

# A new $|V_{us}|$ determination from kaon and tau decays

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**Abstract:** The determination of the CKM matrix element  $|V_{us}|$  provides a critical test of Standard Model's first-row unitarity. In this work,  $|V_{us}|$  is extracted from leptonic and semileptonic kaon decays ( $K_{\ell 2}$  and  $K_{\ell 3}$ ), as well as exclusive  $\tau \rightarrow K\nu_\tau$  and semileptonic  $\tau \rightarrow K\pi\nu_\tau$  decays. For the semileptonic channels, phase-space integrals are evaluated using standard form-factor parameterizations. For the decay  $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$  a fit to the spectrum from Belle is performed by incorporating energy-dependent Breit-Wigner resonance descriptions for the participating  $K^*(892)$  and  $K^*(1410)$  resonances. The individual extraction methods yield compatible values for  $|V_{us}|$  within uncertainties. Combining all determinations gives a global weighted average of  $|V_{us}| = 0.22434(29)$ , consistent with current world averages. The corresponding unitarity test reveals a persistent deviation from unity at the  $2.5\sigma$  level, indicating a lingering tension with Standard Model predictions.

**Keywords:** CKM matrix elements, kaon and tau decays, hadronic form factors.

**SDGs:** Quality education, Industry, innovation and infrastructure.

## I. INTRODUCTION

The Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix provides a cornerstone for testing the electroweak sector of the Standard Model (SM) of particle physics [1, 2]. Within this framework, testing the unitarity of the first row via the relation  $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$  constitutes one of the most stringent precision tests against new physics scenarios. While the heavy-quark mixing element  $|V_{ub}|$  is exceptionally small ( $\sim 3 \times 10^{-3}$  [3]) and its contribution to this sum is practically negligible, the precision of the unitarity test is heavily dictated by the values of  $|V_{ud}|$  and  $|V_{us}|$ . Modern experimental extractions uncover a persistent tension with the theoretical expectation of unity [3]:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.99885(34). \quad (1)$$

This discrepancy, accumulating a significance of more than  $3\sigma$ , has stimulated intense efforts within the particle physics community to systematically clean up the determination of the CKM matrix elements. Given that  $|V_{ud}|$  is tightly constrained by superallowed nuclear  $\beta$  decays,  $|V_{ud}| = 0.97367(32)$  [3], the primary focus for refining this unitarity test lies in reducing the lingering systematic and parameterization uncertainties that plague  $|V_{us}|$ .

To establish a robust and comprehensive picture, it is vital to analyze and cross-examine independent physical processes governed by different systematics. Historically, the most competitive avenues for extracting  $|V_{us}|$  rely on leptonic ( $K_{\ell 2}$ ) and semileptonic ( $K_{\ell 3}$ ) kaon decays. These channels, require highly precise descriptions of non-perturbative QCD interactions, which are encoded into hadronic decay constants and form factors. Complementary to the kaon sector, semileptonic  $\tau$  decays—such as the exclusive  $\tau \rightarrow K\nu_\tau$  and the broader  $\tau \rightarrow K\pi\nu_\tau$  decays—offer an advantageous environment to probe  $s \rightarrow u$  current quark transitions.

A major theoretical challenge arises when dealing with multi-body final states, such as ( $K_{\ell 3}$ ) decays and the  $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$ , where the physical spectrum is heavily influenced by intermediate hadronic resonance poles like the  $K^*(892)$  and  $K^*(1410)$ . While traditional Taylor expansions can characterize the form factors near zero momentum transfer, they inherently fail to capture the complex, energy-dependent phase space across the entire physical region where resonance effects dominate.

In this work, we address these challenges by conducting a systematic extraction of  $|V_{us}|$  across a diverse set of decay channels. For the semileptonic modes, we evaluate the phase-space integrals using advanced form-factor parameterizations. For the  $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$  decay mode, we perform a customized fit to the experimental spectrum measured by the Belle collaboration. This is achieved by implementing an energy-dependent Breit-Wigner framework for the vector form factor that accounts for both the primary  $K^*(892)$  and a sub-dominant  $K^*(1410)$  pole contribution, along with an interpolated scalar form factor to safely handle the low-energy region. Ultimately, by assessing the statistical compatibility of these varied leptonic, semileptonic, and tau-based avenues, we compute a new global weighted average for  $|V_{us}|$  to re-evaluate the current status of first-row CKM unitarity.

