Casimir forces between inclusions in an out of equilibrium fluid

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Abstract: At present time, the nature of the forces in out-equilibrium systems is still far to be well understood. It is thought that fluctuations appearing in these systems can play a crucial role. In the present report, we develop a computational model to systematically analyze the nature of interactions arising in an out-equilibrium system, consisting in two phosphor-bronze spheres surrounded by a bath of poppy seeds, subjected to an oscillatory forcing along one dimension. An experimental study of this system has been recently carried out in *Universidad de Navarra* and their results are not well explained yet. Here we show an important anisotropy in strength, the existence of short-range depletion forces and long-range attractive Casimir-like forces.

I. INTRODUCTION

According to modern physics, the vacuum state is by no means a simple empty space, and it is a mistake to think of any physical vacuum as some absolutely empty [1]. In agreement to quantum field theory, vacuum contains virtual particles which are in a continuous state of creation and destruction [2]. Therefore, the energy density of the ground-state is different to zero [3, 4]. In 1948, Hendrik Casimir realised that a physical force arises between two close parallel uncharged conductive plates in the vacuum [5]. In a classical description, the lack of an external field means that there is no field between the plates and so, no force would be measured between them. However, quantum field theory predicts that the presence of virtual particles affects the energy density of vacuum. In particular, it predicts that between the two plates, only those virtual photons whose wavelengths fit a whole number of times into the gap should affect the energy density. So, the energy density decreases as the plates are moved closer, and this implies that there is a force that makes they be together.
Although photons are not the only particles that affect the energy density of vacuum, they have the most relevant effect. In general, all bosons produce an attractive force between the plates while fermions make a repulsive contribution. [6]

While Casimir forces have a purely quantum origin, there are lots of analogous interactions in classical systems. In 1978, Fisher and De Gennes showed the existence of Casimir-like forces induced by fluctuations in systems close to the critical point [7]. The strength of Casimir forces is proportional to the driving energy of fluctuations and thus proportional to the temperature in classical systems.
Recently, this type of forces have been identified in many of non-equilibrium systems as a result of fluctuations. In these systems, fluctuations arise from energy input, either from an external field or from individual particles. Some examples of these systems include charged fluids, nematic fluids, active matter or hydrodynamic fluctuations in granular fluids. Casimir forces are responsible of many important interactions in biophysics. For example, they allow aggregation of red blood cells; this cells are negatively charged and therefore they are subjected to repulsive Coulomb interaction. However, attractive Casimir force stabilize the rouleaux formation against the explosion which would take place if only the electrical repulsion was existent [8].

In this work, we focus on hydrodynamic fluctuations in granular fluids. Granular materials are conglomerations of discrete macroscopic solid particles with sizes large enough to prevent thermal motion fluctuations. A granular material is characterised by a loss of energy due to the dissipative nature of forces acting on interacting grains, such as inelastic collisions and friction [9].

This work is inspired by a recent experiment, performed in *Universidad de Navarra*, that consists in a horizontal rectangular tray connected to an electromechanical shaker that provides a sinusoidal vibration along the X direction. Above the tray there are two types of particles: phosphor-bronze spheres (1.5mm diameter and 8.8gcm^{-3} density) and poppy seeds which were kidney shaped (1.06 mm diameter and a 0.2 gcm^{-3} density)[10]. Here we develop a computational model to analyze the nature of this forces, going to regimes that cannot be achieved experimentally, and clarify the relevance of Casimir mechanism in this interactions.

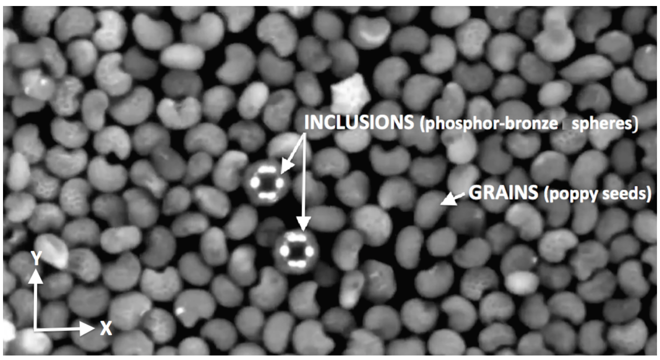

FIG. 1: Image of the experiment realized in *Universidad de Navarra.* We can see two phosphor-bronze spheres surrounded by lots of poppy seeds.[10]

