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ARTICLE

Examining the errors made by high school students while applying the Pythagorean Theorem

Analisando os erros cometidos por estudantes do ensino médio ao aplicar o Teorema de Pitágoras

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Abstract

This study aimed to identify the errors made by tenth-grade students while working on the Pythagorean Theorem and analyze the factors contributing to these errors. The selected sample was the tenth-grade students of Alhosn school in the Emirate of Abu Dhabi, United Arab Emirates, in the academic year 2022/2023. There were 67 samples of the work done by the students, taken as the object of the research. The data collection used were interviews, questionnaires, and tests. To understand the errors that students can make in the Pythagorean Theorem, these errors have been classified into six categories. The study results revealed that most of the students' errors were conceptual, data, response, strategy, calculation, and conclusion errors, respectively.

Palabras clave: Errors. Pythagorean Theorem. Mathematics Education. Mathematics teachers' training. Problem solving.

Resumo

Neste estudo, objetivamos identificar os erros cometidos por alunos do décimo ano ao aplicarem o Teorema de Pitágoras, além de analisar os fatores que contribuem para esses erros. A amostra selecionada foi constituída pelos alunos do décimo ano da escola Alhosn, no Emirado de Abu Dhabi, Emirados Árabes Unidos, no ano letivo de 2022/2023. O objeto da pesquisa consistiu em 67 amostras do trabalho realizado pelos alunos. Para a coleta de dados, utilizamos entrevistas, questionários e testes. Para entender os erros que os alunos podem cometer quando trabalham com o Teorema de Pitágoras, esses erros foram classificados em seis categorias. Os resultados do estudo revelaram que a maioria dos erros dos alunos foram erros conceituais, de dados, de resposta, de estratégia, de cálculo e de conclusão, respectivamente.

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Palavras-chave: Erros. Teorema de Pitágoras. Educação matemática. Treinamento de professores de matemática. Resolução de problemas.

1 Introduction

Trigonometry is an essential mathematical subject employed in other disciplines such as algebraic, geometric, and graphical thinking. It allows students to develop their cognitive abilities, including problem-solving through reasoning and improving their abilities (Sulistyaningsih; Purnomo; Purnomo, 2021). Literature shows that trigonometry is very difficult for middle school students and is one of the main reasons for failure in science and mathematics, where students make errors compared to other aspects of mathematics (Nizeyimana *et al*., 2023). Errors in solving the Pythagorean theorem Problems are caused by many basic factors that affect students' ability to properly apply mathematical concepts. Of these factors, improper procedures and methods to reach the correct answer because some students rely on unfair or incorrect solution strategies leading to the wrong conclusion (Retnawati, 2020). In addition, carelessness and haste are major obstacles when solving mathematical problems because some students ignore checking the correctness of the calculation or fail to follow the solution stages with sufficient attention (Alfitri *et al*., 2023).

In addition, the weak understanding of trigonometry concepts is an important factor, as some students find it difficult to understand geometric relations associated with the Pythagorean theorem, which negatively affects the accuracy of their solutions (Alfitri *et al*., 2023; Hidayati, 2020). The inability to convert word problems into a mathematical model and strategy is an additional challenge because students have to face difficulty in translating word problems into mathematical equations that reflect the correct relationship between the elements of the triangle (Wardhani; Argaswari, 2022; Sundayana; Parani, 2023). Inadequate understanding of trigonometric concepts, as a result of inappropriate teaching and learning methods used by teachers to teach mathematics, is an important factor leading to these errors (Obeng *et al.,* 2024). Accordingly, addressing these factors requires enhancing conceptual understanding, developing mathematical thinking skills, and following an organized and accurate approach to solving mathematical problems.

Error analysis in mathematics is more than just a means of assessing performance; it is a powerful tool for understanding how students think when solving problems and help uncover alternative ideas that may hinder effective learning. Research shows that errors are not just individual mistakes but reflect common thinking patterns among students, which, if not addressed, can lead to cumulative learning difficulties (Fuchs *et al*., 2021). By analyzing errors, students can become more aware of their thought processes, improve their ability to learn independently, develop critical thinking skills, and improve their problem-solving strategies (Seah; Horne, 2020). In addition, providing an interactive learning environment where students discuss their errors can help to gain a deeper understanding of mathematical concepts rather than relying on memorizing rules (Uesato *et al*., 2022). In this study, we focus on common errors in solving Pythagorean theorem problems, not only to identify them but also to understand their underlying causes, contributing to new insights on how to improve students' mathematical understanding and enhance their problem-solving abilities.

Understanding the types and causes of these errors is crucial for educators to design effective teaching strategies, provide targeted feedback, and help students improve their problem-solving skills. This paper focuses on a particular concept from trigonometry: understanding the Pythagorean theorem. We address the following research question: What kind of errors make tenth graders of a school in Abu Dhabi in solving Pythagorean problems?

We hope to gain some knowledge on understanding students' most common errors when solving trigonometry problems focused on using the Pythagorean theorem to contribute to the mathematics education community. To gain a more comprehensive insight into the errors made by students when applying the Pythagorean theorem, this study delved into the specific errors encountered. Six distinct errors were identified during students' engagement with the problem-solving test of the Pythagorean theorem. In the next sections, we will present our theoretical framework and state-of-the-art research contributions to studying students' errors when solving Pythagorean problems.