This work is structured as follows: Section II outlines the theoretical framework, defining the decay widths, phase-space integrals, and form-factor parameterizations for each channel. Section III presents the experimental data analysis, the spectral fitting procedure for the Belle data, and the resulting extractions of both  $|V_{us}|$  and the underlying resonance properties. Finally, we present our conclusions in Section IV.

## II. THEORETICAL FRAMEWORK

Before defining the specific theoretical expressions that we use to extract  $V_{us}$ , we define the branching ratio (BR), which accounts for the probability of a parent particle  $i$  decaying into a specific final state  $f$ :

$$BR(i \rightarrow f) = \frac{\Gamma(i \rightarrow f)}{\Gamma(i)}, \quad (2)$$

where  $\Gamma(i \rightarrow f)$  represents the partial decay width of the specific channel and  $\Gamma(i)$  is the total decay width of the parent particle  $i$ . The total decay width is inversely proportional to the particle's mean lifetime  $\tau_i$ , given by  $\Gamma_i = \tau_i^{-1}$ . In appendix A we collect the branching ratios and mean lives from the PDG [3] that we will use for our analysis.

### A. $|V_{us}|$ determination from leptonic kaon decays

Within the Standard Model, we extract  $|V_{us}|$  from the ratio of the photon-inclusive leptonic decay widths of kaons and pions,  $K^\pm \rightarrow \ell^\pm \nu$  ( $K_{\ell 2}$ ) and  $\pi^\pm \rightarrow \ell^\pm \nu$  ( $\pi_{\ell 2}$ ), which is given by [4]:

$$\frac{\Gamma(K^\pm \rightarrow \ell^\pm \nu_\ell)}{\Gamma(\pi^\pm \rightarrow \ell^\pm \nu_\ell)} = \frac{|V_{us}|^2 f_K^2 m_K (1 - m_\ell^2/m_K^2)^2}{|V_{ud}|^2 f_\pi^2 m_\pi (1 - m_\ell^2/m_\pi^2)^2}, \quad (3)$$

where  $f_K$  and  $f_\pi$  denote the the kaon and pion decay constants, respectively. For our work, we take the numerical value for  $f_K/f_\pi = 1.1934(19)$  from the Flavour Lattice Averaging Group (FLAG) review [5]. In the ratio, hadronic uncertainties stemming from short-distance universal radiative effects cancel out.

### B. $|V_{us}|$ determination from $\tau \rightarrow K \nu_\tau$ decays

An analogous approach can be applied within the tau lepton sector to extract  $|V_{us}|$ . Taking the ratio of the exclusive tau decay into a kaon over its decay into a pion yields [7]:

$$\frac{BR(\tau^\pm \rightarrow K^\pm \nu_\tau)}{BR(\tau^\pm \rightarrow \pi^\pm \nu_\tau)} = \frac{|V_{us}|^2 f_K^2 (m_\tau^2 - m_K^2)^2}{|V_{ud}|^2 f_\pi^2 (m_\tau^2 - m_\pi^2)^2}. \quad (4)$$

Alternatively, the standalone  $|V_{us}|$  determination from  $\tau^\pm \rightarrow K^\pm \nu_\tau$  also gives a competitive value. The required theoretical expression reads [7]:

$$BR(\tau^- \rightarrow K^- \nu_\tau) = \frac{G_F^2 \tau_\tau}{16\pi\hbar} f_K^2 |V_{us}|^2 m_\tau^3 \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 S_{EW}, \quad (5)$$

where  $G_F = 1.1663788(6) \times 10^{-5} \text{GeV}^{-2}$  is the Fermi coupling constant determined from muon decays [3],  $f_K = 155.7(3) \text{ MeV}$  is the kaon decay constant [6], and  $S_{EW} = 1.0232(3)$  is a short-distance electroweak radiative correction [4].

