KLEMS database average hourly wage rates in h- and l-sectors for each country and year. We find that the wage rates tend to be very similar across these industries in sample countries. This serves as a further confirmation that our classification of human capital types is rather horizontal.

[23] Clearly, $Y = Y_h + Y_l$ in the first/base year. Therefore, the first sample year is dropped in estimations.

For each country, the values of $\overline{\omega^{Y}_{Y_h}}$ and $\overline{\left(1-\omega^{Y}_{Y_h}\right)}$ are obtained using the geometric averages of compensation shares of h- and l-sectors:

$$\overline{\omega^{Y}_{Y_h}} = \overline{\left(\frac{p_h Y_h}{Y}\right)},$$
$$\overline{\left(1-\omega^{Y}_{Y_h}\right)} = \overline{\left(\frac{p_l Y_l}{Y}\right)}.$$

After obtaining the estimate of ε_1 and $g(t)$ we use equation

$$\gamma_1 = \overline{\left(\frac{p_h Y_h}{Y}\right)} \left(\frac{\exp\left(\frac{1}{T}\sum_{t=1}^{T} g(t)\right) \bar{Y}_h}{\bar{Y}} \right)^{-\frac{\varepsilon_1 - 1}{\varepsilon_1}} \tag{24}$$

to obtain the estimate of γ_1.

The estimations of equations (21)-(23) are carried using non-linear seemingly unrelated regressions routine in STATA. Tables 48 and 49 summarize our estimation results. The results depend on the initial value of the elasticity that we specify. Usually, the non-linear estimator converges to a point less than one for ε_1 when we specify initial value that is less than 1. It converges to a point greater than 1 otherwise. The less than 1 values of ε_1 are preferable in terms of the Root Square Mean Error.

Appendix - Employment Shares in Industries and Specific Human Capital

In the main text we offer between and within industry decomposition of the trends in the share of specific human capital. As a complementary exercise, we check the explanatory power of employment in industries which use specific human capital the most.

For each country and year, we compute employment shares in industries where the share of specific human capital is higher than the 50th, 70th, and 90th percentiles of its industry-level distribution in the country and year. Table 40 offers correlations of the employment shares in these industries with the share of specific human capital. Table 41 offers the results from regressions where the dependent variable is the share of specific human capital and explanatory variables are the employment shares in these industries. The employment shares in these industries appear to be important explanatory variables and account for approximately 70 percent of the variation in the share of specific human capital. A similar inference holds if we smooth the series of the share of specific human capital using third degree polynomial approximation.

Data Appendix

Data Appendix - Tables

Table 16: *ANOVA for the Coefficient of Variation (CV)*

Source	Partial SS	df	MS	F	P-stat
Model	9183.221	71	129.341	1011.860	0.000
Occupation	9036.305	25	361.452	2827.700	0.000
Country	134.210	28	4.793	37.500	0.000
Year	9.765	18	0.543	4.240	0.000
Residual	1499.263	11729	0.128		
Total	10682.484	11800	0.905		

Note: This table reports the results from an ANOVA exercise for the Coefficient of Variation (CV). The variation in the data are at occupation-country-year level, and we perform the ANOVA exercise along each of these dimensions. Number of Obs. = 11801; Adj. R-squared = 0.859.

Table 17: *ANOVA for Generalized Entropy Index (GE)*

Source	Partial SS	df	MS	F	P-stat
Model	46033.661	71	648.361	791.990	0.000
Occupation	45108.672	25	1804.347	2204.060	0.000
Country	858.189	28	30.650	37.440	0.000
Year	61.641	18	3.424	4.180	0.000
Residual	9601.894	11729	0.819		
Total	55635.555	11800	4.715		

Note: This table reports the results from an ANOVA exercise for the Generalized Entropy Index (with parameter 2; GE). The variation in the data are at occupation-country-year level, and we perform the ANOVA exercise along each of these dimensions. Number of Obs. = 11801; Adj. R-squared = 0.826.

Table 18: *ANOVA for the Gini Index*

Source	Partial SS	df	MS	F	P-stat
Model	177.780	71	2.504	1146.640	0.000
Occupation	174.207	25	6.968	3191.010	0.000
Country	3.246	28	0.116	53.080	0.000
Year	0.162	18	0.009	4.110	0.000
Residual	25.613	11729	0.002		
Total	203.393	11800	0.017		

Note: This table reports the results from an ANOVA exercise for the Gini Index. The variation in the data are at occupation-country-year level, and we perform the ANOVA exercise along each of these dimensions. Number of Obs. = 11801; Adj. R-squared = 0.873.

Table 19: *ANOVA for the Herfindahl Index (HI)*

Source	Partial SS	df	MS	F	P-stat
Model	734.778	71	10.349	789.010	0.000
Occupation	720.155	25	28.806	2196.200	0.000
Country	13.391	28	0.478	36.460	0.000
Year	1.274	18	0.071	5.400	0.000
Residual	153.842	11729	0.013		
Total	888.620	11800	0.075		

Note: This table reports the results from an ANOVA exercise for the Herfindahl Index (HI). The variation in the data are at occupation-country-year level, and we perform the ANOVA exercise along each of these dimensions. Number of Obs. = 11801; Adj. R-squared = 0.826.

Table 20: *ANOVA for the Theil Index*

Source	Partial SS	df	MS	F	P-stat
Model	4687.007	71	66.014	986.790	0.000
Occupation	4595.087	25	183.803	2747.520	0.000
Country	84.047	28	3.002	44.870	0.000
Year	6.391	18	0.355	5.310	0.000
Residual	784.647	11729	0.067		
Total	5471.654	11800	0.464		

Note: This table reports results from an ANOVA exercise for the Theil Index. The variation in the data are at occupation-country-year level, and we perform the ANOVA exercise along each of these dimensions. Number of Obs. = 11801; Adj. R-squared = 0.856.

Table 21: *Rank Correlations Among Concentration Measures*

Variable	1	2	3	4
1 Coefficient of Variation (CV)				
2 Generalized Entropy Index (GE)	1.000			
3 Gini Index	0.987	0.987		
4 Herfindahl Index (HI)	0.999	0.999	0.987	
5 Theil Index	0.994	0.994	0.998	0.994

Note: This table offers pairwise rank correlations among the concentration measures computed for the within-occupation employment shares distribution. The measures are coefficient of variation (CV), Generalized Entropy (with parameter 2; GE) index, Gini index, Theil index, and Herfindahl index (HI). The variation in the data are at occupation-country-year level. Number of Obs. = 11802; All correlations are significant at 1% level.

Table 22: *The Employment Shares of Occupations in Years*

Occupation (ISCO-88)	Year 1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
11	0.010	0.010	0.009	0.009	0.007	0.007	0.006	0.005	0.005	0.005	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004
12	0.028	0.030	0.031	0.037	0.038	0.039	0.042	0.043	0.041	0.042	0.041	0.040	0.041	0.041	0.041	0.042	0.043	0.042	0.041
13	0.069	0.050	0.047	0.045	0.038	0.039	0.041	0.038	0.037	0.037	0.035	0.036	0.037	0.037	0.036	0.034	0.034	0.032	0.031
21	0.023	0.024	0.025	0.023	0.025	0.027	0.026	0.027	0.028	0.028	0.028	0.028	0.030	0.031	0.032	0.032	0.033	0.027	0.027
22	0.020	0.021	0.022	0.021	0.020	0.021	0.019	0.020	0.019	0.019	0.018	0.018	0.019	0.020	0.019	0.019	0.019	0.021	0.021
23	0.043	0.044	0.045	0.043	0.042	0.041	0.041	0.042	0.042	0.043	0.042	0.042	0.043	0.044	0.043	0.042	0.043	0.045	0.046
24	0.033	0.033	0.034	0.032	0.036	0.038	0.038	0.039	0.040	0.042	0.042	0.044	0.047	0.048	0.050	0.052	0.053	0.053	0.053
31	0.027	0.027	0.028	0.030	0.032	0.034	0.035	0.034	0.036	0.036	0.036	0.036	0.036	0.037	0.037	0.037	0.037	0.033	0.033
32	0.019	0.020	0.020	0.021	0.023	0.025	0.025	0.025	0.025	0.026	0.027	0.027	0.027	0.026	0.026	0.026	0.025	0.027	0.027
33	0.010	0.009	0.009	0.012	0.013	0.013	0.012	0.013	0.012	0.012	0.012	0.012	0.013	0.013	0.013	0.013	0.013	0.013	0.013
34	0.055	0.058	0.059	0.059	0.063	0.067	0.066	0.068	0.069	0.070	0.072	0.073	0.076	0.077	0.078	0.079	0.081	0.081	0.081
41	0.106	0.119	0.116	0.110	0.105	0.097	0.088	0.089	0.086	0.085	0.085	0.084	0.080	0.080	0.078	0.077	0.076	0.074	0.072
42	0.019	0.018	0.019	0.021	0.021	0.021	0.020	0.020	0.020	0.020	0.019	0.020	0.019	0.020	0.020	0.020	0.020	0.019	0.019
51	0.073	0.076	0.078	0.079	0.080	0.083	0.079	0.082	0.081	0.081	0.083	0.084	0.083	0.084	0.087	0.087	0.088	0.090	0.092
52	0.066	0.053	0.053	0.053	0.054	0.054	0.053	0.055	0.057	0.059	0.059	0.059	0.056	0.056	0.057	0.057	0.056	0.056	0.057
61	0.056	0.059	0.059	0.059	0.057	0.055	0.053	0.050	0.054	0.046	0.049	0.047	0.047	0.045	0.041	0.039	0.037	0.039	0.038
71	0.065	0.059	0.059	0.058	0.053	0.055	0.057	0.057	0.056	0.056	0.055	0.056	0.056	0.058	0.060	0.061	0.061	0.055	0.052
72	0.064	0.058	0.056	0.056	0.055	0.059	0.060	0.058	0.057	0.057	0.055	0.053	0.053	0.052	0.051	0.051	0.050	0.047	0.046
73	0.010	0.009	0.009	0.009	0.009	0.009	0.008	0.008	0.008	0.008	0.007	0.007	0.007	0.006	0.006	0.006	0.006	0.005	0.005
74	0.034	0.033	0.033	0.034	0.030	0.029	0.031	0.029	0.030	0.028	0.027	0.026	0.025	0.024	0.023	0.022	0.021	0.020	0.020
81	0.018	0.010	0.010	0.009	0.009	0.012	0.013	0.012	0.012	0.012	0.012	0.012	0.011	0.011	0.011	0.011	0.011	0.010	0.010
82	0.035	0.034	0.036	0.034	0.037	0.037	0.037	0.036	0.036	0.037	0.038	0.036	0.035	0.035	0.034	0.034	0.033	0.030	0.030
83	0.038	0.040	0.039	0.040	0.039	0.044	0.047	0.046	0.047	0.046	0.046	0.046	0.046	0.046	0.046	0.047	0.046	0.046	0.045
91	0.049	0.052	0.053	0.053	0.051	0.052	0.055	0.055	0.056	0.057	0.057	0.058	0.058	0.056	0.056	0.056	0.056	0.056	0.057
92	0.012	0.010	0.010	0.008	0.006	0.006	0.008	0.007	0.008	0.007	0.008	0.008	0.008	0.007	0.008	0.007	0.007	0.006	0.007
93	0.030	0.031	0.031	0.031	0.025	0.026	0.027	0.027	0.029	0.028	0.027	0.028	0.029	0.028	0.028	0.030	0.029	0.026	0.025

Note: This table reports the share of workers in each occupation (2-digit ISCO-88) out of total employment, averaged across countries. See Table 3 for the names of occupations.

Table 23: *Correlations for the Share of Specific Human Capital*

Country	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1 Austria																												
2 Belgium	0.45																											
3 Bulgaria	0.37	0.42																										
4 Croatia	0.63	0.65	0.77																									
5 Cyprus	0.60	0.41	0.89	0.71																								
6 Czech Repulic	0.58	0.01	0.67	0.88	0.54																							
7 Denmark	0.04	0.35	0.50	0.58	0.43	0.10																						
8 Estonia	0.68	0.33	0.72	0.42	0.66	0.58	-0.13																					
9 Finland	0.81	0.29	0.24	0.86	0.63	0.23	0.05	0.49																				
10 France	0.75	0.69	0.70	0.80	0.66	0.57	0.18	0.54	0.67																			
11 Germany	0.88	0.72	0.43	0.80	0.67	0.38	0.10	0.56	0.91	0.50																		
12 Greece	0.94	0.72	0.50	0.86	0.72	0.50	-0.05	0.64	0.90	0.49	0.95																	
13 Hungary	0.01	-0.30	0.66	0.75	0.82	0.71	-0.26	0.77	0.75	-0.17	-0.19	-0.13																
14 Iceland	0.86	0.52	0.21	0.40	0.31	0.43	0.15	0.58	0.77	0.74	0.90	0.91	-0.18															
15 Ireland	0.52	0.75	0.21	0.21	0.29	0.00	0.01	0.24	0.51	0.52	0.76	0.83	-0.57	0.69														
16 Italy	0.98	0.64	0.42	0.73	0.65	0.57	-0.03	0.66	0.85	0.50	0.94	0.97	-0.07	0.90	0.75													
17 Latvia	0.67	0.35	0.84	0.83	0.75	0.81	0.33	0.74	0.56	0.83	0.72	0.75	0.83	0.61	0.36	0.69												
18 Lithuania	0.67	0.38	0.52	0.69	0.74	0.29	0.21	0.54	0.89	0.65	0.90	0.83	0.71	0.68	0.11	0.72	0.70											
19 Luxembourg	0.86	0.73	0.66	0.86	0.83	0.52	-0.22	0.69	0.89	-0.14	0.56	0.71	-0.02	0.83	0.60	0.59	0.83	0.87										
20 Netherlands	0.45	0.75	0.58	0.26	0.59	-0.01	0.16	0.32	0.64	0.47	0.76	0.81	-0.22	0.51	0.82	0.69	0.50	0.59	0.65									
21 Norway	0.02	0.02	0.40	0.16	0.02	0.19	0.26	0.16	-0.03	0.13	0.13	0.10	-0.34	0.09	0.59	0.11	0.03	-0.37	0.10	0.30								
22 Poland	0.79	0.12	0.43	0.44	0.58	0.74	-0.22	0.76	0.73	0.43	0.64	0.76	0.83	0.49	0.70	0.76	0.67	0.60	0.86	0.08	0.40							
23 Portugal	0.82	0.56	0.22	0.60	0.60	0.11	0.05	0.50	0.92	0.45	0.87	0.80	0.01	0.80	0.50	0.83	0.48	0.90	0.38	0.63	-0.19	0.70						
24 Slovakia	0.10	0.42	0.61	0.75	0.34	0.55	0.19	0.33	-0.38	0.17	-0.03	0.03	0.23	-0.02	0.00	0.09	0.41	-0.14	0.02	-0.23	0.08	0.40	-0.31					
25 Slovenia	0.83	0.49	0.63	0.73	0.79	0.51	0.19	0.72	0.89	0.79	0.84	0.88	0.20	0.73	0.37	0.84	0.84	0.88	0.96	0.66	0.04	0.85	0.82	0.03				
26 Spain	0.86	0.71	0.73	0.85	0.86	0.56	0.05	0.72	0.88	0.49	0.93	0.97	-0.04	0.81	0.78	0.93	0.89	0.88	0.68	0.85	0.20	0.83	0.78	0.10	0.95			
27 Sweden	0.72	0.29	0.54	0.74	0.74	0.22	0.16	0.59	0.90	0.75	0.84	0.84	0.68	0.70	0.41	0.75	0.65	0.90	0.84	0.71	-0.07	0.51	0.89	-0.28	0.82	0.85		
28 Switzerland	0.09	-0.02	0.76	0.30	0.69	0.42	0.26	0.46	-0.02	0.18	-0.02	0.06	0.06	-0.09	0.13	0.11	0.40	0.00	0.17	0.23	0.53	0.60	-0.16	0.42	0.18	0.27	0.06	
29 UK	0.74	0.78	0.42	0.76	0.68	0.08	0.10	0.44	0.89	0.47	0.94	0.93	-0.27	0.81	0.82	0.86	0.55	0.95	0.66	0.86	0.01	0.32	0.84	-0.25	0.76	0.91	0.92	-0.09

Note: This table offers pairwise correlations among the shares of specific human capital in sample countries. The variation in the data is at country-year level. See Table 3 for the assignment of occupations into specific and general human capital types.

Table 24: *ANOVA for the Share of Specific Human Capital*

Source	Partial SS	df	MS	F	P-stat
Model	1.225	46	0.027	39.630	0.000
Country	1.117	28	0.040	59.420	0.000
Year	0.208	18	0.012	17.210	0.000
Residual	0.275	409	0.001		
Total	1.499	455	0.003		

Note: This table reports the results from an ANOVA exercise for the share of specific human capital. The variation in the data are at country-year level, and we perform the ANOVA exercise along each of these dimensions. Number of Obs. = 456; Adj. R-squared = 0.796. See Table 3 for the assignment of occupations into specific and general human capital types.

Table 25: *Rank Correlations for the Share of Specific Human Capital in Industries*

Country	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1 Austria																												
2 Belgium	0.97																											
3 Bulgaria	0.93	0.93																										
4 Croatia	0.94	0.94	0.97																									
5 Cyprus	0.94	0.94	0.92	0.92																								
6 Czech Republic	0.95	0.95	0.95	0.95	0.92																							
7 Denmark	0.97	0.97	0.93	0.95	0.97	0.94																						
8 Estonia	0.94	0.93	0.93	0.91	0.93	0.94	0.94																					
9 Finland	0.94	0.93	0.91	0.92	0.90	0.91	0.94	0.92																				
10 France	0.94	0.94	0.93	0.90	0.91	0.93	0.93	0.93	0.95																			
11 Germany	0.97	0.96	0.93	0.94	0.94	0.95	0.97	0.94	0.95	0.94																		
12 Greece	0.96	0.97	0.95	0.94	0.95	0.96	0.97	0.96	0.94	0.96	0.97																	
13 Hungary	0.97	0.97	0.95	0.96	0.96	0.96	0.98	0.96	0.95	0.94	0.97	0.98																
14 Iceland	0.89	0.88	0.84	0.84	0.89	0.84	0.91	0.86	0.90	0.87	0.91	0.88	0.89															
15 Ireland	0.86	0.86	0.83	0.84	0.83	0.89	0.86	0.84	0.89	0.87	0.87	0.86	0.87	0.81														
16 Italy	0.95	0.94	0.93	0.93	0.92	0.94	0.94	0.93	0.95	0.95	0.96	0.97	0.95	0.89	0.87													
17 Latvia	0.93	0.92	0.95	0.93	0.91	0.94	0.93	0.93	0.89	0.91	0.90	0.93	0.94	0.79	0.82	0.91												
18 Lithuania	0.94	0.94	0.97	0.94	0.91	0.95	0.93	0.95	0.91	0.92	0.93	0.96	0.95	0.83	0.83	0.94	0.94											
19 Luxembourg	0.95	0.96	0.91	0.91	0.92	0.94	0.94	0.91	0.92	0.95	0.96	0.96	0.94	0.89	0.86	0.93	0.88	0.91										
20 Netherlands	0.92	0.92	0.87	0.89	0.91	0.93	0.93	0.89	0.90	0.90	0.90	0.91	0.93	0.82	0.92	0.89	0.90	0.88	0.89									
21 Norway	0.92	0.92	0.92	0.93	0.90	0.89	0.94	0.90	0.97	0.93	0.94	0.93	0.94	0.90	0.87	0.94	0.88	0.90	0.91	0.89								
22 Poland	0.93	0.91	0.93	0.93	0.91	0.92	0.93	0.92	0.95	0.92	0.96	0.94	0.95	0.89	0.87	0.96	0.89	0.92	0.90	0.87	0.94							
23 Portugal	0.93	0.94	0.89	0.88	0.88	0.90	0.92	0.92	0.94	0.95	0.94	0.95	0.93	0.88	0.85	0.92	0.86	0.90	0.94	0.87	0.93	0.88						
24 Slovakia	0.93	0.92	0.92	0.91	0.90	0.97	0.92	0.93	0.91	0.92	0.93	0.93	0.94	0.83	0.92	0.92	0.91	0.92	0.91	0.93	0.89	0.91	0.88					
25 Slovenia	0.94	0.94	0.97	0.94	0.93	0.95	0.94	0.92	0.92	0.93	0.95	0.96	0.95	0.88	0.84	0.95	0.92	0.95	0.93	0.87	0.92	0.94	0.89	0.91				
26 Spain	0.96	0.96	0.94	0.92	0.92	0.95	0.95	0.95	0.95	0.96	0.96	0.98	0.96	0.89	0.86	0.96	0.91	0.95	0.96	0.89	0.93	0.91	0.97	0.92	0.94			
27 Sweden	0.94	0.93	0.91	0.89	0.90	0.90	0.94	0.93	0.97	0.96	0.95	0.95	0.94	0.92	0.87	0.95	0.87	0.90	0.93	0.88	0.96	0.93	0.96	0.89	0.92	0.96		
28 Switzerland	0.95	0.93	0.91	0.92	0.94	0.92	0.96	0.92	0.94	0.95	0.93	0.94	0.95	0.87	0.86	0.93	0.93	0.91	0.91	0.92	0.92	0.93	0.89	0.91	0.92	0.92	0.93	
29 UK	0.96	0.95	0.91	0.92	0.92	0.91	0.96	0.94	0.97	0.95	0.97	0.96	0.96	0.91	0.88	0.95	0.90	0.91	0.94	0.91	0.96	0.94	0.96	0.91	0.92	0.96	0.97	0.94

Note: This table offers the rank correlations among the shares of specific human capital in industries (1-digit NACE) in sample countries and years. In a country, the variation in the data is at industry-year level. All correlations are significant at 1% level. See Table 3 for the assignment of occupations into specific and general human capital types.

Table 26: *Industry-Level Decomposition of the Trends in the Share of Specific Human Capital for Each Year*

		Between Industries				Within Industries				Total			
	Obs.	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max
Year													
1993	11	-0.007	0.010	-0.019	0.009	-0.012	0.089	-0.262	0.1	-0.019	0.091	-0.281	0.083
1994	12	-0.003	0.003	-0.008	0.002	0.003	0.010	-0.006	0.03	0.000	0.012	-0.013	0.033
1995	12	-0.003	0.004	-0.008	0.007	-0.003	0.009	-0.030	0.01	-0.006	0.011	-0.037	0.008
1996	14	-0.001	0.005	-0.013	0.006	-0.004	0.007	-0.024	0.01	-0.006	0.009	-0.028	0.006
1997	18	0.004	0.032	-0.012	0.132	0.000	0.007	-0.007	0.02	0.005	0.031	-0.011	0.125
1998	22	-0.002	0.007	-0.019	0.020	-0.005	0.027	-0.119	0.02	-0.007	0.031	-0.138	0.035
1999	25	-0.003	0.005	-0.015	0.010	0.000	0.006	-0.017	0.01	-0.003	0.008	-0.017	0.018
2000	26	-0.004	0.004	-0.013	0.008	0.000	0.012	-0.030	0.03	-0.004	0.011	-0.038	0.016
2001	28	-0.005	0.015	-0.059	0.006	0.000	0.009	-0.016	0.02	-0.005	0.016	-0.055	0.014
2002	28	-0.001	0.005	-0.012	0.010	0.000	0.013	-0.037	0.03	-0.001	0.015	-0.041	0.033
2003	28	-0.001	0.008	-0.018	0.015	-0.003	0.008	-0.021	0.02	-0.004	0.011	-0.027	0.018
2004	29	-0.002	0.009	-0.017	0.035	-0.005	0.016	-0.055	0.01	-0.006	0.021	-0.070	0.043
2005	29	-0.002	0.006	-0.018	0.010	0.000	0.005	-0.011	0.01	-0.002	0.008	-0.022	0.019
2006	29	-0.001	0.006	-0.018	0.021	-0.002	0.006	-0.016	0.01	-0.003	0.009	-0.021	0.028
2007	29	-0.002	0.003	-0.011	0.004	-0.002	0.009	-0.040	0.01	-0.004	0.010	-0.044	0.014
2008	29	-0.003	0.004	-0.014	0.008	-0.003	0.006	-0.024	0.01	-0.006	0.008	-0.027	0.007
2009	29	-0.008	0.007	-0.028	0.001	0.001	0.009	-0.019	0.03	-0.007	0.009	-0.027	0.010
2010	29	-0.002	0.004	-0.015	0.007	0.001	0.01	-0.013	0.02	0.000	0.008	-0.013	0.024

Note: This table offers the basic statistics for between- and within-industry decomposition of changes in the share of specific human capital (1) for each year. Columns 1-4 and 5-8 offer the basic statistics for between- and within-industry components. Columns 9-12 offer the basic statistics for total change in the share of specific human capital. Figures 4 and 2 show that there are spikes in the share of specific human capital. These spikes can stem from sample imperfections and imperfections in sampling weights at this level of disaggregation. For example, in our data there are large persistent changes in the number of employees in occupations 82 and 93 in between 2000 and 2001 in the Czech Republic. Such imperfections can bias these decompositions in ambiguous directions. The effects of such biases are likely to be alleviated when we take country and/or year averages. In Appendix - Treatment of Spikes in the Shares of Human Capital we attempt to mitigate these imperfections.

Table 27: *The Share of Workers in Education Fields and Countries*

	Education Field (ISCED-97)									
	0	1	2	3	4	5	6	7	8	9
Country										
Austria	0.062	0.043	0.034	0.249	0.017	0.366	0.055	0.063	0.108	0.003
Belgium	0.072	0.069	0.057	0.227	0.071	0.289	0.019	0.120	0.061	0.015
Bulgaria	0.187	0.051	0.022	0.129	0.014	0.445	0.036	0.047	0.040	0.053
Croatia	0.150	0.037	0.018	0.212	0.026	0.339	0.029	0.051	0.137	
Cyprus	0.063	0.054	0.117	0.371	0.063	0.212	0.006	0.043	0.079	0.007
Czech Republic	0.040	0.037	0.024	0.177	0.021	0.510	0.052	0.052	0.087	0.000
Denmark	0.021	0.047	0.061	0.304	0.044	0.274	0.040	0.155	0.054	0.000
Estonia	0.251	0.048	0.032	0.150	0.025	0.311	0.042	0.044	0.097	0.001
Finland	0.104	0.028	0.043	0.187	0.027	0.317	0.050	0.133	0.111	0.000
France	0.008	0.010	0.075	0.317	0.071	0.315	0.051	0.094	0.050	0.009
Germany	0.037	0.057	0.038	0.272	0.029	0.364	0.031	0.097	0.076	0.000
Greece	0.400	0.023	0.053	0.154	0.045	0.183	0.016	0.070	0.054	0.002
Hungary	0.101	0.093	0.017	0.198	0.022	0.427	0.037	0.059	0.072	0.000
Iceland	0.154	0.088	0.059	0.164	0.037	0.290	0.025	0.087	0.096	0.004
Ireland	0.481	0.037	0.041	0.182	0.070	0.124	0.024	0.074	0.049	0.003
Italy	0.047	0.047	0.072	0.303	0.070	0.250	0.025	0.052	0.047	0.087
Latvia	0.304	0.052	0.040	0.137	0.024	0.291	0.045	0.049	0.059	0.001
Lithuania	0.186	0.056	0.030	0.153	0.033	0.331	0.053	0.060	0.097	0.002
Luxembourg	0.086	0.061	0.079	0.304	0.071	0.178	0.017	0.069	0.114	0.023
Netherlands	0.071	0.077	0.045	0.288	0.037	0.210	0.039	0.138	0.084	0.011
Norway	0.148	0.105	0.117	0.209	0.061	0.133	0.033	0.102	0.038	0.054
Poland	0.084	0.041	0.027	0.182	0.048	0.389	0.079	0.048	0.101	0.003
Portugal	0.062	0.080	0.188	0.263	0.160	0.124	0.014	0.069	0.037	0.129
Slovakia	0.045	0.042	0.015	0.163	0.020	0.516	0.062	0.048	0.088	0.001
Slovenia	0.050	0.050	0.023	0.265	0.016	0.397	0.036	0.056	0.107	0.001
Spain	0.255	0.056	0.050	0.236	0.062	0.188	0.016	0.089	0.036	0.012
Sweden	0.103	0.085	0.044	0.197	0.028	0.280	0.026	0.145	0.069	0.023
Switzerland	0.105	0.049	0.040	0.251	0.033	0.309	0.053	0.086	0.075	
UK	0.004	0.038	0.060	0.148	0.062	0.133	0.010	0.076	0.037	0.432

Note: This table offers for each education field (1-digit ISCED-97) the yearly average share of workers who have their highest degree in that education field out of total employment. There are no observations for Croatia and Switzerland for education field 9. The data for education fields contains a few missing observations because some respondents do not report their highest level of education. The data for education fields are available for the period of 2003–2010. See Table 11 for the names of education fields.

