Optimal Fiscal Policy in Overlapping Generations Models*

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Abstract

This paper provides, from a theoretical and quantitative point of view, an explanation of why taxes on capital returns are high by analyzing the optimal fiscal policy in an economy with intergenerational redistribution. For this purpose, the government is modeled explicitly and can choose (and commit to) an optimal tax policy in order to maximize society's welfare. In an infinitely lived economy with heterogeneous agents, the long run optimal capital tax is zero. If heterogeneity is due to the existence of overlapping generations, this result in general is no longer true. I provide a sufficient conditions for zero capital, and show that a general class of preferences, commonly used in the macro and public finance literature, violate this condition. For a version of the model, calibrated to the US economy, the main results are: first, if the government is restricted to use proportional taxes across generations, the model can account for the observed capital and labor income taxes. Second, if the government can use specific taxes for each generation, then the age profile capital tax pattern implies subsidizing asset returns of the younger generations and taxing at higher rates the asset returns of the older ones.

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1 Introduction

The standard view of economists is that capital returns should not be taxed at all. In Lucas' words: "*I believed that the single most desirable change in the U.S. tax structure would be the taxation of capital gains as ordinary income. I now believe that neither capital gains nor any source of income from capital should be taxed at all. My earlier view was based on what I viewed as the best available economic analysis, but of course I think my current view is based on a better economic analysis",* (Lucas (1990)).

This view is built on a well-established theory of optimal fiscal policy. In standard neoclassical growth models, with infinitely-lived consumers, the optimal policy predicts that capital taxes should be zero in the long run and important welfare gains can be realized by implementing this policy. This result was first shown by Chamley (1986). Several papers have extended this result to more complex economies that include heterogeneous consumers (Judd (1985)), endogenous growth (Jones, Manuelli and Rossi (1997)), aggregate shocks (Chari, Christiano and Kehoe (1994)), and open economies (Razin and Sandka (1995)). In each case the result is the same, capital returns should not be taxed at all.

However the observed data for most OECD economies show that capital taxes are different from zero. For instance in the US the average capital tax over the period $1965 - 1995$ is around 35%, and in the U.K and Germany it is around 37% and 23.5% respectively.

At this point theory and data seem to be mutually exclusive. Why might governments choose to tax capital returns at high rates? Can we find a model where the optimal policy implies capital taxes different from zero? Is this model consistent with the observed capital taxation?

The main contribution of this paper is to show that when we consider a finite lifetime economy, in the tradition of Auerbach and Kotlikoff (1987), for the class of utility functions commonly used in the macro and public finance literature the optimal policy implies a tax rate on capital returns different from zero. Moreover, for some plausible choices of parameter values the optimal policy is consistent with the average capital taxes observed for most OECD countries.

This result contrasts with previous results in the extensive literature on optimal fiscal policy in overlapping generations economies, for example Pestieau (1974), Atkinson and Sandmo (1980), Atkinson and Stiglitz (1980), and more recently Chari and Kehoe (1999). These papers focus their analysis on the steady state and find restrictive conditions for zero capital taxes in the long run. These conditions cannot be extended to economies where agents live more than two periods.

This result together with the standard results in infinite-lived consumers models suggested other explanations to account for the observed high capital taxation: incomplete markets (Aiyagari (1995) and Domeij and Heathcote (2000)), time consistency issues (Klein and Rios-Rull (2000) and Phelan and Stachetti (2001)), or information problems (Golosov, Kocherlakota and Tsyvinski (2001)). One of the main problems in these papers is that fiscal policy plays two roles: finance government expenditure and substitute for missing markets (insurance, lack of commitment or information problems). This raises the question why the government cannot implement some other type of policy to achieve this goal, and set capital taxes to zero. In this paper markets are complete, and the focus is on the distortions due to taxation.

This paper provides theoretical foundations as well as quantitative results for the optimal fiscal policy in finite lifetime environments. The main theoretical result is that the optimal tax on capital returns is in general different from zero, both in the transition path and in the long run. I provide a sufficient condition for the zero capital tax result and show that a general class of preferences violates this condition.

To provide some intuition it is useful to compare this result with the optimal policy in an infinitely-lived consumers economy. In a dynamic economy there is certain equivalence between tax instruments. For example, a tax system that uses positive capital taxes to finance government expenditure is equivalent to a tax system that uses an ever-increasing consumption tax and an increasing labor subsidy. In an infinitely-lived consumer economy, if preferences are separable and exhibit some degree of substitutability, an increasing consumption tax creates an important distortion on the relative price between today's consumption and consumption at period T. Then given that individuals want to smooth consumption, they prefer a constant consumption tax than an ever-increasing consumption tax.

In contrast, if individuals live a finite number of periods, as in an overlapping generations model, the distortions associated with this policy are not that important because for a given generation today's consumption and period T consumption are not perfectly substitutable. Hence the effect of capital distortions is much smaller and not necessarily bigger than distortions caused by other taxes.

In general taxes on capital returns are different from zero unless we make some assumption either on the lifetime horizon or on the set of instruments available to the government. The optimal capital tax can be zero if we consider either a two period OG model where the old generation does not supply labor (this is the standard framework used in the referenced literature[1]), or the government is allowed to use age-dependent taxes. In these two cases the uniform tax property ensures zero capital taxes, both in the transition path and in the long run. Restrictions on the set of tax instruments that the government can use play an important role in the determination of the optimal policy. This point is emphasized in the quantitative results.

For a version of the model calibrated to the US economy the key findings are: first, if the government uses proportional taxes across ages, the optimal capital tax is as high as in the US economy. Second, if the government can use age-dependent

[1] In a simple two period model, where only the young generations supply labor, the standard assumption for the set of fiscal instruments available implies that labor income tax affects young generations while capital taxes affect old generations in the economy. Therefore, under certain assumptions on the utility function (satisfy unitary expenditure elasticity), it can be shown that cumulative distortions of the capital income taxes on the intertemporal decisions make young generations prefer a static distortion due to labor taxes. If we allow generations to work in all periods, then the previous result is no longer true even with the same type of preferences.

proportional taxes, then the age profile capital tax pattern implies a subsidy on the asset returns of the younger generations and a tax on the asset holdings of the older ones. Hence, constraints on the set of instruments that the government can use are quantitatively too important to be ignored.

At this point two papers deserve special comments. Escolano (1992) showed, using a quantitative analysis, that in overlapping generations economies, positive capital taxes can be optimal and the efficiency loss of the current fiscal system in the US economy is not quantitatively relevant. In his model for a particular value of the government discount factor, the optimal tax on capital returns is zero. This paper has two shortcomings, first it does not derive sufficient conditions to check whether this is a general result or not. Second, the quantitative exercise implies that the government only has access to proportional taxes across ages. Independent work by Erosa and Gervais (2000) considers a similar economy and derives some of the results presented here for the case of age-dependent taxes.

The paper is organized as follows. Section 2 describes the environment and defines a competitive equilibrium, and section 3 defines the government problem and derives sufficient conditions for the zero capital tax result. Section 4 describes the calibration process and the main quantitative findings under different tax arrangements. Finally, section 5 concludes.

