Inverse amplitude method for the perturbative electroweak symmetry breaking sector: The singlet Higgs portal as a study case

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We explore the use of the inverse amplitude method for unitarization of scattering amplitudes to derive the existence and properties of possible new heavy states associated with perturbative extensions of the electroweak breaking sector of the Standard Model starting from the low-energy effective theory. We use a toy effective theory generated by integrating out a heavy singlet scalar and compare the pole mass and width of the unitarized amplitudes with those of the original model. Our results show that the inverse amplitude method reproduces correctly the singlet mass up to factors of $\mathcal{O}(1\text{–}3)$, but its width is overestimated.

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I. INTRODUCTION

The discovery of a new particle [1,2] resembling the Standard Model (SM) Higgs boson marks the beginning of the direct study of the electroweak symmetry breaking sector (EWSB). The complete characterization of the EWSB requires the precise measurement of the Higgs couplings, as well as the search for new states. In this work we analyze what we can learn from the observation of departures from the SM predictions for the Higgs couplings in the case that no new state is observed. As is well known, anomalous Higgs couplings lead to rapid growth of the scattering amplitudes with energy, leading to partial-wave unitarity violation [3]. Our goal is to verify how well unitarization procedures, more specifically the inverse amplitude method (IAM) [4–8], predict the existence and properties of possible new states associated with perturbative extensions of the SM.

Here we consider the simplest extension of the SM symmetry breaking system; i.e., we add a real singlet scalar field that is not charged under the SM gauge group. Despite its simplicity, this extension of the SM can have an impact in the Higgs physics at the LHC [9–12], as well as offers an interesting candidate for a portal to a hidden sector [13–16]. We assume that this singlet field is too heavy to be produced so we integrate it out to obtain the low-energy effective theory.

The IAM is based on dispersion relations to unitarize the perturbative partial-wave amplitudes even in the presence of coupled channels, and it has been applied with success to describe low-energy hadronic physics [4–8]. This method has also been extensively used to study strongly interacting EWSB sectors and models exhibiting a heavy Higgs [17–21]. In this work we apply the IAM to the effective theory generated by integrating out a heavy singlet scalar, and we compare its predictions to the original model parameters. In Sec. II we derive the corresponding effective Lagrangian up to $\mathcal{O}(p^4)$ and, after briefly reviewing the elements of the IAM relevant for our calculations in Sec. III, we present our results and draw our conclusions in Sec. IV. In particular, we show that for this toy model, the IAM indicates correctly that only the $I = 0$ and $J = 0$ channel exhibits a resonance, reproducing the singlet mass up to factors of $\mathcal{O}(1\text{–}3)$ even for relatively weak couplings. Its width, however, is systematically overestimated.

II. EFFECTIVE LAGRANGIAN FOR A HEAVY SINGLET HIGGS PORTAL

Our starting point is the SM scalar sector extended by a real singlet scalar field S,

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$$\mathcal{L}(\Phi, S) = (D^\mu\Phi)^\dagger(D_\mu\Phi) + \frac{1}{2}(\partial_\mu S)(\partial^\mu S) - V(\Phi, S), \quad (1)$$

where Φ stands for the SM scalar doublet and

$$V(\Phi, S) = -\mu_H^2|\Phi|^2 + \lambda|\Phi|^4 - \frac{\mu_S^2}{2}S^2 + \frac{\lambda_S}{4}S^4 + \frac{\lambda_m}{2}|\Phi|^2S^2. \quad (2)$$

For simplicity, we have imposed a Z_2 symmetry to forbid linear and cubic terms in S. We concentrate on a scenario in which the S develops a vacuum expectation value (VEV) v_S. Presently, a new heavy scalar is allowed, provided the ratio of the SM VEV (v) to v_S is small and the mixing between the mass eigenstates is small [22].

As we will show below, in order to conveniently parametrize the low-energy effective Lagrangian, it is easier to write the SM Higgs doublet as

$$\Phi = U\begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix}, \quad (3)$$

where U is a function of the goldstone bosons ω_i,

$$U = \exp\left[\frac{i\omega\cdot\tau}{v}\right], \quad (4)$$

and τ_i are the Pauli matrices. Therefore, we can write Eq. (1) as

$$\begin{aligned}\mathcal{L}(H, S) = {} & \frac{1}{2}(\partial_\mu H)(\partial^\mu H) + \frac{1}{2}(\partial_\mu S)(\partial^\mu S) \\ & + \frac{(v+H)^2}{4}\mathrm{Tr}[(D^\mu U)(D_\mu U)^\dagger] - \frac{1}{2}M_H^2H^2 \\ & - \frac{1}{2}M_S^2S^2 - \lambda_m v v_s HS \\ & - \left[\lambda_S v_s S^3 + \frac{\lambda_S}{4}S^4 + \frac{\lambda_m}{2}v_s H^2 S \right. \\ & \left. + \frac{\lambda_m}{4}(2vH + H^2)S^2 + \lambda v H^3 + \frac{\lambda}{4}H^4\right], \end{aligned} \quad (5)$$

with $M_H^2 = 2\lambda v^2$, $M_S^2 = 2\lambda_S v_s^2$. We have traded the mass parameters μ_H^2 and μ_S^2 for the VEVs using the minimization conditions $\mu_H^2 = \lambda v^2 + \frac{\lambda_m}{2}v_s^2$ and $\mu_S^2 = \lambda_S v_s^2 + \frac{\lambda_m}{2}v^2$. The covariant derivative of U takes the form

$$D_\mu U \equiv \partial_\mu U + \frac{i}{2}gW_\mu^a\tau_a U - \frac{ig'}{2}B_\mu U\tau_3. \quad (6)$$

The two mass eigenstates H_1, and S_1 exhibit a doublet-singlet mixing due to the presence of the HS term in Eq. (5),

$$H_1 = \cos\theta H + \sin\theta S \quad \text{and} \quad S_1 = \cos\theta S - \sin\theta H, \quad (7)$$

with the lighter state (H_1) identified with the recently discovered 125 GeV Higgs particle. The mixing angle θ and masses are given by [23]

$$\sin^2\theta = \frac{4y^2}{4y^2 + (1 - x^2 + \sqrt{(1-x^2)^2 + 4y^2})^2} \quad (8)$$

$$M_{H_1,S_1}^2 = \frac{M_S^2}{2}\left(1 + x^2 \mp \sqrt{(1-x^2)^2 + 4y^2}\right) \quad (9)$$

with $x \equiv M_H/M_S$ and $y \equiv \lambda_m v/(2\lambda_S v_S)$.