2 Literature Review

Researchers have explored specific recurring error types in the context of various studies examining the classification of errors made by students when applying the Pythagorean theorem. Some of these studies focused on errors, such as confusion between the concepts of hypotenuse and side or inaccuracies in calculating the side length of a triangle. Simultaneously, other studies delved into the analysis of broader error classifications, including data errors, concept errors, strategy errors, calculation errors, and careless errors.

According to the research conducted by Rudi, Suryadi, and Rosjanuardi (2020), who investigated school misconceptions about the Pythagorean Theorem, the findings indicated that students need help to grasp the definitions, explain symbols or notations associated with

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mathematical concepts, and interpret mathematical objects. Data were collected through students' answers to the test and interviews. Twenty-five students who learned the Pythagorean theorem participated in this study, and only four of them participated in the interviews. On the other hand, when solving problems related to applying the Pythagorean theorem, students demonstrated proficiency in describing procedures, algorithms, and techniques for addressing questions. Nurmeidina and Rafidiyah (2019) conducted a study examining students' difficulties in solving trigonometry problems. The results show that the students have difficulty understanding the information given to solve the problems. They make many errors in applying trigonometric concepts to answer the questions because they do not correctly calculate the results of angle comparison. Besides, they incorrectly determine the angle of contrast between the angles obtained.

According to Arivina and Jailani (2022), students need help interpreting language, misusing data (incorrect use of the data available in the problem), or distorting theory definitions, mainly when dealing with the Pythagorean Theorem. Ahmad, Al Yakin, and Sarbi (2018) proposed a more inclusive categorization of student errors related to the Pythagorean Theorem, identifying five types of errors: failure of process skills, carelessness or inaccuracies, misunderstanding of problems, errors in the use of notation, and misconceptions of concepts. This classification is consistent with the findings of Veloo, Krishnasamy, and Wan Abdullah (2015).

A research investigation by Yunis Sulistyorini (2018) examined errors in solving geometric problems among pseudo-thinking students, revealing several challenges. These identified errors encompassed misunderstandings in measuring a line segment (some students failed to recognize that the total length of a segment is the sum of two sub-segments), an inability to recognize that triangles should be right-angled, and errors in applying the Pythagorean theorem and trigonometric ratios. Moreover, students exhibited challenges in employing triangle congruence to substantiate the congruence of measurements for two angles.

Several studies have delved into a more detailed and comprehensive classification of student errors associated with the Pythagorean Theorem. For instance, Taamneh; Díez Palomar; Mallart Solaz (2024) identified five types of misconceptions: (1) data errors; (2) concept errors; (3) strategy errors; (4) calculation errors; and (5) careless errors. Also, Fahrudin and Pramudya (2019) identified five types of student errors: data errors, concept errors, strategic errors, calculation errors, and conclusion errors. Similarly, Supardi (2021) identified four misconception types: word use error, visual mediator error, narrative error, and routine error. In the same vein, scholars have employed the NEA classification (Newman Error Analysis) to

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classify some of the errors made by students. This classification distinguishes errors in reading, comprehension, transformation, process skills, and encoding (Hutapea; Suryadi; Nurlaelah, 2015; Sari; Wutsqa, 2019; Zulyanty; Mardia, 2022).

Indicator	Name of the error
Indicator 1	Finding the unknown side length in a right-angled triangle
Indicator 2	Determine the hypotenuse of a right-angled triangle
Indicator 3	Determining the lengths of the sides in a right-angled triangle (45º -45º- 90º)
Indicator 4	Determining the lengths of the sides in a right-angled triangle (30º -60º - 90º)
Indicator 5	Distinguishing a right-angled triangle from other triangles
Indicator 6	The geometric meaning of the Pythagorean theorem
Indicator 7	Verify that a triple is a Pythagorean triple
Indicator 8	Correct understanding of the Pythagorean theorem based on the given information
Indicator 9	Finding the trigonometric ratios of a right-angled triangle (sin, cos, tan, cot, sec, csc)
Indicator 10	Finding the value of the angle using the inverse of the trigonometric functions (arcsin, arccos, arctan)
Indicator 11	Finding the area of a right-angled triangle

Frame 1 - List of potential errors
Source: Author elaboration (2023)

According to most of those studies, the most common error that students make when solving Pythagorean problems is related to the modelling phase of the problem (analyze the problem and transform it into a mathematical, theoretical, or visual model that helps in understanding and solving it). Most students deal with the algorithms, showing a need for more understanding of the relationships between the hypothenuse and the legs of a right triangle. In addition, previous studies also provide evidence that students also need help with solving the tasks. Some students need help with notation, or they cannot correctly label the sides of the triangle or use the algorithms.

Drawing on this previous research, we aim to investigate how tenth graders in a school in Abu Dhabi solve Pythagorean problems to identify what errors they make (if any) and whether they are consistent with previous research. In addition, we expect to develop a more detailed taxonomy of students’ errors to provide a methodological tool to the scientific community for studying students’ errors when solving this kind of Pythagorean problem. This classification will contribute to a deep understanding of students' misconceptions while dealing with the Pythagorean theorem problems, enabling researchers and teachers to analyze these errors more accurately. As a result, it can support the development of more effective teaching strategies in mathematics education, contributing to students' learning and understanding of basic mathematical concepts.