2 The economy

The model is a standard overlapping generations production economy with two goods, a consumption-capital good and labor. Agents live $I \geq 2$ periods and each cohort is populated by a continuum of identical households, represented by the interval $[0,1]$. Without loss of generality, the population is assumed to be stationary and its total size is constant.[2]

There is a representative firm that produces aggregate output Y_t using a constant returns to scale production function $F(K_t, L_t)$, using aggregate capital K_t and aggregate labor L_t as primary inputs. Labor is measured in efficiency units. The production function $F : R^2_+ \rightarrow R_+$ is strictly concave, monotone, homogeneous of degree one and continuously differentiable, with partial derivatives $F_1(K_t, L_t) > 0$ and $F_2(K_t, L_t) > 0$. Furthermore for all $K, L \in R^2_+$ $F(0, L) = F(K, 0) = 0$. Capital depreciates each period at a constant rate $\delta \in (0, 1)$ and there is no exogenous technological change. These assumptions imply that in competitive factor markets firms will make zero profits, hence it is unnecessary to specify firms' ownership. Then, each period prices are determined by the marginal products, that is

$$r_t = F_1(K_t, L_t) - \delta, \tag{1}$$

$$w_t = F_2(K_t, L_t), \tag{2}$$

where r_t denotes the interest rate net of depreciation and w_t is the wage rate per efficiency unit of labor. Let C_t and L_t denote aggregate consumption and labor

[2]This is not an important assumption for the basic results and simplifies notation.

respectively

$$C_t = \sum_{i=1}^{I} c_t^i \qquad \forall t, \tag{3}$$

$$L_t = \sum_{i=1}^{I} \epsilon^i \ell_t^i \qquad \forall t, \tag{4}$$

where c_t^i denotes consumption of an individual of age i at time t, ϵ^i denote her efficiency units, and ℓ_t^i is hours worked.

The government in this economy finances an exogenous sequence of expenditure $\{G\}_{t=0}^{\infty}$ using proportional capital taxes θ_t, labor taxes τ_t and debt D_t. The government intertemporal budget constraint is

$$G_t + R_t D_t \leq \tau_t w_t L_t + \theta_t r_t K_t + D_{t+1} \qquad \forall t, \tag{5}$$

where R_t denotes the return on government debt.[3] Let $\pi = \{\tau_{\ell_t}, \theta_t\}_{t=0}^{\infty}$ be a tax policy consisting of an infinite sequence of proportional taxes. Notice that the government debt has not been included as part of the policy. Given the initial government debt D_0 and the sequence $\{G\}_{t=0}^{\infty}$, we can back up the sequence of debt $\{D\}_{t=1}^{\infty}$.

The economy resource constraint at each time t is given by

$$C_t + K_{t+1} - (1-\delta)K_t + G_t \leq F(K_t, L_t) \qquad \forall t. \tag{6}$$

Households in this economy have standard preferences defined over a stream of consumption and labor $\{c_t^i, \ell_t^i\}_{t=0}^{\infty}$, and are represented by a time separable utility function

$$\sum_{i=1}^{I} \beta^{i-1} U(c_t^i, \ell_t^i) \qquad \forall t, \tag{7}$$

where $\beta > 0$ is the subjective discount factor. The utility function $U : R_+^2 \to R_+$ is C^2, strictly concave, increasing in consumption $U_c(c_t^i, \ell_t^i) > 0$, decreasing in labor $U_\ell(c_t^i, \ell_t^i) < 0$. The agents are endowed at each period with a time invariant vector of efficiency units of labor $\epsilon = (\epsilon^1, ..., \epsilon^I)$. One unit of time of an individual of age i can be transformed into ϵ^i units of input in the production function. At each period, taking prices and taxes as given, individuals choose consumption, labor supply, and asset holdings. Households' decisions face a sequence of budget and non-negativity constraints:

$$c_t^i + a_{t+1}^{i+1} \leq (1-\tau_t) w_t \epsilon^i \ell_t^i + (1 + r_t(1-\theta_t)) a_t^i \qquad 1 \leq i \leq I \quad \forall t, \tag{8}$$

$$a_t^1 = 0, \quad 0 \leq \ell_t^i \leq 1, \quad c_t^i \geq 0 \qquad \forall t.$$

Households are born with zero assets and accumulate wealth a_{t+1}^{i+1} in three forms: buying government debt of one period maturity and lending to households or firms. Government debt and capital are perfect substitutes for the individual investment decisions.

[3] Without loss of generality I have abstracted from consumption taxes, because in this model they are redundant.

At the initial period, $t = 0$, the stock of capital and debt is distributed among the initial, s, generations (individuals of age 2 to I). Let $\overline{a}_0^s$ be the initial endowment of wealth of generation s. The period 0 budget constraint is given by

$$c_0^s + a_1^{s+1} \leq (1 - \tau_0) w_0 \epsilon^s \ell_0^s + (1 + r_0(1 - \theta_0))\overline{a}_0^s \qquad 2 \leq s \leq I, \tag{9}$$

Intertemporal trade between generations to smooth consumption over the life-cycle is allowed. [4] Market clearing conditions in the capital markets imply:

$$K_{t+1} = \sum\nolimits_{i=1}^{I} a_{t+1}^i - D_{t+1} \qquad \forall t, \tag{10}$$

Next we proceed by defining the notion of competitive equilibrium.

Definition 1 (Competitive Equilibrium): *Given a tax policy π and a sequence of government expenditure $\{G_t\}_{t=0}^{\infty}$, a competitive equilibrium in this economy is a sequence of individual allocations $\{\{c_t^i, \ell_t^i, a_{t+1}^i\}_{i=1}^{I}\}_{t=0}^{\infty}$, production plans $\{K_t, L_t\}_{t=0}^{\infty}$, government debt $\{D_{t+1}\}_{t=0}^{\infty}$, and relative prices $\{r_t, w_t, R_t\}_{t=0}^{\infty}$, such that:*

1. *Consumers born at time $t \geq 1$ maximize* (7) *subject to* (8). *Similarly consumers born at $t \leq 0$ maximize utility subject to* (9).
2. *In the production sector* (1) *and* (2) *are satisfied for all t.*
3. *Factor markets clear and* (4) *and* (10) *hold.*
4. *The government budget constraint* (5) *is satisfied.*
5. *Feasibility* (6) *is satisfied for all t.*

3 Government problem

Let's consider the policy problem faced by the government. Suppose that there exists a benevolent government that chooses the optimal policy π^*, to maximize the welfare of all (present and future) generations subject to given constraints. These constraints imply that the present value government budget constraint must be satisfied and the allocation associated with the optimal policy has to belong to the subset of policies for which a competitive equilibrium exists.

The optimal fiscal policy might cause time-consistency problems because the government might have incentives to deviate from the optimal policy once it has been announced and taken into account by the agents. This time-consistency issue has been shown in Kydland and Prescott (1977) and Calvo (1978). In this paper we abstract from these issues and it is assumed that the government can

[4] This assumption implies that credit markets are complete, in the sense that agents are not credit constrained. Aiyagari (1995) shows that in an economy with incomplete markets the optimal capital tax is positive. The basic intuition works as follows. Because of the incomplete insurance markets, there is a precautionary motive for accumulating capital. In addition, the possibility of being borrowing constrained in the future leads to some additional savings. These facts increase the capital stock. Then, a positive capital tax is needed to reduce capital accumulation and equalize the interest rate of the economy to the rate of time preference.

commit to future policies. This *commitment technology* is modelled in the timing of the decisions and it works as follows: first the government chooses a policy π at time zero, second agents choose optimal allocations taking as given the policy and prices.[5]

The government is benevolent and values all present and future generations in the economy. The government objective function is defined as a weighted sum of each generation's lifetime utility[6]. Therefore the government assigns a non-negative weight to all individuals when born. Let $\omega_t \in R_+$ be the relative weight of a generation born at time t. The sequence of weights $\{\omega_t\}_{t=-(I-1)}^{\infty}$ is bounded above by a large positive constant $\Gamma < \infty$. Formally the government objective function is

$$W(\{c_t^i, l_t^i\}) = \sum\nolimits_{t=0}^{\infty} \sum\nolimits_{i=1}^{I} \omega_{t+1-i} \left(\beta^{i-1} U(c_t^i, l_t^i)\right). \tag{11}$$

To find an asymptotic steady state for the government problem it is convenient to impose some structure on the sequence of weights[7], such as

$$\lim_{t \to \infty} \frac{\omega_t}{\omega_{t+1}} = \frac{1}{\lambda} > 1.$$

where λ can be view as a subjective valuation of present and future generations. If the government valuation of future generations is high, then λ is close to one.

