

Composite Models of Dynamical Dark Energy: Λ XCDM and ω XCDM

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Abstract: Although the Standard Λ CDM Model works well, it suffers from serious theoretical problems (eg. the Cosmological Constant Problem and the Cosmic Coincidence Problem) as well as phenomenological problems, such as the cosmological tensions (Hubble Tension and growth tension). In this work, we analyze the Λ XCDM model, a composite Dark Energy (DE) model involving a variable cosmological term (Running Vacuum) and an additional dark energy field X, which can play the role of "phantom matter" (different from phantom DE) [4, 8]. Through the analytical resolution of the cosmological equations and the study of the density evolution and the effective equation of state (ω_e), we demonstrate that the model can alleviate the Coincidence Problem thanks to the existence of a local maximum in the density ratio $r = \rho_D/\rho_m$, without need for fine-tuning, and leading the Universe towards a Turning Point of expansion instead of a Big Rip singularity. Finally, we show that the model's dynamics exhibits a type of crossing of the phantom divide which is fully compatible with the recent releases of DESI 2024 data [7, 10, 11].

Keywords: Dark Energy, Λ XCDM, Running Vacuum, Cosmological Coincidence Problem, Phantom Crossing, DESI.

SDGs: SDG 4 (Quality Education), SDG 9 (Industry, Innovation and Infrastructure), SDG 10 (Reduced Inequalities).

I. INTRODUCTION

The Cosmological Concordance model, or Standard Λ CDM model offers us an accurate picture of our Universe. Despite the phenomenological success of the model [1, 2], it suffers from serious theoretical pathologies that cast doubt on its validity like the Cosmological Constant Problem, and in particular the Coincidence Problem, and observational tensions such as the Hubble Tension.

The Cosmological Coincidence Problem arises from the distinct dynamical evolution of the cosmic components. While matter behaves as dust ($\rho_m \propto a^{-3}$), the vacuum energy density remains constant. Consequently, the ratio $r \equiv \rho_\Lambda/\rho_m$ grows strictly as $r \propto a^3$, tending to infinity. The fact that observers exist precisely in the brief epoch where $r \sim \mathcal{O}(1)$ implies an extreme fine-tuning of initial conditions ($\rho_{vac}^{teo}/\rho_{vac}^{obs} \sim 10^{-120}$), a coincidence the Standard Model cannot explain naturally [3].

In addition to these theoretical problems, nowadays the Standard Model faces serious observational challenges that were not obvious when the paper of reference [4] was published (2006). While WMAP data supported Λ CDM, recent high-precision Planck data and local Hubble measurements have exposed the Hubble tension [5, 6]. This suggests that Λ CDM model could be insufficiently accurate when it comes to describing the expansion of the Universe, motivating the need for models with dynamic DE such as the model we revisit in this paper, Λ XCDM.

The model we re-examine here is an extension of the

Standard Model that modifies the physics of DE sector. Λ XCDM is founded on the premise that the total DE is made out of two dynamical components: a variable cosmological term, Λ , and a dynamical "Cosmon" X, possibly interacting with Λ but not with matter. In the first case, the variability of Λ may be related to the running under quantum effects, whereas X might be identified with some fundamental field whose specific nature is not critical to reach the analytical solutions of the model. The energy transference between these two components allows us to explain the Coincidence Problem. Recently, DESI collaboration 2024 [7] has found evidence suggesting that DE is dynamical and that there is a crossing of the phantom divide from the phantom region to the quintessence one. The LXCDM adapts to both of these conditions and for this reason became fashionable again, beyond its impact on the coincidence problem. Advanced reconstruction analyses of this data support the hypothesis that DE could be composed of two interacting components.

In this paper, we aim to study the cosmological viability of the Λ XCDM model. This general framework serves as the theoretical basis for recent simplified extensions (e.g ω XCDM, Ref.[8]) that specifically exploit "phantom matter" to alleviate the Hubble and growth tensions. Aligning with this phenomenology, in our analysis we focus exclusively on the regime where X behaves as Phantom Matter $\rho_X < 0$. By deriving analytical and numerical solutions (Ref. [4]) for the densities and ratio evolution, we will discuss the relevance of these results in light of the new spectroscopic data from DESI. We will assess if the running vacuum models like the one presented fit the evidence better with a dynamical DE than Λ CDM.