In this scenario, the heavier scalar S_1 is unstable and decays via its mixing with the doublet or the singlet-doublet direct coupling in Eq. (1). For $M_{S1} \geq 2m_{\mathrm{top}}$, the S_1 width is given by

$$\begin{aligned}\Gamma_{S_1} = {} & \Gamma(S_1 \to W^+W^-) + \Gamma(S_1 \to ZZ) + \Gamma(S_1 \to t\bar{t}) + \Gamma(S_1 \to H_1H_1) \\ = {} & \frac{g^2M_{S_1}^3}{128\pi M_W^2}\sin^2\theta\left[2\left(1 - x_W + \frac{3}{2}x_W^2\right)\sqrt{1-x_W} + \left(1 - x_Z + \frac{3}{2}x_Z^2\right)\sqrt{1-x_Z} + 3x_t\sqrt{1-x_t}\right] + \frac{\tilde{\lambda}^2}{32\pi M_{S1}}\sqrt{1-x_{H_1}}, \end{aligned} \quad (10)$$

where $x_i = \frac{4m_i^2}{M_{S_1}^2}$ and $\tilde{\lambda}/2$ is the coefficient of the $S_1H_1^2$ term obtained after we rotate Eq. (5) to the mass basis. Here, we are interested in the scenario where S_1 is heavy compared with H_1 which allows us to approximate Eq. (10) by

$$\Gamma_{S_1} = \frac{M_{S_1}}{256\pi}\Delta\left[2\left(1 - x_W + \frac{3}{2}x_W^2\right)\sqrt{1-x_W} + \left(1 - x_Z + \frac{3}{2}x_Z^2\right)\sqrt{1-x_Z} + 3x_t\sqrt{1-x_t} + \sqrt{1-x_{H_1}}\right], \quad (11)$$

where we have defined the parameter

$$\Delta = \frac{\lambda_m^2}{\lambda_S} \quad \text{so} \quad \Delta \frac{v^2}{2M_{S_1}^2} \simeq \sin^2\theta. \tag{12}$$

In the regime in which S_1 is very heavy, we can integrate it out and generate a low-energy effective Lagrangian. Since H_1 is not a doublet field component, the corresponding effective Lagrangian cannot be expressed in terms of higher-dimension operators obtained in the linear representation of the electroweak symmetry breaking with a doublet scalar. As we will show below, it can, instead, be matched to an effective chiral Lagrangian with a light Higgs.[1]

We integrate out the S_1 field to obtain the tree-level effective action using the approach of Ref. [24]: the tree-level effective action is obtained by solving the equation of motion (EOM) and inserting the solution into the action. In order to do so, we recast Eq. (5) in the mass basis as

$$\mathcal{L}(H_1) + \frac{1}{2} S_1[-\partial^\mu\partial_\mu - M_{S_1}^2 - R]S_1 + S_1 B + \Delta\mathcal{L}(H_1, S_1), \tag{13}$$

with

$$\begin{aligned} B &= \frac{1}{4}\mathrm{Tr}[(D^\mu U)(D_\mu U)^\dagger](H_1 \sin 2\theta + 2v \sin\theta) + \frac{1}{4} H_1^3[\lambda_m \sin 2\theta \cos 2\theta + 2 \sin 2\theta(\lambda_S \sin^2\theta - \lambda \cos^2\theta)] \\ &\quad + \frac{1}{2} H_1^2[-3 \sin 2\theta(\lambda v \cos\theta + \lambda_S v_S \sin\theta) + \lambda_m \sin 2\theta(v \cos\theta + v_S \sin\theta) - \lambda_m(v \sin^3\theta + v_S \cos^3\theta)] \\ R &= -\frac{1}{2}\mathrm{Tr}[(D^\mu U)(D_\mu U)^\dagger]\sin^2\theta + \frac{1}{4} H_1^2\left[2\lambda_m\left(\cos^2 2\theta - \frac{1}{2}\sin^2 2\theta\right) + 3\sin^2 2\theta(\lambda + \lambda_S)\right] \\ &\quad + H_1[\lambda_m(v \cos^3\theta - v_S \sin^3\theta) + 3 \sin 2\theta(\lambda v \sin\theta - \lambda_S v_S \cos\theta) + \sin 2\theta \lambda_m(v_S \cos\theta - v \sin\theta)]. \end{aligned} \tag{14}$$

$\Delta\mathcal{L}$ contains the nonquadratic terms $H_1 S_1^3$, S_1^3 and S_1^4.

The linearized solution to the EOM for the field S_1 yields

$$S_{1C} = \frac{1}{\partial_\mu\partial^\mu + M_{S1}^2 + R} B. \tag{15}$$

Replacing S_1 by S_{1C} in Eq. (13), one obtains

$$\mathcal{L}_{\mathrm{eff}}(H_1) = \mathcal{L}(H_1) + \frac{1}{2} B S_{1C} + \Delta\mathcal{L}(H_1, S_{1C}). \tag{16}$$

Now we expand the effective Lagrangian (16) up to four derivatives and keep only terms up to dimension six, which allows us to match the resulting chiral effective Lagrangian to that of Refs. [25,26],[2]

$$\begin{aligned} \mathcal{L}_{\mathrm{eff}}(H_1) &= \frac{1}{2}(\partial_\mu H_1)(\partial^\mu H_1) - \frac{1}{2} M_{H_1}^2 H_1^2 + c_C \mathcal{P}_C(H_1) \\ &\quad + c_H \mathcal{P}_H(H_1) + c_6 \mathcal{P}_6(H_1) + c_7 \mathcal{P}_7(H_1) - V(H_1), \end{aligned} \tag{17}$$

where

$$\begin{aligned} \mathcal{P}_C(H_1) &= \frac{v^2}{4}[\mathrm{Tr}(D^\mu U)(D_\mu U)^\dagger]\mathcal{F}_C(H_1), \\ \mathcal{P}_H(H_1) &= \frac{1}{2}(\partial^\mu H_1)(\partial_\mu H_1)\mathcal{F}_H(H_1), \\ \mathcal{P}_6(H_1) &= [\mathrm{Tr}(D^\mu U)(D_\mu U)^\dagger]^2 \mathcal{F}_6(H_1), \\ \mathcal{P}_7(H_1) &= [\mathrm{Tr}(D^\mu U)(D_\mu U)^\dagger]\partial_\nu \partial^\nu \mathcal{F}_7(H_1), \end{aligned} \tag{18}$$

and

$$c_i \mathcal{F}_i(H_1) \equiv c_i + a_i \frac{H_1}{v} + b_i \left(\frac{H_1}{v}\right)^2 + d_i \left(\frac{H_1}{v}\right)^3 + e_i \left(\frac{H_1}{v}\right)^4. \tag{19}$$

We present in Table I the lowest nonzero order in v/M_{S_1} coefficients defining the functions $\mathcal{F}$ in Eq. (19). Within our approximation, the H_1 potential is given by

$$V(H_1) = \left(\lambda v - \frac{\lambda_m^2 v}{4\lambda_S}\right) H_1^3 + \left(\frac{\lambda}{4} - \frac{\lambda_m^2}{16\lambda_S}\right) H_1^4 + \mathcal{O}\left(\frac{1}{M_{S_1}^2}\right). \tag{20}$$

It is interesting to notice that the operators generated at order p^4 by the integration of S_1 modify the Higgs interactions with electroweak gauge-boson pairs ($\mathcal{P}_c$)

[1] Alternatively, if one integrates out the field S, ignoring the corrections due to mixing, one can match the resulting Lagrangian to an effective expansion in terms of higher-dimension operators involving the remaining doublet field Φ as is shown in the Appendix.

