

Experimental Evidence of Accelerated Seismic Release without Critical Failure in Acoustic Emissions of Compressed Nanoporous Materials

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The total energy of acoustic emission (AE) events in externally stressed materials diverges when approaching macroscopic failure. Numerical and conceptual models explain this accelerated seismic release (ASR) as the approach to a critical point that coincides with ultimate failure. Here, we report ASR during soft uniaxial compression of three silica-based (SiO₂) nanoporous materials. Instead of a singular critical point, the distribution of AE energies is stationary, and variations in the activity rate are sufficient to explain the presence of multiple periods of ASR leading to distinct brittle failure events. We propose that critical failure is suppressed in the AE statistics by mechanisms of transient hardening. Some of the critical exponents estimated from the experiments are compatible with mean field models, while others are still open to interpretation in terms of the solution of frictional and fracture avalanche models.

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The mechanical deformation and failure of materials is a well-documented case of avalanche dynamics [1–33]. The energy of mechanical avalanches is partially released in elastic waves that can be detected by means of acoustic emission (AE) measurement [34]. Several studies suggested the presence of a phase transition associated with the ultimate failure point [18–22,35] which could, in theory, be monitored and forecast by means of the statistical analysis of the preceding AE activity [6,36–38] and be used for hazard assessment. AE signals recorded during mechanical tests usually display a scale-free distribution of energies (E) close to a power law: $D(E)dE \sim E^{-\varepsilon}dE$ with exponent $1 \lesssim \varepsilon \lesssim 2.5$. Three different relationships are often reported between this scale-free phenomenon and the proximity to failure: (i) The exponent ε in AE can decrease before failure [39–44]. (ii) The rate of energy released over time in AE experiments [45–49] diverges as a power law with an exponent m with respect to the time of failure t_c :

$$dE/dt(t) \propto (t_c - t)^{-m}, \quad (1)$$

a phenomenon called accelerated seismic release (ASR) [50]. (iii) The characteristic scales of the avalanches depend on the distance to failure [25–28]. This latter observation supports the well-established idea that failure occurs due to the divergence of correlation lengths at a critical point

[15,20,57,58]. This so-called critical failure hypothesis predicts a generalized homogeneous distribution of event energies:

$$D(E;f)dE = E^{-\varepsilon}\mathcal{D}(Ef^\beta)dE = f^{\beta\varepsilon}\tilde{\mathcal{D}}(Ef^\beta)dE, \quad (2)$$

where $\mathcal{D}(x)$ and $\tilde{\mathcal{D}}(x)$ are scaling functions, $f \equiv 1 - t/t_c$ is the time to failure, and β is a characteristic exponent of the model.

While the exponent decrease (i) is currently not understood from a model perspective, ASR (ii) and critical failure (iii) are well reproduced by most micromechanical models [15–17,37,57]. Since all statistical n -moments diverge at failure as $\langle E^n \rangle \sim f^{(\varepsilon-1-n)\beta}$ and the activity rate (dN/dt) is constant in most micromechanical models, ASR (ii) is a natural outcome of critical failure:

$$dE/dt(f) = \langle E \rangle(f) dN/dt(f) \sim f^{(\varepsilon-2)\beta}. \quad (3)$$

Although ASR is assumed as a signature of criticality [52,57], its connection with Eq. (2) is rarely tested with AE. Here, we analyze the AE during the approach to failure of nanoporous materials under soft uniaxial compression. We prove that ASR (ii) can appear in the absence of progressive exponent changes (i) or critical failure (iii). We estimate the experimental exponents m [Eq. (1)], ε [Eq. (2)], and γ ,

TABLE I. Sample details: cross-sectional area A , height h , compression rate dP/dt , number N of recorded signals above threshold Th .

	Area A (mm ²)	Height h (mm)	Driving rate dP/dt (kPa/s)	Th (dB)	N
Vycor (V32)	17.0	5.65	5.7	23	34 138
Gelsil (G26)	46.7	6.2	0.7	26	5 412
Sands. (SR2)	17.0	4.3	2.4	23	27 271

relating the characteristic E of an event with its duration T through the conditional average:

$$\langle E|T \rangle \propto T^\gamma, \quad (4)$$

and interpret them in terms of the mean field solutions of fracture and frictional avalanches.