II. THEORETICAL FRAMEWORK AND FORMALISM

Motivated by QFT in curved spacetime, we regard ρ_Λ as a gravitational vacuum coupling so we expect that it also follows an analogous RG equation, where its variation is determined by a parameter μ that acts as an effective vacuum β -function of RG equations. In an FLRW space-time, the characteristic energy scale associated with gravitational curvature and global dynamics is the Hubble rate $H(t)$. Therefore, we assume the association $\mu \sim H$.

Under this approach, in the context of the Running Vacuum framework in QFT [9], arguments based on general covariance and dimensional analysis indicate that the functional dependence of the vacuum energy density must necessarily contain H^2 terms, and powers of this, excluding odd powers. The resulting renormalization group equation is:

$$H \frac{d\rho_\Lambda}{dH} = \frac{3\nu}{4\pi} M_P^2 H^2 \quad (1)$$

The parameter $\nu \propto m_i^2/M_P^2$ serves as a dimensionless coefficient (related to the RG β -function) that measures running “intensity”, receiving contributions from heavy fields at the Grand Unified Theory scale ($M_i \sim M_{GUT} \approx 10^{16}$ GeV). Physically, (1) implies that vacuum energy is not a rigid constant but responds dynamically to the global expansion rate of the Universe. For a more rigorous derivation of this expression and the full expansion details, we refer the reader to [9].

The Λ XCDM model assumes a Universe composed of four principal components: matter and radiation ($\rho_m = \rho_M + \rho_R$) and a DE sector formed by a variable Λ and a Cosmon field, X , ($\rho_D = \rho_\Lambda + \rho_X$). This formulation arises from an internal consistency requirement of General Relativity. When covariance divergence is applied to Einstein Equations, the conservation of the total energy-momentum tensor ($\nabla^\nu T_{\nu\mu}^{tot} = 0$) is required by the Bianchi identity ($\nabla^\mu G_{\mu\nu} \equiv 0$).

In the Standard Model, where cosmological term is fixed ($\rho_\Lambda = \text{const.}$), this condition is trivially satisfied by conserving matter independently. In this paper, we treat matter and radiation as standard conserved fluids ($\nabla^\mu T_{\mu\nu}^{(m,r)} = 0$), according to physical observation. As a result, if we want to satisfy the total Bianchi identity, ρ_D must also be conserved. If we assume a scenario of running vacuum where ρ_Λ evolves with time ($\dot{\rho}_\Lambda \neq 0$) and a covariant conservation of energy-momentum tensor in a homogeneous and isotropic FLRW Universe typically leads to a matter-vacuum interaction, often expressed as:

$$\dot{\rho} + 3H(\rho + P) = -\frac{\dot{\Lambda}}{8\pi G} \quad (2)$$

This generates a contradiction with the premise of our model, as it implies an energy exchange between the matter and the DE sectors. In the Λ XCDM framework, we strictly impose the conservation of matter ($\dot{\rho}_m + \alpha_m \rho_m = 0$). Consequently, the term $\dot{\rho}_\Lambda$ must vanish from the matter equation. This is possible because the variation of the vacuum is compensated by the variation of the Cosmon X . Thus, the interaction is confined entirely within the dark sector, leading to the following conservation law:

$$\dot{\rho}_\Lambda + \dot{\rho}_X + 3H(1 + \omega_X)\rho_X = 0 \quad (3)$$

since $\omega_\Lambda = -1$. This equation shows that the Cosmon X acts as a compensating component that preserves the global conservation of the dark sector despite the vacuum dynamics.