Table 28: *The Share of Workers in Education Fields and Years*

Year	Education Field (ISCED-97) 0	1	2	3	4	5	6	7	8	9
2003	0.127	0.050	0.049	0.220	0.039	0.309	0.038	0.074	0.074	0.098
2004	0.125	0.055	0.054	0.219	0.047	0.281	0.035	0.079	0.070	0.052
2005	0.110	0.053	0.053	0.217	0.047	0.296	0.035	0.077	0.080	0.042
2006	0.139	0.052	0.055	0.215	0.046	0.287	0.035	0.075	0.073	0.033
2007	0.139	0.052	0.054	0.225	0.047	0.304	0.036	0.076	0.075	0.029
2008	0.125	0.051	0.053	0.222	0.046	0.296	0.035	0.076	0.075	0.028
2009	0.123	0.058	0.052	0.228	0.042	0.291	0.035	0.084	0.074	0.026
2010	0.126	0.059	0.051	0.228	0.041	0.287	0.034	0.087	0.074	0.027

Note: This table offers for each education field (1-digit ISCED-97) country-averaged share of workers who have their highest degree in that education field out of total employment. The data for education fields contains a few missing observations because some respondents do not report their highest level of education. See Table 11 for the names of education fields.

Table 29: *Within-Education Field Share Across Occupations*

Occupations (ISCO-88)	Education Field (ISCED-97) 0	1	2	3	4	5	6	7	8	9
11	0.005	0.006	0.007	0.009	0.007	0.004	0.006	0.002	0.010	0.004
12	0.041	0.047	0.054	0.094	0.078	0.054	0.045	0.026	0.038	0.050
13	0.047	0.020	0.036	0.044	0.032	0.041	0.066	0.015	0.045	0.050
21	0.017	0.010	0.017	0.016	0.228	0.103	0.013	0.004	0.009	0.030
22	0.003	0.004	0.002	0.002	0.043	0.002	0.081	0.294	0.002	0.009
23	0.014	0.544	0.180	0.019	0.114	0.012	0.016	0.018	0.017	0.038
24	0.035	0.045	0.189	0.182	0.062	0.017	0.022	0.038	0.020	0.058
31	0.036	0.008	0.037	0.016	0.095	0.091	0.025	0.015	0.038	0.041
32	0.017	0.008	0.006	0.006	0.019	0.004	0.037	0.304	0.009	0.018
33	0.016	0.123	0.016	0.004	0.006	0.003	0.003	0.019	0.008	0.015
34	0.131	0.047	0.124	0.197	0.078	0.045	0.052	0.046	0.081	0.091
41	0.141	0.030	0.074	0.160	0.065	0.038	0.040	0.026	0.070	0.085
42	0.036	0.008	0.018	0.034	0.014	0.006	0.008	0.007	0.032	0.019
51	0.112	0.037	0.046	0.046	0.035	0.038	0.043	0.123	0.296	0.099
52	0.077	0.018	0.043	0.082	0.031	0.030	0.038	0.017	0.057	0.045
61	0.023	0.005	0.012	0.010	0.008	0.021	0.275	0.005	0.025	0.017
71	0.039	0.005	0.014	0.007	0.009	0.130	0.028	0.002	0.025	0.047
72	0.027	0.003	0.014	0.006	0.013	0.140	0.022	0.002	0.024	0.046
73	0.006	0.001	0.036	0.002	0.003	0.007	0.002	0.005	0.003	0.014
74	0.017	0.003	0.010	0.006	0.006	0.034	0.013	0.002	0.019	0.021
81	0.008	0.001	0.004	0.003	0.006	0.018	0.010	0.001	0.008	0.013
82	0.030	0.004	0.021	0.014	0.014	0.044	0.025	0.005	0.024	0.035
83	0.038	0.004	0.012	0.010	0.010	0.063	0.060	0.003	0.071	0.045
91	0.054	0.015	0.021	0.023	0.016	0.028	0.033	0.016	0.049	0.078
92	0.005	0.001	0.001	0.001	0.001	0.004	0.017	0.001	0.004	0.004
93	0.027	0.004	0.008	0.009	0.008	0.025	0.023	0.003	0.019	0.028

Note: This table offers for each education field (1-digit ISCED-97) the country-year average share of workers in each occupation (2-digit ISCO-88) who have their highest degree in that field, out of total number of workers who have their highest degree in that field. The data for education fields contains a few missing observations because some respondents do not report their highest level of education. See Table 3 for the list of occupations and Table 11 for education fields.

Table 30: *ANOVA for the Within-Education Field Share Across Human Capital Types*

Source	Partial SS	df	MS	F	P-stat
Model	209.159	54	3.873	458.970	0.000
Specific Human Capital	60.044	1	60.044	7114.940	0.000
Specific Human Capital x Education Field	151.269	9	16.808	1991.630	0.000
Education Field	0.062	9	0.007	0.810	0.604
Country	0.023	28	0.001	0.100	1.000
Year	0.000	7	0.000	0.010	1.000
Residual	35.132	4163	0.008		
Total	244.291	4217	0.058		

Note: This table reports the results from an ANOVA exercise for the share of workers in each highest-degree education field-human capital type cell out of total number of workers who have their highest degree in that education field. The variation in the data are at human capital type-education field-country-year level, and we perform the ANOVA exercise along each of these dimensions and the interaction of education fields and human capital types. Specific Human Capital is a dummy variable which is equal to 1 for specific human capital and 0 for general human capital. The data for education fields are available for the period of 2003–2010. Number of obs = 4218; Adj. R-squared = 0.854. See Table 3 for the list of occupations and Table 11 for education fields.

Table 31: *Skill-Levels Across Countries*

Country	Skill-level (ISCED-97 0-2; 3-4; 5-6) Low-skilled	Medium-skilled	Highly-skilled
Austria	0.189	0.645	0.166
Belgium	0.275	0.382	0.343
Bulgaria	0.157	0.586	0.256
Croatia	0.190	0.616	0.194
Cyprus	0.290	0.391	0.320
Czech Republic	0.066	0.789	0.145
Denmark	0.196	0.511	0.293
Estonia	0.102	0.554	0.343
Finland	0.196	0.464	0.341
France	0.282	0.455	0.263
Germany	0.148	0.576	0.277
Greece	0.452	0.361	0.187
Hungary	0.152	0.656	0.192
Iceland	0.417	0.328	0.255
Ireland	0.319	0.376	0.305
Italy	0.479	0.403	0.117
Latvia	0.127	0.640	0.232
Lithuania	0.078	0.562	0.360
Luxembourg	0.355	0.392	0.253
Netherlands	0.255	0.450	0.295
Norway	0.141	0.524	0.335
Poland	0.102	0.705	0.193
Portugal	0.749	0.134	0.116
Slovakia	0.055	0.796	0.149
Slovenia	0.186	0.620	0.193
Spain	0.532	0.195	0.273
Sweden	0.175	0.526	0.299
Switzerland	0.174	0.548	0.278
UK	0.296	0.400	0.305

Note: This table offers for each country the share of workers in each level of highest attained education out of total employment, which we have averaged over years. There are three levels of highest attained education: pre-primary to lower-secondary (low-skilled; ISCED-97 0-2), secondary to post-secondary non-tertiary (medium-skilled; ISCED-97 3-4), and tertiary (highly-skilled; ISCED-97 5-6).

Table 32: *Skill-Levels Across Years*

Year	Skill-level (ISCED-97 0-2; 3-4; 5-6) Low-skilled	Medium-skilled	Highly-skilled
1992	0.512	0.324	0.164
1993	0.458	0.344	0.199
1994	0.443	0.349	0.208
1995	0.422	0.380	0.198
1996	0.362	0.425	0.213
1997	0.323	0.449	0.228
1998	0.272	0.508	0.220
1999	0.274	0.490	0.235
2000	0.266	0.493	0.242
2001	0.256	0.508	0.236
2002	0.246	0.516	0.238
2003	0.240	0.515	0.245
2004	0.229	0.511	0.260
2005	0.220	0.513	0.268
2006	0.219	0.510	0.271
2007	0.217	0.507	0.276
2008	0.212	0.504	0.284
2009	0.203	0.501	0.296
2010	0.196	0.500	0.304

Note: This table offers for each year the share of workers in each level of highest attained education out of total employment, which we have averaged across countries. There are three levels of highest attained education: pre-primary to lower-secondary (low-skilled; ISCED-97 0-2), secondary to post-secondary non-tertiary (medium-skilled; ISCED-97 3-4), and tertiary (highly-skilled; ISCED-97 5-6).

Table 33: *Concentrations of 1-digit ISCO-88 Occupations*

1-digit ISCO-88: Occupation Name Occupations	CV	GE	Gini	HI	Theil	Concentrated
1: Legislators, Senior Officials and Managers	1.229	0.744	0.564	0.163	0.597	0
2: Professionals	1.440	0.983	0.641	0.193	0.771	0
3: Technicians and Associate Professionals	1.066	0.535	0.538	0.137	0.525	0
4: Clerks	1.025	0.494	0.526	0.132	0.500	0
5: Service Workers and Shop and Market Sales Workers	1.723	1.408	0.705	0.247	0.980	1
6: Skilled Agricultural and Fishery Workers	3.460	5.677	0.888	0.787	2.220	1
7: Craft and related Trades Workers	2.033	1.933	0.766	0.313	1.233	1
8: Plant and Machine Operators and Assemblers	2.061	2.019	0.755	0.324	1.200	1
9: Elementary Occupations	0.956	0.443	0.485	0.125	0.434	0

Note: This table offers the assignment of 1-digit ISCO-88 occupations into "Concentrated" and "Not Concentrated" groups. For each country, year, and occupation, we compute 5 concentration measures (CV, GE, Gini, HI, Theil) for the distribution of within-occupation share across industries. We average the values of concentration measures across countries and years. The country- and year-averaged values of the concentration measures are offered in columns 2-6 of this table. Clearly, the values of concentration measures on average are lower than the values of concentration measures for 2-digit ISCO-88 offered in Table 3. We define dummy variables for each of the concentration measures which are equal to 1 for the values of the concentration measures that are higher than their medians. We take the average of these dummy variables and define Concentrated dummy variable which is equal to 1 if the average is greater than 0.5, and to 0 otherwise. Column 7 offers the value of this dummy variable for each occupation. We call an occupation Concentrated if Concentrated dummy variable is equal to 1, Not Concentrated otherwise. Similarly to 2-digit occupations, concentration measures for 1-digit occupations vary a lot across occupations and much less across countries and years and have very high rank correlations. We perform a similar assignment into the Concentrated group for each country and year. This assignment has almost no year and country variation and is highly correlated with the assignment offered in this table ($\rho = 0.896$).

Table 34: *Skill-Levels of Concentrated and Not Concentrated 1-Digit ISCO-88 Occupations*

		Concentrated				Not Concentrated					
Level of Education	Obs.	Mean	SD	Min	Max	Mean	SD	Min	Max	Diff. in Means	SE
Low (ISCED-97 0-2)	445	0.371	0.215	0.037	0.951	0.185	0.12	0.028	0.623	0.187***	(0.012)
Medium (ISCED-97 3-4)	445	0.561	0.211	0.045	0.942	0.424	0.126	0.17	0.708	0.137***	(0.012)
High (ISCED-97 5-6)	445	0.062	0.045	0.003	0.26	0.385	0.096	0.156	0.674	-0.324***	(0.005)

Note: This table offers basic statistics for the share of workers with low-, medium-, and high-level skills/education who have Concentrated 1-digit occupations out of total employment in Concentrated occupations. It also offers these basic statistics for the share of workers who have Not Concentrated occupations. The data are for all countries and years. There are three levels of highest attained education: pre-primary to lower-secondary (low-skilled; ISCED-97 0-2), secondary to post-secondary non-tertiary (medium-skilled; ISCED-97 3-4), and tertiary (highly-skilled; ISCED-97 5-6). The last two columns of the table use two-sided t-tests to test the significance of differences in means. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. These results are sensitive to the inclusion of the median concentrated occupations, Professionals, in the group of not Concentrated occupations. If these occupations are included in the group of Concentrated occupations then education levels are more uniform. See Table 33 for the assignment of occupations into Concentrated and Not Concentrated groups.

Table 35: *Concentrations of 3-digit ISCO-88 Occupations*

3-digit ISCO-88 Occupations	CV	GE	Gini	HI	Theil	Concentrated	2-digit ISCO-88	Specific
111	3.067	4.705	0.850	0.663	1.961	1	11	1
114	3.216	4.932	0.898	0.701	2.184	1	11	1
119	3.229	4.887	0.881	0.673	1.997	1	11	1
121	1.331	0.861	0.616	0.172	0.717	0	12	0
122	1.246	0.765	0.582	0.160	0.641	0	12	0
123	1.392	0.973	0.629	0.186	0.766	0	12	0
131	1.736	1.490	0.699	0.251	0.991	0	13	0
211	2.133	2.182	0.789	0.337	1.349	0	21	0
212	1.653	1.284	0.711	0.236	0.988	0	21	0
213	1.957	1.867	0.736	0.299	1.135	0	21	0
214	1.801	1.562	0.727	0.260	1.065	0	21	0
221	1.808	1.599	0.743	0.266	1.154	0	22	1
222	3.215	4.871	0.890	0.678	2.059	1	22	1
223	3.768	6.665	0.926	0.907	2.537	1	22	1
231	3.620	6.206	0.917	0.849	2.427	1	23	1
232	3.853	6.971	0.927	0.943	2.605	1	23	1
233	3.826	6.885	0.930	0.933	2.609	1	23	1
234	3.105	4.559	0.900	0.640	2.129	1	23	1
235	2.847	4.008	0.842	0.569	1.790	1	23	1
241	1.623	1.294	0.667	0.226	0.881	0	24	0
242	2.652	3.416	0.846	0.492	1.722	0	24	0
243	2.860	3.990	0.876	0.590	1.969	1	24	0
244	1.700	1.508	0.691	0.253	1.000	0	24	0
245	2.410	2.772	0.831	0.413	1.572	0	24	0
246	2.554	3.053	0.865	0.469	1.772	0	24	0
247	2.989	4.371	0.848	0.614	1.857	1	24	0
311	1.584	1.199	0.677	0.215	0.885	0	31	0
312	1.622	1.279	0.684	0.224	0.930	0	31	0
313	1.882	1.731	0.766	0.283	1.259	0	31	0
314	3.117	4.672	0.887	0.659	2.071	1	31	0
315	2.100	2.188	0.750	0.341	1.238	0	31	0
321	1.992	1.936	0.761	0.309	1.243	0	32	1
322	2.769	3.665	0.854	0.525	1.756	1	32	1
323	3.791	6.747	0.925	0.915	2.542	1	32	1
331	3.632	6.261	0.920	0.850	2.459	1	33	1
332	3.334	5.325	0.906	0.740	2.271	1	33	1
333	3.171	4.822	0.897	0.674	2.139	1	33	1
334	2.640	3.432	0.830	0.496	1.666	0	33	1
341	1.850	1.655	0.745	0.272	1.135	0	34	0
342	1.831	1.609	0.729	0.266	1.074	0	34	0
343	1.266	0.817	0.586	0.166	0.649	0	34	0
344	2.884	3.999	0.850	0.567	1.783	1	34	0
345	3.775	6.681	0.927	0.923	2.562	1	34	0
346	2.756	3.761	0.844	0.533	1.750	1	34	0
347	2.083	2.106	0.786	0.330	1.330	0	34	0
411	1.144	0.634	0.563	0.143	0.585	0	41	0
412	1.606	1.477	0.643	0.248	0.886	0	41	0
413	1.836	1.598	0.744	0.265	1.131	0	41	0
414	2.719	3.658	0.830	0.530	1.697	0	41	0

Table 35 – (Continued)

3-digit ISCO-88 Occupations	CV	GE	Gini	HI	Theil	Concentrated	2-digit ISCO-88	Specific
419	1.190	0.681	0.567	0.148	0.592	0	41	0
421	2.195	2.374	0.792	0.362	1.385	0	42	0
422	1.424	1.012	0.642	0.191	0.814	0	42	0
512	2.820	3.847	0.831	0.548	1.690	1	51	0
513	3.015	4.361	0.877	0.614	1.954	1	51	0
514	3.409	5.508	0.897	0.756	2.200	1	51	0
516	2.602	3.336	0.817	0.483	1.595	0	51	0
521	3.728	6.507	0.918	0.890	2.445	1	52	1
522	3.477	5.706	0.899	0.783	2.227	1	52	1
523	3.687	6.350	0.925	0.913	2.498	1	52	1
611	3.047	4.564	0.862	0.638	1.936	1	61	1
612	3.651	6.318	0.917	0.862	2.445	1	61	1
613	3.880	7.066	0.929	0.957	2.641	1	61	1
614	3.588	6.058	0.915	0.837	2.385	1	61	1
615	3.726	6.516	0.927	0.907	2.539	1	61	1
711	2.343	2.563	0.826	0.408	1.511	0	71	1
712	3.212	4.880	0.879	0.679	2.018	1	71	1
713	2.826	3.855	0.833	0.549	1.704	1	71	1
714	2.718	3.559	0.848	0.512	1.721	0	71	1
721	2.673	3.402	0.850	0.493	1.710	0	72	0
722	3.261	5.088	0.876	0.704	2.054	1	72	0
723	2.135	2.185	0.784	0.338	1.326	0	72	0
724	1.662	1.325	0.710	0.230	0.990	0	72	0
731	2.778	3.743	0.857	0.533	1.812	1	73	1
732	3.509	5.840	0.913	0.797	2.347	1	73	1
733	3.037	4.454	0.879	0.627	2.015	1	73	1
734	3.335	5.277	0.899	0.728	2.185	1	73	1
741	2.975	4.230	0.887	0.598	2.033	1	74	1
742	3.219	4.982	0.887	0.692	2.095	1	74	1
743	3.187	4.939	0.886	0.687	2.094	1	74	1
744	3.689	6.405	0.926	0.863	2.491	1	74	1
811	3.137	4.741	0.893	0.671	2.130	1	81	1
812	3.843	6.932	0.929	0.936	2.605	1	81	1
813	3.931	7.243	0.935	0.971	2.698	1	81	1
814	3.549	5.978	0.914	0.821	2.386	1	81	1
815	3.482	5.769	0.903	0.790	2.288	1	81	1
816	1.959	1.931	0.753	0.308	1.228	0	81	1
817	3.842	6.926	0.929	0.931	2.593	1	81	1
821	3.624	6.196	0.912	0.843	2.385	1	82	1
822	3.712	6.506	0.924	0.888	2.519	1	82	1
823	3.662	6.350	0.923	0.864	2.502	1	82	1
824	3.340	5.356	0.905	0.749	2.272	1	82	1
825	3.471	5.682	0.906	0.782	2.262	1	82	1
826	3.180	4.963	0.887	0.690	2.129	1	82	1
827	3.483	5.793	0.908	0.795	2.324	1	82	1
828	3.386	5.484	0.895	0.754	2.204	1	82	1
829	2.936	4.117	0.851	0.581	1.804	1	82	1
831	3.321	5.313	0.887	0.730	2.154	1	83	0
832	2.287	2.533	0.765	0.383	1.289	0	83	0
833	1.885	1.714	0.755	0.279	1.168	0	83	0
911	2.906	4.340	0.862	0.606	1.963	1	91	0
912	1.489	1.035	0.640	0.205	0.754	0	91	0

Table 35 – (Continued)

3-digit ISCO-88 Occupations	CV	GE	Gini	HI	Theil	Concentrated	2-digit ISCO-88	Specific
913	1.342	0.891	0.616	0.175	0.719	0	91	0
914	1.608	1.302	0.666	0.227	0.897	0	91	0
915	1.413	1.022	0.620	0.192	0.759	0	91	0
916	1.720	1.581	0.679	0.265	0.991	0	91	0
921	3.352	5.375	0.889	0.742	2.161	1	92	1
931	2.974	4.307	0.854	0.606	1.864	1	93	0
932	2.796	3.824	0.838	0.546	1.739	1	93	0
933	1.918	1.757	0.766	0.286	1.229	0	93	0

Note: This table offers the assignment of 3-digit ISCO-88 occupations into "Concentrated" and "Not Concentrated" groups. The assignment is performed in the following manner. For each country and year, we drop occupations which comprise 1 percent of observations in the sample of employed individuals and occupations which have less than 10 percent employment in the corresponding 2-digit code. We also drop occupation which are coded only at 1- and 2-digit levels. Further, for each country, year, and occupation, we compute 5 concentration measures (CV, GE, Gini, HI, Theil) for the distribution of within-occupation share across industries. We average the values of concentration measures across countries and years. The country- and year-averaged values of the concentration measures are offered in columns 2-6 of this table. Clearly, the values of concentration measures on average are higher than the values of concentration measures for 2-digit ISCO-88 offered in Table 3. We define dummy variables for each of the concentration measures which are equal to 1 for the values of the concentration measures that are higher than their medians. We take the average of these dummy variables and define Concentrated dummy variable which is equal to 1 if the average is greater than 0.5, and to 0 otherwise. Column 7 offers the value of this dummy variable for each occupation. We call an occupation Concentrated if Concentrated dummy variable is equal to 1, Not Concentrated otherwise. In columns 8 and 9 we offer the corresponding 2-digit ISCO-88 codes and their assignment into specific and general human capital types from Table 3. These assignments are highly correlated ($\rho = 0.639$). Similarly to 2-digit occupations, concentration measures for 3-digit occupations vary a lot across occupations and much less across countries and years and have very high rank correlations. We perform a similar assignment into Concentrated group for each country and year. This assignment has almost no year and country variation and is highly correlated with the assignment offered in this table ($\rho = 0.744$). Bulgaria, Poland, and Slovenia are excluded from the sample of countries because we do not have 3-digit ISCO-88 for them.

Table 36: *Skill-Levels of Concentrated and Not Concentrated 3-Digit ISCO-88 Occupations*

		Concentrated				Not Concentrated					
Level of Education	Obs.	Mean	SD	Min	Max	Mean	SD	Min	Max	Diff. in Means	SE
Low (ISCED-97 0-2)	405	0.320	0.190	0.039	0.864	0.241	0.151	0.032	0.738	0.080***	(0.012)
Medium (ISCED-97 3-4)	405	0.451	0.182	0.046	0.842	0.486	0.145	0.168	0.797	-0.035***	(0.012)
High (ISCED-97 5-6)	405	0.222	0.076	0.065	0.402	0.267	0.086	0.071	0.535	-0.045***	(0.006)

Note: This table offers basic statistics for the share of workers with low-, medium-, and high-level skills/education who have Concentrated 3-digit occupations out of total employment in Concentrated occupations. It also offers these basic statistics for the share of workers who have Not Concentrated occupations. There are three levels of highest attained education: pre-primary to lower-secondary (low-skilled; ISCED-97 0-2), secondary to post-secondary non-tertiary (medium-skilled; ISCED-97 3-4), and tertiary (highly-skilled; ISCED-97 5-6). The last two columns of the table uses two-sided t-test to test the significance of differences in means. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. See Table 35 for the assignment of occupations into Concentrated and Not Concentrated groups. The data are for all years and countries, except Bulgaria, Poland and Slovenia.

Table 37: *Concentrations of Smoothed 2-digit ISCO-88 Occupations*

2-digit ISCO-88: Occupation Name	CV	GE	Gini	HI	Theil	Specific (= 1; General = 0)
11: Legislators and Senior Officials	3.151	4.844	0.872	0.677	2.049	1
12: Corporate Managers	1.167	0.675	0.550	0.149	0.563	0
13: General Managers	1.709	1.436	0.694	0.245	0.968	0
21: Physical, Mathematical, and Engineering Science Professionals	1.720	1.429	0.691	0.244	0.946	0
22: Life Science and Health Professionals	3.023	4.324	0.860	0.611	1.853	1
23: Teaching Professionals	3.711	6.466	0.917	0.882	2.444	1
24: Other Professionals	1.351	0.872	0.635	0.174	0.747	0
31: Physical and Engineering Science Associate Professionals	1.405	0.948	0.624	0.183	0.731	0
32: Life Science and Health Associate Professionals	3.107	4.597	0.862	0.645	1.907	1
33: Teaching Associate Professionals	3.034	4.444	0.878	0.627	2.000	1
34: Other Associate Professionals	1.173	0.653	0.584	0.146	0.622	0
41: Office Clerks	1.069	0.544	0.545	0.132	0.535	0
42: Customer Services Clerks	1.508	1.110	0.664	0.203	0.855	0
51: Personal and Protective Services Workers	1.716	1.456	0.706	0.247	0.995	0
52: Models, Salespersons, and Demonstrators	3.509	5.798	0.902	0.798	2.254	1
61: Market-oriented Skilled Agricultural and Fishery Workers	3.484	5.781	0.896	0.797	2.253	1
71: Extraction and Building Trades Workers	2.920	4.042	0.839	0.574	1.737	1
72: Metal, Machinery, and Related Trades Workers	2.063	2.026	0.766	0.320	1.217	0
73: Precision, Handicraft, Printing, and Related Trades Workers	3.052	4.449	0.867	0.627	1.915	1
74: Other Craft and Related Trades Workers	3.050	4.474	0.874	0.631	1.964	1
81: Stationary-plant and Related Operators	3.003	4.333	0.853	0.611	1.853	1
82: Machine Operators and Assemblers	3.434	5.573	0.888	0.769	2.167	1
83: Drivers and Mobile-plant Operators	1.977	1.861	0.737	0.299	1.109	0
91: Sales and Services Elementary Occupations	1.101	0.588	0.546	0.138	0.546	0
92: Agricultural, Fishery, and Related Laborers	3.386	5.479	0.895	0.758	2.214	1
93: Labourers in Mining, Construction, Manufacturing, and Transport	1.872	1.672	0.746	0.275	1.133	0

Note: We obtain smoothed employment shares of occupations within each industry-country pair using 3rd degree polynomial fit. These smoothed shares are then used to compute concentration measures and to assign 2-digit ISCO-88 occupations into specific and general human capital types. For each country, year, and occupation, we compute 5 concentration measures (CV, GE, Gini, HI, Theil) for the distribution of smoothed within-occupation share across industries. We average the values of concentration measures across countries and years. The country- and year-averaged values of the concentration measures are offered in columns 2-6 of this table. We define dummy variables for each of the concentration measures which are equal to 1 for the values of the concentration measures that are higher than or equal to their medians. We take the average of these dummy variables and define Specific dummy variable which is equal to 1 if the average is greater than 0.5, and to 0 otherwise. Column 7 offers the value of this dummy variable for each occupation. We call an occupation specific human capital if Specific dummy variable is equal to 1, geneal human capital otherwise. This exercise yields the same assignment of occupations into specific and general types of human capital as in Table 3.

