Synchrotron emission in stellar systems: the case of GRS 1915+105

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Abstract: We present the model proposed by van der Laan (1966) to describe the evolution of the flux emitted by a spherical cloud in adiabatic expansion whose electrons follow a power law $N(E) \propto E^{-p}$. The model is tested in order to apply it to the light curves corresponding to the data obtained on May 15, 1997 from observation on the X-ray binary star system GRS 1915+105.

Key words: Synchrotron emission, van der Laan model, GRS 1915+105.

I. INTRODUCTION

Electrons accelerated under a magnetic field emit radiation, namely synchrotron radiation when their velocities are relativistic. Considering a uniform spherical clould of radius R with a flux of electrons at relativistic energies following an isotropic velocity distribution and an energy distribution $N(E) \propto E^{-p}$, for energies E between $E_1(t)$ and $E_2(t)$, the model predicts that the cloud is initially compact, making it optically thick at all radio frequencies. The spectral index α is defined as $\alpha = \frac{1-p}{2}$.

The flux radiated by the cloud depends on the frequency at which it is observed due to the dependance of optical depth —a measure of how much radiation is absorbed — on frequency:

$$\tau_\nu = \left(\frac{\nu}{\nu_1}\right)^{-(p+4)/2} \quad (1)$$

Where ν_1 is the frequency at which the optical depth is 1. Then, the radiated flux for a given frequency looks like:

$$I_\nu = S(\nu_1)\left(\frac{\nu}{\nu_1}\right)^{5/2}\left[1 - e^{-\left(\frac{\nu}{\nu_1}\right)^{-\frac{p+4}{2}}}\right] \quad (2)$$

Thus,

$$\text{for } \nu \ll 1,\ I_\nu \propto \nu^{5/2}$$
$$\text{and for } \nu \gg 1,\ I_\nu \propto \nu^{\frac{1-p}{2}}$$

By solving $\left(\frac{dI_\nu}{d\nu}\right) = 0$, we obtain ν_m, the frequency at which the radiated flux is maximum. This frequency ν_m is such that it satisfies

$$e^{\tau_m} = 1 + \frac{p+4}{5}\tau_m$$

p	τ_m
1.2	0.078
1.4	0.152
1.6	0.223
1.8	0.290
2.0	0.354

p	τ_m
2.2	0.416
2.4	0.475
2.6	0.532
2.8	0.586
3.0	0.639

TABLE I: Values for τ_m depending on p.

where τ_m stands for τ_{ν_m} (see table I).

Additionally, we have that

$$\frac{\nu_1}{\nu_m} = \tau_m^{\frac{2}{p+4}}$$

II. VAN DER LAAN MODEL

The adiabatic expansion model for the evolution with time of a spherical cloud of radius $R = R(t)$ implies, as is natural, that the angular diameter evolves like

$$\theta = \theta_0\left(\frac{R}{R_0}\right), \quad (3)$$

While the magnetic flux is conserved:

$$B = B_0\left(\frac{R}{R_0}\right)^{-2}, \quad (4)$$

And, since the relativistic gas cools adiabatically, we obtain

$$E = E_0\left(\frac{R}{R_0}\right)^{-1}, \quad (5)$$

Additionally, the number of particles inside the cloud is assumed to remain constant.

The fraction R/R_0 that appears on (3), (4) and (5) will be denoted by ρ, the relative radius.

In the model proposed by van der Laan in 1964 [2], the maximum flux density depends on the radius as

$$\nu_m(\rho) \propto \rho^{-(4p+6)/(p+4)} \quad (6)$$

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In particular, $\nu_m(\rho) = \nu_{m_0}\rho^{-(4p+6)/(p+4)}$, where $\nu_{m_0} = \nu_m(\rho = 1)$.

