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Full nonlocality beyond maximally entangled scenarios

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Nonlocality is a quantum phenomenon that enables many different technology applications. Full nonlocality, arguably its strongest form, is of particular interest. However, identifying exactly which states can exhibit this property remains an open problem. Recent work established a connection between the phenomenon and antidistinguishability. Building upon that result, we propose a specific protocol with particular measurement settings in order to identify states that exhibit full nonlocality extending beyond the maximally entangled scenario. Consequently, this work derives new conditions for a family of non-maximally entangled states in d -dimensions narrowing the discussion to 5-dimensional states when considering explicit measurements.

Keywords: Bell nonlocality, Full nonlocality, Antidistinguishability.

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1 Introduction

The term nonlocality was first introduced in 1964 by John Bell [1] in his response to the Einstein, Podolsky, and Rosen paper published 29 years prior [2]. Bell demonstrated, using a thought experiment, that in specific quantum settings, the correlations presented could not be explained by a theory of local variables. In this case, the setting was described as nonlocal. Later, in 1969, a way to experimentally demonstrate nonlocality was derived mathematically via violations of Bell inequalities [3]. Following a 46-year interval, the first loophole-free experimental proofs were conducted [4, 5]. Since then, nonlocality has been used in many device-independent technological applications, among them quantum random number generation [6], self-testing [7, 8], and quantum key distribution [9, 10].

However, while violating a Bell inequality indicates the presence of nonlocality, the magnitude of that violation does not necessarily quantify it. In fact, in some cases, non-maximally entangled states, which do not achieve the Tsirelson bound, can show more nonlocality than maximally entangled states [11]. In order to quantify nonlocality, it is possible to write the joint probability distribution of measuring two outputs of a bipartite quantum system as a combination of a local and a nonlocal component. When the local component has weight zero, the state is said to be fully nonlocal.

Full nonlocality, arguably the strongest form of nonlocality, can surpass other nonlocal strategies. This is demonstrated through games such as the Peres-Mermin square [12] or all-versus-nothing proofs [13]. This effect has been proved for maximally entangled states of two d -dimensional quantum systems [14]. However, it is not trivial to identify which non-maximally entangled states can show full nonlocality. The advantage of having states that exhibit full nonlocality while not being maximally entangled is that they offer alternative candidates for experimental implementation, which may be significantly easier to prepare in a laboratory setting.

Another valuable quantum phenomenon is the antidistinguishability of quantum states [15, 16], which consists of measuring a given set of states and being able to identify which preparation has not occurred. It has been studied due to its importance to the foundations of quantum theory, but it also shows advantages in quantum communication protocols [17].

A connection between antidistinguishability and full nonlocality was first established in [18]; moreover, it was proven that any pure entangled state can exhibit full nonlocality in the many-copy scenario. In this work, we focus on finding fully nonlocal states at the single copy level. With this approach, we show the existence of a family of states, extending beyond the maximally entangled scenario, that can show full nonlocality. This result establishes new and more general conditions for fully nonlocal states when considering specific measurements in the Bell scenario.

This work is structured in the following way: Section 2 provides a brief review of Bell nonlocality, extending the notion to full nonlocality, which is illustrated through the explanation of the Peres-Mermin square. Section 3 is dedicated to the explanation of antidistinguishability and its connection to full nonlocality, while Section 4 presents the same example of the Peres-Mermin square now using the antidistinguishability argument. Finally, Section 5 summarizes the main results, starting with a concrete example which is later generalized for d -dimensional systems with arbitrary coefficients. The discussion is then narrowed to 5-dimensional systems when considering explicit measurement settings.

2 Bell nonlocality

In a so called Bell experiment, illustrated in Fig. 1, two scientists, Alice and Bob, are spatially separated, and each receive a quantum system of an entangled pair produced by the same source. They both choose and apply local quantum measurements to their designated system. Alice measures an observable associated with x and obtains an outcome a , while Bob chooses to measure y and obtains an outcome b . This scenario is typically considered to be non-signaling, that is, Alice's measurement choices will not affect Bob's probability distribution and vice versa. Mathematically, $p(b|xy) = p(b|y)$ and $p(a|xy) = p(a|x)$.

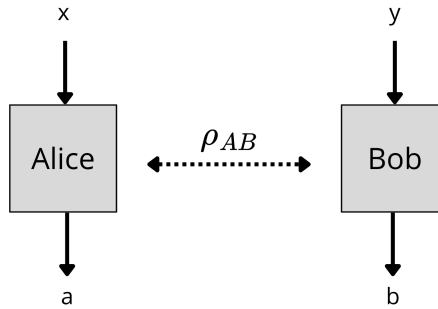


Figure 1: Illustration of the Bell experiment setup. Alice and Bob each receive a correlated quantum system from the same source. They perform measurements labeled as x and y and obtain results a and b .

Assuming multiple runs of this same experiment, the data obtained can be used to estimate a probability distribution $p(ab|xy)$ for the measurements. According to the Born rule, for a bipartite quantum state,

$$p_Q(ab|xy) = \text{Tr}[(A_{a|x} \otimes B_{b|y})\rho_{AB}], \quad (1)$$

where we denoted Alice's (Bob's) measurement by $A_{a|x}$ ($B_{b|y}$).

The most general quantum measurements are called positive operator valued measurements (POVMs). In order for a set of operators $\{M_i\}$ to define a POVM, it must fulfill

$$M_i : \mathcal{H} \rightarrow \mathcal{H}, \quad M_i \succeq 0, \quad \forall i \quad \text{s.t.} \quad \sum_i M_i = \mathbb{1}. \quad (2)$$

If the estimated probability distribution derived by the experiment does not satisfy

$$p(ab|xy) = p(a|x)p(b|y), \quad (3)$$

this means that the outcomes of the measurements of Alice and Bob are not independent from each other. In fact, this result is experimentally verifiable when the two scientists receive spin 1/2 particles and choose to measure the spin in different directions, say S_x and S_z for Alice and $(S_z + S_x)/\sqrt{2}$ and $(S_z - S_x)/\sqrt{2}$ for Bob.

It was believed by Einstein, Podolski and Rosen [2], that this phenomenon was a consequence of some “hidden variable” λ that could account for the correlation between the two particles. It could be a simple relation derivative from a past interaction, for example, a feature of the source that distributed the particles. In this case, we should be able to write

$$p(ab|xy) = p(a|x, \lambda)p(b|y, \lambda). \quad (4)$$

Taking into account all possible values of λ and their probability distribution $q(\lambda)$, the following equation should be satisfied:

$$p(ab|xy) = \int_{\Lambda} d\lambda q(\lambda) p(a|x, \lambda) p(b|y, \lambda). \quad (5)$$

This is called the condition of locality.

However, it was proven experimentally that some scenarios in quantum mechanics, such as the one cited above, do not obey this assumption [1]. This aspect of quantum mechanics is called Bell nonlocality and can be detected using inequalities, such as the CHSH [3].

2.1 Full nonlocality

Full nonlocality, equivalent to quantum pseudo-telepathy [19], is arguably the strongest form of nonlocality. Non-local games are one way to illustrate this effect. These scenarios are said to present quantum pseudo-telepathy if the players can win with certainty using entanglement, while players who can't use it don't always win. It is important to notice that it does not involve the exchange of information between the parties, in fact, it removes the need for them to exchange classical information [20, 21]. Differently from other non-local games, where winning is still bounded by statistics, quantum pseudo-telepathy was experimentally proved to allow players to win with probability 1 when playing the non-local version of the Peres-Mermin magic square game [22].

The magic square game was first introduced to simplify the understanding and demonstration of Bell's theorem and has since been extended to demonstrate quantum pseudo-telepathy [12, 23]. The game is defined by the following rules: Alice and Bob need to fill in a 3×3 square, with each grid element being assigned a value ± 1 . Alice must fill her grid such that the product of the three values in each row is always $+1$ and Bob needs to fill in his grid such that the multiplication of all values of his column is always -1 . Charlie, the referee, chooses for every round of the game a random row to Alice and a random column to Bob. They have to tell Charlie the chosen values for each grid slot. They win the round if they both assign the same value for the slot where the designated row and column overlap. They are allowed to communicate before the game starts, that is, they can agree on a strategy beforehand, but they cannot communicate after Charlie assigns the chosen row and columns. Even if they choose the best strategy beforehand, all possible combinations have at least one conflict.