[2] Notice that despite the UV theory being fully perturbative, the effective low-energy Lagrangian can be written as a theory more characteristic of strongly interacting or composite electroweak theories with a light scalar simply because these theories allow for enough freedom to account for the nondoublet nature of the light scalar.

TABLE I. Leading order in v/M_{S_1} coefficients defining the functions $\mathcal{F}$ in Eq. (19).

	c	a	b	d	e
$\mathcal{P}_C(h)$	1	$2-\frac{\lambda_m^2 v^2}{2\lambda_S M_{S_1}^2}$	$1-\frac{\lambda_m^2 v^2}{\lambda_S M_{S_1}^2}$	$-\frac{\lambda_m^2 v^2}{2\lambda_S M_{S_1}^2}$	$\frac{\lambda_m^2(9\lambda_m^2-16\lambda\lambda_S-10\lambda_m\lambda_S)v^4}{16\lambda_S^2 M_{S_1}^4}$
$\mathcal{P}_H$	0	0	$\frac{\lambda_m^2 v^2}{4\lambda_S M_{S_1}^2}$	0	0
$\mathcal{P}_6$	$\frac{\lambda_m^2 v^4}{16\lambda_S M_{S_1}^4}$	$\frac{\lambda_m^2 v^4}{8\lambda_S M_{S_1}^4}$	$\frac{\lambda_m^2 v^4}{16\lambda_S M_{S_1}^4}$	0	0
$\mathcal{P}_7$	0	0	$\frac{\lambda_m^2 v^4}{8\lambda_S M_{S_1}^4}$	0	0

and quartic electroweak gauge-boson vertices ($\mathcal{P}_6$), as well as introduce a rescaling of all Higgs couplings to SM particles ($\mathcal{P}_H$).

III. WEAK GAUGE BOSON SCATTERING AND ITS UNITARIZATION USING THE INVERSE AMPLITUDE METHOD

The low-energy effective Lagrangian in Eq. (17) implies a modification of the gauge boson scattering with respect to the SM expectation, leading to unitarity violation at high energies. In this respect, two of the operators generated are most relevant for this discussion: $\mathcal{P}_C(h)$ and $\mathcal{P}_6(h)$. $\mathcal{P}_C(h)$ determines the H_1 couplings to gauge boson pairs, in particular the term in a_C, and leads to a correction to the contribution of the virtual H_1 exchange required for unitarity. $\mathcal{P}_6(h)$, in particular the term in c_6, gives a contact four gauge boson coupling.[3]

For example, the scattering amplitude at tree level for longitudinal gauge bosons is given by

$$A(W_L^+W_L^- \to Z_LZ_L) = A(W_L^+W_L^- \to Z_LZ_L)_{\rm SM} + \left(-\frac{1}{4}(a_C^2-4)\frac{v^2}{(s-M_{H_1}^2)} + 8a_6\right)\frac{(s-2M_W^2)(s-2M_Z^2)}{v^4}, \qquad (21)$$

where $\sqrt{s}$ is the center-of-mass energy. As we can see, the term associated with a_C grows as s at high energy, while the one containing a_6 exhibits growth with s^2, and hence leads to violation of partial-wave unitarity.

The inverse amplitude method (IAM) [4] is an approach, based on dispersion relations, that allows for the full unitarization of the partial-wave amplitudes. The IAM was originally developed for chiral perturbation theory for mesons [5–8], and it was also applied to the unitarization of the one-loop weak gauge boson scattering amplitudes without a light Higgs resonance [17]. Most recently IAM has been applied in the context of effective Lagrangians with a light Higgs [18–21], mostly with the aim of inferring information about the possible existence of heavier resonances associated with EWSB expected in composite models with a new strongly interacting sector. Let us briefly summarize this approach.

The rigorous derivation of the IAM is valid only for one or several channels of particle pairs all with equal masses [18]. In order to apply the IAM to the longitudinal electroweak gauge boson scattering, one has to work in the isospin symmetry approximation, i.e. setting $c_w \to 1 (M_Z \to M_W \equiv M)$. In this case, one can define the longitudinally polarized weak-gauge boson scattering amplitudes as

$$A^{abcd}(p_1,p_2,p_3,p_4) \equiv A(W_L^a(p_1)W_L^b(p_2) \to W_L^c(p_3)W_L^d(p_4)), \qquad (22)$$

where a, b, c, d label the third component of the isospin-one triplet with values in the range 1,2,3 which are related to the charged states as $W_L^\pm \equiv |1,\pm1\rangle = (W_L^1 \pm iW_L^2)/\sqrt{2}$, and $W_L^3 \equiv |1,0\rangle$. Isospin symmetry implies

$$A^{abcd}(p_1,p_2,p_3,p_4) = \delta^{ab}\delta^{cd}A_1(p_1,p_2,p_3,p_4) + \delta^{ac}\delta^{bd}A_2(p_1,p_2,p_3,p_4) + \delta^{ad}\delta^{bc}A_3(p_1,p_2,p_3,p_4), \qquad (23)$$

so only three of these amplitudes are independent, though related by crossing symmetry, and the corresponding scattering amplitudes in the charge basis satisfy

