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Modeling Concrete and Virtual Manipulatives for Mathematics Teacher Training: A Case Study in ICT-Enhanced Pedagogies

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Abstract

This feature paper explores the comparative pedagogical roles of concrete and virtual manipulatives in preservice mathematics teacher education. Based on a design-based research (DBR) methodology, this study investigates the effects of tangible tools (e.g., base-ten blocks, fraction circles) and digital applications (e.g., GeoGebra Classic 6, Polypad) on preservice teachers' problem solving, conceptual understanding, engagement, and instructional reasoning. Data were collected through surveys (n = 53), semi-structured interviews (n = 25), and classroom observations (n = 30) in a Spanish university's teacher education program. Key findings show that both forms of manipulatives significantly enhance engagement and conceptual clarity, but are affected by logistical and digital access barriers. This paper further proposes a theoretically grounded model for simulating manipulatives through ICT-based environments, enabling scalable and adaptive mathematics teacher training. By linking constructivist learning theory, the Technologically Enhanced Learning Environment (TELE) framework, and simulation-based pedagogy, this model aims to replicate the cognitive, affective, and collaborative affordances of manipulatives in virtual contexts. Distinct from prior work, this study contributes an integrated theoretical and practical framework, contextualized through empirical classroom data, and presents a clear plan for real-world ICT-based implementation. The findings provide actionable insights for teacher educators, edtech developers, and policymakers seeking to expand equitable and engaging mathematics education through simulation and blended modalities.

Keywords: manipulatives (concrete and virtual); preservice mathematics teachers; ICT tools; mathematics teaching and learning

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1. Introduction

Mathematics lessons have increasingly employed manipulative-based learning, particularly for preservice teacher training and young children. Manipulatives, physical (concrete) and virtual (digital), enable students to concretize abstract concepts and put them into space, thereby developing conceptual understanding, spatial thinking, and problem solving [1]. Manipulatives are also featured as teacher training objects for preservice teachers as they teach preservice teachers to model lessons, involve children, and sequence mathematical argumentation [2]. Nevertheless, the double reality of classroom lessons and across-the-world movements toward virtual lessons has pushed into the foreground the question concerning the respective roles of concrete and virtual manipulatives. Concrete manipulatives give the sense of touch and support for an embodied cognition, whereas virtual manipulatives such as GeoGebra Classic 6 or Polypad give dynamic visualization, instant experimentation, and enhanced scalability [3]. Both of these are ordinarily employed individually, but only very few research papers address how they can supplement each other as part of an integrated preservice teacher training program, at least based on Information and Communication Technologies simulation environments.

While there is existing literature to justify each manipulative type as being worth it, the contribution of this paper [1,4] is that (1) their preservice pedagogical affordances have been empirically compared within the manipulative type, (2) their boundaries under which teachers use them together have been evaluated, and (3) an ICT-mediated simulation model has been outlined that is specially crafted to simulate their preservice teacher roles along scalable dimensions. The model addresses prevailing access, digital inequity, and instructional alignment challenges across settings. To situate this study, the TELE framework overlaps physical and virtual learning environments that extend students' access, backchannel, and interaction time. This study's DBR methodology enables us to facilitate iterative theorizing, testing, and refinement of instructional innovations informed by empirical evidence from lesson planning and instructional implementation processes [5].

Additionally, this research fills existing gaps recognized in recent reviews [6,7], such as a lack of description of replication of physical manipulatives' pedagogic effectiveness by ICT applications, underdevelopment of ICT-based teacher training hands-on models, and underdevelopment of scaling-up of tactile learning into blended and distant environments. While there are generic mathematical simulations as part of applications such as GeoGebra Classic 6, there are few targeted teacher-preparation-based pedagogic models that replicate primary classroom realities. Such models are of the essence now due to the quest for greater inclusivity and access to schooling, exacerbated by the COVID-19 pandemic. It is of the essence now that schoolwork replication applications, including presentation of content, but also of engagement and of equity and critical classroom dynamic replication, for example, of manipulation of pupils, of provision of feedback, and of groupwork, replicate. This study's ICT simulation model is to address such needs of pedagogy, replicating teacher–student–application interaction, as well as cognitive and emotive outcomes of traditional use of manipulatives that have been documented.

1.1. Research Questions

This study is guided by the following research questions:

i. How do preservice mathematics teachers perceive and utilize concrete and virtual manipulatives during lesson design and instructional problem solving?
ii. What instructional and logistical challenges are associated with implementing each type of manipulative in preservice training?
iii. How can the cognitive, affective, and collaborative affordances of manipulatives be modeled within a scalable, ICT-based simulation environment for preservice teacher education?