Besides, the value of the flux at frequency ν_m is

$$S_m(\rho) \propto \rho^{-(7p+3)/(p+4)} \tag{7}$$

Thus, one can easily observe that the spectral curve does not change its shape but moves down and to the left in the $\log S(\nu) - \log \nu$ representation, to lower flux densities and lower frequencies.
In practical terms, this means that observing any given source at one frequency will show a quick increase in the flux density at first. After reaching its maximum, the decrease will be softer. Simultaneous observation at a lower frequency will show the same fractional rate of increase, but the maximum will be reached later and have a smaller value.

The flux density, relative to its maximum value, is:

$$S(\nu,\rho) \propto \left(\frac{\nu}{\nu_m}\right)^{5/2} \rho^3 \, \frac{1 - \exp\left[-\tau_m \left(\frac{\nu}{\nu_m}\right)^{-\frac{p+4}{2}} \rho^{-(2p+3)}\right]}{1 - \exp\left(-\tau_m\right)} \tag{8}$$

We can observe some examples of such behaviour in Figures 1, 2 and 3:

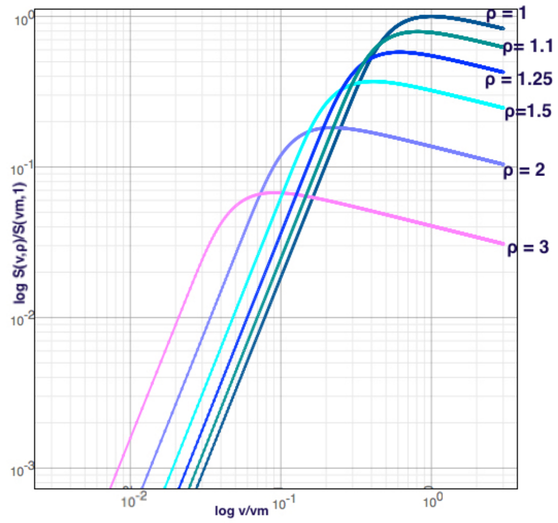

Figure 1: For $p = 1.5$ and different values for ρ, both figures depict how the flux density evolves with ν, for some values of ρ, in log-log representation.

In order to study the flux density dependance on frequency and time, we need to define the time dependance of the expansion velocity.
Assuming that the cloud expands linearly with time ($R = R_0 + vt$), since the source we are going to study (the X-ray binary star system GRS 1915+105 [6]) is assumed to expand at $\simeq 0.2c$, from observation at larger

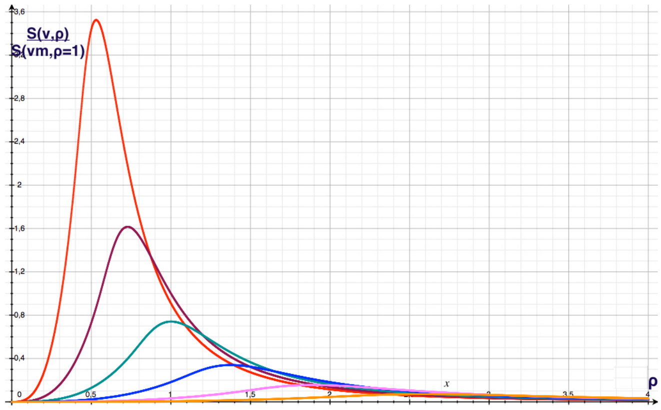

Figure 2: For $p = 1.5$, the figure shows how the flux at different frequencies evolves with ρ. From top to bottom, the values for ν/ν_m are 2, 1, 1/2, 1/4, 1/8 and 1/16, respectively.

