Forest wildfires and cellular automata

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Abstract: The purpose of this work is to study the properties of wildfire propagation trough the comparison of two different theoretical models: percolation and self-organised criticality. Computer simulations for the two models were performed using a cellular automaton in a 2D lattice using Python. Results show how the percolation model presents a phase transition and SOC behaviour establishes a characteristic frequency of appearance for each fire size that follows a power law.

I. INTRODUCTION

Although forest wildfire propagation has been studied by many authors, theoretical results can hardly be applied as exact models to describe the natural phenomena. Two classes of relatively simple models are used to describe forest fire propagation and occurrence[1]: Self-Organised Criticality (SOC) and percolation.

The percolation model suggests that wildfires behave as a fractal object[2] with a dimensionality d that correlates the burnt trees S and a characteristic size R in the following way[3][4]:

$$S \sim R^d \tag{1}$$

The probability of bond between two sites p establishes the regime that the propagation of the fire will follow. At the critical value $p = p_c$ a phase transition occurs. Percolation deals with the clusters formed.

In a SOC situation, continuous burn and regrowth of the trees maintains the forest in a stationary state where the total number of trees fluctuates around a constant value[5].

The model proposes a universal equation for the frequency f of wildfires with burnt trees (or area) S:

$$f(S) \sim S^{-\alpha} \tag{2}$$

Both models can not be used simultaneously, since percolation determines the propagation of the wildfire in a full forest depending on p, and SOC establishes a frequency-area statistical law followed by fires in the same forest over time.

In both cases computer simulations are performed in a 2D lattice using a cellular automaton. Cellular automata are algorithms that study the dynamic evolution of a discrete system.

II. PERCOLATION

The percolation model treats the propagation of the wildfire as a critical phenomenon. This means that a phase transition will exist at a certain critical value of p and critical exponents can be associated with it.

Numerical results may vary with the use of different lattice geometry, but the physical principles of the phenomena will not be altered[6].

In this case a two-dimensional squared lattice is used, with an $L \times L$ size. Every site can exist in three states: live tree, burning tree and burnt tree, or ash. A live tree has the ability to get caught on fire, and a tree on fire will become ash. Neighbouring trees are defined as those directly above or below or at the sides, so that every tree has four neighbours, as shown in Fig. 1.

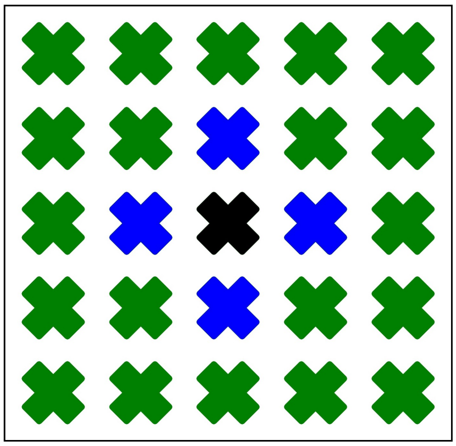

FIG. 1: First nearest neighbours in blue of the black site.

The propagation of the fire is determined by the bond probability p, which dictates if a live tree will get caught on fire if one of its neighbours is on fire.

Percolation occurs at $p_c = 0.5$[7], where an infinite cluster will begin to form. Above the percolation threshold, the wildfire propagates up to the limits of the forest leaving few unburnt trees. Below the threshold the cluster behaves as a fractal and the propagation will stop spontaneously.

It is not uncommon that wildfires generate high convective winds so that the propagation becomes three-dimensional[3], but this case will not be discussed in this work, so the spread is treated like a localised surface phenomenon.

A. Modelling and simulation

The simulation will be performed with Python, but the cellular automaton is independent of the language used. The size of the lattice used is 1000×1000 trees.

The forest starts in a state where every site has a live tree. The fire is started in the first iteration, or at the beginning of time, when the tree in the middle of the forest is ignited (in this case there are four trees, but it does not matter which tree gets burned first).

In the next iteration the trees neighbouring the tree on fire are subject to catch on fire, so they are tested with a probability p. If the test turns out favourable, the selected live tree will pass onto the second state and become a burning tree. If the test is unfavourable, the tree remains as a live tree.

This is done for all the trees that are near a burning tree, and if a live tree has more than one neighbour burning it will be tested for ignition multiple times.

Once every tree has been tested, all the trees that were on fire at the beginning of the iteration are converted to ash, so they are no longer capable of catching on fire and propagating it. In the next iteration its burning neighbours will have no effect on the ashed trees. An example is shown in Fig. 2.