We limit our analysis to the three silica (SiO₂)–based materials studied in Ref. [5]: natural red sandstone (SR2, $\Phi = 17\%$ porosity) extracted from Arran Isle (United Kingdom) and two artificial porous silica glasses, Gelsil (Gel26, $\Phi = 36\%$) and Vycor (V32, $\Phi = 40\%$). Experimental details are found in Ref. [5] and summarized in Table I. Samples are compressed without lateral confinement at a steady quasistatically slow loading rate $dP/dt \sim 1$ kPa/s, equivalent to a strain rate $(d\epsilon/dt) \sim 10^{-5}$ s⁻¹ during quasielastic deformation. The sample height (h) is measured over time with a laser extensometer, and the AE is recorded by a piezoelectric transducer attached to the upper compression plate. Individual AE events are identified by thresholding the acoustic signal $V(t)$, defining the hitting time t_{AE} and duration D_{AE} of each AE event. The AE energy of each event is computed as $E_{AE} \propto \int_{t_{AE}}^{t_{AE}+D_{AE}} |V(t)|^2 dt$.

Figure 1 shows the relations between AE energy (E_{AE}) and duration (D_{AE}) in a density map, and the conditional averages $\langle D_{AE} \rangle(E_{AE})$. The experimental data are compared

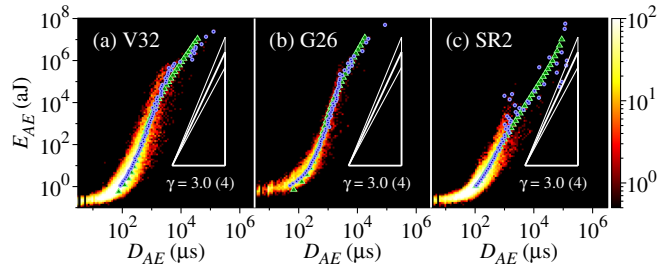


FIG. 1. Histograms (color-coded) of AE events in the duration-energy (D_{AE} , E_{AE}) space. Blue dots: Conditional averages $\langle D_{AE} \rangle(E_{AE})$. Green triangles: Numerical solutions of $E_{AE}(D_{AE})$ consistent with Eq. (4) (see main text for details), with $\gamma = 3.0(4)$ for V32, $\gamma = 3.4(4)$ for G26, and $\gamma = 3.2(4)$ for SR2.

to a nonstochastic model considering a scale-free avalanche profile [Eq. (4)] and the best value of γ found by inspection (see Supplemental Material [59]). Within error bars (± 0.4), all values are compatible with $\gamma = 3$, as predicted by mean field (MF) models [60,61]. The density clouds fill narrow stripes around the conditional average values as expected by Eq. (4).

The activity rate—the number of AE events per time unit—is nonstationary, as is also reported in Refs. [4–9]. Figure 2(a) shows the mechanical evolution expressed as a decrease in sample height $[h(t)]$ and the cumulative number of AE events $[N(t)]$ for the experiment V32. Figure 2(b) shows the activity rate (dN/dt) and the decrease in height (dh/dt) evaluated in intervals of uniaxial pressure $\Delta P = 100$ kPa (converted from t by dP/dt in Table I). We identify several sharp drops in h (five in Fig. 2), with a short characteristic temporal span $\Delta t_c \approx 0.1$ s (or $\Delta P \approx 100$ Pa), at pressure values P_c^k . These so-called strain drops are outliers to an otherwise smooth strain evolution, as observed in the dh/dP profile, and match a simultaneous increase of AE activity (dN/dP) and strong AE events. The events at P_c^k resemble brittle failure, a typical outcome of internal weakening or progressive damage in MF micro-mechanical models [10,62]. Brittle failure events are macroscopic by definition. Thus, during a loading cycle, a single (not multiple) brittle event is expected in these models. Here, however, the material recovers the stiffness during the intervals $P_c^k < P < P_c^{k+1}$ (Fig. 2). This can be explained by hardening, as reported in compression experiments [12], due to the accommodation of the stress field. The presence of both weakening and hardening localizes damage in brittle events that can correspond to spallation, correcting boundary defects [63] or be arrested due to stress heterogeneities [64]. An ultimate