### C. $|V_{us}|$ determination from semileptonic kaon decays $K_{\ell 3}$

Semileptonic kaon  $K \rightarrow \pi \ell \nu_\ell$  decays ( $K_{\ell 3}$ ) offer another historically robust pathway for determining  $|V_{us}|$ , benefiting from a wealth of highly precise independent experimental channels. The expression for the decay width governing these decays is [4]:

$$\Gamma_{K\ell 3} \equiv \Gamma(K \rightarrow \pi \ell \nu_\ell) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{EW} \times \left(|V_{us}| f_+^{K^0 \pi^-}(0)\right)^2 I_{K\ell} (1 + \delta_{SU(2)}^{K\pi}), \quad (6)$$

where  $C_K$  is a Clebsch-Gordan coefficient ( $C_{K^0} = 1$  and  $C_{K^\pm} = 1/\sqrt{2}$ ) [4],  $f_+^{K^0 \pi^-}(0) = 0.9698(17)$  is the  $K^0 \rightarrow \pi^-$  vector form factor at zero momentum transfer [5],  $\delta_{SU(2)}^{K\pi}$  is the isospin-breaking correction for  $K_{\ell 3}$  decays ( $\delta_{SU(2)}^{K^0 \pi^-} = 0$  and  $\delta_{SU(2)}^{K^\pm \pi^0} = 0.029(4)[4]$ ). The term  $I_{K\ell}$  is the phase-space integral given by [4]:

$$I_{K\ell} = \int_{m_\ell^2}^{(m_K - m_\pi)^2} dq^2 \frac{1}{m_K^8} \lambda^{3/2} \left(1 + \frac{m_\ell^2}{2q^2}\right) \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \times \left(|\tilde{f}_+(q^2)|^2 + \frac{3m_\ell^2 \Delta_{K\pi}^2}{(2q^2 + m_\ell^2)\lambda} |\tilde{f}_0(q^2)|^2\right), \quad (7)$$

where  $q^2 = (p_K - p_\pi)^2 = (p_\ell + p_\nu)^2$  is the momentum transferred to the lepton pair,  $\lambda = (q^2 - (m_K + m_\pi)^2)(q^2 - (m_K - m_\pi)^2)$ ,  $\Delta_{K\pi} = m_K^2 - m_\pi^2$ , and the  $q^2$  functions  $\tilde{f}_+(q^2)$  and  $\tilde{f}_0(q^2)$  are, respectively, the normalized vector and scalar form factors, corresponding to the exchange of  $J^P = 1^-$  and  $J^P = 0^+$  particles.

#### 1. Form factors parametrization

Hadronic form factors model the internal, non-perturbative QCD structure of hadrons during weak interactions. For idealized, point-like particles, the transition amplitude would be entirely determined by weak electroweak vertex couplings. However, because hadrons are composite bound states of quarks and gluons, strong-interaction dynamics alter systematically the transition probabilities.

Given the smallness of  $q^2$  in  $K_{\ell 3}$  decays, form factor parameterizations based on a Taylor expansions are most widely used. For our analysis, we adopt the one from the FlaviaNet Working Group [4]:

$$\tilde{f}_{+,0}^{\text{Taylor}}(q^2) = 1 + \lambda'_{+,0} \frac{q^2}{m_{\pi^\pm}^2} + \frac{1}{2} \lambda''_{+,0} \left(\frac{q^2}{m_{\pi^\pm}^2}\right)^2 + \dots, \quad (8)$$

where the parameters  $\lambda'_{+,0}$  are the slope and curvature of the form factor. These values are found to be [4]:

$$\lambda'_+ = 2.502(112) \cdot 10^{-2}, \quad \lambda''_+ = 1.57(48) \cdot 10^{-3}, \quad \lambda'_0 = 1.334(141) \cdot 10^{-2}. \quad (9)$$

If we use single pole-like parameterizations we will obtain similar results, again due to the small  $q^2$  available in these reactions.

#### D. $|V_{us}|$ determination from $\tau \rightarrow K\pi\nu_\tau$ decays

For a  $|V_{us}|$  determination from  $\tau \rightarrow K\pi\nu_\tau$  we rely on the channel  $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$  as it is the one measured by the Belle Collaboration [9]. The theoretical expression for the differential decay distribution as function of the invariant mass of the kaon-pion system  $\sqrt{s}$  is expressed as [10]:

$$\frac{d\Gamma(\tau \rightarrow K\pi\nu_\tau)}{d\sqrt{s}} = \frac{G_F^2 m_\tau^3}{32\pi^3 s} |V_{us} f_+^{K\pi}(0)|^2 S_{EW} \left(1 - \frac{s}{m_\tau^2}\right)^2 \times \left[ \left(1 + 2\frac{s}{m_\tau^2}\right) q_{K\pi}^3(s) |\tilde{f}_+(s)|^2 + \frac{3\Delta_{K\pi}^2}{4s} q_{K\pi}(s) |\tilde{f}_0(s)|^2 \right], \quad (10)$$