Drawing on the literature review, we collected a list of errors that previous scholars have raised in their research. Most of them had similarities. For this reason, we started our study by creating an analytical list of errors and their definitions, identifying their commonalities, and aiming to obtain a systematic list of errors to analyse the data we collected later in our research. In Frame 2, we collect all the primary types of errors raised by the review of the previous research, and we discuss them, looking for commonalities to create the taxonomy. We selected scientific papers using a systematic literature review procedure, looking for papers in the main scientific repositories, including *Web of Science*, *Scopus* and *Google Scholar*.

Previous Research	**Type of Errors**	**Correspondence with our Classification**
Hanggara; Agustyaningrum; Ismayanti, (2024)	Conceptual errors Procedural errors Principal errors	Conceptual error Strategy error Concept error
Sekgoma (2023)	Calculations error Misidentification error Interpretation error	Calculations error Conceptual error Conclusion error
Arivina and Jailani(2022)	Misinterpret language Misuse data Distort the theorem definition	Data error Data error Conceptual error
Setiawan (2022)	Misconception error Miscalculation error Fact errors	Conceptual errors Calculations errors Conceptual error
Supardi (2021)	Word use error Visual mediator error Narrative error	Data error Data error Conceptual error
Fahrudin; Pramudya (2019)	Data error Conceptual error Strategy error Calculation error Conclusion error	Data error Conceptual error Strategy error Calculation error Conclusion error
Hutapea; Suryadi; Nurlaelah, (2015); Sari; Wutsqa (2019); Zulyanty; Mardia (2022)	Decoding Comprehension Transformation Process skill Encoding	Data error Conceptual error Conceptual error Calculation error Calculation error
Veloo; Krishnasamy; Wan Abdullah (2015)	Conceptual errors Careless errors Problem-solving errors Value errors	Conceptual errors Careless errors Strategy error Calculation error

Frame 2 – Regrouping previous classifications' research on students ‘errors with our classification
Source: Author elaboration (2023)

3 Methods

This study employed mixed methods. The qualitative part aims to understand the behaviour, perception, and actions of the sample selected. This qualitative analysis involves describing these phenomena using words. In particular, the focus is on analyzing students' errors in trigonometry and identifying the underlying factors that lead to these errors. Qualitative

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analysis in this study included (a) semi-structured interviews with students to understand how they thought while solving problems related to the Pythagorean theorem and (b) classroom observations to monitor their interactions during the problem-solving in order to identify and classify the types of errors they analyzed in later tests. These instruments helped to achieve a broad view of the causes of nature and errors. This quantitative analysis allowed us to calculate and present the percentage distribution of student errors for each type.

3.1 Participants

This study was conducted at Al Hosn Secondary School in the Emirate of Abu Dhabi, United Arab Emirates. The research sample consisted of 67 tenth-grade students divided into two classes: the first class had 34 students representing the experimental group, and the second class had 33 students representing the control group.

3.2 Data Collection Tools

Students' works for their solutions were collected using their written responses to the pre-tests, where students were asked to write down the steps of their solution and explain their justifications when answering the questions. Interviews with students after the test and semi-structured interviews were conducted with a group of students to explore the reasons for their errors and how they justified their solutions, which provided a deeper understanding of their thinking strategies.

By analyzing students' works while working on Pythagorean Theorem problems and semi-structured student interviews, we came up with a specific *list of errors* that students may (or may not) make when solving problems related to the Pythagorean theorem (Frame 3). We define a data error as the error students commit when they write down existing problem data or mistranslate it. Conceptual error occurs when students incorrectly determine the formula, theorem, or definition to answer the problem (i.e., students do not write formulas or theorems, misuse them, etc.). Strategy error occurs when a student uses inappropriate procedures and methods to work at the correct level to solve a mathematical problem or when students try to work at the correct level but choose inappropriate data. Calculation errors happen when students give or miswrite signs of mathematical operation. Students incorrectly count math operations such as adding, subtracting, multiplying, and dividing. Conclusion errors occur when students determine conclusions incorrectly or do not draw conclusions. Finally, a response error happens

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when a student fails to offer a correct response, along with a valid explanation for the problem, or if he chooses not to address the problem at all.

Types of Errors	Indicator
Data Error	Students are writing down existing problem data wrongly. Students are translating existing problems wrongly.
Concept Error	Students are determining the formula or theorem or definition to answer the problem incorrectly. Students do not write formulas or theorems or definitions to answer problems.
Strategy Error	Students try to operate at the right level on a problem but use procedures or methods that are not appropriate. Students try to operate at the right level but choose inappropriate data information.
Calculation Error	Students are giving or writing signs of mathematical operation incorrectly. Students are wrong in counting maths operations such as adding, subtracting, multiplying, and dividing wrongly.
Conclusion Error	Students are determining conclusions incorrectly Students do not write conclusions

Frame 3 – Indicator of error' type
Source: Author elaboration (2023)

For each type of error, we defined a list of indicators to collect evidence in the students' worksheets. This non-exhaustive list of indicators may be expanded in further research steps.