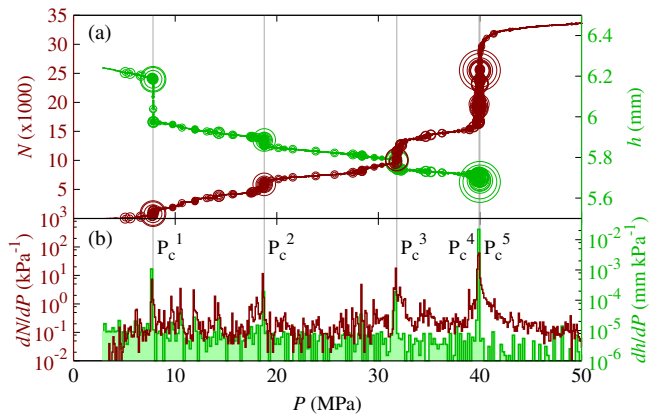


FIG. 2. Mechanical response and AE sequence for experiment on Vycor (V32). (a) Cumulative number of events N (dark red) and height evolution h (light green) in experiment V32 as a function of uniaxial pressure P . The size of the circles depends on the AE energy (size $\sim E_{AE}^{0.25}$). (b) Mean AE activity rate dN/dt (dark red histograms) and strain rate dh/dt (light green histograms) in intervals of $\Delta P = 100$ kPa. Vertical gray lines: P_c^k .

system-sized failure event collapsing the whole sample is observed in all experiments (P_c^5 in Fig. 2 has an associated $\Delta h \sim 5$ mm).

We study how the statistics of AE events are modified close to the most prominent stress drops by evaluating $\langle E_{AE} \rangle$, ε and dE_{AE}/dt in short stress intervals correlated with the distance to each strain drop: $f_k := 1 - P/P_c^k$. We select P_c^k as the onset of each strain drop, identified with a precision of 0.01 s (equivalent to $\delta f_k \sim 10^{-6} - 10^{-5}$) and compare the results to Eq. (2) where $\mathcal{D}$ is an exponential cutoff:

$$D(E; E_m, E_c, \varepsilon) dE = E^{-\varepsilon} \frac{E_c^{\varepsilon-1} \exp(-\frac{E}{E_c})}{\Gamma(1 - \varepsilon, \frac{E_m}{E_c})} dE. \quad (5)$$

Here, $\Gamma(a, x)$ is the incomplete gamma function and E_m is the lower boundary of the distribution. E_c is the characteristic scale of the exponential cutoff and, according to critical failure, should be proportional to $f_k^{-\beta}$ [Eq. (2)]. We truncate the distribution at the lower boundary $E_m = 1$ aJ to avoid resolution artifacts distorting the power law for low energies.

We inquire if the strain drops at P_c^k can be interpreted as independent failure events, identified by at least one of the three trademarks mentioned earlier. Figures 3(a)–3(c) show

the exponents $\hat{\varepsilon}(f_k)$ estimated by maximum likelihood inside the interval 1–1000 aJ [65] (overhat denotes estimation), compared to the global estimated exponent (gray line). Figures 3(d)–3(f) show the mean energy of individual AE events ($\langle E_{AE} \rangle(f_k)$ in dots) compared to the solution to Eq. (5) (triangles) with $\hat{\varepsilon}(f_k)$ from Figs. 3(a)–3(c) and stationary $\hat{E}_c$ (gray lines). The lower panels [Figs. 3(g)–3(i)] show the rate of energy released by all events in temporal intervals ($dE_{AE}/dP(f_k)$ in dots). In Figs. 3(g)–3(i), since some avalanches last longer than the evaluation intervals close to failure, their AE energy is split into intervals of 1 ms in order to increase the temporal resolution. The exponent $\hat{\varepsilon}(f_k)$ is almost stationary except for a few low values in the last intervals before P_c^k . Since all $\hat{\varepsilon}(f_k) < 2$, critical failure expects a divergence in $\langle E_{AE} \rangle$ when $f_k \rightarrow 0$. As first reported in Vycor [4], $\langle E_{AE} \rangle(f_k)$ is instead almost stationary and compatible with a finite and constant $\hat{E}_c$ (see E_{AE} distributions in the Supplemental Material [59]). Only the last intervals prior to failure show higher $\langle E_{AE} \rangle(f_k)$, close to the 90% confidence interval limit. Despite the stationary $\langle E_{AE} \rangle$, all data sets exhibit a steady increase in dE_{AE}/dt starting far from failure [Figs. 3(g)–(i)], as predicted by ASR [Eq. (1)] considering $m \sim 1.0$ (thin gray lines). Thus, we observe ASR, even when avalanches are noncritical.