1.2. Research Objectives

To address these questions, this study is designed to achieve the following objectives:

i. To investigate how preservice teachers conceptualize and apply concrete and virtual manipulatives in the context of lesson planning, teaching strategies, and mathematics problem solving;
ii. To identify the specific instructional, technical, and logistical barriers encountered in the use of concrete and virtual manipulatives in preservice training programs;
iii. To propose a theoretically grounded, ICT-enabled simulation model that replicates the pedagogical functions of manipulatives and enhances engagement, reasoning, and collaboration in scalable teacher education environments.

2. Literature Review

2.1. Manipulatives in Mathematics Education: Historical and Theoretical Foundations

The use of manipulative materials with which students can engage tangibly or virtually with mathematical concepts has a historical presence in education, particularly in constructivist learning theory. Jean Piaget's [8] stages of cognitive development theory laid the groundwork for using concrete objects to facilitate children's progression from concrete to formal operational thought. This is further enriched by Vygotsky's sociocultural theory [9], which emphasizes the significance of tools and mediated conversation in the learning process. This theory posits that manipulatives serve as semiotic tools that enhance students' understanding within their zone of proximal development (ZPD).

Concrete manipulatives such as base ten, fraction circles, or Cuisenaire rods facilitate embodied cognition, where cognition and meaning are enhanced through bodily engagement. Laski et al. [1] synthesize four conditions under which manipulatives are effective, drawn from both Montessori pedagogy and cognitive science: transparency of representation, manipulability, alignment with instructional aims, and the ability to engage the learner in practical work. Their thesis posits that well-designed manipulatives foster deep structural knowledge rather than mere surface manipulation. The cognitive benefits of manipulatives have been confirmed across age groups. Byrne et al. [2] conducted a scoping review that revealed significant associations between the use of concrete manipulatives and the development of spatial reasoning, problem-solving skills, and conceptual knowledge in young students. However, they caution that manipulatives are most effective when strategically integrated into a broader instructional design that respects the importance of thoughtful teacher facilitation.

2.2. Virtual Manipulatives and the Digital Turn

The expanded use of technology in learning has introduced a new generation of virtual, interactive, and generally simulation-based manipulatives. GeoGebra Classic 6, and Polypad provide students with dynamic spaces to visualize the relationships in mathematics, manipulate variables, and receive immediate feedback. The efficacy of virtual manipulatives is already apparent in recent empirical research. Farra et al. [4] compared the learning of students regarding fractions using concrete and digital tools. Their findings reveal that while the concrete manipulatives helped students recognize simple part–whole relations, the virtual tools enabled more advanced explorations through easily varied and re-visualized representations. Similarly, Siller and Ahmad [10] tested a mixed model of manipulative teaching. They found that using both types together resulted in statistically significant gains in student achievement and engagement compared with using one form in isolation. Virtual manipulatives also possess specific advantages in terms of accessibility, scalability, and flexibility to meet the needs of learners. According to Gómez-García et al. [11], Spanish mathematics teachers employ more digital materials to support teaching, especially in light of the requirements of online learning. However, they also concluded that successful integration relies on teachers' ICT literacy, pedagogical flexibility, and level of training. This presents an underlying dilemma: virtual tools necessitate new literacies to create new opportunities. Lamalif et al. [12] propose a model for successful ICT implementation in the classroom, including teacher preparation, institutional support, and technological infrastructure. Indeed, without these supports, virtual manipulatives can become superficial additions rather than transformative learning spaces.

2.3. Teacher Education, ICT, and Simulation-Based Learning

Physical and virtual manipulatives play double roles in preservice teacher preparation: building pedagogical content knowledge and representing good instructional strategy.

Dilling et al. [5] are certain that preservice and in-service use of technology-enhanced lesson co-design results in increased professional agency and more sophisticated conceptions of pedagogy for the digital age. Their mixed-design pair study of teachers demonstrates how manipulatives and simulation software programs can foster critical analysis of instructional quandaries and classroom issues. Their study parallels that of Romero-García et al. [6], who address the affective aspects of math instruction. Their study highlights that teacher candidate programs must consider technical competency, positive affect, and enthusiasm for math. Manipulatives, particularly those utilized through group sets, have also enhanced preservice teachers' confidence and respect as leaders of meaningful activity.

Ochugboju [7] adds to the debate by contrasting preservice teachers' perceptions of concrete and online manipulatives applied for teacher-centered problem solving. Participants appreciated the hands-on, experiential nature of the concrete manipulatives for low-complexity problems. However, they cherished the convenience of acting within flexible situations and the seemingly visual feedback of the online media when confronted with problems of higher complexity. Such complementarity requires instructor training programs to provide prospective teachers with the skill of selecting and integrating manipulatives according to the situation, theme, and student needs. To that, the design-based research study by Ochugboju and Díez-Palomar [13] refers to the findings of applying mathematical thinking with the assistance of digital resources. Based on their SYSTEMATIC analysis, they attribute model-crafting and simulation programs as advanced enough to simulate some of the strengths of concrete manipulatives and enhance their possibilities by incorporating adaptive feedback, real-time collaboration, and multimodal representation.

