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Discrete analysis of dividend payments in a non-life insurance portfolio

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Abstract

The process of free reserves in a non-life insurance portfolio as defined in the classical model of risk theory is modified by the introduction of dividend policies that set maximum levels for the accumulation of reserves. The first part of the work formulates the quantification of the dividend payments via the expectation of their current value under different hypotheses. The second part presents a solution based on a system of linear equations for discrete dividend payments in the case of a constant dividend barrier, illustrated by solving a specific case.

Keywords: dividend policies, expected present value

JEL Classification: G22

1 Introduction

The present study has two objectives. One is to formalize the dividend payment policies for a non-life insurance portfolio under different hypotheses, and the other is to obtain the expectation of the current value of the dividend payments in the discrete case.

The classical model analyses the solvency of non-life insurance portfolios using the probability of ruin as the criterion (ruin being taken to be when the reserves, $R(t)$, become negative). An alternative approach that modifies the classical method by proposing the pay-out of part of the reserves in the form of dividends is found in the literature.

In the classical model of the theory of ruin (Beard et al. (1984), Bowers et al. (1987), Daykin et al. (1994), Gerber (1979), Grandell (1990), Panjer(1992)), the probability of ruin of an insurance company is calculated by modeling the total cost of the claims through a compound Poisson process, i.e., by considering that the claim frequency follows a discrete Poisson distribution of parameter λ . The use of a Poisson process implies that the time between consecutive claims has an exponential distribution with mean $\frac{1}{\lambda}$.

The calculation of the probability of ruin , $\psi(u) = P[R(t) < 0]$, under this hypothesis depends on the initial level of reserves $R(0) = u$, with $R(t) = u + c \cdot t - S(t)$ where $S(t)$ is the aggregate claim process up to time t, and c is the intensity of the premium income, with the restriction $c > \lambda \cdot E[z]$ where $E[z]$ is the mean amount of a claim.

As an alternative model to this classical process of risk theory, the actuarial literature includes approaches which propose the pay-out of part of the reserves in the form of dividends (Bülhmann (1970), Gerber (1981), Paulsen (1997), Siegl and Tichy (1996, 1999)). I.e., these are modifications of the classical model with the introduction of different policies that determine the form in which the dividends are paid out.

The technical basis for proposing the control of reserves has its origin in the critique of De Finneti (1957) that, under the classical hypotheses of the process of risk, the level of reserves $R(t)$

will tend to infinity as t tends to infinity with probability unity. Hence, dividend policies are a form of controlling this unbounded growth in $R(t)$.

Also, the payment of dividends is in itself a reasonable objective for a risk portfolio manager, since these amounts may be interpreted as earnings surpluses whose end could be to cover other risk portfolios with shortfalls. As dividends paid out to shareholders, their role as incentives for the capture of the initial capital (the initial level of the reserves u) is unquestionable.

Out of this justification of a dividend policy in the management of an insurance portfolio, there arises the need to quantify the part of the reserves that are to be paid out as dividends. Indeed, the quantification of these dividends is an indispensable measure for the evaluation of dividend payment policies introduced into the model.

Paying out dividends also affects the probability of ruin. Evidently, a cap on the level of accumulation of reserves leads to a greater chance of ruin. The reason is that claims which do not provoke negative levels of reserves in the classical model, in the dividend payment model may lead to ruin since the level of reserves declines because of the pay-outs.

The form in which the control over the level of reserves is specified is by means of introducing dividend barriers, represented formally by $b(t)$.

These barriers are functional expressions which depend on the time variable t. The actuarial literature includes constant (Bülhmann (1970)) and linear (Alegre et al.(2001), Gerber (1981), Siegl and Tichy (1996, 1999)) dividend barriers, setting the maximum level of the reserves for the entire temporal horizon.

In the constant dividend barrier case, it is easily shown that the ultimate (infinite time) probability of ruin is unity (Eigido dos Reis (1999)). There would therefore be no sense in using the probability of ruin in this case as a criterion for the comparison or choice between barriers.

Section 2 describes the various hypotheses put forward in the actuarial literature on dividend payments. We include the definition of new variables that allows us to improve and reformulate the model in discrete dividend payments. The payment strategies are represented as functional expressions $b(t)$ which act as a cap to the accumulation of reserves, so that $R(t)$ is not allowed to surpass the level given by $b(t)$, with the differences being paid out in the form of dividends.

