

Ultracold atoms in an optical lattice: One-dimensional phase diagram

Author: Judit Serralta Vidal, jserravi120@alumnes.ub.edu
Facultat de Física, Universitat de Barcelona, Diagonal 645, 08028 Barcelona, Spain.

Advisors: Montserrat Guilleumas Morell and Héctor Briongos Merino

Abstract: In this project, the ground-state phase diagram of the one-dimensional Extended Bose-Hubbard Model is systematically characterized using the DMRG algorithm via the Julia `ITensors` library. Benchmark simulations with only on-site interactions, validate the implementation by reproducing the conventional Superfluid to Mott Insulator transition. Incorporating nearest-neighbor interactions at unit filling and employing finite-size scaling at a fixed value of the ratio between on-site interactions and tunneling strength equal to 5, we map three distinct insulating regimes. We confirm that weak nearest-neighbor interactions maintain the Mott phase, while intermediate coupling stabilizes the topological Haldane Insulator, identified by a non-local string order parameter. Finally, strong interactions drive the system into a crystalline Density Wave phase via spontaneous translational symmetry breaking.

Keywords: Bose-Hubbard model, tensor networks, DMRG, Haldane Insulator, Mott Insulator, Nearest-neighbor interaction.

SDGs: This work is related to sustainable-development goals 4, 7 and 9 (see page 6).

I. INTRODUCTION

Over the past few decades, ultracold atomic systems have become one of the key platforms for studying quantum many-body phenomena [1]. When an ensemble of bosons is cooled down to extremely low temperatures, their associated de Broglie wavelength increases until the atomic wave-functions start to overlap. In this degenerate regime, the system can no longer be described as a collection of independent particles; instead, collective quantum behaviors emerge, giving rise to a rich variety of strongly correlated phases [2].

In order to manipulate these systems, optical lattices are commonly used. These consist of periodic potentials generated by standing waves of laser light, creating a lattice structure where atoms are trapped at the potential minima. One of the main advantages of these platforms is the high degree of experimental control they provide. In particular, the tunneling rate can be tuned by adjusting the laser intensity, while the strength of interatomic interactions can also be independently controlled by other techniques [3]. This allows for the exploration of different physical regimes in a highly controlled way. For this reason, these setups act as excellent quantum simulators for studying many-body systems.

In these types of lattices, the behavior of the system at near-zero temperatures is governed by a competition between different physical parameters, giving rise to quantum phase transitions. Depending on the dominating mechanism, the ground state of the bosons reorganises itself into completely different structures. When particles can move freely through the lattice, the Superfluid (SF) phase takes place, where the atoms are delocalized over the whole system, achieving a global phase coherence and large particle number fluctuations at each site. However, if the repulsion between atoms occupying the same node becomes strong enough, hopping is suppressed and the

system enters the Mott Insulator (MI) phase, characterized by a fixed integer number of bosons per site and the absence of number fluctuations [4].

The aim of this project is to investigate the emergence of exotic phases in one-dimensional lattices when long-range repulsive interactions between nearest neighbors are taken into account. On one hand, this repulsion can break the translational symmetry of the lattice, forcing the atoms into an alternating occupation pattern known as the Density Wave (DW) phase. Additionally, the system can host a Haldane Insulator (HI) phase, a topological phase characterized by the absence of local order but the presence of a hidden string order [5]. In particular, we explore the appearance of these different physical regimes by studying the transitions between the SF, MI, DW, and HI phases, identifying their phase boundaries through various observables.

II. THEORETICAL FRAMEWORK AND NUMERICAL IMPLEMENTATION

A. Extended Bose-Hubbard Model

To describe the behavior of ultracold bosons in a one-dimensional optical lattice at zero temperature, we employ the Extended Bose-Hubbard Model (EBHM) [6]. For a linear chain of L sites and N particles under Open Boundary Conditions (OBC), the Hamiltonian is expressed as:

$$H = -t \sum_{i=1}^{L-1} (\hat{b}_i^\dagger \hat{b}_{i+1} + \text{h.c.}) + \frac{U}{2} \sum_{i=1}^L \hat{n}_i(\hat{n}_i - 1) - \mu \sum_{i=1}^L \hat{n}_i + V \sum_{i=1}^{L-1} \hat{n}_i \hat{n}_{i+1}, \quad (1)$$

where $\hat{b}_i^\dagger$ and $\hat{b}_i$ represent the bosonic creation and annihilation operators at site i , while $\hat{n}_i = \hat{b}_i^\dagger \hat{b}_i$ denotes the particle number operator.