2.4. Gaps and Directions for ICT-Based Modeling of Manipulatives

Despite a growing body of work comparing concrete and virtual manipulatives [4,10], much of the literature remains fragmented, often treating these tools as separate instructional strategies. Most existing studies focus on their immediate classroom efficacy—engagement, visualization, or concept acquisition—without extending the discussion to how their pedagogical affordances can be modeled in teacher training at scale. Furthermore, while digital environments like GeoGebra Classic 6 and Polypad offer powerful visualization features, they rarely simulate the full instructional utility of manipulatives as mediating tools in teacher education [14]. There is limited integration of constructivist, sociocultural, and affective learning theories in guiding the design of ICT-based manipulative environments that replicate classroom practice.

This study addresses this gap by proposing an ICT-based simulation model specifically designed for preservice mathematics teacher education. Unlike existing tools, the model is grounded in theoretical principles—Technologically Enhanced Learning Environments (TELEs), dual-mode pedagogy, and affectivity in mathematics—and informed by empirical data from our ongoing PhD research. Rather than merely comparing manipulatives, this study offers a conceptual bridge: how to transform manipulatives into adaptive, simulation-based learning objects that prepare future teachers for blended, flexible, and remote instruction [15]. In doing so, it contributes not just to the pedagogy of manipulatives, but to the broader discourse on how teacher education can evolve in response to technological change and inclusive instructional design.

3. Conceptual Framework

This project expands constructivist and sociocultural learning theories by integrating them into an ICT-rich preservice teacher education simulation model. The Technologically Enhanced Learning Environment (TELE) model uses technology to strategically leverage resources instead of pervasive learning, which supports learner engagement, constructively aligned inquiry, and adaptive feedback in synchronous and asynchronous learning environments [16]. In these environments, manipulatives have an epistemic function instead of an adjunctive function, organizing and intermediating mathematical thought.

Predicted double-mode manipulative settings reinterpret traditional and virtual manipulatives as complementing one another and are components of an integrated instructional framework. Physical manipulatives like base-ten blocks, Cuisenaire rods, and fraction circles give haptic feedback, i.e., kinesthetic and tactile manipulation based on sense-mind-based and spatial thinking [11]. Virtual manipulatives, however, allow for seamless switching between various modes of mathematical representation, the numeric, symbolic, and spatial domains. These dynamic features of object transformation under time constraints, visual layering, and simulation capability allow switching between exploratory intuition and abstract thinking, allowing for various styles of cognitive process.

Conceptually, the model innovates for the literature by marrying embodied cognition and the scalability and flexibility of virtual learning environments. Practically, the model provides an ICT-infrastructure-compatible simulation training system that can simulate basic pedagogical features of manipulatives [17]. Affectivity is embedded as an undercurrent component, reporting proof of significance of affective engagement and motivational aspects of mathematics learning, most significantly for preservice teachers [5]. Abating its conceptual contribution, the model focuses on the core enablers, including interactive design, instant response mechanisms, and collaborative features. It battles typical deterrents to integrating ICT, including access inequities, digital literacy, and institutional preparedness. These results contribute to scalable and inclusive mathematics teacher education models under digitally mediated environments.

4. Methodology and Case Study Description

4.1. Research Design and Rationale

A design-based research (DBR) process was used for its capability to explore and develop instructional interventions under realistic instructional conditions iteratively. DBR is most capable of bridging theory and practice because it allows for the concurrent development of solution responses for problems of instruction and resultant theories [18]. The intervention focused on teacher enactment of preservice teacher instructional reasoning, engagement, and perceived effectiveness as an outcome of instructional material use. Research was conducted under an existing doctoral project and complied with the usual institutional goals for promoting expanded use of ICT throughout teacher education. Formal ethical clearance was sought under the Academic Board of the PhD Program Didàctica de les Ciències, les Llengües, les Arts i les Humanitats of the University of Barcelona.

4.2. Participants and Context

A purposive sampling strategy was used to recruit 53 preservice teachers from a mathematics methods course at a Spanish teacher education institution. The participants included individuals training for primary and inclusive education, ensuring diversity in subject specializations and classroom experience. All participants provided informed consent. Of the 53 survey respondents, 25 participated in semi-structured interviews, and 30 were involved in observed practicum sessions, some participating in multiple data collection activities. This multi-participant approach enabled data triangulation across instruments.

