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A revision of the revaluation index of Spanish pensions

Oriol Roch

Manuela Bosch- Príncep

Isabel Morillo

Daniel Vilalta

Universitat de Barcelona

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Abstract: This article reviews the methodological aspects of the revaluation index of Spanish pensions developed following Law 23/2013 which regulates the sustainability factor and revaluation index of the Social Security pension system. From a gradual breakdown of the elements that make up the revaluation index, an exposition is given of the formal and implementation problems it involves. Finally, its use is illustrated with numerical results.

JEL Codes: H55.

Keywords: revaluation index, automatic balancing mechanisms, Spanish Social Security System.

Oriol Roch
Universitat de Barcelona

Manuela Bosch-Príncep
Universitat de Barcelona

Isabel Morillo
Universitat de Barcelona

Daniel Vilalta
Universidad de Alcalá

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1 Introduction

Over the last fifteen years, many European countries have believed a profound reform of their state pension system to be necessary (Börsch-Supan (2012)). Strong demographic pressure, an acute economic crisis, and internal imbalances have affected the sustainability of state pension systems. Among the measures taken, we would highlight the parametric changes in the system, i.e., those changes that affect one or several variables without affecting their structure. Examples are raising the retirement age or increasing the number of years in employment used to compute the base pension. Also, in some cases the reforms have involved the incorporation of a sustainability factor so as to automatically modify certain aspects of the pension system.

The sustainability factors fall within the category of so-called "automatic balancing mechanisms". The underlying idea is to link one or more parameters of the system with the evolution of an exogenous variable. This may be life expectancy, the variation of the ratio of pensioners to contributors, or GDP growth (see Vidal-Meliá et al. (2009) or Sakamoto (2008)). For example, in Denmark retirement age is linked to life expectancy, whereas in Portugal and Finland it is the base pension which is affected by variations in this same variable. In Germany, however, the revaluation of the pensions is affected by the variation of the ratio between pensioners and contributors (Börsch-Supan et al. (2003)). The case of Sweden is more complex as the automatic balancing mechanism is based on the result of the actuarial balance of the system (Settergren (2001)). In this way, the mechanism restores balance to the system without legislative intervention.

In the Spanish case, with the progressive aging of the population, the problems that the state pension system is facing are no different from those affecting most European countries. According to the demographic projections included in "The 2012 Ageing Report" (European Comission (2012)), it is estimated that the 7.2 million pensioners in 2015 will become 14.7 million in 2050. Also, the long-term demographic projections of the National Statistics Institute 2012-2052 estimate that life expectancy at the age of 65 is increasing by about sixteen months every ten years. Thus the projections indicate that the system will have to provide pensions to more beneficiaries for longer time. On the other hand, birth rates are decreasing. The rate of growth of revenue from contributions is therefore slowing. According to the same European Commission's projections, the ratio of contributors to pensioners will fall from 2.32 in 2015 to 1.28 in 2050. For these reasons, in recent years several studies have highlighted the need to implement reforms in the system (see for example Conde-Ruiz and Alonso (2004), Jimeno et al. (2008), Díaz-Giménez and Díaz-Saavedra (2009), Sánchez-Martín (2010)).

The biggest reform in the Social Security system since the restoration of democracy was with Law 27/2011 of 1 August. This addressed the updating and modernization of the Social Security system. It was passed after the approval of the Recommendation Report of the Parliamentary Commission on the Toledo Pact, and based on the social and economic agreement for growth, employment, and guarantee of pensions signed by the Government and employer (CEOE, CEPYME) and worker (CCOO, UGT) organizations on 2 February 2011. The reform gradually establishes (from 2013, with transition periods to 2027) significant parametric changes. These include raising the retirement age from 65 to 67 (except in long-period contribution retirements which remain at 65), increasing by two years the number of qualifying years to be eligible for a full pension (37 from the previous 35), and increasing from 15 to 25 the number of years computed for the calculation of the base pension. As a novelty, it establishes the implementation of a sustainability factor. According to Article 8, in order to maintain the proportionality between the contributions to the system and the expected benefits of the system, as well as to guarantee its sustainability, it is agreed that from 2027 the fundamental parameters of the system will be reviewed. This review will respond to the differences in the evolution of life expectancy at the age of 67 in the year the review is carried out and the life expectancy at the age of 67 in 2027. This reform is consistent with the recommendations of the "White Paper: An Agenda for Adequate, Safe, and Sustainable Pensions" of 2012, and with other reports framed within the 2020 European Strategy for the coordination of the economic policies of the members of the European Union. According to the 2011-2014 Stability Programme, these legislative modifications would mean a saving suppose for the system equivalent to 1.4% of GDP in 2030, 2.8% in 2040, and 3.5% in 2050.