The government problem of choosing the optimal policy is solved using the so-called *primal approach,* as in Lucas and Stokey (1983) and Chari and Kehoe (1999). One way to think of it is having the government choosing directly from the set of implementable allocations given a tax policy π. Then from the allocations it is possible to back out policies and prices from the competitive equilibrium. The set of implementable allocations is characterized by the period resource constraint and an implementability constraint for each generation. The implementability constraint takes into account that changes in the policy will affect agents' decisions, therefore prices and government revenues. These constraints are constructed by substituting the households' budget constraints, and firms' first-order conditions, into the households' decision rules. The next proposition shows how to characterize the set of implementable allocations for a given class of policy π.

Proposition 1 (Set of Implementable Allocations): *Given a tax policy π any competitive equilibrium allocation $x = \{\{c_t^i, l_t^i\}_{i=1}^{I}, K_{t+1}\}_{t=0}^{\infty}$ satisfies the period resource constraint:*

$$\sum\nolimits_{i=1}^{I} c_t^i + K_{t+1} - (1-\delta)K_t + \overline{G}_t = F(K_t, \sum\nolimits_{i=1}^{I} \epsilon^i \ell_t^i) \qquad \forall t, \tag{12}$$

[5]If a *commitment technology* is not available there are two ways to go, one is to find mechanisms that substitute for commitment as in Chari and Kehoe (1990) or Stokey (1991), or solve the case where neither commitment or reputation mechanism are operative as in Klein and Ríos-Rull (1999) or Phelan and Stacchetti (1999).

[6]In the early literature, see Samuelson (1958) or Diamond (1965), society's welfare was represented by the utility of a representative generation in steady state.

[7]Atkinson and Sandmo (1980) derive, using the so-called *dual approach*, the optimal capital tax in the steady state for different government discount factors. It can be easily shown that all are particular cases of this formulation.

implementability constraints for all newborn generations:

$$\sum_{i=1}^{I} \beta^{i-1} \left(c_{t+i-1}^{i} U_{c_{t+i-1}^{i}} + \ell_{t+i-1}^{i} U_{\ell_{t+i-1}^{i}} \right) = 0 \qquad t \geq 0, \tag{13}$$

implementability constraints for the initial old generations at $t = 0$:

$$\sum_{i=s}^{I} \beta^{i-s} \left(c_{i-s}^{i} U_{c_{i-s}^{i}} + \ell_{i-s}^{i} U_{\ell_{i-s}^{i}} \right) = U_{c_0^s} a_0^s \quad s = 2, ..., I, \tag{14}$$

marginal rates of substitution between consumption and labor, and consumption today and tomorrow are equal across consumers:

$$\frac{U_{c_t^1}}{U_{l_t^1}} \epsilon^1 = \ldots = \frac{U_{c_t^I}}{U_{l_t^I}} \epsilon^I \qquad \forall t, i, \tag{15}$$

$$\frac{U_{c_t^1}}{U_{c_{t+1}^2}} = \ldots = \frac{U_{c_t^{I-1}}}{U_{c_{t+1}^I}} \qquad \forall t, i. \tag{16}$$

Furthermore, given allocations that satisfies (12), (13), (14), (15), *and* (16), *we can construct a tax policy $\pi = \{\tau_t, \theta_t\}_{t=0}^{\infty}$, a sequence of government debt $\{D_t\}_{t=0}^{\infty}$ and relative prices $\{r_t, w_t, R_t\}_{t=0}^{\infty}$, that together with the allocation x, constitute a competitive equilibrium.*

Proof. We first proceed by showing that the allocations in a competitive equilibrium must satisfy (12), (13), (14), (15), and (16). Condition (12) is straightforward from substituting the labor market clearing condition (4) into (6). To derive the implementability constraint for each generation we have to use the first-order conditions of the consumer's problem

$$\beta^i U_{c_t^i} = \alpha_t^i \qquad \forall t, i, \tag{17}$$

$$\beta^i U_{l_t^i} = -\alpha_t^i (1 - \tau_t) w_t \epsilon^i \qquad \forall t, i, \tag{18}$$

$$\alpha_t^i = \alpha_{t+1}^{i+1} \left[1 + r_{t+1}(1 - \theta_{t+1}) \right] \qquad \forall t, i, \tag{19}$$

where α_t^i denotes the Lagrange multiplier of age i budget constraint. To derive the implementability constraint we have to multiply (17), (18) by their respective control variables and then add them up for all i. Then substituting in the resulting expression the households' budget constraint and using (19) we derive the implementability constraint for the newborn generations (13). For the initial generations in the economy at time $t = 0$ the distribution of asset holdings appears on the right hand side of (14).

If the government is restricted to use the same proportional taxes for all generations the set of implementable allocations needs to include constraints (15), and (16).

Now we prove the second part of Proposition 1. From the aggregate capital stock, K_t, and the aggregate labor supply, L_t, we construct the relative prices using the firm's first-order conditions (1) and (2).

To derive the policy $\pi = \{\tau_t, \theta_t\}_{t=0}^{\infty}$ we substitute the allocations $\{\{c_t^i, l_t^i\}_{i=1}^{I}\}_{t=0}^{\infty}$ and the equilibrium prices $\{r_t, w_t,\}_{t=0}^{\infty}$ into households' first-order conditions as follows

$$1 - \tau_t = -\frac{U_{l_t^i}}{U_{c_t^i}\epsilon^i F_{L_t}} \qquad \forall t, i, \tag{20}$$

$$1 - \theta_{t+1} = \frac{1}{F_{K_t} - \delta}\left[\frac{U_{c_t^i}}{\beta U_{c_{t+1}^{i+1}}} - 1\right] \qquad \forall t, i. \tag{21}$$

The tax rates are set to satisfy the consumers' first-order conditions. The return on debt holdings for all t is determined by an arbitrage argument, $R_t = 1 + r_t(1 - \theta_t)$. Substituting $U_{c_t^i}$ and $U_{\ell_t^i}$ from the consumers problem into (13), (14) we obtain the intertemporal budget constraint of each household (for simplicity written in Arrow-Debreu terms):

$$\sum\nolimits_{i=0}^{I-1} p_{t+i}\left[c_{t+i}^i - (1 - \tau_{t+i})w_{t+i}\epsilon^i \ell_{t+i}^i\right] = 0 \qquad \forall i. \tag{22}$$

The sequence of government debt $\{D_t\}_{t=0}^{\infty}$ is adjusted to satisfy the desired capital-output ratio, and is obtained using market clearing in the capital market

$$D_{t+1} = \sum\nolimits_{i=1}^{I} a_{t+1}^i - K_{t+1}. \tag{23}$$

This expression does not impose any restrictions on the sign of the government debt, therefore it might be negative. If feasibility and the households budget constraint are satisfied, then the government budget constraint is also satisfied. ■

It is important to note that if an allocation $x = \{\{c_t^i, l_t^i\}_{i=1}^{I}, K_{t+1}\}_{t=0}^{\infty}$ satisfies feasibility and each generation's implementability constraint, that does not imply that the marginal rates of substitution across consumers are equal. To see this, suppose that the government can use age-dependent taxes, that is $\pi^i = \{\{\tau_t^i, \theta_t^i\}_{i=1}^{I}\}_{t=0}^{\infty}$, then if an allocation x satisfies feasibility and the implementability constraint, the marginal rates of substitution do not need to be equal across generations at a given point in time. The constraints in the set of taxes that the government can use will play an important role to determine the optimal fiscal policy but are often not considered.