Four different terms can be distinguished in Eq. (1), each of them playing a distinct role in the phases of the system. First of all, the hopping amplitude (t) governs the quantum tunneling of bosons between adjacent sites, promoting particle delocalization and driving the system into the SF phase. Conversely, we have the on-site interaction ($U > 0$), which energetically penalizes multiple occupation of the same site, a mechanism that suppresses particle mobility and leads to the MI phase. Furthermore, the chemical potential (μ) controls the total number of particles in the lattice. Finally, the nearest-neighbor interaction ($V > 0$) introduces a repulsive force between atoms on adjacent sites, being the key parameter responsible for the emergence of the DW and HI phases [7]. Throughout this work, all parameters are taken to be positive, and each of them can be independently controlled in experimental optical lattice setups.

B. Tensor Networks and DMRG Implementation

To investigate the phase diagram for different values of the parameters, the system's ground state must be determined for each Hamiltonian configuration. The traditional approach involves a direct product of the Hilbert spaces of each site; however, the total dimension scales exponentially with both the maximum number of particles allowed per site and the total number of sites [4]. This exponential growth makes exact calculations impossible for large systems due to computational limits.

To overcome this issue, we apply the Density Matrix Renormalization Group (DMRG) algorithm [8], which compresses the many-body wave function into a Matrix Product State (MPS). This approach is highly efficient for our one-dimensional system because, instead of storing the massive global state tensor, the MPS decomposes it into a chain of smaller, interconnected local tensors assigned to each lattice site [9]. By breaking down the global state this way, the method drastically reduces the memory required to store the wave function, transforming the computational complexity from exponential to polynomial.

These simulations are implemented using the `ITensors` library in the Julia programming language, which natively incorporates all the mechanisms of tensor networks and DMRG algorithms [10]. It automatically performs the ansatz minimization to find the ground state once the parameters of the Hamiltonian are introduced. The precision and convergence of the results are managed through key variables, such as the number of sweeps over the lattice, the maximum dimension of the tensor, the error truncation limit and the noise term, which prevents the algorithm from getting trapped in local energy minima.

C. Physical Observables

To characterize the different quantum phases of the EBHM and delimit their transition boundaries, we compute several observables over the ground state obtained via DMRG.

First, the transition between the superfluid phase and the insulating regimes is identified by the local particle number variance at each site j , as in Ref. [11]:

$$\Delta n_j = \langle \hat{n}_j^2 \rangle - \langle \hat{n}_j \rangle^2. \quad (2)$$

This quantity is directly proportional to the compressibility of the system. In a compressible superfluid phase, quantum fluctuations are significant, yielding $\Delta n_j > 0$. Conversely, the insulating phases are incompressible, meaning that these fluctuations are suppressed ($\Delta n_j \rightarrow 0$) in the zero-temperature limit.

Furthermore, the specific nature of each insulating phase is revealed by two-point correlation functions in the long-range limit ($r \rightarrow \infty$). To isolate and identify the Density Wave phase, we define the correlator [7]:

$$C_{\text{DW}}(r) = (-1)^r \langle \delta \hat{n}_j \delta \hat{n}_{j+r} \rangle, \quad (3)$$

where $\delta \hat{n}_j = \hat{n}_j - \bar{n}$ (with $\bar{n} = 1$). The corresponding DW order parameter is defined as:

$$\mathcal{O}_{\text{DW}} \equiv \lim_{r \rightarrow \infty} C_{\text{DW}}(r). \quad (4)$$

Due to the alternating spatial ordering, $\mathcal{O}_{\text{DW}}$ takes finite values in the Density Wave phase, while it decays exponentially to zero in the remaining phases. In practice, numerical simulations can induce a global negative sign. To account for this effect, our analysis evaluates the absolute value $|\mathcal{O}_{\text{DW}}|$.