Although Law 27/2011 established the implementation of the sustainability factor in 2027, the severe economic tensions that followed added pressure for an earlier implantation. The major pension surplus became a deficit. It became necessary to use two reserve funds, the Social Security Reserve Fund and the Prevention and Rehabilitation Fund of the Mutual Insurance Societies of Work Accidents and Occupational Diseases, to cover pension expenditure. The fall in employment, and therefore in contributions, showed the need for additional measures to ensure the financial stability and soundness of the system. Thus, the law Ley Orgánica 2/2012 of 27 April of budgetary stability and financial sustainability establishes in its Article 18 that, in the case of anticipating a long term deficit in the state pension system, the Government will review the system, implementing automatically the sustainability factor in the terms and conditions provided in the Law 27/2011. This is intended to balance future expenditures with the contributions following the deterioration of the proportion between contributors and pensioners. As noted by Ferreras (2013), these modifications were consistent with the position expressed by the European Council when, in the 10 July 2012 session, it recommended "to ensure that retirement age is rising in line with the life expectancy when regulating the sustainability factor foreseen in the recent pension reform".

The law Real Decreto-Ley 5/2013 of 15 March, dealing with measures to favour the continuity of the working life of older workers and to promote active aging, in its ninth additional disposition, calls upon the Government to create an independent committee of experts, with the purpose of preparing a report on the sustainability factor to be sent to the Toledo Pact Commission. After the presentation by the Government of a preliminary report on the sustainability factor in the Social Security system, the Council of Ministers of 12 April 2013 agreed to the creation of the committee of experts, whose report was submitted on 7 June 2013.

The Committee of Experts Report, which was composed of twelve academics of recognized prestige, proposes a sustainability factor of two components. The first is an Intergenerational Equity Factor (IEF) designed to protect the system from an increase in life expectancy. According to this proposal, the initial retirement pension is adjusted so that the amount received throughout the life of a pensioner who has just entered the system is equivalent to that of another pensioner with identical characteristics who entered the system earlier (and therefore, possibly with a lower life expectancy). Thus, the IEF would be applied only to new pensioners to affect the determination of the initial amount of their pension. As a correction would be applied to new pensioners if their life expectancy increases, this would produce as a result an increase in the intergenerational equity of the system. The idea behind the IEF is to compensate the longer life expectancy of the pensioners by lowering their initial pension. This component was not completely novel in Europe because it was in line with Portugal's Social Security System reforms of 2010.

The second factor is the Annual Revaluation Factor (ARF). This factor affects the revaluation of all pensions. It is calculated so that it ensures the budgetary equilibrium of the system. It would establish a revaluation to be applied to the system's existing pensioners so that revenues and expenditures are in balance. Therefore, in accordance with the Committee of Experts' proposal, the revaluation of the pension is detached from the Consumer Price Index (CPI), contrary to how it had been calculated before, and instead is based on the ARF. It is important to note that the proposed ARF, unlike the IEF, lies outside the remit of the Law 27/2011 since it establishes a mechanism that is not directly dependent on the evolution of life expectancy, but is linked to an economic factor – the economic balance of the system.

Based on the Committee of Experts Report, on 13 November 2013 the Government presented the draft of the law regulating the sustainability factor and the revaluation index of the Social Security pension system. In this draft, the sustainability factor is established from the concepts and ideas set out in the IEF. The ARF is also adopted by way of the new figure of the revaluation index.

In contrast to the consensus reached in Law 27/2011, in this case there were discrepancies in the opinions of the social agents that it affected. On 12 September 2013, the UGT trade union presented its analysis and assessment of the Government's proposal for the creation of the sustainability factor for the pension system and the revaluation index. The report criticizes, among other things, the decrease in the initial pension in real terms, and the non-discrimination between different life expectancies according to social condition, work activity, or health of the future pensioners, as well as the subjectivity in applying the calculation formula with the incorporation of estimates of the variables. On 24 September 2013, the CCOO trade union presented its assessment of the draft. This criticized the proposed approach, based on the containment of spending, and put forward a battery of alternative measures with which to act on the pension system. Finally, on 26 September, the Spanish Economic and Social Council published its opinion of the draft. In this, they raised doubts about a reform proposal made without the consensus of the social agents involved after only a year having elapsed from the passage of Law 27/2011. It considered it appropriate to study alternative ways of financing, and raised the possibility of including in the automatic balancing mechanisms other variables that impact revenues such as those related to economic activities, the evolution of employment, and work participation. Finally, they advise the Government to reconsider the application of the new revaluation index since it fails to ensure the maintenance of the purchasing power of the pensions.

Finally, on 26 December 2013 the Government passed Law 23/2013 regulating the Sustainability Factor and the Revaluation Index of the Social Security Pension System, thereby modifying Articles 48 and 163 of the revised General Law on Social Security providing legislative recognition of the revaluation index and the sustainability factor, respectively. Also, it establishes upper and lower bounds for the revaluation of the pensions to ensure their nominal increase.

The present paper has the following structure. Section 2 describes the previous revaluation model and the current pension reform. Section 3 analyses the mathematical aspects and formulas of the model, highlighting the drawbacks with their formalization and implementation. In Sections 4 and 5, we shall discuss some extensions of the basic model. Section 6 gives examples of its use with real data from the Spanish Social Security System. Finally, Section 7 presents our conclusions.