In a representative agent economy, the government has incentives to tax heavily the initial stock of capital. To avoid this problem it is commonly assumed that the government takes as given the initial capital tax. In this type of economy the government faces an infinite sequence of implementability constraints, because the economy is populated over time by an infinite number of individual that live a finite number of periods. Then, taxing the initial distribution of asset holdings $\{\overline{a}_0^s\}_{s=0}^{I}$ is equivalent to tax an inelastically supplied factor and has intergenerational redistributive effects, therefore it is assumed that the government takes as given the initial capital tax $\{\theta_0\}$. Dropping this assumption does not affect the main results of the paper because capital taxes at $t = 0$ cannot be used to mimic lump-sum taxes beyond $t = 1$.

The government's optimal taxation problem is to choose x from the set of implementable allocations such that the utilitarian objective function is maximized. Formally the Ramsey allocation problem is,

$$\max_{\{\{c_t^i,\ell_t^i\}_{i=1}^I,K_{t+1}\}_{t=0}^\infty} \sum_{t=0}^{\infty} \sum_{i=1}^{I} \omega_{t+1-i} \left[\beta^{i-1} U(c_t^i, \ell_t^i)\right], \tag{24}$$

$$s.t. \quad \sum_{i=1}^{I} c_t^i + K_{t+1} - (1-\delta)K_t + \overline{G}_t = F(K_t, \sum_{i=1}^{I} \epsilon^i \ell_t^i) \qquad \forall t, \tag{25}$$

$$\sum_{i=1}^{I} \beta^{i-1} \left(c_{t+i-1}^i U_{c_{t+i-1}^i} + \ell_{t+i-1}^i U_{\ell_{t+i-1}^i}\right) = 0 \qquad t \geq 0, \tag{26}$$

$$\sum_{i=s}^{I} \beta^{i-s} \left(c_{i-s}^i U_{c_{i-s}^i} + \ell_{i-s}^i U_{\ell_{i-s}^i}\right) = U_{c_0^s} a_0^s \quad s = 2, ..., I, \tag{27}$$

$$\frac{U_{c_t^i}}{U_{c_{t+1}^{i+1}}} = ... = \frac{U_{c_t^{I-1}}}{U_{c_{t+1}^I}} \qquad \forall t, \quad i = 1, ..., I, \tag{28}$$

$$\frac{U_{c_t^1}}{U_{\ell_t^1}} \epsilon^1 = ... = \frac{U_{c_t^I}}{U_{\ell_t^I}} \epsilon^I \qquad \forall t, \quad i = 1, ..., I. \tag{29}$$

where the initial distribution of wealth $a_{s,0}$, $K_0 = \overline{K} > 0$ and $\{G_t\}_{t=0}^\infty$ are given and $c_t^i \geq 0$, $\ell_t^i \in (0, 1)$.

The allocation x that solves the Ramsey allocation problem is constrained efficient, in the sense that there exists no other constrained efficient allocation x' that Pareto dominates the optimal.

To solve the government problem I follow a particular approach assuming that the government has access to a complete set of age-dependent taxes (that is dropping the additional constraints on the marginal rates of substitution), and then verify under what conditions capital taxes will be zero. If capital taxes are not zero in the less constrained problem, then we cannot expect them to be zero for the age-independent tax case. Then, it is useful to redefine the objective function by introducing the implementability constraint on it, and use the associated Lagrange multiplier as co-state variable. Let η_{t-i} be the Lagrange multiplier of the implementability constraint for the agent born in period $t-i$. Let's define

$$V(c_t^i, \ell_t^i, \eta_{t-i}) = U(c_t^i, \ell_t^i) + \eta_{t-i}(c_t^i U_{c_t^i} + \ell_t^i U_{\ell_t^i}). \tag{30}$$

The modified Ramsey Allocation Problem can be written as follows:

$$\max_{\{\{c_t^i,\ell_t^i\}_{i=1}^I,K_{t+1}\}_{t=0}^\infty} \sum_{t=0}^{\infty} \sum_{i=1}^{I} \omega_{t+1-i} \left[\beta^{i-1} V(c_t^i, \ell_t^i, \eta_{t-i})\right] - \sum_{s=2}^{I} \eta_{1-s} U_{c_0^s} a_0^s, \tag{31}$$

$$s.t. \quad \sum_{i=1}^{I} c_t^i + K_{t+1} - (1-\delta)K_t + \overline{G}_t = F(K_t, \sum_{i=1}^{I} \epsilon^i \ell_t^i) \qquad \forall t. \tag{32}$$

This problem has a similar structure to an optimal planning problem, except that the period utility function $U(\cdot)$ has been replaced by a "pseudo" utility function $V(\cdot)$. If the government has access to lump-sum taxes, the implementability constraint will not be binding, $\eta_{t-i} = 0$, and it will not be optimal to use distortionary taxes and the economy would achieve a full efficient allocation. Let μ_t be the Lagrange multiplier of the resource constraint, then the first-order necessary conditions at $t > 0$ are defined by

$$
\begin{array}{lrl}
[c_t^i] & \omega_{t+1-i}\beta^{i-1}V_{c_t^i} - \mu_t & = 0, \\
[c_t^{i+1}] & \omega_{t-i}\beta^{i}V_{c_t^{i+1}} - \mu_t & = 0, \\
[\ell_t^i] & \omega_{t+1-i}\beta^{i-1}V_{\ell_t^i} + \mu_t F_{L_t}\epsilon^i & = 0, \\
[K_{t+1}] & -\mu_t + \mu_{t+1}(1-\delta+F_{K_t}) & = 0,
\end{array}
$$

together with the transversality condition for the optimal capital path:

$$\lim_{t\to\infty} \mu_t K_{t+1} = 0. \tag{33}$$

Throughout the paper I assume that the solution of the Ramsey allocation problem exists and that the time paths of the solutions converge to a steady state. Neither of these assumptions is innocuous. Notice that since the set of allocations that satisfy the implementability constraint might fail to be convex, the first-order conditions are necessary but not sufficient (for a detailed discussion of nonconvexities, see Lucas and Stokey (1983)). Rearranging terms, we have:

$$\omega_{t+1-i}V_{c_t^i} = \omega_{t+2-i}V_{c_{t+1}^i}(1-\delta+F_{K_{t+1}}) \qquad \forall i,t, \tag{34}$$

$$\frac{V_{\ell_t^i}}{V_{c_t^i}} = -F_{L_t}\epsilon^i \qquad \forall i,t, \tag{35}$$

$$V_{c_t^i} = \frac{\omega_{t-i}}{\omega_{t+1-i}}\beta V_{c_t^{i+1}}, \qquad \forall i,t. \tag{36}$$

Notice that Equation (34) is slightly different from the competitive equilibrium Euler equation. In this case the government equates the derivative of the objective function of a newborn generation at different times. Equation (35) is the intratemporal condition between consumption and labor, that determines the amount of effective hours worked by each generation at a given period t. Finally, equation (36) is the static redistributive condition, and implies that the government will assign consumption among two different generations according to the ratio of their relative weights.