II. METHODS

A. Equations of motion

As previously mentioned, our simulations reproduce the dynamics of a bi-dimensional system subjected to an external oscillating force. We define two different types of particles: inclusions and grains, corresponding to the phosphor-bronze spheres and poppy seeds, respectively. We fixed the diameter of inclusions to 1.5mm and the mean diameter of grains to 1mm. To prevent crystallization the diameter of grains oscillate between 0.8mm and 1.2mm. In addition, we set other magnitudes such as mass, frequency of tray oscillations or friction coefficients, in order to accurately reproduce the experiment realized in *Universidad de Navarra*. Another important magnitude to lay down is the increment of time between two consecutive steps. We considered this time equal to $10^{-4}\ t_0$, where t_0 is the period of tray oscillations. For all the simulations there were 2 inclusions and 592 grains in a 25mm x 25mm box, that correspond to a surface fraction of 0.75.

Grains were modelled as discs subjected to friction with an oscillatory tray and to collisions and friction with inclusions and other grains. They were also subjected to an aleatory force which reproduces in some way the effect of shaking and movement in the vertical direction that may translated into random in-plain motion. This term also takes into account the non-uniformity of the grains. Therefore, we obtain the following equations of motion for grains:

$$\frac{d\vec{p_i}}{dt} = -\gamma_i\left(\frac{\vec{p_i}}{m_i} - \vec{v}_s\right) + \sum_{j=1} \vec{f}_{ij}^{INT} + \vec{f}_i^{RAND} \qquad (1)$$

where m corresponds to the mass of particles, γ represents the dissipative constant of particles with the ground, $\vec{v}s = 2\pi A_0 f sin(2\pi f t)\hat{x}$ symbolizes the velocity of the tray (f indicates the oscillatory frequency of the tray and A_0 indicates the amplitude of oscillations), $\vec{f}_{ij}^{INT}$ accounts for the interaction between particle i and j (this interaction has a dissipative and a repulsive contribution), and $\vec{f}_i^{RAND}$ is the random force which is defined in order to fulfil fluctuation-dissipation theorem.

Regarding inclusions, we modelled them as discs subjected to friction with the ground and collisions and friction with inclusions and grains. Inclusions were also subjected to a random force which has the same effect as in grains. However, inclusions were not affected by the oscillations of the tray. That is because we wanted to model inclusions as spheres, and we approach the movement of a sphere as the movement of a disc that is not affected by the movement of the ground. Therefore, we obtained the following equations of motion for inclusions:

$$\frac{d\vec{p_i}}{dt} = -\gamma_i\frac{\vec{p_i}}{m_i} + \sum_{j=1} \vec{f}_{ij}^{INT} + \vec{f}_i^{RAND} \qquad (2)$$

To solve all the equations of motion (we have one equation for each particle) we used a leapfrog algorithm [11]. We performed a discretization of time in steps of length Δt and we integrated it using calculations in intermediate steps of time $t \pm \frac{1}{2}\Delta t$.

Once we had set out the equations of motion and we had exposed a way to solve them, we could start our simulations. First of all, we wanted to test our model. In order to check it, we can paid attention to the global movement of inclusions and grains. Our equations of motion imposes that the global movement of grains is caused by tray oscillations, and the movement of inclusions is caused by particle impacts. Therefore, we expected a delay in the movement of inclusions relative to grains, and a delay in the movement of grains relative to the tray. In addition, we expected a decrease in the oscillation amplitude of inclusions relative to grains, and grains in relation to the tray.

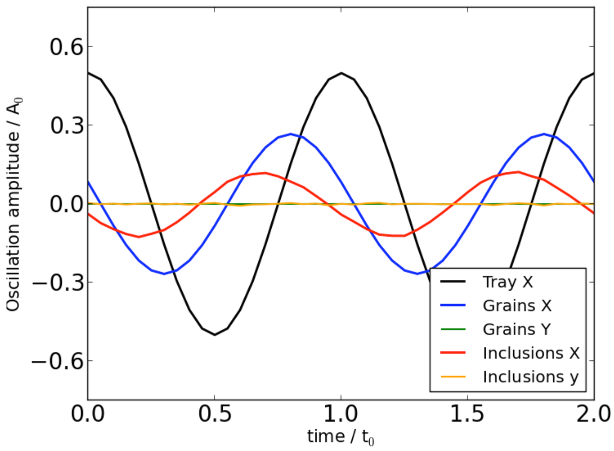

FIG. 2: Representation of the movement of the tray and the displacement of grains and inclusions in X and Y directions as a function of time. A_0 and t_0 are the amplitude and period of tray oscillations.