$$\begin{aligned}
A^{\pm\mp00}(p_1,p_2,p_3,p_4) &\equiv A(W_L^\pm(p_1)W_L^\mp(p_2) \to W_L^3(p_3)W_L^3(p_4)) = A_1(p_1,p_2,p_3,p_4)\\
A^{\pm0\pm0}(p_1,p_2,p_3,p_4) &\equiv A(W_L^\pm(p_1)W_L^3(p_2) \to W_L^\pm(p_3)W_L^3(p_4)) = A_2(p_1,p_2,p_3,p_4)\\
A^{\pm00\pm}(p_1,p_2,p_3,p_4) &\equiv A(W_L^\pm(p_1)W_L^3(p_2) \to W_L^3(p_3)W_L^\pm(p_4)) = A_3(p_1,p_2,p_3,p_4)\\
A^{\pm\pm\pm\pm}(p_1,p_2,p_3,p_4) &\equiv A(W_L^\pm(p_1)W_L^\pm(p_2) \to W_L^\pm(p_3)W_L^\pm(p_4)) = A_2(p_1,p_2,p_3,p_4) + A_3(p_1,p_2,p_3,p_4)\\
A^{0000}(p_1,p_2,p_3,p_4) &\equiv A(W_L^3(p_1)W_L^3(p_2) \to W_L^3(p_3)W_L^3(p_4)) = A_1(p_1,p_2,p_3,p_4) + A_2(p_1,p_2,p_3,p_4)\\
&\qquad + A_3(p_1,p_2,p_3,p_4)\\
A^{\pm\mp\pm\mp}(p_1,p_2,p_3,p_4) &\equiv A(W_L^\pm(p_1)W_L^\mp(p_2) \to W_L^\pm(p_3)W_L^\mp(p_4)) = A_1(p_1,p_2,p_3,p_4) + A_2(p_1,p_2,p_3,p_4)\\
A^{\pm\mp\mp\pm}(p_1,p_2,p_3,p_4) &\equiv A(W_L^\pm(p_1)W_L^\mp(p_2) \to W_L^\mp(p_3)W_L^\pm(p_4)) = A_1(p_1,p_2,p_3,p_4) + A_3(p_1,p_2,p_3,p_4).
\end{aligned} \qquad (24)$$

[3] $\mathcal{P}_6(H)$ without the Higgs terms corresponds to the L_5 operator in Refs. [27–29] or $\mathcal{O}_5$ in Refs. [20,21], while a_C and b_C correspond, respectively, to the coefficients $2a$ and b of, for example, Refs. [21,30].

At this point, we project these amplitudes in the isospin basis because the isospin symmetry implies that

$$\langle I, m|S|I'm'\rangle = T_I \delta_{II'}\delta_{mm'}. \tag{25}$$

Using the composition of isospin representations, for example in our convention [31]

$$|0,0\rangle = (W_L^+W_L^- + W_L^-W_L^+ + W_L^3W_L^3)/\sqrt{3},$$
$$|1,0\rangle = (W_L^+W_L^- - W_L^-W_L^+)/\sqrt{2},$$
$$|2,2\rangle = W_L^+W_L^+,$$

and the relations in Eq. (24), one can express the three isospin amplitudes as

$$\begin{aligned}
T_0 &= \langle 00|S|00\rangle \\
&= 3A^{+-00}(p_1,p_2,p_3,p_4) + A^{++++}(p_1,p_2,p_3,p_4), \\
T_1 &= \langle 10|S|10\rangle \\
&= 2A^{+-+-}(p_1,p_2,p_3,p_4) - 2A^{+-00}(p_1,p_2,p_3,p_4) \\
&\quad - A^{++++}(p_1,p_2,p_3,p_4), \\
T_2 &= \langle 20|S|20\rangle = \langle 22|S|22\rangle = A^{++++}(p_1,p_2,p_3,p_4).
\end{aligned} \tag{26}$$

Defining $s=(p_1+p_2)^2$, $t=(p_1-p_3)^2=-\frac{1}{2}(s-4M^2)(1-\cos\theta)$, $u=(p_1-p_4)^2=-\frac{1}{2}(s-4M^2)(1+\cos\theta)$, with θ the scattering angle in the center of mass, we expand the isospin amplitudes in partial waves as

$$T_I = 16\pi\sum_j (2J+1)P_J(\cos\theta)t_{IJ}, \tag{27}$$

where the $P_J(x)$ are the Legendre polynomials.

Let us assume that we know the isospin partial-wave amplitudes perturbatively as

$$t_{IJ} = t_{IJ}^{(0)} + t_{IJ}^{(2)} + \ldots, \tag{28}$$

where $t_{IJ}^{(0)}$ and $t_{IJ}^{(2)}$ are, respectively, the leading-order (LO) and next-to-leading-order (NLO) contributions in the chiral expansion. Then the IAM approximation [4,5] of the full amplitude is

$$t_{IJ} \simeq t_{IJ}^{\mathrm{IAM}} = \frac{t_{IJ}^{(0)}}{1 - t_{IJ}^{(2)}/t_{IJ}^{(0)}} = \frac{(t_{IJ}^{(0)})^2}{t_{IJ}^{(0)} - t_{IJ}^{(2)}}, \tag{29}$$

which, by construction, satisfies the unitarity constraint, $|t_{IJ}| \le 1$.

In general, one has to deal with the possibility of coupled channels [7]. For instance, in the case of chiral Lagrangians applied to EWSB [18,19], the processes $W_L^+W_L^- \to hh$ and $hh \to hh$ also contribute to the partial wave $I=J=0$. If we define

$$\begin{aligned}
A^{+-HH}(p_1,p_2,p_3,p_4) &= A(W_L^+(p_1)W_L^-(p_2) \to h(p_3)h(p_4)), \\
A^{HHHH}(p_1,p_2,p_3,p_4) &= A(h(p_1)h(p_2) \to h(p_3)h(p_4)),
\end{aligned} \tag{30}$$

their corresponding projections in the $I=0$ channel are

$$\begin{aligned}
T_{H,0} &= \sqrt{3}A^{+-HH}(p_1,p_2,p_3,p_4), \\
T_{HH,0} &= A^{HHHH}(p_1,p_2,p_3,p_4),
\end{aligned} \tag{31}$$

so the relevant partial-wave amplitudes are

$$t_{(H)H,00} = \frac{1}{64\pi}\int_{-1}^{1} d(\cos\theta) T_{(H)H,0}. \tag{32}$$

Including all the 00 channels, one can group the corresponding perturbatively expanded amplitudes in a matrix form in the basis of states (WW, HH) as

$$\begin{aligned}
M_{00} &= M_{00}^{(0)} + M_{00}^{(2)} + \ldots \\
&\equiv \begin{pmatrix} t_{00}^{(0)} & t_{H,00}^{(0)} \\ t_{H,0}^{(0)} & t_{HH,00}^{(0)} \end{pmatrix} + \begin{pmatrix} t_{00}^{(2)} & t_{H,00}^{(2)} \\ t_{H,0}^{(2)} & t_{HH,00}^{(2)} \end{pmatrix} + \ldots.
\end{aligned} \tag{33}$$