Table 38: *Industry-Level Decomposition of the Smoothed Trends in the Share of Specific Human Capital for Each Country*

		Between Industries				Within Industries				Total			
Country	Obs.	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max
Austria	15	-0.001	0.006	-0.009	0.011	-0.003	0.006	-0.010	0.008	-0.004	0.009	-0.019	0.009
Belgium	17	-0.001	0.005	-0.010	0.009	-0.001	0.001	-0.003	0.003	-0.002	0.005	-0.012	0.006
Bulgaria	10	-0.003	0.006	-0.010	0.011	0.000	0.005	-0.006	0.009	-0.003	0.008	-0.016	0.012
Croatia	8	-0.001	0.009	-0.011	0.015	0.000	0.001	-0.001	0.001	-0.001	0.009	-0.011	0.015
Cyprus	11	-0.001	0.005	-0.012	0.006	-0.005	0.005	-0.012	0.001	-0.005	0.007	-0.016	0.003
Czech Republic	13	-0.002	0.004	-0.009	0.005	0.001	0.005	-0.004	0.012	-0.001	0.006	-0.010	0.009
Denmark	17	0.000	0.004	-0.008	0.006	0.000	0.001	-0.001	0.002	0.000	0.004	-0.008	0.007
Estonia	13	-0.003	0.013	-0.036	0.018	0.001	0.006	-0.006	0.015	-0.002	0.011	-0.021	0.013
Finland	13	-0.003	0.006	-0.018	0.005	-0.003	0.004	-0.007	0.007	-0.006	0.007	-0.022	0.008
France	17	-0.002	0.004	-0.007	0.004	-0.002	0.003	-0.006	0.001	-0.004	0.005	-0.012	0.005
Germany	17	-0.001	0.005	-0.008	0.008	-0.001	0.002	-0.002	0.003	-0.002	0.005	-0.009	0.011
Greece	17	-0.004	0.007	-0.020	0.011	-0.001	0.002	-0.003	0.003	-0.005	0.007	-0.022	0.009
Hungary	13	-0.002	0.004	-0.009	0.003	-0.001	0.001	-0.002	0.002	-0.003	0.004	-0.009	0.003
Iceland	15	-0.002	0.012	-0.017	0.024	0.000	0.003	-0.007	0.004	-0.002	0.013	-0.018	0.029
Ireland	17	-0.008	0.025	-0.080	0.015	-0.006	0.010	-0.026	0.012	-0.014	0.032	-0.106	0.024
Italy	17	-0.003	0.008	-0.020	0.011	-0.003	0.005	-0.009	0.006	-0.006	0.009	-0.019	0.012
Latvia	12	-0.003	0.012	-0.021	0.024	-0.004	0.008	-0.015	0.012	-0.007	0.014	-0.029	0.013
Lithuania	12	-0.007	0.018	-0.054	0.017	0.001	0.008	-0.005	0.022	-0.006	0.013	-0.032	0.012
Luxembourg	17	-0.003	0.006	-0.017	0.006	-0.001	0.003	-0.005	0.010	-0.005	0.005	-0.012	0.006
Netherlands	17	-0.002	0.003	-0.007	0.006	-0.002	0.002	-0.007	0.000	-0.003	0.004	-0.013	0.005
Norway	14	-0.002	0.006	-0.012	0.007	0.002	0.002	-0.001	0.004	-0.001	0.007	-0.012	0.011
Poland	10	-0.006	0.008	-0.027	0.001	0.001	0.002	-0.004	0.003	-0.004	0.007	-0.024	0.002
Portugal	17	-0.004	0.011	-0.032	0.016	0.000	0.003	-0.003	0.007	-0.004	0.014	-0.033	0.023
Slovakia	12	-0.002	0.006	-0.011	0.011	0.002	0.004	-0.001	0.010	0.000	0.007	-0.012	0.013
Slovenia	14	-0.004	0.010	-0.023	0.015	-0.001	0.006	-0.007	0.015	-0.004	0.009	-0.025	0.011
Spain	17	-0.004	0.006	-0.015	0.010	-0.001	0.001	-0.002	0.001	-0.005	0.006	-0.016	0.008
Sweden	13	-0.001	0.005	-0.011	0.007	-0.001	0.002	-0.004	0.005	-0.002	0.006	-0.014	0.006
Switzerland	14	-0.003	0.008	-0.021	0.012	0.002	0.007	-0.017	0.016	-0.001	0.006	-0.011	0.006
UK	17	-0.001	0.002	-0.004	0.003	-0.001	0.002	-0.003	0.003	-0.002	0.002	-0.007	0.002

Note: This table offers the basic statistics for between- and within-industry decomposition of changes in the smoothed share of specific human capital (1) for each country. Columns 1-4 and 5-8 offer the basic statistics of between- and within-industry components. Columns 9-12 offer the basic statistics of total change in the share of specific human capital. Appendix - Treatment of Spikes in the Shares of Human Capital describes the construction of the smoothed share of specific human capital.

Table 39: *Industry-Level Decomposition of the Smoothed Trends in the Share of Specific Human Capital for Each Year*

		Between Industries				Within Industries				Total			
	Obs.	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max
Year													
1994	12	-0.009	0.024	-0.080	0.016	-0.001	0.009	-0.026	0.010	-0.010	0.032	-0.106	0.023
1995	12	-0.004	0.015	-0.048	0.016	-0.001	0.005	-0.015	0.004	-0.005	0.020	-0.063	0.020
1996	14	-0.002	0.007	-0.025	0.008	-0.001	0.005	-0.013	0.008	-0.003	0.011	-0.038	0.009
1997	17	-0.005	0.008	-0.023	0.004	0.000	0.007	-0.012	0.016	-0.004	0.007	-0.024	0.009
1998	22	-0.007	0.011	-0.036	0.012	-0.001	0.007	-0.017	0.015	-0.007	0.010	-0.033	0.004
1999	25	-0.007	0.011	-0.054	0.005	0.001	0.007	-0.012	0.022	-0.006	0.010	-0.032	0.012
2000	26	-0.004	0.006	-0.020	0.008	-0.001	0.005	-0.011	0.008	-0.005	0.008	-0.017	0.013
2001	28	-0.003	0.008	-0.027	0.015	-0.001	0.005	-0.010	0.012	-0.004	0.009	-0.024	0.009
2002	28	-0.001	0.006	-0.019	0.015	-0.002	0.003	-0.008	0.005	-0.002	0.007	-0.018	0.008
2003	28	0.001	0.006	-0.014	0.015	-0.002	0.003	-0.009	0.004	-0.001	0.007	-0.019	0.011
2004	29	0.002	0.007	-0.019	0.017	-0.001	0.003	-0.009	0.003	0.001	0.007	-0.025	0.013
2005	29	-0.001	0.005	-0.015	0.011	-0.001	0.003	-0.010	0.004	-0.002	0.006	-0.019	0.012
2006	29	-0.005	0.006	-0.021	0.001	-0.002	0.003	-0.008	0.003	-0.007	0.006	-0.029	0.001
2007	29	-0.007	0.005	-0.017	0.001	-0.001	0.003	-0.008	0.004	-0.008	0.005	-0.020	-0.001
2008	29	-0.006	0.004	-0.012	0.002	-0.001	0.003	-0.011	0.007	-0.007	0.005	-0.023	-0.001
2009	29	0.004	0.008	-0.015	0.024	-0.001	0.004	-0.015	0.009	0.004	0.008	-0.012	0.029
2010	29	0.000	0.007	-0.013	0.015	0.001	0.005	-0.015	0.012	0.001	0.009	-0.020	0.024

Note: This table offers the basic statistics for between- and within-industry decomposition of changes in the smoothed share of specific human capital (1) for each year. Columns 1-4 and 5-8 offer the basic statistics of between- and within-industry components. Columns 9-12 offer the basic statistics of total change in the share of specific human capital. Appendix - Treatment of Spikes in the Shares of Human Capital describes the construction of the smoothed share of specific human capital.

Table 40: *Correlations Among the Share of Specific Human Capital and Employment Shares in Industries with high Share of Specific Human Capital Employment*

	1	2	3
1 The Share of Specific Human Capital			
2 Employment Share in P50 Industries	0.6882		
3 Employment Share in P70 Industries	0.3967	0.3156	
4 Employment Share in P90 Industries	0.7938	0.5899	0.4451

Note: This table offers pairwise correlations among the share of specific human capital and employment shares in industries where the employment share of specific human capital is higher than its 50th, 70th, and 90th percentile in industries (within years and countries). Employment Share in P[] Industries is the share of employment in industries where the employment share of specific human capital is higher than its []th percentile. All correlations are significant at 10% level.

Table 41: *Regression Results for the Share of Specific Human Capital and Employment Shares in Industries with high Share of Specific Human Capital Employment*

	(1)	(2)	(3)	(4)
Employment Share in P50 Industries	0.351***	0.621***	0.334***	0.621***
	(0.060)	(0.134)	(0.060)	(0.139)
Employment Share in P70 Industries	0.022	-0.087	0.023	-0.06
	(0.060)	(0.142)	(0.061)	(0.169)
Employment Share in P90 Industries	0.807***	0.538***	0.813***	0.471**
	(0.094)	(0.156)	(0.095)	(0.223)
Country Fixed Effects	N	Y	N	Y
Year Fixed Effects	N	N	Y	Y
Obs.	443	443	443	443
R2	0.707	0.893	0.710	0.896

Note: In regressions reported in this table, the dependent variable is the share of specific human capital. Employment Share in P[] Industries is the share of employment in industries where the employment share of specific human capital is higher than its []th percentile. The variation in the data is at country-year level. Regressions are estimated using the OLS method. Robust standard errors are in parentheses. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level.

Table 42: *Assignment of Occupations (ISCO-88) into Groups Requiring Abstract, Manual, and Routine Skills*

Occupations (ISCO-88)	Abstract (A); Manual (M); Routine (R)	Specific (1); General (0)
11	A	1
12	A	0
13	A	0
21	A	0
22	A	1
23	A	1
24	A	0
31	A	0
32	A	1
33	A	1
34	A	0
41	R	0
42	R	0
51	M	0
52	M	1
61	M	1
71	M	1
72	M	0
73	R	1
74	R	1
81	R	1
82	R	1
83	M	0
91	R	0
92	M	1
93	M	0

Note: This table offers the assignment of 2-digit ISCO-88 occupations into abstract (A), manual (M), and routine (R) types. This assignment is performed using the 5 groups of occupations in Table 2 of Autor and Dorn (2013). The third column identifies specific and general human capital occupations. See Table 3 for the definitions of occupations.

Table 43: *Basic Statistics for the Share of Employment in Abstract, Manual, and Routine Occupations*

	Obs.	Mean	SD	Min	Max
Share of Abstract Occupations	457	0.361	0.058	0.198	0.519
Share of Manual Occupations	457	0.241	0.040	0.129	0.340
Share of Routine Occupations	457	0.378	0.058	0.218	0.538
Share of Specific Human Capital	457	0.353	0.058	0.222	0.581

Note: This table offers the basic statistics for the shares of employment in occupations requiring abstract, manual, and routine skills out of total employment. It also offers basic statistics for the share of specific human capital. The variation in the data is at country-year level. See Table 42 for the assignment of occupations into abstract, manual, and routine occupations and into specific and general human capital types.

Table 44: *Correlations Among the Shares of Employment in Abstract, Manual, and Routine Occupations and the Share of Specific Human Capital*

	Within and Between Years 1	2	3	Between Years 1	2	3
1 Share of Abstract Occupations						
2 Share of Manual Occupations	-0.457*			-0.946*		
3 Share of Routine Occupations	-0.696*	-0.137*		-0.638*	0.683*	
4 Share of Specific Human Capital	-0.651*	0.014	0.789*	-0.912*	0.921*	0.863*

Note: This table offers pairwise correlations among the shares of employment in occupations requiring abstract, manual, and routine skills and the share of specific human capital. In the first panel, variation in the data is at country-year level, and the number of observations is 457. In the second panel, we take country averages. The variation in the data is at year level, and the number of observations is 19. * indicates significance at the 1% level. See Table 42 for the assignment of occupations into abstract, manual, and routine occupations and into specific and general human capital types.

Table 45: *Assignment of Occupations (ISCO-88) into High-paying, Middling, and Low-paying*

Occupations (ISCO-88)	High-paying (H); Middling (M); Low-paying (L)	Specific (1); General (0)
11		1
12	H	0
13	H	0
21	H	0
22	H	1
23		1
24	H	0
31	H	0
32	H	1
33		1
34	H	0
41	M	0
42	M	0
51	L	0
52	L	1
61		1
71	M	1
72	M	0
73	M	1
74	M	1
81	M	1
82	M	1
83	M	0
91	L	0
92		1
93	L	0

Note: This table offers the correspondence between High-paying (H), Middling (M), and Low-paying (L) 2-digit ISCO-88 occupations and specific and general human capital occupations. The assignment of occupations into High-paying, Middling, and Low-paying is based on Table 1 of Goos et al. (2014). Some of the cells of column 2 are left blanc because Goos et al. (2014) drop these occupations. The third column identifies specific and general human capital occupations. See Table 3 for the definitions of occupations.

Table 46: *Basic Statistics for the Share of Employment in High-paying, Middling, and Low-paying Occupations*

	Obs.	Mean	SD	Min	Max
Share of High-paying Occupations	457	0.255	0.046	0.113	0.353
Share of Middling Occupations	457	0.195	0.034	0.133	0.291
Share of Low-paying Occupations	457	0.311	0.045	0.198	0.408

Note: This table offers the basic statistics for the share of High-paying, Middling, and Low-paying occupations in total employment. See Table 43 for the basic statistics of the share of specific human capital. The variation is at country-year level. See Table 45 for the assignment of occupations into High-paying, Middling, and Low-paying.

Table 47: *Correlations Among the Shares of Employment in High-paying, Middling, and Low-paying Occupations and the Share of Specific Human Capital*

	Within and Between Years			Between Years		
	1	2	3	1	2	3
1 Share of High-paying Occupations						
2 Share of Middling Occupations	-0.1014			0.8876*		
3 Share of Low-paying Occupations	-0.4985*	-0.2675*		-0.8605*	-0.8787*	
4 Share of Specific Human Capital	-0.6604*	-0.1745*	0.3595*	-0.8999*	-0.8642*	0.9580*

Note: This table offers pairwise correlations among the shares of High-paying, Middling, and Low-paying occupations out of total employment and the share of specific human capital. In the first panel, variation in the data is at country-year level, and the number of observations is 457. In the second panel, we take country averages. The variation in the data is at year level, and the number of observations is 19. * indicates significance at the 1% level. See Table 45 for the correspondence between High-paying, Middling, and Low-paying occupations and specific and general human capital occupations.

Table 48: *Estimated values of* ε_1 *and* γ_1 *- Initial* $\varepsilon_1 \in (0,1)$

Country	Sample Period	ε_1	δ_1	δ_2	δ_3	γ_1	RMSE EQ. (21)	RMSE EQ. (22)	RMSE EQ. (23)
Austria	1971–2007	0.317***	-0.013***	0.000***	0.000	0.390	0.004	0.006	0.017
Belgium	1971–2007	0.430***	-0.005***	0.000*	0.000	0.461	0.004	0.009	0.020
Cyprus	1996–2007	0.578***	-0.005***	0.001***	0.000	0.410	0.001	0.014	0.022
Czech Republic	1996–2007	0.642***	-0.002	0.000**	0.000	0.631	0.002	0.008	0.020
Denmark	1971–2007	0.458***	0.003***	-0.000***	0.000***	0.482	0.005	0.008	0.018
Estonia	1996–2007	0.751***	0.022***	-0.003***	0.000***	0.611	0.011	0.015	0.032
Finland	1971–2007	0.316***	-0.001	-0.000***	0.000***	0.412	0.003	0.006	0.017
France	1971–2007	0.333***	-0.007***	0.000***	0	0.328	0.004	0.006	0.014
Germany	1971–2007	0.475***	-0.007***	0.000***	-0.000**	0.497	0.004	0.008	0.021
Greece	1971–2007	0.554***	-0.003***	-0.000**	0.000***	0.528	0.004	0.009	0.023
Hungary	1992–2007	0.264	-0.016***	0.002***	-0.000***	0.354	0.004	0.014	0.040
Ireland	1971–2007	0.538***	0.004***	-0.001***	0.000***	0.607	0.008	0.014	0.035
Italy	1971–2007	0.279***	-0.001	-0.000***	0.000***	0.364	0.005	0.005	0.018
Latvia	1996–2007	0.263*	0	0.001	0	0.226	0.004	0.014	0.027
Lithuania	1996–2007	0.395***	0.006***	-0.001	0	0.539	0.005	0.015	0.041
Luxembourg	1971–2007	0.456***	-0.056***	0.002***	-0.000***	0.184	0.042	0.048	0.098
Netherlands	1971–2007	0.541***	-0.002**	-0.000**	0.000***	0.532	0.007	0.007	0.018
Poland	1996–2007	0.112***	0	0	0	0.105	0.006	0.010	0.019
Portugal	1971–2007	0.664***	0	-0.000***	0.000***	0.649	0.007	0.013	0.038
Slovakia	1996–2007	0.747***	0	0	0.000**	0.707	0.002	0.013	0.039
Slovenia	1996–2007	0.446***	0	0	0	0.507	0.000	0.008	0.022
Spain	1971–2007	0.336***	-0.009***	0.000***	0	0.430	0.003	0.007	0.018
Sweden	1971–2007	0.401***	-0.007***	0.000***	0	0.352	0.004	0.008	0.018
UK	1971–2007	0.439***	0	-0.000***	0.000***	0.441	0.009	0.011	0.030
Australia	1971–2007	0.543***	-0.001	-0.000***	0.000***	0.504	0.010	0.012	0.027
Japan	1974–2007	0.293***	-0.003***	0	0.000**	0.389	0.004	0.006	0.021
Korea	1971–2007	0.544***	0.005***	-0.000***	0.000***	0.685	0.010	0.008	0.029
US	1978–2007	0.413***	-0.007***	0	0.000***	0.362	0.004	0.008	0.017

Note: This table reports the results from the estimation of equations (21)-(23). Columns (8)-(10) report the Root Mean Square Error of the fit for each equation. The estimations are carried using non-linear seemingly unrelated regressions routine in STATA and initial values of ε_1 are selected from $(0,1)$. Adjusted R^2 of the first equations is greater than 0.99 in all estimations. Standard errors are robust to arbitrary heteroscedasticity and autocorrelation. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. We compute the value of γ_1 from (24) using the estimated value of ε_1. This is the reason why no significance level is attached to γ_1.

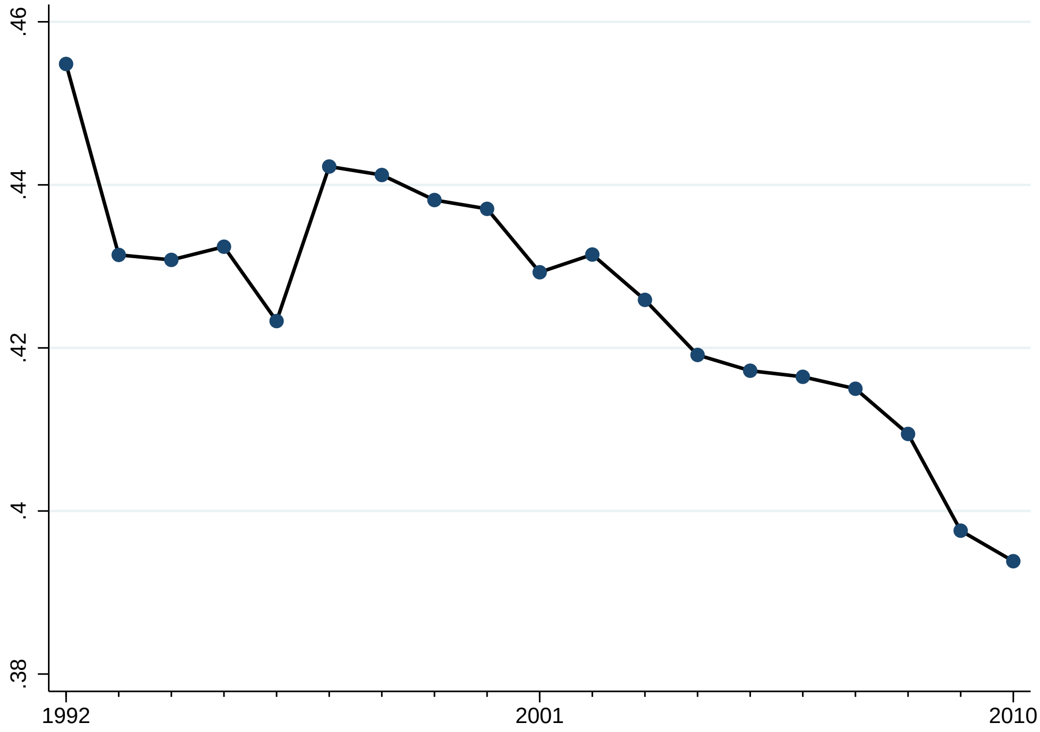
Table 49: *Estimated values of ε_1 and γ_1 - Initial $\varepsilon_1 > 1$*

Country	Sample Period	ε_1	δ_1	δ_2	δ_3	γ_1	RMSE EQ. (21)	RMSE EQ. (22)	RMSE EQ. (23)
Austria	1971–2007	1.559***	-0.005***	0	0	0.792	0.021	0.011	0.030
Belgium	1971–2007	1.490***	0	0	0	0.788	0.018	0.012	0.032
Cyprus	1996–2007	1.153***	0.004***	0	0	0.648	0.004	0.016	0.023
Czech Republic	1996–2007	1.058***	0	0	0	0.749	0.003	0.008	0.022
Denmark	1971–2007	1.548***	0.002	-0.000**	0.000***	0.783	0.016	0.011	0.023
Estonia	1996–2007	1.098***	0.057***	-0.007***	0.000***	0.682	0.020	0.020	0.042
Finland	1971–2007	1.404***	0.002	-0.000***	0.000***	0.809	0.014	0.010	0.031
France	1971–2007	1.525***	-0.004***	0	0	0.770	0.014	0.010	0.021
Germany	1971–2007	1.739***	0	0	0	0.810	0.016	0.011	0.027
Greece	1971–2007	1.344***	0	0	0	0.791	0.015	0.012	0.035
Hungary	1992–2007	1.158***	0.008	-0.001	0	0.765	0.007	0.018	0.058
Ireland	1971–2007	1.279***	0.003	-0.001***	0.000***	0.773	0.016	0.016	0.043
Italy	1971–2007	1.436***	0.003	-0.000***	0.000***	0.795	0.020	0.012	0.037
Latvia	1996–2007	1.096***	0.010***	-0.001	0.000**	0.701	0.006	0.019	0.041
Lithuania	1996–2007	1.230***	0.024**	-0.003**	0.000**	0.790	0.011	0.017	0.054
Luxembourg	1971–2007	2.931***	0.011	-0.001**	0.000**	0.737	0.082	0.095	0.111
Netherlands	1971–2007	1.421***	0	0	0.000**	0.779	0.019	0.012	0.026
Poland	1996–2007	1.140***	0.041***	-0.005***	0.000***	0.768	0.012	0.014	0.042
Portugal	1971–2007	1.336***	-0.001	-0.000**	0.000***	0.784	0.012	0.013	0.039
Slovakia	1996–2007	1.151***	0.004	0	0	0.785	0.005	0.013	0.042
Slovenia	1996–2007	1.165***	0.002	0	0	0.763	0.002	0.009	0.026
Spain	1971–2007	1.568***	-0.005***	0	0	0.804	0.019	0.010	0.029
Sweden	1971–2007	1.246***	-0.004**	0	0	0.736	0.013	0.015	0.033
UK	1971–2007	1.562***	0.006	-0.001**	0.000***	0.782	0.032	0.020	0.047
Australia	1971–2007	1.466***	-0.004	0	0	0.767	0.021	0.014	0.026
Japan	1974–2007	1.494***	0.004*	-0.000**	0.000***	0.788	0.016	0.013	0.035
Korea	1971–2007	1.260***	0.005*	-0.000***	0.000***	0.829	0.019	0.009	0.036
US	1978–2007	1.462***	-0.003*	0	0.000*	0.744	0.017	0.013	0.025

Note: This table reports the results from the estimation of equations (21)-(23). Columns (8)-(10) report the Root Mean Square Error of the fit for each equation. The estimations are carried using non-linear seemingly unrelated regressions routine in STATA and initial values of ε_1 are selected above 1. Adjusted R^2 of the first equations is greater than 0.99 in all estimations. Standard errors are robust to arbitrary heteroscedasticity and autocorrelation. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. We compute the value of γ_1 from (24) using the estimated value of ε_1. This is the reason why no significance level is attached to γ_1.

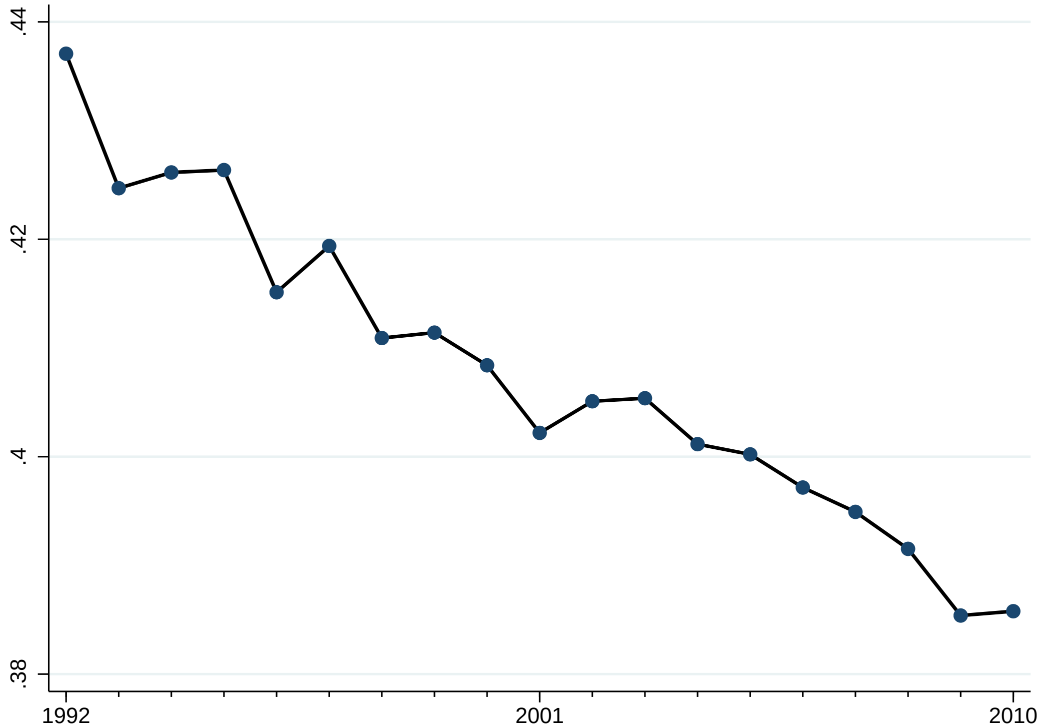
Data Appendix - Figures

Figure 6: *The Average Employment Share of Concentrated 1-digit ISCO-88 Occupations in Sample Countries*



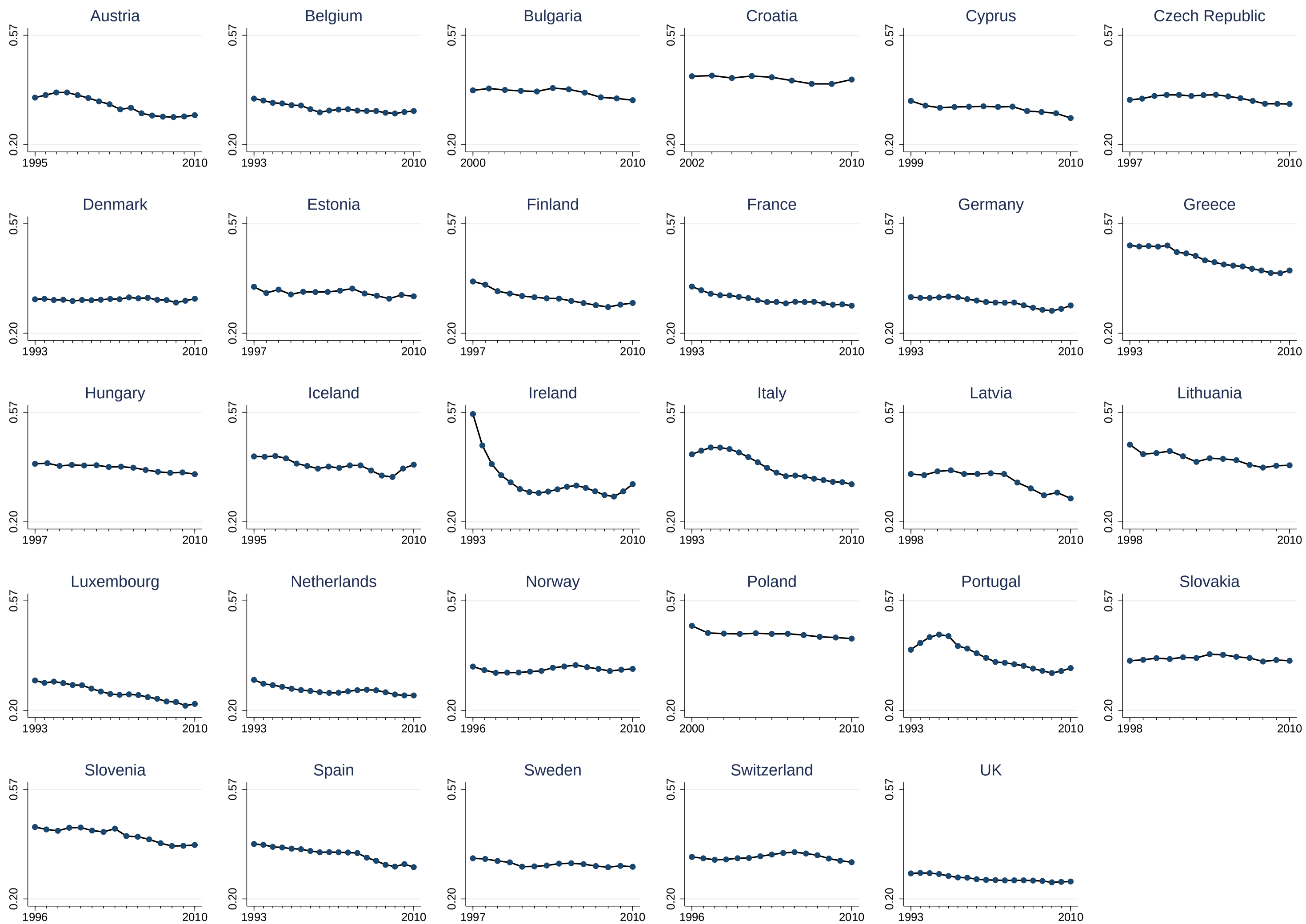
Note: This figure offers country-averaged value of the share of Concentrated 1-digit ISCO-88 occupations. See Table 33 for the assignment of occupations into Concentrated and Not Concentrated groups. The coefficient in front of time trend t is highly significant and negative in regressions of a form $\textit{Share of Concentrated 1-digit Occupations}_{c,t} = \alpha + \beta t + \eta_{c,t}$.