The proof is as follows: assume the entries for the square are

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}. \quad (6)$$

Based on Alice's rules, $(abc)(def)(ghi) = 1$, but based on Bob's rules, $(adg)(beh)(cfi) = -1$, which is a contradiction. So, for all possible classical strategies, the best winning probability is $8/9$, as illustrated in Fig. 2. However, they can agree on a quantum strategy based on measurements beforehand. The perfect strategy is possible if they share entangled pairs of qubits. For N rounds of the game, they need $2N$ pairs of entangled qubits.

The idea is that Alice and Bob share a pair of entangled Bell states,

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \otimes \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle), \quad (7)$$

+1	+1	+1
+1	-1	-1
-1	+1	?

Figure 2: Illustration of the Peres-Mermin magic square using only classical strategies. Image source: Wikimedia Commons (CC0 1.0) [24].

and fill each of the squares with products of the Pauli matrices, like shown below

$$\begin{bmatrix} I \otimes Z & Z \otimes I & Z \otimes Z \\ X \otimes I & I \otimes X & X \otimes X \\ X \otimes Z & Z \otimes X & Y \otimes Y \end{bmatrix}. \quad (8)$$

Each player measures according to the corresponding eigenbasis of the operators in the corresponding row (for Alice) or column (for Bob) [25]. This is further elaborated in Section 4. Notice that this can only be done because all measurements in a row or column commute. To see that explicitly, notice how for any Pauli σ_i , $\sigma_i \otimes \sigma_i$ always commutes with $\sigma_j \otimes \sigma_j$, because the anti commutation happens twice. Using this preestablished strategy, Alice's measurements will always multiply to +1 and Bob's to -1.

One way to quantify these nonlocal correlations is to consider the Bell scenario once again. Given a shared state ρ_{AB} and a set of local measurements for each party, we decompose the probability distribution of the data between a local and a nonlocal content,

$$p(ab|xy) = q \cdot p_L(ab|xy) + (1 - q) \cdot p_{NL}(ab|xy). \quad (9)$$

The local term, p_L , needs to satisfy Eq. (5), while the nonlocal one, p_{NL} , does not satisfy Eq. (5), but must satisfy non-signaling. Identifying which decomposition maximizes q , we find the local content of $p(ab|xy)$. If $q = 0$, ρ is said to generate fully nonlocal correlations; in other words, the initial state is fully nonlocal. Since no local strategy allows the players to win the Peres-Mermin square game with probability 1, such strategy cannot appear in the decomposition (9), confirming that the correlations are fully nonlocal.

3 Antidistinguishability

A finite set of states $\{\rho_i\}_{i \in [n]}$ is said to be antidistinguishable if there exists a measurement $\{M_j\}_{j \in [k]}$ such that $\forall j$ there is at least one ρ_i satisfying $\text{Tr}(M_j \rho_i) = 0$ [15]. Considering rank-one POVM elements $M_j = |\phi_j\rangle\langle\phi_j|$ and pure states $\rho_i = |\psi_i\rangle\langle\psi_i|$, this is equivalent to writing

$$\text{Tr}[M_j \rho_i] = \text{Tr}[|\phi_j\rangle\langle\phi_j| \psi_i\rangle\langle\psi_i|] = 0. \quad (10)$$

Evaluating the trace, we get

$$\langle\phi_j|\psi_i\rangle\langle\psi_i|\phi_j\rangle = |\langle\psi_i|\phi_j\rangle|^2 = 0, \quad (11)$$

which is true if, and only if,

$$\langle\psi_i|\phi_j\rangle = 0. \quad (12)$$

To visualize it consider three states:

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle), \quad (13)$$

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |2\rangle), \quad (14)$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \quad (15)$$

and the measurement setting,

$$M_0 = |0\rangle\langle 0|; M_1 = |1\rangle\langle 1|; M_2 = |2\rangle\langle 2|, \quad (16)$$

such that

$$\sum_{j=0}^2 M_j = \mathbb{1}. \quad (17)$$

If you receive the outcome M_0 , you cannot know whether the initial state was $|\psi_1\rangle$ or $|\psi_2\rangle$; however, you know with certainty that it was not $|\psi_0\rangle$. In other words, if the outcome M_i occurs, we know for a fact that $|\psi_i\rangle$ was not prepared. This phenomenon is sometimes called state exclusion or the weak notion of antidistinguishability [26], depending on the chosen reference.

In order to identify which set of states could be antidistinguishable, Johnston, Russo and Sikora [16] proved the following theorem, which is a generalization of the work of Caves, Fuchs and Schack [27]:

Theorem 1. ([44, Cor. 5.3]). *A set of n pure quantum states $\{|\psi_1\rangle, |\psi_2\rangle, \dots, |\psi_n\rangle\}$ is antidistinguishable if*

$$\|G\|_F := \sqrt{\sum_{i=1}^n \sum_{j=1}^n |\langle \psi_i | \psi_j \rangle|^2} \leq \frac{n}{\sqrt{2}}. \quad (18)$$

Here, $G_{ij} = \langle \psi_i | \psi_j \rangle$ is the Gram matrix associated with the states $\{|\psi_i\rangle\}_{i \in [n]}$, and $\|G\|_F$ is its Frobenius norm. In particular, the set is antidistinguishable if $|\langle \psi_i | \psi_j \rangle| \leq \sqrt{(n-2)/(2n-2)}$ for all pairs $i \neq j$.

Furthermore, it is possible to check computationally if a set of states is antidistinguishable when the following semidefinite program yields zero [16]:

$$\min_{\{M_i\}_i} \sum_i \text{Tr}[M_i \rho_i] \quad \text{s.t.} \quad \sum_i M_i = \mathbb{1}, M_i \succeq 0. \quad (19)$$

3.1 Connecting full nonlocality and antidistinguishability

A proof of the connection between antidistinguishability and full nonlocality is presented in [18]:

Theorem 2. *A bipartite state is fully nonlocal if and only if there exist measurements for Alice such that all the sets $\rho_{\alpha}^B := \{\rho_{a=\alpha_1|x=1}^B, \dots, \rho_{a=\alpha_n|x=n}^B\}$ of post-measurement states are antidistinguishable.*

It can be explained as follows: Considering a Bell scenario with inputs $x \in [n]$ and $y \in [n']$, and a probability distribution given by Eq. (9), we assume that Alice and Bob

agree on a deterministic local strategy before the start of the experiment. In this case, we can rewrite the local component of the probability distribution as

$$p_L(ab|xy) = \sum_{\vec{\alpha}, \vec{\beta}} p(\vec{\alpha}\vec{\beta}) D_{\vec{\alpha}\vec{\beta}}(ab|xy), \quad (20)$$

where $\vec{\alpha} = \{\alpha_1, \dots, \alpha_n\}$ and $\vec{\beta} = \{\beta_1, \dots, \beta_{n'}\}$ correspond to the strategy in which Alice outputs $a = \alpha_x$ and Bob outputs $b = \beta_y$, respectively. Under these circumstances, once the input is chosen, the output is fixed, with probability being either 0 or 1. For that reason,

$$D_{\vec{\alpha}\vec{\beta}}(ab|xy) := \delta_{\alpha_x, a} \delta_{\beta_y, b}. \quad (21)$$

To illustrate the following steps, we will restrict the argument to $n = n' = 2$. Consider the Popescu-Rolich (PR) Box [28, 29]: Given two inputs, $x \in \{0, 1\}$ and $y \in \{0, 1\}$, the outputs $a \in \{0, 1\}$ and $b \in \{0, 1\}$ are such that $a \oplus b = x \cdot y$. The probability tables conditioned on each possible input, $p_{PR}(ab|xy)$, are shown below.

$$p_{PR}(ab|0,0) = \begin{array}{c|cc} & b=0 & b=1 \\ \hline a=0 & 1/2 & 0 \\ a=1 & 0 & 1/2 \end{array} \quad p_{PR}(ab|0,1) = \begin{array}{c|cc} & b=0 & b=1 \\ \hline a=0 & 1/2 & 0 \\ a=1 & 0 & 1/2 \end{array}$$

$$p_{PR}(ab|1,0) = \begin{array}{c|cc} & b=0 & b=1 \\ \hline a=0 & 1/2 & 0 \\ a=1 & 0 & 1/2 \end{array} \quad p_{PR}(ab|1,1) = \begin{array}{c|cc} & b=0 & b=1 \\ \hline a=0 & 0 & 1/2 \\ a=1 & 1/2 & 0 \end{array}$$