III. RESULTS AND DISCUSSION

The dynamical resolution of the Λ XCDM model is based on a closed system of three equations that describe geometry, conservation and the nature of vacuum. Our starting point is the following closed system of equations:

$$\frac{d\rho_X}{dz} + \frac{d\rho_\Lambda}{dz} = \frac{\alpha_X \rho_X}{1+z} \quad (4)$$

$$\frac{d\rho_\Lambda}{dz} = \frac{3\nu}{8\pi} M_P^2 \frac{dH^2}{dz} \quad (5)$$

$$\frac{dH^2}{dz} = \frac{8\pi G}{3} \frac{\alpha_m \rho_m + \alpha_X \rho_X}{1+z} + 2H_0^2 \Omega_k^0 (1+z) \quad (6)$$

where $\rho_D = \rho_\Lambda + \rho_X$ and ω_e is the effective EOS parameter of the model Λ XCDM, $\omega_e = P_D/\rho_D = -1 + (1 + \omega_X)\rho_X/\rho_D$.

The first equation is a consequence of the Bianchi identity applied to a globally conserved dark sector. The second equation arises from solving (1) which we saw was motivated by QFT in curved spacetime. The third and final equation needed to solve the system is the first Friedmann equation, which imposes the geometric constraint where we will consider a flat Universe ($\Omega_k^0 = \frac{k}{H_0^2}, k = 0$).

Substituting the second and the third into the first equation and using

$$\Omega_i \equiv \frac{\rho_i}{\rho_c^0} \quad \sum \Omega_i = 1 \quad i = \Lambda, X, m \quad (7)$$

where $\rho_c^0 = \frac{3H_0^2}{8\pi G}$ is the critical density at the present, we get the decoupled equation for Ω_X

$$\frac{d\Omega_X}{dz} - (1 - \nu) \frac{\alpha_X \Omega_X}{1+z} = -\nu \frac{\alpha_m \Omega_m}{1+z} \quad (8)$$

Assuming a constant EOS parameter ω_X and $\nu \neq 0$, the general solution is obtained through standard integration

methods. By grouping all redshift-dependent terms into an auxiliary function $F(z)$, we arrive at the final analytical expression.

$$\Omega_X(z) = (1 - \nu)F(z) - \Omega_m^0(1 + z)^{\alpha_m} \quad (9)$$

$$F(z) = \frac{\Omega_m^0(\alpha_m - \alpha_X)}{\alpha_m - \alpha_X(1 - \nu)}(1 + z)^{\alpha_m} \quad (10)$$

$$+ \left[\frac{1 - \Omega_\Lambda^0}{1 - \nu} - \frac{\Omega_m^0(\alpha_m - \alpha_X)}{\alpha_m - \alpha_X(1 - \nu)} \right] (1 + z)^{\alpha_X(1 - \nu)}$$

where $\alpha_i = 3(1 + \omega_i)$ $i = \Lambda, X, m$.

From which we can easily find:

$$\frac{H^2(z)}{H_0^2} = \frac{\Omega_\Lambda^0 - \nu}{1 - \nu} + F(z) \quad (11)$$

$$\Omega_\Lambda = \frac{\Omega_\Lambda^0 - \nu}{1 - \nu} + \nu F(z) \quad (12)$$

A. Phenomenological constraints imposed by Nucleosynthesis (BBN) and analysis of the asymptotic behavior of r and stability

The existence of running vacuum ($\rho_\Lambda \propto \nu H^2$) modifies the Friedmann equation and, therefore, the expansion history of the Universe $H(z)$. This is especially crucial in the early Universe, during the radiation-dominated era. Big Bang Nucleosynthesis (BBN) ($z \sim 10^9$) is extremely sensitive to the expansion rate. Since the cosmic expansion rate determines the precise moment of neutron freeze-out, any significant contribution from the running vacuum ($\rho_\Lambda \sim \nu H^2$) would shift this decoupling epoch, thereby altering the primordial abundances of light elements. To preserve BBN predictions, the ratio of dark energy to radiation must remain below a critical threshold. The model predicts that the ratio stabilizes at a constant asymptotic value given by the expression:

$$|r_e| = \left| \frac{\varepsilon}{\omega_R - \omega_X + \varepsilon} \right| < 0.1 \quad (13)$$

where $\omega_r = 1/3$ is the EOS parameter for radiation and ε is an effective parameter defined by the running coefficient as $\varepsilon \equiv \nu(1 + \omega_X)$.