Figure 3 illustrates how ASR [Eq. (1)] is more general than critical failure [Eq. (2)]. This result can be reproduced

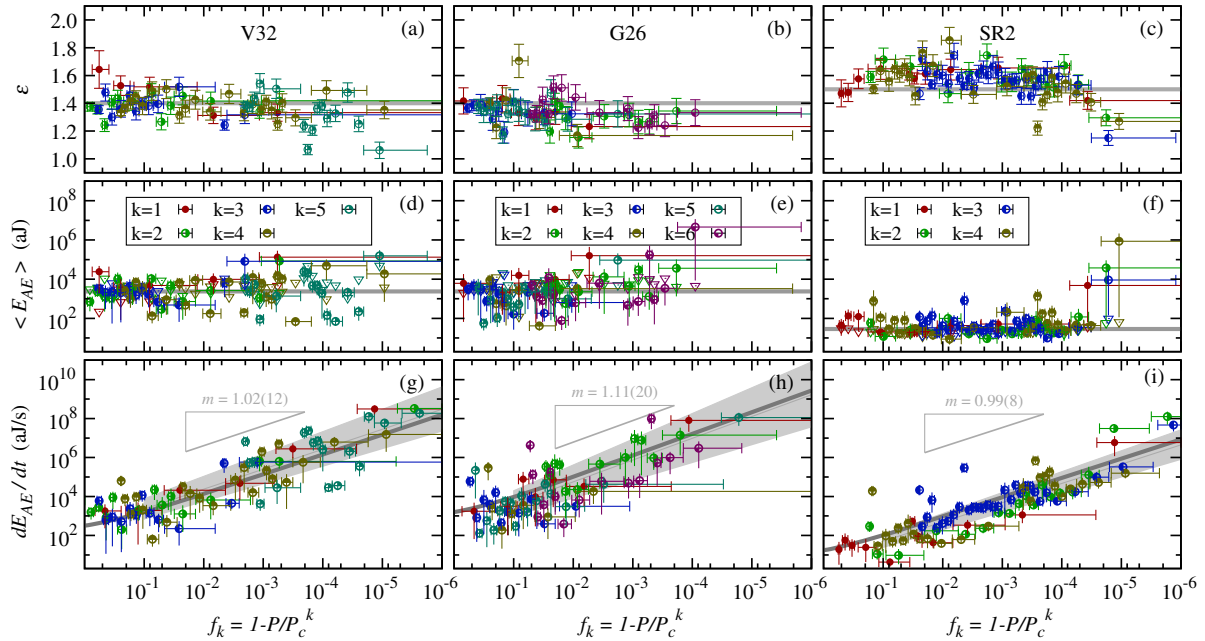


FIG. 3. Statistical variations with distance to strain drops P_c^k . The color scheme identifies the index k . (a)–(c) Exponent $\hat{\varepsilon}(f_k)$ from Eq. (2) estimated within the interval (1.0–1000 aJ). (d)–(f) Mean energy per signal $\langle E_{AE} \rangle(f_k)$; expected mean value according to $D(E; E_m, E_c, \hat{\varepsilon}(f_k))$ (triangles) with $E_c = 10^6$ aJ (10^4 aJ for SR2); expected value from the global exponent (gray line). (g)–(i) Rate of AE energy dE_{AE}/dt . Thin gray line: Exponent m fitted by least squares within $10^{-6} < f_k < 10^{-1}$. Thick gray line: A correction as expected by critical failure $D(E; E_m, E_m f_k^{\beta(\hat{\varepsilon}, \hat{m})}, \hat{\varepsilon}(f_k))$ with global $\hat{\varepsilon}$ and estimated $\hat{m}$. The f_k intervals of evaluation grow exponentially and have an imposed minimum size of $n = 100$ signals ($n = 50$ for G26). X-error bars: Integration interval. Y-error bars: 90% bootstrap interval in (d)–(i) and likelihood standard deviation in (d)–(f). Hard lower threshold imposed at $E_m = 1.0$ aJ.