That said, Alice and Bob can agree on a deterministic approach: Alice will always output $a = 0$ while Bob will output $b = y$. Now, the probabilities are given by a different set of tables:

$$D_{\vec{\alpha}\vec{\beta}}(ab|0,0) = \begin{array}{c|cc} & b=0 & b=1 \\ \hline a=0 & 1 & 0 \\ a=1 & 0 & 0 \end{array} \quad D_{\vec{\alpha}\vec{\beta}}(ab|0,1) = \begin{array}{c|cc} & b=0 & b=1 \\ \hline a=0 & 0 & 1 \\ a=1 & 0 & 0 \end{array}$$

$$D_{\vec{\alpha}\vec{\beta}}(ab|1,0) = \begin{array}{c|cc} & b=0 & b=1 \\ \hline a=0 & 1 & 0 \\ a=1 & 0 & 0 \end{array} \quad D_{\vec{\alpha}\vec{\beta}}(ab|1,1) = \begin{array}{c|cc} & b=0 & b=1 \\ \hline a=0 & 0 & 1 \\ a=1 & 0 & 0 \end{array}$$

One can notice that if $p(ab|xy) = 0$, for some a, b, x and y , then it is necessary that any local strategy must also satisfy $D_{\vec{\alpha}\vec{\beta}}(ab|xy) = 0$. However, in the PR box scenario, we have $p(a = 0, b = 1|x = 0, y = 1) = 0$, nevertheless, $D_{\vec{\alpha}\vec{\beta}}(a = 0, b = 1|x = 0, y = 1) = 1$, as

highlighted in red. This means that this specific local strategy could not have been used to reproduce the correlations of the PR box. Applying the same procedure for all zeroes in a distribution, if every possible local strategy violates at least one zero condition, then no local strategy can contribute to the distribution. In this case, the local content is zero and we know that $p(ab|xy)$ is fully nonlocal.

In order to connect full nonlocality with the concept of antidistinguishability, we go back to the bipartite state ρ_{AB} shared between the two parties. After Alice performs one measurement $x \in [n]$ on her side, getting the outcome $a \in [m_A]$, Bob's post-measurement state can be written as

$$\rho_{a|x}^B = \frac{\text{Tr}_A[(A_{a|x} \otimes \mathbb{1})\rho_{AB}]}{\text{Tr}[(A_{a|x} \otimes \mathbb{1})\rho_{AB}]}.$$
 (22)

Now, we fix Alice's outcomes for every local measurement, assuming $\vec{\alpha}$ such that the set of post-measurement states $\rho_{\alpha}^B := \{\rho_{a=\alpha_1|x=1}^B, \dots, \rho_{a=\alpha_n|x=n}^B\}$ is antidistinguishable. In addition to that, we assume Bob picks a specific measurement setting, labeled as $y_{\vec{\alpha}}$, that can perfectly antidistinguish the states. His POVM elements are denoted as $\{B_{b|y_{\vec{\alpha}}}\}_{b \in [n]}$.

This means that

$$\text{Tr}[\rho_{a=\alpha_x|x}^B B_{b=x|y_{\vec{\alpha}}}] = 0, \forall x \in [n].$$
 (23)

Rewriting it, it is possible to notice that

$$\text{Tr}[\rho_{a=\alpha_x|x}^B B_{b=x|y_{\vec{\alpha}}}] = p_Q(b=x|x, y_{\vec{\alpha}}, a=\alpha_x) = 0,$$
 (24)

where p_Q indicates a quantum correlation. Using Bayes' rule,

$$p(ab|xy) = p(b|axy) \cdot p(a|xy),$$
 (25)

and adding the result found above, it is possible to conclude that

$$p_Q(a=\alpha_x, b=x|x, y_{\vec{\alpha}}) = 0, \forall x \in [n].$$
 (26)

Now, suppose there is a deterministic local strategy $D_{\alpha\beta}$ in the decomposition of p_Q . Under this strategy, if Alice gets x as input, she must always output $a = \alpha_x$. Since Bob's measurement choices are defined in order to satisfy antidistinguishability conditions, his output must be inside the set $[n]$. Let Bob's outcome be $b = x_0$. In this case, if $x = x_0$ and $y = y_{\vec{\alpha}}$, we have that

$$D_{\vec{\alpha}\vec{\beta}}(a=\alpha_{x_0}, b=x_0|x=x_0, y_{\vec{\alpha}}) = 1.$$
 (27)

However, this contradicts Eq. (26). There cannot be any local strategy where $D_{\vec{\alpha}\vec{\beta}}(a=\alpha_x, b=x|x, y_{\vec{\alpha}}) = 1$ in the decomposition of p_Q . This implies that the shared state ρ_{AB} has no local content, hence, it is fully nonlocal. The proof of the converse can be found in [18].

4 Peres-Mermin square using the antidistinguishability argument

Now that the connection between full nonlocality and antidistinguishability has been presented, the same example of the Peres-Mermin square explained in Section 2.1 can be reevaluated, now applying the antidistinguishability argument. Using the measurements introduced in the matrix (8), and the state in Eq. (7), it is possible to calculate the simultaneous eigenstates of the observables in each of the rows and columns [25].

Once again, Alice and Bob share a pair of entangled Bell states,

$$\begin{aligned} |\psi\rangle &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \\ &= \frac{1}{2}(|0000\rangle + |0011\rangle + |1100\rangle + |1111\rangle). \end{aligned} \quad (28)$$

Now we regroup the Hilbert space by pairing the first qubit of each Bell state as subsystem A , and the second qubit of each Bell state as subsystem B :

$$|a_1 b_1\rangle \otimes |a_2 b_2\rangle = |a_1 a_2\rangle_A \otimes |b_1 b_2\rangle_B. \quad (29)$$

Through this ordering,

$$|\psi\rangle = \frac{1}{2}(|00\rangle_A |00\rangle_B + |01\rangle_A |01\rangle_B + |10\rangle_A |10\rangle_B + |11\rangle_A |11\rangle_B). \quad (30)$$

Using the binary identification,

$$|00\rangle \rightarrow |0\rangle, \quad |01\rangle \rightarrow |1\rangle, \quad |10\rangle \rightarrow |2\rangle, \quad |11\rangle \rightarrow |3\rangle, \quad (31)$$

we obtain

$$|\psi\rangle = \frac{1}{2}(|00\rangle + |11\rangle + |22\rangle + |33\rangle). \quad (32)$$

Given this notation, the simultaneous eigenstates of the observables in the three rows of Matrix (8), that is, Alice's measurements, are given in Table 1. For simplicity, $+$ was used to denote the eigenvalue $+1$ and $-$ to denote -1 . Similarly for Bob, the eigenvalues of the measurements in his three columns can be written explicitly, as shown in Table 2.

Notice that the post-measurement states on Bob's side are $|\psi_{a|x}^B\rangle = |A_{a|x}\rangle$ such that for every local strategy $\vec{\alpha}$ of Alice, the corresponding set of post-measurement states can be antidistinguished by one of Bob's measurements.

To visualize how this implies full nonlocality, suppose Alice chooses the strategy

$$\vec{\alpha} = [\alpha_1 = + + +, \alpha_2 = - + -, \alpha_3 = + + +]. \quad (33)$$

The corresponding post-measurement states will be $|0\rangle$, $|0\rangle + |1\rangle - |2\rangle - |3\rangle$ and $|0\rangle - |1\rangle - |2\rangle - |3\rangle$, which can be antidistinguished by Bob's measurements $B_{y=2}$. Considering α_1 , if Bob measures his second column ($y = 2$), the probability of him obtaining the outcome $b = - - -$ or $b = - + +$ is zero. For any α_i , Bob's measurement will rule out at least one outcome.

Following this same logic for Alice's entire local deterministic strategy, we can reconstruct the reduced probability table for p_Q , shown in Table 3. By focusing only on the relevant outcomes, it becomes clear that the local strategy $\vec{\alpha}$ is forbidden from appearing in the decomposition of p_Q . Since the set of post-measurement states corresponding to every local deterministic strategy of Alice can be antidistinguished by one of Bob's measurements, it follows that p_Q is fully nonlocal.