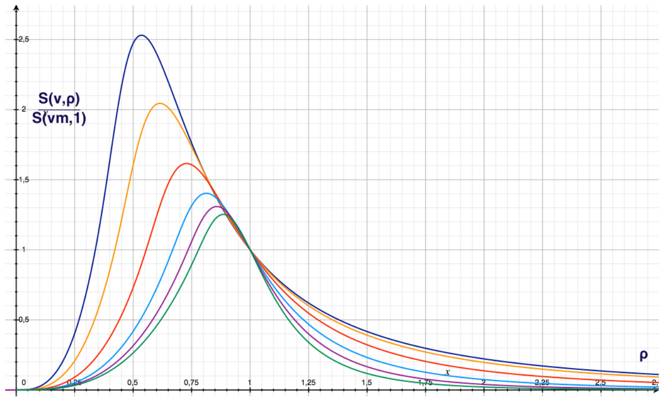

Figure 3: For $\nu = \nu_m$, the figure depicts the evolution of the flux density with ρ at different frequencies, given different values for p. From top to bottom, the values for p are 1.1, 1.2, 1.5, 2.0, 2.5 and 3.0, respectively.

scale ejection, we have:

$$\rho(t) = \frac{R}{R_0} = 1 + \frac{0.2\,c\,t}{R_0} \tag{9}$$

The maximum value for the flux at a given frequency follows

$$S_m(\nu_2) = S_m(\nu_1)\left(\frac{\nu_1}{\nu_2}\right)^{-\frac{7p+3}{4p+6}} \tag{10}$$

Equivalently,

$$S_m(\lambda_2) = S_m(\lambda_1)\left(\frac{\lambda_2}{\lambda_1}\right)^{-\frac{7p+3}{4p+6}} \tag{11}$$

Whatsmore, this maximum will be reached at

$$t_{m,\lambda} \propto \lambda^{\frac{p+4}{4p+6}} \tag{12}$$

Relations expressed by equations 11 and 12 can be applied to any source whose spectrum is variable and known over a fraction of the expansion age. Then, future spectral changes become predictable (although for later times they depend on the expansion mode).

$\lambda = 2$cm, for a span of one hour), when the values for p are around 1.5.

With the help from the relations in Section II, first we will model one of the flares from Figure 4. Then, by subtracting the *contamination* (the nonintrinsic value added by the contiguous flare to the one we are studying), we will be able to obtain the values for $S_{m,\lambda}$ and $t_{m,\lambda}$ at each frequency, which will allow us to obtain the value for p.