Section 3 deals with the analysis of the dividend payments when the model is modified to have a constant dividend barrier $b(t) = b$, assuming discrete payments, and presents a method for solving such problems. The expectation of the present value of the dividend payments has been obtained (Bülhmann (1970)) assuming a special distribution for the total cost in a period. In Section 4 we obtain, for any discrete total cost distribution, this expectation, using a system of linear equations, that is solved using its matrix form.

2 Quantification of the dividend payments

Independently of which dividend policy that one may determine, its quantification depends on a series of hypotheses concerning the moment that dividends are to be paid and until when, and the measure used to evaluate those dividends.

The magnitude chosen to evaluate surplus reserves is the expected present value of the amounts to be paid out.

With respect to the time period during which dividend payments are to be made, there are two possible hypotheses:

- Hypothesis 1: The process is taken to end at the moment that ruin occurs. In this case, we shall formalize the expected present value of the dividend payments up to that instant as W. This will be a function of the level of reserves and of some of the parameters defining the barrier which represents the dividend policy.

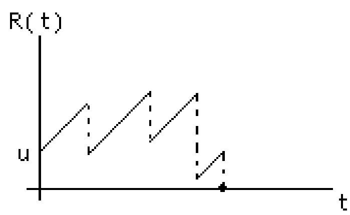
- Hypothesis 2. The process does not end when ruin occurs, i.e., recovery of the process is allowed when there are negative levels of reserves. Under this hypothesis, the expected present value of the dividends is represented by V. This function does not depend separately on u and the barrier parameters, but is a function of a single variable, d, defined as the difference between the level of reserves and the levels set as caps in the dividend policies.

Payment of dividends may be discrete or continuous:

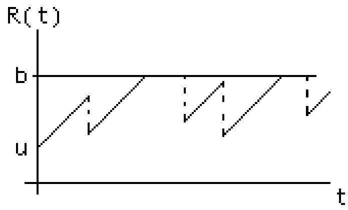
- We take a discrete dividend policy to be that which makes the pay-outs at given times , t_i for $i = 1, 2, 3, ...$, as long as $b(t_i) < R(t_i)$, without regard for whether at any intermediate time the level of reserves surpasses the cap represented by the dividend barrier.
- In a continuous dividend policy, pay-outs are made continuously $\forall t \in (0, \infty)$, whenever there is a surplus of reserves.

In the Theory of Ruin, two types of barrier are considered (Eigido dos Reis (1999)):

1. Absorbing barriers: In this situation, the process stops when the level of the reserves reaches the value established in the barrier. Graphically, the most usual case of imposing an absorbing barrier at zero to control ruin is schematically as follows:



2. Reflecting barriers: This type of barrier modifies the process so that when the cap of the reserves is reached, $R(t)$ is maintained at the level of the barrier until the occurrence of the next claim. With a constant barrier $b(t) = b$, this may be represented graphically as follows:



To calculate the expectation of the current value of the dividends, there is only sense in using the reflecting barriers. This is because we consider that, each time the reserves reach the level of the barrier, all income from contributions from that instant until the occurrence of the next claim is devoted to dividend payments.

In the calculation of W, the process is limited below by an absorbing barrier at the level $R(t) = 0$, since ruin is taken to terminate the process, and above by a reflecting barrier at $b(t) = b$. In the calculation of V, only the reflecting barrier at $b(t) = b$ appears.

We shall now present the reformulation of the model on the basis of the definitions that exist for dividend payments in the discrete cases.

2.1 Discrete dividend payments

Let τ be the instant of ruin, and consider the equidistant times t_i for $i = 1, 2, 3, ...$ with $t_0 = 0$, the time unit being one year.