B. Distribution function and forces between inclusions

To start the simulation, first we place the two inclusions at a prescribed distance and orientation with respect to the driving force of the substrate, and distribute randomly the grains in the remaining area, ensuring that there is no overlap. Subsequently, we follow the tie evolution of the system in response to the applied external oscillation. The computer simulations provide us with detailed information; we know the positions and velocities for each configuration of the system.
In order to understand the nature of the forces arising between inclusions in granular media, we will make two types of measures:

On the one hand, we make statistics analysis of the separation between inclusions to obtain P(D). We know that in equilibrium P(D) is related to the effective interaction with a Boltzman factor. In general, this does not have to be true, however we can use this method to know if we can establish a kind of effective equilibrium in the system.

On the other hand, we want to directly compute the forces between inclusions. This method is a direct mechanic measure and there is no doubt that it allows us to know the force between inclusions. Because we are not working in equilibrium conditions, we cannot suppose that this is an isotropic force and we want to understand the nature of this possible anisotropy.
We can define the force between inclusions as the vector difference between the net force acting to each inclusion. We can divide this force into two contributions: the radial contribution (F_r^R), which is the force in the direction of the line joining the two inclusions, and the tangential force (F_r^T), which is the force in the perpendicular direction. Moreover, we can calculate the force acting to the center of mass of both inclusions. We define this force as the vector sum between the net force acting to each inclusion. We can also divide this force in two contributions: the contribution in the direction of tray oscillations (F_{CM}^X), and the perpendicular direction (F_{CM}^Y).
To compute the force, we can set free inclusions and then make an statistical analysis of the forces obtained for each distance D, or we can fix the positions of inclusions and do series of measurements. It is important to note that even when fixing the positions of inclusions, velocity keeps being different from zero. We use the second method because it is computationally more efficient.

In order to know what part of the force is due to the fact that the system is out of equilibrium, it is useful to do simulations of the system when it is in equilibrium. Next section explains how we did it.

C. Equilibrium system

Equilibrium statistical physics provides us a relation between the distribution function P(D) and the force between inclusions $\vec{F}_r$. This relation is the following:

$$P(D) = \frac{\int \delta(r-D) e^{-\beta U(D)}\, d\Omega}{\int e^{-\beta U(D)}\, d\Omega} = 2\pi D \frac{e^{-\beta U(D)}}{\int e^{-\beta U(D)}\, d\Omega} \tag{3}$$

The factor 2πD in eq. (3) takes into account that there are more possible states for a particle to stay to a distance D_1 than to stay to a distance D_2, if $D_1 > D_2$.

Because of the nature of the interaction between particles in granular materials, we know that these types of materials are intrinsically out of equilibrium. However, we can use the relation supplied by equilibrium statistical physics to know if we can establish an effective equilibrium in our system.

Before studying the relation between the theoretical results obtained with equilibrium statistical physics and the behaviour of our system, we wanted to modify our system to transform it into a system in equilibrium. To do so, we needed to disconnect tray oscillations and the friction between particles. As mentioned, in equations (1) and (2) we have a random term, now this term played the role of temperature and it was the only cause of movement in the system in equilibrium.
So the equations of motion for grains and inclusions are the following:

$$\frac{d\vec{p_i}}{dt} = -\gamma_i \frac{\vec{p_i}}{m_i} + \sum_{j=1} \vec{f}_{ij}^{INT_{eq}} + \vec{f}_i^{RAND} \tag{4}$$

It is important to note that $\vec{f}_{ij}^{INT_{eq}}$ is different than $\vec{f}_{ij}^{INT}$, because in equilibrium we have ignored dissipation.
In this case, the force between inclusions are from entropic nature. To compute this type of forces we will need a lot of statistics.
At this point, we could compute forces between inclusions and the distribution function for the new equilibrating system. We expected compatibility between the results obtained with equilibrium statistical physics and the results obtained with the modified system. This comparison will be very useful to later analyze of our original system.

III. RESULTS AND DISCUSSION

First of all, we show the results of the measurement of the force between inclusions as a function of the distance between them (D). As we said in the previous section, we have four different contributions of the force: F_r^R, F_r^T, F_{CM}^X and F_{CM}^Y.
We measured the force in four different cases. First, when inclusions form an angle of 0^o with the direction of tray oscillations. Second, when the angle is 45^o and third when the angle is 90^o. Finally, we measured the force in the equilibrating system. In this case we do not have a preferred direction because we don't have tray oscillations. So we don't need to analyse different angles.