	$ A_{a x=1}\rangle$	$ A_{a x=2}\rangle$	$ A_{a x=3}\rangle$
$a = + + +$	$ 0\rangle$	$ 0\rangle + 1\rangle + 2\rangle + 3\rangle$	$ 0\rangle - 1\rangle - 2\rangle - 3\rangle$
$a = + - -$	$ 2\rangle$	$ 0\rangle - 1\rangle + 2\rangle - 3\rangle$	$ 0\rangle + 1\rangle - 2\rangle + 3\rangle$
$a = - + -$	$ 1\rangle$	$ 0\rangle + 1\rangle - 2\rangle - 3\rangle$	$ 0\rangle - 1\rangle + 2\rangle + 3\rangle$
$a = - - +$	$ 3\rangle$	$ 0\rangle - 1\rangle - 2\rangle + 3\rangle$	$ 0\rangle + 1\rangle + 2\rangle - 3\rangle$

Table 1: Alice's simultaneous unnormalized eigenstates $|A_{a|x}\rangle$ for each row x and outcome a . The labels $+$ and $-$ correspond to the eigenvalues $+1$ and -1 .

	$ B_{b y=1}\rangle$	$ B_{b y=2}\rangle$	$ B_{b y=3}\rangle$
$b = - - -$	$ 1\rangle - 3\rangle$	$ 2\rangle - 3\rangle$	$ 1\rangle - 2\rangle$
$b = - + +$	$ 1\rangle + 3\rangle$	$ 2\rangle + 3\rangle$	$ 1\rangle + 2\rangle$
$b = + - +$	$ 0\rangle - 2\rangle$	$ 0\rangle - 1\rangle$	$ 0\rangle - 3\rangle$
$b = + + -$	$ 0\rangle + 2\rangle$	$ 0\rangle + 1\rangle$	$ 0\rangle + 3\rangle$

Table 2: Bob’s simultaneous unnormalized eigenstates $|B_{b|y}\rangle$ for each column y and outcome b . The labels $+$ and $-$ correspond to the eigenvalues $+1$ and -1 .

		$y = 2$			
		$- - -$	$- + +$	$+ - +$	$+ + -$
$x = 1$	$+ + +$	0	0	$\times$	$\times$
$x = 2$	$- + -$	$\times$	$\times$	0	$\times$
$x = 3$	$+ + +$	$\times$	$\times$	$\times$	0

Table 3: A section of the joint probability table p_Q for Bob’s measurement $y = 2$, evaluating Alice’s specific local deterministic strategy $\vec{\alpha} = [+ + +, - + -, + + +]$. The zeros demonstrate the forbidden outcomes that rule out this strategy, while the probabilities marked with $\times$ are irrelevant to the antidistinguishability argument.

5 Results

5.1 Starting example

Most examples of full nonlocality found previously are not explicit for non-maximally entangled states and remain limited [18]. Because of that, we propose a new approach starting from a set of states that are known to be antidistinguishable, such as the ones presented in Tables 1 and 2. From that, our motivation for the subsequent steps start with one explicit example: Considering a non-maximally entangled bipartite state

$$|\Psi_{AB}\rangle = \frac{1}{2}|00\rangle + \frac{1}{2}|11\rangle + \frac{1}{2}|22\rangle + \frac{1}{2\sqrt{2}}|33\rangle + \frac{1}{2\sqrt{2}}|44\rangle, \quad (34)$$

Alice applies three measurements on her system. That is, $(|A_{i|j}\rangle\langle A_{i|j}| \otimes \mathbb{1}_B)$, where i denotes the choice of measurement and j labels the round. Since states of the form $(|i\rangle \pm |j\rangle)/\sqrt{2}$ appear in the Peres-Mermin square and are known to be antidistinguishable¹, we define the measurements such that the post-measurement states are always of this form. To do so, we fix the first measurement with the following POVM elements:

$$\begin{aligned} |A_{1|1}\rangle &= \sqrt{1/2}|0\rangle + \sqrt{1/2}|1\rangle, \\ |A_{2|1}\rangle &= \sqrt{1/2}|0\rangle - \sqrt{1/2}|1\rangle, \\ |A_{3|1}\rangle &= 1/2|2\rangle + \sqrt{1/2}|3\rangle, \\ |A_{4|1}\rangle &= 1/2|2\rangle - \sqrt{1/2}|3\rangle, \\ |A_{5|1}\rangle &= 1/2|2\rangle + \sqrt{1/2}|4\rangle, \\ |A_{6|1}\rangle &= 1/2|2\rangle - \sqrt{1/2}|4\rangle, \end{aligned} \quad (35)$$

¹Due to the symmetry of the Peres-Mermin square, Alice’s post-measurement states can be antidistinguished by Bob’s measurements and vice versa. For simplicity, we focus on the form of Bob’s measurements.

where

$$\sum_i |A_{i|1}\rangle\langle A_{i|1}| = \mathbb{1}_5. \quad (36)$$

Similarly, for the second and third measurement settings,

$$\begin{aligned} |A_{1|2}\rangle &= \sqrt{1/2}|0\rangle + \sqrt{1/2}|2\rangle, \\ |A_{2|2}\rangle &= \sqrt{1/2}|0\rangle - \sqrt{1/2}|2\rangle, \\ |A_{3|2}\rangle &= 1/2|1\rangle + \sqrt{1/2}|3\rangle, \\ |A_{4|2}\rangle &= 1/2|1\rangle - \sqrt{1/2}|3\rangle, \\ |A_{5|2}\rangle &= 1/2|1\rangle + \sqrt{1/2}|4\rangle, \\ |A_{6|2}\rangle &= 1/2|1\rangle - \sqrt{1/2}|4\rangle; \end{aligned} \quad (37)$$

$$\begin{aligned} |A_{1|3}\rangle &= \sqrt{1/2}|1\rangle + \sqrt{1/2}|2\rangle, \\ |A_{2|3}\rangle &= \sqrt{1/2}|1\rangle - \sqrt{1/2}|2\rangle, \\ |A_{3|3}\rangle &= 1/2|0\rangle + \sqrt{1/2}|3\rangle, \\ |A_{4|3}\rangle &= 1/2|0\rangle - \sqrt{1/2}|3\rangle, \\ |A_{5|3}\rangle &= 1/2|0\rangle + \sqrt{1/2}|4\rangle, \\ |A_{6|3}\rangle &= 1/2|0\rangle - \sqrt{1/2}|4\rangle. \end{aligned} \quad (38)$$

Since each one of the trios of post-measurement states is antidistinguishable, it follows that the state in Eq. (34) is fully nonlocal, i.e., has zero local content.

More explicitly, each combination of measurements in Alice's side generates a triple that can be antidistinguished by one of these 5 bases:

$$\begin{aligned} |B_{1|1}\rangle &= |0\rangle, \\ |B_{2|1}\rangle &= |1\rangle, \\ |B_{3|1}\rangle &= |2\rangle, \\ |B_{4|1}\rangle &= |3\rangle, \\ |B_{5|1}\rangle &= |4\rangle; \end{aligned} \quad (39)$$

$$\begin{aligned} |B_{1|2}\rangle &= 1/2(|0\rangle + |1\rangle + |2\rangle + |3\rangle), \\ |B_{2|2}\rangle &= 1/2(|0\rangle + |1\rangle - |2\rangle - |3\rangle), \\ |B_{3|2}\rangle &= 1/2(|0\rangle - |1\rangle + |2\rangle - |3\rangle), \\ |B_{4|2}\rangle &= 1/2(|0\rangle - |1\rangle - |2\rangle + |3\rangle), \\ |B_{5|2}\rangle &= |4\rangle; \end{aligned} \quad (40)$$

$$\begin{aligned}
|B_{1|3}\rangle &= 1/2(-|0\rangle + |1\rangle + |2\rangle + |3\rangle), \\
|B_{2|3}\rangle &= 1/2(|0\rangle - |1\rangle + |2\rangle + |3\rangle), \\
|B_{3|3}\rangle &= 1/2(|0\rangle + |1\rangle - |2\rangle + |3\rangle), \\
|B_{4|3}\rangle &= 1/2(|0\rangle + |1\rangle + |2\rangle - |3\rangle), \\
|B_{5|3}\rangle &= |4\rangle;
\end{aligned} \tag{41}$$

$$\begin{aligned}
|B_{1|4}\rangle &= 1/2(|0\rangle + |1\rangle + |2\rangle + |4\rangle), \\
|B_{2|4}\rangle &= 1/2(|0\rangle + |1\rangle - |2\rangle - |4\rangle), \\
|B_{3|4}\rangle &= 1/2(|0\rangle - |1\rangle + |2\rangle - |4\rangle), \\
|B_{4|4}\rangle &= 1/2(|0\rangle - |1\rangle - |2\rangle + |4\rangle), \\
|B_{5|4}\rangle &= |3\rangle;
\end{aligned} \tag{42}$$

$$\begin{aligned}
|B_{1|5}\rangle &= 1/2(-|0\rangle + |1\rangle + |2\rangle + |4\rangle), \\
|B_{2|5}\rangle &= 1/2(|0\rangle - |1\rangle + |2\rangle + |4\rangle), \\
|B_{3|5}\rangle &= 1/2(|0\rangle + |1\rangle - |2\rangle + |4\rangle), \\
|B_{4|5}\rangle &= 1/2(|0\rangle + |1\rangle + |2\rangle - |4\rangle), \\
|B_{5|5}\rangle &= |3\rangle.
\end{aligned} \tag{43}$$