Let D_{t_i} be the dividends paid out at t_i for $i = 1, 2, 3, ...$

$$D_{t_i} = Max\left\{\left(R^*\left(t_i\right) - b\left(t_i\right)\right), 0\right\}$$

The sum of the dividend payments in an interval $[0, t]$ is

$$SD(t) = D_{t_1} + D_{t_2} + ... + D_{t_s}$$

where $t_s = Max\left\{t_i / t_i \leq t\right\}.$

Thus, the equation for the level of reserves $R(t)$ in the discrete dividend payments modified model has the form

$$R(t) = u + c \cdot t - S(t) - SD(t) \tag{1}$$

where $S(t)$ is the aggregate of claims in the period $[0, t]$, and c is the annual intensity of contributions.

The process of reserves $R(t)$ in (1) is defined for all t, but, since in the discrete case our interest is in the level of reserves at times t_i for $i = 1, 2, 3, ...$, we shall focus on the analysis of $R(t_i)$ defined as the level of reserves at t_i after any dividend payments have been made. Thus,

$$R(t_i) = u + c \cdot t_i - S(t_i) - SD(t_i)$$

and the level of reserves before dividend payments can be defined as

$$R^*(t_i) = u + c \cdot t_i - S(t_i) - SD(t_{i-1}) \tag{2}$$

where both the claims and the dividend payments at t_i act as jumps in the reserve process.

Equation (2) can be written as

$$R^*(t_i) = R(t_{i-1}) + c \cdot (t_i - t_{i-1}) - (S(t_i) - S(t_{i-1}))$$

Let v be a constant financial discount rate for all the periods. Then the expected present value of the dividend payments, assuming that the process does not end in ruin, is

$$V = E\left[\sum_{i=1}^{\infty} D_{t_i} \cdot v^{t_i}\right]$$

and assuming that there are dividend payments only up to the instant of ruin:

$$W = E\left[\sum_{i=1}^{k} D_{t_i} \cdot v^{t_i}\right] \text{ siendo } k = Max\left\{i \in N \;/\; t_i \leq \tau\right\}$$

3 Dividend policy in the discrete case

We shall now generalize the calculation of the expectation of the current value of the dividends, following the approach of Bülhmann (1970) for the calculation of $W(u,b)$ in a modified model with a constant dividend barrier $b(t)=b$.

Bühlmann (1970) proposed a system of finite difference equations, considering the situation at time t_1

$$W(u,b)=\begin{cases} 0 & \text{si } u<0 \\ v\cdot\sum\limits_{j=-\infty}^{\infty} W(u+j,b)\cdot P\left[c\cdot t_1-S(t_1)=j\right] & \text{si } 0\leq u\leq b \\ u-b+W(b,b) & \text{si } u>b \end{cases}$$

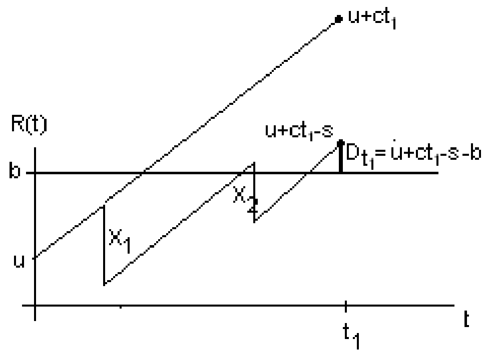
and solving the system for the particular case in which the variation in the reserves is dichotomous, taking only the values $j=-1$ and $j=1$, with probabilities p and $1-p$. Since the only random factor considered in the model is the occurrence of claims, the case that Bühlmann calculated implies that the claims in a given period can only take the values $(c\cdot t_1+1)$ and $(c\cdot t_1-1)$, with probabilities p and $1-p$.

To generalize the calculation of $W(u,b)$, we shall analyse the situation of the process at time t_1 :

The dividend payments will depend on whether $R^*(t_1)$ is greater or lesser than the level of the barrier $b(t_1)$, with $S(t_1)=s$ being the aggregate of claims in the period $]0,t_1]$. Hence,

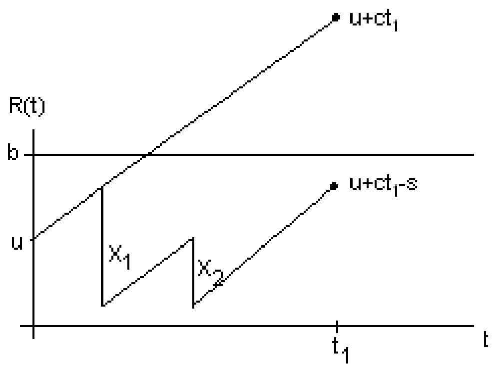
$$R^*(t_1)=u+c\cdot t_1-s$$

Case 1 $R^*(t_1)=u+c\cdot t_1-s$ *is greater than the level of the barrier* $b(t_1)$. *Graphically for example, for a constant dividend barrier* $b(t_i)=b$,