Analogously, the topological Haldane Insulator phase is detected through a non-local string correlator [5]:

$$C_{\text{string}}(r) = \langle \delta \hat{n}_j e^{i\pi \sum_{j \leq k < j+r} \delta \hat{n}_k} \delta \hat{n}_{j+r} \rangle. \quad (5)$$

The underlying hidden order is captured by the string order parameter,

$$\mathcal{O}_{\text{string}} \equiv \lim_{r \rightarrow \infty} C_{\text{string}}(r), \quad (6)$$

which remains finite within the HI phase (and coexists in the DW phase) but vanishes completely in the conventional Mott Insulator phase.

From a practical standpoint, since numerical simulations are performed on finite systems, the order parameters defined in Eq. (4) and Eq. (6) cannot be accessed directly in the thermodynamic limit. Instead, the correlators are evaluated at large but finite distances and plotted as a function of $1/r$, allowing a linear extrapolation to $1/r \rightarrow 0$, that effectively minimizes finite-size corrections.

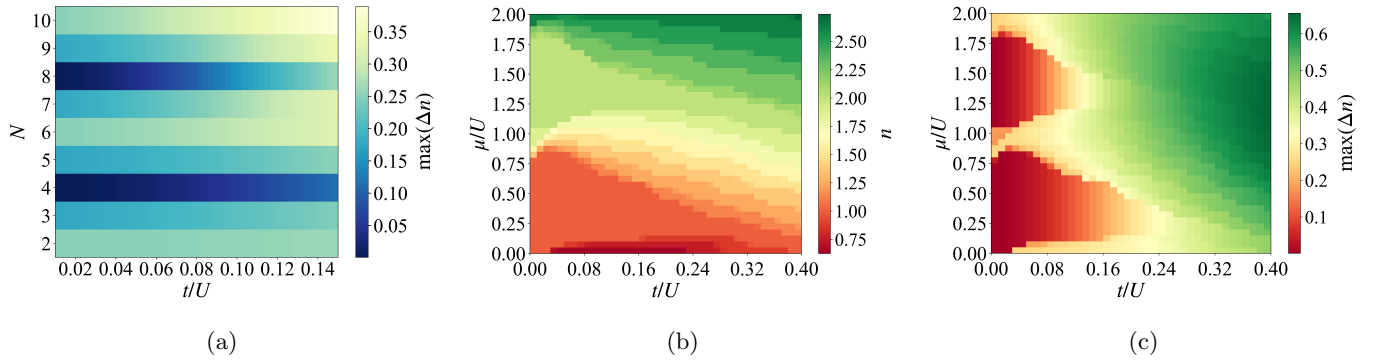


FIG. 1: a) Maximum local density fluctuations, $\max(\Delta n)$, as a function of the dimensionless ratio t/U and the number of bosons N , for a system of $L = 4$ sites, with $V = 0$ and $\mu = 0$. b) Average occupation density, $n = N/L$, and c) maximum local density fluctuations, $\max(\Delta n)$, as a function of the dimensionless parameters t/U and μ/U , for a system of $L = 8$ sites with $V = 0$ and $\mu \neq 0$.

III. RESULTS

A. MI-SF transition with $V/U = 0$

We have studied the Superfluid to Mott Insulator (SF-MI) transition, with $V/U = 0$ (only on-site interactions), as an initial benchmark. The simulation was performed by fixing the chemical potential to zero ($\mu/U = 0$), which allows the system to maintain a constant number of particles. For this preliminary test, we considered a lattice of $L = 4$ sites with the on-site interaction fixed to unity ($U = 1$). From this baseline, we studied the evolution of the system as a function of the dimensionless ratio t/U by gradually varying the tunneling parameter for different numbers of bosons, specifically within the range $N \in [2, 10]$.

The colormap in Fig. 1(a) shows two clearly distinguishable behaviors. Horizontal bands of vanishing compressibility are identifiable at $N = 4$ and $N = 8$, where the maximum local density fluctuation defined in Eq. (2) drops strictly to zero. This phenomenon is a direct consequence of the commensurability between the number of bosons and the lattice size ($n = N/L$): these specific values correspond to integer fillings per site ($n = 1$ and $n = 2$, respectively). When the filling is commensurate and the on-site interaction U dominates over the tunneling t , the bosons become localized and the system turns incompressible, thereby suppressing density fluctuations. This behavior constitutes the characteristic signature of the Mott Insulator phase. In contrast, for incommensurate values of N , the fluctuations remain significantly higher across the entire t/U range, indicating that the system cannot structure a regular Mott state and remains in the Superfluid phase. This initial figure thus provides the first solid evidence that the implemented model successfully captures the physics underlying the SF-MI transition [12].