by introducing microscopical mechanisms of transient hardening such as rheology damage [66,67], rate-and-state-dependent friction [68], or viscoelasticity [38,69,70] into models that would otherwise exhibit critical failure [61,62,70]. Transient hardening acts as an effective dissipation [61,62,71] preventing criticality [62,70,72,73] and introduces temporal scales to the model reproducing the foreshock and aftershock sequences [61,69,74]. The latter are perceivable in Fig. 2(a) after P_c^5 , for example, and reported in Refs. [4,5] and the Supplemental Material [59].

Some of the last intervals preceding P_c^k exhibit a significant decrease of $\hat{\varepsilon}$ [see Fig. 3(c)] and an increase in $\langle E_{AE} \rangle$ even higher than the expectation from Eq. (5) and the estimated $\hat{\varepsilon}$. Such intervals might contain superposition of events [75], artifacts due to the signal clipping of large avalanches, or strong AE related to brittle failure. As discussed in Ref. [61], brittle events can follow particular statistical laws. Some experiments of rock fracture report instead a progressive decrease in $\hat{\varepsilon}$ far from failure [1,39,43,76,77], but this is not a universal feature [48], and it is also inconsistent with models [20]. Anisotropic stresses are known to affect ε in structural phase transitions [78], which might or might not play a role in rock fracture [48]. The small size of our samples, close to the width of localization bands in sandstones [48,79], might prevent any band-related anisotropy. Finally, several brittle events might commonly appear under uniaxial compression, since similar results were reported at constant stress [80]. Simulations can reproduce multifragmentation from dynamic fracture [81] or localized weakening bands in a predominantly hardening process [14,20].

Both friction and different fracture mechanisms are involved in mechanical failure under compression [24,82]. We compare the experimental values of ε , γ , and m to the MF solutions of pure fracture and frictional models with transient hardening. We consider the MF stick-slip model [10,60,83,84] as a prototype for frictional avalanches and the democratic fiber bundle model [37] for fracture (see Supplemental Material [59], which includes Refs. [85–87]). The collection of MF exponents [10,61,88] is shown in Table II. The critical exponents [Eqs. (2) and (4)] are defined in terms of the size (S) of the avalanche from the relations

$$D(S; f) dS = S^{-\kappa} \mathcal{D}_S(S f^{1/\sigma}) dS; \quad \langle S|T \rangle \sim T^{1/\sigma \nu \zeta}. \quad (6)$$

In MF models, the exponents κ , $\sigma \nu \zeta$, ε , and γ are universal and invariant under transient hardening [10,61]. Given the broad regime with $\langle D_{AE} \rangle \sim E_{AE}^{1/\gamma}$ (Fig. 1), we assume $E_{AE} \propto E$. The estimated exponents ε and γ determine the values of κ and $\sigma \nu \zeta$, as shown in Table II. While $\sigma \nu \zeta$ and β are MF, κ and ε are higher but close to MF, below 2 standard deviations in V32 and G26, and 3 standard deviations in SR2, which might indicate the relevance of long-ranged elastic interactions.

TABLE II. Top three rows: Fitted exponents as represented in Figs. 3(g)–3(i), Figs. 2(a)–2(c), and Fig. 3(a)–3(c), compared to the MF exponents for slip and fracture. Bottom six rows: Fundamental exponents estimated from MF theory. The superscripts a [Eq. (7)] and b [Eq. (3)] denote two different interpretations of ASR in terms of MF theory (see text).