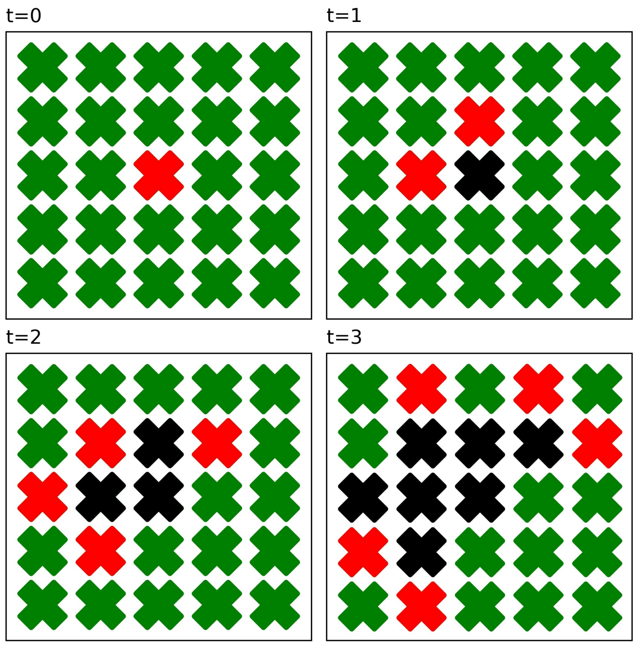

FIG. 2: Possible first four steps for a fire.

This is the end of one iteration. In the next iteration the time will increase one unity and the steps are going to be the same.

The simulation will stop when there no longer are any trees on fire, so they all are live trees or burned trees. If the cluster reaches one of the edges before the fire has ended all the burning trees are automatically converted to ash and the simulation stops. This means that the fire has percolated for the specific size of the studied system. An example is shown in Fig. 3.

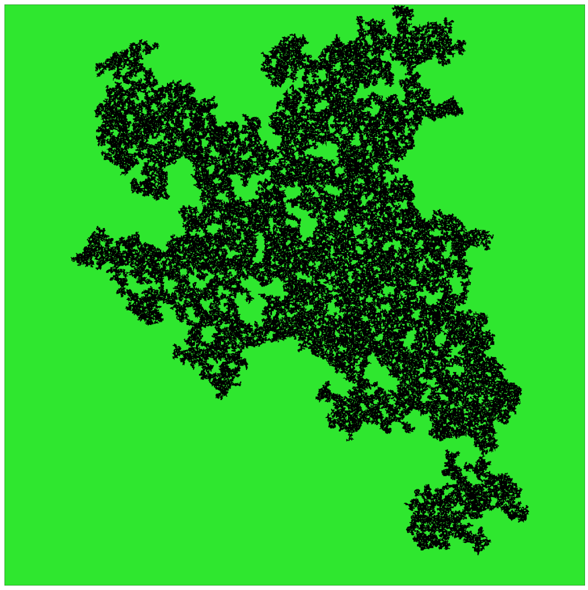

FIG. 3: Example of a percolated fire.

In order to be able to calculate the critical exponents and the fractal dimensionality of the percolating fire 13000 fires have been simulated with probabilities varying from $p = 0.45$ up to $p = p_c = 0.5$, the percolation threshold.

B. Results

In this section the dimensionality and the critical exponents of the phase transition are going to be discussed and calculated.

1. *Dimensionality*

As written in Eq. 1, dimensionality d is the relation between burnt area (or trees) and a characteristic size, in this case, the gyration radius R_s, which is defined in the following way[4]:

$$R_s^2 = \frac{1}{S}\sum_{i=1}^{S} |r_i - r_{cm}|^2 \tag{3}$$

Where r_{cm} is the centre of mass and is defined like usual:

$$r_{cm} = \frac{1}{S}\sum_{i=1}^{S} r_i \tag{4}$$

Eq. 1 is only valid for $\epsilon = 0$, where $\epsilon = |p - p_c|$, so the fit for d is only done with data from fires with such probability.

The value obtained is $d = 1.904 \pm 0.011$. Comparing it with the theoretical value[4], $d = 91/48 \sim 1.896$, it is observed that it is statistically compatible.

2. *Critical exponents*

The critical behaviour is observed near the percolation threshold, where $p \to p_c$. In this regime, the total number of burnt trees is given by the following equation[4]:

$$S \sim |p - p_c|^{-\gamma} \tag{5}$$

In this case, data used is from fires with $\epsilon > 0$, the case with $\epsilon = 0$ has an infinite cluster size so it is not possible to fit it.

The result obtained from the simulation is $\gamma = 2.38 \pm 0.04$. Data is shown in Fig. 4. The theoretical value[4] is $\gamma = 43/18 \sim 2.39$, which is also statistically compatible.