Once the analytical solution for Ω_D has been obtained and the constraint on the ratio r established based on physical considerations, we can explicitly construct $r(z) \equiv \rho_D(z)/\rho_m(z)$. The mathematical structure of this function is revealing, as its dependence on redshift is not simply monotonic, but rather presents critical points that are decisive for the model's viability. Taking into account that in the matter-dominated era we can approximate

$\Omega_m \approx 1 - \Omega_D$, we can study the behavior of this function.

$$r_M = \frac{\Omega_\Lambda^0 - \nu}{(1 - \nu)\Omega_M^0(1 + z)^3} + \frac{\varepsilon}{\omega_X - \varepsilon} + R_M(z) \quad (14)$$

where

$$R_M(z) = \left[\frac{1 - \Omega_\Lambda^0}{\Omega_M^0(1 - \nu)} - \frac{\omega_X}{\omega_X - \varepsilon} \right] (1 + z)^{3(\omega_X - \varepsilon)} \quad (15)$$

In the limiting case of the standard model ($\nu = 0$), the dark energy density remains constant while matter dilutes with expansion as $(1 + z)^3$. This causes the ratio to grow unboundedly towards the future ($z \rightarrow -1$), tending to infinity as matter dilutes. This reduces to a matter dilution term, leading to a de Sitter-like Universe with perpetual expansion.

In our model, $R_M(z)$ is crucial as it acts as a dynamic regulator. Unlike the standard case, where vacuum dominance is monotonic, here the interaction allows the ratio to exhibit local extrema. The relevance of this term lies in its ability to alter the sign of the total density of the Universe. The dynamic dependence of $R_M(z) \propto (1 + z)^{3(\omega_X - \varepsilon)}$ allows the ratio to evolve towards negative values in the cosmic future ($z < 0$). Therefore, $R_M(z)$ acts as the mathematical mechanism driving dark energy towards the negative density regime (AdS-like vacuum), satisfying the stopping condition $r(z_s) = -1$, halting the expansion at a Turning Point ($H = 0$) rather than a singularity. Initially, matter dilution dominates, causing $r(z)$ to grow as the Universe expands (a behavior we observe today). As we advance towards the future ($z < 0$), the term $R_M(z)$ (which induces negativity) gains relative weight. This slows down the growth of the ratio. To transition from increasing positive values to negative values, the function $r(z)$ is mathematically required to reach a local maximum point, z_{max} . At this point, the trend reverses: dark energy stops gaining relative ground and begins to decay rapidly towards the AdS-like regime.

We can locate this exact point by imposing the extremum condition on the derivative of the analytical solution with respect to redshift where we find z_{max} as a function of the model free parameters (ν, ω_X)

$$1 + z_{max} = \left[\frac{\Omega_\Lambda^0 - \nu}{\omega_X(\Omega_X^0 + \nu\Omega_M^0) - \varepsilon(1 - \Omega_\Lambda^0)} \right]^{\frac{1}{3(1 + \omega_X - \varepsilon)}} \quad (16)$$

The fact that the evolution of the ratio exhibits a maximum rather than an infinite divergence is the definitive key to understanding the coincidence problem present in the standard model. In this scenario, the ratio $r(z)$ remains bounded from above by $r(z_{max})$. This implies that the current epoch is not special; rather, we are living through the moment of maximum dark energy dominance.

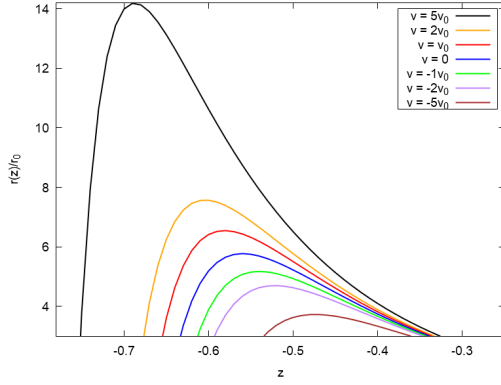


FIG. 1: $\frac{r(z)}{r_0}$ for $\omega_X = -1.85$, $\Omega_\Lambda^0 = 0.75$ and different values of ν . Comparison of the maximum ratio is shown. We define the loop coefficient as $\nu_0 = \frac{1}{12\pi}$ as following [4].