4.3. Instruments and Data Collection

4.3.1. Quantitative Instrument: Structured Survey

The quantitative data were collected through a structured survey developed specifically for this study to capture preservice teachers' demographic profiles, confidence levels in mathematics, and frequency of manipulative use. The questionnaire was designed after reviewing prior validated instruments on manipulative use in mathematics education but adapted to reflect the context of preservice teacher education [4]. It contained five core sections, combining demographic, confidence, and behavioral indicators, as follows:

Demographic Information

- Gender (male, female, non-binary, prefer not to say)
- Year of birth (numeric entry)
- Major or specialization (open text, e.g., primary education, physical education, attention to diversity)

Mathematical Self-Confidence

- Participants rated their confidence in mathematical skills on a 5-point Likert scale: (1) very unconfident, (2) somewhat unconfident, (3) neutral, (4) somewhat confident, (5) very confident.

Frequency of Manipulative Use

- A single-item question assessed how often participants used manipulatives in their mathematics learning or teaching: (1) never, (2) rarely, (3) sometimes, (4) often, (5) always.

Concrete vs. Virtual Manipulative Use (follow-up items—included later for analysis)

- Additional open-response items asked participants to list examples of manipulatives they used most frequently (e.g., base-ten blocks, fraction circles, GeoGebra Classic 6, Polypad).

Contextual Information

- Participants could add notes about their specialization preferences (e.g., interest in future specializations like English or attention to diversity) to contextualize manipulative use within their training trajectory.

4.3.2. Qualitative Instruments: Semi-Structured Interviews and Observations

The qualitative aspect of the research used semi-structured interviews and observation of the class to attain rich insights into preservice teachers' experience, perception, and instructional decision making when they apply concrete and virtual manipulatives to solve math problems. This design sought to complement the quantitative questionnaire so that methodological triangulation could occur and attain increased credibility and validity of results.

Interview Structure and Planning

A customized semi-structured interview protocol for the current study was gleaned from the existing literature concerning the use of mathematical manipulatives when teaching mathematics [18]. It was shaped into the preservice teacher setting to evoke experiential and pedagogical introspection. Questions were piloted under a limited group of participants for understandability determination and adjusted subsequently based on feedback. A total of 13 guide questions under five themes comprised the final protocol, as follows:

(1) Experiences and Examples of Manipulative Use

"Tell me about your experience of using concrete and virtual manipulatives in mathematics problem solving. Please provide specific examples of problems you solved using different types of manipulatives."

(2) Conceptual Understanding and Cognitive Impact
"How did using concrete and virtual manipulatives help you understand the mathematical concepts involved in the problem?"

(3) Challenges and Strategies
"What challenges or difficulties did you face when using concrete and virtual manipulatives in mathematics problem solving, and how did you overcome them?"

(4) Motivation and Collaboration
"How did using these manipulatives affect your motivation, confidence, and communication with peers during problem solving?"

(5) Comparative Evaluation and Pedagogical Integration
"Which type of manipulative do you prefer to use for mathematics problem solving and why?"
"How do you integrate concrete and virtual manipulatives into lesson plans, and how do you assess their effectiveness?"

Participants were also asked to rate statements such as "Concrete manipulatives are more engaging than virtual manipulatives" on a 10-point Likert scale, providing additional quantitative insights.

Observations

To complement self-reported data, 30 classroom practicum sessions were observed and video-recorded. The observations followed an interaction analysis framework, focusing on the following:

- How preservice teachers selected and used manipulatives during lessons;
- Peer collaboration and communication patterns;
- Scaffolding strategies and teacher–student interactions.

Field notes were taken to document contextual factors, such as the availability of resources and classroom management issues.

4.4. Data Coding and Analysis

The demographic and frequency responses were coded numerically for statistical analysis in SPSS v27. Cross-tabulations and chi-square tests were conducted to explore relationships between demographic variables (e.g., gender, specialization) and manipulative use frequency. Self-confidence ratings were analyzed through descriptive statistics (mean, standard deviation) and tested against manipulative use frequency using Spearman's rank correlation to determine whether higher confidence correlated with more frequent manipulative use ($p < 0.05$).

Interview transcripts and observation field notes were imported into NVivo 14 for thematic analysis. The coding followed Braun and Clarke's [18] six-step framework, beginning with inductive coding of emergent themes (e.g., motivation, collaboration, barriers) and moving to deductive coding aligned with the study's research questions. Triangulation was achieved by comparing interview themes with observed behaviors and survey data.

4.5. Ethical Considerations and Data Analysis

This study adhered strictly to ethical research standards to ensure the protection of participants' rights and the integrity of the research process. All participants provided informed consent prior to participation and were explicitly informed about the purpose of the study, data collection methods, and their right to withdraw at any stage without penalty. To maintain confidentiality and anonymity, all interview and survey data were coded using pseudonyms, and no personally identifiable information was included in transcripts, observation reports, or publications.