Moreover, the bands of zero fluctuation observed in Fig. 1(a) suggest a richer structure in the full parameter space, which becomes apparent once the chemical poten-

tial is treated as a free variable. The natural next step was therefore to release μ/U and explore the complete parameter space ($\mu/U, t/U$). To do so, and in order to minimize finite-size effects while improving the resolution of the phase boundaries, the lattice size was scaled up to $L = 8$ sites for the complete phase diagram simulations.

In Fig. 1(b) the average occupation density $n = N/L$ is represented as a function of t/U and μ/U . Two distinct regions are clearly visible where the density is frozen at an integer value ($n = 1$ orange and $n = 2$ light green), forming the structures known as Mott lobes [4]. This phenomenon has a clear energetic origin: when the on-site interaction U strongly dominates over the hopping t , adding or removing a particle in the system has an energetic cost, known as the Mott gap. As long as the chemical potential μ is not large enough to overcome this gap, the system remains locked in a state of integer occupation, reflecting the incompressibility characteristic of the Mott Insulator phase. Outside these lobes, the density varies continuously with the parameters, indicating that the system is in the Superfluid phase. Furthermore, the lobes are observed to narrow progressively as t/U increases, eventually disappearing entirely once the hopping is large enough to close the Mott gap and drive the system irreversibly into the Superfluid phase [13].

The complementary picture is provided by Fig. 1(c), which maps the maximum density fluctuation over the same parameter space. The fluctuation minima coincide precisely with the Mott lobes of Fig. 1(b), directly confirming that the suppression of fluctuations is the signature of the Mott gap [13]. Moreover, the band of zero fluctuation at $\mu/U = 0$ recovers exactly the horizontal cross-sections of Fig. 1(a) scaled to a larger system, demonstrating the internal consistency of our results across different system sizes.

B. MI-HI-DW transition with $V/t \neq 0$

Once the general behavior of the model was validated at zero nearest-neighbor interaction ($V/t = 0$), the study was extended to explore the exotic insulating phases of the EBHM that emerge upon incorporating V/t . In particular, we studied the transitions among the Mott Insulator, Haldane Insulator and Density Wave phases. For this purpose, following the parameters established in Ref. [7], the system was fixed at unitary filling ($n = 1$) by considering a lattice of $L = 200$ sites with a constant number of bosons $N = 200$, while maintaining the chemical potential at zero ($\mu/t = 0$). This significantly larger system size, compared to the small lattices used in Fig. 1, is essential to properly capture long-range correlation behaviors. Accordingly, the hopping amplitude was set as the unit of energy ($t = 1$) and the on-site interaction was fixed at $U/t = 5$, ensuring that the nearest-neighbor interaction ratio V/t remained the only free parameter tuned throughout the simulations.

To evaluate the order parameters, the origin of the correlation functions in Eq. (3) and Eq. (5) was fixed at $j = 60$ to avoid boundary effects. The spatial evolution of $C_{DW}(r)$ and $C_{string}(r)$ was monitored within the range $r \in [50, 80]$ sites. This specific window was selected to ensure that the correlation functions had reached their asymptotic scaling regime, free from short-range transients. This procedure was systematically applied across a range of nearest-neighbor interaction between $V/t \in [2.6, 3.8]$. The spatial decay of these correlators as a function of $1/r$ is displayed in Fig. 2 and Fig. 3, where the distinct colored lines represent the progression of increasing V/t values across the scanned interval.

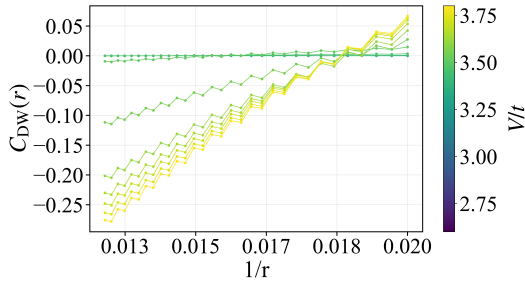


FIG. 2: Density Wave correlator $C_{DW}(r)$ as a function of the inverse distance $1/r$ at $U/t = 5$ for different values of the nearest-neighbor interaction V/t .