Figure 7: *The Average Employment Share of Concentrated 3-digit ISCO-88 Occupations in Sample Countries*



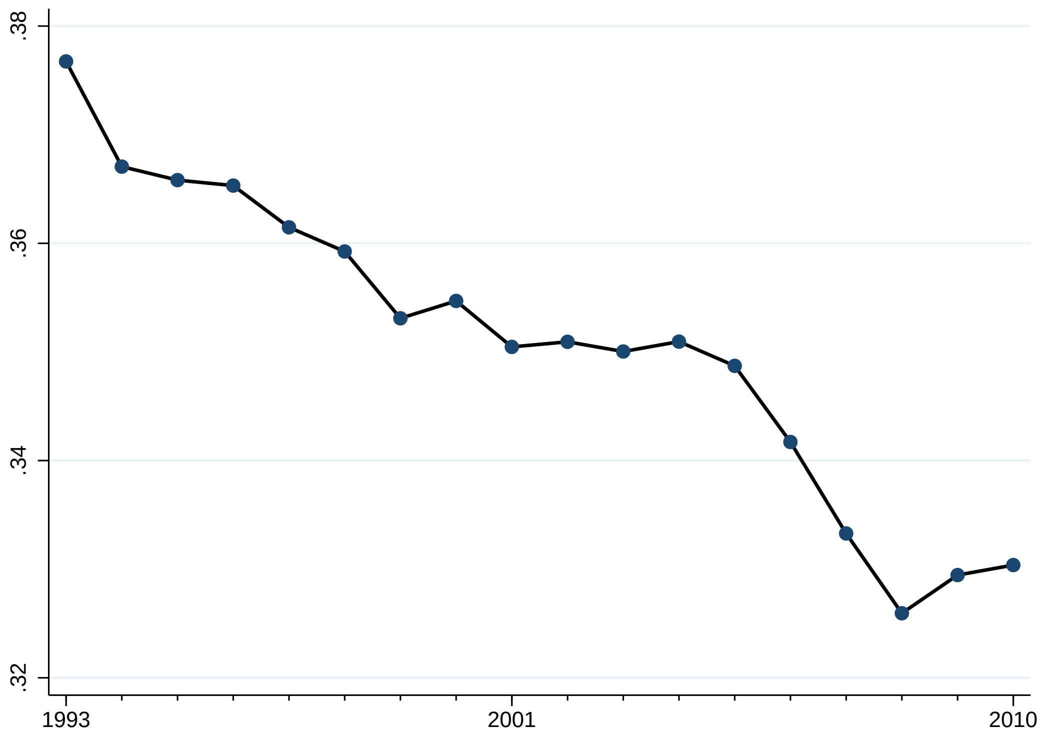
Note: This figure offers country-averaged value of the share of Concentrated 3-digit ISCO-88 occupations. See Table 35 for the assignment of occupations into Concentrated and Not Concentrated groups. The coefficient in front of time trend t is highly significant and negative in regressions of a form $\textit{Share of Concentrated 3-digit Occupations}_{c,t} = \alpha + \beta t + \eta_{c,t}$. Bulgaria, Poland, and Slovenia are excluded from the sample of countries because we do not have 3-digit ISCO-88 for them.

Figure 8: *The Smoothed Share of Specific Human Capital in Sample Countries*



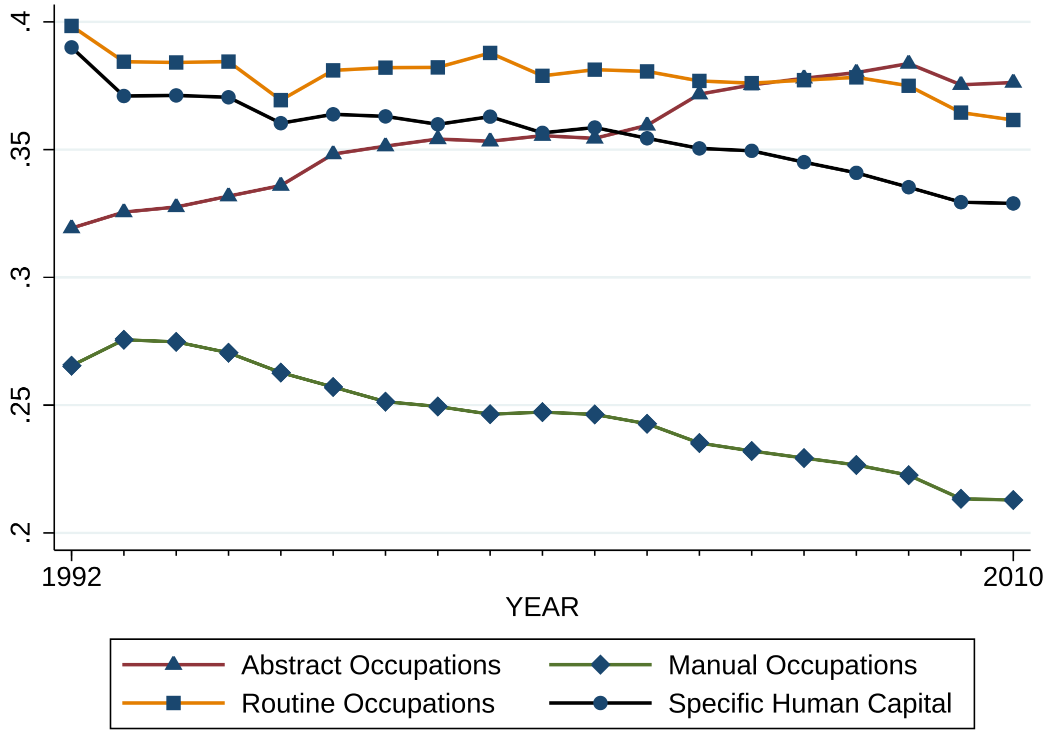
Note: We fit 3rd degree polynomials on employment shares of occupations within each industry-country pair to smooth the share of specific human capital. This figure offers the smoothed share of workers in specific human capital occupations out of total employment in each country in our sample. See Table 37 for the assignment of occupations into specific and general human capital types using smoothed shares.

Figure 9: *The Average Smoothed Employment Share of Specific Human Capital in Sample Countries*



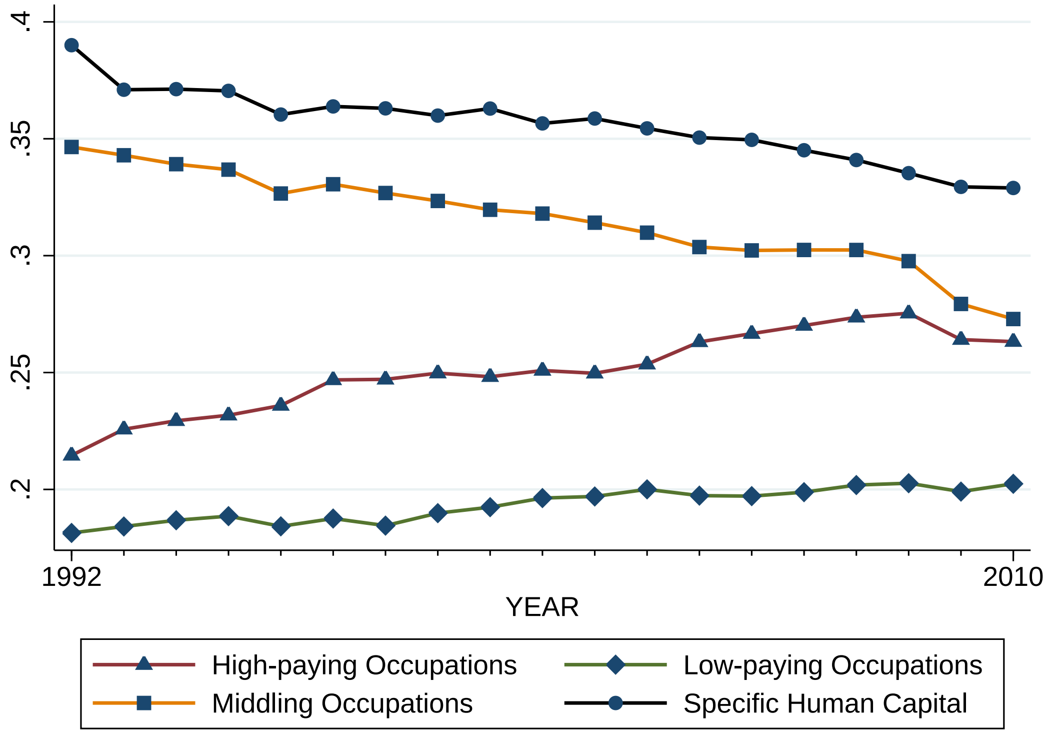
Note: This figure offers country-averaged value of the smoothed share of specific human capital. See Table 37 for the assignment of occupations into specific and general human capital types using smoothed shares. The coefficient in front of time trend t is highly significant and negative in regressions of a form $\textit{Smoothed Share of Specific Human Capital}_{c,t} = \alpha + \beta t + \eta_{c,t}$.

Figure 10: *The Average Employment Shares in Abstract, Manual, and Routine Occupations in Sample Countries*



Note: This figure offers the shares of employment in occupations requiring abstract, manual, and routine skills averaged over the sample countries. It also offers the share of specific human capital averaged over the sample countries. See Table 42 for the assignment of occupations into abstract-, manual-, and routine-skills occupations and into specific and general human capital types.

Figure 11: *The Average Employment Shares in High-paying, Middling, and Low-paying Occupations in Sample Countries*



Note: This figure offers the shares of employment in High-paying, Middling, and Low-paying occupations averaged over the sample countries. It also offers the share of specific human capital averaged over the sample countries. See Table 45 for the assignment of occupations into High-paying, Middling, and Low-paying groups and into specific and general human capital types.

Proofs Appendix

Proof of Proposition 1: We use $F_1()$ to denote

$$F_1\left(u_h^g, H_g, Y_m\right) = \left(u_h^g\right)^{1-\frac{\varepsilon_2-1}{\varepsilon_2}}\left(1-u_h^g\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4-1} - \mathbb{E}\left[\left(\frac{\lambda_{Y_h}}{\lambda_{Y_l}}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\frac{\gamma_2}{\gamma_4}\frac{\gamma_1}{1-\gamma_1}$$
$$\times\left(H_g\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}-\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4}\left[\gamma_2\left(u_h^g H_g\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \left(1-\gamma_2\right)Y_m^{\frac{\varepsilon_2-1}{\varepsilon_2}}\right]^{\frac{\varepsilon_2}{\varepsilon_2-1}\frac{\varepsilon_1-1}{\varepsilon_1}-1}.$$

According to (17) we have that $F_1\left(u_h^g, H_g, Y_m\right) \equiv 0$, and the partial derivatives of $F_1\left(u_h^g, H_g, Y_m\right)$ are given by

$$\frac{\partial F_1\left(u_h^g, H_g, Y_m\right)}{\partial u_h^g} = \frac{1}{u_h^g}\left(u_h^g\right)^{1-\frac{\varepsilon_2-1}{\varepsilon_2}}\left(1-u_h^g\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4-1}$$
$$\times\left[1-\frac{\varepsilon_1-1}{\varepsilon_1}+\left(\frac{\varepsilon_1-1}{\varepsilon_1}-\frac{\varepsilon_2-1}{\varepsilon_2}\right)\omega_{Y_m}^{Y_h}+\left(1-\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4\right)\frac{u_h^g}{1-u_h^g}\right],$$
$$\frac{\partial F_1\left(u_h^g, H_g, Y_m\right)}{\partial H_g} = -\frac{1}{H_g}\left(u_h^g\right)^{1-\frac{\varepsilon_2-1}{\varepsilon_2}}\left(1-u_h^g\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4-1}$$
$$\times\left[-\left(\frac{\varepsilon_1-1}{\varepsilon_1}-\frac{\varepsilon_2-1}{\varepsilon_2}\right)\omega_{Y_m}^{Y_h}+\frac{\varepsilon_1-1}{\varepsilon_1}\left(1-\gamma_4\right)\right],$$
$$\frac{\partial F_1\left(u_h^g, H_g, Y_m\right)}{\partial Y_m} = -\frac{1}{Y_m}\left(u_h^g\right)^{1-\frac{\varepsilon_2-1}{\varepsilon_2}}\left(1-u_h^g\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4-1}\left(\frac{\varepsilon_1-1}{\varepsilon_1}-\frac{\varepsilon_2-1}{\varepsilon_2}\right)\omega_{Y_m}^{Y_h}.$$

Therefore, using the Implicit Function theorem we have that

$$\frac{\partial u_h^g}{\partial H_g} = \frac{u_h^g}{H_g}\frac{-\left(\frac{\varepsilon_1-1}{\varepsilon_1}-\frac{\varepsilon_2-1}{\varepsilon_2}\right)\omega_{Y_m}^{Y_h}+\frac{\varepsilon_1-1}{\varepsilon_1}\left(1-\gamma_4\right)}{1-\frac{\varepsilon_1-1}{\varepsilon_1}+\left(\frac{\varepsilon_1-1}{\varepsilon_1}-\frac{\varepsilon_2-1}{\varepsilon_2}\right)\omega_{Y_m}^{Y_h}+\left(1-\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4\right)\frac{u_h^g}{1-u_h^g}},$$
$$\frac{\partial u_h^g}{\partial Y_m} = \frac{u_h^g}{Y_m}\frac{\left(\frac{\varepsilon_1-1}{\varepsilon_1}-\frac{\varepsilon_2-1}{\varepsilon_2}\right)\omega_{Y_m}^{Y_h}}{1-\frac{\varepsilon_1-1}{\varepsilon_1}+\left(\frac{\varepsilon_1-1}{\varepsilon_1}-\frac{\varepsilon_2-1}{\varepsilon_2}\right)\omega_{Y_m}^{Y_h}+\left(1-\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4\right)\frac{u_h^g}{1-u_h^g}}.$$

Multiplying the denominators and numerators of these expressions by $\varepsilon_1\varepsilon_2$ gives

$$\frac{\partial u_h^g}{\partial H_g} = \frac{u_h^g}{H_g}\frac{-\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h}+\varepsilon_2\left(\varepsilon_1-1\right)\left(1-\gamma_4\right)}{\varepsilon_2+\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h}+\varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}}, \tag{25}$$

$$\frac{\partial u_h^g}{\partial Y_m} = \frac{u_h^g}{Y_m}\frac{\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h}}{\varepsilon_2+\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h}+\varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}}, \tag{26}$$

$$\frac{\partial u_h^g}{\partial H_s} = \left(1-\omega_K^{Y_m}\right)\frac{Y_m}{H_s}\frac{\partial u_h^g}{\partial Y_m}. \tag{27}$$

The denominator in (25) and (26) is positive. This can be easily checked noticing that the denominator increases with ε_1 and is positive for the limiting value $\varepsilon_1 = 0$. Therefore, u_h^g increases (declines) with Y_m if $\varepsilon_1 > \varepsilon_2$ ($\varepsilon_2 > \varepsilon_1$). This implies that u_h^g increases (declines) with K and H_s if $\varepsilon_1 > \varepsilon_2$ ($\varepsilon_2 > \varepsilon_1$) since Y_m increases with these inputs. In turn, u_h^g increases (declines) with H_g if the numerator in (25) is positive (negative). It is sufficient to have $\varepsilon_2 > \varepsilon_1 > 1$ in order the numerator to be positive and $1 > \varepsilon_1 > \varepsilon_2$ in order it to be negative. Moreover, if $\gamma_4 = 1$ then the numerator is positive (negative) if $\varepsilon_2 > \varepsilon_1$ ($\varepsilon_1 > \varepsilon_2$).

The ratio Y_h/Y_l increases with K. To show this we note that Y_m increases with K and evaluate the sign of the following partial derivative.

$$\frac{\partial}{\partial Y_m}\frac{Y_h}{Y_l} = \frac{Y_l\frac{\partial Y_h}{\partial Y_m} - Y_h\frac{\partial Y_l}{\partial Y_m}}{(Y_l)^2}.$$

The sign of the numerator in this expression is the same as the sign of

$$\left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial Y_m} + \omega_{Y_m}^{Y_h}\frac{1}{Y_m} + \gamma_4\frac{u_h^g}{1-u_h^g}\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial Y_m}.$$

We denote

$$d = \varepsilon_1\left(1-u_h^g\right) + \varepsilon_2\left[\varepsilon_1 - (\varepsilon_1-1)\gamma_4\right]u_h^g + (\varepsilon_1-\varepsilon_2)\gamma_4 u_h^g.$$

Using the expression for $\partial u_h^g/\partial Y_m$ it can be shown that d has the same sign as the partial derivative of Y_h/Y_l with respect to Y_m. It can be easily shown that d increases with ε_1 and is equal to zero when $\varepsilon_1 = 0$. Therefore, Y_h/Y_l increases with K.

In Appendix - Numerical Exercises we confirm these results with numerical exercises where we use (16) instead of (17).

Proof of Proposition 2: For brevity, we use Λ and $F_2\left(\right)$ to denote

$$\begin{aligned}\Lambda &= \mathbb{E}\left[\left(\lambda_{Y_h}/\lambda_{Y_l}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\\ &= \exp\left(\frac{\varepsilon_1-1}{\varepsilon_1}\left(\mu_{z_h}-\mu_{z_l}\right) + \frac{1}{2}\left(\frac{\varepsilon_1-1}{\varepsilon_1}\right)^2\left(\sigma_{z_h}^2+\sigma_{z_l}^2\right)\right),\end{aligned}$$

and

$$\begin{aligned}F_2\left(u_h^g,\Lambda\right) = {} & \left(u_h^g\right)^{1-\frac{\varepsilon_2-1}{\varepsilon_2}}\left(1-u_h^g\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4-1} - \frac{\gamma_2}{\gamma_4}\frac{\gamma_1}{1-\gamma_1}\left(H_g\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}-\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4}\\ & \times\left[\gamma_2\left(u_h^g H_g\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + (1-\gamma_2)Y_m^{\frac{\varepsilon_2-1}{\varepsilon_2}}\right]^{\frac{\varepsilon_2}{\varepsilon_2-1}\frac{\varepsilon_1-1}{\varepsilon_1}-1}\Lambda.\end{aligned}$$

The partial derivatives of $F_2\left(u_h^g,\Lambda\right)$ are given by

$$\begin{aligned}\frac{\partial F_2\left(u_h^g,\Lambda\right)}{\partial u_h^g} &= \frac{\partial}{\partial u_h^g}F_1\left(u_h^g,H_g,Y_m\right),\\ \frac{\partial F_2\left(u_h^g,\Lambda\right)}{\partial\Lambda} &= -\left(u_h^g\right)^{1-\frac{\varepsilon_2-1}{\varepsilon_2}}\left(1-u_h^g\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4-1}\frac{1}{\Lambda}.\end{aligned}$$

Therefore,

$$\frac{\partial u_h^g}{\partial\Lambda} = \frac{u_h^g}{\Lambda}\frac{\varepsilon_1\varepsilon_2}{\varepsilon_2 + (\varepsilon_1-\varepsilon_2)\omega_{Y_m}^{Y_h} + \varepsilon_2\left[\varepsilon_1-(\varepsilon_1-1)\gamma_4\right]\frac{u_h^g}{1-u_h^g}}. \tag{28}$$

This implies that u_h^g increases with Λ. Therefore, u_h^g increases with μ_{z_h} and declines with μ_{z_l} when h- and l-goods are gross substitutes ($\varepsilon_1 > 1$). It declines with μ_{z_h} and increases with μ_{z_l} when h- and l-goods are gross complements ($1 > \varepsilon_1$). Clearly, it also increases with σ_{z_h} when $\varepsilon_1 > 1$ and with σ_{z_l} when $1 > \varepsilon_1$.

In Appendix - Numerical Exercises we confirm these results with numerical exercises where we use (16) instead of (17). However, according to our numerical results u_h^g does not increase with σ_{z_l} when $\varepsilon_1 > 1$ and with σ_{z_h} when $1 > \varepsilon_1$ if we use (16) instead of (17).

Proof of Proposition 3: Consider the derivative of the relative (inverse) demand for general human capital (18) with respect to K. It is given by

$$\begin{aligned}\frac{\partial \tilde{w}_g}{\partial K} &= \frac{\gamma_2}{1-\gamma_2}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} H_s \frac{\partial}{\partial K}\frac{1}{1-\omega_K^{Y_m}} \\ &+ \frac{\gamma_2}{1-\gamma_2}\frac{1}{1-\omega_K^{Y_m}}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} H_s \frac{\partial}{\partial K}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}} \\ &+ \frac{\gamma_2}{1-\gamma_2}\frac{1}{1-\omega_K^{Y_m}}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}} H_s \frac{\partial}{\partial K}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}},\end{aligned}$$

where

$$\begin{aligned}\frac{\partial}{\partial K}\frac{1}{1-\omega_K^{Y_m}} &= \frac{\varepsilon_3-1}{\varepsilon_3}\frac{\omega_K^{Y_m}}{1-\omega_K^{Y_m}}\frac{1}{K}, \\ \frac{\partial}{\partial K}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}} &= -\frac{1}{\varepsilon_2}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}}\frac{1}{u_h^g}\omega_K^{Y_m}\frac{Y_m}{K}\frac{\partial u_h^g}{\partial Y_m}, \\ \frac{\partial}{\partial K}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} &= -\frac{\varepsilon_2-1}{\varepsilon_2}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}\frac{1}{Y_m}\omega_K^{Y_m}\frac{Y_m}{K}.\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{\partial \tilde{w}_g}{\partial K} &= \tilde{w}_g \omega_K^{Y_m}\frac{1}{K}\left[\frac{\varepsilon_3-1}{\varepsilon_3} - \frac{1}{\varepsilon_2}\frac{1}{u_h^g}Y_m\frac{\partial u_h^g}{\partial Y_m} - \frac{\varepsilon_2-1}{\varepsilon_2}\right] \\ &= \tilde{w}_g \omega_K^{Y_m}\frac{1}{K}\left\{\frac{\varepsilon_3-1}{\varepsilon_3} - \frac{\varepsilon_2-1}{\varepsilon_2}\right. \\ &\left. -\frac{1}{\varepsilon_2}\frac{(\varepsilon_1-\varepsilon_2)\,\omega_{Y_m}^{Y_h}}{\varepsilon_2 + (\varepsilon_1-\varepsilon_2)\,\omega_{Y_m}^{Y_h} + \varepsilon_2\left[\varepsilon_1 - (\varepsilon_1-1)\,\gamma_4\right]\frac{u_h^g}{1-u_h^g}}\right\}.\end{aligned} \tag{29}$$

A sufficient condition to have $\partial \tilde{w}_g/\partial K > 0$ then is

$$\varepsilon_3 > 1 \geq \varepsilon_2 > \varepsilon_1.$$

A sufficient condition to have $\partial \tilde{w}_g/\partial K < 0$ is

$$\varepsilon_1 > \varepsilon_2 \geq 1 > \varepsilon_3.$$

Clearly, when $\varepsilon_2 = 1$, $\partial \tilde{w}_g/\partial K > 0$ if and only if $\varepsilon_3 > 1 > \varepsilon_1$.

The derivative of the demand function with respect to H_g is given by

$$\frac{\partial \tilde{w}_g}{\partial H_g} = \frac{\gamma_2}{1-\gamma_2}\frac{1}{1-\omega_K^{Y_m}}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} H_s \frac{\partial}{\partial H_g}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}}, \tag{30}$$

where

$$\frac{\partial}{\partial H_g}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}} = -\frac{1}{\varepsilon_2}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}}\frac{1}{H_g}$$
$$\times\left\{\frac{\varepsilon_2+\varepsilon_2\left[\varepsilon_1-(\varepsilon_1-1)\gamma_4\right]\frac{u_h^g}{1-u_h^g}+\varepsilon_2(\varepsilon_1-1)(1-\gamma_4)}{\varepsilon_2+(\varepsilon_1-\varepsilon_2)\omega_{Y_m}^{Y_h}+\varepsilon_2\left[\varepsilon_1-(\varepsilon_1-1)\gamma_4\right]\frac{u_h^g}{1-u_h^g}}\right\}.$$

Clearly,

$$\frac{\partial}{\partial H_g}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}} < 0,$$

which implies that

$$\frac{\partial \tilde{w}_g}{\partial H_g} < 0.$$

The derivative of the demand function with respect to H_s is given by

$$\begin{aligned}\frac{\partial \tilde{w}_g}{\partial H_s} &= \tilde{w}_g\frac{1}{H_s}+\frac{\gamma_2}{1-\gamma_2}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}H_s\frac{\partial}{\partial H_s}\frac{1}{1-\omega_K^{Y_m}} \\ &+\frac{\gamma_2}{1-\gamma_2}\frac{1}{1-\omega_K^{Y_m}}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}H_s\frac{\partial}{\partial H_s}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}} \\ &+\frac{\gamma_2}{1-\gamma_2}\frac{1}{1-\omega_K^{Y_m}}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}}H_s\frac{\partial}{\partial H_s}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}},\end{aligned}$$

where

$$\begin{aligned}\frac{\partial}{\partial H_s}\frac{1}{1-\omega_K^{Y_m}} &= -\frac{\varepsilon_3-1}{\varepsilon_3}\frac{\omega_K^{Y_m}}{1-\omega_K^{Y_m}}\frac{1}{H_s}, \\ \frac{\partial}{\partial H_s}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}} &= -\frac{1}{\varepsilon_2}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}}\frac{1}{u_h^g}\left(1-\omega_K^{Y_m}\right)\frac{Y_m}{H_s}\frac{\partial u_h^g}{\partial Y_m}, \\ \frac{\partial}{\partial H_s}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} &= -\frac{\varepsilon_2-1}{\varepsilon_2}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}\frac{1}{Y_m}\left(1-\omega_K^{Y_m}\right)\frac{Y_m}{H_s}.\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{\partial \tilde{w}_g}{\partial H_s} &= \tilde{w}_g\frac{1}{H_s}\left[1-\frac{\varepsilon_3-1}{\varepsilon_3}\omega_K^{Y_m}-\frac{1}{\varepsilon_2}\left(1-\omega_K^{Y_m}\right)\frac{Y_m}{u_h^g}\frac{\partial u_h^g}{\partial Y_m}-\frac{\varepsilon_2-1}{\varepsilon_2}\left(1-\omega_K^{Y_m}\right)\right] \\ &= \tilde{w}_g\frac{1}{H_s}\left\{1-\frac{\varepsilon_3-1}{\varepsilon_3}\omega_K^{Y_m}-\frac{\varepsilon_2-1}{\varepsilon_2}\left(1-\omega_K^{Y_m}\right)-\frac{1}{\varepsilon_2}\left(1-\omega_K^{Y_m}\right)\right. \\ &\left.\times\frac{(\varepsilon_1-\varepsilon_2)\omega_{Y_m}^{Y_h}(1-u_h^g)}{\varepsilon_2(1-u_h^g)+(\varepsilon_1-\varepsilon_2)\omega_{Y_m}^{Y_h}(1-u_h^g)+\varepsilon_2\left[\varepsilon_1-(\varepsilon_1-1)\gamma_4\right]u_h^g}\right\}.\end{aligned} \tag{31}$$

This expression declines as the term $(\varepsilon_3-1)/\varepsilon_3$ increases. Clearly, $(\varepsilon_3-1)/\varepsilon_3$ increases with ε_3 and tends to 1 as ε_3 tends to $+\infty$. Therefore,

$$\frac{\partial \tilde{w}_g}{\partial H_s} > \left.\frac{\partial \tilde{w}_g}{\partial H_s}\right|_{\varepsilon_3=+\infty}.$$

We take the maximum of ε_3 to obtain

$$\left.\frac{\partial \tilde{w}_g}{\partial H_s}\right|_{\varepsilon_3=+\infty} = \tilde{w}_g \frac{1}{H_s}\left(1-\omega_K^{Y_m}\right) \\ \times \frac{1}{\left(1-u_h^g\right)} \frac{1+\left(\varepsilon_1-1\right)\left(1-\gamma_4\right) u_h^g}{\varepsilon_2+\left(\varepsilon_1-\varepsilon_2\right) \omega_{Y_m}^{Y_h}+\varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right) \gamma_4\right] \frac{u_h^g}{1-u_h^g}}.$$

This is positive, which means that

$$\frac{\partial \tilde{w}_g}{\partial H_s}>0.$$

In Appendix - Numerical Exercises we confirm these results with numerical exercises where we use (16) instead of (17).