In this study, grounded theory was used to analyze the errors made by secondary school students when applying the Pythagorean theorem. Deductive and inductive approaches were combined to ensure accurate classification of errors while allowing for discovering new patterns that may not be included in traditional classifications. The analysis began with an initial list of errors drawn from previous literature. However, using an inductive approach, data from student worksheets, semi-structured interviews, and classroom observations were analyzed, which helped identify emerging errors that were not previously anticipated. This approach was chosen for its flexibility in exploring individual differences in mathematical thinking and its ability to gain a deeper understanding of the nature and causes of errors, which contributes to the development of more effective instructional strategies to correct these errors and better support student learning.

3.3 Analysis Process

Based on these errors, shown in Frame 1, a pre-test was created to identify the mistakes made by Grade 10-th students at Al Hosn School (Frame 4). The pre-test consisted of 8 questions, most of which were multiple-choice, with a rationale provided for each of these answers. The first question pertains to determining the area of the square formed on the

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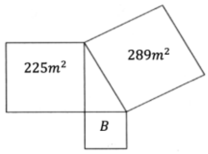
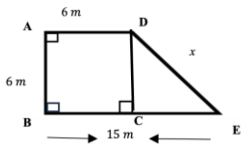
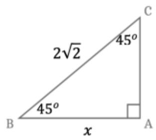
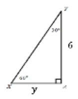
hypotenuse, while the second question involves confirming the idea that a Pythagorean triangle has a right angle. Following this, the third question entails finding the length of the hypotenuse in a right-angled triangle, and the fourth question involves determining the length of an unknown side in a right-angled triangle (45º - 45º - 90º). Additionally, the fifth question revolves around finding the length of an unknown side in a right-angled triangle (30º - 60º - 90º). Moving forward, the sixth question focuses on calculating the area of a rectangle, while the seventh question aims to ascertain the unknown angle in a right-angled triangle. Finally, the eighth question involves identifying the type of triangle based on the length of its sides.

ID	Question	Mathematical Objective / Concept
01	Find the area of square B in the below figure. (justify your answer). 	The geometric meaning of the Pythagorean theorem.
02	Which of these sides form a right-angle triangle? (justify your answer). (a) 5, $\sqrt{8}$, 33 (b) 8, 15, 17 (c) $\sqrt{2}$, $\sqrt{2}$, 2 (d) 4, 8, $\sqrt{32}$	Verify that the triple is a Pythagorean triple.
03	The diagram below shows a trapezium ABCD, find the length of hypotenuse of triangle DCE to the nearest tenth.  (a) 16.2 m; (b) 8.5 m; (c) 13.7 m; (d) 10.8	Finding the unknown side length in a right-angled triangle. Determine the hypotenuse of a right-angled triangle.
04	Find the unknown side length in the right-angle triangle to the nearest whole num-ber? (justify your answer).  (a) $2\sqrt{2}$; (b) 4; (c) $4\sqrt{2}$; (d) 2	Determining the lengths of the sides in a right-angled triangle (45 - 45 - 90).
05	What is the value of the length y in the right-angle triangle? 	Determining the lengths of the sides in a right-angled triangle (30 - 60 - 90).

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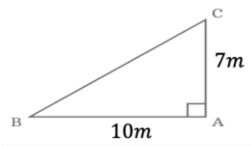
	(a) $6\sqrt{3}$; (b) $2\sqrt{3}$; (c) 6; (d) 12	
06	The high of the triangle is 17 cm less than the length of its base, and the length of the hypotenuse is 25 cm, find the area of this triangle. (justify your answer).	Correct understanding of the Pythago-rean theorem based on the given in-formation. Finding the area of the right-angled triangle.
07	Find the measure of < B to the nearest degree.  (a) 44 m; (b) 55 m; (c) 46 m; (d) 35 m	Finding the trigonometric ratios of a right-angled triangle (sin, cos, tan, cot, sec, csc). Finding the value of the angle using the inverse of the angle.
08	The sides of a figure have 12 cm, 18 cm, and 20 cm, what type of figure is it. (justify your answer). (a) Right triangle (b) Acute triangle (c) Obtuse triangle (d) Not triangle.	Distinguishing a right-angled triangle from other triangles.

Frame 4 – Pre- test details according to mathematical concepts.
Source: Author elaboration(2023)

To analyze the data on the test, students received two points for selecting the correct answer, accompanied by a valid justification. Those who offered the correct answer without justification or provided an incorrect answer with a proper justification received one point. However, if a student neither selected the correct answer nor provided appropriate mathematical justification, they were awarded zero marks. For instance, a student solving for the hypotenuse length without justifying the answer would earn one mark, while two marks would be granted if the correct justification was provided; otherwise, the score would be zero.

4 Results

4.1 General overview of the results

The Tables 1 and 2 show the results of the two groups (experimental and control) participating in the pre-test. Table 1 shows the initial analysis set (The arithmetic means and standard deviations for each of the eight test questions), and the difference in the error level between the two groups (experimental and control groups) was determined for each error type.