As displayed in Fig. 2 and Fig. 3, the long-range behavior of both correlators exhibits a robust linear scaling with respect to $1/r$ within the asymptotic region. This linear dependence allows for a reliable finite-size scaling extrapolation toward the thermodynamic limit ($1/r \rightarrow 0$). By evaluating the vertical intercept of these linear regressions, we precisely determine the infinite-system asymptotic values of the order parameters $\mathcal{O}_{DW}$ and $\mathcal{O}_{string}$, effectively removing finite-size corrections.

The resulting profiles of these extrapolated order pa-

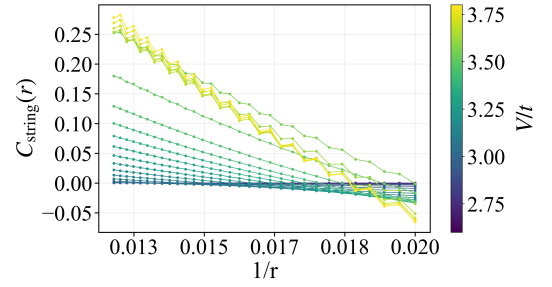


FIG. 3: Non-local String correlator $C_{string}(r)$ as a function of the inverse distance $1/r$ at $U/t = 5$ for different values of the nearest-neighbor interaction V/t .

rameters as a function of V/t are compiled in Fig. 4, providing a direct mapping of the system's phase diagram [7]. In this figure, the red squared markers track the local density-wave order parameter $|\mathcal{O}_{DW}|$, while the blue circular markers track the non-local string order parameter $\mathcal{O}_{string}$.

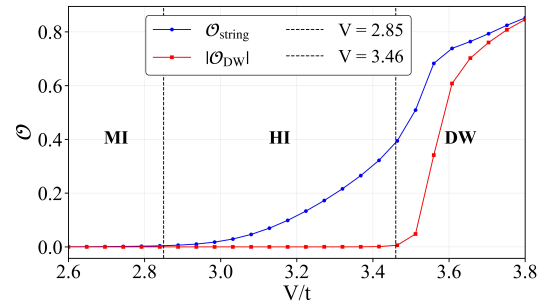


FIG. 4: Extrapolated values of the Density Wave ($\mathcal{O}_{DW}$) and String ($\mathcal{O}_{string}$) order parameters as a function of V/t at $U/t = 5$. The vertical dashed black lines mark the phase transitions between the three regimes (MI, HI and DW).

By examining the simultaneous evolution of both extrapolated order parameters as a function of V/t , we can clearly distinguish three physically distinct regimes, explicitly verifying the stabilization of different quantum insulating phases in the one-dimensional EBHM at unitary filling.

For low values of nearest-neighbor interaction specifically below the critical threshold $V/t \approx 2.85$, both $|\mathcal{O}_{DW}|$ and $\mathcal{O}_{string}$ vanish strictly within the thermodynamic limit. In this region, the extrapolated observables remain bounded at residual numerical values of order of 10^{-4} or lower. This behavior indicates that the system possesses neither spatial nor topological long-range order, a regime completely consistent with the conventional Mott Insulator (MI) phase. Here, the physics is dominated by the strong local on-site repulsion ($U/t = 5$), which penalizes fluctuations, localizes exactly one boson per lattice site, and prevents the establishment of non-local correlations.

As the nearest-neighbor interaction enters the inter-

mediate regime, bounded between $V/t \approx 2.85$ and $V/t \approx 3.46$, the system undergoes a topological transition into the Haldane Insulator (HI) phase. The critical boundaries are precisely identified as the points where the extrapolated observables distinctively lift off from the numerical noise floor. In this phase, while the density-wave order parameter remains strictly zero, the non-local string order parameter continuously rises to finite values, growing from $\mathcal{O}_{\text{string}} \approx 0.003$ at $V/t = 2.84$ to $\mathcal{O}_{\text{string}} \approx 0.32$ at $V/t = 3.41$. This selective activation of the non-local order, while the local density profile remains completely uniform, constitutes the definitive signature of a hidden topological order, stabilizing due to the delicate competitive balance between U/t and V/t .