Proof of Proposition 4: We use μ_x and σ_x^2 to denote the mean and variance of a variable x. Since the shocks $\{\lambda\}$ are log-normal their means, variances, and coefficients of variation are given by

$$\begin{aligned} \mu_\lambda &= \exp\left(\mu_z+\frac{1}{2}\sigma_z^2\right), \\ \sigma_\lambda^2 &= \left(\exp\left(\sigma_z^2\right)-1\right)\mu_z^2, \\ \frac{\sigma_\lambda}{\mu_\lambda} &= \left(\exp\left(\sigma_z^2\right)-1\right)^{\frac{1}{2}}. \end{aligned}$$

To prove the proposition we rewrite (2) in the following manner

$$Y^{\frac{\varepsilon_1-1}{\varepsilon_1}} = \gamma_1\left(\frac{Y_h}{\lambda_h}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} \lambda_Y^{\frac{\varepsilon_1-1}{\varepsilon_1}} \lambda_h^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \left(1-\gamma_1\right)\left(\frac{Y_l}{\lambda_l}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} \lambda_Y^{\frac{\varepsilon_1-1}{\varepsilon_1}} \lambda_l^{\frac{\varepsilon_1-1}{\varepsilon_1}}.$$

This implies that the variance of $Y^{\frac{\varepsilon_1-1}{\varepsilon_1}}$ is given by

$$\begin{aligned} V\left[Y^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right] =& \left[\gamma_1\left(\frac{Y_h}{\lambda_h}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^2 \\ & \times\left(\mathbb{E}\left[\lambda_h^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^2 V\left[\lambda_Y^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]+V\left[\lambda_h^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right] \mathbb{E}\left[\lambda_Y^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^2+V\left[\lambda_h^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right] V\left[\lambda_Y^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\right) \\ & +\left[\left(1-\gamma_1\right)\left(\frac{Y_l}{\lambda_l}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^2 \\ & \times\left(\mathbb{E}\left[\lambda_l^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^2 V\left[\lambda_Y^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]+V\left[\lambda_l^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right] \mathbb{E}\left[\lambda_Y^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^2+V\left[\lambda_l^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right] V\left[\lambda_Y^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\right) \\ & +2\left[\gamma_1\left(\frac{Y_h}{\lambda_h}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\left[\left(1-\gamma_1\right)\left(\frac{Y_l}{\lambda_l}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right] \mathbb{E}\left[\lambda_h^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right] \mathbb{E}\left[\lambda_l^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right] V\left[\lambda_Y^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]. \end{aligned}$$

Further, we use the Delta Method to obtain the variance of $f(x)=x^{\frac{\varepsilon_1-1}{\varepsilon_1}}$. It is given by

$$V\left[x^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]=\left[\left(\frac{\varepsilon_1-1}{\varepsilon_1}\right) \mathbb{E}[x]^{\frac{-1}{\varepsilon_1}}\right]^2 V[x].$$

Therefore, the variance of final output can be rewritten as

$$
\begin{aligned}
V[Y] = {} & \left[\mathbb{E}[Y]^{\frac{1}{\varepsilon_1}}\right]^2 \left[\gamma_1 \left(\frac{Y_h}{\lambda_h}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^2 \left(\mathbb{E}\left[\lambda_Y^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^2 + V\left[\lambda_Y^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\right) \left[\mathbb{E}[\lambda_h]^{\frac{-1}{\varepsilon_1}}\right]^2 V[\lambda_h] \\
& + \left[\mathbb{E}[Y]^{\frac{1}{\varepsilon_1}}\right]^2 \left[(1-\gamma_1)\left(\frac{Y_l}{\lambda_l}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^2 \left(\mathbb{E}\left[\lambda_Y^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^2 + V\left[\lambda_Y^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\right) \left[\mathbb{E}[\lambda_l]^{\frac{-1}{\varepsilon_1}}\right]^2 V[\lambda_l] \\
& + \left[\mathbb{E}[Y]^{\frac{1}{\varepsilon_1}}\right]^2 \left[\left[\gamma_1\left(\frac{Y_h}{\lambda_h}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\mathbb{E}\left[\lambda_h^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right] + \left[(1-\gamma_1)\left(\frac{Y_l}{\lambda_l}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\mathbb{E}\left[\lambda_l^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\right]^2 \\
& \times \left[\mathbb{E}[\lambda_Y]^{\frac{-1}{\varepsilon_1}}\right]^2 V[\lambda_Y].
\end{aligned}
$$

We use Ψ_1 and Ψ_2 to denote

$$
\begin{aligned}
\Psi_1 &= \left[\mathbb{E}[Y]^{\frac{1}{\varepsilon_1}}\right]^2 \left(\mathbb{E}\left[\lambda_Y^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^2 + V\left[\lambda_Y^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\right), \\
\Psi_2 &= \left[\frac{\mathbb{E}[Y]}{\mathbb{E}[\lambda_Y]}\right]^{\frac{2}{\varepsilon_1}} \left[\left[\gamma_1\left(\frac{Y_h}{\lambda_h}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\mathbb{E}\left[\lambda_h^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right] + \left[(1-\gamma_1)\left(\frac{Y_l}{\lambda_l}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\mathbb{E}\left[\lambda_l^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\right]^2 V[\lambda_Y] \\
&= \left[\frac{\mathbb{E}[Y]}{\mathbb{E}[\lambda_Y]}\right]^{\frac{2}{\varepsilon_1}} \left[\gamma_1\mathbb{E}\left[Y_h^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right] + (1-\gamma_1)\,\mathbb{E}\left[Y_l^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\right]^2 V[\lambda_Y].
\end{aligned}
$$

Finally, we rewrite the variance of final output as

$$
\sigma_Y^2 = \Psi_1 \left\{\left[\gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^2 \left(\frac{\sigma_{\lambda_h}}{\mu_{\lambda_h}}\right)^2 + \left[(1-\gamma_1)\,\mu_{Y_l}^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^2 \left(\frac{\sigma_{\lambda_l}}{\mu_{\lambda_l}}\right)^2\right\} + \Psi_2. \tag{32}
$$

In order to prove the proposition we consider how μ_{Y_h} and μ_{Y_l} change under the variation in H_g or H_s which keeps μ_Y constant. It is clear that μ_{Y_h} and μ_{Y_l} move in opposite directions under such a variation.

The derivatives of μ_{Y_h} and μ_{Y_l} with respect to H_g and H_s are given by

$$
\frac{d\mu_{Y_h}}{dH_g} = \mu_{Y_h}\left(1-\omega_{Y_m}^{Y_h}\right)\left(\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_g} + \frac{1}{H_g}\right), \tag{33}
$$

$$
\frac{d\mu_{Y_h}}{dH_s} = \mu_{Y_h}\left[\left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_s} + \omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{1}{H_s}\right], \tag{34}
$$

and

$$
\frac{d\mu_{Y_l}}{dH_g} = \mu_{Y_l}\gamma_4\left(\frac{1}{H_g} - \frac{1}{1-u_l^g}\frac{\partial u_h^g}{\partial H_g}\right), \tag{35}
$$

$$
\frac{d\mu_{Y_l}}{dH_s} = -\mu_{Y_l}\gamma_4\frac{1}{1-u_l^g}\frac{\partial u_h^g}{\partial H_s}. \tag{36}
$$

The following two remarks follow from these derivatives without imposing the condition that expected output stays constant. The derivatives with respect to H_g imply that μ_{Y_l} and μ_{Y_h} increase with H_g unless the corner case $\varepsilon_2 = 0$ holds. When $\varepsilon_2 = 0$, μ_{Y_l} increases with H_g but μ_{Y_h} does not depend on it because H_s and H_g are complements.

Therefore, in case when $\varepsilon_2 = 0$ and $1 > \varepsilon_1$ ($\varepsilon_1 > 1$) the contribution of $\sigma^2_{z_l}$ (or $\sigma^2_{\lambda_l}$) to σ^2_Y relative to the contribution of $\sigma^2_{z_h}$ (or $\sigma^2_{\lambda_h}$) declines (increases) with H_g. The derivatives with respect to H_s imply that μ_{Y_h} increases with H_s but μ_{Y_l} declines with it when, for example, $\varepsilon_1 > \varepsilon_2$. Therefore, in case when $\varepsilon_1 > \varepsilon_2$ and $1 > \varepsilon_1$ ($\varepsilon_1 > 1$) the contribution of $\sigma^2_{z_l}$ (or $\sigma^2_{\lambda_l}$) to σ^2_Y relative to the contribution of $\sigma^2_{z_h}$ (or $\sigma^2_{\lambda_h}$) increases (declines) with H_s.

The partial derivatives of μ_Y with respect to μ_{Y_h} and μ_{Y_l} are given by

$$\frac{\partial \mu_Y}{\partial \mu_{Y_h}} = \frac{\mu_Y}{\mu_{Y_h}} \hat{\omega}^Y_{Y_h}, \tag{37}$$

$$\frac{\partial \mu_Y}{\partial \mu_{Y_l}} = \frac{\mu_Y}{\mu_{Y_l}} \left(1 - \hat{\omega}^Y_{Y_h}\right), \tag{38}$$

where we use $\hat{\omega}^Y_{Y_h}$ to denote

$$\hat{\omega}^Y_{Y_h} = \frac{\mathbb{E}\left[\omega^Y_{Y_h} Y\right]}{\mathbb{E}\left[Y\right]}. \tag{39}$$

In order the expected output to stay constant, we have to have that

$$\frac{\partial \mu_Y}{\partial H_g} dH_g + \frac{\partial \mu_Y}{\partial H_s} dH_s = 0. \tag{40}$$

Using the above expressions this condition can be rewritten as

$$\begin{aligned} 0 = &\left\{ \left[\hat{\omega}^Y_{Y_h}\left(1 - \omega^{Y_h}_{Y_m}\right) - \gamma_4\left(1 - \hat{\omega}^Y_{Y_h}\right)\frac{u^g_l}{1 - u^g_l}\right]\frac{H_g}{u^g_h}\frac{\partial u^g_h}{\partial H_g} \right.\\ &\left. + \left[\gamma_4\left(1 - \hat{\omega}^Y_{Y_h}\right) + \hat{\omega}^Y_{Y_h}\left(1 - \omega^{Y_h}_{Y_m}\right)\right]\right\}\frac{1}{H_g} dH_g \\ &+ \left\{ \left[\hat{\omega}^Y_{Y_h}\left(1 - \omega^{Y_h}_{Y_m}\right) - \left(1 - \hat{\omega}^Y_{Y_h}\right)\gamma_4 \frac{u^g_h}{1 - u^g_l}\right]\frac{H_s}{u^g_h}\frac{\partial u^g_h}{\partial H_s} \right.\\ &\left. + \hat{\omega}^Y_{Y_h}\omega^{Y_h}_{Y_m}\left(1 - \omega^{Y_m}_K\right)\right\}\frac{1}{H_s} dH_s. \end{aligned}$$

Further, we plug for $\partial u^g_h / \partial H_g$ and $\partial u^g_h / \partial Y_m$ from (25) and (26) to obtain

$$\begin{aligned} \frac{1}{H_g} dH_g = &-\frac{1}{H_s} dH_s \\ &\times \frac{\omega^{Y_h}_{Y_m}\left(1 - \omega^{Y_m}_K\right)\left\{\hat{\omega}^Y_{Y_h}\varepsilon_1 + \left[\hat{\omega}^Y_{Y_h}\varepsilon_1\left(\varepsilon_2 - 1\right)\left(1 - \gamma_4\right) - \left(\varepsilon_1 - \varepsilon_2\right)\gamma_4\right] u^g_h\right\}}{\hat{\omega}^Y_{Y_h}\left(1 - \omega^{Y_h}_{Y_m}\right)\varepsilon_2\varepsilon_1\left(1 - \gamma_4\right) + \gamma_4\left(1 - \hat{\omega}^Y_{Y_h}\right)\omega^{Y_h}_{Y_m}\varepsilon_1 + \gamma_4\left(1 - \omega^{Y_h}_{Y_m}\right)\varepsilon_2}. \end{aligned} \tag{41}$$

Given that all production functions are increasing in factor inputs we need to have that

$$\hat{\omega}^Y_{Y_h}\varepsilon_1 + \left[\hat{\omega}^Y_{Y_h}\varepsilon_1\left(\varepsilon_2 - 1\right)\left(1 - \gamma_4\right) - \left(\varepsilon_1 - \varepsilon_2\right)\gamma_4\right] u^g_h > 0, \tag{42}$$

so that dH_g and dH_s have different signs.

The total variation of the mean of Y_h is given by

$$\frac{1}{\mu_{Y_h}} d\mu_{Y_h} = \left(1 - \omega^{Y_h}_{Y_m}\right)\frac{1}{u^g_h} du^g_h + \left(1 - \omega^{Y_h}_{Y_m}\right)\frac{1}{H_g} dH_g + \omega^{Y_h}_{Y_m}\left(1 - \omega^{Y_m}_K\right)\frac{1}{H_s} dH_s. \tag{43}$$

Clearly, according to (40) this is positive (negative) if the total variation of μ_{Y_l},

$$\frac{1}{\mu_{Y_l}} d\mu_{Y_l} = \gamma_4 \left(\frac{1}{H_g} dH_g - \frac{1}{1-u_l^g} du_h^g \right), \tag{44}$$

is negative (positive).

Using (25)-(27), (41), (44) and the total variation of u_h^g,

$$du_h^g = \frac{\partial u_h^g}{\partial H_g} dH_g + \frac{\partial u_h^g}{\partial H_s} dH_s,$$

it can be shown that

$$\begin{aligned}
&\frac{1}{\gamma_4}\frac{1}{\mu_{Y_l}} d\mu_{Y_l} \frac{1}{\frac{1}{H_s} dH_s} \\
&= -\frac{1}{1-u_l^g} \frac{\varepsilon_2 + (\varepsilon_1 - \varepsilon_2)\,\omega_{Y_m}^{Y_h}}{\varepsilon_2 + (\varepsilon_1 - \varepsilon_2)\,\omega_{Y_m}^{Y_h} + \varepsilon_2 \left[\varepsilon_1 - (\varepsilon_1 - 1)\,\gamma_4\right] \frac{u_h^g}{1-u_h^g}} \\
&\times \frac{\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\left\{\hat{\omega}_{Y_h}^{Y}\varepsilon_1 + \left[\hat{\omega}_{Y_h}^{Y}\varepsilon_1 (\varepsilon_2 - 1)(1-\gamma_4) - (\varepsilon_1 - \varepsilon_2)\,\gamma_4\right] u_h^g\right\}}{\hat{\omega}_{Y_h}^{Y}\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_2\varepsilon_1 (1-\gamma_4) + \gamma_4\left(1-\hat{\omega}_{Y_h}^{Y}\right)\omega_{Y_m}^{Y_h}\varepsilon_1 + \gamma_4\varepsilon_2\left(1-\omega_{Y_m}^{Y_h}\right)} \\
&- \frac{u_h^g}{1-u_l^g} \frac{\left(1-\omega_K^{Y_m}\right)\omega_{Y_m}^{Y_h}(\varepsilon_1 - \varepsilon_2)}{\varepsilon_2 + (\varepsilon_1 - \varepsilon_2)\,\omega_{Y_m}^{Y_h} + \varepsilon_2 \left[\varepsilon_1 - (\varepsilon_1 - 1)\,\gamma_4\right] \frac{u_h^g}{1-u_h^g}}.
\end{aligned}$$

The sign of this expression is equivalent to the sign of the following sum

$$-\left\{\varepsilon_1 \omega_{Y_m}^{Y_h}\left(1-u_h^g\right) + \varepsilon_2\left[1 + u_h^g(\varepsilon_1 - 1)(1-\gamma_4) - \omega_{Y_m}^{Y_h}\left(1-u_h^g\right)\right]\right\}.$$

It turns out that the interior of the curly brackets is exactly the denominator in (25). Therefore,

$$\frac{d\mu_{Y_l}}{dH_s} < 0,$$

and higher H_s reduces μ_{Y_l} and increases μ_{Y_h}. This implies that when $1 > \varepsilon_1$ ($\varepsilon_1 > 1$) higher H_s reduces (increases) the contribution of $\sigma_{z_h}^2$ (or $\sigma_{\lambda_h}^2$) to σ_Y^2 and increases (reduces) the contribution of $\sigma_{z_l}^2$ (or $\sigma_{\lambda_l}^2$) to σ_Y^2.[24]

Consider two countries which produce the same expected output but have different amounts of H_s and H_g. This result means that if $1 > \varepsilon_1$ ($\varepsilon_1 > 1$) in the country where H_s is higher the volatility of final output because of shocks to h-sector is lower (higher). In turn, the volatility of final output because of shocks to l-sector is higher (lower).

This result also implies that in the country where H_s is higher the volatility of final output is higher, for example, if either $1 > \varepsilon_1$ and $\sigma_{\lambda_l}^2 > \sigma_{\lambda_h}^2 = \sigma_\lambda^2 = 0$ or $\varepsilon_1 > 1$ and $\sigma_{\lambda_h}^2 > \sigma_{\lambda_l}^2 = \sigma_\lambda^2 = 0$. It is lower, for example, if either $1 > \varepsilon_1$ and $\sigma_{\lambda_h}^2 > \sigma_{\lambda_l}^2 = \sigma_\lambda^2 = 0$ or $\varepsilon_1 > 1$ and $\sigma_{\lambda_l}^2 > \sigma_{\lambda_h}^2 = \sigma_\lambda^2 = 0$.[25]

[24] This result does not hold in the corner case when $\varepsilon_1 = 0$. In that case, $d\mu_{Y_l}/dH_s = 0$.

[25] These corner cases can be generalized, and the following is also true. In the country where H_s is higher the volatility of final output is higher, for example, if either $1 > \varepsilon_1$ and $\sigma_{\lambda_l}^2 >> \sigma_{\lambda_h}^2$ and $\sigma_{\lambda_l}^2 >> \sigma_\lambda^2$ or $\varepsilon_1 > 1$ and $\sigma_{\lambda_h}^2 >> \sigma_{\lambda_l}^2$ and $\sigma_{\lambda_h}^2 >> \sigma_\lambda^2$. It is lower, for example, if either $1 > \varepsilon_1$ and $\sigma_{\lambda_h}^2 >> \sigma_{\lambda_l}^2$ and $\sigma_{\lambda_h}^2 >> \sigma_\lambda^2$ or $\varepsilon_1 > 1$ and $\sigma_{\lambda_l}^2 >> \sigma_{\lambda_h}^2$ and $\sigma_{\lambda_l}^2 >> \sigma_\lambda^2$.

In these exercises we consider variations of H_g and H_s which keep $\mathbb{E}[Y]$ constant. According to the expression for Ψ_2, the condition that $\sigma_\lambda^2 = 0$ can be dropped if $\mathbb{E}\left[Y^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right]$ also stays constant or changes very marginally.

In Appendix - Numerical Exercises we confirm these results with numerical exercises where we use (16) instead of (17). Moreover, our numerical exercises suggest that $\mathbb{E}\left[Y^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right]$ changes very little with the variations of H_g and H_s which keep $\mathbb{E}[Y]$ constant.

Proof of Corollary 2: To prove the corollary, we denote

$$\tilde{\omega}_{Y_h}^{Y} = \frac{\gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1 - \gamma_1)\, \mu_{Y_l}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}. \tag{45}$$

For a real number x inequalities $\omega_{Y_h}^{Y} > x$ and $\omega_{Y_h}^{Y} < x$ imply $\tilde{\omega}_{Y_h}^{Y} > x$ and $\tilde{\omega}_{Y_h}^{Y} < x$, respectively. The multiplier of the coefficient of variation of λ_h is higher than the the multiplier of the coefficient of variation of λ_l if $\tilde{\omega}_{Y_h}^{Y} > 1/2$. Moreover, $\tilde{\omega}_{Y_h}^{Y}$ increases with $\gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}$ and declines with $(1 - \gamma_1)\, \mu_{Y_l}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}$.

Suppose now that $\sigma_\lambda^2 = 0$ and let $\varepsilon_1 > 1$ and

$$\sigma_{z_h}^2 > \sigma_{z_l}^2, \tag{46}$$

i.e., $\sigma_{\lambda_h}/\mu_{\lambda_h} > \sigma_{\lambda_l}/\mu_{\lambda_l}$. Further, suppose that the share of expected Y_h is higher than or equal to the share of expected Y_l:

$$\tilde{\omega}_{Y_h}^{Y} \geq \frac{1}{2}. \tag{47}$$

In such a case, the volatility of final output is higher in the country where H_s higher. The volatility of final output is also higher in case when $1 > \varepsilon_1$ and the inverses of (46) and (47) hold.

If

$$(1 - \gamma_1)\, \mu_{Y_l}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \geq \gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}},$$

and $1 > \varepsilon_1$ and $\sigma_{\lambda_l} > \sigma_{\lambda_h}$ (equivalently $\sigma_{z_l} > \sigma_{z_h}$) then, again, the volatility of final output is higher in the country where H_s higher.

In these exercises we consider variations of H_g and H_s which keep $\mathbb{E}[Y]$ constant. According to the expression for Ψ_2, the condition that $\sigma_\lambda^2 = 0$ can be dropped if $\mathbb{E}\left[Y^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right]$ also stays constant or changes very marginally.

In Appendix - Numerical Exercises we confirm these results with numerical exercises where we use (16) instead of (17). Moreover, our numerical exercises suggest that $\mathbb{E}\left[Y^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right]$ changes very little with the variations of H_g and H_s which keep $\mathbb{E}[Y]$ constant.

Proof of Propositions 5: First, we consider the case when $\sigma_{z_h}^2 > \sigma_{z_l}^2 = \sigma_{z_Y}^2 = 0$ so that $\sigma_{\lambda_h}^2 > \sigma_{\lambda_l}^2 = \sigma_{\lambda_Y}^2 = 0$. The standard deviation of final output in this case is given by

$$\sigma_Y = \mathbb{E}\left[\lambda_Y^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right] \gamma_1 \frac{\sigma_{\lambda_h}}{\mu_{\lambda_h}} \mu_Y^{\frac{1}{\varepsilon_1}} \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}.$$

Therefore, its partial derivatives with respect to H_s and H_g are given by

$$\frac{\partial \sigma_Y}{\partial H_g} = \sigma_Y \left[\left(1-\hat{\omega}^Y_{Y_h}\right) \frac{1}{\varepsilon_1} \frac{1}{\mu_{Y_l}} \left(\frac{\partial \mu_{Y_l}}{\partial H_g} - \frac{\mu_{Y_l}}{\mu_{Y_h}} \frac{\partial \mu_{Y_h}}{\partial H_g} \right) + \frac{1}{\mu_{Y_h}} \frac{\partial \mu_{Y_h}}{\partial H_g} \right],$$
$$\frac{\partial \sigma_Y}{\partial H_s} = \sigma_Y \left[\left(1-\hat{\omega}^Y_{Y_h}\right) \frac{1}{\varepsilon_1} \frac{1}{\mu_{Y_l}} \left(\frac{\partial \mu_{Y_l}}{\partial H_s} - \frac{\mu_{Y_l}}{\mu_{Y_h}} \frac{\partial \mu_{Y_h}}{\partial H_s} \right) + \frac{1}{\mu_{Y_h}} \frac{\partial \mu_{Y_h}}{\partial H_s} \right].$$

The ratio of the partial derivatives is given by

$$\frac{\frac{\partial \sigma_Y}{\partial H_g}}{\frac{\partial \sigma_Y}{\partial H_s}} = \frac{\left(1-\hat{\omega}^Y_{Y_h}\right) \frac{1}{\varepsilon_1} \frac{1}{\mu_{Y_l}} \left(\mu_{Y_h} \frac{\partial \mu_{Y_l}}{\partial H_g} - \mu_{Y_l} \frac{\partial \mu_{Y_h}}{\partial H_g} \right) + \frac{\partial \mu_{Y_h}}{\partial H_g}}{\left(1-\hat{\omega}^Y_{Y_h}\right) \frac{1}{\varepsilon_1} \frac{1}{\mu_{Y_l}} \left(\mu_{Y_h} \frac{\partial \mu_{Y_l}}{\partial H_s} - \mu_{Y_l} \frac{\partial \mu_{Y_h}}{\partial H_s} \right) + \frac{\partial \mu_{Y_h}}{\partial H_s}}, \tag{48}$$

Using (33)-(36), this ratio can be rewritten as

$$\begin{aligned}
\frac{\frac{\partial \sigma_Y}{\partial H_g}}{\frac{\partial \sigma_Y}{\partial H_s}} = & \left\{ \left(1-\hat{\omega}^Y_{Y_h}\right) \left[\gamma_4 \left(\frac{1}{H_g} - \frac{1}{1-u^g_h} \frac{\partial u^g_h}{\partial H_g} \right) - \left(1-\omega^{Y_h}_{Y_m}\right) \left(\frac{1}{H_g} + \frac{1}{u^g_h} \frac{\partial u^g_h}{\partial H_g} \right) \right] \right. \\
& \left. + \varepsilon_1 \left(1-\omega^{Y_h}_{Y_m}\right) \left(\frac{1}{H_g} + \frac{1}{u^g_h} \frac{\partial u^g_h}{\partial H_g} \right) \right\} \\
& \times \left(- \left(1-\hat{\omega}^Y_{Y_h}\right) \left\{ \gamma_4 \frac{1}{1-u^g_h} \frac{\partial u^g_h}{\partial H_s} + \left[\omega^{Y_h}_{Y_m} \left(1-\omega^{Y_m}_K\right) \frac{1}{H_s} + \left(1-\omega^{Y_h}_{Y_m}\right) \frac{1}{u^g_h} \frac{\partial u^g_h}{\partial H_s} \right] \right\} \right. \\
& \left. + \varepsilon_1 \left[\omega^{Y_h}_{Y_m} \left(1-\omega^{Y_m}_K\right) \frac{1}{H_s} + \left(1-\omega^{Y_h}_{Y_m}\right) \frac{1}{u^g_h} \frac{\partial u^g_h}{\partial H_s} \right] \right)^{-1},
\end{aligned}$$

where

$$\begin{aligned}
\frac{1}{H_g} + \frac{1}{u^g_h} \frac{\partial u^g_h}{\partial H_g} = & \frac{1}{H_g} \frac{\varepsilon_2 \left[(\varepsilon_1 - 1)(1-\gamma_4) + 1 \right]}{1-u^g_h} \\
& \times \frac{1}{\varepsilon_2 + (\varepsilon_1 - \varepsilon_2)\omega^{Y_h}_{Y_m} + \varepsilon_2 \left[\varepsilon_1 - (\varepsilon_1 - 1)\gamma_4 \right] \frac{u^g_h}{1-u^g_h}},
\end{aligned} \tag{49}$$

$$\begin{aligned}
\frac{1}{H_g} - \frac{1}{1-u^g_h} \frac{\partial u^g_h}{\partial H_g} = & \frac{1}{H_g} \frac{1}{1-u^g_h} \\
& \times \frac{\varepsilon_2 + (\varepsilon_1 - \varepsilon_2)\omega^{Y_h}_{Y_m}}{\varepsilon_2 + (\varepsilon_1 - \varepsilon_2)\omega^{Y_h}_{Y_m} + \varepsilon_2 \left[\varepsilon_1 - (\varepsilon_1 - 1)\gamma_4 \right] \frac{u^g_h}{1-u^g_h}},
\end{aligned} \tag{50}$$

$$\begin{aligned}
& \omega^{Y_h}_{Y_m} \left(1-\omega^{Y_m}_K\right) \frac{1}{H_s} + \left(1-\omega^{Y_h}_{Y_m}\right) \frac{1}{u^g_h} \frac{\partial u^g_h}{\partial H_s} \\
& = \frac{1}{H_s} \left(1-\omega^{Y_m}_K\right) \omega^{Y_h}_{Y_m} \frac{\varepsilon_1 + \varepsilon_2 \left[\varepsilon_1 - (\varepsilon_1 - 1)\gamma_4 \right] \frac{u^g_h}{1-u^g_h}}{\varepsilon_2 + (\varepsilon_1 - \varepsilon_2)\omega^{Y_h}_{Y_m} + \varepsilon_2 \left[\varepsilon_1 - (\varepsilon_1 - 1)\gamma_4 \right] \frac{u^g_h}{1-u^g_h}},
\end{aligned} \tag{51}$$

$$\begin{aligned}
& \gamma_4 \left(\frac{1}{H_g} - \frac{1}{1-u^g_h} \frac{\partial u^g_h}{\partial H_g} \right) - \left(1-\omega^{Y_h}_{Y_m}\right) \left(\frac{1}{H_g} + \frac{1}{u^g_h} \frac{\partial u^g_h}{\partial H_g} \right) \\
& = \frac{1}{H_g} \frac{1}{1-u^g_h} \frac{\left[\varepsilon_2 + (\varepsilon_1 - \varepsilon_2)\omega^{Y_h}_{Y_m} \right] \gamma_4 - \left(1-\omega^{Y_h}_{Y_m}\right) \varepsilon_2 \left[(\varepsilon_1 - 1)(1-\gamma_4) + 1 \right]}{\varepsilon_2 + (\varepsilon_1 - \varepsilon_2)\omega^{Y_h}_{Y_m} + \varepsilon_2 \left[\varepsilon_1 - (\varepsilon_1 - 1)\gamma_4 \right] \frac{u^g_h}{1-u^g_h}}.
\end{aligned} \tag{52}$$

Plugging these expressions back into the ratio of the partial derivatives of standard deviations gives the following expression.

$$\begin{aligned}
\frac{\frac{\partial\sigma_Y}{\partial H_g}}{\frac{\partial\sigma_Y}{\partial H_s}} &= \frac{H_s}{H_g}\frac{1}{\left(1-\omega_K^{Y_m}\right)\omega_{Y_m}^{Y_h}}\times \\
&\left(\left(1-\hat{\omega}_{Y_h}^{Y}\right)\left\{\gamma_4\left[\varepsilon_2+\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h}\right]-\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_2\left[\left(\varepsilon_1-1\right)\left(1-\gamma_4\right)+1\right]\right\}\right. \\
&\left.+\varepsilon_1\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_2\left[\left(\varepsilon_1-1\right)\left(1-\gamma_4\right)+1\right]\right) \\
&\times\left(-\left(1-\hat{\omega}_{Y_h}^{Y}\right)\left\{\varepsilon_1\left(1-u_h^g\right)+\gamma_4\left(\varepsilon_1-\varepsilon_2\right)u_h^g+\varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right)\gamma_4\right]u_h^g\right\}\right. \\
&\left.+\varepsilon_1\left\{\varepsilon_1\left(1-u_h^g\right)+\varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right)\gamma_4\right]u_h^g\right\}\right)^{-1}.
\end{aligned}$$

Let for example $\varepsilon_1=\varepsilon_2=\gamma_4=1$. In such a case, the ratio of the partial derivatives is given by

$$\frac{\frac{\partial\sigma_Y}{\partial H_g}}{\frac{\partial\sigma_Y}{\partial H_s}}=\frac{H_s}{H_g}\frac{1}{1-\omega_K^{Y_m}}\frac{1-\gamma_1\left(1-\gamma_2\right)}{\gamma_1\left(1-\gamma_2\right)}.$$

This can be greater or lower than 1 depending on parameter values, which implies that, in general, $\frac{\partial\sigma_Y}{\partial H_g}/\frac{\partial\sigma_Y}{\partial H_s}$ [as well as the ratio of elasticities $\left(\frac{\partial\sigma_Y}{\partial H_g}H_g\right)/\left(\frac{\partial\sigma_Y}{\partial H_s}H_s\right)$] can be greater or lower than 1.