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Table 1 – Mean and Standard Deviation (n = 67)

	Experimental Group		Control Group	
Variable	**Mean**	**SD**	**Mean**	**SD**
Q1	0.88	1.0	0.67	0.92
Q2	0.79	0.84	1.33	0.85
Q3	0.68	0.77	1	0.90
Q4	0.32	0.53	0.12	0.33
Q5	0.47	0.56	0.27	0.45
Q6	0.03	0.17	0.09	0.29
Q7	0.47	0.51	0.58	0.50
Q8	0.24	0.43	0.12	0.41
Percent average	0.49	0.60	0.52	0.58

Source: Author elaboration (2023)

According to Table 1, the highest percentage of errors committed by students in both the experimental and control groups was in the sixth question (finding the trigonometric ratios of a right-angled triangle (sin, cos, tan, cot, sec, csc)), with an arithmetic mean of 0.03 and 0.09 for the two groups, respectively, and finding the value of the angle using the inverse of the angle. The lowest percentage of errors committed by the experimental group was in the first question: the geometric meaning of the Pythagorean theorem (mean = 0.88), and the second question verified that the triple is a Pythagorean triple for the control group (mean = 1.33).

When we focus on the type of errors and compare the results obtained by the two groups, we can see that the proficiency level in solving the problems is similar: the experimental group obtains slightly better results than the control group (see Table 2). However, for both groups, the data suggest that students usually provide the correct answer without additional justification or explanation. If we focus on the errors made by the students, then for the experimental and control groups, the most common errors were conceptual and data errors. In the experimental group, students committed data errors at a rate of 36.4%, compared to 32.2% in the control group. Regarding conceptual errors, the experimental group had a rate of 54.8%, whereas the control group recorded a higher rate of 61.4%. In contrast, students more often applied the Pythagorean theorem, calculation, and data errors (finding the area of a right-angled triangle, determining the lengths of the sides in a right-angled triangle (45º-45- 90º), and determining the lengths of the sides in a right-angled triangle (30º -60º-90º). In contrast, the students' data revealed errors from carelessness, making up a substantial share of their overall mistakes. These errors were primarily characterized by failing to answer questions and lacking suitable justifications for their solutions.

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Table 2 – Percentage of students' errors (Number of the students/percent)

	Experimental Group						Control Group					
Variable	**Data**	**Concept Error**	**Strategy Error**	**Calculation Error**	**Conclusion Error**	**Careless Error**	**Data Error**	**Concept Error**	**Strategy Error**	**Calculation Error**	**Conclusion Error**	**Careless Error**
Q1	13 38.2%	23 67.6%	8 23.5%	6 17.6%	0 0%	2 5.9%	20 60.6%	31 93.9%	16 48.5%	4 12.1%	0 0%	1 3%
Q2	15 44.1%	28 82.4%	2 5.9%	0 0%	8 23.5%	2 5.9%	2 6.1%	24 72.7%	0 0%	5 15.2%	3 9.1%	5 15.2%
Q3	19 55.9%	15 44.1%	7 20.6%	4 11.8%	0 0%	9 26.5%	8 24.2%	19 57.6%	8 24.2%	3 9.1%	0 0%	9 27.3%
Q4	8 23.5%	13 38.2%	4 11.8%	2 5.9%	0 0%	20 58.8%	15 45.5%	18 54.5%	10 30.3%	1 3%	1 3%	15 45.5%
Q5	10 29.4%	22 64.7%	3 8.8%	5 14.7%	3 8.8%	12 35.3%	10 30.3%	15 45.5%	4 12.1%	1 3%	2 6.1%	17 51.5%
Q6	21 61.8%	16 47.1%	8 23.5%	5 14.7%	0 0%	11 32.4%	22 66.7%	16 48.5%	4 12.1%	10 30.3%	0 0%	11 33.3%
Q7	3 8.8%	18 52.9%	1 2.9%	0 0%	2 5.9%	15 44.1%	3 9.1%	19 57.6%	0 0%	0 0%	0 0%	13 39.4%
Q8	10 29.4%	14 41.2%	0 0%	0 0%	6 17.6%	18 52.9%	5 15.2%	20 60.6%	0 0%	1 3%	4 12.1%	12 36.4%
Percent Average	36.4%	54.8%	12.1%	8.1%	7%	32.7%	32.2%	61.4%	15.9%	9.5%	3.8%	31.5%

Source: Author elaboration (2023)

Now, we provide a sample to illustrate the different types of errors in the students' work collected in our study.

4.2 Data error

Data error occurs when students write incorrectly the data existing in the problem or wrongly represent the data in the figure. Figure 1 illustrates a sample of errors the students most commonly make. In Figure 1 left, the student made an error in translating the data in the problem when he made the length of one side equal 2, but the correct response is that the length is 2 meters less than its hypotenuse; also, he made the same error in the other side, he supposes this

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side equal 4, whereas the length of this side is 4 meters less than the hypotenuse. The student in the problem had the correct assumption for the hypotenuse, which is x. The student needed to understand how to form equations. In Figure 1, right, the student needed to correctly determine the triangle's height value; he supposed the height was 17 cm instead of 17 cm less than the length of its base. Another error the student made in this problem was that he didn't find the area of this triangle; maybe he forgot that or didn't know the meaning and formula of area. The errors of this solution are related to the correct understanding of the Pythagorean theorem based on the given information and misconceptualizing the area of the right-angle triangle.