In this case, the dividend payments in $t_1, D_{t_1} = SD(t_1)$, are positive, with their amount being the difference between $R^*(t_1)$ and the barrier $b(t_1)$, i.e. $D_{t_1} = u + c \cdot t_1 - s - b(t_1)$. Also, for the calculation of $W(u,b)$ the calculated future dividends must be discounted to t_1, and are $W(b(t_1), b(t_1))$

Case 2 *$R^*(t_1)$ is less than or equal to the level of the barrier $b(t_1)$, independently of what happened in the interval $]0, t_1]$. Graphically, for a constant dividend barrier, $b(t_i) = b$*



In this case, for the calculation of $W(u,b)$, we must discount $W(u + c \cdot t_1 - s, b(t_1))$

4 Constant barrier: calculation of $W(u,b)$

In this section, we shall focus on the study of the constant dividend barrier $b(t) = b$. To determine the expression for the expected present value of the dividend payments, we shall formalize the two cases described in the previous section, by setting up a system of linear equations. Using a linear system, we avoid to solve a differential equation system of order $b+1$.

The solution of the problem involves considering the random variable of the total accumulated claims in the first period as a discrete random variable, and the hypothesis that all monetary magnitudes $(u, b, c, ...)$ are multiples of some given unit. Neither of these conditions implies any major restriction on the validity of the model: in the case of the monetary magnitudes, we simply have to change the reference unit, and in the case of the claims, we shall just have to previously discretize the random variable if it is not already discrete.

For simplicity, we shall write $P[S(t_1) = s] = P_s$ and $F_s(x) = P[s \leq x]$, and redefine c as $c \cdot t_1$.

According to the initial level of reserves u, such that $u \leq b$, one can define $b+1$ equations for the calculation of $W(u,b)$ with $u = 0, ..., b$..

- **If the initial level of reserves coincides with the barrier level,** $u = b$

  First, let us consider the case in which the total of claims s coincides with the premium income c. At t_1 therefore, the new level of reserves is $b + c - s = b$, hence

$$W(b,b) \cdot P_c \tag{3}$$

  In those cases when the amount of claims s lies in the interval $[0, c-1]$, there will be dividend payments, since $b+c-s$ is greater than b, with $D_{t_1} = c - s$, so that the new level of reserves will be $u = b$. Hence

$$\sum_{s=0}^{c-1} W(b+c-s, b) \cdot P_s = \sum_{s=0}^{c-1} \left(W(b,b) + (c-s)\right) \cdot P_s$$

or, equivalently,

$$\sum_{s=0}^{c-1} \left(W\left(b,b\right) + \left(c-s\right)\right) \cdot P_s = W\left(b,b\right) \cdot \sum_{s=0}^{c-1} P_s + \sum_{s=0}^{c-1} \left(c-s\right) \cdot P_s = \qquad (4)$$
$$= W\left(b,b\right) \cdot F_s\left(c-1\right) + \sum_{s=0}^{c-1} \left(c-s\right) \cdot P_s$$

Finally, let us consider the cases in which the aggregate claims amount s lies in the interval $[c+1, b+c]$. The level of reserves at t_1 , $b+c-s$, is less than b. Hence

$$\sum_{s=c+1}^{b+c} W\left(b+c-s,b\right) \cdot P_s$$

Applying the change of variable $r = s-c$, one has

$$\sum_{s=c+1}^{b+c} W\left(b+c-s,b\right) \cdot P_s = \sum_{r=1}^{b} W\left(b-r,b\right) \cdot P_{r+c} \qquad (5)$$

We can therefore write $W\left(b,b\right)$ as the discounted sum of $\left(3\right), \left(4\right)$ and $\left(5\right)$

$$W\left(b,b\right) = v \cdot \left[W\left(b,b\right) \cdot F_s\left(c\right) + \sum_{s=0}^{c-1} \left(c-s\right) \cdot P_s + \sum_{s=1}^{b} W\left(b-s,b\right) \cdot P_{s+c}\right] \qquad (6)$$

- **If the initial level of reserves is below the barrier by less than c units,** $b-c < u < b$.