As shown in FIG.3, for all cases, there is a minimum in the relative radial force at D=1,5r_0, followed by a maximum at D= $2,3r_0$. After this maximum, we can observe minimums and maximums alternate one after another. The position and magnitude of these successive maximums and minimums vary slightly for each configuration.
In addition, we can see that only in the case in which the angle is 45^o there exists a relevant tangential force between inclusions. This tangential force only appear in the out-equilibrium system and it makes inclusions tend to align with the oscillatory direction.
Regarding the forces in the center of mass, we can see,

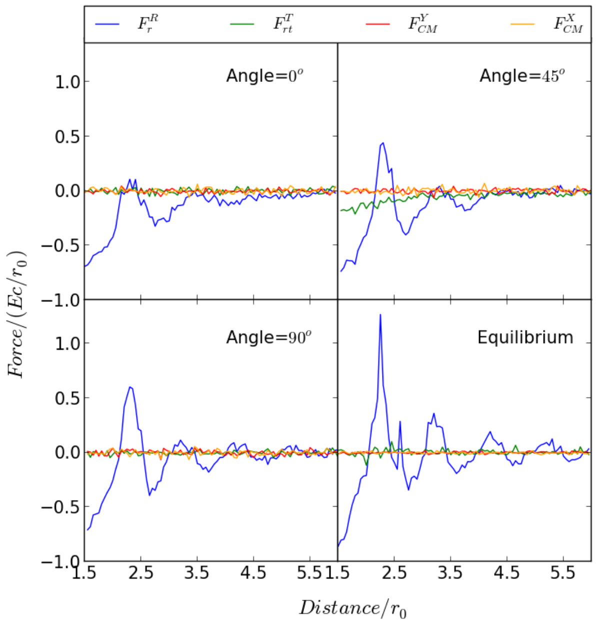

FIG. 3: Forces between inclusions, multiplied by r_0 and divided by the kinetic energy of grains, as a function of D/r_0, where r_0 is the mean radius of grains. We measured the kinetic energy for out-equilibrium system and for equilibrium system and we obtain the following relation: Ec(out_equilibrium)/Ec(equilibrium) = 7.4. To obtain the forces in out-equilibrium systems we, did 20 measures per cycle during 500 cycles, for each distance. We calculated the force every 0,05r_0. In the equilibrating system we did 20 measures per cycle during 5.000 cycles for each distance.

as expected, that there is no relevant contribution at any distance. We can use the noise observed in these cases to get an idea about the error in our measures.
Comparing the case of equilibrium with the other three cases, we can conclude that, as the angle increases, more similarities appear. This performance is logical if we think that the movement of an inclusion is less affected by the presence of another inclusion when both inclusions form a large angle respect to the oscillating direction.

Secondly, we show the results of the measurement of the distribution function in the four cases studied. As we said, to compute P(D) we set inclusions free and next we recounted the number of times that two inclusions are at distance D. In out-equilibrium systems, we expect that P(D) depends on the angle between inclusions and X axis (tray oscillations are in this direction). For that reason, we computed P(D) when the angle is 0^o,45^o and 90^o. However, we can not compute P(D) for a particular angle because we don't have got enough statistics for a specific angle. We need to compute P(D) for an interval of angles around the angle that we want to measure. Therefore, to compute P(D) when the angle is 0^o,45^o and 90^o we computed P(D) when the angle is into the interval (0^o-10^o),(40^o-50^o) and (80^o-90^o). In the case of the system in equilibrium, we used all the angles to compute P(D).

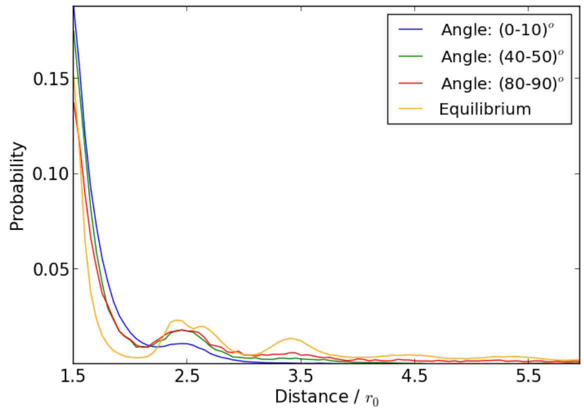

FIG. 4: Distribution function for the four cases we have studied: when the angle between inclusions and the tray oscillations is 0^o,45^o and 90^o, and the equilibrium case. To obtain P(D) we did 20 measures per cycle during 500.000 cycles.