The unitarized matrix amplitude matrix in this case is [18]

$$M_{00}^{\mathrm{IAM}} = M_{00}^{(0)}(M_{00}^{(0)} - M_{00}^{(2)})^{-1}M_{00}^{(0)}, \tag{34}$$

so the unitarized amplitude for the $WW \to WW$ channel is the (1,1) entry of the matrix above and reads

$$t_{00}^{\mathrm{IAM}} = M_{00}^{\mathrm{IAM}}(1,1) = \frac{(t_{00}^{(0)})^2 - t_{H,00}^{(0)}\frac{t_{H,00}^{(0)}(t_{00}^{(0)}+t_{00}^{(2)}) - 2t_{H,00}^{(2)}t_{00}^{(0)}}{t_{HH,00}^{(0)} - t_{H,00}^{(2)}}}{t_{IJ}^{(0)} - t_{IJ}^{(2)} - \frac{(t_{H,00}^{(0)} - t_{H,00}^{(2)})^2}{t_{HH,00}^{(0)} - t_{H,00}^{(2)}}}, \tag{35}$$

which clearly reduces to Eq. (29) if the amplitude of the mixed channel ($t_{H,00}$) vanishes.

Besides being a method for unitarization of the amplitudes, the combination of terms appearing in the denominator of the IAM amplitude allows for the possibility of having poles in the second Riemann sheet for some regions of the parameter space. When they are close enough to the physical region, those poles can be interpreted as resonances. An alternative approach [21] to identify these resonances appearing in the unitarized amplitudes is to search for values of the center-of-mass energy ($\sqrt{s_{\mathrm{pole}}}$) for which the real part of the denominator of the IAM amplitude t_{IJ}^{IAM} vanishes, and then one identifies the mass of the resonance as $M_R^2 \equiv s_{\mathrm{pole}}$. Expanding the amplitude near the pole as

$$t_{IJ}^{\rm IAM}(s) \propto \frac{1}{(s-M_R^2)+i\sqrt{s}\Gamma_R}, \qquad (36)$$

one can also derive the value of the resonance width as $\Gamma_R \propto {\rm Im}[t_{IJ}^{\rm IAM}(s)]$.

IV. RESULTS AND CONCLUSIONS

Next we apply the IAM to unitarize the gauge boson scattering amplitudes obtained in the effective Lagrangian derived for the heavy singlet Higgs portal model Eq. (17). We will then search for poles in the corresponding unitarized amplitudes and reconstruct the properties of the inferred "resonance(s)." In what follows, we will focus on the lowest J partial-wave amplitudes for each isospin channel, i.e. t_{00}, t_{11}, and t_{20}.

Technically, the mass and width of the "reconstructed" resonance are obtained by searching for poles in the denominator of t_{IJ}^{IAM} in Eq. (29), i.e. by solving

$$t_{IJ}^{(0)}(M_R^2) - {\rm Re}t_{IJ}^{(2)}(M_R^2) = 0 \quad \text{and}$$
$$\Gamma_R = -\frac{1}{M_R}\frac{{\rm Im}t_{IJ}^{(2)}(M_R^2)}{\frac{d(t_{IJ}^{(0)}(s)-{\rm Re}t_{IJ}^{(2)}(s))}{ds}\Big|_{s=M_R^2}}. \qquad (37)$$

In principle, for $IJ = 00$ we should consider the coupled channels, which, as discussed in the previous section, are relevant to the $WW \to WW$ scattering if $t_{H,00}$ is not too small. For large s, $t_{H,00}$ is proportional to $s[b_C - (a_C/2)^2]$ [19] and for the effective Lagrangian in Eq. (17) this coefficient takes the value $b_C - (a_C/2)^2 = -\frac{\Delta}{2}\frac{v^2}{M_{S_1}^2}$ which is assumed to be small in the effective Lagrangian expansion. So the inclusion of the coupled channels represents a small correction which, for simplicity, we neglect in the following, and we search for the resonances in the $IJ = 00$ channel as in Eq. (37).[4] The effect of the $WW \to hh$ channel is, nevertheless, taken into account in the evaluation of ${\rm Im}t_{00}^{(2)}$ (see Eq. (39) below).

In this work, we evaluate tree-level amplitudes using FeynArts [32] with the anomalous Higgs interactions from the Lagrangian Eq. (17) introduced using FeynArts [33] and take the exact isospin limit. Our results agree with the expressions in the literature [20] in the corresponding limits.

In order to organize the perturbative expansion of the t_{IJ}, we follow the counting in terms of powers of p that is characteristic of chiral Lagrangians [34]. In this expansion, the tree-level contributions from the Higgs anomalous couplings, $a_C - 2$, and $b_C - 1$, are counted as being part of $t_{IJ}^{(0)}$, i.e. $\mathcal{O}(p^2)$, and therefore their corresponding loop contributions must be included in $t_{IJ}^{(2)}$ since they are $\mathcal{O}(p^4)$. At present, the full calculation of the loop amplitude $W_L W_L \to W_L W_L$ in the presence of the anomalous couplings is lacking in the literature. In Ref. [35] the corresponding loop amplitude has been obtained using the equivalence theorem [36,37] and given in the approximation of massless external particles. That calculation contains the correct divergent pieces, required for renormalization of the anomalous couplings, but it represents only an approximation to the finite part of the loop amplitude. In particular, the divergent parts of the loop amplitude cancel against the renormalization of some of the tree-level couplings of the $\mathcal{O}(p^4)$ operators defined at some renormalization scale μ_R. This is the case for c_6 which then at a scale $\sqrt{s}$ becomes

$$c_6(s) \simeq c_6(\mu_R^2) - \frac{1}{24}\frac{1}{4\pi}\left[\left(1-\frac{a_C^2}{4}\right)^2 + \frac{3}{2}\left(\left(1-\frac{a_C^2}{4}\right)^2 - (1-b_C)^2\right)^2\right]\log\frac{s}{\mu_R^2}. \qquad (38)$$

In our calculations, we will take the renormalization scale as the mass of the heavy scalar $\mu_R = M_{S_1}$. Thus, when extrapolating the amplitudes to scales $s \sim M_{S_1}^2$, we can approximate $c_6(s) \simeq c_6(M_{S_1}^2)$ with $c_6(M_{S_1}^2)$ given in Eq. (18) and in Table I.[5]

The remaining finite part of the loop amplitude from both the SM and the anomalous values of a_C and b_C has to be included in $t^{(2)}$. In order to estimate the uncertainty of our final results associated with the approximations used in the evaluation of the finite part of this loop amplitude, we have performed our calculations both with and without including it in the evaluation of ${\rm Re}(t^{(2)})$. We will refer to these two calculations as $\mathcal{O}(p^4)$-1loop and $\mathcal{O}(p^4)$-tree, respectively. With respect to the amplitude ${\rm Im}t_{00}^{(2)}(s)$, it could be obtained by the application of the cutting rules to the corresponding approximated one-loop amplitude of Ref. [35]. Alternatively, we follow the approach in Refs. [20,21] and obtain the relevant imaginary part by perturbative application of the optical theorem,

$${\rm Im}t_{00}^{(2)}(s) = \frac{2p}{\sqrt{s}}|t_{00}^{(0)}(s)|^2 + \frac{2p_H}{\sqrt{s}}|t_{H,00}^{(0)}(s)|^2, \qquad (39)$$

where p (p_H) is the modulus of the three-momentum of the gauge bosons (H_1 pairs) in the center of mass.