In case when $\sigma_{z_l}^2>\sigma_{z_h}^2=\sigma_{z_Y}^2=0$ (so that $\sigma_{\lambda_l}^2>\sigma_{\lambda_h}^2=\sigma_{\lambda_Y}^2=0$), the standard deviation of final output is given by

$$\sigma_Y=\mathbb{E}\left[\lambda_Y^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\left(1-\gamma_1\right)\frac{\sigma_{\lambda_l}}{\mu_{\lambda_l}}\mu_Y^{\frac{1}{\varepsilon_1}}\mu_{Y_l}^{\frac{\varepsilon_1-1}{\varepsilon_1}}.$$

Therefore, the partial derivatives of the standard deviation of final output are given by

$$\begin{aligned}
\frac{\partial\sigma_Y}{\partial H_g}&=\sigma_Y\left[\frac{1}{\varepsilon_1}\hat{\omega}_{Y_h}^{Y}\frac{1}{\mu_{Y_h}}\left(\frac{\partial\mu_{Y_h}}{\partial H_g}-\frac{\mu_{Y_h}}{\mu_{Y_l}}\frac{\partial\mu_{Y_l}}{\partial H_g}\right)+\frac{1}{\mu_{Y_l}}\frac{\partial\mu_{Y_l}}{\partial H_g}\right], \\
\frac{\partial\sigma_Y}{\partial H_s}&=\sigma_Y\left[\frac{1}{\varepsilon_1}\hat{\omega}_{Y_h}^{Y}\frac{1}{\mu_{Y_h}}\left(\frac{\partial\mu_{Y_h}}{\partial H_s}-\frac{\mu_{Y_h}}{\mu_{Y_l}}\frac{\partial\mu_{Y_l}}{\partial H_s}\right)+\frac{1}{\mu_{Y_l}}\frac{\partial\mu_{Y_l}}{\partial H_s}\right].
\end{aligned}$$

The ratio of the partial derivatives is given by

$$\frac{\frac{\partial\sigma_Y}{\partial H_g}}{\frac{\partial\sigma_Y}{\partial H_s}}=\frac{\frac{1}{\varepsilon_1}\hat{\omega}_{Y_h}^{Y}\frac{1}{\mu_{Y_h}}\left(\mu_{Y_l}\frac{\partial\mu_{Y_h}}{\partial H_g}-\mu_{Y_h}\frac{\partial\mu_{Y_l}}{\partial H_g}\right)+\frac{\partial\mu_{Y_l}}{\partial H_g}}{\frac{1}{\varepsilon_1}\hat{\omega}_{Y_h}^{Y}\frac{1}{\mu_{Y_h}}\left(\mu_{Y_l}\frac{\partial\mu_{Y_h}}{\partial H_s}-\mu_{Y_h}\frac{\partial\mu_{Y_l}}{\partial H_s}\right)+\frac{\partial\mu_{Y_l}}{\partial H_s}}, \tag{53}$$

where the partial derivatives of μ_{Y_h} and μ_{Y_l} are given by (33)-(36). This implies that the above ratio can be rewritten as

$$\frac{\frac{\partial\sigma_Y}{\partial H_g}}{\frac{\partial\sigma_Y}{\partial H_s}}=\frac{\hat{\omega}_{Y_h}^{Y}\frac{1}{\varepsilon_1}\left(1-\omega_{Y_m}^{Y_h}\right)\left(\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_g}+\frac{1}{H_g}\right)+\left(1-\hat{\omega}_{Y_h}^{Y}\frac{1}{\varepsilon_1}\right)\gamma_4\left(\frac{1}{H_g}-\frac{1}{1-u_l^g}\frac{\partial u_h^g}{\partial H_g}\right)}{\hat{\omega}_{Y_h}^{Y}\frac{1}{\varepsilon_1}\left[\left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_s}+\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{1}{H_s}\right]-\left(1-\hat{\omega}_{Y_h}^{Y}\frac{1}{\varepsilon_1}\right)\gamma_4\frac{1}{1-u_l^g}\frac{\partial u_h^g}{\partial H_s}},$$

where the expressions in brackets are given by (49)-(51). Therefore, ratio of the partial derivatives can be further rewritten as

$$\frac{\frac{\partial\sigma_Y}{\partial H_g}}{\frac{\partial\sigma_Y}{\partial H_s}} = \frac{H_s}{H_g}\frac{1}{\left(1-\omega_K^{Y_m}\right)\omega_{Y_m}^{Y_h}}\left\{\hat{\omega}_{Y_h}^{Y}\frac{1}{\varepsilon_1}\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_2\left[\left(\varepsilon_1-1\right)\left(1-\gamma_4\right)+1\right]\right.$$
$$\left.+\left(1-\hat{\omega}_{Y_h}^{Y}\frac{1}{\varepsilon_1}\right)\gamma_4\left[\varepsilon_2+\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h}\right]\right\}$$
$$\times\left(\hat{\omega}_{Y_h}^{Y}\frac{1}{\varepsilon_1}\left\{\varepsilon_1\left(1-u_h^g\right)+\varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right)\gamma_4\right]u_h^g\right\}\right.$$
$$\left.-\left(1-\hat{\omega}_{Y_h}^{Y}\frac{1}{\varepsilon_1}\right)\gamma_4\left(\varepsilon_1-\varepsilon_2\right)u_h^g\right)^{-1}.$$

Let for example $\varepsilon_1=\varepsilon_2=\gamma_4=1$. In such a case, the above expression reduces, again, to

$$\frac{\frac{\partial\sigma_Y}{\partial H_g}}{\frac{\partial\sigma_Y}{\partial H_s}} = \frac{H_s}{H_g}\frac{1}{1-\omega_K^{Y_m}}\frac{1-\gamma_1\left(1-\gamma_2\right)}{\gamma_1\left(1-\gamma_2\right)}.$$

This [as well as the ratio of elasticities $\left(\frac{\partial\sigma_Y}{\partial H_g}H_g\right)/\left(\frac{\partial\sigma_Y}{\partial H_s}H_s\right)$] can be greater or lower than 1 depending on parameter values.

In case when $\sigma_z=\sigma_\lambda=0$, the variance of final output is given by

$$\sigma_Y^2=\left[\mathbb{E}\left[\lambda_Y^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\mu_Y^{\frac{1}{\varepsilon_1}}\gamma_1\mu_{Y_h}^{\frac{\varepsilon_1-1}{\varepsilon_1}}\frac{\sigma_{\lambda_h}}{\mu_{\lambda_h}}\right]^2+\left[\mathbb{E}\left[\lambda_Y^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\mu_Y^{\frac{1}{\varepsilon_1}}\left(1-\gamma_1\right)\mu_{Y_l}^{\frac{\varepsilon_1-1}{\varepsilon_1}}\frac{\sigma_{\lambda_l}}{\mu_{\lambda_l}}\right]^2.$$

We denote

$$x_1=\mathbb{E}\left[\lambda_Y^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\mu_Y^{\frac{1}{\varepsilon_1}}\gamma_1\mu_{Y_h}^{\frac{\varepsilon_1-1}{\varepsilon_1}}\frac{\sigma_{\lambda_h}}{\mu_{\lambda_h}},$$
$$x_2=\mathbb{E}\left[\lambda_Y^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\mu_Y^{\frac{1}{\varepsilon_1}}\left(1-\gamma_1\right)\mu_{Y_l}^{\frac{\varepsilon_1-1}{\varepsilon_1}}\frac{\sigma_{\lambda_l}}{\mu_{\lambda_l}}.$$

According to the previous results, the partial derivatives of x_1 and x_2 and σ_Y^2 are given by

$$\frac{\partial x_1}{\partial H_g}=x_1\left[\left(1-\hat{\omega}_{Y_h}^{Y}\right)\frac{1}{\varepsilon_1}\frac{1}{\mu_{Y_l}}\left(\frac{\partial\mu_{Y_l}}{\partial H_g}-\frac{\mu_{Y_l}}{\mu_{Y_h}}\frac{\partial\mu_{Y_h}}{\partial H_g}\right)+\frac{1}{\mu_{Y_h}}\frac{\partial\mu_{Y_h}}{\partial H_g}\right],$$
$$\frac{\partial x_1}{\partial H_s}=x_1\left[\left(1-\hat{\omega}_{Y_h}^{Y}\right)\frac{1}{\varepsilon_1}\frac{1}{\mu_{Y_l}}\left(\frac{\partial\mu_{Y_l}}{\partial H_s}-\frac{\mu_{Y_l}}{\mu_{Y_h}}\frac{\partial\mu_{Y_h}}{\partial H_s}\right)+\frac{1}{\mu_{Y_h}}\frac{\partial\mu_{Y_h}}{\partial H_s}\right],$$
$$\frac{\partial x_2}{\partial H_g}=x_2\left[\hat{\omega}_{Y_h}^{Y}\frac{1}{\varepsilon_1}\frac{1}{\mu_{Y_h}}\left(\frac{\partial\mu_{Y_h}}{\partial H_g}-\frac{\mu_{Y_h}}{\mu_{Y_l}}\frac{\partial\mu_{Y_l}}{\partial H_g}\right)+\frac{1}{\mu_{Y_l}}\frac{\partial\mu_{Y_l}}{\partial H_g}\right],$$
$$\frac{\partial x_2}{\partial H_s}=x_2\left[\hat{\omega}_{Y_h}^{Y}\frac{1}{\varepsilon_1}\frac{1}{\mu_{Y_h}}\left(\frac{\partial\mu_{Y_h}}{\partial H_s}-\frac{\mu_{Y_h}}{\mu_{Y_l}}\frac{\partial\mu_{Y_l}}{\partial H_s}\right)+\frac{1}{\mu_{Y_l}}\frac{\partial\mu_{Y_l}}{\partial H_s}\right].$$

and

$$\frac{\partial\sigma_Y^2}{\partial H_g}=2x_1^2\left[\left(1-\hat{\omega}_{Y_h}^{Y}\right)\frac{1}{\varepsilon_1}\frac{1}{\mu_{Y_l}}\left(\frac{\partial\mu_{Y_l}}{\partial H_g}-\frac{\mu_{Y_l}}{\mu_{Y_h}}\frac{\partial\mu_{Y_h}}{\partial H_g}\right)+\frac{1}{\mu_{Y_h}}\frac{\partial\mu_{Y_h}}{\partial H_g}\right]$$
$$+2x_2^2\left[\hat{\omega}_{Y_h}^{Y}\frac{1}{\varepsilon_1}\frac{1}{\mu_{Y_h}}\left(\frac{\partial\mu_{Y_h}}{\partial H_g}-\frac{\mu_{Y_h}}{\mu_{Y_l}}\frac{\partial\mu_{Y_l}}{\partial H_g}\right)+\frac{1}{\mu_{Y_l}}\frac{\partial\mu_{Y_l}}{\partial H_g}\right],$$
$$\frac{\partial\sigma_Y^2}{\partial H_s}=2x_1^2\left[\left(1-\hat{\omega}_{Y_h}^{Y}\right)\frac{1}{\varepsilon_1}\frac{1}{\mu_{Y_l}}\left(\frac{\partial\mu_{Y_l}}{\partial H_s}-\frac{\mu_{Y_l}}{\mu_{Y_h}}\frac{\partial\mu_{Y_h}}{\partial H_s}\right)+\frac{1}{\mu_{Y_h}}\frac{\partial\mu_{Y_h}}{\partial H_s}\right]$$
$$+2x_2^2\left[\hat{\omega}_{Y_h}^{Y}\frac{1}{\varepsilon_1}\frac{1}{\mu_{Y_h}}\left(\frac{\partial\mu_{Y_h}}{\partial H_s}-\frac{\mu_{Y_h}}{\mu_{Y_l}}\frac{\partial\mu_{Y_l}}{\partial H_s}\right)+\frac{1}{\mu_{Y_l}}\frac{\partial\mu_{Y_l}}{\partial H_s}\right].$$

The ratio of the partial derivatives then is given by

$$\frac{\frac{\partial\sigma_Y^2}{\partial H_g}}{\frac{\partial\sigma_Y^2}{\partial H_s}} = \left\{ \left(1-\hat{\omega}_{Y_h}^Y\right) \frac{1}{\varepsilon_1} \left(\frac{1}{\mu_{Y_l}} \frac{\partial\mu_{Y_l}}{\partial H_g} - \frac{1}{\mu_{Y_h}} \frac{\partial\mu_{Y_h}}{\partial H_g} \right) + \frac{1}{\mu_{Y_h}} \frac{\partial\mu_{Y_h}}{\partial H_g} \right.$$
$$\left. + \left(\frac{x_2}{x_1}\right)^2 \left[\hat{\omega}_{Y_h}^Y \frac{1}{\varepsilon_1} \left(\frac{1}{\mu_{Y_h}} \frac{\partial\mu_{Y_h}}{\partial H_g} - \frac{1}{\mu_{Y_l}} \frac{\partial\mu_{Y_l}}{\partial H_g} \right) + \frac{1}{\mu_{Y_l}} \frac{\partial\mu_{Y_l}}{\partial H_g} \right] \right\}$$
$$\times \left\{ \left(1-\hat{\omega}_{Y_h}^Y\right) \frac{1}{\varepsilon_1} \left(\frac{1}{\mu_{Y_l}} \frac{\partial\mu_{Y_l}}{\partial H_s} - \frac{1}{\mu_{Y_h}} \frac{\partial\mu_{Y_h}}{\partial H_s} \right) + \frac{1}{\mu_{Y_h}} \frac{\partial\mu_{Y_h}}{\partial H_s} \right.$$
$$\left. + \left(\frac{x_2}{x_1}\right)^2 \left[\hat{\omega}_{Y_h}^Y \frac{1}{\varepsilon_1} \left(\frac{1}{\mu_{Y_h}} \frac{\partial\mu_{Y_h}}{\partial H_s} - \frac{1}{\mu_{Y_l}} \frac{\partial\mu_{Y_l}}{\partial H_s} \right) + \frac{1}{\mu_{Y_l}} \frac{\partial\mu_{Y_l}}{\partial H_s} \right] \right\}^{-1},$$

where

$$\left(\frac{x_2}{x_1}\right)^2 = \left[\frac{1-\gamma_1}{\gamma_1} \left(\frac{\mu_{Y_l}}{\mu_{Y_h}}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} \frac{\mu_{\lambda_h}}{\mu_{\lambda_l}} \frac{\sigma_{\lambda_l}}{\sigma_{\lambda_h}} \right]^2.$$

Let for example $\varepsilon_1 = \varepsilon_2 = \gamma_4 = 1$. In such a case, the above expression reduces, again, to

$$\frac{\frac{\partial\sigma_Y}{\partial H_g}}{\frac{\partial\sigma_Y}{\partial H_s}} = \frac{H_s}{H_g} \frac{1}{1-\omega_K^{Y_m}} \frac{1-\gamma_1(1-\gamma_2)}{\gamma_1(1-\gamma_2)}.$$

This [as well as the ratio of elasticities $\left(\frac{\partial\sigma_Y}{\partial H_g}H_g\right) / \left(\frac{\partial\sigma_Y}{\partial H_s}H_s\right)$] can be greater or lower than 1 depending on parameter values. In general, the magnitude of $\frac{\partial\sigma_Y^2}{\partial H_g} / \frac{\partial\sigma_Y^2}{\partial H_s}$ depends on parameter values since so does the magnitude of this ratio when either of σ_{λ_h} and σ_{λ_l} is zero (i.e., either of σ_{z_h} and σ_{z_l} is zero).

In Appendix - Numerical Exercises we confirm the results from Proposition 5 with numerical exercises where we use (16) instead of (17).

Proof of Propositions 6: In case the marginal products of H_s and H_g are equal then the following condition needs to hold

$$\frac{1}{\mu_Y}\frac{\partial\mu_Y}{\partial H_s} = \frac{1}{\mu_Y}\frac{\partial\mu_Y}{\partial H_g}, \tag{54}$$

where

$$\frac{1}{\mu_Y}\frac{\partial\mu_Y}{\partial H_g} = \hat{\omega}_{Y_h}^Y \left(1-\omega_{Y_m}^{Y_h}\right) \left(\frac{1}{H_g} + \frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_g}\right) + \left(1-\hat{\omega}_{Y_h}^Y\right)\gamma_4 \left(\frac{1}{H_g} - \frac{1}{1-u_h^g}\frac{\partial u_h^g}{\partial H_g}\right),$$
$$\frac{1}{\mu_Y}\frac{\partial\mu_Y}{\partial H_s} = \hat{\omega}_{Y_h}^Y \left[\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{1}{H_s} + \left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_s}\right] - \left(1-\hat{\omega}_{Y_h}^Y\right)\gamma_4\frac{1}{1-u_h^g}\frac{\partial u_h^g}{\partial H_s}.$$

These partial derivatives can be rewritten as

$$\frac{1}{\mu_Y}\frac{\partial\mu_Y}{\partial H_g} = \left(1-\hat{\omega}^Y_{Y_h}\right)\left[\gamma_4\left(\frac{1}{H_g}-\frac{1}{1-u^g_h}\frac{\partial u^g_h}{\partial H_g}\right)-\left(1-\omega^{Y_h}_{Y_m}\right)\left(\frac{1}{H_g}+\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial H_g}\right)\right] \quad (55)$$
$$+\left(1-\omega^{Y_h}_{Y_m}\right)\left(\frac{1}{H_g}+\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial H_g}\right),$$

$$\frac{1}{\mu_Y}\frac{\partial\mu_Y}{\partial H_s} = -\left(1-\hat{\omega}^Y_{Y_h}\right)\left\{\gamma_4\frac{1}{1-u^g_h}\frac{\partial u^g_h}{\partial H_s}\right. \quad (56)$$
$$\left.+\left[\omega^{Y_h}_{Y_m}\left(1-\omega^{Y_m}_K\right)\frac{1}{H_s}+\left(1-\omega^{Y_h}_{Y_m}\right)\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial H_s}\right]\right\}$$
$$+\left[\omega^{Y_h}_{Y_m}\left(1-\omega^{Y_m}_K\right)\frac{1}{H_s}+\left(1-\omega^{Y_h}_{Y_m}\right)\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial H_s}\right].$$

In case when $\sigma^2_{z_h} > \sigma^2_{z_l} = \sigma^2_z = 0$ (i.e., $\sigma^2_{\lambda_h} > \sigma^2_{\lambda_l} = \sigma^2_\lambda = 0$), the ratio of partial derivatives of standard deviations is given by (48),

$$\frac{\frac{\partial\sigma_Y}{\partial H_g}}{\frac{\partial\sigma_Y}{\partial H_s}} = \frac{\left(1-\hat{\omega}^Y_{Y_h}\right)\frac{1}{\varepsilon_1}\frac{1}{\mu_{Y_l}}\left(\mu_{Y_h}\frac{\partial\mu_{Y_l}}{\partial H_g}-\mu_{Y_l}\frac{\partial\mu_{Y_h}}{\partial H_g}\right)+\frac{\partial\mu_{Y_h}}{\partial H_g}}{\left(1-\hat{\omega}^Y_{Y_h}\right)\frac{1}{\varepsilon_1}\frac{1}{\mu_{Y_l}}\left(\mu_{Y_h}\frac{\partial\mu_{Y_l}}{\partial H_s}-\mu_{Y_l}\frac{\partial\mu_{Y_h}}{\partial H_s}\right)+\frac{\partial\mu_{Y_h}}{\partial H_s}},$$

where the partial derivatives of μ_{Y_h} and μ_{Y_l} are given by (33)-(36). It has been shown previously that

$$\frac{\frac{\partial\sigma_Y}{\partial H_g}}{\frac{\partial\sigma_Y}{\partial H_s}} = \left\{\left(1-\hat{\omega}^Y_{Y_h}\right)\left[\gamma_4\left(\frac{1}{H_g}-\frac{1}{1-u^g_h}\frac{\partial u^g_h}{\partial H_g}\right)-\left(1-\omega^{Y_h}_{Y_m}\right)\left(\frac{1}{H_g}+\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial H_g}\right)\right]\right.$$
$$\left.+\varepsilon_1\left(1-\omega^{Y_h}_{Y_m}\right)\left(\frac{1}{H_g}+\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial H_g}\right)\right\}$$
$$\times\left(-\left(1-\hat{\omega}^Y_{Y_h}\right)\left\{\gamma_4\frac{1}{1-u^g_h}\frac{\partial u^g_h}{\partial H_s}+\left[\omega^{Y_h}_{Y_m}\left(1-\omega^{Y_m}_K\right)\frac{1}{H_s}+\left(1-\omega^{Y_h}_{Y_m}\right)\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial H_s}\right]\right\}\right.$$
$$\left.+\varepsilon_1\left[\omega^{Y_h}_{Y_m}\left(1-\omega^{Y_m}_K\right)\frac{1}{H_s}+\left(1-\omega^{Y_h}_{Y_m}\right)\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial H_s}\right]\right)^{-1}.$$

Therefore, according to (54) if $\varepsilon_1 = 1$ the partial derivatives of the volatilities are equal, $\frac{\partial\sigma_Y}{\partial H_g} = \frac{\partial\sigma_Y}{\partial H_s}$.

The ratio of the partial derivatives has the following form

$$X = \frac{x_1+\alpha x_2}{x_3+\alpha x_4},$$

where we have replaced ε_1 with α. The derivative of this ratio with respect to α is

$$\frac{\partial X}{\partial\alpha} = \frac{x_2x_3-x_1x_4}{\left(x_3+\alpha x_4\right)^2}.$$

We denote

$$d = x_2x_3-x_1x_4.$$

For the ratio of the partial derivatives of the volatility, d is given by

$$
\begin{aligned}
d = & -\left(1-\omega_{Y_m}^{Y_h}\right)\left(\frac{1}{H_g}+\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_g}\right)\left(1-\hat{\omega}_{Y_h}^{Y}\right) \\
& \times\left\{\gamma_4\frac{1}{1-u_h^g}\frac{\partial u_h^g}{\partial H_s}+\left[\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{1}{H_s}+\left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_s}\right]\right\} \\
& -\left(1-\hat{\omega}_{Y_h}^{Y}\right)\left[\gamma_4\left(\frac{1}{H_g}-\frac{1}{1-u_h^g}\frac{\partial u_h^g}{\partial H_g}\right)-\left(1-\omega_{Y_m}^{Y_h}\right)\left(\frac{1}{H_g}+\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_g}\right)\right] \\
& \times\left[\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{1}{H_s}+\left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_s}\right].
\end{aligned}
$$

If d is negative then for $\varepsilon_1 > 1$ we have that $1 > \frac{\partial \sigma_Y^2}{\partial H_g}/\frac{\partial \sigma_Y^2}{\partial H_s}$ and for $1 > \varepsilon_1$ we have that $\frac{\partial \sigma_Y^2}{\partial H_g}/\frac{\partial \sigma_Y^2}{\partial H_s} > 1$.

We denote

$$
\begin{aligned}
\tilde{d} = & \left(\frac{1}{H_g}-\frac{1}{1-u_h^g}\frac{\partial u_h^g}{\partial H_g}\right)\left[\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{1}{H_s}+\left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_s}\right] \\
& +\frac{1}{1-u_h^g}\left(1-\omega_{Y_m}^{Y_h}\right)\left(\frac{1}{H_g}+\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_g}\right)\frac{\partial u_h^g}{\partial H_s}.
\end{aligned}
$$

It is clear that $d < 0$ when $\tilde{d} > 0$. To check the sign of $\tilde{d}$ we plug the partial derivatives of u_h^g into $\tilde{d}$ and multiply $\tilde{d}$ by $H_s H_g$ and divide it to $\left(1-\omega_K^{Y_m}\right)$, $\omega_{Y_m}^{Y_h}$, and

$$
\frac{1}{1-u_h^g}\frac{1}{\left\{\varepsilon_2+\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h}+\varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}\right\}^2}
$$

to obtain the following expression

$$
\begin{aligned}
\hat{d} = & \left[\varepsilon_2+\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h}\right]\left\{\varepsilon_1+\varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}\right\} \\
& +\frac{u_h^g}{1-u_h^g}\left(\varepsilon_1-\varepsilon_2\right)\left(1-\omega_{Y_m}^{Y_h}\right)\left[\varepsilon_2+\varepsilon_2\left(\varepsilon_1-1\right)\left(1-\gamma_4\right)\right].
\end{aligned}
$$

The right-hand side of this expression apparently has the same sign as $\tilde{d}$. Moreover, clearly it is positive since it is equal to

$$
\varepsilon_1\left[\varepsilon_1\omega_{Y_m}^{Y_h}+\varepsilon_2\left(1-\omega_{Y_m}^{Y_h}\right)\right]+\frac{u_h^g}{1-u_h^g}\varepsilon_1\varepsilon_2\left[1+\left(\varepsilon_1-1\right)\left(1-\gamma_4\right)\right] > 0.
$$

Therefore, if $\varepsilon_1 > 1$ the ratio $\frac{\partial \sigma_Y}{\partial H_g}/\frac{\partial \sigma_Y}{\partial H_s}$ is less than one and if $1 > \varepsilon_1$ the ratio $\frac{\partial \sigma_Y}{\partial H_g}/\frac{\partial \sigma_Y}{\partial H_s}$ is greater than one.