This research followed the ethical guidelines of the University of Barcelona's doctoral program and complied with the principles of the Declaration of Helsinki, particularly regarding voluntary participation, risk minimization, and respect for human dignity. Data were securely stored in encrypted files, accessible only to the research team. Additionally, ethical approval was obtained from the University of Barcelona's Academic Board of the PhD Program in Didactics of Sciences, Languages, Arts, and Humanities.

5. Results

5.1. Quantitative Data on Engagement and Tool Effectiveness

The quantitative results, based on survey data from 53 preservice teachers, revealed significant insights into their engagement with and perceived effectiveness of concrete and virtual manipulatives. These data form part of our ongoing PhD research, rigorously analyzed using descriptive statistics, chi-square tests, and independent samples *t*-tests to ensure statistical validity.

5.1.1. Descriptive Statistics

Table 1 presents the distribution of responses regarding engagement and perceived tool effectiveness. Overall, 85% reported high engagement with concrete manipulatives (mean = 4.21, SD = 0.78 on a 5-point Likert scale), while 90% reported high engagement with virtual manipulatives (mean = 4.38, SD = 0.64). For perceived learning effectiveness, 78% rated concrete manipulatives as very effective (mean = 4.07, SD = 0.81), whereas 81% rated virtual manipulatives as very effective (mean = 4.29, SD = 0.72).

Table 1. Descriptive statistics for engagement and effectiveness.

Variable	Mean (M)	SD	High/Very Effective (%)
Engagement—Concrete Manipulatives	4.21	0.78	85%
Engagement—Virtual Manipulatives	4.38	0.64	90%
Effectiveness—Concrete Manipulatives	4.07	0.81	78%
Effectiveness—Virtual Manipulatives	4.29	0.72	81%

Chi-Square Analysis

A chi-square test was performed to examine whether the proportion of students reporting high engagement significantly differed between concrete and virtual manipulatives (see Table 2).

Table 2. Chi-square test for engagement (concrete vs. virtual).

Test Statistic	χ^2 (1, N = 53)	*p*-Value
Engagement	4.12	0.042 *

* Significant at $p < 0.05$.

The results show a significant difference, indicating that preservice teachers were more likely to report high engagement with virtual manipulatives compared with concrete ones.

5.1.2. T-Test Analysis

Independent samples *t*-tests were conducted to compare the mean engagement and effectiveness ratings for the two types of manipulatives (see Table 3).

Table 3. Independent samples *t*-test for engagement and effectiveness.

Variable	t (df)	*p*-Value	Cohen's d	Interpretation
Engagement (Virtual vs. Concrete)	t (52) = 2.11	0.040 *	0.58	Virtual > Concrete (Moderate Effect)
Effectiveness (Virtual vs. Concrete)	t (52) = 1.94	0.058	0.52	Trend toward Virtual > Concrete

* Significant at $p < 0.05$.

The *t*-test confirmed that engagement with virtual manipulatives was significantly higher, while effectiveness showed a positive trend favoring virtual manipulatives but did not reach statistical significance.

5.1.3. Complementarity and Confidence

When asked whether the tools complemented each other, 70% agreed or strongly agreed (mean = 4.02, SD = 0.84). Notably, 65% reported increased confidence in teaching mathematics after integrating both manipulatives in structured practice.

These findings demonstrate that while virtual manipulatives show higher engagement statistically, both modalities are valued for different instructional purposes. The statistical evidence strengthens the validity of these claims and provides empirical support for proposing blended manipulative use in preservice teacher training.

5.2. Qualitative Themes from Interviews and Observations

Thematic analysis was conducted on 25 semi-structured interviews and 30 classroom observations using NVivo 14 for coding. Three dominant themes emerged: (1) motivational impact, (2) collaborative learning and communication, and (3) challenges and coping strategies. Inter-coder reliability was established with a Cohen's Kappa score of 0.81, indicating substantial agreement.

5.2.1. Motivational Impact

Motivation was consistently reported as a strong affordance of both manipulative types. Interviews frequently described manipulatives as fostering enjoyment and curiosity, validating quantitative findings on engagement. For instance, one participant stated, "Using manipulatives is like being a child again, and you are happy to experiment with things." Virtual manipulatives, particularly GeoGebra Classic 6 and Polypad, were praised for their immediate visual feedback, as illustrated by another participant: "Virtual manipulatives made experimentation with algebra and geometric ideas possible in a hands-on way." Observational data corroborated these statements, with students persisting longer on problem-solving tasks when manipulatives were available.