The equation for $u = b-x$, when $x = 1, ..., c-1$ results from taking into account that the new level of reserves at t_1 is

$$b-x+c-s \qquad (7)$$

The expression $\left(7\right)$ is greater than b if

$$b-x+c-s > b \Rightarrow s < c-x$$

and therefore leads to dividend payment. This is reflected in the sum

$$\sum_{s=0}^{c-(x+1)} W\left(b-x+c-s,b\right) \cdot P_s$$

where $(c-s-x)$ would have to be paid out, leaving the new level of reserves at b. Hence

$$\sum_{s=0}^{c-(x+1)} \left(\left(c-s-x\right)+W\left(b,b\right)\right)\cdot P_s = W\left(b,b\right)\cdot F_s\left(c-\left(x+1\right)\right)+\sum_{s=0}^{c-(x+1)}\left(c-s-x\right)\cdot P_s \quad (8)$$

The other situation is when (7) is less than b, i.e.,

$$b-x+c-s<b \Rightarrow s>c-x$$

In this case, there will be no dividend payment. Also, so as not to cause ruin, one must have that

$$b-x+c-s\geq 0 \Rightarrow s\leq b-x+c$$

Hence, the amount of s has to lie in the interval $[c-x+1, c-x+b]$. Hence

$$\sum_{s=c-x+1}^{c-x+b} W\left(b-x+c-s,b\right)\cdot P_s \quad (9)$$

With the change of variable$r=s-(c-x)\Rightarrow s=r+(c-x)$, expression (9) becomes

$$\sum_{r=1}^{b} W\left(b-r,b\right)\cdot P_{r+c-x} \quad (10)$$

Lastly, it can be assumed that

$$b-x+c-s=b\Rightarrow s=c-x$$

in which case one has

$$W\left(b,b\right)\cdot P_{c-x} \quad (11)$$

Grouping together (8), (10) and (11) ,one then has

$$W\left(b-x,b\right)=v\cdot\left[W\left(b,b\right)\cdot F_s\left(c-x\right)+\sum_{s=0}^{c-(x+1)}\left(c-s-x\right)\cdot P_s+\sum_{s=1}^{b}W\left(b-s,b\right)\cdot P_{s+c-x}\right] \quad (12)$$

- **The initial level of reserves is below the barrier by at least c units,** $0\leq u\leq b-c<b$.

Now, for $u=b-x$, when $x=c,c+1,...,b$, the new level of reserves is $b-x+c-s$, which is therefore always less than b given the values of x. There is therefore no dividend payment:

$$W\left(b-x,b\right)=v\cdot\sum_{s=0}^{b+(c-x)}W\left(b-x+c-s,b\right)\cdot P_{s} \tag{13}$$

where the top of the sum limits the case in which the new level of reserves is negative,

$$b-x+c-s\geq 0\Rightarrow s\leq b+\left(c-x\right)$$

From these three cases, grouping together the expressions $(6),(12)$ and (13), one has the following system of equations:

For $u=b$

$$W\left(b,b\right)=v\cdot\left[W\left(b,b\right)\cdot F_{s}\left(c\right)+\sum_{s=0}^{c-1}\left(c-s\right)\cdot P_{s}+\sum_{s=1}^{b}W\left(b-s,b\right)\cdot P_{s+c}\right]$$

For $u=b-x$, if $x=1,...,c-1$

$$W\left(b-x,b\right)=v\cdot\left[W\left(b,b\right)\cdot F_{s}\left(c-x\right)+\sum_{s=0}^{c-(x+1)}\left(c-s-x\right)\cdot P_{s}+\sum_{s=1}^{b}W\left(b-s,b\right)\cdot P_{s+c-x}\right]$$

For $u=b-x$, if $x=c,c+1,...,b$

$$W\left(b-x,b\right)=v\cdot\sum_{s=0}^{b+(c-x)}W\left(b-x+c-s,b\right)\cdot P_{s}$$