In case when $\sigma_{z_l}^2 > \sigma_{z_h}^2 = \sigma_z^2 = 0$ (i.e., $\sigma_{\lambda_l}^2 > \sigma_{\lambda_h}^2 = \sigma_\lambda^2 = 0$), the ratio of partial derivatives of standard deviations is given by (53),

$$
\frac{\frac{\partial \sigma_Y}{\partial H_g}}{\frac{\partial \sigma_Y}{\partial H_s}} = \frac{\frac{1}{\varepsilon_1}\hat{\omega}_{Y_h}^{Y}\frac{1}{\mu_{Y_h}}\left(\mu_{Y_l}\frac{\partial \mu_{Y_h}}{\partial H_g}-\mu_{Y_h}\frac{\partial \mu_{Y_l}}{\partial H_g}\right)+\frac{\partial \mu_{Y_l}}{\partial H_g}}{\frac{1}{\varepsilon_1}\hat{\omega}_{Y_h}^{Y}\frac{1}{\mu_{Y_h}}\left(\mu_{Y_l}\frac{\partial \mu_{Y_h}}{\partial H_s}-\mu_{Y_h}\frac{\partial \mu_{Y_l}}{\partial H_s}\right)+\frac{\partial \mu_{Y_l}}{\partial H_s}},
$$

where the partial derivatives of μ_{Y_h} and μ_{Y_l} are given by (33)-(36). It can be shown that

$$\begin{aligned}\frac{\frac{\partial\sigma_Y}{\partial H_g}}{\frac{\partial\sigma_Y}{\partial H_s}} = &\left\{-\hat{\omega}^Y_{Y_h}\left[\gamma_4\left(\frac{1}{H_g}-\frac{1}{1-u^g_l}\frac{\partial u^g_h}{\partial H_g}\right)-\left(1-\omega^{Y_h}_{Y_m}\right)\left(\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial H_g}+\frac{1}{H_g}\right)\right]\right.\\
&\left.+\varepsilon_1\gamma_4\left(\frac{1}{H_g}-\frac{1}{1-u^g_l}\frac{\partial u^g_h}{\partial H_g}\right)\right\}\\
&\times\left\{\hat{\omega}^Y_{Y_h}\left[\left(1-\omega^{Y_h}_{Y_m}\right)\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial H_s}+\omega^{Y_h}_{Y_m}\left(1-\omega^{Y_m}_K\right)\frac{1}{H_s}+\gamma_4\frac{1}{1-u^g_l}\frac{\partial u^g_h}{\partial H_s}\right]\right.\\
&\left.+\varepsilon_1\left(-\gamma_4\frac{1}{1-u^g_l}\frac{\partial u^g_h}{\partial H_s}\right)\right\}^{-1}.\end{aligned}$$

Clearly, according to (54), the partial derivatives of the volatilities are equal, $\frac{\partial\sigma_Y}{\partial H_g}=\frac{\partial\sigma_Y}{\partial H_s}$, if $\varepsilon_1=1$.

Similar to the previous case, we compute d for the ratio of partial derivatives of volatilities:

$$\begin{aligned}d = &\gamma_4\left(\frac{1}{H_g}-\frac{1}{1-u^g_l}\frac{\partial u^g_h}{\partial H_g}\right)\\
&\times\hat{\omega}^Y_{Y_h}\left[\left(1-\omega^{Y_h}_{Y_m}\right)\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial H_s}+\omega^{Y_h}_{Y_m}\left(1-\omega^{Y_m}_K\right)\frac{1}{H_s}+\gamma_4\frac{1}{1-u^g_l}\frac{\partial u^g_h}{\partial H_s}\right]\\
&+\hat{\omega}^Y_{Y_h}\left[\left(1-\omega^{Y_h}_{Y_m}\right)\left(\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial H_g}+\frac{1}{H_g}\right)-\gamma_4\left(\frac{1}{H_g}-\frac{1}{1-u^g_l}\frac{\partial u^g_h}{\partial H_g}\right)\right]\\
&\times\gamma_4\frac{1}{1-u^g_l}\frac{\partial u^g_h}{\partial H_s}.\end{aligned}$$

It can be shown that

$$\frac{1}{\gamma_4\hat{\omega}^Y_{Y_h}}d=\frac{1}{H_g}\left(1-\omega^{Y_h}_{Y_m}\right)\frac{1}{1-u^g_l}\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial H_s}+\omega^{Y_h}_{Y_m}\left(1-\omega^{Y_m}_K\right)\frac{1}{H_s}\left(\frac{1}{H_g}-\frac{1}{1-u^g_l}\frac{\partial u^g_h}{\partial H_g}\right).$$

We use the expressions for the derivatives of u^g_h to get

$$d=\frac{\gamma_4\varepsilon_1\frac{1}{H_gH_s}\frac{\hat{\omega}^Y_{Y_h}\omega^{Y_h}_{Y_m}\left(1-\omega^{Y_m}_K\right)}{1-u^g_l}}{\varepsilon_2+\left(\varepsilon_1-\varepsilon_2\right)\omega^{Y_h}_{Y_m}+\varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right)\gamma_4\right]\frac{u^g_h}{1-u^g_h}},$$

which implies that $d>0$. Therefore, when $\varepsilon_1>1$ the ratio $\frac{\partial\sigma_Y}{\partial H_g}/\frac{\partial\sigma_Y}{\partial H_s}$ is greater than 1 and if $1>\varepsilon_1$ the ratio $\frac{\partial\sigma_Y}{\partial H_g}/\frac{\partial\sigma_Y}{\partial H_s}$ is less than 1.

In Appendix - Numerical Exercises we confirm the results from Proposition 6 with numerical exercises where we use (16) instead of (17).

Proof of Proposition 7: We ignore that λ, λ_h, and λ_l are stochastic and drop expectation operators everywhere. The total changes of ω^g_h and ω_h are given by

$$\begin{aligned}\frac{1}{\omega^g_h\left(1-\omega^g_h\right)}d\omega^g_h&=\frac{1}{H_g}dH_g-\frac{1}{H_s}dH_s+\frac{1}{u^g_h}du^g_h,\\
\frac{1}{\omega_l}d\omega_l&=\frac{H_s}{H_g+H_s}\left(\frac{1}{H_g}dH_g-\frac{1}{H_s}dH_s\right)-\frac{1}{1-u^g_h}du^g_h.\end{aligned}$$

Given that supply fixes the ratio of wages the total variation of wages should satisfy:

$$0=\frac{\partial\frac{w_g}{w_s}}{\partial K}dK+\left(\frac{\partial u_h^g}{\partial\lambda_{Y_l}}d\lambda_{Y_l}+\frac{\partial u_h^g}{\partial\lambda_{Y_h}}d\lambda_{Y_h}\right)\frac{\partial\frac{w_g}{w_s}}{\partial u_h^g}+\frac{\partial\frac{w_g}{w_s}}{\partial H_g}dH_g+\frac{\partial\frac{w_g}{w_s}}{\partial H_s}dH_s. \tag{57}$$

The partial derivatives of relative wages are given by (29), (30), and (31).

From the demand and supply of specific human capital it also follows that

$$\lambda_H=\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{\omega_{Y_h}^{Y}\lambda_Y\left[\gamma_1Y_h^{\frac{\varepsilon_1-1}{\varepsilon_1}}+(1-\gamma_1)Y_l^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^{\frac{\varepsilon_1}{\varepsilon_1-1}}}{H_s}.$$

Therefore,

$$\begin{aligned}0=&\lambda_H\frac{1}{\omega_{Y_m}^{Y_h}}d\omega_{Y_m}^{Y_h}-\lambda_H\frac{1}{1-\omega_K^{Y_m}}d\omega_K^{Y_m}+\lambda_H\frac{1}{\omega_{Y_h}^{Y}}d\omega_{Y_h}^{Y}-\lambda_H\frac{1}{H_s}dH_s\\&+\lambda_H\omega_{Y_h}^{Y}\frac{1}{Y_h}dY_h+\lambda_H\left(1-\omega_{Y_h}^{Y}\right)\frac{1}{Y_l}dY_l.\end{aligned}$$

From (44) and (43) it follows that this expression can be rewritten in the following manner:

$$\begin{aligned}0=&\lambda_H\frac{1}{\omega_{Y_m}^{Y_h}}d\omega_{Y_m}^{Y_h}-\lambda_H\frac{1}{1-\omega_K^{Y_m}}d\omega_K^{Y_m}+\lambda_H\frac{1}{\omega_{Y_h}^{Y}}d\omega_{Y_h}^{Y}-\lambda_H\frac{1}{H_s}dH_s\\&+\lambda_H\omega_{Y_h}^{Y}\left[\left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{u_h^g}du_h^g+\left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{H_g}dH_g+\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{1}{H_s}dH_s\right.\\&\left.+\omega_{Y_m}^{Y_h}\omega_K^{Y_m}\frac{1}{K}dK\right]+\lambda_H\left(1-\omega_{Y_h}^{Y}\right)\gamma_4\left(-\frac{1}{1-u_h^g}du_h^g+\frac{1}{H_g}dH_g\right).\end{aligned} \tag{58}$$

Since only K changes, we.use (26), (5), (11), and (12) to evaluate the partial derivatives of the shares:

$$\begin{aligned}\frac{\partial\omega_{Y_h}^{Y}}{\partial K}&=\frac{\varepsilon_1-1}{\varepsilon_1}\omega_{Y_h}^{Y}\left(1-\omega_{Y_h}^{Y}\right)\left[\omega_{Y_m}^{Y_h}\omega_K^{Y_m}\frac{1}{K}+\left(1-\omega_{Y_m}^{Y_h}+\gamma_4\frac{u_h^g}{1-u_h^g}\right)\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial K}\right],\\\frac{\partial\omega_{Y_m}^{Y_h}}{\partial K}&=\frac{\varepsilon_2-1}{\varepsilon_2}\omega_{Y_m}^{Y_h}\left(1-\omega_{Y_m}^{Y_h}\right)\left(\omega_K^{Y_m}\frac{1}{K}-\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial K}\right),\\\frac{\partial\omega_K^{Y_m}}{\partial K}&=\frac{\varepsilon_3-1}{\varepsilon_3}\omega_K^{Y_m}\left(1-\omega_K^{Y_m}\right)\frac{1}{K}.\end{aligned}$$

The expression for the total variation in the demand for specific human capital (58) can be rewritten as:

$$\begin{aligned}0=&\frac{K}{\omega_{Y_m}^{Y_h}}\frac{\partial\omega_{Y_m}^{Y_h}}{\partial K}-\frac{\omega_K^{Y_m}}{1-\omega_K^{Y_m}}\frac{K}{\omega_K^{Y_m}}\frac{\partial\omega_K^{Y_m}}{\partial K}+\frac{K}{\omega_{Y_h}^{Y}}\frac{\partial\omega_{Y_h}^{Y}}{\partial K}-\frac{K}{H_s}\frac{dH_s}{dK}\\&+\omega_{Y_h}^{Y}\left[\left(1-\omega_{Y_m}^{Y_h}\right)\frac{K}{u_h^g}\frac{\partial u_h^g}{\partial K}+\left(1-\omega_{Y_m}^{Y_h}\right)\frac{K}{H_g}\frac{dH_g}{dK}+\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{K}{H_s}\frac{dH_s}{dK}\right]\\&+\omega_{Y_m}^{Y_h}\omega_K^{Y_m}\omega_{Y_h}^{Y}+\left(1-\omega_{Y_h}^{Y}\right)\gamma_4\left(-\frac{u_h^g}{1-u_h^g}\frac{K}{u_h^g}\frac{\partial u_h^g}{\partial K}+\frac{K}{H_g}\frac{dH_g}{dK}\right).\end{aligned}$$

The sum of the first three terms is given by

$$X_3 = \left[\frac{\varepsilon_2 - 1}{\varepsilon_2}\left(1 - \omega_{Y_m}^{Y_h}\right) - \frac{\varepsilon_3 - 1}{\varepsilon_3} + \frac{\varepsilon_1 - 1}{\varepsilon_1}\left(1 - \omega_{Y_h}^{Y}\right)\omega_{Y_m}^{Y_h}\right]\omega_K^{Y_m}$$
$$+ \left[\frac{\varepsilon_1 - 1}{\varepsilon_1}\left(1 - \omega_{Y_h}^{Y}\right)\left(1 - \omega_{Y_m}^{Y_h} + \gamma_4 \frac{u_h^g}{1 - u_h^g}\right) - \frac{\varepsilon_2 - 1}{\varepsilon_2}\left(1 - \omega_{Y_m}^{Y_h}\right)\right]\frac{K}{u_h^g}\frac{\partial u_h^g}{\partial K}.$$

This implies that when $\varepsilon_2 = \gamma_4 = 1$ and $\varepsilon_3 = +\infty$ the total variation can be rewritten as:

$$0 = \left[\frac{\varepsilon_1 - 1}{\varepsilon_1}\left(1 - \omega_{Y_h}^{Y}\right)\omega_{Y_m}^{Y_h} + \omega_{Y_h}^{Y}\omega_{Y_m}^{Y_h} - 1\right]\omega_K^{Y_m} + \frac{K}{u_h^g}\frac{\partial u_h^g}{\partial K}$$
$$\times \left[\frac{\varepsilon_1 - 1}{\varepsilon_1}\left(1 - \omega_{Y_h}^{Y}\right)\left(1 - \omega_{Y_m}^{Y_h} + \frac{u_h^g}{1 - u_h^g}\right) + \omega_{Y_h}^{Y}\left(1 - \omega_{Y_m}^{Y_h}\right) - \left(1 - \omega_{Y_h}^{Y}\right)\frac{u_h^g}{1 - u_h^g}\right]$$
$$- \left[1 - \omega_{Y_h}^{Y}\omega_{Y_m}^{Y_h}\left(1 - \omega_K^{Y_m}\right)\right]\frac{K}{H_s}\frac{dH_s}{dK} + \left(1 - \omega_{Y_h}^{Y}\omega_{Y_m}^{Y_h}\right)\frac{K}{H_g}\frac{dH_g}{dK}.$$

On the other hand, from (18) and (20) it follows that the total variation of wages is zero. This, together with (29), (30), (31), and (57) implies that when $\varepsilon_2 = \gamma_4 = 1$ and $\varepsilon_3 = +\infty$ the following holds:

$$0 = \omega_K^{Y_m} - \frac{K}{H_g}\frac{dH_g}{dK} + \left(1 - \omega_K^{Y_m}\right)\frac{K}{H_s}\frac{dH_s}{dK}.$$

We solve for the percentage change of H_s using this expression and the one above:

$$\frac{K}{H_s}\frac{dH_s}{dK} = \left(\varepsilon_1 - 1\right)\omega_{Y_m}^{Y_h}\frac{1 - \omega_{Y_h}^{Y}\omega_{Y_m}^{Y_h}}{\left(\varepsilon_1 - 1\right)\omega_{Y_m}^{Y_h} + \frac{1}{1 - u_h^g}}.$$

Therefore,

$$\frac{K}{H_g}\frac{dH_g}{dK} - \frac{K}{H_s}\frac{dH_s}{dK} = \omega_K^{Y_m}\frac{\frac{1}{1 - u_h^g} + \left(\varepsilon_1 - 1\right)\omega_{Y_m}^{Y_h}\omega_{Y_h}^{Y}\omega_{Y_m}^{Y_h}}{\left(\varepsilon_1 - 1\right)\omega_{Y_m}^{Y_h} + \frac{1}{1 - u_h^g}},$$
$$\frac{K}{H_g}\frac{dH_g}{dK} - \frac{K}{H_s}\frac{dH_s}{dK} + \frac{K}{u_h^g}\frac{du_h^g}{dK} = \omega_K^{Y_m}\frac{\frac{1}{1 - u_h^g} + \left(\varepsilon_1 - 1\right)\omega_{Y_m}^{Y_h}\omega_{Y_h}^{Y}\omega_{Y_m}^{Y_h} + \left(\varepsilon_1 - 1\right)\omega_{Y_m}^{Y_h}}{\left(\varepsilon_1 - 1\right)\omega_{Y_m}^{Y_h} + \frac{1}{1 - u_h^g}}.$$

In terms of changes in the shares then we have that

$$\frac{K}{\omega_h^g\left(1 - \omega_h^g\right)}\frac{\partial \omega_h^g}{\partial K} = \frac{K}{H_g}\frac{dH_g}{dK} - \frac{K}{H_s}\frac{dH_s}{dK} + \frac{K}{u_h^g}\frac{\partial u_h^g}{\partial K}$$
$$= \frac{\omega_K^{Y_m}\left\{\frac{1}{1 - u_h^g} + \left(\varepsilon_1 - 1\right)\left(1 - \gamma_2\right)\left[1 + \omega_{Y_h}^{Y}\left(1 - \gamma_2\right)\right]\right\}}{\left(\varepsilon_1 - 1\right)\left(1 - \gamma_2\right) + \frac{1}{1 - u_h^g}},$$

and

$$\frac{K}{\omega_l}\frac{\partial \omega_l}{\partial K} = \frac{\omega_K^{Y_m}\left\{\frac{H_s}{H_g + H_s}\left[\frac{1}{1 - u_h^g} + \left(\varepsilon_1 - 1\right)\left(1 - \gamma_2\right)\left(1 - \gamma_2\right)\omega_{Y_h}^{Y}\right] - \frac{u_h^g}{1 - u_h^g}\left(\varepsilon_1 - 1\right)\left(1 - \gamma_2\right)\right\}}{\left(\varepsilon_1 - 1\right)\left(1 - \gamma_2\right) + \frac{1}{1 - u_h^g}},$$

where we have replaced $\omega_{Y_m}^{Y_h}$ by $(1-\gamma_2)$. Clearly, $\partial\omega_h^g/\partial K$ is positive at least for

$$\varepsilon_1 > \frac{(1-\gamma_2)\left[1+\omega_{Y_h}^{Y}(1-\gamma_2)\right]-1}{(1-\gamma_2)\left[1+\omega_{Y_h}^{Y}(1-\gamma_2)\right]}.$$

In turn, if

$$\frac{H_s}{H_g+H_s}(1-\gamma_2)\,\omega_{Y_h}^{Y}-\frac{u_h^g}{1-u_h^g}>0,$$

then in order for both $\partial\omega_h^g/\partial K$ and $\partial\omega_l/\partial K$ to be positive it is sufficient to have

$$\varepsilon_1 > \frac{(1-\gamma_2)\left[1+\omega_{Y_h}^{Y}(1-\gamma_2)\right]-1}{(1-\gamma_2)\left[1+\omega_{Y_h}^{Y}(1-\gamma_2)\right]}.$$

However, if

$$\frac{H_s}{H_g+H_s}(1-\gamma_2)\,\omega_{Y_h}^{Y}-\frac{u_h^g}{1-u_h^g}<0,$$

then in order for $\partial\omega_l/\partial K$ to be positive it is sufficient to have

$$\frac{\frac{H_s}{H_g+H_s}\frac{1}{1-u_h^g}+(1-\gamma_2)\left[\frac{u_h^g}{1-u_h^g}-\frac{H_s}{H_g+H_s}(1-\gamma_2)\,\omega_{Y_h}^{Y}\right]}{(1-\gamma_2)\left[\frac{u_h^g}{1-u_h^g}-\frac{H_s}{H_g+H_s}(1-\gamma_2)\,\omega_{Y_h}^{Y}\right]}>\varepsilon_1.$$

When

$$\frac{H_s}{H_g+H_s}(1-\gamma_2)\,\omega_{Y_h}^{Y}-\frac{u_h^g}{1-u_h^g}<0,$$

these limits have the following relationship

$$\frac{\frac{H_s}{H_g+H_s}\frac{1}{1-u_h^g}+(1-\gamma_2)\left[\frac{u_h^g}{1-u_h^g}-\frac{H_s}{H_g+H_s}(1-\gamma_2)\,\omega_{Y_h}^{Y}\right]}{(1-\gamma_2)\left[\frac{u_h^g}{1-u_h^g}-\frac{H_s}{H_g+H_s}(1-\gamma_2)\,\omega_{Y_h}^{Y}\right]}>1>\frac{(1-\gamma_2)\left[1+\omega_{Y_h}^{Y}(1-\gamma_2)\right]-1}{(1-\gamma_2)\left[1+\omega_{Y_h}^{Y}(1-\gamma_2)\right]}.$$

We define function $\mathbb{I}(.,.)$ as

$$\mathbb{I}(x,y)=\begin{cases} x & if \;\; x>0,\\ y & otherwise.\end{cases}$$

In general, in order for $\partial\omega_h^g/\partial K$ and $\partial\omega_l/\partial K$ to be positive it is sufficient to have ε_1 higher than

$$\mathbb{I}\left(\frac{(1-\gamma_2)\left[1+\omega_{Y_h}^{Y}(1-\gamma_2)\right]-1}{(1-\gamma_2)\left[1+\omega_{Y_h}^{Y}(1-\gamma_2)\right]},0\right)$$

and lower than

$$\mathbb{I}\left(\frac{\frac{H_s}{H_g+H_s}\frac{1}{1-u_h^g}+(1-\gamma_2)\left[\frac{u_h^g}{1-u_h^g}-\frac{H_s}{H_g+H_s}(1-\gamma_2)\,\omega_{Y_h}^{Y}\right]}{(1-\gamma_2)\left[\frac{u_h^g}{1-u_h^g}-\frac{H_s}{H_g+H_s}(1-\gamma_2)\,\omega_{Y_h}^{Y}\right]},+\infty\right).$$

Clearly, this interval includes both lower than 1 and greater than 1 values of ε_1.

Proof of Proposition 8: The partial derivatives of Y with respect to sectoral shocks are given by

$$\frac{\partial Y}{\partial \lambda_h} = \frac{\partial Y}{\partial Y_h}\left[\frac{\partial Y_h}{\partial \lambda_h} + \frac{\partial Y_h}{\partial u_h^g}\frac{\partial u_h^g}{\partial \lambda_h}\right] - \frac{\partial Y}{\partial Y_l}\frac{\partial Y_l}{\partial u_l^g}\frac{\partial u_h^g}{\partial \lambda_h},$$

$$\frac{\partial Y}{\partial \lambda_l} = \frac{\partial Y}{\partial Y_h}\frac{\partial Y_h}{\partial u_h^g}\frac{\partial u_h^g}{\partial \lambda_l} + \frac{\partial Y}{\partial Y_l}\left[\frac{\partial Y_l}{\partial \lambda_l} - \frac{\partial Y_l}{\partial u_l^g}\frac{\partial u_h^g}{\partial \lambda_l}\right].$$

We use (3), (4), (6), (13), (10), (14), and (28) to derive the elasticities of final output with respect to sectoral shocks. The elasticities are given by

$$\frac{\lambda_h}{Y}\frac{\partial Y}{\partial \lambda_h} = \frac{\varepsilon_1 \omega_{Y_m}^{Y_h}\omega_{Y_h}^{Y} + \omega_{Y_h}^{Y}\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_1\varepsilon_2 + \varepsilon_2\left[\varepsilon_1\omega_{Y_h}^{Y} - (\varepsilon_1 - 1)\gamma_4\right]\frac{u_h^g}{1-u_h^g}}{\varepsilon_2 + (\varepsilon_1-\varepsilon_2)\,\omega_{Y_m}^{Y_h} + \varepsilon_2\left[\varepsilon_1 - (\varepsilon_1-1)\gamma_4\right]\frac{u_h^g}{1-u_h^g}},$$

$$\frac{\lambda_l}{Y}\frac{\partial Y}{\partial \lambda_l} = 1 - \frac{\lambda_h}{Y}\frac{\partial Y}{\partial \lambda_h}.$$

Both these elasticities should be greater than zero, which implies that they are bounded between 0 and 1. Therefore, in equilibrium the following two restrictions should hold

$$\varepsilon_1 \omega_{Y_m}^{Y_h}\omega_{Y_h}^{Y} + \omega_{Y_h}^{Y}\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_1\varepsilon_2 + \varepsilon_2\left[\varepsilon_1\omega_{Y_h}^{Y} - (\varepsilon_1-1)\gamma_4\right]\frac{u_h^g}{1-u_h^g} > 0, \tag{59}$$

$$\varepsilon_1 \omega_{Y_m}^{Y_h}\omega_{Y_h}^{Y} + \omega_{Y_h}^{Y}\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_1\varepsilon_2 - \varepsilon_2\varepsilon_1\left(1-\omega_{Y_h}^{Y}\right)\frac{u_h^g}{1-u_h^g} < \varepsilon_2 + (\varepsilon_1-\varepsilon_2)\,\omega_{Y_m}^{Y_h}. \tag{60}$$

In order to evaluate the partial derivatives of the elasticities with respect to stocks of human capital types I consider the partial derivatives of shares.using (5) and (11). The partial derivatives of shares are given by

$$\frac{\partial \omega_{Y_h}^{Y}}{\partial H_s} = \frac{\partial \omega_{Y_h}^{Y}}{\partial Y_h}\left[\frac{\partial Y_h}{\partial Y_m}\frac{\partial Y_m}{\partial H_s} + \frac{\partial Y_h}{\partial u_h^g}\frac{\partial u_h^g}{\partial H_s}\right] - \frac{\partial \omega_{Y_h}^{Y}}{\partial Y_l}\frac{\partial Y_l}{\partial u_l^g}\frac{\partial u_h^g}{\partial H_s},$$

$$\frac{\partial \omega_{Y_h}^{Y}}{\partial H_g} = \frac{\partial \omega_{Y_h}^{Y}}{\partial Y_h}\left[\frac{\partial Y_h}{\partial H_g} + \frac{\partial Y_h}{\partial u_h^g}\frac{\partial u_h^g}{\partial H_g}\right] + \frac{\partial \omega_{Y_h}^{Y}}{\partial Y_l}\left[\frac{\partial Y_l}{\partial H_g} - \frac{\partial Y_l}{\partial u_l^g}\frac{\partial u_h^g}{\partial H_g}\right],$$

and

$$\frac{\partial \omega_{Y_m}^{Y_h}}{\partial H_s} = \frac{\partial \omega_{Y_m}^{Y_h}}{\partial Y_m}\frac{\partial Y_m}{\partial H_s} + \frac{\partial \omega_{Y_m}^{Y_h}}{\partial u_h^g}\frac{\partial u_h^g}{\partial H_s},$$

$$\frac{\partial \omega_{Y_m}^{Y_h}}{\partial H_g} = \frac{\partial \omega_{Y_m}^{Y_h}}{\partial H_g} + \frac{\partial \omega_{Y_m}^{Y_h}}{\partial u_h^g}\frac{\partial u_h^g}{\partial H_g}.$$

Clearly, according to the formulas of the shares and production functions it is the case that

$$\frac{\partial \omega_{Y_h}^{Y}}{\partial Y_h} = \frac{\varepsilon_1 - 1}{\varepsilon_1}\frac{1}{Y_h}\omega_{Y_h}^{Y}\left(1-\omega_{Y_h}^{Y}\right),$$

$$\frac{\partial \omega_{Y_h}^{Y}}{\partial Y_l} = -\frac{\varepsilon_1 - 1}{\varepsilon_1}\frac{1}{Y_l}\omega_{Y_h}^{Y}\left(1-\omega_{Y_h}^{Y}\right),$$

and

$$\frac{\partial \omega_{Y_m}^{Y_h}}{\partial Y_m} = \frac{\varepsilon_2 - 1}{\varepsilon_2} \frac{1}{Y_m} \omega_{Y_m}^{Y_h} \left(1 - \omega_{Y_m}^{Y_h}\right),$$
$$\frac{\partial \omega_{Y_m}^{Y_h}}{\partial u_h^g} = -\frac{\varepsilon_2 - 1}{\varepsilon_2} \frac{1}{u_h^g} \omega_{Y_m}^{Y_h} \left(1 - \omega_{Y_m}^{Y_h}\right),$$
$$\frac{\partial \omega_{Y_m}^{Y_h}}{\partial H_g} = -\frac{\varepsilon_2 - 1}{\varepsilon_2} \frac{1}{H_g} \omega_{Y_m}^{Y_h} \left(1 - \omega_{Y_m}^{Y_h}\right).$$

In turn, the partial derivatives of u_h^g are given by (25) and (27). Therefore, the partial derivatives of the shares are given by

$$\frac{\partial \omega_{Y_h}^{Y}}{\partial H_s} = \frac{\varepsilon_1 - 1}{\varepsilon_1} \frac{\omega_{Y_h}^{Y}}{H_s} \left(1 - \omega_{Y_h}^{Y}\right) \left\{ \omega_{Y_m}^{Y_h} \left(1 - \omega_K^{Y_m}\right) + \left[\left(1 - \omega_{Y_m}^{Y_h}\right) + \gamma_4 \frac{u_h^g}{1 - u_h^g} \right] \frac{H_s}{u_h^g} \frac{\partial u_h^g}{\partial H_s} \right\},$$
$$\frac{\partial \omega_{Y_h}^{Y}}{\partial H_g} = \frac{\varepsilon_1 - 1}{\varepsilon_1} \frac{\omega_{Y_h}^{Y}}{H_g} \left(1 - \omega_{Y_h}^{Y}\right) \left\{ \left(1 - \omega_{Y_m}^{Y_h} - \gamma_4\right) + \left[\left(1 - \omega_{Y_m}^{Y_h}\right) + \gamma_4 \frac{u_h^g}{1 - u_h^g} \right] \frac{H_g}{u_h^g} \frac{\partial u_h^g}{\partial H_g} \right\},$$

and

$$\frac{\partial \omega_{Y_m}^{Y_h}}{\partial H_s} = \frac{\varepsilon_2 - 1}{\varepsilon_2} \left(1 - \omega_{Y_m}^{Y_h}\right) \frac{\omega_{Y_m}^{Y_h}}{H_s} \left[\left(1 - \omega_K^{Y_m}\right) - \frac{H_s}{u_h^g} \frac{\partial u_h^g}{\partial H_s} \right],$$
$$\frac{\partial \omega_{Y_m}^{Y_h}}{\partial H_g} = -\frac{\varepsilon_2 - 1}{\varepsilon_2} \left(1 - \omega_{Y_m}^{Y_h}\right) \frac{\omega_{Y_m}^{Y_h}}{H_g} \left[1 + \frac{H_g}{u_h^g} \frac{\partial u_h^g}{\partial H_g} \right].$$