	V32	G26	SR2	Slip MF	Fracture MF
γ	3.0 (4)	3.4 (4)	3.2 (4)	3	3
ε	1.40 (5)	1.40 (5)	1.50 (5)	4/3	4/3
m	1.02 (13)	1.11 (20)	0.99 (8)	1 ^a 2 ^b	1/2 ^a 1 ^b
$\sigma \nu \zeta$	0.50 (6)	0.45 (6)	0.48 (5)	1/2	1/2
κ	1.60 (8)	1.62 (8)	1.76 (8)	3/2	3/2
σ^a	0.40 (9)	0.34 (9)	0.24 (8)	1/2	1
σ^b	0.88 (12)	0.80 (16)	0.76 (7)	1/2	1
β^a	3.7 $\pm$ 0.8	4.6 $\pm$ 1.2	6.3 $\pm$ 2.1	3	3/2
β^b	1.67 (24)	1.83 (37)	2.00 (25)	3	3/2

The MF solutions of friction and fracture are similar, but they differ in the values of $1/\sigma$ and β related to the approach to failure (see the Supplemental Material [59]). Furthermore, the interpretation of m in terms of the MF exponents is unclear when transient hardening is present. According to MF models, the exponent m defining the seismic energy released [Eq. (1)] is modified by transient hardening. Following Eq. (6), the mean size in models with critical failure diverges as $\langle S \rangle(f) \sim f^{(\kappa-2)/\sigma}$, and thus $dS/dt \sim f^{(\kappa-2)/\sigma}$. Under slow driving, dS/dt is invariant under transient hardening [61]. Considering the constant $\langle E \rangle(f)$ observed in Figs. 3(d)–3(f), the MF model assumes that $\langle S \rangle(f)$ is also constant. Thus, dS/dt diverges due to the divergence of dN/dt and, instead of Eq. (3), we have

$$dE/dt(f) = \langle E \rangle(f) dN/dt(f) \sim f^{\frac{\kappa-2}{\sigma}}. \quad (7)$$

This interpretation of $dE/dt(f)$ derived from MF theory is presented with superscripts a in Table II. The experimental $m = (2 - \kappa)/\sigma \approx 1$ coincides with the MF model of frictional avalanches. However, the values of $1/\sigma \sim 2.5$ –4 and $\beta \sim 4$ –6 are higher than the MF predictions of both models.

The relation between m and the fundamental exponents is discussed in MF theory, but not in models with local interactions, where transient hardening is known to affect the exponents [69,89]. An alternative hypothesis is that ASR [Eq. (3)] is invariant under transient hardening. Then, $m = (2 - \varepsilon)\beta \approx 1$ is compatible with the fracture MF model, and the exponents $\sigma \sim 0.8$ and $\beta \sim 1.8$ are between both models, and notably closer to fracture (superscript b in Table II). The presence of brittle events denoting damage and related to fracture is consistent with this interpretation. Rock fracture experiments at low confining pressure [24] are dominated by tensile fracture (not shear) AE events, a phenomenon related to dilatancy, and also reproduced in numerical simulations [90].

In conclusion, sharp strain drops with massive AE events denoting brittle failure are identified during the compression of nanoporous materials. Instead of critical failure, we find that $\langle E_{\text{AE}} \rangle$ is stationary, and accelerated seismic release (ASR) is exclusively observed in the activity rate (dN_{AE}/dt). Experiments under strain driving reported similar results [48], but failure precedes the divergence time of ASR [t_c in Eq. (1)], especially in materials with low porosity ($\Phi \lesssim 10\%$) [49]. Many theoretical models expect avalanche criticality at failure due to the divergence of correlation lengths [15–17,37,57]. This criticality can be prevented by dissipation [70,72,73], the dynamic weakening or hardening of the material [10,62], or the combined effect [71]. In particular, the ASR and the lack of criticality reported here, together with the temporal correlations reported in Ref. [5], can be reproduced by transient hardening [61]. In our experiment, an effective transient hardening can be caused by one or several internal micro-mechanical processes such as viscoelasticity [69,91], friction between crack surfaces [74], stress corrosion [92], diffusion of internal fluids [93,94], etc. In contrast, externally measured slip avalanches usually scale to failure and appear unperturbed by transient hardening [25–28]. Analytic solutions of MF models allow us to interpret the experimental results in terms of critical exponents. While the interpretation of the ASR [Eq. (1)] and its associated exponents remains an open question, other exponents are consistent with MF theory. A remaining challenge for the future is to validate this extension of MF models to noncritical failure through new micromechanical experiments able to control the potential mechanisms of transient hardening and dissipation.

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