where  $q_{K\pi}(s) = \frac{\sqrt{(s-(m_K+m_\pi)^2)(s-(m_K-m_\pi)^2)}}{2\sqrt{s}}$ . The function  $\tilde{f}_+(s)$  in Eq. (10) is the vector form factor, which account for the effects of vector resonance exchanges, *i.e.* for the effects of the  $K^*(892)$ . We describe it by means of a usual Breit-Wigner parametrization,

$$\tilde{f}_+(s) \equiv \frac{f_+(s)}{f_+(0)} = \frac{m_{K^*}^2}{m_{K^*}^2 - s - im_{K^*}\Gamma_{K^*}(s)}, \quad (11)$$

where  $\Gamma_{K^*}(s)$  is the energy-dependent width:

$$\Gamma_{K^*}(s) = \Gamma_{K^*} \frac{s}{m_{K^*}^2} \frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}^3(m_{K^*}^2)} \theta(s - (m_K + m_\pi)^2), \quad (12)$$

with  $\sigma_{K\pi}(s) = 2q_{K\pi}(s)/\sqrt{s}$  and  $\Gamma_{K^*}$  is the total width of the  $K^*$ . Given the available phase space in this decay (due to the large mass of the tau lepton), this process can be sensitive to a second resonance, the  $K^{*'}(1410)$  appearing at  $\sqrt{s} \approx 1410$  MeV. We add this new resonance contribution through [8]:

$$\tilde{f}_+(s) = \frac{m_{K^*}^2 + \gamma s}{m_{K^*}^2 - s - im_{K^*}\Gamma_{K^*}(s)} - \frac{\gamma s}{m_{K^{*'}}^2 - s - im_{K^{*'}}\Gamma_{K^{*'}}(s)}, \quad (13)$$

where the new parameter  $\gamma$  controls the relative weight of the  $K^{*'}$  resonance with respect to  $K^*$ , and  $\Gamma_{K^{*'}}(s)$  is the  $K^{*'}$  energy-dependent width:

$$\Gamma_{K^{*'}}(s) = \Gamma_{K^{*'}} \frac{s}{m_{K^{*'}}^2} \frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}^3(m_{K^{*'}}^2)} \theta(s - (m_K + m_\pi)^2), \quad (14)$$

where  $\Gamma_{K^{*'}}$  is the total  $K^{*'}$  width.

The function  $\tilde{f}_0(s)$  in Eq. (10) is the scalar form factor that accounts for the effects of a scalar resonance exchanges, *e.g.* the  $\kappa(700)$  [3]. It is not obvious that the

functional form of  $\tilde{f}_0(s)$  follows a Breit-Wigner, we therefore use the more sophisticated results obtained from a coupled-channel analysis from Ref. [11]. In any case, the scalar contribution is subleading.

### III. RESULTS

Employing Eqs. (3) and (4) together with the associated decay widths we obtain two values for the relation  $\frac{|V_{us}|}{|V_{ud}|}$ . For a  $|V_{us}|$  extraction from one-meson tau decays we use Eq. (5). We provide a graphical account of these three results in the  $|V_{us}|$ - $|V_{ud}|$  plane in figure 1. The

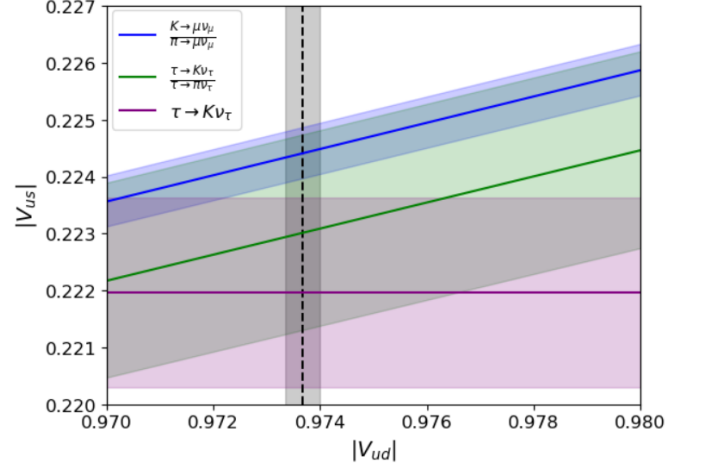


FIG. 1:  $|V_{us}|(|V_{ud}|)$  relation for leptonic and tau to meson decays. The black dashed line represents the  $|V_{ud}|$  PDG value.