Plugging the values of $\partial u_h^g / \partial H_g$ and $\partial u_h^g / \partial H_s$ gives

$$\frac{\partial \omega_{Y_h}^{Y}}{\partial H_s} = \frac{\omega_{Y_h}^{Y}}{H_s} \left(1 - \omega_{Y_h}^{Y}\right) \left(1 - \omega_K^{Y_m}\right) \omega_{Y_m}^{Y_h} \tag{61}$$
$$\times \frac{(\varepsilon_1 - 1) \left\{ 1 + \left[\varepsilon_2 (1 - \gamma_4) + \gamma_4\right] \frac{u_h^g}{1 - u_h^g} \right\}}{\varepsilon_2 + (\varepsilon_1 - \varepsilon_2) \omega_{Y_m}^{Y_h} + \varepsilon_2 \left[\varepsilon_1 - (\varepsilon_1 - 1) \gamma_4\right] \frac{u_h^g}{1 - u_h^g}},$$
$$\frac{\partial \omega_{Y_h}^{Y}}{\partial H_g} = \frac{\varepsilon_1 - 1}{\varepsilon_1} \left(1 - \omega_{Y_h}^{Y}\right) \frac{\omega_{Y_h}^{Y}}{H_g} \tag{62}$$
$$\times \left\{ \frac{\left(1 - \omega_{Y_m}^{Y_h}\right) \left[\varepsilon_2 + \varepsilon_2 (\varepsilon_1 - 1)(1 - \gamma_4)\right] - \left[\varepsilon_2 + (\varepsilon_1 - \varepsilon_2) \omega_{Y_m}^{Y_h}\right] \gamma_4}{\varepsilon_2 + (\varepsilon_1 - \varepsilon_2) \omega_{Y_m}^{Y_h} + \varepsilon_2 \left[\varepsilon_1 - (\varepsilon_1 - 1) \gamma_4\right] \frac{u_h^g}{1 - u_h^g}} \frac{1}{1 - u_h^g} \right\},$$

and

$$\frac{\partial \omega_{Y_m}^{Y_h}}{\partial H_s} = \frac{\varepsilon_2 - 1}{\varepsilon_2} \left(1 - \omega_{Y_m}^{Y_h}\right) \frac{\omega_{Y_m}^{Y_h}}{H_s} \left(1 - \omega_K^{Y_m}\right) \tag{63}$$
$$\times \frac{\varepsilon_2 + \varepsilon_2 (\varepsilon_1 - 1)(1 - \gamma_4) u_h^g}{\varepsilon_2 + (\varepsilon_1 - \varepsilon_2) \omega_{Y_m}^{Y_h} + \varepsilon_2 \left[\varepsilon_1 - (\varepsilon_1 - 1) \gamma_4\right] \frac{u_h^g}{1 - u_h^g}} \frac{1}{1 - u_h^g},$$
$$\frac{\partial \omega_{Y_m}^{Y_h}}{\partial H_g} = -\frac{\varepsilon_2 - 1}{\varepsilon_2} \left(1 - \omega_{Y_m}^{Y_h}\right) \frac{\omega_{Y_m}^{Y_h}}{H_g} \tag{64}$$
$$\times \frac{\varepsilon_2 + \varepsilon_2 (\varepsilon_1 - 1)(1 - \gamma_4)}{\varepsilon_2 + (\varepsilon_1 - \varepsilon_2) \omega_{Y_m}^{Y_h} + \varepsilon_2 \left[\varepsilon_1 - (\varepsilon_1 - 1) \gamma_4\right] \frac{u_h^g}{1 - u_h^g}} \frac{1}{1 - u_h^g}.$$

Let i be either s or g. This implies that the partial derivative of $\frac{\lambda_{Y_h}}{Y}\frac{\partial Y}{\partial \lambda_{Y_h}}$ with respect to H_i is given by

$$\frac{\partial}{\partial H_i}\left(\frac{\lambda_{Y_h}}{Y}\frac{\partial Y}{\partial \lambda_{Y_h}}\right) = \frac{\partial}{\partial H_i}\frac{\varepsilon_1 \omega_{Y_m}^{Y_h}\omega_{Y_h}^{Y} + \omega_{Y_h}^{Y}\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_1\varepsilon_2 + \varepsilon_2\left[\varepsilon_1\omega_{Y_h}^{Y} - (\varepsilon_1 - 1)\gamma_4\right]\frac{u_h^g}{1-u_h^g}}{\varepsilon_2 + (\varepsilon_1-\varepsilon_2)\omega_{Y_m}^{Y_h} + \varepsilon_2\left[\varepsilon_1 - (\varepsilon_1-1)\gamma_4\right]\frac{u_h^g}{1-u_h^g}}.$$

The numerator of this expression is given by

$$\begin{aligned}
X_{1,H_i} = {} & \left[\varepsilon_1\omega_{Y_m}^{Y_h} + \varepsilon_1\varepsilon_2\left(1-\omega_{Y_m}^{Y_h}\right) + \varepsilon_2\varepsilon_1\frac{u_h^g}{1-u_h^g}\right] \\
& \times\left\{\varepsilon_2 + (\varepsilon_1-\varepsilon_2)\omega_{Y_m}^{Y_h} + \varepsilon_2\left[\varepsilon_1-(\varepsilon_1-1)\gamma_4\right]\frac{u_h^g}{1-u_h^g}\right\}\frac{\partial}{\partial H_i}\omega_{Y_h}^{Y} \\
& + \varepsilon_1\omega_{Y_h}^{Y}(1-\varepsilon_2)\left\{\varepsilon_2 + (\varepsilon_1-\varepsilon_2)\omega_{Y_m}^{Y_h} + \varepsilon_2\left[\varepsilon_1-(\varepsilon_1-1)\gamma_4\right]\frac{u_h^g}{1-u_h^g}\right\}\frac{\partial}{\partial H_i}\omega_{Y_m}^{Y_h} \\
& - (\varepsilon_1-\varepsilon_2)\left\{\varepsilon_1\omega_{Y_m}^{Y_h}\omega_{Y_h}^{Y} + \omega_{Y_h}^{Y}\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_1\varepsilon_2 + \varepsilon_2\left[\varepsilon_1\omega_{Y_h}^{Y} - (\varepsilon_1-1)\gamma_4\right]\frac{u_h^g}{1-u_h^g}\right\}\frac{\partial}{\partial H_i}\omega_{Y_m}^{Y_h} \\
& + \left\{\left[\varepsilon_1\omega_{Y_h}^{Y} - (\varepsilon_1-1)\gamma_4\right]\left[\varepsilon_2 + (\varepsilon_1-\varepsilon_2)\omega_{Y_m}^{Y_h}\right]\right. \\
& \left. - \varepsilon_1\omega_{Y_h}^{Y}\left[\varepsilon_1-(\varepsilon_1-1)\gamma_4\right]\left[\omega_{Y_m}^{Y_h} + \left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_2\right]\right\}\varepsilon_2\frac{\partial}{\partial H_i}\frac{u_h^g}{1-u_h^g}.
\end{aligned}$$

After some algebra, the numerator can be expressed as

$$\begin{aligned}
H_i X_{1,H_i} = {} & \left[\varepsilon_1\omega_{Y_m}^{Y_h} + \varepsilon_1\varepsilon_2\left(1-\omega_{Y_m}^{Y_h}\right) + \varepsilon_2\varepsilon_1\frac{u_h^g}{1-u_h^g}\right] \\
& \times\left\{\varepsilon_2 + (\varepsilon_1-\varepsilon_2)\omega_{Y_m}^{Y_h} + \varepsilon_2\left[\varepsilon_1-(\varepsilon_1-1)\gamma_4\right]\frac{u_h^g}{1-u_h^g}\right\}H_i\frac{\partial}{\partial H_i}\omega_{Y_h}^{Y} \\
& - \varepsilon_2(\varepsilon_1-1)\left(\varepsilon_1\omega_{Y_h}^{Y}\left[1-u_h^g(1-\gamma_4)\right] + \left\{\varepsilon_2\left[\varepsilon_1\omega_{Y_h}^{Y}(1-\gamma_4)+\gamma_4\right] - \varepsilon_1\gamma_4\right\}u_h^g\right) \\
& \times\frac{1}{1-u_h^g}H_i\frac{\partial}{\partial H_i}\omega_{Y_m}^{Y_h} \\
& - (\varepsilon_1-1)\left\{\left[\varepsilon_1\omega_{Y_h}^{Y}(1-\gamma_4)+\gamma_4\right]\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_2 + \varepsilon_1\omega_{Y_m}^{Y_h}\gamma_4\left(1-\omega_{Y_h}^{Y}\right)\right\}\varepsilon_2 H_i\frac{\partial}{\partial H_i}\frac{u_h^g}{1-u_h^g}.
\end{aligned}$$

Clearly, when $\varepsilon_2 = \gamma_4 = 1$ the numerator can be rewritten as

$$\begin{aligned}
H_i X_{1,H_i} = {} & \varepsilon_1\frac{1}{1-u_h^g}\left[(\varepsilon_1-1)\omega_{Y_m}^{Y_h} + \frac{1}{1-u_h^g}\right]H_i\frac{\partial\omega_{Y_h}^{Y}}{\partial H_i} \\
& - (\varepsilon_1-1)\left[1-\omega_{Y_m}^{Y_h} + \varepsilon_1\omega_{Y_m}^{Y_h}\left(1-\omega_{Y_h}^{Y}\right)\right]\frac{1}{1-u_h^g}\frac{1}{1-u_h^g}H_i\frac{\partial u_h^g}{\partial H_i},
\end{aligned}$$

which means that

$$H_g X_{1,H_g} = -(\varepsilon_1 - 1) \frac{\omega_{Y_m}^{Y_h} \frac{1}{\left(1-u_h^g\right)^2}}{(\varepsilon_1 - 1)\,\omega_{Y_m}^{Y_h} + \frac{1}{1-u_h^g}} \tag{65}$$

$$\times \left\{ \varepsilon_1 \left[(\varepsilon_1 - 1)\,\omega_{Y_m}^{Y_h} + \frac{1}{1-u_h^g} \right] \omega_{Y_h}^{Y} \left(1 - \omega_{Y_h}^{Y}\right) - (\varepsilon_1 - 1) \left[1 - \omega_{Y_m}^{Y_h} + \varepsilon_1 \omega_{Y_m}^{Y_h} \left(1 - \omega_{Y_h}^{Y}\right) \right] u_h^g \right\},$$

$$H_s X_{1,H_s} = (\varepsilon_1 - 1) \frac{\left(1 - \omega_K^{Y_m}\right) \omega_{Y_m}^{Y_h} \frac{1}{\left(1-u_h^g\right)^2}}{(\varepsilon_1 - 1)\,\omega_{Y_m}^{Y_h} + \frac{1}{1-u_h^g}} \tag{66}$$

$$\times \left\{ \varepsilon_1 \left[(\varepsilon_1 - 1)\,\omega_{Y_m}^{Y_h} + \frac{1}{1-u_h^g} \right] \omega_{Y_h}^{Y} \left(1 - \omega_{Y_h}^{Y}\right) - (\varepsilon_1 - 1) \left[1 - \omega_{Y_m}^{Y_h} + \varepsilon_1 \omega_{Y_m}^{Y_h} \left(1 - \omega_{Y_h}^{Y}\right) \right] u_h^g \right\}.$$

The difference between the elasticities $\Delta_{\lambda_h} = H_s X_{1,H_s} - H_g X_{1,H_g}$ is given by

$$\Delta_{\lambda_h} = (\varepsilon_1 - 1) \frac{1}{\left(1-u_h^g\right)^2} \left[\frac{\left(2 - \omega_K^{Y_m}\right) \omega_{Y_m}^{Y_h}}{1 + (\varepsilon_1 - 1)\,\omega_{Y_m}^{Y_h} + \frac{u_h^g}{1-u_h^g}} \right]$$

$$\times \left\{ \varepsilon_1 \left[(\varepsilon_1 - 1)\,\omega_{Y_m}^{Y_h} + \frac{1}{1-u_h^g} \right] \omega_{Y_h}^{Y} \left(1 - \omega_{Y_h}^{Y}\right) - (\varepsilon_1 - 1) \left[1 - \omega_{Y_m}^{Y_h} + \varepsilon_1 \omega_{Y_m}^{Y_h} \left(1 - \omega_{Y_h}^{Y}\right) \right] u_h^g \right\}.$$

A similar exercise for elasticities with respect to λ_l gives $\Delta_{\lambda_l} = -\Delta_{\lambda_h}$. If $1 > \varepsilon_1$ then Δ_{λ_h} is negative and Δ_{λ_l} is positive. Therefore, the elasticity of final output with respect to shocks λ_h (λ_l) increases less (more) with a marginal (percentage) increase in H_s than with a marginal (percentage) increase in H_g if $1 > \varepsilon_1$.

To analyze the case when $\varepsilon_1 > 1$, we denote by d the following expression

$$d = \varepsilon_1 \left[(\varepsilon_1 - 1)\,\omega_{Y_m}^{Y_h} + \frac{1}{1-u_h^g} \right] \omega_{Y_h}^{Y} \left(1 - \omega_{Y_h}^{Y}\right) - (\varepsilon_1 - 1) \left[\left(1 - \omega_{Y_m}^{Y_h}\right) + \varepsilon_1 \omega_{Y_m}^{Y_h} \left(1 - \omega_{Y_h}^{Y}\right) \right] u_h^g \tag{67}$$

and make use of the following function

$$F_3\left(u_h^g\right) = u_h^g \left(1 - u_h^g\right)^{-1} - \frac{\gamma_2 \gamma_1}{1 - \gamma_1} \left(\frac{Y_h}{Y_l} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}.$$

The equilibrium value of u_h^g solves $F_3\left(u_h^g\right) = 0$ when $\varepsilon_2 = \gamma_4 = 1$. At the equilibrium value of u_h^g, it is the case that

$$\frac{\partial F_3\left(u_h^g\right)}{\partial u_h^g} = \frac{1}{1-u_h^g} \left[1 + \frac{u_h^g}{1-u_h^g} - \frac{\varepsilon_1 - 1}{\varepsilon_1} \left(\gamma_2 + \frac{u_h^g}{1-u_h^g} \right) \right] > 0.$$

In turn, F_3 is a smooth function and $F_3\left(\omega_{Y_h}^{Y}\right) > 0$. Therefore,

$$\omega_{Y_h}^{Y} > u_h^g$$

and

$$d > \varepsilon_1 \left[(\varepsilon_1 - 1)\, \omega_{Y_m}^{Y_h} + \frac{1}{1 - u_h^g} \right] u_h^g \left(1 - \omega_{Y_h}^{Y}\right) - (\varepsilon_1 - 1) \left[\left(1 - \omega_{Y_m}^{Y_h}\right) + \varepsilon_1 \omega_{Y_m}^{Y_h} \left(1 - \omega_{Y_h}^{Y}\right) \right] u_h^g$$
$$= \left[\frac{1 - \omega_{Y_h}^{Y}}{1 - u_h^g} - \frac{\varepsilon_1 - 1}{\varepsilon_1} \left(1 - \omega_{Y_m}^{Y_h}\right) \right] \varepsilon_1 u_h^g.$$

From (60) and the expression above it follows that

$$u_h^g > \frac{(\varepsilon_1 - 1)\left(1 - \omega_{Y_m}^{Y_h}\right) - \varepsilon_1 \left(1 - \omega_{Y_h}^{Y}\right)}{(\varepsilon_1 - 1)\left(1 - \omega_{Y_m}^{Y_h}\right)}$$

and

$$d > \left[\frac{1 - \omega_{Y_h}^{Y}}{1 - u_h^g} - \frac{\varepsilon_1 - 1}{\varepsilon_1} \left(1 - \omega_{Y_m}^{Y_h}\right) \right] \varepsilon_1 u_h^g > 0.$$

This implies that Δ_{λ_h} is positive and Δ_{λ_l} is negative for $\varepsilon_1 > 1$. It also implies that, the elasticity of final output with respect to shocks λ_h (λ_l) increases more (less) with a marginal (percentage) increase in H_s than with a marginal (percentage) increase in H_g.

It is also possible to derive inference for the levels of change of the elasticity of final output, for example, when $\varepsilon_2 = 0$. In such a case, when $1 > \varepsilon_1$ ($\varepsilon_1 > 1$) the elasticity of final output with respect to λ_h declines (increases) with H_s and increases (declines) with H_g. The elasticity of final output with respect to λ_l increases (declines) with H_s and declines (increases) with H_g.

Proof of Proposition 9: The total change of the elasticities is given by

$$d\left(\frac{\lambda_{Y_i}}{Y}\frac{\partial Y}{\partial \lambda_{Y_i}}\right) = \frac{\partial}{\partial H_s}\left(\frac{\lambda_{Y_i}}{Y}\frac{\partial Y}{\partial \lambda_{Y_i}}\right) dH_s + \frac{\partial}{\partial H_g}\left(\frac{\lambda_{Y_i}}{Y}\frac{\partial Y}{\partial \lambda_{Y_i}}\right) dH_g. \tag{68}$$

We consider variation in H_s and H_g such that (expected) final output stays constant. This variation satisfies (41), where $\hat{\omega}_{Y_h}^{Y} = \omega_{Y_h}^{Y}$. We again set $\varepsilon_2 = \gamma_4 = 1$. The total variation of elasticities (68) for λ_h is a ratio where the numerator is given by

$$\begin{aligned} X_{\lambda_h} = & \left\{ \varepsilon_1 \frac{1}{1 - u_h^g} \left[1 + (\varepsilon_1 - 1)\, \omega_{Y_m}^{Y_h} + \frac{u_h^g}{1 - u_h^g} \right] \frac{\partial \omega_{Y_h}^{Y}}{\partial H_g} \right. \\ & \left. - (\varepsilon_1 - 1) \left[\left(1 - \omega_{Y_m}^{Y_h}\right) + \varepsilon_1 \omega_{Y_m}^{Y_h} \left(1 - \omega_{Y_h}^{Y}\right) \right] \frac{\partial}{\partial H_g} \frac{u_h^g}{1 - u_h^g} \right\} dH_g \\ & + \left\{ \varepsilon_1 \frac{1}{1 - u_h^g} \left[1 + (\varepsilon_1 - 1)\, \omega_{Y_m}^{Y_h} + \frac{u_h^g}{1 - u_h^g} \right] \frac{\partial \omega_{Y_h}^{Y}}{\partial H_s} \right. \\ & \left. - (\varepsilon_1 - 1) \left[\left(1 - \omega_{Y_m}^{Y_h}\right) + \varepsilon_1 \omega_{Y_m}^{Y_h} \left(1 - \omega_{Y_h}^{Y}\right) \right] \frac{\partial}{\partial H_s} \frac{u_h^g}{1 - u_h^g} \right\} dH_s, \end{aligned}$$

and the denominator is positive. For λ_l we have that $X_{\lambda_l} = -X_{\lambda_h}$. Using $\varepsilon_2 = \gamma_4 = 1$ and (41), where $\hat{\omega}^Y_{Y_h} = \omega^Y_{Y_h}$, and (65) together with (66), X_{λ_h} can be rewritten as

$$
\begin{aligned}
X_{\lambda_h}\frac{1}{dH_s}\frac{H_s}{H_g} &= (\varepsilon_1 - 1)\frac{\omega^{Y_h}_{Y_m}\left(1-\omega^{Y_m}_K\right)}{\left(1-u^g_h\right)^2\left[(\varepsilon_1-1)\,\omega^{Y_h}_{Y_m} + \frac{1}{1-u^g_h}\right]} \\
&\times\left\{1 + \frac{\left[\omega^Y_{Y_h}\varepsilon_1 - (\varepsilon_1-1)\,u^g_h\right]\omega^{Y_h}_{Y_m}}{\left(1-\omega^{Y_h}_{Y_m}\right) + \varepsilon_1\omega^{Y_h}_{Y_m}\left(1-\omega^Y_{Y_h}\right)}\right\} \\
&\times\left\{\varepsilon_1\left[(\varepsilon_1-1)\,\omega^{Y_h}_{Y_m} + \frac{1}{1-u^g_h}\right]\omega^Y_{Y_h}\left(1-\omega^Y_{Y_h}\right)\right. \\
&\left. - (\varepsilon_1-1)\left[\left(1-\omega^{Y_h}_{Y_m}\right) + \varepsilon_1\omega^{Y_h}_{Y_m}\left(1-\omega^Y_{Y_h}\right)\right]u^g_h\right\}.
\end{aligned}
$$

According to the restriction (59) it is sufficient to look at the sign of the term in the second curly brackets. This term is identical to d in (67), which is positive for any value of ε_1. Therefore, the elasticity of final output with respect to shocks λ_h (λ_l) is lower (higher) in the country where H_s is higher if $1 > \varepsilon_1$. If $\varepsilon_1 > 1$ or $\varepsilon_1 = +\infty$ then the elasticity of final output with respect to shocks λ_h (λ_l) is higher (lower) in the country where H_s is higher.

Proof of Proposition 10: The wage rates of specific and general human capital are given by (9) and (10). Let i be either h or l. The difference between elasticities of wage rates with respect to shocks $\{\lambda_i\}$ are given by the following equation

$$
\begin{aligned}
&\lambda_i\left(\frac{\partial}{\partial\lambda_i}\ln w_s - \frac{\partial}{\partial\lambda_i}\ln w_g\right) \\
&= \lambda_i\left[\frac{\partial}{\partial\lambda_i}\ln\omega^{Y_h}_{Y_m} + \frac{\partial}{\partial\lambda_i}\ln\left(1-\omega^{Y_m}_K\right) - \frac{\partial}{\partial\lambda_i}\ln u^g_h - \frac{\partial}{\partial\lambda_i}\ln\left(1-\omega^{Y_h}_{Y_m}\right)\right].
\end{aligned}
$$

In this equation

$$
\begin{aligned}
\frac{\partial}{\partial\lambda_i}\ln\left(1-\omega^{Y_m}_K\right) &= 0, \\
\frac{\partial}{\partial\lambda_i}\ln\omega^{Y_h}_{Y_m} &= -\frac{\varepsilon_2-1}{\varepsilon_2}\left(1-\omega^{Y_h}_{Y_m}\right)\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial\lambda_i}, \\
\frac{\partial}{\partial\lambda_i}\ln\left(1-\omega^{Y_h}_{Y_m}\right) &= \frac{\varepsilon_2-1}{\varepsilon_2}\omega^{Y_h}_{Y_m}\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial\lambda_i},
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial}{\partial\lambda_h}\ln u^g_h &= \frac{\varepsilon_1-1}{\varepsilon_1}\frac{1}{\lambda_h}\frac{\varepsilon_1\varepsilon_2}{\varepsilon_2 + (\varepsilon_1-\varepsilon_2)\,\omega^{Y_h}_{Y_m} + \varepsilon_2\left[\varepsilon_1 - (\varepsilon_1-1)\,\gamma_4\right]\frac{u^g_h}{1-u^g_h}}, \\
\frac{\partial}{\partial\lambda_l}\ln u^g_h &= -\frac{\varepsilon_1-1}{\varepsilon_1}\frac{1}{\lambda_l}\frac{\varepsilon_1\varepsilon_2}{\varepsilon_2 + (\varepsilon_1-\varepsilon_2)\,\omega^{Y_h}_{Y_m} + \varepsilon_2\left[\varepsilon_1 - (\varepsilon_1-1)\,\gamma_4\right]\frac{u^g_h}{1-u^g_h}}.
\end{aligned}
$$

Therefore,

$$
\lambda_i\left(\frac{\partial}{\partial\lambda_i}\ln w_s - \frac{\partial}{\partial\lambda_i}\ln w_g\right) = -\left(\frac{\varepsilon_2-1}{\varepsilon_2}+1\right)\frac{\lambda_i}{u^g_h}\frac{\partial u^g_h}{\partial\lambda_i}.
$$

This implies that the sign of the difference between elasticities of wage rates for λ_h is the opposite of the sign for λ_l. Moreover, the elasticity of w_s with respect to λ_h (λ_l) is greater (lower) than the elasticity of w_g with respect to λ_h (λ_l) either when $1 > \varepsilon_1$ and $\varepsilon_2 > 1/2$ or when $\varepsilon_1 > 1$ and $1/2 > \varepsilon_2$. The elasticity of w_s with respect to λ_h (λ_l) is lower (greater) than the elasticity of w_g with respect to λ_h (λ_l) either when $1 > \varepsilon_1$ and $1/2 > \varepsilon_2$ or when $\varepsilon_1 > 1$ and $\varepsilon_2 > 1/2$. If $\varepsilon_2 = 1$ then the elasticity of w_s with respect to λ_h (λ_l) is greater (lower) than the elasticity of w_g with respect to λ_h (λ_l) when $1 > \varepsilon_1$. The elasticity of w_s with respect to λ_h (λ_l) is lower (greater) than the elasticity of w_g with respect to λ_h (λ_l) when $\varepsilon_1 > 1$.

Appendix - Numerical Exercises

We set $\gamma_2 = \gamma_3 = 0.5$, $\gamma_4 = 1$ and $\varepsilon_2 = 1.01$ in all numerical exercises in order to limit the space of parameter values. We use 3 values for γ_1: 0.453, 0.611, and 0.768. These values are the averages of γ_1 from Tables 48 and 49 when $1 > \varepsilon_1$, $1 > \varepsilon_1$ or $\varepsilon_1 > 1$, and $\varepsilon_1 > 1$. We use numerical integration routines for log-normally distributed variables to solve for the share of general human capital in h-sector from (16). The number of nodes in numerical integration is set to 7. In all numerical exercises, we divide the intervals of parameter values into 5 equidistant points and form a multidimensional grid using all possible values. For propositions 4 and 6, we set $\varepsilon_3 = 0.9$.

For Propositions 1 and 2 we use the following intervals for parameter values:

$H_g, Y_m \in (0.1, 10)$;	$\varepsilon_1 \in (0.4, 1.4)$
$\mu_{z_h}, \mu_{z_l} \in (0.1, 3)$;	$\sigma_{z_h}, \sigma_{z_l} \in (0.1, 1)$

We compute the value of u_h^g and check how it changes with model parameters. Our numerical results imply that (17), which is our approximation of (16), delivers correct results for H_g, Y_m, μ_{z_h}, and μ_{z_l}. It delivers correct results for σ_{z_h} when $\varepsilon_1 > 1$. It also delivers correct results for σ_{z_l} when $1 > \varepsilon_1$ with a limited number of discrepancies (around 5 percent of the evaluations).

For Proposition 3, we normalize everything by H_s which we set to equal to 1. This is possible only when $\gamma_4 = 1$. Given that λ_h and λ_l are log-normal, their mean and variance increase with μ_{z_h} and μ_{z_l}. To limit the space of parameter values, we set $\sigma_{z_h} = \sigma_{z_l} = 0.5$. For the reminder of the parameters we use the following intervals:

$H_g, K \in (0.1, 10)$;	$\varepsilon_1, \varepsilon_3 \in (0.4, 1.4)$
$\mu_{z_h}, \mu_{z_l} \in (0.1, 4)$	

Further, we compute u_h^g and $\mathbb{E}[w_g] / \mathbb{E}[w_s]$ using (16) and (18). We check how the latter changes with H_g and K. Our numerical results imply that (17), which is our approximation of (16), delivers correct results.

For Propositions 4 and 6, we normalize everything by K setting it to be equal to 1. Similar to the previous exercise, we also set $\sigma_{z_h} = \sigma_{z_l} = 0.5$. For the reminder of the parameters, we use the following intervals:

$H_g, H_s \in (0.1, 10)$;	$\varepsilon_1 \in (0.4, 1.4)$
$\mu_{z_h}, \mu_{z_l} \in (0.1, 4)$	

For Proposition 4, we compute u_h^g using (16) and compute the values of Y_h/λ_h and Y_l/λ_l. Further, we compute the required change of H_g for a given change H_s that keeps expected output constant. The change in H_s is set to 0.01 in percentage terms. After computing the required change in H_g, we recompute the values of u_h^g, Y_h/λ_h and Y_l/λ_l for the new values of H_s and H_g. We check that Y_h/λ_h increases and Y_l/λ_l declines as H_s increases and H_g declines. For $1 > \varepsilon_1$ ($\varepsilon_1 > 1$) higher values of Y_i/λ_i imply lower (higher) coefficient in front of λ_i in Y for $i = l, h$. Moreover, we check that $\mathbb{E}\left[Y^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right]$ changes very little with the variations of H_g and H_s which keep $\mathbb{E}[Y]$ constant.

For Proposition 6, we compute u_h^g using (16) and the expected value of Y for $(1 + d) H_s$, where we set $d = 0.01$. We also compute u_h^g and the value of H_g which keeps the expected value of Y constant for H_s. We check that Y_h/λ_h increases more with H_s and Y_l/λ_l increases more with H_g.