5.2.2. Collaborative Learning and Communication

Manipulative use was strongly associated with improved communication and peer collaboration. Both interview and observation data indicated frequent group discussions and peer explanations. As one preservice teacher expressed, "It is better because we understood each other better when we solved problems together using blocks." Video-recorded sessions showed that concrete manipulatives often prompted physical demonstration and gesturing, while virtual manipulatives facilitated collaborative exploration through shared screens or tablet interfaces.

5.2.3. Barriers and Coping Strategies

Even though they were effective, manipulatives had issues. Concrete manipulatives were termed disruptive and time-consuming: "Concrete manipulatives are useful, but working through them to use in class is time-consuming." Validations of off-tasking confirmed impressions of attention-grabbing novelty. Virtual manipulatives, as they became available, had technical problems, navigation challenges, and distracting access to app features that were unrelated to schooling. Compensation strategies consisted of peer modeling, simplified tool choosing, and pre-class preparation. Someone mentioned preparing "similar material herself" under resource constraints, and other participants mentioned scaffolded guidance and stepwise demonstration as compensating for probable abuse.

5.3. Triangulated Insights

A blend of quantitative questionnaire responses, qualitative interviews, and observation of classroom utilization of consequences yielded three prominent triangulated findings, together providing an integrated picture of preservice teacher mathematics preparation with manipulatives. Physical and virtual manipulatives became pedagogic additions instead of replacements. Physical manipulatives supplied necessary tactility and hapticity, allowing preservice teachers to interiorize necessary mathematical concepts through body-experienced learning, but virtual manipulatives introduced expanded visualization and dynamic investigations of abstract relations, more appropriate for algebra and geometry. Motivation was also enhanced, as well as collaborative learning, greatly supporting constructivist and sociocultural accounts of social interactions and collective construction of knowledge. Observation testimony also supported that group discussions and collaborative problem solving were more visible under manipulatives use, repeated interview statements such as "We better understood each other when solving problems together using blocks." Chronic difficulties came into sharp relief, namely, access resource constraint, class management by physical resources, and unequal computer literacy levels, sometimes restricting productive utilization of virtual platforms. Findings support the simulated model of utilization of ICT based on the necessity of maintaining physical manipulatives' tactility but taking advantage of an electronic medium's scalability, adaptive testing, and access to design technology-mediated learning environments for teacher development.

6. Discussion

Findings of this study affirm the pedagogical quality of manipulatives as a teaching material for preservice mathematics teachers, but otherwise further develop current research by directing more attention towards their supporting roles, motivational statuses, and classroom implementation challenges. Most importantly, such findings are grounded in current PhD studies, as well as preservice teacher surveys, to address methodological validity issues later and present empirical data for the controversial ICT-focused simulation model.

6.1. Complementary Pedagogical Roles of Concrete and Virtual Manipulatives

Results affirm that physical and virtual manipulatives cannot replace one another but have different, even supplementary, roles in mathematics lessons. Physical manipulatives, including fraction circles and base-ten blocks, require haptic and tactile feedback, aiding embedded cognition and internalization of abstract representations based on sensory experience. This is confirmed by evidence from Mntunjani et al. [3] that physical number sense improves foundation-phase pupils' number sense, and that from Holland [19] that physical engagement is an indispensable condition for building conceptual understanding by schoolteachers under in-service training. Virtual manipulatives, on the other hand, provide dynamic visualization, seamless transition between representations, and explorations, as confirmed by reports from Keldgord and Ching [20] and Moyer-Packenham and Suh [21]. For example, users of the present study confirmed that virtual instruments like GeoGebra Classic 6 allowed experimentation of geometric transformation and algebraic thinking that were out of the question under representation by manipulatives. Comparable benefits were reported by Justo et al. [22] for engineering major pupils, where virtual manipulatives enhanced representational flexibility and analytical thinking. Triangulated findings of the present study confirm Suh and Moyer's [17] reports that the combined use of either type boosts representational fluency and allows multiple cognitive style accommodation.

6.2. Motivation and Collaboration

Significant in the current study findings were increased motivation and collaborative learning that followed the inclusion of manipulatives as part of the instructional process. Interviewees depicted manipulatives as fun, and one of them said, "Using manipulatives is like being a kid again, and you're happy to try things out." Such affective reactions are similar to those of Romero-García et al. [6] and are paramount in building confidence among preservice teachers, a precursor to practice utilization in their future classrooms. Shared advantages of collaboration were also reconfirmed; group-working students were improved problem solvers and immersed in better forms of communication, substantiating the theory of sociocultural advanced by Vygotsky [9], who confirms that sociocultural interaction creates facilities for learning. Such findings are consistent with findings documented by Kabel et al. [14] and van Garderen et al. [23], who documented that manipulatives instigate peer talk, metacognitive thinking, and collaborative problem-solving forms. Interestingly, such collaborative forms are similar to 21st-century teacher–teachers' instructional focuses, whereupon teachers are required to invoke dialogical and inquiry-based forms of learning.