For the initial s generations at $t = 0$, the first-order necessary conditions are different from the previous ones, given the initial distribution of asset holdings. The first-order condition with respect to consumption-labor is given by

$$\frac{V_{\ell_0^s} - \eta_{1-s}\left[U_{c_0^s\ell_0^s}\left((1+F_{K_0}(1-\theta_{s0})a_0^s\right) + U_{c_0^s}F_{K_0\ell_0^s}(1-\theta_{s0})a_0^s\right]}{V_{c_0^s} - \eta_{1-s}U_{c_0^s c_0^s}\left((1+F_{K_0}(1-\theta_{s0})a_0^s\right)} = -F_{L_0}\epsilon^s, \tag{37}$$

and the redistributive first-order condition is

$$\frac{V_{c_0^s} - \eta_{1-s}U_{c_0^s c_0^s}\left((1+F_{K_0}(1-\theta_{s0})a_0^s\right)}{V_{c_0^{s+1}} - \eta_{2-s}U_{c_0^{s+1}c_0^{s+1}}\left((1+F_{K_0}(1-\theta_{s+1,t})a_0^{s+1}\right)} = \beta\frac{\omega_{s+1}}{\omega_s}. \tag{38}$$

This redistributive condition does not appear in the competitive equilibrium first-order conditions, but it is very useful to derive the optimal capital taxes. Updating (36) one period and substituting in (34) we obtain

$$V_{c_t^i} = \beta V_{c_{t+1}^{i+1}}(1-\delta+F_{K_{t+1}}) \qquad \forall i,t, \tag{39}$$

This new expression is very similar to a life-cycle Euler equation, but instead of having the marginal utility with respect to consumption it has the derivative of $V(\cdot)$.

To derive the optimal tax policy π^* we combine the first-order conditions of the Ramsey Allocation Problem together with the competitive equilibrium first-order conditions. The resulting expression is the optimal capital tax rate

$$\theta_{t+1}^* = \frac{1}{\beta r_{t+1}}\left[\frac{V_{c_t^i}}{V_{c_{t+1}^{i+1}}} - \frac{U_{c_t^i}}{U_{c_{t+1}^{i+1}}}\right] \qquad \forall i,t. \tag{40}$$

We use the same procedure to determine the expression for the optimal labor tax rate

$$\tau_t^* = 1 - \frac{U_{c_t^i}}{U_{\ell_t^i}}\frac{V_{\ell_t^i}}{V_{c_t^i}} \qquad \forall i,t. \tag{41}$$

It is important to remark that we are solving a relaxed version of the Ramsey problem, hence we cannot expect that taxes on capital returns are zero unless two conditions are satisfied: first, the ratio $V_{c_t^i}/V_{c_{t+1}^{i+1}}$ is equal to the solution of the competitive equilibrium $U_{c_t^i}/U_{c_{t+1}^{i+1}}$, and second, the additional constraints on marginal utilities are also satisfied. Hence in these type of economies the optimal capital tax is affected by the distortions in the labor market implied by the restrictions on the set of taxes π.

If we drop the age subscripts from the first-order conditions of the Ramsey problem, the associated expression for the optimal tax policy is equal to the one obtained in an infinite-lived consumer economy. Chamley (1986) proves two important results. First, for a general class of utility functions capital taxes should be zero in the long run (consumption is constant, therefore $U_{c_t} = U_{c_{t+1}}$). Second, for a particular class of functions, that satisfy $V_{c_t}/V_{c_{t+1}} = U_{c_t}/U_{c_{t+1}}$, the optimal capital income taxes are zero after a finite number of periods. The conditions for the zero capital result in the transition path are generally viewed as an application of the uniform commodity taxation principle (see Atkinson and Stiglitz (1980)), that specifies conditions under which taxing all goods at the same rate is optimal.

In overlapping generations economies the previous results are not generally true. The next proposition provides sufficient conditions that preferences need to satisfy for the zero capital income tax result.

Proposition 2: *If the government has access to proportional distortionary taxes across agents, then taxes on capital income are zero for $t \geq 2$ if preferences satisfy:*

$$\theta_t = 0 \Rightarrow \frac{c_t^i U_{cc_t^i} + \ell_t^i U_{\ell c_t^i}}{U_{c_t^i}} = \frac{\ell_t^i U_{\ell\ell_t^i} + c_t^i U_{c\ell_t^i}}{U_{\ell_t^i}} \qquad \forall t > 1 \tag{42}$$

Proof. We need to show that if preferences satisfy this property, then the solution of the less constrained Ramsey problem is also a solution to the more constrained problem and capital income taxes are zero from $t \geq 2$. We can rewrite condition (42) as follows

$$c_t^i U_{cc_t^i} + \ell_t^i U_{\ell c_t^i} = A U_{c_t^i}, \tag{43}$$

$$\ell U_{\ell\ell} + c_t^i U_{c\ell_t^i} = A U_{\ell_t^i}. \tag{44}$$

Now let's consider the first-order conditions of the Ramsey problem with respect to c_t^i

$$(1+\eta_{t-i})U_{c_t^i} + \eta_{t-i}\left[c_t^i U_{cc_t^i} + \ell_t^i U_{\ell c_t^i}\right] = \alpha_t,$$

where η_t and α_t denote the Lagrange multipliers of the implementability constraint of a generation born at period t and the period resource constraint respectively. Substituting Equation (43) in the first order conditions, we can rewrite the first-order conditions as

$$U_{c_t^i}(1+\eta_{t-i}(1+A)) = \alpha_t, \tag{45}$$

since this equations holds for time t and $t+1$, then we can derive

$$\frac{U_{c_t^i}}{U_{c_{t+1}^{i+1}}} = 1 + F_{K_t} - \delta.$$

This condition is sufficient to ensure zero capital taxes in an infinitely-lived consumers model, but is these type of economies it does not guarantee that the additional constraints are satisfied. Now we want to show that condition (43) together with condition (44) are sufficient to ensure that the solution of the less constrained problem is a solution of the more constrained problem. Using the same argument, consider the first-order conditions of the Ramsey problem with respect to ℓ_t^i,

$$(1+\eta_{t-i})U_{\ell_t^i} + \eta_{t-i}\left[\ell U_{\ell\ell} + c_t^i U_{c\ell_t^i}\right] = -\alpha_t F_{L_t}\epsilon^i$$

substituting (44) we have

$$U_{\ell_t^i}(1+\eta_{t-i}(1+A)) = -\alpha_t F_{L_t}\epsilon^i. \tag{46}$$

Since Equation (46) holds for all generations at a given period t, then the marginal rates of substitution between consumption and labor are equal across generations.