[4] We have also verified that if we artificially set $b_C = a_C^2$ in our calculations, the reconstructed value of mass and width found in the $IJ = 00$ channels are very similar to those obtained with the correct value.

[5] The same loops generate a coefficient for the operator $[{\rm Tr}(D^\mu U)(D^\nu U)^\dagger][{\rm Tr}(D_\mu U)(D_\nu U)^\dagger]$. Such an operator is not generated by integrating out S_1 at the order given in Eq. (17). Thus, we will take the corresponding renormalized coefficient to be zero in our calculations.

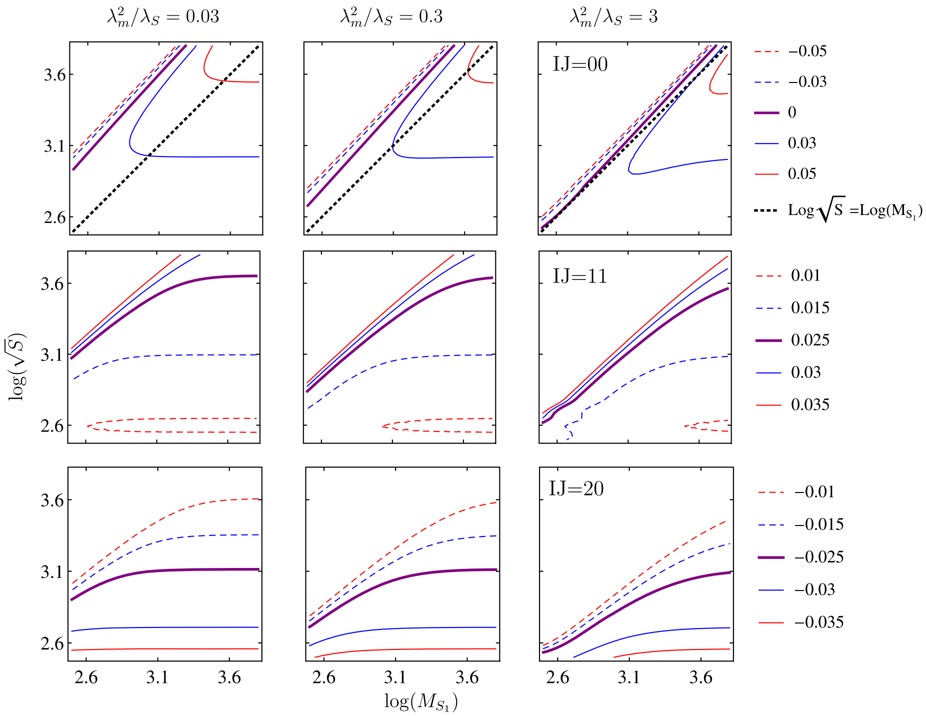

FIG. 1. Contours of the functions $\mathrm{Re}[t_{IJ}^{(0)}(s) - t_{IJ}^{(2)}(s)]$ in the plane $(\sqrt{S}, M_{S_1})$ for three characteristic values of the relevant coupling ratio $\Delta = \lambda_m^2/\lambda_S$ and for the three isospin channels $IJ = 00$ (upper panels), $IJ = 11$ central panels, and $IJ = 20$ (lower panels).

In summary:

(i) $t_{IJ}^{(0)}$ is the $\mathcal{O}(p^2)$ isospin amplitude which contains the tree-level contributions from the SM and the Higgs anomalous couplings, $a_C - 2$ and $b_C - 1$.

(ii) $\mathrm{Re}t_{IJ}^{(2)}$ is the real part of the $\mathcal{O}(p^4)$ isospin amplitude which contains the anomalous tree-level amplitude generated by c_6 only and the $\mathcal{O}(p^4)$-tree calculation. In what we call the $\mathcal{O}(p^4)$-1loop calculation, it includes as well the real part of the one-loop amplitudes generated by the SM and the Higgs anomalous couplings in the approximations given in Ref. [35].

(iii) $\mathrm{Im}t_{IJ}^{(2)}$ is calculated from the optical theorem.

We first look for the presence of physical poles in the isospin amplitudes $t_{IJ}^{\mathrm{IAM}}(s)$ as a function of the relevant parameters of the effective Lagrangian: the coupling ratio Δ and the mass scale M_{S_1} which determine the values of all relevant anomalous couplings entering the $WW \to WW$ scattering, in particular, a_C, b_C and c_6. One must notice that for the simplified potential in Eq. (2), the condition that the electroweak breaking minimum is a global minimum sets an upper bound for $\Delta < 4\lambda \simeq 0.6$; see Ref. [12] for a recent analysis of the bounds with a more general potential. Nevertheless, in what follows, we will extend our study to larger values of Δ to illustrate the results in stronger coupled scenarios.

We show in Fig. 1 contours of the real part of the denominator of the $\mathcal{O}(p^4)$-1loop functions $t_{IJ}^{\mathrm{IAM}}(s)$, i.e. $\mathrm{Re}(t_{IJ}^{(0)} - t_{IJ}^{(2)})$, for $IJ = 00$ (upper panels), 11 (central panels), and 20 (lower panels) in the $s \otimes M_{S_1}$ plane and for three characteristic values of $\Delta = 0.03$, 0.3, and 3. Therefore, this figure illustrates that for no value of Δ do the functions $\mathrm{Re}(t_{11}^{(0)} - t_{11}^{(2)})$ and $\mathrm{Re}(t_{20}^{(0)} - t_{20}^{(2)})$ present a zero in the physical plane, while $\mathrm{Re}(t_{00}^{(0)} - t_{00}^{(2)})$ as a function of s always possesses a zero for any value of Δ and M_{S1}. In other words, the effective theory after unitarization by the IAM method is compatible with the presence of one possible physical scalar resonance in the zero-isospin channel and none in any other spin-isospin channels, which is in agreement with the original full theory that has a scalar S_1 state in the physical spectrum and no other heavy states.