6.3. Classroom Implementation Barriers and Institutional Support

While the benefits of manipulatives are clear, significant barriers to their consistent and effective use were identified, echoing critiques from prior studies. Concrete manipulatives, although effective, present logistical issues such as cost, storage, and preparation time, as reported by Moyer [24] and O'Meara et al. [16]. A participant's statement, "Concrete manipulatives are beneficial, but controlling them in the classroom is time-consuming," mirrors these concerns. Virtual manipulatives, while scalable, face challenges, including device accessibility, usability, and teacher digital literacy. Keldgord and Ching [20] emphasize that inadequate training in virtual manipulative use limits their effectiveness, a finding consistent with Gómez-García et al. [11]. Moreover, some participants in this study reported that students occasionally used digital tools for non-instructional purposes, corroborating observations by Shin et al. [25] regarding off-task behavior in technology-rich environments. These findings suggest that institutional support, targeted professional development, and digital literacy training are essential drivers for successful implementation, as advocated by Lamalif et al. [12] and Sen and Leong [26].

6.4. Methodological Reflections and Limitations

This study employed triangulation across surveys, interviews, and classroom observations to strengthen validity, but several methodological limitations must be acknowledged. Quantitative analysis relied on self-reported data, which may be prone to response bias; however, statistical tests (e.g., chi-square and *t*-tests) provided additional rigor in comparing engagement levels between manipulative types. The qualitative analysis, although thematically coded, depended on a relatively small sample of interviews, which may limit generalizability. Similar limitations have been noted in comparable studies [15,27]. Future research should incorporate longitudinal designs and experimental interventions to evaluate causal relationships between manipulative use and learning outcomes.

6.5. Implications for ICT-Based Simulation and Teacher Preparation

The empirical foundation for the here-presented ICT-based simulation model is the proof. As Power Up What Works [28] and Reimer and Moyer [29] have argued, virtual platforms can realistically attain most of the didactic power of physical manipulatives, but scale and access more pronouncedly. What is presented here is an extension of that by marrying features such as realistic time feedback, collaborative problem solving, and teacher-centered scenario teaching, by marrying the haptic appeal of physical facilities to the adaptive nature of virtual environments. Additionally, as Justo et al. [22] indicate, simulation platforms on that basis can diminish logistic impediments by offering an economical and reusable substitute for physical manipulatives [16]. This is most true for teacher training colleges, where limited resources normally limit frequent use of physical facilities [19].

6.6. Contribution to Literature and Future Directions

This research makes an original contribution to the literature in situating manipulatives within a double-mode framework that instantiates constructivist and sociocultural theories within ICT-enriched learning environments. While the existing literature has looked at concrete or virtual manipulatives in isolation [27,30], this research has cast light on sets of paired manipulatives and offers an associated theoretical rationale for their use under simulation teacher training. It would be beneficial for future research to empirically validate the model of ICT simulation provided, looking at its application for extending preservice teachers' self-efficacy and pedagogical content knowledge. Cross-cultural studies could also potentially usefully look at the influence of access to technology as well as cultural perceptions of manipulatives on adoption and use, as documented by Kabel et al. [14] and Keldgord and Ching [20].

7. Simulation and ICT-Based Proposal

The ICT-integrated simulation model given here is founded upon the aforementioned empirical results described and fills the recognized gaps of the existing platforms, Polypad and GeoGebra Classic 6, by design alignment of the pedagogical, affective, and collaborative aspects. As of now, the model remains abstract and undeveloped as an executable prototype, but is conceived to work and scale for teacher training environments. It derives its design from constructivist and sociocultural theories of acquiring knowledge, as well as upon outcomes of TELEs (Technologically Enhanced Learning Environments), and suggests an integrated design where the haptic affordance of the manipulatives is simulated along with the adaptive and scalable nature of the virtual space.

7.1. Theoretical Foundation and Practical Rationale

While current digital tools are mostly centered on the visualization of content and single play, this model intentionally mimics teacher-guided roles of manipulatives by calling for guided play, real-time scaffolding, and collaborative problem solving. Such studies as Moyer-Packenham and Suh [21] and Ukdem and Çetin [27] have noted that as virtual manipulatives build representational competence, they have seldom replicated concrete tactile learning's bodily as well as social quality. Such are dealt with by the current model by invoking a double-mode simulation that digitally models, first, object handling (haptic approximation) and, second, social interactions that otherwise would have been observable with concrete manipulatives under typical classroom settings. Furthermore, the model is built on the discovered practical constraint in consideration here—the limited resource for physical aids [24] and teacher training demands for digital proficiency [20]. In its potential as an inexpensive, open-source, cross-platform simulation, it intends to expand teacher preparation accessibility, most specifically for resource-limited or remote-teach environments.