At the initial period $t=1$, capital and labor taxes are different from zero because the implementability constraints of the initial generations include the initial distribution of capital stock,

$$\theta_1 = \frac{1}{\beta(F_{K_1}-\delta)}\left[\frac{(1+\eta_s)U_{c_0^s} + \eta_{s-i}\left(c_t^s U_{c_t^s c_t^s} + \ell_t^s U_{\ell_t^s c_t^s}\right) - \eta_{1-s}U_{c_0^s c_0^s}a_0^s}{(1+\eta_s)U_{c_1^{s+1}} + \eta_{s-i}\left(c_{t+1}^{s+1}U_{cc_{t+1}^{s+1}} + \ell_t^s U_{\ell c_{t+1}^{s+1}}\right)} - \frac{U_{c_0^s}}{U_{c_1^{s+1}}}\right], \tag{47}$$

where $\eta_{1-s} U_{c_0^s c_0^s} a_0^s$ prevents capital taxes from being zero. At $t = 0$ the initial capital taxes are given, $\overline{\theta}_0$. ■

The class of utility functions that satisfies this property are of the form

$$U(c, \ell) = W(G(c, \ell))$$

where $c = (c^1, ..., c^I)$, $\ell = (\ell^1, ..., \ell^I)$ and G is homothetic with respect to both arguments consumption and labor ($G_1 > 0$, $G_{11} < 0$, $G_2 < 0$ and $G_{22} < 0$). An example of this class of preferences is given by

$$U(c, \ell) = \frac{(c\ell^{-\gamma})^{1-\sigma}}{1-\sigma}$$

where $\gamma \in (0, 1)$ and $\sigma \geq \frac{1+\gamma}{\gamma}$. Under this class of preferences the optimal policy implies set all taxes equal to zero. The optimal plan involves collecting tax revenues in excess in the initial periods, then with these claims against the private sector the government finances government expenditure with the interest earnings on government capital, after setting all taxes equal to zero. From the government budget constraint in steady state,

$$G \leq \tau w L + \theta r K + (1 - R)B$$

given that $\tau = \theta = 0$ and $R = \lambda^{-1}$, hence $B < 0$. In this case there is nothing special about the finite-horizon of the individuals. With the additional conditions on the cross derivatives with respect to labor, the optimal tax on labor income for an infinitely-lived consumer is also zero.

At this point it is important to mention that standard preferences commonly used in the macro and public finance literature like

$$U(c, \ell) = \frac{c^{1-\sigma}}{1-\sigma} - v(\ell) \tag{48}$$

or

$$U(c, \ell) = \frac{(c^{\gamma}(1-\ell)^{1-\gamma})^{1-\sigma}}{1-\sigma} \tag{49}$$

do not satisfy the sufficient condition. Therefore, in these cases the optimal tax on capital income is different from zero even though the preferences satisfy the uniform commodity tax property. It is important to remark that ℓ denotes labor and not leisure. If we redefine the objective function then the associated implementability constraint has to include $(1 - \ell)U_\ell$ instead of ℓU_ℓ.

This result improves the existing literature in two ways. First, it considers a general model where individuals live I periods, and second it analyzes the optimal policy on the transition path. The analysis of the tax policy in two period OLG economies ignores the additional constraints, and in a more general framework the conditions for zero capital tax are much more restrictive. This result contrasts with the class of models with infinitely-lived consumers where the optimal capital tax generally converges to zero after a finite number of periods.

Then why is the uniform commodity tax result not sufficient to guarantee zero capital taxes in this class of model? The answer is intergenerational heterogeneity.

This factor plays an important role in this result, because the government faces additional constraints if taxes have to be equal across generations at each period t, (i.e. marginal rates of substitution need to be equal across consumers). In this case the solution of the less constrained problem is not a solution to the more constrained problem. This is not true in an economy with heterogeneous infinitely-lived consumers. Chari and Kehoe (1999) show that the additional constraints are satisfied if the production function $F(\cdot)$ has the form $F(K,L) = F(K, H(L))$, where $H(\cdot)$ is some function. In an overlapping generations economy this argument does not apply because the properties of the zero capital tax result depend on $U(\cdot)$ and not on $F(\cdot)$. In the numerical simulations it will be clear that these additional constraints have important quantitative effects.

In general taxes on capital returns are different from zero unless we make some assumption either on the lifetime horizon or on the set of instruments available to the government. The next proposition states that the additional set of constraints play a crucial role to determine the optimal fiscal policy in this type of economies.

Proposition 3: *In this environment the uniform commodity tax result is a sufficient condition to ensure zero capital taxes if:*

1) The government has access to age-dependent taxes, or

2) I=2 and the old generation does not supply labor.

Proof. If the government can use age-specific taxes the Ramsey allocation drops the additional constraints, therefore the optimal policy for utility functions of the form (48) and (49) imply zero capital taxes from period two onwards. In this case labor taxes are different from zero. Equivalently in a two period OLG economy where the old generation does not supply labor, the government problem does not include additional constraints. The young generation supplies labor in the market while the old generation supplies capital. ■

In a different environment the previous result might not be true. Consider a standard two period OLG model but suppose the young generations have an endowment, m_t, which is not taxed at all. This is almost equivalent to considering an exogenously fixed labor supply. Then, the new implementability for a newborn generation is

$$c_t^1 U_{c_t^1} + \ell_t^1 U_{\ell_t^1} + \beta c_{t+1}^2 U_{c_{t+1}^2} = m_t U_{c_t^1}$$

Then even if preferences satisfy the uniform commodity taxation property, the Ramsey taxes on c_t^1 and c_{t+1}^2 are not uniform, that implies capital taxes different from zero. The previous example is useful because it highlights the relation between taxes on consumption and capital, and gives some of the intuition of why it might be optimal to tax capital. In an overlapping generations model, the distortions associated with this policy are not that important because for a given generation today's consumption and period T consumption are not perfectly substitutable. Hence the effect of capital distortions is much smaller and not necessarily bigger than distortions caused by other taxes like labor income taxes.

This model predicts that in general taxes on capital returns are different from zero. In the next section I explore if the model can account for the observed taxes for the US economy for some plausible parameter values.

4 Quantitative Results

4.1 Calibration

This section describes the choice of the functional forms for the numerical simulations and the calibration process. The utility function is a standard CRRA

$$U(c,\ell) = \frac{\left(c^{\gamma}(1-\ell)^{1-\gamma}\right)^{1-\sigma} - 1}{1-\sigma}$$

where $\gamma \in (0,1)$ is the consumption share in the utility function and σ denotes the inverse of the intertemporal elasticity of substitution. In the benchmark economy σ is set equal to 1 as in Chari and Kehoe (1999), that is a logarithmic utility function. This utility function does not satisfy the sufficient conditions for zero capital taxes if individuals live more than two periods. The technology is a constant returns to scale Cobb-Douglas production function

$$F(K,N) = K^{\alpha}N^{1-\alpha}$$

Given these functional forms the parameters in the model have been calibrated so that the steady state of the competitive equilibrium matches selected aggregates of US economy.

For computational simplicity it is assumed that a period in the economy is 6 years, and agents live up to 10 periods. Therefore, the model can be interpreted as one in which individuals are born economically at age 20 and live up to 74 years. This assumption does not affect the quantitative results. In this economy there is no mandatory retirement and without loss of generality it is assumed that population growth is zero.

The empirical evidence shows that hours worked are not constant over the life-cycle. Ghez and Becker (1975) and Juster and Stafford (1991) find that households allocate one third of their discretionary time in market activities. Setting $\gamma = 0.4$ in the model implies that individuals work an average of 33% of their time endowment over the life-cycle.