In conclusion, considering a setting with 3 measurements on Alice's side and 5 on Bob's, since all the 216 possible combinations of states are antidistinguishable by one of the bases presented, the state given by Eq. (34) is fully nonlocal.

5.2 Generalizing the 5D case

Now, in order to generalize the example presented above and find other fully nonlocal states, we consider a 5 level bipartite system with arbitrary coefficients λ_i :

$$|\Psi\rangle_{AB} = \sqrt{\lambda_0}|00\rangle + \sqrt{\lambda_1}|11\rangle + \sqrt{\lambda_2}|22\rangle + \sqrt{\lambda_3}|33\rangle + \sqrt{\lambda_4}|44\rangle, \tag{44}$$

where $\sum_{i=0}^4 \lambda_i = 1$, due to normalization. Again, we consider three settings for Alice. In order to keep the simplicity and convenience of the post measurement states of the form $(|i\rangle \pm |j\rangle)/\sqrt{2}$, the following measurements are proposed for the first setting:

$$\begin{aligned}
|A_{1|1}\rangle &= \sqrt{2\lambda_1}|0\rangle + \sqrt{2\lambda_0}|1\rangle, \\
|A_{2|1}\rangle &= \sqrt{2\lambda_1}|0\rangle - \sqrt{2\lambda_0}|1\rangle, \\
|A_{3|1}\rangle &= \sqrt{2\lambda_3}|2\rangle + \sqrt{2\lambda_2}|3\rangle, \\
|A_{4|1}\rangle &= \sqrt{2\lambda_3}|2\rangle - \sqrt{2\lambda_2}|3\rangle, \\
|A_{5|1}\rangle &= \sqrt{2\lambda_4}|2\rangle + \sqrt{2\lambda_2}|4\rangle, \\
|A_{6|1}\rangle &= \sqrt{2\lambda_4}|2\rangle - \sqrt{2\lambda_2}|4\rangle;
\end{aligned} \tag{45}$$

where the constants $\sqrt{2}$ were added to preserve post measurement state normalization. Insuring that these measurements fulfill POVM properties,

$$\begin{aligned} \sum_i |A_{i|1}\rangle \langle A_{i|1}| &= 4\lambda_1 |0\rangle \langle 0| + 4\lambda_0 |1\rangle \langle 1| + 4\lambda_3 |2\rangle \langle 2| \\ &+ 4\lambda_2 |3\rangle \langle 3| + 4\lambda_4 |2\rangle \langle 2| + 4\lambda_2 |4\rangle \langle 4| = 1_5, \end{aligned} \quad (46)$$

which implies,

$$\lambda_0 = \lambda_1 = \lambda_2 = \frac{1}{4}, \quad (47)$$

and

$$\lambda_3 + \lambda_4 = \frac{1}{4}. \quad (48)$$

The second and third settings of measurements are defined by

$$\begin{aligned} |A_{1|2}\rangle &= \sqrt{2\lambda_2} |0\rangle + \sqrt{2\lambda_0} |2\rangle, \\ |A_{2|2}\rangle &= \sqrt{2\lambda_2} |0\rangle - \sqrt{2\lambda_0} |2\rangle, \\ |A_{3|2}\rangle &= \sqrt{2\lambda_3} |1\rangle + \sqrt{2\lambda_1} |3\rangle, \\ |A_{4|2}\rangle &= \sqrt{2\lambda_3} |1\rangle - \sqrt{2\lambda_1} |3\rangle, \\ |A_{5|2}\rangle &= \sqrt{2\lambda_4} |1\rangle + \sqrt{2\lambda_1} |4\rangle, \\ |A_{6|2}\rangle &= \sqrt{2\lambda_4} |1\rangle - \sqrt{2\lambda_1} |4\rangle; \end{aligned} \quad (49)$$

$$\begin{aligned} |A_{1|3}\rangle &= \sqrt{2\lambda_2} |1\rangle + \sqrt{2\lambda_1} |2\rangle, \\ |A_{2|3}\rangle &= \sqrt{2\lambda_2} |1\rangle - \sqrt{2\lambda_1} |2\rangle, \\ |A_{3|3}\rangle &= \sqrt{2\lambda_3} |0\rangle + \sqrt{2\lambda_0} |3\rangle, \\ |A_{4|3}\rangle &= \sqrt{2\lambda_3} |0\rangle - \sqrt{2\lambda_0} |3\rangle, \\ |A_{5|3}\rangle &= \sqrt{2\lambda_4} |0\rangle + \sqrt{2\lambda_0} |4\rangle, \\ |A_{6|3}\rangle &= \sqrt{2\lambda_4} |0\rangle - \sqrt{2\lambda_0} |4\rangle; \end{aligned} \quad (50)$$

and give out the same conditions for the arbitrary coefficients. Since the post measurement states match those of the previous example, the measurements on Bob's side required to antidistinguish them remain identical.

This can be extended to dimension 6 by adding the following elements in the three rounds, respectively:

$$\begin{aligned} |A_{7|1}\rangle &= \sqrt{2\lambda_5} |2\rangle + \sqrt{2\lambda_2} |5\rangle, \\ |A_{8|1}\rangle &= \sqrt{2\lambda_5} |2\rangle - \sqrt{2\lambda_2} |5\rangle, \\ |A_{7|2}\rangle &= \sqrt{2\lambda_5} |1\rangle + \sqrt{2\lambda_1} |5\rangle, \\ |A_{8|2}\rangle &= \sqrt{2\lambda_5} |1\rangle - \sqrt{2\lambda_1} |5\rangle, \\ |A_{7|3}\rangle &= \sqrt{2\lambda_5} |0\rangle + \sqrt{2\lambda_0} |5\rangle, \\ |A_{8|3}\rangle &= \sqrt{2\lambda_5} |0\rangle - \sqrt{2\lambda_0} |5\rangle. \end{aligned} \quad (51)$$

Similarly, the conditions for the state to be antidistinguishable, and consequently fully nonlocal, are

$$\lambda_0 = \lambda_1 = \lambda_2 = \frac{1}{4}, \quad (52)$$

and

$$\lambda_3 + \lambda_4 + \lambda_5 = \frac{1}{4}. \quad (53)$$

Extending it to more dimensions, using the same procedure presented above, it is possible to see that every state of the form

$$|\Psi_{AB}\rangle = \frac{1}{4}(|00\rangle + |11\rangle + |22\rangle) + \sum_{i=3}^{d-1} \lambda_i |ii\rangle, \quad (54)$$

such that $\sum_{i=3}^{d-1} \lambda_i = 1/4$, is fully nonlocal. Note that, in this case, it is easy to check that the post measurement states can be antidistinguishable, however, Bob's measurements must be adapted in order to antidistinguish them.