results show that the uncertainties in the three determinations of  $|V_{us}|$  are dominated by the uncertainties in the corresponding branching ratios. In particular, the leptonic determination  $|V_{us}^{K_{\ell 2}}| = 0.2244(5)$  is the most precise and consistent with the PDG value (see Fig. 1). On the other hand, we can observe that the determination from the decay  $\tau \rightarrow K\nu_\tau$ ,  $|V_{us}| = 0.2220(17)$ , shows a slight deviation with respect to that from the leptonic decay  $K_{\ell 2}$ .

A  $|V_{us}|$  determination from  $K_{\ell 3}$  decays requires the integration of the phase space function in Eq. (7) for each of the six possible decay channels, *i.e.*  $K^+ \rightarrow \pi^0 \ell^+ \nu_\ell$ ,  $K_{S,L}^0 \rightarrow \pi^- \ell^+ \nu_\ell$  ( $\ell = e, \mu$ ). This value is then inserted into (6) to extract  $|V_{us}|$ . We do not provide here the individual extractions but rather a graphical account of the determinations in figure 2. One can observe that the extractions from  $K_L^0$  decays are the most precise ones while the determination from  $K_S^0 \rightarrow \pi^\pm \mu \nu_\mu$  carries a large uncertainty, due to the poor precision of the measured branching ratio. A weighted averaged result from  $K_{\ell 3}$  yields  $|V_{us}^{K_{\ell 3}}| = 0.2245(4)$ , which is compatible with the value reported by the  $|V_{us}^{PDG}| = 0.22431(85)$  [3].

The  $|V_{us}|$  determination from  $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$  decay is slightly more involved. The differential decay rate of

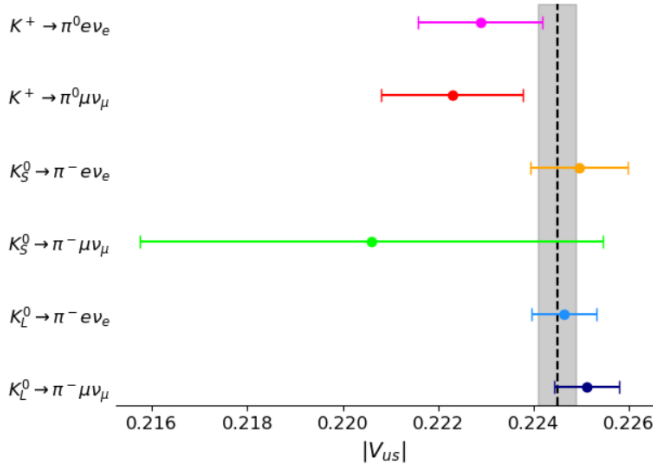


FIG. 2:  $|V_{us}|$  determination for each kaon semileptonic decay and its weighted average value shown in a black dashed line.

Eq. (10) is related to the measured number of events by the Belle Collaboration through [9]:

$$\frac{dN_{events}}{d\sqrt{s}} = \frac{1}{3} \frac{N_{events} \Delta_{bin}}{\Gamma_{\tau} BR(\tau^{-} \rightarrow K_S^0 \pi^{-} \nu_{\tau})} \frac{d\Gamma(\tau^{-} \rightarrow K_S^0 \pi^{-} \nu_{\tau})}{d\sqrt{s}}, \quad (15)$$

where  $N_{events} = 531113$  is the total number of events measured,  $\Gamma_{\tau}$  is the inverse lifetime of the tau,  $\Delta_{bin} = 0.0115$  GeV/bin is the bin width, and  $BR(\tau^{-} \rightarrow K_S^0 \pi^{-} \nu_{\tau})$  is the measured branching ratio. The fourth, fifth and sixth points were excluded from the fit following the recommendation from the experimental group, as they are difficult to bring in accord with theoretical descriptions, and the points beyond  $\sqrt{s} = 1.48$  GeV were also neglected in the fit, as the data above this energy carry large uncertainties and can distort a precision determination of  $|V_{us}|$ , which is our main objective.