Finally, for strong nearest-neighbor interactions exceeding the second critical point at $V/t \approx 3.46$, the system experiences another quantum phase transition toward the Density Wave (DW) phase. The density-wave order parameter $|\mathcal{O}_{\text{DW}}|$ abruptly lifts off from zero, shifting to a substantial magnitude of $|\mathcal{O}_{\text{DW}}| \approx 0.34$ at $V/t = 3.56$, and approaching an asymptotic saturation value of 0.85 at $V = 3.80$. This phenomenon marks the spontaneous breaking of the lattice translational symmetry. In this regime, the repulsive force exerted by the nearest-neighbor interaction V/t becomes strong enough to overcome the local localization energy of the Mott state. Consequently, the bosons reorganize into an alternating spatial structure (resembling a ...2-0-2-0... pattern), locking the system into a crystalline DW state where the local density-wave order coexists with a finite value of the non-local string correlator.

IV. CONCLUSIONS

In this work, the ground-state phase diagram of the one-dimensional Extended Bose-Hubbard Model

(EBHM) has been successfully characterized using the DMRG algorithm via the `ITensors` library in Julia.

First, simulations at $V/U = 0$ validated the model's consistency by replicating the Superfluid to Mott Insulator transition, where the suppression of local density fluctuations precisely defined the Mott lobes in the $(\mu/U, t/U)$ parameter space [4]. Second, by performing a finite-size scaling extrapolation to the thermodynamic limit ($1/r \rightarrow 0$) at $U/t = 5$, we mapped the extended insulating regimes under non-zero nearest-neighbor interactions ($V/t \neq 0$). For weak interactions ($V/t \lesssim 2.85$), the system remains in the conventional Mott Insulator phase. In the intermediate regime ($2.85 \lesssim V/t \lesssim 3.46$), the competition stabilizes the topological Haldane Insulator (HI) phase, uniquely identified by a finite non-local string order parameter while the density profile remains uniform. Finally, for strong interactions ($V/t \gtrsim 3.46$), nearest-neighbor repulsion overcomes Mott localization, spontaneously breaking lattice translational symmetry and forcing the bosons into a crystalline Density Wave (DW) phase.

As future research lines, it would be highly interesting to compute the neutral or charge gap to provide an independent determination of the phase boundaries [7], or to extend this tensor network methodology to the study of quasi-one-dimensional geometries.

Acknowledgments

I would like to thank my advisors Muntsa Guilleumas and Héctor Briongos for their guidance, support, and for providing me with all the necessary resources and tools to carry out this project. Also, a heartfelt thank you to my family and friends for supporting me through this journey.

-
- [1] I. Bloch, J. Dalibard, and W. Zwerger, *Many-body physics with ultracold gases*, Reviews of Modern Physics **80**, 885 (2008).
 - [2] Pitaevskii, L. P. *Bose-Einstein condensation*. Clarendon Press, Oxford New York, 2003. ISBN 978 0198507192.
 - [3] T. Mishra, B. K. Sahoo, and R. V. Pai, *Phase separated charge density wave phase in two species extended Bose-Hubbard model*, Phys. Rev. A **78**, 013632 (2008).
 - [4] I. Morera Navarro, *Exotic phases of matter in low dimensional lattices: from quantum liquids to kinetic magnetism*, Ph.D. thesis, Universitat de Barcelona (2023).
 - [5] E. G. Dalla Torre, E. Berg, and A. Altman, *Hidden order in 1D Bose-Hubbard models*, Phys. Rev. Lett. **97**, 260401 (2006).
 - [6] R. V. Pai and R. Pandit, *The one-dimensional extended Bose-Hubbard model*, Proc. Indian Acad. Sci. (Chem. Sci.) **115**, 721 (2003).
 - [7] D. Rossini and R. Fazio, *Phase diagram of the extended Bose-Hubbard model*, New J. Phys. **14**, 065012 (2012).
 - [8] S. R. White, *Density matrix formulation for quantum renormalization groups*, Phys. Rev. Lett. **69**, 2863 (1992).
 - [9] R. Orús, *A practical introduction to tensor networks: Matrix product states and projected entangled pair states*, Ann. Phys. **349**, 117–158 (2014).
 - [10] M. Fishman, S. R. White, and E. M. Stoudenmire, *The ITensor software library for tensor networks*, SciPost Phys. Codebases, 4 (2022).
 - [11] R. Kraus, K. Biedroń, J. Zakrzewski, and G. Morigi, *Superfluid phases induced by dipolar interactions*, Phys. Rev. A **102**, 053315 (2020).
 - [12] G. G. Batrouni, V. Rousseau, R. T. Scalettar, M. Rigol, A. Muramatsu, P. J. H. Denteneer, and M. Troyer, *Mott Domains of Bosons Confined on Optical Lattices*, Phys. Rev. Lett. **89**, 117203 (2002).
 - [13] M. P. A. Fisher, P. B. Weichman, G. Grinstein, and D. S. Fisher, *Boson localization and the superfluid-insulator transition*, Phys. Rev. B **40**, 546 (1989).