5.3 5D case using the pentagon geometry

With the aim of finding even more general conditions for fully nonlocal states, once again using arbitrary coefficients λ_i , with $i = \{0, 1, 2, 3, 4\}$, the shared state between Alice and Bob is of the form

$$|\Psi\rangle_{AB} = \sqrt{\lambda_0} |00\rangle + \sqrt{\lambda_1} |11\rangle + \sqrt{\lambda_2} |22\rangle + \sqrt{\lambda_3} |33\rangle + \sqrt{\lambda_4} |44\rangle. \quad (55)$$

Still working on a 5-dimensional system, we start from a pentagon geometry to define Alice's measurements in order to maximize symmetry and find a pattern, as illustrated in Fig. 3. However, we now introduce arbitrary constants N_i to relax the conditions on the coefficients λ_i , while ensuring that the post measurement states remain antidistinguishable.

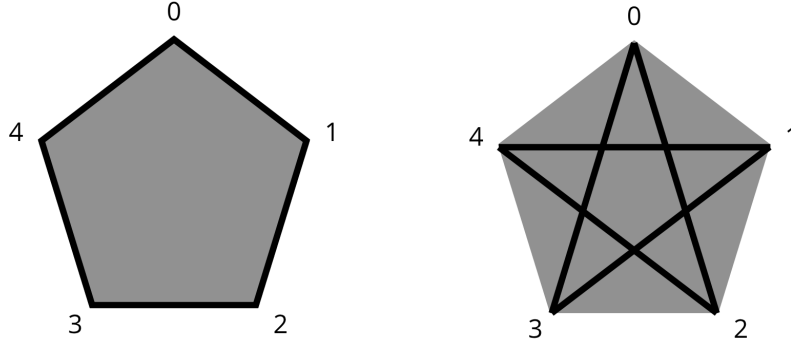


Figure 3: The left pentagon illustrates the formulation of the first and second rounds of measurements, where the POVMs are constructed by connecting adjacent vertices. The right pentagon shows the construction for the third and fourth rounds, which are instead formed by connecting non-consecutive edges of the pentagon.

By constraining Alice's post measurement states to the form $(|i\rangle \pm |j\rangle)/\sqrt{2}$, in a 5-dimensional system, we obtain 20 different state vectors. Because a valid POVM requires its elements to sum to the identity, these vectors can be grouped into two complete measurement settings. However, because any combination of these two post measurement states fail to be antidistinguishable, we introduce complex phases to generate two additional measurement settings, expanding the total number of post measurement states.

In this case, we need four measurements to achieve antidistinguishability. The chosen measurements for the first round are given by the direct connection between the edges of

the pentagon:

$$\begin{aligned}
|A_{1|1}\rangle &= (\sqrt{\lambda_1}|0\rangle + \sqrt{\lambda_0}|1\rangle)\sqrt{N_1}, \\
|A_{2|1}\rangle &= (\sqrt{\lambda_1}|0\rangle - \sqrt{\lambda_0}|1\rangle)\sqrt{N_1}, \\
|A_{3|1}\rangle &= (\sqrt{\lambda_2}|1\rangle + \sqrt{\lambda_1}|2\rangle)\sqrt{N_2}, \\
|A_{4|1}\rangle &= (\sqrt{\lambda_3}|1\rangle - \sqrt{\lambda_2}|2\rangle)\sqrt{N_2}, \\
|A_{5|1}\rangle &= (\sqrt{\lambda_3}|2\rangle + \sqrt{\lambda_2}|3\rangle)\sqrt{N_3}, \\
|A_{6|1}\rangle &= (\sqrt{\lambda_3}|2\rangle - \sqrt{\lambda_2}|3\rangle)\sqrt{N_3}, \\
|A_{7|1}\rangle &= (\sqrt{\lambda_4}|3\rangle + \sqrt{\lambda_3}|4\rangle)\sqrt{N_4}, \\
|A_{8|1}\rangle &= (\sqrt{\lambda_4}|3\rangle - \sqrt{\lambda_3}|4\rangle)\sqrt{N_4}, \\
|A_{9|1}\rangle &= (\sqrt{\lambda_4}|0\rangle + \sqrt{\lambda_0}|4\rangle)\sqrt{N_5}, \\
|A_{10|1}\rangle &= (\sqrt{\lambda_4}|0\rangle - \sqrt{\lambda_0}|4\rangle)\sqrt{N_5},
\end{aligned} \tag{56}$$

so that the post measurement states are always of the form $(|i\rangle \pm |j\rangle)/\sqrt{2}$.

On the second round, with the addition of complex phases, we have the following POVM elements:

$$\begin{aligned}
|A_{1|2}\rangle &= (\sqrt{\lambda_1}|0\rangle + \sqrt{\lambda_0}i|1\rangle)\sqrt{N_1}, \\
|A_{2|2}\rangle &= (\sqrt{\lambda_1}|0\rangle - \sqrt{\lambda_0}i|1\rangle)\sqrt{N_1}, \\
|A_{3|2}\rangle &= (\sqrt{\lambda_2}|1\rangle + \sqrt{\lambda_1}i|2\rangle)\sqrt{N_2}, \\
|A_{4|2}\rangle &= (\sqrt{\lambda_3}|1\rangle - \sqrt{\lambda_2}i|2\rangle)\sqrt{N_2}, \\
|A_{5|2}\rangle &= (\sqrt{\lambda_3}|2\rangle + \sqrt{\lambda_2}i|3\rangle)\sqrt{N_3}, \\
|A_{6|2}\rangle &= (\sqrt{\lambda_3}|2\rangle - \sqrt{\lambda_2}i|3\rangle)\sqrt{N_3}, \\
|A_{7|2}\rangle &= (\sqrt{\lambda_4}|3\rangle + \sqrt{\lambda_3}i|4\rangle)\sqrt{N_4}, \\
|A_{8|2}\rangle &= (\sqrt{\lambda_4}|3\rangle - \sqrt{\lambda_3}i|4\rangle)\sqrt{N_4}, \\
|A_{9|2}\rangle &= (\sqrt{\lambda_4}|0\rangle + \sqrt{\lambda_0}i|4\rangle)\sqrt{N_5}, \\
|A_{10|2}\rangle &= (\sqrt{\lambda_4}|0\rangle - \sqrt{\lambda_0}i|4\rangle)\sqrt{N_5}.
\end{aligned} \tag{57}$$

The coefficients N_i have to satisfy the same conditions as before.

For the third and fourth rounds, the measurements are defined by the connection of alternated edges of the pentagon, forming a star shaped graph. The explicit choices are

$$\begin{aligned}
|A_{1|3}\rangle &= (\sqrt{\lambda_2}|0\rangle + \sqrt{\lambda_0}|2\rangle)\sqrt{N_6}, \\
|A_{2|3}\rangle &= (\sqrt{\lambda_2}|0\rangle - \sqrt{\lambda_0}|2\rangle)\sqrt{N_6}, \\
|A_{3|3}\rangle &= (\sqrt{\lambda_4}|2\rangle + \sqrt{\lambda_2}|4\rangle)\sqrt{N_7}, \\
|A_{4|3}\rangle &= (\sqrt{\lambda_4}|2\rangle - \sqrt{\lambda_2}|4\rangle)\sqrt{N_7}, \\
|A_{5|3}\rangle &= (\sqrt{\lambda_4}|1\rangle + \sqrt{\lambda_1}|4\rangle)\sqrt{N_8}, \\
|A_{6|3}\rangle &= (\sqrt{\lambda_4}|1\rangle - \sqrt{\lambda_1}|4\rangle)\sqrt{N_8}, \\
|A_{7|3}\rangle &= (\sqrt{\lambda_3}|1\rangle + \sqrt{\lambda_1}|3\rangle)\sqrt{N_9}, \\
|A_{8|3}\rangle &= (\sqrt{\lambda_3}|1\rangle - \sqrt{\lambda_1}|3\rangle)\sqrt{N_9}, \\
|A_{9|3}\rangle &= (\sqrt{\lambda_3}|0\rangle + \sqrt{\lambda_0}|3\rangle)\sqrt{N_{10}}, \\
|A_{10|3}\rangle &= (\sqrt{\lambda_3}|0\rangle - \sqrt{\lambda_0}|3\rangle)\sqrt{N_{10}},
\end{aligned} \tag{58}$$