The  $\chi^2$  function minimised is:

$$\chi^2 = \sum_i^{86} \left( \frac{N_{events,i}^{th} - N_{events,i}^{exp}}{\sigma_{N_{events,i}^{exp}}} \right)^2, \quad (16)$$

where  $N_{events,i}^{exp}$  and  $\sigma_{N_{events,i}^{exp}}$  are, respectively, the experimental number of events and the corresponding uncertainties in the  $i$ -th bin. The result of our  $\chi^2$  fit is displayed in figure 3 compared to the measured by Belle. A very good agreement with experimental data is observed for all data points. As seen, the  $K\pi$  spectrum is dominated by the contribution of the  $K^*(892)$  vector resonance, whose peak is clearly visible, while the scalar form factor contributes mainly only in the low-energy region.

The result of our fit yields:

$$\begin{aligned} |V_{us}| &= 0.2245(29), \quad \gamma = 0.054(11), \\ m_{K^*} &= 0.89525(19) \text{ GeV}, \quad \Gamma_{K^*} = 0.04686(42) \text{ GeV}, \\ m_{K^{*'}} &= 1.4718(144) \text{ GeV}, \quad \Gamma_{K^{*'}} = 0.120(35) \text{ GeV}. \end{aligned} \quad (17)$$

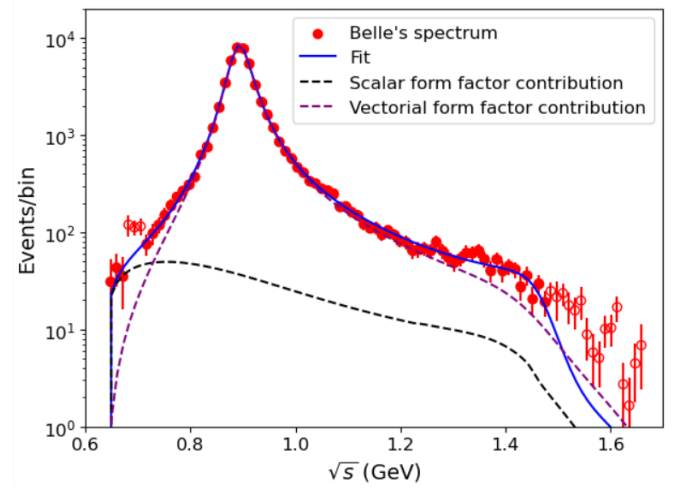


FIG. 3: Belle  $\tau^{-} \rightarrow K_S^0 \pi^{-} \nu_{\tau}$  measurement (red points) [9] as compared to our fit (blue solid line). Empty circles correspond to data points which have not been included in the analysis. The individual vector and scalar contributions are represented by the purple and black dashed lines, respectively.

The value obtained for  $|V_{us}|$  is compatible with the PDG determination although it has a large error associated. In addition to the  $|V_{us}|$  determination, we also determine the physical pole mass and width of the  $K^*$  and  $K^{*'}$  resonances. The fitted parameters  $m_{K^{*(r)}}$  and  $\Gamma_{K^{*(r)}}$  are model mass and width parameters, and their values do not directly correspond the physical ones. In order to obtain the physical parameters, *i.e.* the ones appearing in the PDG [3], one has to look for a zero of the denominator of vector form factor in the complex energy plane [8], *i.e.*:

$$M_R^2 - s - iM_R\Gamma_R(s) = 0, \quad (18)$$

where  $R$  is the resonance ( $K^*$  or  $K^{*'}$ ), and the solution of this equation is  $\sqrt{s} = M_R - i\Gamma_R/2$ . For both of our vector resonances we have obtained the following pole parameters:

$$\begin{aligned} M_{K^*} &= 0.89216(19) \text{ GeV}, \quad \Gamma_{K^*} = 0.0459(4) \text{ GeV}, \\ M_{K^{*'}} &= 1.466(14) \text{ GeV}, \quad \Gamma_{K^{*'}} = 0.118(33) \text{ GeV}. \end{aligned} \quad (19)$$

The value obtained for the physical mass  $M_{K^*}$  is very close to the value reported by the PDG, while its width is slightly below, but compatible within uncertainties. On the other hand, the values obtained for the  $M_{K^{*'}}$  mass and width differ significantly from those reported by the PDG because not all the experimental point sensitive to this resonance were included in the fit.