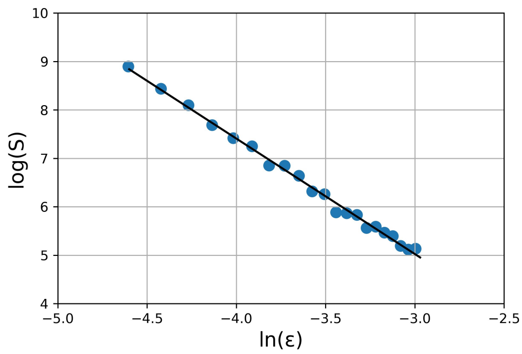

FIG. 4: Plot of the logarithm of the cluster size over the logarithm of ϵ.

The correlation length is also associated with a critical exponent with a similar equation[7]:

$$\xi = |p - p_c|^{-\nu} \tag{6}$$

The same procedure is applied to calculate ν and the result is $\nu = 1.33 \pm 0.04$, with a statistically compatible theoretical value[4] of $\nu = 4/3 \sim 1.33$. Data is shown in Fig. 5.

The rest of the critical exponents can be computed through hyperscaling relations. In this case the equations used are[4]:

$$D\nu = \gamma + 2\beta = 2 - \alpha$$
$$\frac{1}{d} = \sigma\nu$$
$$\gamma = \frac{3 - \tau}{\sigma}$$

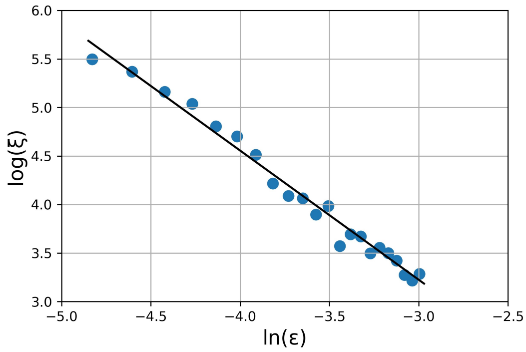

FIG. 5: Plot of the logarithm of the correlation distance over the logarithm of ϵ.

The following table presents both all theoretical[4] and computed critical exponents and dimensionality:

	Theoretical	Empirical
d	$91/48 \sim 1.896$	1.9044 ± 0.011
ν	$4/3 \sim 1.33$	1.33 ± 0.04
γ	$43/18 \sim 2.39$	2.38 ± 0.04
α	$-2/3 \sim -0.66$	-0.66 ± 0.08
β	$5/36 \sim 0.14$	0.14 ± 0.06
σ	$36/91 \sim 0.396$	0.394 ± 0.015
τ	$187/91 \sim 2.05$	2.06 ± 0.05

TABLE I: Theoretical and computed values of the critical exponents and the dimensionality.

All values are within the margins of errors, so the simulation has proven to be successful.

III. SELF-ORGANISED CRITICALITY

A forest in a SOC state maintains its total number of trees fluctuating around a constant value. The number of trees burned and regrown over time cancels each other.

Again, a two-dimensional squared grid is used and the fires are considered a surface phenomenon. The lattice has a significantly larger size $L \times L$ than the one used for the percolation model in order to avoid finite-size effects[5]. Site states and neighbourhood are defined in the same way as in the previous section.

The fire propagation is no longer subject to the probability of bonding p but to the distribution of trees in the forest. The constant wildfires and regrowth of the trees create a pattern where the propagation of a new fire started in a random spot by lightning[1] is subject to the density and distribution of trees in its surroundings.

The probability of bond p is high and it is maintained constant during the whole simulation, in this case set at $p = 0.7$. In this way the extinction of a fire is unlikely when the density of trees in its surroundings is high[5].

This model suggests that the (noncumulative) frequency f of forest fires of size S follows a universal power law[1]:

$$f(S) \sim S^{-\alpha} \tag{7}$$

This implies that there will not be a characteristic size associated with the wildfires. Small fires (even when only one tree is burned), called "spot fires", are expected to be the most common fire size[1]. They occur when the fuel content of neighbouring trees is very small so fire will unlikely propagate.

A. Modelling and simulation

The simulation is performed also with Python, in the same environment than the previous one. The main difference is the size of the lattice, in this case being 10000×10000.

Since the first part of the simulation has considerable fluctuations in the number of total trees, the initial state of the forest is not very relevant to it. In this case every site has a $p = 0.5$ chance of having a live tree. Sites without a tree remain blank.

The cellular automaton used consists of three steps:

1. Beginning of the fire:

   A random site on the map is chosen. If the site contains a live tree, the tree will catch on fire and the simulation proceeds to the propagation of the fire.

   If the site is empty, the fire is not started and the propagation step is omitted, going directly to the growth step.

2. Propagation of the fire:

   This step uses the same code for the percolation simulation, with a few differences. Here, the fire propagates with a higher probability, $p = 0.7$, and the system has periodic boundary conditions, so a fire can propagate through the limit of the grid to the other side of it.

   This step stops only when there no longer are any trees on fire, and it always leads to the growth step.