4.1 Matrix form of the system

It can be readily verified that the generalization of the system presented in the previous subsection, and defined by equations $(6),(12)$ and (13), can be written in matrix form:

$$v\cdot A\cdot\overline{w}+v\cdot D=\overline{w} \tag{14}$$

where A is the matrix of coefficients made up of different submatrices:

$$A=\begin{pmatrix} M_{1} & M_{2}\\ M_{3} & M_{4}\end{pmatrix}$$

with:

- M_1 is a vector of $(c+1)$ components

$$M_1 = \begin{pmatrix} F_s(c) \\ F_s(c-1) \\ F_s(c-2) \\ \vdots \\ F_s(0) \end{pmatrix}$$

- M_2 is a matrix of order $(c+1) \times b$

$$M_2 = \begin{pmatrix} P_{c+1} & P_{c+2} & P_{c+3} & \cdots & P_{c+b} \\ P_c & P_{c+1} & P_{c+2} & \cdots & P_{c+b-1} \\ P_{c-1} & P_c & P_{c+1} & \cdots & P_{c+b-2} \\ \vdots & \vdots & \vdots & & \vdots \\ P_1 & P_2 & P_3 & \cdots & P_b \end{pmatrix}$$

- M_3 is a null vector of $(b-c)$ components

- M_4 is a matrix of order $(b-c) \times b$

$$M_4 = \begin{pmatrix} P_0 & P_1 & P_2 & P_3 & \cdots & \cdots & \cdots & P_{b-1} \\ 0 & P_0 & P_1 & P_2 & \cdots & \cdots & \cdots & P_{c+b-1} \\ 0 & 0 & P_0 & P_1 & \cdots & \cdots & \cdots & P_{c+b-2} \\ 0 & 0 & 0 & P_0 & \cdots & \cdots & \cdots & P_{c+b-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & P_0 & \cdots & P_c \end{pmatrix}$$

The matrix A is therefore a square matrix of order $(b+1)$,

$$A = \begin{pmatrix} F_s(c) & P_{c+1} & P_{c+2} & P_{c+3} & \cdots & \cdots & & P_{c+b} \\ F_s(c-1) & P_c & P_{c+1} & P_{c+2} & \cdots & \cdots & & P_{c+b-1} \\ F_s(c-2) & P_{c-1} & P_c & P_{c+1} & \cdots & \cdots & & P_{c+b-2} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \cdots & & \cdots \\ F_s(0) & P_1 & P_2 & P_3 & \cdots & \cdots & & P_b \\ 0 & P_0 & P_1 & P_2 & \cdots & \cdots & \cdots & P_{b-1} \\ 0 & 0 & P_0 & P_1 & \cdots & \cdots & \cdots & P_{c+b-1} \\ 0 & 0 & 0 & P_0 & \cdots & \cdots & \cdots & P_{c+b-2} \\ 0 & 0 & 0 & 0 & \cdots & \cdots & \cdots & P_{c+b-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & P_0 & \cdots & P_c \end{pmatrix}$$

The vector of independent terms D is of order $(b+1) \times 1$, formed by c first elements different from zero, and the remaining $b+1-c$ elements equal to zero:

$$D = \begin{pmatrix} \sum_{s=0}^{c-1} (c-s) \cdot P_s \\ \sum_{s=0}^{c-2} (c-s-1) \cdot P_s \\ \sum_{s=0}^{c-3} (c-s-2) \cdot P_s \\ \vdots \\ P_0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

The vector of unknowns is:

$$\overline{w} = \begin{pmatrix} W(b,b) \\ W(b-1,b) \\ W(b-2,b) \\ \vdots \\ W(b-c,b) \\ W(b-c-1,b) \\ \vdots \\ W(0,b) \end{pmatrix}$$

The solution of system (14) is,

$$\overline{w} = [I - v \cdot A]^{-1} \cdot v \cdot D$$

As the espectral norm of the matrix $v \cdot A$ is less than one, $[I - v \cdot A]$ is regular.

The specific case indicated in Section 3, where the aggregate of claims in a period can only take the values $(c+1)$ and $(c-1)$, with probabilities p and $q = 1-p$, can be solved by means of a system of finite difference equations (Bülhmann (1970)), and alternatively by applying the matrix system presented in the previous subsection.