In FIG.4 we can see that there exists a similarity between all the cases, but there are some differences between them. For example, maximums and minimums are more pronounced in the equilibrium system. If we consider the three out-equilibrium cases, we can see that, when the angle is larger, maximums and minimums are more pronounced. So, as we saw when we paid attention to forces, for large angles, the behaviour of the system is more similar to the equilibrium system. Another factor that attracted our attention is the fact that in the equilibrating system a maximum appeared at D=2,6r_0 which had not an equivalent maximum in out-equilibrium system.

At this point, we are ready to do the comparison between P(D) and F(D). As we said, equilibrium statistical physics provides us the relation shown in Eq.3. In this equation, appears the potential U(D). We can obtain this potential by numerically integrate the force from $1,5/r_0$ to D/r_0.
Furthermore, we do not know the value of the denominator in Eq.3. However, this term is just a constant that will have not a relevant effect. If we name this constant A and make some transformations in Eq.3, we can finally obtain the following equality:

$$U(D) = \frac{1}{\beta} ln[\frac{N(D)}{2\pi D N_{TOTAL}}] + A \quad (5)$$

We can take A=0 if we impose U(∞)=$K_B T ln[\frac{P(\infty)}{2\pi\infty}]$=0. Thereby, if we plot U(D) and $K_B T ln[\frac{P(D)}{2\pi D}]$ in the same graphic, we expect to obtain two equivalent curves in the case of the equilibrium system.

As shown in FIG.5, there exists a strong connection between the behaviour of U(D) and $K_B T ln[\frac{P(D)}{2\pi D}]$ in the equilibrium system. In the other three cases, due to non-

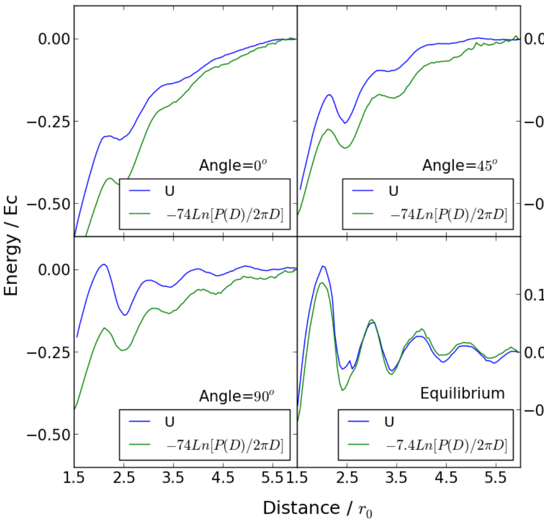

FIG. 5: U(D) and $K_BTln[\frac{P(D)}{2\pi D}]$ divided by the mean kinetic energy of grains, as a function of the distance between inclusions. In the equilibrium case we have a proportional constant of 7.4 between this two factor which corresponds to K_BT. In the out-equilibrium system this factor is larger because of the energy input induced by tray oscillations.

equilibrium properties, this two factors are less correlated. So we can not easily define an effective temperature in these cases. In addition, we can see that in the equilibrium system the potential is an oscillating function around zero, whereas in the out-equilibrium system there is an oscillating function with a decay due to an attractive potential. In equilibrium, only depletion force acts on inclusions. However, in non-equilibrium conditions we observe another contribution that makes the potential be attractive. We can conclude that this attractive contribution is due to Casimir-like forces.

IV. CONCLUSIONS

In this work we present an effective computational model to analyze an experiment recently realized in *Universidad de Navarra*. This experiment consists in two inclusions surrounded by a bath of grains, subjected to an oscillatory forcing along X axis. Nowadays, the nature of forces in out-equilibrium systems is still far to be well understood. Since granular materials are intrinsically out of equilibrium, we are interested in this experiment to better understand this type of forces.

Here we show the existence of short-range depletion force and long-range attractive Casimir-like force between inclusions. It is important to note that we find a tangential contribution in the force that tends to align inclusions with the direction of tray oscillations.

In addition, when comparing with an equivalent equilibrium system, we realize that the behaviour of the forces when the angle between inclusions and X axis is larger is more similar to that observed in the equilibrium system. This performance is logical if we think that the movement of an inclusion is more affected by the presence of another when they are aligned with tray oscillation direction.

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