Overall, coming back to the main objective, from all our individual  $|V_{us}|$  determinations reported previously we obtain the following weighted average:

$$|V_{us}| = 0.22434(29). \quad (20)$$

In figure 4, we graphically compare the averaged value in Eq. (20) (black dashed vertical line in the plot) with

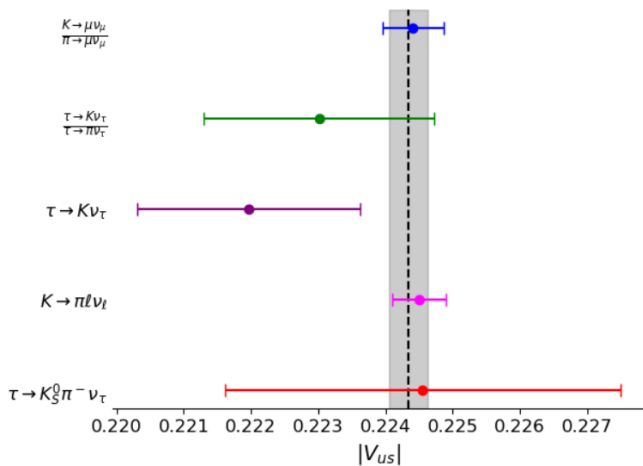


FIG. 4: Summary of the  $|V_{us}|$  determinations obtained in this work from different channels. The black dashed vertical line represents the weighed averaged result from Eq. (20).

the individual extractions. As it can be observed, this value is dominated by the extraction from semileptonic  $K_{\ell 3}$  decays and the leptonic  $K_{\ell 2}$  ratio due the small uncertainties of the measured branching ratios. Even so, computing the  $|V_{us}|$  value for each decay gives more consistency to our average value.

Finally, we are now in a position to test the CKM unitarity of the first row. Using our  $|V_{us}|$  determination from Eq. (20) we obtain:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.99838(63), \quad (21)$$

which shows a deviation of about  $2.5\sigma$  from unity.

#### IV. CONCLUSIONS

In this work, we have determined the CKM matrix element  $|V_{us}|$  from leptonic and semileptonic kaon and tau decays. In particular, we have employed the purely leptonic ratio of branching ratios  $K_{\ell 2}/\pi_{\ell 2}$ , the one-meson

tau decays  $\tau \rightarrow K\nu_\tau$  and  $\tau \rightarrow K\nu_\tau/\tau \rightarrow \pi\nu_\tau$  and the semileptonic kaon  $K \rightarrow \pi\ell\nu_\ell$  and tau  $\tau^- \rightarrow K_S^0\pi^-\nu_\tau$  decays. For the semileptonic ones, we developed a description of the corresponding vector and scalar form factors that allowed us to extract  $|V_{us}|$  and analyse the  $\tau^- \rightarrow K_S^0\pi^-\nu_\tau$  decay distribution experimental data from the Belle Collaboration.

As it can be seen in Fig. 4, the different determination of  $|V_{us}|$  are, in general, compatible within uncertainties. The most precise determinations are obtained from  $K \rightarrow \ell\nu_\ell$  and  $K \rightarrow \pi\ell\nu_\ell$  decays due to the precise experimental data while the tau-based determinations exhibit larger uncertainties. Combining all our individual determinations on a weighted average we have obtained  $|V_{us}| = 0.22434(29)$  [20], which is the main result of this work. This value is found to be in agreement with the PDG reported value  $|V_{us}^{PDG}| = 0.22431(85)$  [3].

In addition, from the experimental data analysis of the decay  $\tau^- \rightarrow K_S^0\pi^-\nu_\tau$  we have extracted the physical parameters of the participating  $K^*(892)$  and  $K^{*'}(1410)$  vector resonances [19]. The pole parameters obtained for the  $K^*$  resonance are compatible with the values reported by the PDG, whereas the  $K^{*'}$  parameters remain less constrained due to their larger uncertainties.

Overall, the results support a consistent determination of  $|V_{us}|$  from several independent processes. Future improvements could be achieved through more precise branching-ratio measurements, the inclusion of additional Standard Model corrections, and more sophisticated form-factor descriptions incorporating higher resonances and dispersive approaches.