The discount factor β is chosen together with the intertemporal elasticity of substitution to match the observed capital-output ratio of the economy. Given that in the benchmark economy $\sigma = 1$, setting $\beta = 0.99$ matches the observed average capital-output ratio of 2.4.

In the technology, the capital share is set $\alpha = 0.33$. The depreciation rate is set to $\delta = 0.08$ to match the observed average investment-output ratio of 16.1%. The capital-output ratio together with the depreciation rate imply a gross interest rate of 5.6%. The preferences and technology parameters are displayed in the next table:

Table 1: Preferences and technology parameters

β	γ	σ	α	δ
0.99	0.4	1.0	0.33	0.08

The labor earnings age profile $\{\epsilon_j\}$ is derived using PSID data. In the model, labor services are homogeneous, so there is a single wage per efficiency unit of labor. Hence, $\{\epsilon_j\}$ is chosen to match the age profile of average wages in the cross-section of US data.

The government consumption is set to 19%, which corresponds to the average of the last decade. In determining taxation on factor earnings and consumption I follow the methodology of Mendoza, Tesar and Razin (1994). They develop consistent measures of the effective tax rate on factors' income for OECD countries. The competitive economy is calibrated using the average tax rates since 1965. These taxes are reported in table 2:

Table 2: Effective tax rates

θ	τ_c	τ_l
35%	5%	24%

Therefore the net of taxes interest rate is 3.6%. In steady state if the level of debt is positive, that implies that the government must have a surplus. Given G/Y and the tax policy π, the ratio D/Y is endogenously determined in the model according. The resulting debt-output ratio in the competitive economy is 24%. This figure is roughly consistent with the average observed in the data since 1965, which is 23%.

A feature of this model is that if the economy converges to the steady state, then it has the modified golden rule property and it is independent of the initial conditions a_0 (see Escolano (1992)).

In the next subsections I quantify the optimal fiscal policy in the long run when the government only has access to proportional taxes across ages and age-independent proportional taxes. The numerical simulations are accompanied by a sensitivity analysis with respect to the parameter values that play an important role in the determination of the optimal policy.

4.2 Proportional taxes

In this section I quantify the optimal fiscal policy in the long run when the government only has access to proportional taxes. In the Ramsey allocation problem, the relative weight that the government places between present and future generations has no counterpart in the competitive economy. Determining this parameter is somehow arbitrary, and the results as we will see are very sensitive to changes in the specified value. I choose this parameter so that the capital-output ratio associated with the optimal policy coincides with the competitive economy. That implies setting $\lambda = 0.947$.

A summary of the results obtained for the case where the government is restricted to use proportional taxes across consumers in presented in table 3.

Table 3: Optimal policy with proportional taxes

	Benchmark	**Optimal Policy**
Capital tax	35.0%	40.8%
Labor tax	24.0%	21.4%
Net interest rate	3.6%	3.3%
Debt/GDP	24.4%	8.7%

There are several important quantitative features of the solution. First, the calibrated model can account for the observed fiscal policy for some plausible choice of parameter values. The optimal capital income tax is 40.8% and the optimal labor income tax is 21.4%. Both values are consistent with the observed time-series, a 35% capital income tax and a 24% labor income tax. Given that the capital-output ratio is the same in both economies, the gross relative prices are also the same but the after tax prices are different. Second, the optimal ratio debt-GDP is smaller than the average value in the data. This lower value reduces the government need of having an important surplus.

The optimal policy and its effect on the after tax relative prices affect households decisions. The reduction of labor taxes and the increase of the net wage rates reduces the fraction of time supplied by young generations in the labor market. The lower net interest rate (after tax and net of depreciation) allows young generations to borrow more resources and is a disincentive for the old generations to accumulate capital.

In the benchmark calibration there are two parameters that could have a large impact on the optimal fiscal policy: the government discount rate and the intertemporal elasticity of substitution. First, all these calculations are done assuming that the capital-output ratio associated to the optimal policy coincides with the competitive equilibrium of the benchmark economy. There is no reason to believe that changes in the fiscal policy will not affect this relationship. Second, the intertemporal elasticity of substitution might play a role to determine the distortions of capital taxes in consumption. To test the sensitivity of these results to the choice of parameter, I performed several additional computations with different parameter choices. The next table describes the effects of varying the government discount rate ($\lambda = 0.947$ for the benchmark economy):

Table 4: Changes in the government discount rate

λ	Capital tax	Labor tax	Int. rate	Net int. rate	Debt/GDP	Capital/GDP
0.93	58.1%	17.9%	7.6%	3.1%	18.1%	2.2
0.94	49.2%	19.8%	6.4%	3.2%	12.8%	2.3
0.95	36.6%	22.1%	5.3%	3.3%	6.8%	2.5
0.96	17.7%	24.7%	4.1%	3.4%	-0.3%	2.7
0.97	0.3%	19.3%	3.1%	3.1%	-30.1%	3.0
0.98	-6.5%	10.4%	2.0%	2.2%	-91.5%	3.3

Changes in the government discount rate have redistributive effects in this economy because it changes the relative weight that the government places between present and future generations. If the government increases λ it lowers the gross interest rate of the economy in steady state given by

$$\frac{1}{\lambda} = 1 + F_K - \delta. \tag{50}$$

From table 4 the following facts can be summarized. First, the optimal tax on capital returns is inversely related with the government discount factor. Keeping fixed individual discount rate, β, if the government increases λ (that means discounts less and it values more future generations) lowers the gross interest rate of the economy in steady state by increasing the capital-output ratio. For some parameter value, $\lambda = 0.97$, the optimal capital income tax is roughly zero, and for higher values negative. This result is consistent with the numerical findings of Escolano (1992), for a different class of preferences. Changes in the government discount factor have a smaller effect on the optimal labor taxes. The optimal level of debt is also inversely related with λ. The decrease of revenues is compensated by a decrease of the level of debt.

The next table summarizes the effects of changes in the intertemporal elasticity of substitution (σ) :

Table 5: Changes in the elasticity of substitution

σ	Capital tax	Labor tax	Net int. rate	Debt/GDP
1.0	40.8%	21.8%	3.3%	8.7%
1.5	23.0%	27.0%	4.3%	9.5%
2.0	8.9%	27.9%	5.1%	2.6%
2.5	3.4%	22.2%	5.4%	-9.7%
3.0	-1.2%	17.7%	5.6%	-18.8%
4.0	-6.9%	10.2%	6.0%	-32.2%

As in the previous case, changes in the intertemporal elasticity of substitution have important effects on the optimal fiscal policy. The optimal capital income tax is highly sensitive to assumptions about the intertemporal elasticity of substitution. Auerbach and Kotlikoff (1987) use 1.5, and some other authors in environments with uncertainty have used values around 2. The optimal tax on capital returns is zero only for a special parametrization of the model economy, in this case when $\sigma \in (2.5, 3.0)$. The optimal labor tax is not very sensitive to variations in this parameter because the consumption and labor decisions are not affected by σ. In general, a lower taxation is compensated with a lower debt-output ratio.

To summarize, if the government uses proportional taxes across ages, the optimal capital tax is consistent with the observed tax for the US economy. The optimal fiscal policy is very sensitive to variations of the government discount rate and the intertemporal elasticity of substitution. The calibrated model can account for the observed capital taxes for some plausible choice of parameter values. The conditions for zero capital taxes in steady state are much more restrictive that in the infinitely-lived agent model. In the next section we study the optimal policy when the government has access to age-dependent proportional taxes.