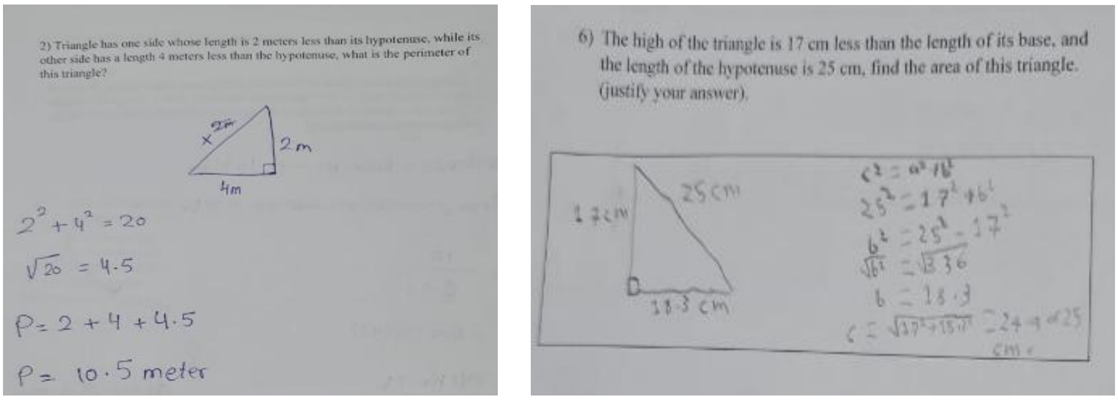

Figure 1 – Data Error
Source: Author elaboration (2023)

4.3 Conceptual error

Another set of errors that are common among students is conceptual errors. Misconceptions occur when students are wrong or do not write formulas, theorems, or definitions to answer the problems. In Figure 2 left, the student uses the cosine to figure out the length of the side shown in the picture with an x. However, s/he is incorrectly using the formula: instead of writing the value for the hypotenuse in the denominator, s/he is doing it oppositely. In the second example (Figure 2 right), the student assumes that 12, 18, and 20 are Pythagorean triples; thus, s/he uses the Pythagorean theorem to prove (or not) that those numbers cannot be the sides of a triangle.

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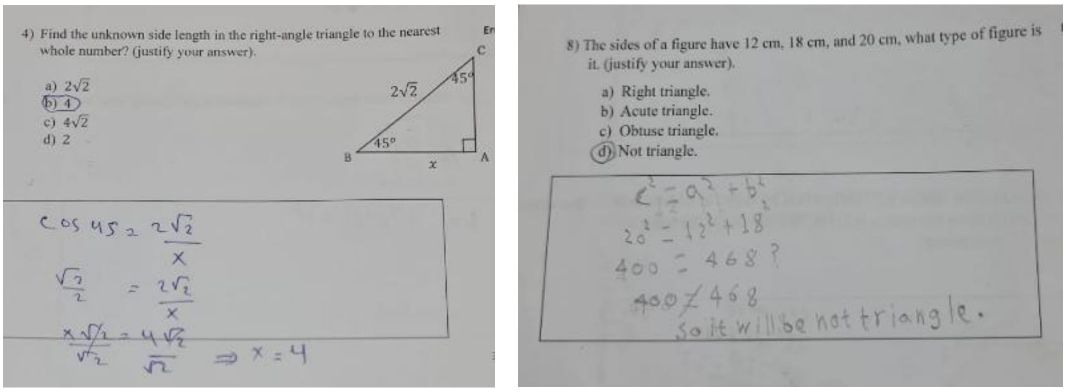

Figure 2 – Conceptual Error
Source: Author elaboration (2023)

What happens is that when s/he uses the Pythagorean formula ($c^2 = a^2 + b^2$) the result is the sum of the lengths of the two legs of the right triangle is not equivalent to the square of the hypotenuse. Hence, s/he concludes that those numbers do not correspond to a triangle. However, to build a triangle, the sum of two sides must be greater than the third one (triangular inequality: a+b>c; b+c>a; a+c>b). The triplet of this problem satisfies that requirement; hence, 12, 18, and 20 form a triangle. Again, the student's answer is wrong in this case because s/he is generalizing a theorem (and ignoring other rules).

4.4 Strategy error

This type of error occurs when a student uses inappropriate procedures and methods to work at the correct level to solve a mathematical problem or when students try to work at the correct level but choose inappropriate data. In Figure 3 left, the student fails in the method: he must calculate ½·x· $(x - 17)^2$, after calculating the value of x. Moreover, the student did not calculate the value of x because he tried to work on the problem appropriately, but he struggled in algebra, analyzing the square of the binomial (. The student believed that the analysis of the algebraic expression is done by squaring both x and the number 17, forgetting the middle term of the expression 34x.

In Figure 3 right, the student tried to work out the question correctly, writing down the law of the Pythagorean theorem but made a data error when he assumed the height to be 17 cm instead of (x – 17).