For the sake of illustration, we also present in the upper panels of Fig. 1 the line corresponding to $\sqrt{s} = M_{S_1}$ for comparison with the zero value contour which determines the position of the resonance $s = M_R$. As seen in this figure, the larger the value of Δ the closer the two lines; i.e., the reconstructed mass of the IAM resonance is closer to the real mass of the scalar of the full theory for stronger couplings. The results in the figure correspond to the $\mathcal{O}(p^4)$-1loop calculation, but the same qualitative results hold for the $\mathcal{O}(p^4)$-tree calculation.

We further quantify this comparison in Fig. 2 where the upper panels depict the ratio of the reconstructed scalar pole mass M_R over the S_1 mass as a function of Δ and M_{S_1} for the $\mathcal{O}(p^4)$-1loop calculation (left upper panel) and $\mathcal{O}(p^4)$-tree calculation (right upper panel). As seen in these panels, the masses agree within a factor $\mathcal{O}(1-3)$, even for very small couplings independent of whether the approximate one-loop or tree amplitudes are included in the calculation.

In order to verify that the scalar pole found can be interpreted as a physical state, we also compute its width as

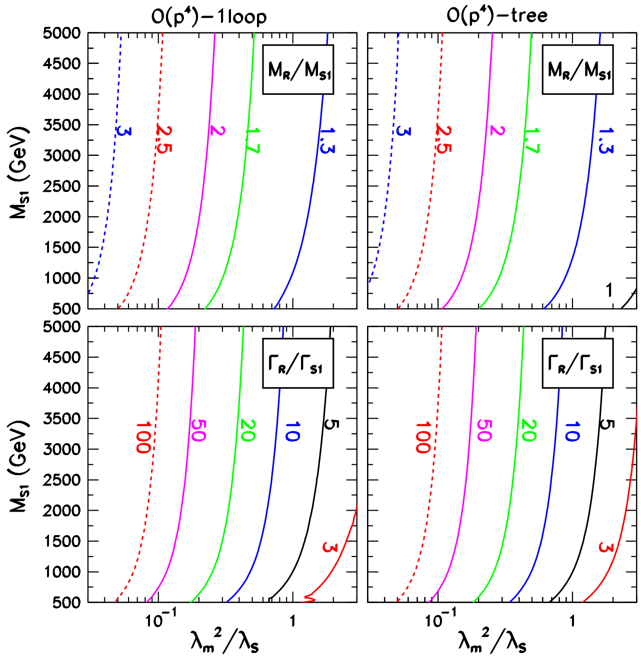

FIG. 2. Upper panels: Contours of the ratio of the mass of the resonance found in the t_{00} channel, M_R, to the mass of the integrated out scalar, M_{S_1}, versus the relevant ratio of Yukawa couplings $\Delta = \lambda_m^2/\lambda_S$ and M_{S_1}. Lower panels: Contours the ratio of the width of the resonance found in the t_{00} channel, Γ_R, to the width of the scalar, Γ_{S_1} in the plane $\Delta \otimes M_{S1}$.

in Eq. (37). We find that for all values of the model parameters Δ and M_{S_1}, the reconstructed width is positive, so the interpretation of the amplitude pole as a physical scalar state with a mass relatively close to the real scalar mass M_{S_1} holds. Notwithstanding, when compared with the perturbatively computed S_1 width in Eq. (11), we find that the reconstructed width is considerably larger as seen in the lower panels in Fig. 2, particularly for the more weakly interacting scenarios. This is somehow not unexpected. The IAM method was built to unitarize strong interaction amplitudes for which the resonance-dominance approximation holds and the amplitude near the pole of a resonance is fully determined by the resonance mass and width. However, for weakly interacting scenarios, such as that used here for illustration, the violation of unitarity is relatively mild and the full amplitude, even near the new state, contains a non-negligible "continuous" contribution from the SM. So the unitarization used in Eqs. (28) and (29) with the full SM contribution included in the reconstructed amplitude as part of the resonance amplitude does not seem to be optimum.

In summary, in this work we have explored the capability of the inverse amplitude method for unitarization of scattering amplitudes to predict the properties of possible new heavy states associated with perturbative electroweak breaking extensions of the SM, using as a starting point the unitarity violating amplitudes of the low-energy effective theory. We have used as a study case that of the singlet Higgs portal. First, in Sec. II we derived the effective Lagrangian obtained after integrating out the heavier scalar while leaving the lighter scalar, a mixture of the doublet and singlet states. We showed that in this case the effective Lagrangian can be matched to that of a chiral expansion which we write up to $\mathcal{O}(p^4)$. With this effective Lagrangian in hand, we obtained the relevant unitarity violating amplitudes. Working in the isospin approximation, we used the IAM method to reconstruct unitarized amplitudes and search for possible physical poles in these amplitudes. The results in Sec. IV show that only the unitarized spin scalar zero-isospin amplitude presents poles in the physical plane, in agreement with the full theory which has only one additional heavy scalar. We also find that the IAM reconstructs correctly the scalar singlet mass up to factors of $\mathcal{O}(1–3)$ even for relatively weak couplings. Nevertheless, its width is systematically overestimated. It remains an open question whether this is symptomatic of the applicability of the IAM for unitarization of weakly coupled pertubative scenarios.

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APPENDIX: EFFECTIVE LAGRANGIAN AFTER INTEGRATING OUT THE S FIELD

Our starting point is the SM extended by the addition of a real singlet scalar field S as given in Eqs. (1) and (2). Below the scale at which S acquires a VEV v_s, the scalar potential can be written as [23]

$$V(\Phi, S) = -\tilde{\mu}_H^2|\Phi|^2 + \lambda|\Phi|^4 + \frac{M_S^2}{2}S^2 + v_s\lambda_S S^3 + \frac{\lambda_S}{4}S^4 + \lambda_m v_s|\Phi|^2 S + \frac{\lambda_m}{2}|\Phi|^2 S^2, \tag{A1}$$

with $\tilde{\mu}_H^2 = \mu_H^2 - (\lambda_m v_s^2)/2$ and $M_S^2 = 2\lambda_S v_S^2$. Now we rewrite this Lagrangian as $\mathcal{L} = \mathcal{L}(\Phi) + \Delta\mathcal{L}(\Phi, S)$ with

$$\Delta\mathcal{L} = \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}M_S^2 S^2 - A|\Phi|^2 S - \frac{1}{2}k|\Phi|^2 S^2 - \frac{1}{3!}\mu S^3 - \frac{1}{4!}\tilde{\lambda}_S S^4, \tag{A2}$$

where

$$\mu = 6\lambda_S v_S, \quad \tilde{\lambda}_S = 6\lambda_S, \quad k = \lambda_m, \quad A = \lambda_m v_S. \tag{A3}$$