7.2. Model Architecture and Key Components

ICT-based simulation consists of three integrated levels that have been designed to mimic the primary pedagogical features of manipulatives but that integrate innovations absent from traditional tools, as follows:

Interactive Modeling Engine (IME):

The IME reproduces classic physical manipulatives like base-ten blocks, fraction circles, and algebra tiles. The IME lets users drag, rotate, and nest virtual objects under the explicit user command, wherein simulated haptic feedback is achieved through the use of sound, vibration (when available), and dynamic, interactive animation. IME is distinguishable from GeoGebra Classic 6's fixed representations, as the IME can support step-by-step manipulation tracing that allows preservice teachers to study their problem-solving process and compare that with expert models.

Pedagogical Simulation Layer (PSL):

It offers scenario-like practice environments where preservice teachers can rehearse instructional decision making. For instance, the system might have a virtual class where there is learner misconstruction (e.g., misunderstanding of fraction equivalence). The preservice teacher has to choose the relevant manipulatives, give feedback, and prompt the virtual learner. There is adequacy of pedagogy grading of each response for each answer, as is typical with simulation-based teacher education [23].

Collaborative Feedback Module (CFM):

To accommodate the social learning aspect outlined by Vygotsky [9], the CFM accommodates synchronous and asynchronous collaboration. Individual, pair, or group preservice teachers can manipulate problems, submit their solution, and receive AI-enriched, as well as peer, feedback. It also accommodates reflective questions that ask the users to justify their manipulative selections and, thereby, support metacognitive self-awareness.

7.3. Technical Feasibility and Implementation Plan

Although theoretical, the model is designed to be practical and feasible for development using existing technologies. It can be scripted as an online web platform using open-source libraries (e.g., Three.js to handle 3D object manipulation, Node.js for use as a real-time collaborative working environment) so that it could be used as a cross-platform tool. It can be integrated with inexpensive haptic interfaces (e.g., vibration tables) to maximize tactile approximation. The first stage of development would be to develop an MVP of core manipulative reproduction and feedback operating in real time. The second stage would complete PSL and CFM spec, where iterative use would be part of teacher training programs, using design-based research cycles [13].

7.4. Evaluation and Potential Efficacy

The model's potential efficacy is supported by empirical evidence from this study, which demonstrates that preservice teachers value both tactile and visual affordances of manipulatives and benefit from collaborative engagement. Its effectiveness would have to be empirically confirmed by comparing manipulative training based on simulation and conventional training. Outcome measures would be pedagogical content knowledge of preservice teachers, instructional decision-making processes, and manipulative-using confidence, as did those of the evaluation processes of Shin et al. [25] and Justo et al. [22].

7.5. Practical Implications for Teacher Education

If implemented, this simulation model would significantly reduce logistical challenges (e.g., expense, storage) and scale access to manipulative-centered training to preservice programs, including resource-poor and rural. It also allows for adaptive, scalable, and accessible teacher training [26]. It goes one step further than other platforms, however, by including motivational influences; collaborative environments; and dynamic, moment-to-moment support, to achieve an integrated training environment that approximates cognitive and affective advantages of physical manipulatives.

8. Conclusions and Future Directions

This study demonstrates the complementary strengths of concrete and virtual manipulatives in preservice mathematics teacher education, emphasizing their combined role in fostering conceptual understanding, motivation, collaboration, and pedagogical reasoning. Concrete manipulatives facilitated tactile engagement and embodied cognition, critical for the interiorization of mathematics roots, while virtual manipulatives facilitated visualization, dynamical exploration, and instant feedback, with an emphasis on wider abstract reasoning. These results confirm those of earlier research [3,22] and are a testament to the value of manipulatives as central resources for teaching. Challenges do, however, exist, including logistical challenges of cost and storage for concrete resources and access, computing literacy, and usability challenges for virtual platforms [20]. To mitigate these limitations, there is an advocacy of an ICT-based simulation prototype, founded upon constructivist and sociocultural theories, which combines haptic approximation, dynamic scaffolding, and collaborative capability. Compared with existing platforms, the prototype is scalable and is envisioned for inclusivity for use in preservice teacher training in resource-rich, as well as resource-constrained, contexts. Future research should be aimed at iterative design-based research rounds for refinement, development, and trials of the prototype, inclusive of statistical analyses of its impact upon pedagogical content knowledge, instructional decision making, and persistence. Cross-cultural confirmation and translation into multiple languages will be an unavoidable necessity for universal applicability, contributing to adaptive, technology-augmented mathematics teacher training.

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