3. Growth:

   Every iteration a number $\theta = 2.5 \cdot 10^{-4} L^2$ of sites are randomly selected. If the site is empty, a live tree is placed in it. If the site is already occupied by a tree, it remains in its current state.

Once the growth stage has ended, the simulation returns to the first step and tries to ignite another fire.

Since the initial state of the forest is not that of a SOC situation the system needs a number of iterations before reaching SOC, as shown in Fig. 6, where the system becomes stable at approximately 40000 wildfires. After the total number of live trees in the forest has reached a stable value with small oscillations (in comparison with the total number of sites in the forest), the size of generated fires is ruled by Eq. 7.

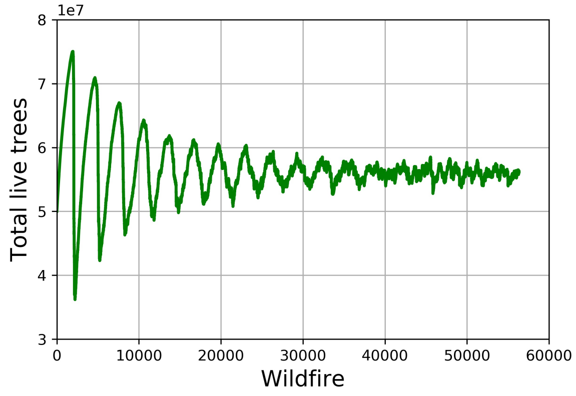

FIG. 6: Stabilisation of the total number of live trees in the forest.

The number of iterations done in this simulation is 100000, with a total of 56345 fires generated. The fires considered to be in the SOC state are the last 18346, which are shown in Fig. 7.

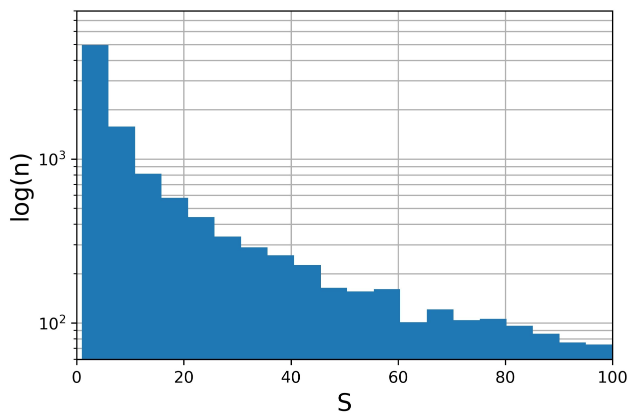

FIG. 7: Logarithmic histogram of the amount of fires n as a function of their size S computed in the SOC regime.

B. Results

To estimate the value of the exponent α, the Clauset method has been used[8]. The following equations determine the exponents value and its uncertainty:

$$\alpha = 1 + \frac{1}{\ln {}^{g}/_{a}} \tag{8}$$

$$g = n^{-1} \sum_{i=1}^{n} \ln S_i$$

$$\sigma_\alpha = \frac{\alpha - 1}{\sqrt{n}} \tag{9}$$

With a being a lower cutoff, a value of S above which the power law is considered valid (in this case, $a = 1$), S_i the value of the burnt trees for every fire simulated and n the number of fires generated with a SOC behaviour.

The result obtained is $\alpha = 1.2165 \pm 0.0016$. The theoretical value is $\alpha = 1.3$, but values from 1 to 1.3 are apparently considered acceptable[1].

IV. DISCUSSION

Although significant results have been obtained from both theoretical models, both cases can not be treated together.

In the percolation model fires have a characteristic length and scale until the percolation threshold is achieved, at $p = p_c = 0.5$, where the fire percolates and it no longer has a finite length, but it tends to grow infinitely. The model is studied only in forests that are completely packed, so every spot has a live tree.

Percolation theory aims to study the flow of the fire, which presents a phase transition at p_c that changes the behaviour of the fire.

The SOC behaviour does not present a characteristic length since it is characterised by a power law, and relies on burned areas to determine the size of every new fire. The constant burn and regrowth of the forest is achieved in a way such that the total number of trees in the forest remains somewhat constant.

The SOC model predicts the frequency at which fires of a certain size will appear in a forest over time. The followed law is described in Eq. 2.

V. CONCLUSION

The aim of this work was to study wildfire propagation with the use of two models, percolation and self-organised criticality, and the following comparison between them, to try to get a better understanding of the phenomena that occur during forest fires.

The conducted simulations, with the use of cellular automata, show how each model excels in explaining a characteristic behaviour for each case: percolation theory presents fires that propagate according to the bond probability p, which shows critical behaviour at $p_c = 0.5$, and SOC theory determines the frequency of appearance of fires of a certain size.

However, using one of these models to try to predict other characteristics of wildfires results in a failure. Both models can not be applied in the same framework and present conceptual problems that need to be addressed in order to develop a more generalised theory.

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