4.3 Age-dependent taxes

This section studies from a quantitative point of view the optimal fiscal policy with an unrestricted set of fiscal instruments. Now the government can choose age-dependent proportional taxes. Formally that implies dropping the additional constraints on the marginal rates of substitution in the government problem. Next figure presents a summary of the numerical results for the optimal policy in the benchmark economy:

[Insert Figure 1]

We can summarize these characteristics as follows. First, for the benchmark case ($\sigma = 1$), the age-specific capital tax is constant across households and equal to zero. This result is consistent with Proposition 3, this type of utility function satisfies the sufficient conditions for zero capital taxes if the government can use age-specific taxes.[8] Second, the age specific labor income taxes are large and differ across households. For this particular parametrization labor taxes are the only source of revenues to finance government expenditure. The ratio debt-GDP associated to the optimal policy is 45.1%, almost double than the average observed in the data from 1965-1996.

As in the previous section, the quantitative analysis is accompanied by a sensitivity analysis with respect to the government discount rate and the intertemporal elasticity of substitution. Variations of the government discount rate, λ, do not affect the optimal tax on capital returns as long as the utility function satisfies the sufficient conditions. Hence, this result is independent of the relative weight that the government places on present and future generations. Changes in the discount rate affect the interest rate in steady state, the higher is λ, the higher

[8] In this case the additional conditions that restrict the marginal rates of substitution between consumption today and tomorrow are not binding. In order to satisfy the sufficient conditions for zero capital taxation we must allow labor taxes to differ across agents.

are wages in the economy. Therefore at a higher wage the financial needs of the government can be satisfied with a lower labor income tax. The level of debt is adjusted to satisfy the desired capital-output ratio. For a higher values of the intertemporal elasticity of substitution the optimal capital tax across generations is different from zero. Figure 2 displays the optimal age-specific taxes for different values of σ.

[**Insert Figure 2**]

Summarizing the effects of changes in intertemporal elasticity of substitution. First, the distribution of capital taxes differs over the life cycle. For the parametrized version of the model, if the government can use a complete set of instruments, then the age profile capital tax pattern implies subsidizing asset returns of the young generations in the economy and taxing at high rates the asset returns of old generations. Changes in the elasticity of substitution do not affect the average capital income tax over the life cycle, which is zero. Second, notice that an increase in σ does not substantially affect the distribution of capital taxes across generations, but it lowers the labor-specific taxes for all ages.. Finally, both labor income taxes and the ratio debt-output are decreasing in σ. For $\sigma = 1.5$, the debt-output ratio is 23.1% and for $\sigma = 2$ only 3.9%. For higher values of σ, this ratio is negative.

5 Conclusions

This paper provides, from a theoretical and quantitative point of view, an explanation of why the observed taxes on capital returns are larger than what standard theory predicts, not taxing capital at all. Using a stylized economy with intergenerational redistribution, I derive a sufficient condition to check whether the optimal policy implies zero capital taxes, and I show that commonly used utility functions in the macro and public finance literature violate this condition. Therefore, when heterogeneity is due to an intergenerational factor, we should expect taxes on capital returns to be different from zero. For a version of the model, calibrated to the US economy, the main results are: first, if the government is restricted to use proportional taxes across generations, the model can account for the observed capital and labor income taxes. Second, if the government can use specific taxes for each generation, then the age profile capital tax pattern implies subsidizing asset returns of the younger generations and taxing at higher rates the asset returns of the older ones.

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Figure 1: Capital and labor age-dependent taxes

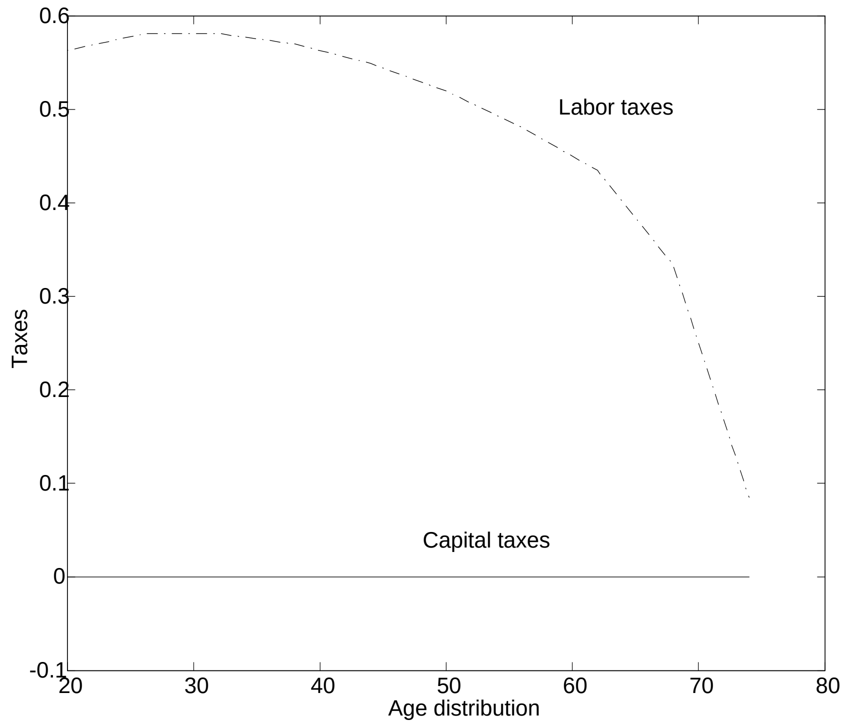
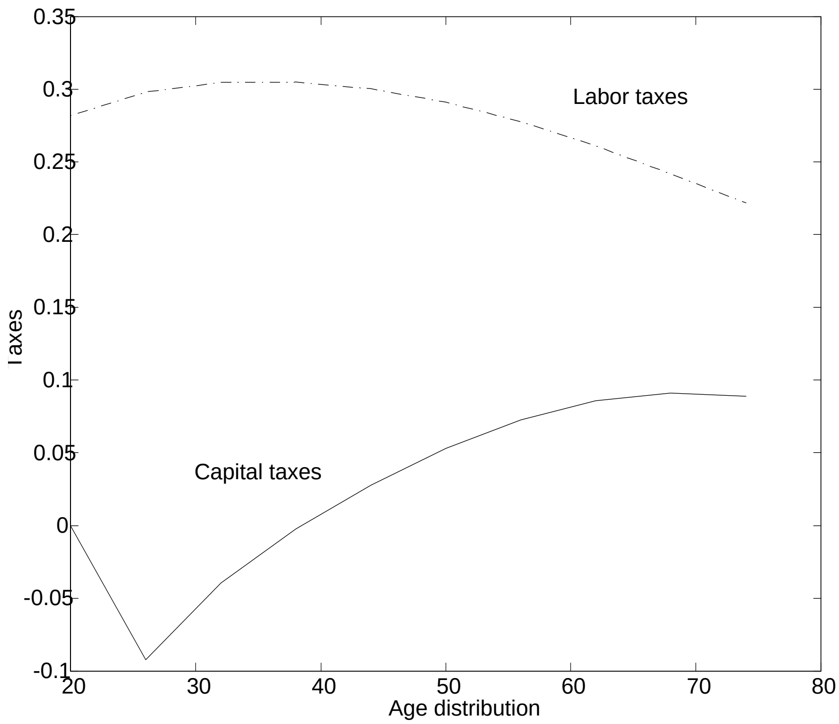


Figure 2: Sensitivity analysis

Case $\sigma = 2$



Case $\sigma = 3$