In order to apply the tree-level integration procedure described in Sec. II for the singlet field S, we must solve the EOM for the S field at lowest order leading to

$$S_C = \frac{1}{\partial_\mu\partial^\mu + M_S^2 + U} B, \tag{A4}$$

where we have defined

$$B = -A|\Phi^2|U = k|\Phi^2|. \tag{A5}$$

Introducing S_C in Eq. (A2) and keeping the terms up to order dimension eight, one obtains the following anomalous interactions:

$$\Delta\mathcal{L}_{\rm eff} = \frac{A^2}{2M_S^2}|\Phi|^4 + \frac{A^2}{2M_S^4}\partial_\mu|\Phi|^2\partial^\mu|\Phi|^2 + \frac{A^2}{2M_S^4}\left(\frac{A\mu}{3M_S^2} - k\right)|\Phi|^6 + \frac{A^2}{2M_S^6}\left(-\frac{A^2\tilde{\lambda}_S}{12M_S^2} + k^2 - \frac{A\mu k}{M_S^2}\right)|\Phi|^8 + \frac{2A^2}{M_S^6}\left(\frac{A\mu}{2M_S^2} - k\right)|\Phi^2|\partial_\mu|\Phi|^2\partial^\mu|\Phi|^2 + \frac{A^2}{2M_S^6}\partial_\mu\partial^\mu|\Phi|^2\partial_\nu\partial^\nu|\Phi|^2. \tag{A6}$$

At this point it is interesting to apply the EOM of the doublet field to the last term of the equation above to better observe the emergence of an anomalous quartic coupling between the electroweak gauge bosons. This is possible since the invariance of the physical observables under the associated operator redefinitions is guaranteed as it has been proven that operators connected by the EOM lead to the same S-matrix elements [38]. The EOM for the doublet field reads

$$(D^\mu D_\mu \Phi) = \mu_H^2\Phi - 2\lambda\Phi|\Phi|^2 + F_{\rm ferm}, \tag{A7}$$

where $F_{\rm ferm}$ is a function involving fermionic fields from the Yukawa operators. Moreover, using this EOM we find

$$\partial_\mu\partial^\mu|\Phi|^2 = 2[(D^\mu\Phi)^\dagger(D_\mu\Phi) + \mu_H^2|\Phi|^2 - 2\lambda|\Phi|^4] + \text{terms containing fermionic fields.} \tag{A8}$$

Therefore, altogether we find that for terms involving only scalar and/or gauge bosons,

$$\Delta\mathcal{L}_{\rm eff} = -\Delta\lambda|\Phi|^4 + \frac{f_{\Phi,3}}{M_S^2}\mathcal{O}_{\Phi,3} + \frac{f_{\Phi,5}}{M_S^4}\mathcal{O}_{\Phi,5} + \frac{f_{\Phi,2}}{M_S^2}\mathcal{O}_{\Phi,2} + \frac{f_{\Phi,4}}{M_S^2}\mathcal{O}_{\Phi,4} + \frac{f_{\Phi,6}}{M_S^4}\mathcal{O}_{\Phi,6} + \frac{f_{\Phi,7}}{M_S^4}\mathcal{O}_{\Phi,7} + \frac{f_{S,1}}{M_S^4}\mathcal{O}_{S,1} \tag{A9}$$

with $\Delta\lambda = -\frac{A^2}{2M_S^2}(1 + \frac{4\mu_H^4}{M_S^4}) = -\frac{\lambda_m^2}{4\lambda_S}(1 + \frac{4\mu_H^4}{M_S^4})$ and

$$\mathcal{O}_{\Phi,2} = \frac{1}{2}\partial_\mu|\Phi|^2\partial^\mu|\Phi|^2 \qquad f_{\Phi,2} = \frac{A^2}{M_S^2} = \frac{\lambda_m^2}{2\lambda_S}$$

$$\mathcal{O}_{\Phi,3} = \frac{1}{3}|\Phi|^6 \qquad f_{\Phi,3} = \frac{3A^2}{2M_S^2}\left(\frac{A\mu}{3M_S^2} - k - \frac{16\lambda\mu_H^2}{M_S^2}\right) = -\frac{12\lambda_m^2}{\lambda_S}\frac{\lambda\mu_H^2}{M_S^2}$$

$$\mathcal{O}_{\Phi,4} = (D^\mu\Phi)^\dagger(D_\mu\Phi)|\Phi|^2 \qquad f_{\Phi,4} = \frac{4A^2\mu_H^2}{M_S^4} = \frac{2\lambda_m^2}{\lambda_S}\frac{\lambda\mu_H^2}{M_S^2}$$

$$\mathcal{O}_{\Phi,5} = \frac{1}{4}|\Phi|^8 \qquad f_{\Phi,5} = \frac{2A^2}{M_S^2}\left(-\frac{A^2\tilde{\lambda}_S}{12M_S^2} + k^2 - \frac{Ak\mu}{M_S^2} + 16\lambda^2\right) \simeq (64\lambda^2 - 9\lambda_m^2)\frac{\lambda_m^2}{4\lambda_S}$$

$$\mathcal{O}_{\Phi,6} = \frac{1}{2}|\Phi|^2\partial_\mu|\Phi|^2\partial^\mu|\Phi|^2 \qquad f_{\Phi,6} = \frac{4A^2}{M_S^2}\left(\frac{A\mu}{2M_S^2} - k\right) = \frac{\lambda_m^3}{\lambda_S}$$

$$\mathcal{O}_{\Phi,7} = |\Phi|^2(D^\mu\Phi)^\dagger(D_\mu\Phi)|\Phi|^2 \qquad f_{\Phi,7} = -\frac{8A^2\lambda}{M_S^2} = -4\lambda\frac{\lambda_m^2}{\lambda_S}$$

$$\mathcal{O}_{S,1} = (D^\mu\Phi)^\dagger(D_\mu\Phi)(D^\nu\Phi)^\dagger(D_\nu\Phi) \qquad f_{S,1} = \frac{2A^2}{M_S^2} = \frac{\lambda_m^2}{\lambda_S}.$$

Notice that in the last column we have introduced the relations in Eq. (A3), and we have expanded to the lowest nonzero order in μ_H^2/M_S^2.

The effective Lagrangian in Eq. (A9) also leads to violation of unitarity of the electroweak boson scattering. For example, the $W^+W^- \to ZZ$ amplitude takes the form in Eq. (21) with the identification (at the lowest order in inverse powers of the heavy mass) $a_C = 2 - f_{\Phi,2}\frac{v^2}{M_S^2}$ and $c_6 = \frac{f_{S,1}}{16}\frac{v^4}{M_S^4}$ [39] .

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