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Una nova determinació de  $|V_{us}|$  a partir de desintegracions de kaons i taus

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**Resum:** Determinar l'element  $|V_{us}|$  de la matriu CKM constitueix una prova fonamental del Model Estàndard mitjançant la unitarietat de la primera fila. En aquest treball, s'extreu  $|V_{us}|$  a partir de desintegracions leptòniques i semileptòniques de kaons ( $K_{\ell 2}$  i  $K_{\ell 3}$ ), així com de les desintegracions exclusives  $\tau \rightarrow K\nu_\tau$  i semileptòniques  $\tau \rightarrow K\pi\nu_\tau$ . Per als canals semileptònics, s'avaluen les integrals d'espai de fases utilitzant parametritzacions estàndard dels factors de forma. Per a la desintegració  $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$ , s'ajusta l'espectre mesurat per Belle incorporant descripcions de les ressonàncies de Breit-Wigner dependents de l'energia  $K^*(892)$  i  $K^*(1410)$ . Els diferents mètodes d'extracció de  $|V_{us}|$  proporcionen resultats compatibles dins de les incerteses experimentals i teòriques. Combinant totes les determinacions s'obté una mitjana global ponderada de  $|V_{us}|=0.22434(29)$ , resultats que són compatibles amb les mitjanes mundials actuals. La prova de la unitarietat corresponent revela una desviació respecte la unitat de  $2.5\sigma$ , fet que reafirma la tensió encara existent amb les prediccions del Model Estàndard.

**Paraules clau:** Elements de la matriu CKM, desintegracions de kaons i taus, factors de forma hadrònics.

**ODS:** (4.) Educació de qualitat, (9. Indústria, innovació, infraestructures.)

## Objectius de Desenvolupament Sostenible (ODSs o SDGs)

|                                           |   |                                          |  |
|-------------------------------------------|---|------------------------------------------|--|
| 1. Fi de la es desigualtats               |   | 10. Reducció de les desigualtats         |  |
| 2. Fam zero                               |   | 11. Ciutats i comunitats sostenibles     |  |
| 3. Salut i benestar                       |   | 12. Consum i producció responsables      |  |
| 4. Educació de qualitat                   | X | 13. Acció climàtica                      |  |
| 5. Igualtat de gènere                     |   | 14. Vida submarina                       |  |
| 6. Aigua neta i sanejament                |   | 15. Vida terrestre                       |  |
| 7. Energia neta i sostenible              |   | 16. Pau, justícia i institucions sòlides |  |
| 8. Treball digne i creixement econòmic    |   | 17. Aliança pels objectius               |  |
| 9. Indústria, innovació, infraestructures | X |                                          |  |

## Appendix A

| Observable                                        | Value                       |
|---------------------------------------------------|-----------------------------|
| $BR(K^\pm \rightarrow \mu^\pm \nu_\mu)$           | 0.6356(11)                  |
| $BR(\pi^\pm \rightarrow \mu^\pm \nu_\mu)$         | 0.9998770(4)                |
| $BR(\tau^\pm \rightarrow K^\pm \nu_\tau)$         | $6.97(10) \cdot 10^{-3}$    |
| $BR(\tau^\pm \rightarrow \pi^\pm \nu_\tau)$       | 0.1082(5)                   |
| $BR(K^\pm \rightarrow \pi^0 e^\pm \nu_e)$         | 0.0507(4)                   |
| $BR(K^\pm \rightarrow \pi^0 \mu^\pm \nu_\mu)$     | 0.03352(34)                 |
| $BR(K_S^0 \rightarrow \pi^\pm e^\mp \nu_e)$       | $7.14(6) \cdot 10^{-4}$     |
| $BR(K_S^0 \rightarrow \pi^\pm \mu^\mp \nu_\mu)$   | $4.56(20) \cdot 10^{-4}$    |
| $BR(K_L^0 \rightarrow \pi^\pm e^\mp \nu_e)$       | 0.4055(11)                  |
| $BR(K_L^0 \rightarrow \pi^\pm \mu^\mp \nu_\mu)$   | 0.2704(7)                   |
| $BR(\tau^\pm \rightarrow K_S^0 \pi^\pm \nu_\tau)$ | $4.04(13) \cdot 10^{-3}$    |
| $\tau_{K^\pm}$                                    | 12.379(21) ns               |
| $\tau_{K_S^0}$                                    | $8.954(4) \cdot 10^{-2}$ ns |
| $\tau_{K_L^0}$                                    | 50.99(21) ns                |
| $\tau_{\pi^\pm}$                                  | 26.033(5) ns                |
| $\tau_{\tau^\pm}$                                 | $2.903(5) \cdot 10^{-4}$ ns |

TABLE I: Branching ratios of the decays and mean lifes of the particles used in the calculations. This data are all collected from the PDG source [3].