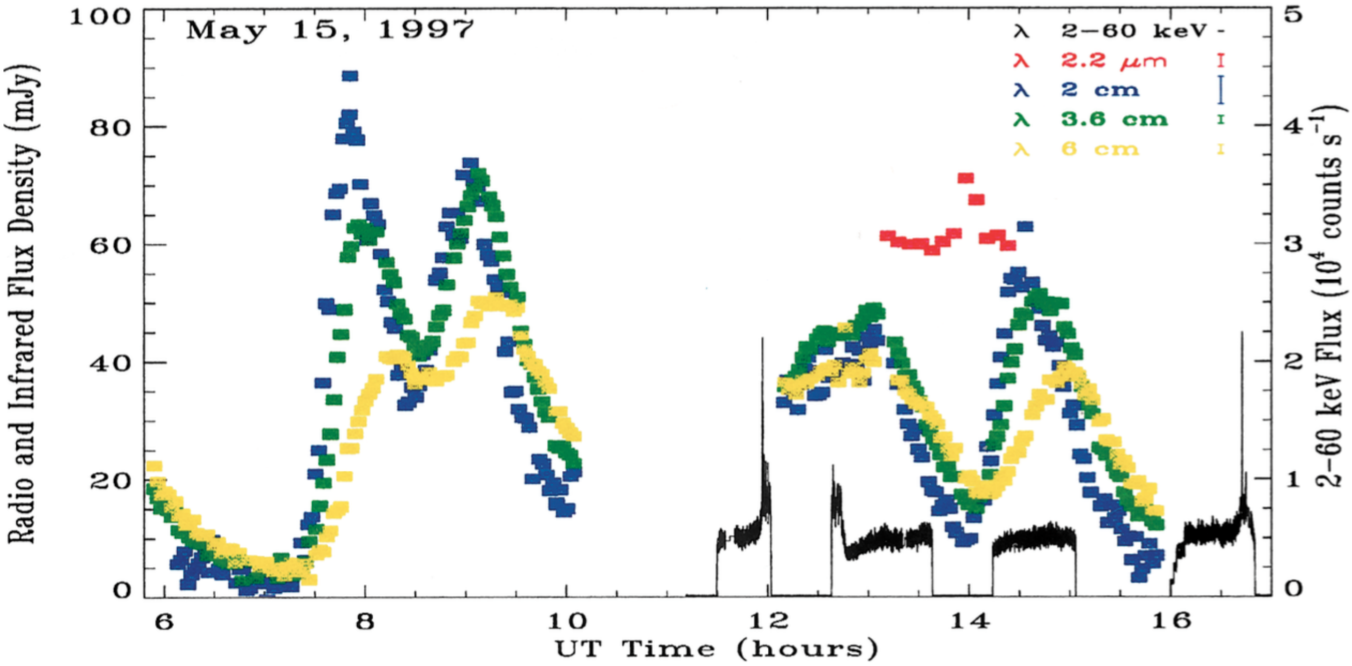

Figure 4: Light curves of GRS 1915+105 at radio wavelenghts 2, 3.6, and 6 cm. obtained on May 15, 1997.[3]

III. APPLYING VAN DER LAAN MODEL TO GRS 1915+105

A. Last flare

We will now seek to model one of the flares from the graph in Figure 4 above, in which the light curves of GRS 1915+105 obtained on May 15, 1997 at radio wavelenghts 2, 3.6, and 6 cm are represented. These observations helped to consolidate the theory of the genesis of expanding clouds of relativistic plasma when GRS 1915+105 recovers from large amplitude dips in the X-ray flux. [3] Although each flare is initially independent of the others, the time lapse between them is short enough for the tail of one to add to the rise of the other flare.

Since the last flare (the one between 14h and 16h UT time) appears to be the most isolated of the four, it is our choice in order to estimate the parameters of those emissions.

As we can see in Figure 5, the model fits best the decline of the last flare of the light curve (at wavelength

Taking a closer look before processing the data [7], we observe that:

λ (cm)	ν (GHz)	$S^*_{m,\nu}$ (mJy)
2	15	62.3
3.6	8.33	51
6	5	39

TABLE II: Data corresponding to the last flare before applying the corrections.

Then, the model predicts that the shape of its increase will be that from Figure 6. Once we subtract what the third flare adds to the last one, we obtain the values in Table III.

B. First flare

Our focus now shifts to the first flare, whose rise is seen to be modelled best by a curve if $p = 2.3$, as can be observed in Figure 8.

Proceeding for this first flare as we did for the latest one —modelling both the first and second flares and sub-

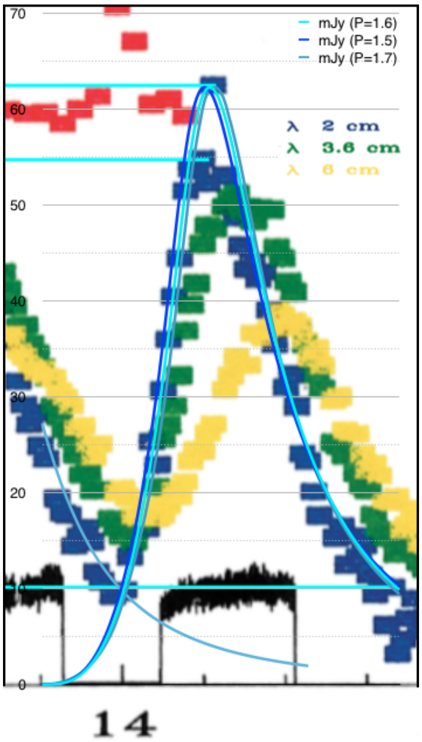

Figure 5: Modelling of the last flare from the graph in Figure 4, at λ=2 cm, for different values of p.