and

$$\begin{aligned}
|A_{1|4}\rangle &= (\sqrt{\lambda_2}|0\rangle + \sqrt{\lambda_0 i}|2\rangle)\sqrt{N_6}, \\
|A_{2|4}\rangle &= (\sqrt{\lambda_2}|0\rangle - \sqrt{\lambda_0 i}|2\rangle)\sqrt{N_6}, \\
|A_{3|4}\rangle &= (\sqrt{\lambda_4}|2\rangle + \sqrt{\lambda_2 i}|4\rangle)\sqrt{N_7}, \\
|A_{4|4}\rangle &= (\sqrt{\lambda_4}|2\rangle - \sqrt{\lambda_2 i}|4\rangle)\sqrt{N_7}, \\
|A_{5|4}\rangle &= (\sqrt{\lambda_4}|1\rangle + \sqrt{\lambda_1 i}|4\rangle)\sqrt{N_8}, \\
|A_{6|4}\rangle &= (\sqrt{\lambda_4}|1\rangle - \sqrt{\lambda_1 i}|4\rangle)\sqrt{N_8}, \\
|A_{7|4}\rangle &= (\sqrt{\lambda_3}|1\rangle + \sqrt{\lambda_1 i}|3\rangle)\sqrt{N_9}, \\
|A_{8|4}\rangle &= (\sqrt{\lambda_3}|1\rangle - \sqrt{\lambda_1 i}|3\rangle)\sqrt{N_9}, \\
|A_{9|4}\rangle &= (\sqrt{\lambda_3}|0\rangle + \sqrt{\lambda_0 i}|3\rangle)\sqrt{N_{10}}, \\
|A_{10|4}\rangle &= (\sqrt{\lambda_3}|0\rangle - \sqrt{\lambda_0 i}|3\rangle)\sqrt{N_{10}}.
\end{aligned} \tag{59}$$

For this choices of measurements, it is possible to prove computationally (19) or analytically (18), that any set of 4 post measurement states, one of each setting, can be antidistinguished. Furthermore, using the conditions for a valid measurement (2), and solving for N , we arrive on the following set of equations:

$$N_1 = \frac{\lambda_1 - \lambda_2 + \lambda_3 - \lambda_4 + \lambda_0}{4\lambda_0\lambda_1}, \tag{60}$$

$$N_2 = \frac{\lambda_1 + \lambda_2 - \lambda_3 + \lambda_4 - \lambda_0}{4\lambda_1\lambda_2}, \tag{61}$$

$$N_3 = \frac{-\lambda_1 + \lambda_2 + \lambda_3 - \lambda_4 + \lambda_0}{4\lambda_2\lambda_3}, \tag{62}$$

$$N_4 = \frac{\lambda_1 - \lambda_2 + \lambda_3 + \lambda_4 - \lambda_0}{4\lambda_3\lambda_4}, \tag{63}$$

$$N_5 = \frac{-\lambda_1 + \lambda_2 - \lambda_3 + \lambda_4 + \lambda_0}{4\lambda_0\lambda_4}. \tag{64}$$

Again, for the POVM to be valid, the constants N_i need to be non-negative. $N_1 \geq 0$ implies

$$\lambda_0 + \lambda_1 + \lambda_3 \geq \lambda_2 + \lambda_4, \tag{65}$$

which is equivalent to writing

$$\lambda_2 + \lambda_4 \leq 1/2, \quad (66)$$

due to

$$\sum_{i=0}^4 \lambda_i = 1. \quad (67)$$

Similarly, using the same condition for the other constants N_i , we obtain

$$\begin{aligned} \lambda_0 + \lambda_3 &\leq 1/2, \\ \lambda_1 + \lambda_4 &\leq 1/2, \\ \lambda_0 + \lambda_2 &\leq 1/2, \\ \lambda_1 + \lambda_3 &\leq 1/2. \end{aligned} \quad (68)$$

The second set of measurements, given by Eq. (59), yields the same conditions. From the third and fourth measurement settings, we obtain the following constraints:

$$\begin{aligned} \lambda_3 + \lambda_4 &\leq 1/2, \\ \lambda_0 + \lambda_1 &\leq 1/2, \\ \lambda_2 + \lambda_3 &\leq 1/2, \\ \lambda_0 + \lambda_4 &\leq 1/2, \\ \lambda_1 + \lambda_2 &\leq 1/2. \end{aligned} \quad (69)$$

This means that, for 5-level quantum systems, any state such that the sum of the two largest Schmidt coefficients is upper bounded by $1/2$ is fully nonlocal.

Since there are 10 possible outcomes for each measurement setting, there are going to be 10^4 different sets of post-measurement states. By using Theorem 1, it can be verified that all combinations are in fact antidistinguishable. However, we would like to construct the antidistinguishing measurements explicitly. This will allow us to reduce the number of measurement settings for Bob from 10000 to 41. In fact, we note that every combination of Alice's post-measurement states can be antidistinguished by the computational basis measurement unless it involves 4 repetitions of the same component. For example, if we choose the measurements $|A_{1|1}\rangle$, $|A_{1|2}\rangle$, $|A_{1|3}\rangle$ and $|A_{1|4}\rangle$, the resulting states on Bob's side will be $(|0\rangle + |1\rangle)/\sqrt{2}$, $(|0\rangle + i|1\rangle)/\sqrt{2}$, $(|0\rangle + |2\rangle)/\sqrt{2}$, and $(|0\rangle + i|2\rangle)/\sqrt{2}$, which we know are antidistinguishable. Still, they cannot be antidistinguished by the computational basis since $|0\rangle$ is not orthogonal to any of them. The total number of combinations that repeat the $|0\rangle$ component in all 4 post-measurement states is $4^4 = 256$. Applying the same logic to the other components, yields $5 \times 256 = 1280$ combinations that cannot be antidistinguished by the computational basis.

In order to find measurements that can antidistinguish the remaining combinations, we begin with one of the specific cases where the computational basis fails. For instance, consider:

$$\begin{aligned} |\psi_1\rangle &= (|0\rangle + |1\rangle)/\sqrt{2}, \\ |\psi_2\rangle &= (|0\rangle + i|1\rangle)/\sqrt{2}, \\ |\psi_3\rangle &= (|0\rangle + |2\rangle)/\sqrt{2}, \\ |\psi_4\rangle &= (|0\rangle + i|2\rangle)/\sqrt{2}. \end{aligned} \quad (70)$$

However, this set of states can be successfully antidistinguished by the following measurement basis:

$$\begin{aligned}
|\phi_1\rangle &= -|0\rangle - |1\rangle + i|2\rangle + |3\rangle, \\
|\phi_2\rangle &= -i|0\rangle - |1\rangle - i|2\rangle - i|3\rangle, \\
|\phi_3\rangle &= i|0\rangle - |1\rangle - i|2\rangle + i|3\rangle, \\
|\phi_4\rangle &= |0\rangle - |1\rangle + i|2\rangle - |3\rangle, \\
|\phi_5\rangle &= |4\rangle;
\end{aligned} \tag{71}$$

$|\phi_1\rangle$ is orthogonal to $|\psi_4\rangle$, $|\phi_2\rangle$ is orthogonal to $|\psi_2\rangle$, $|\phi_3\rangle$ is orthogonal to $|\psi_3\rangle$, $|\phi_4\rangle$ is orthogonal to $|\psi_1\rangle$ and $|\phi_5\rangle$ is orthogonal to all $|\psi_i\rangle$. Furthermore, this basis can be easily modified through unitary operations in order to adapt to other combinations of states. For example, given the set

$$\begin{aligned}
|\psi_1\rangle &= (|0\rangle - |1\rangle)/\sqrt{2}, \\
|\psi_2\rangle &= (|0\rangle - i|1\rangle)/\sqrt{2}, \\
|\psi_3\rangle &= (|0\rangle - |2\rangle)/\sqrt{2}, \\
|\psi_4\rangle &= (|0\rangle - i|2\rangle)/\sqrt{2},
\end{aligned} \tag{72}$$

we simply flip the signs of the components $|1\rangle$ and $|2\rangle$ within the basis. Alternatively, if we have

$$\begin{aligned}
|\psi_1\rangle &= (|0\rangle + |4\rangle)/\sqrt{2}, \\
|\psi_2\rangle &= (|0\rangle + i|4\rangle)/\sqrt{2}, \\
|\psi_3\rangle &= (|0\rangle + |3\rangle)/\sqrt{2}, \\
|\psi_4\rangle &= (|0\rangle + i|3\rangle)/\sqrt{2},
\end{aligned} \tag{73}$$

we can simply exchange $|1\rangle$ with $|4\rangle$ and $|2\rangle$ with $|3\rangle$.