The system of finite difference equations is:

$$W(u,b) = \begin{cases} v \cdot p \cdot W(b,b) + v \cdot p + v \cdot q \cdot W(u-1,b) & u = b \\ v \cdot p \cdot W(u+1,b) + v \cdot q \cdot W(u-1,b) & u < b \end{cases}$$

whose solution is

$$W(u,b) = C_2 \cdot \left(-\frac{1 - r_2 \cdot v \cdot p}{1 - r_1 \cdot v \cdot p} \cdot r_1^u + r_2^u \right)$$

with

$$C_2 = \frac{v^2 \cdot p^2}{\left(r_2^{b-1} - v \cdot q \cdot r_2^{b-2} - v \cdot p \cdot r_2^{b-1} + v^2 \cdot p \cdot q \cdot r_2^{b-2} - v^2 \cdot p \cdot q \cdot r_2^{b-1}\right) - \frac{1 - r_2 \cdot v \cdot p}{1 - r_1 \cdot v \cdot p} \cdot \left(r_1^{b-1} - v \cdot q \cdot r_1^{b-2} - v \cdot p \cdot r_1^{b-1} + v^2 \cdot p \cdot q \cdot r_1^{b-2} - v^2 \cdot p \cdot q \cdot r_1^{b-1}\right)}$$

and r_1 and r_2 the roots of $r^2 - \frac{1}{v \cdot p} \cdot r + \frac{q}{p} = 0$.

To obtain the matrix system, we take into account that the probabilities for the aggregate claims in a period are:

s	P_s
$c+1$	p
$c-1$	$1-p$

(15)

Thus, for instance, taking $b=5$ and $c=1$, the matrix system is:

$$\begin{pmatrix} W(5,5) \\ W(4,5) \\ W(3,5) \\ W(2,5) \\ W(1,5) \\ W(0,5) \end{pmatrix} = \left[I - v \cdot \begin{pmatrix} P_0+P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \\ P_0 & P_1 & P_2 & P_3 & P_4 & P_5 \\ 0 & P_0 & P_1 & P_2 & P_3 & P_4 \\ 0 & 0 & P_0 & P_1 & P_2 & P_3 \\ 0 & 0 & 0 & P_0 & P_1 & P_2 \\ 0 & 0 & 0 & 0 & P_0 & P_1 \end{pmatrix} \right]^{-1} \cdot v \cdot \begin{pmatrix} P_0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

where, according to (15), $P_0 = 1-p$, $P_2 = p$ and $P_i = 0 \; \forall i \neq 1,2$.

4.2 Numerical example

In the following numerical example, the data used for the insurance portfolio are $c = 9$, $\lambda = 3$, and the following distribution for the individual amount of the claims:

x	$\mathbf{P}[X=x]$
1	0.2
2	0.25
3	0.35
4	0.2

Take $v = (1.05)^{-1}$, and $b = 50$.

For the calculation of the probabilities of the total claims in a period, needed to create the matrix of coefficients A, the following recursion relationship is used (Panjer, 1980)

$$f_s(x) = \sum_{y=1}^{s} \left(a + b \cdot \frac{y}{x}\right) \cdot f_x(y) \cdot f_s(x-y) \quad , \quad x = 1, 2, 3, ...$$

The numerical results are obtained from a program of our own elaboration in APL2. The vector of total claim probabilities P_s, $s = 0, 1, ..., 59$ is:

$$P_s = (0.04978, 0.02987, 0.046301, 0.07647, 0.08223, 0.07631, 0.08685, 0.087022, 0.07753, 0.07042, 0.06411, 0.05428, 0.04472, 0.03686, 0.02957, 0.02294, 0.01759, 0.01333, 0.00987, 0.00718, 0.005167, 0.00366, 0.00255, 0.00176, 0.00120, 0.0008086, 0.0005375, 0.0003537, 0.0002303, 0.00014, 0.000094, 0.0000598, 0.000037, 0.000023, 0.000014, 0.0000087, 0, ..., 0)$$