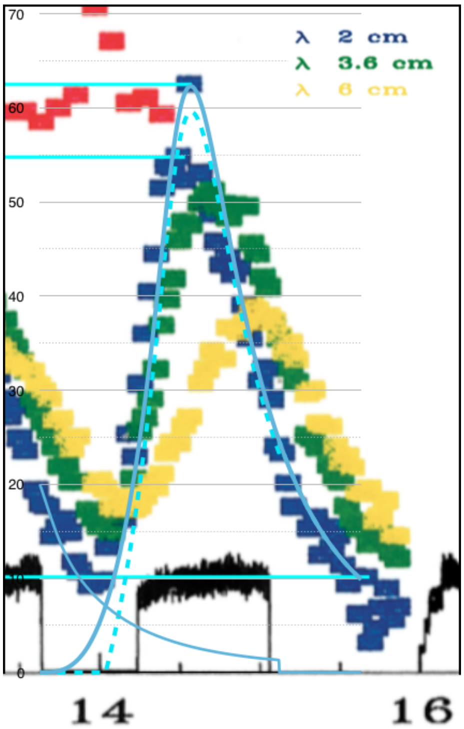

Figure 6: The last flare of the graph, at $\lambda = 2$ cm, before (continuous line) and after (dashed line) applying the corresponding corrections, with $p = 1.5$.

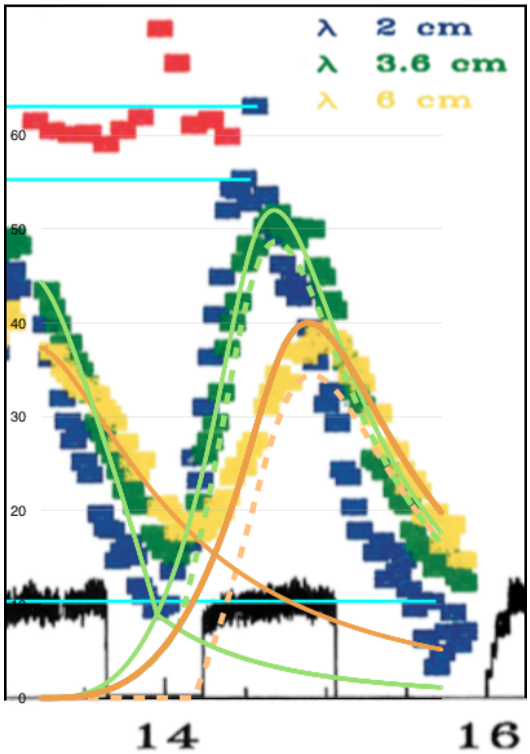

Figure 7: The last flare of the graph, at $\lambda = 3.6$ cm and $\lambda = 6$ cm, before and after applying the corresponding corrections.

λ (cm)	ν (GHz)	$S_{m,\nu}$ (mJy)
2	15	58.2
3.6	8.33	48.7
6	5	34.4

TABLE III: Data corresponding to the last flare, after removing the contamination from the third flare.

stracting the contribution from the second one, although only for $\lambda = 3.6$ cm and 2 this time, since the curve for $\lambda = 6$ cm has too much contamination—, we obtain Figure 9.

λ (cm)	$S_{m,\lambda}$ (mJy)	$t_{m,\lambda}$ (UT time)
2	89	7h 51 min
3.6	61.7	7h 56 min

TABLE IV: Values obtained from the correction of the first flare.