Mathematically, we have seen that z_{max} exists. However, physically, the timing of this occurrence is crucial. If this peak of relative dominance had occurred in the deep past ($z_{max} \gg 0$), it would imply that dark energy was dominant during the era of structure formation. This would have inhibited the gravitational collapse necessary to form galaxies and clusters, contradicting current observations. Therefore, we must demonstrate that, for the model's parameters, the maximum is restricted to the region $z_{max} \leq 0$ (present or future). A detailed dynamical analysis confirming that the Universe transitions to deceleration only in the future can be found in Ref.[4].

B. Dark Energy Dynamics and "Phantom Crossing"

To understand the nature of DE in our model, we must first present the possible theoretical behaviors a cosmological fluid can adopt. The key variable is the equation of state (EOS) parameter, $\omega = P/\rho$, which determines how density evolves with expansion. While standard classifications distinguish between Quintessence and Phantom regimes based on the $\omega = -1$ divide, the phenomenology of Λ XCDM offers a crucial departure from standard Phantom scenarios. In standard models, phantom behavior ($\omega < -1$) implies unbounded energy growth leading to a Big Rip singularity. From this point on, we restrict the analysis to the phantom matter regime ($\rho_X < 0$), corresponding to the ω XCDM realization of the Λ XCDM framework.

We analyze the dynamic evolution of the dark sector by representing its effective equation of state (ω_e) for three benchmark models. Before analyzing the results, it is fundamental to distinguish between the intrinsic EOS parameter of the phantom matter ω_X and the effective EOS parameter of the total dark sector. In our analysis, we fix the internal parameter ω_X to a constant value within

the phantom matter regime. However, the quantity that serves as the cosmological observable is ω_e . While in the standard Λ CDM model this parameter is rigidly constant ($\omega_\Lambda = -1$), the Running Vacuum model predicts a rich and complex evolution that can be tested against future observational data.

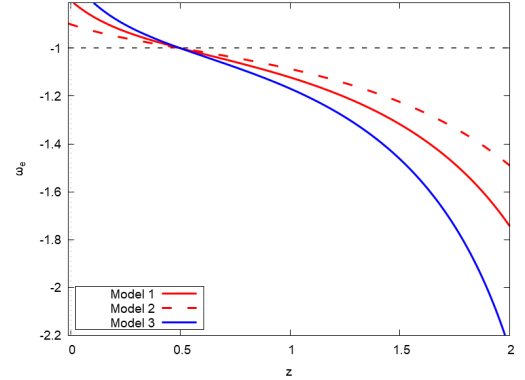


FIG. 2: Comparison of the effective EOS parameter of the Λ XCDM model, as a function of redshift z for the three models presented. The fixed cosmological parameters used for each model are detailed in Table(I) (see Appendix). The horizontal dashed line indicates the phantom divide ($\omega = -1$).

As shown in Fig.(2), despite the intrinsic Phantom matter nature of the Cosmon ($\omega_X < -1, \rho_X < 0$), the effective EOS parameter $\omega_e(z)$ exhibits a crucial evolution. In the past ($z \gtrsim 1$), the system is deep in the Phantom regime ($\omega_X \ll -1$). However, the interaction between the vacuum and phantom matter ($\nu \neq 0$) drives a dynamic evolution that pulls ω_e upwards.

Remarkably, all three scenarios predict a Phantom Crossing from the past towards the present, crossing $\omega_e = -1$ line and entering Quintessence regime $\omega_e > -1$. Crucially, a common feature of all three benchmark models is $\Omega_X < 0$. This specific combination is the mathematical mechanism responsible for the Turning Point scenario. Even though ω_e evolves towards Quintessence regime, the underlying Phantom matter nature of the Cosmon ensures that its negative density will grow in magnitude as the Universe expands (9). Eventually, this negative density will become dominant enough to exactly cancel out the positive matter density and vacuum ($\rho_D + \rho_m = 0$), triggering the Turning point.