Adding phases can be similarly helpful. For instance, if the post-measurement states are

$$\begin{aligned}
|\psi_1\rangle &= (|0\rangle + |1\rangle)/\sqrt{2}, \\
|\psi_2\rangle &= (|0\rangle - i|1\rangle)/\sqrt{2}, \\
|\psi_3\rangle &= (|0\rangle + |2\rangle)/\sqrt{2}, \\
|\psi_4\rangle &= (|0\rangle - i|2\rangle)/\sqrt{2},
\end{aligned} \tag{74}$$

introducing a $-i$ phase in front of $|1\rangle$ and $|2\rangle$ components is equivalent to applying a unitary transformation to the states in Eq. (70). Consequently, the same unitary can be applied directly to the basis. This same process can be applied iteratively until all states are successfully antidistinguished.

By analyzing which bases cover the greatest number of combinations, we can identify a pattern and significantly reduce the number of required measurements on Bob's side. In total, we verified explicitly via an algorithm that each of the 10000 combinations can be antidistinguished by one out of 41 different measurement setting. Of these 41, one is the computational basis and the others can be expressed in terms of 8 *seed* bases (75-82) that can be transformed by unitary operations, which can be seen as relabeling the vectors of the computational basis.

$$\begin{aligned}
|B_{1|1}\rangle &= \frac{1}{2}(|0\rangle - |1\rangle - |2\rangle + i|3\rangle) \\
|B_{2|1}\rangle &= \frac{1}{2}(-i|0\rangle - |1\rangle + |2\rangle - |3\rangle) \\
|B_{3|1}\rangle &= \frac{1}{2}(i|0\rangle - |1\rangle + |2\rangle + |3\rangle) \\
|B_{4|1}\rangle &= \frac{1}{2}(-|0\rangle - |1\rangle - |2\rangle - i|3\rangle) \\
|B_{5|1}\rangle &= |4\rangle
\end{aligned} \tag{75}$$

$$\begin{aligned}
|B_{1|2}\rangle &= \frac{1}{2}(-|0\rangle - |1\rangle + |2\rangle + i|3\rangle) \\
|B_{2|2}\rangle &= \frac{1}{2}(-|0\rangle - i|1\rangle - i|2\rangle - i|3\rangle) \\
|B_{3|2}\rangle &= \frac{1}{2}(-|0\rangle + i|1\rangle + i|2\rangle - i|3\rangle) \\
|B_{4|2}\rangle &= \frac{1}{2}(-|0\rangle + |1\rangle - |2\rangle + i|3\rangle) \\
|B_{5|2}\rangle &= |4\rangle
\end{aligned} \tag{79}$$

$$\begin{aligned}
|B_{1|3}\rangle &= \frac{1}{2}(-|0\rangle - |1\rangle + i|2\rangle + |3\rangle) \\
|B_{2|3}\rangle &= \frac{1}{2}(-i|0\rangle - |1\rangle - i|2\rangle - i|3\rangle) \\
|B_{3|3}\rangle &= \frac{1}{2}(i|0\rangle - |1\rangle - i|2\rangle + i|3\rangle) \\
|B_{4|3}\rangle &= \frac{1}{2}(|0\rangle - |1\rangle + i|2\rangle - |3\rangle) \\
|B_{5|3}\rangle &= |4\rangle
\end{aligned} \tag{76}$$

$$\begin{aligned}
|B_{1|4}\rangle &= \frac{1}{2}(-|0\rangle + |1\rangle + i|2\rangle - |3\rangle) \\
|B_{2|4}\rangle &= \frac{1}{2}(-|0\rangle - i|1\rangle - |2\rangle + |3\rangle) \\
|B_{3|4}\rangle &= \frac{1}{2}(-|0\rangle + i|1\rangle + |2\rangle + |3\rangle) \\
|B_{4|4}\rangle &= \frac{1}{2}(-|0\rangle - |1\rangle - i|2\rangle - |3\rangle) \\
|B_{5|4}\rangle &= |4\rangle
\end{aligned} \tag{80}$$

$$\begin{aligned}
|B_{1|5}\rangle &= \frac{1}{2}(|0\rangle - |1\rangle + |2\rangle + i|3\rangle) \\
|B_{2|5}\rangle &= \frac{1}{2}(-|0\rangle + |1\rangle + |2\rangle + i|3\rangle) \\
|B_{3|5}\rangle &= \frac{1}{2}(|0\rangle + |1\rangle + |2\rangle - i|3\rangle) \\
|B_{4|5}\rangle &= \frac{1}{2}(|0\rangle + |1\rangle - |2\rangle + i|3\rangle) \\
|B_{5|5}\rangle &= |4\rangle
\end{aligned} \tag{77}$$

$$\begin{aligned}
|B_{1|6}\rangle &= \frac{1}{2}(i|0\rangle - |1\rangle + i|2\rangle + i|3\rangle) \\
|B_{2|6}\rangle &= \frac{1}{2}(-i|0\rangle + |1\rangle + i|2\rangle + i|3\rangle) \\
|B_{3|6}\rangle &= \frac{1}{2}(i|0\rangle + |1\rangle + i|2\rangle - i|3\rangle) \\
|B_{4|6}\rangle &= \frac{1}{2}(i|0\rangle + |1\rangle - i|2\rangle + i|3\rangle) \\
|B_{5|6}\rangle &= |4\rangle
\end{aligned} \tag{81}$$

$$\begin{aligned}
|B_{1|7}\rangle &= \frac{1}{2}(i|0\rangle + |1\rangle - |2\rangle - |3\rangle) \\
|B_{2|7}\rangle &= \frac{1}{2}(-i|0\rangle - i|1\rangle - |2\rangle - i|3\rangle) \\
|B_{3|7}\rangle &= \frac{1}{2}(-i|0\rangle + i|1\rangle - |2\rangle + i|3\rangle) \\
|B_{4|7}\rangle &= \frac{1}{2}(i|0\rangle - |1\rangle - |2\rangle + |3\rangle) \\
|B_{5|7}\rangle &= |4\rangle
\end{aligned} \tag{78}$$

$$\begin{aligned}
|B_{1|8}\rangle &= \frac{1}{2}(i|0\rangle - |1\rangle + |2\rangle - |3\rangle) \\
|B_{2|8}\rangle &= \frac{1}{2}(-|0\rangle + |1\rangle - i|2\rangle - |3\rangle) \\
|B_{3|8}\rangle &= \frac{1}{2}(|0\rangle + |1\rangle + i|2\rangle - |3\rangle) \\
|B_{4|8}\rangle &= \frac{1}{2}(-i|0\rangle - |1\rangle - |2\rangle - |3\rangle) \\
|B_{5|8}\rangle &= |4\rangle
\end{aligned} \tag{82}$$

To generate the remaining 32 bases, we use the symmetry of the problem by rotating each component within the pentagon geometry. More specifically, under this cyclic permutation, the component $|0\rangle$ is mapped to $|1\rangle$, $|1\rangle$ becomes $|2\rangle$, and so forth. Repeating this process 4 more times allows the complete set of 41 bases to be fully derived.

In summary, we have shown that any state of the form (55), where the coefficients λ_i obey the conditions given by the set of Eq. (68) and (69), can show full nonlocality in a scenario where Alice has 4 measurement settings and Bob 41.

6 Conclusions

In this work, we have analyzed the concept of nonlocality, narrowing our focus to full nonlocality, which is of special interest in many fields of quantum information. By applying the proof of the connection between full nonlocality and antidistinguishability presented in [18], we propose a new approach to finding fully nonlocal states beyond the maximally entangled scenario.

Working with a 5-dimensional bipartite state and starting from a single explicit example of a non-maximally entangled state showing full nonlocality, we introduced more general measurement settings. From that, it was possible to derive conditions under which non-maximally entangled states can present full nonlocality.

While the proposed protocols can still be improved, these developments are left for future work. In all scenarios considered here, the post-measurement states were limited to the specific $(|i\rangle \pm |j\rangle)/\sqrt{2}$ and $(|i\rangle \pm i|j\rangle)/\sqrt{2}$ forms, which greatly restricts the results. One future goal is to determine the upper bound for the greater Schmidt coefficient λ_i , such that the state continues to exhibit full nonlocality in the single-copy scenario. Other approaches include finding the optimal number of measurements on Bob's side required to antidistinguish a full set of states in d-dimensions.

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