Àtoms ultrafreds en una xarxa òptica: Diagrama de fases unidimensional

Author: Judit Serralta Vidal, jserravi120@alumnes.ub.edu
 Facultat de Física, Universitat de Barcelona, Diagonal 645, 08028 Barcelona, Spain.

Advisors: Montserrat Guilleumas Morell and Héctor Briongos Merino

Resum: En aquest projecte es caracteritza sistemàticament el diagrama de fases de l'estat fonamental del model d'Extended Bose-Hubbard unidimensional. Per fer-ho, s'utilitza l'algorisme DMRG mitjançant la biblioteca **ITensors** de Julia. Primer, s'han dut a terme simulacions de referència tenint en compte només interaccions en el mateix pou, que permeten validar la implementació, ja que reproduïen la transició convencional de Superfluid a Aïllant de Mott. D'altra banda, en incorporar interaccions a primers veïns a ocupació unitària i aplicant l'escalat de mida finita a un valor fixat del quocient entre la interacció de contacte i la intensitat de túnel igual a 5, s'aconsegueixen mapejar tres règims aïllants diferents. Així doncs, es confirma que mentre les interaccions a primers veïns febles mantenen la fase de Mott, l'acoblament intermedi estabilitza l'aïllant topològic de Haldane, identificat per un paràmetre d'ordre de corda no local. Finalment, les interaccions fortes acaben conduint el sistema cap a una fase d'ona de densitat cristal·lina mitjançant el trencament espontani de la simetria de translació.

Paraules clau: Model de Bose-Hubbard, xarxes de tensors, DMRG, Aïllant de Haldane, Aïllant de Mott, Interacció a primers veïns.

ODSs: Aquest TFG està relacionat amb els Objectius de Desenvolupament Sostenible (SDGs)

Objectius de Desenvolupament Sostenible (ODSs o SDGs)

1. Fi de la es desigualtats		10. Reducció de les desigualtats	
2. Fam zero		11. Ciutats i comunitats sostenibles	
3. Salut i benestar		12. Consum i producció responsables	
4. Educació de qualitat	X	13. Acció climàtica	
5. Igualtat de gènere		14. Vida submarina	
6. Aigua neta i sanejament		15. Vida terrestre	
7. Energia neta i sostenible	X	16. Pau, justícia i institucions sòlides	
8. Treball digne i creixement econòmic		17. Aliança pels objectius	
9. Indústria, innovació, infraestructures	X		

El contingut d'aquest TFG de la carrera de Física es relaciona principalment amb l'ODS 9 (Indústria, innovació, infraestructures), ja que l'estudi de sistemes de xarxa quàntics mitjançant l'algorisme numèric DMRG i la llibreria **ITensors** de Julia resulta clau per al desenvolupament i la simulació de noves tecnologies quàntiques. Per aquest mateix motiu, el treball es vincula estretament amb l'ODS 7 (Energia neta i sostenible), atès que la comprensió profunda de les fases aïllants i de la matèria condensada en aquests models és la base per dissenyar futurs materials de transport energètic eficient i sense pèrdues. D'una manera més indirecta, el projecte també es relaciona amb l'ODS 4 (Educació de qualitat), pel fet de ser el resultat del procés d'aprenentatge i formació científica d'un estudiant de física a nivell universitari superior.