C. Connection with recent observations: DESI

Finally, it is essential to contextualize these results within the framework of the most recent observational cosmology. The new spectroscopic data released by the DESI (Dark Energy Spectroscopic Instrument) collaboration in 2024 [7] have provided strong evidence for a dynamical Dark Energy. Recent studies applying non-parametric reconstruction techniques to these data, such as the

Weighted Function Regression method [10, 11], show an intriguing preference for an equation of state that evolves over time. Specifically, these analyses favor, with a statistical significance exceeding 96%, a reconstruction of $\omega_{DE}(z)$ that exhibits a transition from the phantom regime in the past ($z \gtrsim 0.5$) towards quintessence values in recent times ($z \approx 0$). This dynamical behavior is explicitly visible in the right-hand panels of Fig. (4) (see Appendix), which display the evolution of the effective EOS parameter $\omega_e(z)$ showing the Phantom Crossing. By selecting model parameters with these specific characteristics (as shown in our benchmarks), the ω XCDM model successfully replicates the behavior observed in the DESI reconstructions. This demonstrates that our composite model provides a theoretical framework capable of describing the emerging observational evidence. While the Standard Model is rigid and fails to capture this transition, Λ XCDM, and specifically ω XCDM parametrization, naturally accommodates it, thereby lending observational support to the associated Turning Point mechanism.

IV. CONCLUSIONS

We have analyzed the Λ XCDM model as a robust alternative to the Standard Model. Our main conclusions are:

- **Resolution of the Coincidence Problem:** The model naturally generates a local maximum for the density ratio $r(z)$, justifying the current era of comparable densities without requiring the extreme fine-tuning of initial conditions typical of Λ CDM.
- **Mechanism for Phantom Crossing:** By incorporating a Phantom Matter component ($w_X < -1, \rho_X < 0$), the model drives the effective equa-

tion of state to cross the Phantom Divide from the region of phantom to that of quintessence, exactly as preferred by DESI data. This behavior arises from the dynamical interplay between the Running Vacuum and the phantom species, providing a natural mechanism to explain the transition from a phantom-dominated past to a quintessence-like present.

- **The Turning Point Scenario:** The underlying Phantom Matter nature of the Cosmon ensures that the negative energy density will eventually dominate, causing the expansion to halt at a Turning Point, thereby avoiding the scenario of eternal cosmic expansion, which causes the coincidence ratio $r = \rho_D/\rho_m \rightarrow \infty$, and avoiding the Big Rip as well.
- **Observational Consistency:** Recent DESI 2024 analyses favor an evolving Dark Energy sector, a behavior that is naturally reproduced within the ω XCDM realization of the Λ XCDM framework when the dark energy sector includes Phantom Matter ($\rho_X < 0$); showing the Phantom Crossing. Future work will focus on a full statistical confrontation with forthcoming cosmological datasets and on exploring possible extensions of the model's parameter space.

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Models Compostos d'Energia Fosca Dinàmica: Λ XCDM i ω XCDM

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Resum: Tot i que el Model Estàndard Λ CDM funciona bé, presenta problemes teòrics greus (ex. Problema de la Constant Cosmològica i Problema de la Coincidència Còsmica) alhora que problemes fenomenològics, com les tensions cosmològiques (Tensió de Hubble i tensió de creixement). En aquest treball analitzem el model Λ XCDM, un model compost d'Energia Fosca (DE) amb un terme cosmològic variable (Running Vacuum) i un camp de tipus Cosmon, que juga el rol de 'phantom matter' (diferent de la phantom DE) [4]. Mitjançant la resolució analítica de les equacions cosmològiques i l'estudi de l'evolució de les densitats i de l'equació d'estat efectiva, demostrarem que el model alleuja el Problema de la Coincidència gràcies a l'existència d'un màxim local en el quocient de densitats $r = \rho_D / \rho_m$, sense necessitat d'ajust fi (fine-tuning), que porta a l'Univers a una aturada de l'expansió (Turning Point) en lloc d'una singularitat final (Big Rip). Finalment, mostrem que la dinàmica del model presenta un creuament del límit fantasma compatible amb les dades recents de DESI 2024 [10, 11].