IV. TESTING THE VAN DER LAAN MODEL

From the relation (11) that we obtained earlier, namely

$$S_{m,\lambda} \propto \lambda^{-\frac{7p+3}{4p+6}}$$

and the values we have just found, we can also obtain p. In the case of the latest flare, it seems advisable

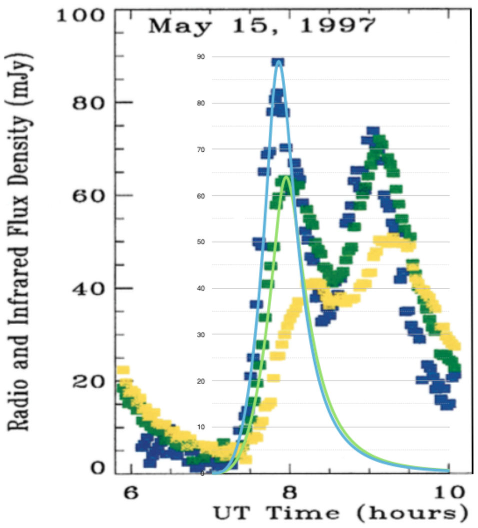

Figure 8: A modelling of the first flare of the graph at λ =2 cm, with $p = 2.3$.

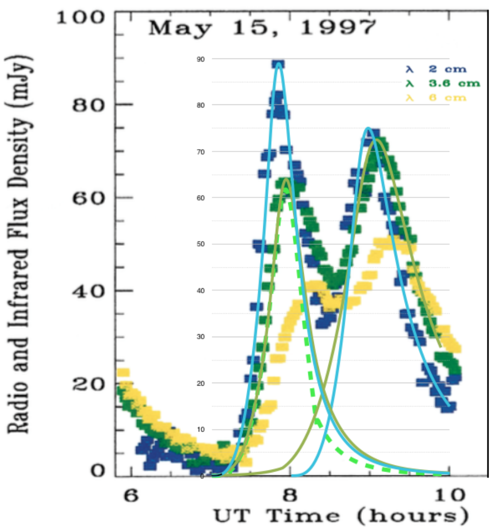

Figure 9: The first flare of the graph, at $\lambda = 3.6$ cm and 2 cm. The continuous and dashed curves represent, respectively, the density flux values before and after applying the corresponding corrections.

to leave aside the values obtained for λ =2 cm due to the uncertainty on whether its isolated maximum value —which is critical to make the adjustment— is actually real. Thus, we obtain a value of p of approximately 0.7, which is far from the value that provided a better modelling of the curve.

Carrying out the same calculations for the values of the first flare (see Table IV), we obtain a value for p of approximately 0.2.

V. DISCUSSION AND CONCLUSIONS

Thus, we have —twice— found values for p satisfying the relations from the van der Laan model which were very different from the ones obtained when trying to model the flux density curves of the source.
Such inconsistency shows the inability of the van der Laan model to describe the emission from this source satisfactorily.

As was mentioned before, the model proposed by van der Laan assumes that the number of particles in the cloud remains constant, thus making it inappropriate when phenomena such as continuous injection occur. Be it the case of continuous ejection or other events like interaction with stellar wind, one can observe that, although the emission from GRS 1915+105 does clearly have the characteristics of synchrotron emission, it does not suit well the model proposed by van der Laan in 1966.

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[2] H. van der Laan, “A model for variable extragalactic radio sources”. Nature **211**: 1131-33 (1966).
[3] I.F. Mirabel, et al. “Accretion instabilities and jet formation in GRS 1915+105”. Astronomy and Astrophysics **330**, L9-L12 (1998).
[4] `https://ned.ipac.caltech.edu/level5/March03/Mirabel/Mirabel_contents.html`
[5] `http://asd.gsfc.nasa.gov/Volker.Beckmann/school/download/Longair_Radiation2.pdf`
[6] “GRS” stands for “GRANAT source”, since it was discovered by this Soviet space observatory. “1915” is the right ascension (19 hours and 15 minutes). Last, “105” indicates its declination (in units of 0.1 degree): 10.5 degrees. This X-ray binary star system, which is in the Milky Way, is a micro-quasar formed by a black hole about 14 times the mass of the sun and a companion star.
[7] The asterisk denotes that these are the values previous to the corrections.