The results for $W(u, 50)$ are:

u	**50**	**49**	**48**	**47**	**46**	**45**	**44**	**43**
$\mathbf{W}(u, 50)$	26.5664	25.7005	24.8523	24.0247	23.2196	22.4376	21.6810	20.9510
u	**42**	**41**	**40**	**39**	**38**	**37**	**36**	**35**
$\mathbf{W}(u, 50)$	20.2453	19.5629	18.9068	18.2716	17.6571	17.0630	16.4886	15.9335
u	**34**	**33**	**32**	**31**	**30**	**29**	**28**	**27**
$\mathbf{W}(u, 50)$	15.3971	14.8785	14.3771	13.8925	13.4237	12.9704	12.5318	12.1075
u	**26**	**25**	**24**	**23**	**22**	**21**	**20**	**19**
$\mathbf{W}(u, 50)$	11.6968	11.2992	10.9141	10.5407	10.1787	9.8272	9.4856	9.1531
u	**18**	**17**	**16**	**15**	**14**	**13**	**12**	**11**
$\mathbf{W}(u, 50)$	8.8290	8.5123	8.2021	7.8975	7.5972	7.3002	7.0052	6.7108
u	**10**	**9**	**8**	**7**	**6**	**5**	**4**	**3**
$\mathbf{W}(u, 50)$	6.4157	6.1022	5.7819	5.4530	5.1139	4.7629	4.3983	4.0207
u	**2**	**1**	**0**					
$\mathbf{W}(u, 50)$	3.6318	3.2308	2.8208					

The greater the initial level of reserves u for a given value of the barrier b, the greater the expected present value of the dividend payments. This is because the moment of ruin is delayed, with it being possible to pay out dividends for a longer time. Also, the first dividends will be paid out closer to time zero, so that their discounted value will be greater.

For the same initial data of x, c, λ and i, we found results for values of the barrier between $b = 10$ and $b = 100$. We can analyse the solutions that were obtained by grouping together some of the results:

- The values obtained for $W(b, b)$ are the following:

b	**10**	**11**	**12**	**13**	**14**	**15**	**16**
W (b, b)	21.5279	22.3576	23.0802	23.7	24.224	24.66	25.025
b	**17**	**18**	**19**	...	...	**30**	**31**
W (b, b)	25.32	25.56	25.76	...	...	26.49	26.509
b	**32**	...	...	**50**	**51**	**52**	...
W (b, b)	26.521	...	...	26.5664	26.5665	26.5667	...
b	...	**91**	**92**	...	...	**99**	
W (b, b)	...	25.56728187	26.56728189			26.56728195	

  One observes that, for values of the initial reserves coinciding with the level of the barrier b, the greater $u = b$ is, the greater will be the expectation of the current value of the dividend payments, since the moment of ruin is delayed, allowing dividends to be paid out over a longer time.

- For $u = 0$, the results for different values of b are the following:

b	**10**	**11**	**12**	**13**	**14**	**15**	**16**
$\mathbf{W}(0,b)$	9.4705	9.4053	9.3032	9.1685	9.0063	8.8217	8.6197
b	**17**	**18**	**19**	...	...	**30**	**31**
$\mathbf{W}(0,b)$	8.4050	8.1815	7.9527	...	...	5.5781	5.3929
b	**32**	...	...	**50**	**51**	**52**	...
$\mathbf{W}(0,b)$	5.2133	...	...	2.8208	2.7260	2.6343	...
b	...	**91**	**92**	...	...	**99**	
$\mathbf{W}(0,b)$	...	0.6942	0.6709			0.5280	

One observes that, for a given value of u, in this case $u = 0$, the greater b is, the smaller is $W(0,b)$. The reason is that it is more difficult to reach the dividend barrier, and hence the amount of dividend payments is less.

5 Conclusion

The present study had a twofold aim. First, we presented the generalization of the calculation of the dividend payments under different hypotheses, looking at the difference between discrete and continuous payment of dividends. It was noticeable the different way in which the choice of the type of payment affected the level of reserves, and therefore the expectation of the current value of the dividends.

In the second part, we introduced a procedure to solve the case of a model modified to include a constant dividend barrier assuming discrete payments. The usefulness of this model is that it allows the evaluation of the dividend payments for a total cost distribution that is not restricted to the dichotomous case dealt with up to now in the actuarial literature.

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