Paraules clau: Energia Fosca, Λ XCDM, Buit Variable, Problema de la Coincidència Cosmològica, Phantom Crossing, DESI

ODS: Aquest TFG està relacionat amb els Objectius de Desenvolupament Sostenible (SDGs)

Objectius de Desenvolupament Sostenible (ODSs o SDGs)

1. Fi de la es desigualtats	10. Reducció de les desigualtats	X
2. Fam zero	11. Ciutats i comunitats sostenibles	
3. Salut i benestar	12. Consum i producció responsables	
4. Educació de qualitat	13. Acció climàtica	
5. Igualtat de gènere	14. Vida submarina	
6. Aigua neta i sanejament	15. Vida terrestre	
7. Energia neta i sostenible	16. Pau, justícia i institucions sòlides	
8. Treball digne i creixement econòmic	17. Aliança pels objectius	
9. Indústria, innovació, infraestructures		

El contingut d'aquest Treball de Final de Grau, adscrit en el context d'un grau universitari de Física, contribueix principalment a l'ODS 4, en particular amb la fita 4.4, ja que contribueix en la consolidació de competències avançades en recerca i física matemàtica. També s'alinea amb l'ODS 9, en fomentar la recerca científica bàsica (fita 9.5) essencial per al progrés del coneixement. Finalment, com a recerca impulsada per una dona, recolza l'ODS 5, promovent la participació femenina en ciència d'alt nivell històricament amb més presència masculina.[12]

[12] Definicions i fites consultades a: <https://www.diba.cat/web/ods/coneix-els-ods>

Appendix A: Model parameters

The numerical values of free parameters for the three benchmark models presented in this work have been optimized based on the finding in Ref.[10].

TABLE I: Numerical values of the free parameters for the three benchmark models used in the analysis.

Model	Ω_m^0	Ω_X^0	ν	ω_X
1	0.325	-0.15	-0.09	-1.9
2	0.350	-0.10	-0.07	-1.65
3	0.375	-0.2	-0.1	-1.94

Appendix B: Phantom and Quintessence Regimes. DESI Results

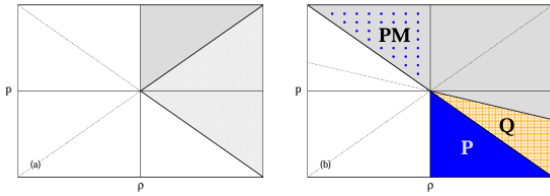


FIG. 3: Classification of Dark Energy regimes in the fluid phase space (ρ, P) . Theoretically, fluids are separated by the Phantom Divide ($\omega = -1$). The region **Q** denotes **Quintessence** ($-1 < \omega < -1/3$), where density dilutes slowly. The region **P** denotes **standard Phantom** behavior ($\omega < -1$), where density grows towards a Big Rip singularity. In contrast, the dotted region **PM** represents **Phantom Matter** that evolves into the negative density half-plane ($\rho < 0$), acting as a "cosmic brake" that halts the expansion at a Turning Point. Figure adapted from Ref.[4].

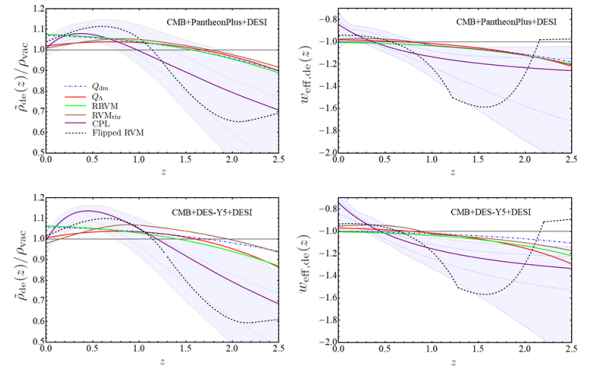


FIG. 4: Evolution of the effective DE density and EoS parameter. The light blue bands show the model-independent reconstruction from DESI data, exhibiting a preference for the Phantom Crossing. Figure adapted from Ref. [11].