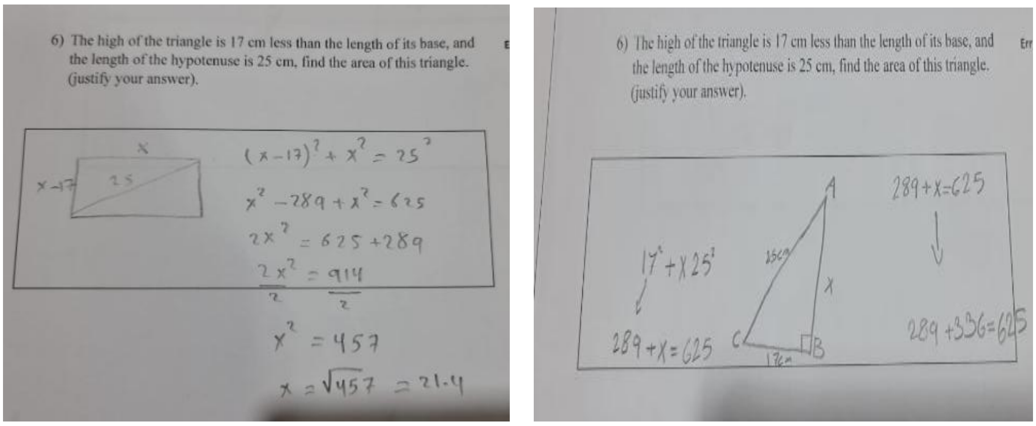

Figure 3 – Strategy Error
Source: Author elaboration (2023)

4.5 Calculation error

Calculation errors occur when students incorrectly give or write signs of mathematical operations or when math operations such as adding, subtracting, multiplying, and dividing are wrong. Figure 4 illustrates a different type of error: calculation error. The student in Figure 4 left makes several mistakes: first, s/he misuses the Pythagorean theorem. In addition (and this is why we coded it as *calculation error*) the result of the square root ($\sqrt{297}$) is not 16.2 (but 17.2). His/her answer is wrong because s/he has translated the problem wrongly, and then s/he makes a mistake in the calculation. The case reported in Figure 4 right used to be common: many students commit several errors in the same problem.

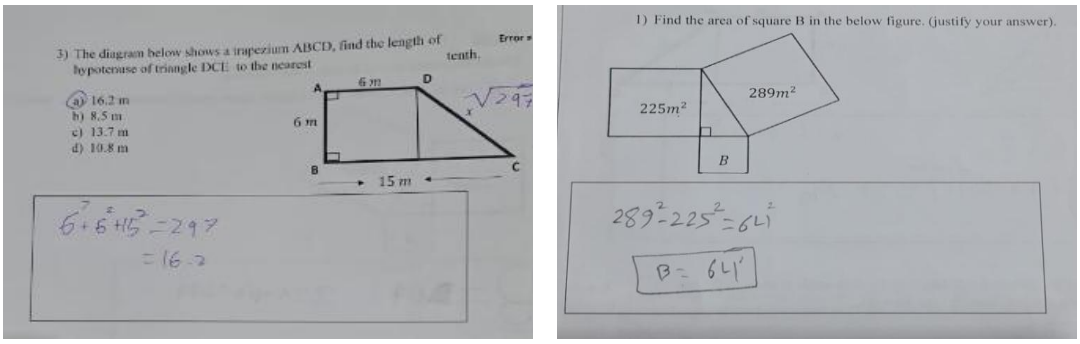

Figure 4 – Calculation Error
Source: Author elaboration (2023)

4.6 Conclusion Error

We also identified errors committed when the students delivered their answers (conclusions) to the problem. We called them *conclusion errors*. This error occurs when

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Students determine conclusions incorrectly or do not write conclusions.

In Figure 5 (left), the student worked correctly. He wrote and applied the law of the Pythagorean theorem and moved in the correct direction, but he made a mistake in assuming that if the square of the hypotenuse length is not equal to the square of the lengths of the two tangents, then these lengths do not represent a triangle. The correct thing is that if the square of the hypotenuse is less than the square of the lengths of the right sides, then the triangle is obtuse. The student in Figure 5 right struggled to solve the question, and this is shown by the incorrect answer to the question. The student made a deduction error when incorrectly assuming that angle 30ºequals 6, so angle 60ºmust equal 12. He tried to apply the Pythagorean theorem through the values he assumed wrongly and rounded the answer √180 to the last choice, 12.

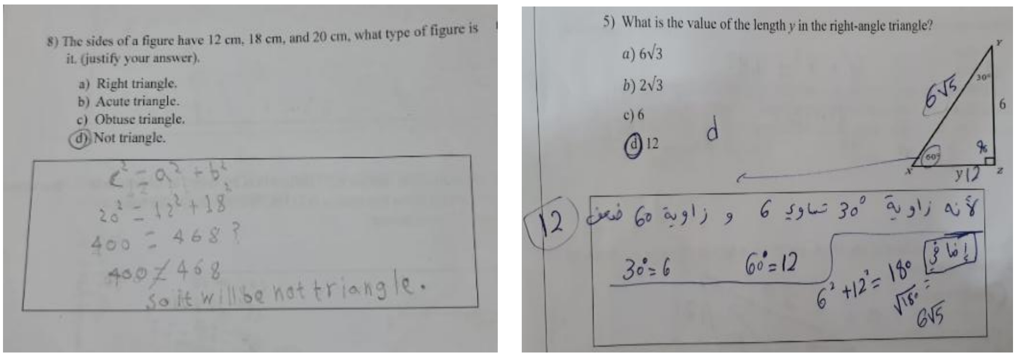

Figure 5 – Conclusion Error
Source: Author elaboration (2023)

4.7 Response Error

Finally, we pinpointed errors when the student either failed to solve the question and provide a valid rationale for their answer or simply left the question unanswered. We referred to these errors as *careless errors*. In Figure 6 the student did not present the correct answer or a justified explanation for the problem. They made a careless error by multiplying the two numbers in the problem without reading or understanding the problem and its requirements. Conversely, in this Figure, Fithe student answered the question without trying to explain their response or justify it.

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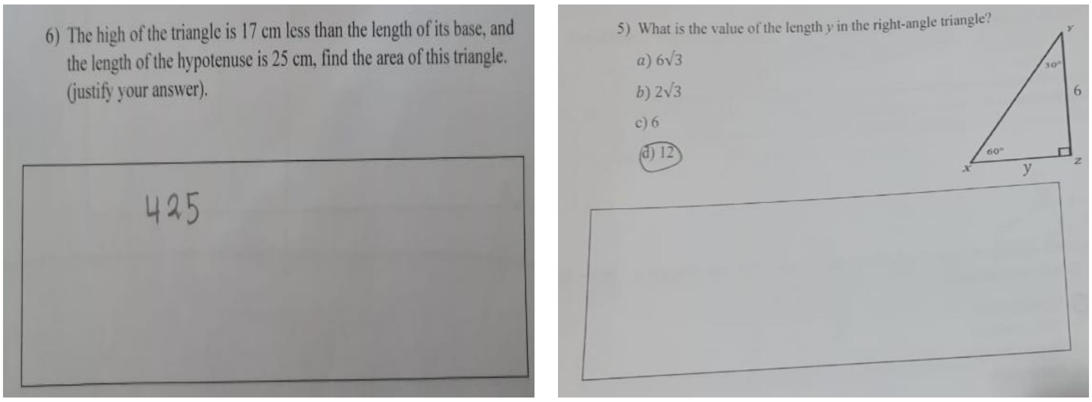

Figure 6 – Response Error
Source: Author elaboration (2023)

5 Discussion and Conclusions

Answering our research question (What are 10th-grade students' errors in a school in Abu Dhabi working with the Pythagorean theorem?), we noticed that those students need help applying trigonometry (the rates and the relationship associated with a right triangle). Consistently with previous studies, our data also suggest that students deal with the modelling phase of the problem. In many cases, the students' main difficulty was understanding the question correctly, where students faced difficulty in converting mathematical problems into a mathematical model (Utami; Dewi; Widodo, 2024, Kusmayadi; Sahara; Fitriana, 2022). Previous research suggests that students need help with mathematical representations, visualization, and symbols. But they also need help understanding the mathematical relations behind a problem (mathematical connection, correct use of the properties, the theorems, definitions, etc.). Applying the Pythagorean theorem and data error are the two most common types of errors. Evidence also suggests that calculation errors appeared in combination with other errors. While analyzing the students' data related to their work on the Pythagorean Theorem, we identified a novel category known as *careless errors*. These errors were conspicuously prevalent in the students' submissions. The student's inclination to commit these errors is rooted in their neglect of the task and lack of enthusiasm for problem-solving. Alternatively, it may be attributed to their inability to adequately resolve the issue and provide a suitable justification for their solutions.

A potential explanation for some of the errors identified is the need for more practice. According to Nuriyansyah (2023), mastering trigonometric concepts requires much practice. Students may need to practice more or only practice memorizing formulas without understanding the underlying concepts. We found evidence consistent with this type of error in

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our data (Taamneh; Díez Palomar; Mallart Solaz, 2024; Fahrudin and Pramudya, 2019). We also identified more general sources for students' errors, including aspects related to the specialized knowledge of teaching mathematics (lack of algebraic skills, difficulty understanding the relationship between trigonometric functions, confusion between different angle measures, inability to recognize patterns, etc.).

In addition, the scientific literature also highlights that some students need help solving problems because they find formulas difficult to memorize (Mursal; Husnianti; Bahar, 2023). The Pythagorean theorem is often introduced as a relationship between the sides and hypotenuse of a right triangle using the formula, which can be an abstract expression disconnected from its real meaning ($a^2=b^2+c^2$).

In further analysis of the data collected, a more detailed description (and explanation) of the errors is needed to figure out actions (supported by the scientific evidence collected) to inform (and support better) in-service and pre-service teachers in their work. Also, a more detailed analysis is needed to expand our discussion to include other learning aspects, such as the dimension of affect (lack of self-confidence, lack of motivation, fear of math etc.). We also recognize the limitations of this study that cannot be generalized.

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Author Contributions

All authors contributed substantially to the conception and planning of the study; to the acquisition, analysis, and interpretation of data; to the writing and critical revision; and approved the final version to be published.

Data Availability

The data generated or analyzed during this study are included in this published article.

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