2 The revaluation of Spanish pensions

As was mentioned in the previous section, one of the objectives of the reform provided in the Law 23/2013 is to rethink the revaluation model of the state pensions by means of a new modification of Article 48 of the revised General Law on Social Security. In particular this would modify the pension revaluation scheme in the Social Security system.

The legislative antecedents to the new reform established a direct link between the CPI and the revaluation of the pensions. Law 24/1972 of 21 June, regarding financing and perfecting the protective actions of the General Regime of the Social Security, foresaw in its Article 5 a periodic revaluation of the pensions by the government. This was proposed by the Labour Minister, taking into consideration among other indicative factors the mean rise in wage levels, the cost of living index, and the general evolution of the economy, as well as the economic possibilities of the Social Security system. This procedure was also reflected in the revised General Law on Social Security of 20 June 1976 in its Article 92. Subsequently, Law 26/1985 of 31 July, on urgent measures for the rationalization of the structure and of the protective action of the Social Security, established in its Article 4 that pensions based on the application of the modifications made to this law would be revalued at the beginning of each year in accordance with the projected CPI for that year. These considerations were reflected in the law Real Decreto-Ley 1/1994 of 20 June approving the revised General Law on Social Security, which includes in its Article 48 the revaluation of the pensions because of retirement and disability in agreement with the projected CPI for that year, while establishing the periodic revaluation by the Government of the remaining pensions recognized by the Social Security system taking into account, among other indicative factors, the mean rise in wage levels, the CPI, and the general evolution of the economy, as well as the economic possibilities of the Social Security system. The deviations that may occur between the inflation that was expected for the year and that which actually occurred are taken into consideration the following year to improve all contributive pensions below the minimum inter-professional wage.

The pre-reform pension revaluation model, effective beginning with the passage of Law 24/1997 of 15 July on the consolidation and rationalization of the Social Security system, established an automatic revaluation of the pensions in accordance to the annual variation of prices, thus ensuring the purchasing power of the pensioners. The mechanism which determined the revaluation of the contributory pensions (including the amount of the minimum pension) was as follows. First, at the beginning of each year, the pensions were revalued in accordance with the projected CPI for that year. Second, if the CPI accumulated between the previous November and the November of the financial year corresponding to the revaluation was greater than projected, then the revaluation was adjusted by that difference. This adjustment was paid in a single payment before 1 April of the subsequent year. In the case that the CPI between the two Novembers was less than expected, the difference was to be absorbed in the revaluation of the following financial year.

The automatic revaluation mechanism introduced in the Law 24/1997 was not, however, always applied. This was due to the need to comply with certain fiscal objectives. For example, the revaluation of the pensions corresponding to 2011 was suspended by the Law 39/2010 of 22 December of the General State Budget. Although it was not applied to the Social Security's minimum pensions, non-contributory pensions, or the (now extinct) non-recurring Compulsory Elderly and Incapacity Insurance pensions, the extraordinary suspension of the pensions marked a break in the guarantee to maintain their real value. The suspension was necessary to achieve the reduction of the State deficit proposed in the law Real Decreto-Ley 8/2010 of 22 May, which required measures to be taken to reduce State expenditure. The estimated saving from freezing the pensions for that year is estimated to have been 1400 million euros according to the Stability Programme 2011-2014.

Subsequently, for 2012, due to the large deficit in the Social Security system and the liquidity tensions at the end of the year corresponding to the ordinary and extraordinary monthly payments, the revaluation of the pensions was again not in agreement with the actual CPI. The law Real Decreto-Ley 28/2012 of 30 November on measures for the consolidation and guarantee of the Social Security system prepared the rescinding of the updating of the pensions in 2012 in accordance with the actual CPI datum. Instead it kept the revaluation of 1% foreseen in the law Real Decreto-Ley 20/2011 of 30 December on urgent measures concerning budgetary, taxation, and financial matters to correct the State deficit. Also, a revaluation of 1% was fixed for 2013, with the exception of pensions of less than 14 000 euros per year, which were to be revalued by 2%.

To avoid distortions in the order of the pensions that could arise from the application of the percentages established by the law Real Decreto-Ley 28/2012, the law Real Decreto-Ley 29/2012 of 28 December dealing with improvement of social protection and management especially for domestic employees, and other socio-economic measures, refines the previous law. It establishes that pensions between 1000.01 euros per month or 14 000.01 euros per year and 1009.90 euros per month or 14 138.60 euros per year will be increased by the amount necessary for the resulting pension to reach the amount of 1020.00 euros per month or 14 280.00 euros per year.

The new Law 23/2013 substantially modifies the mechanism for revaluing the pensions. The Social Security pensions, in their contributory modality (including the amount of the minimum pension), will be increased at the beginning of each year in accordance with the so-called index of revaluation. This index will be published each year in the corresponding General State Budget Law. This new revaluation mechanism is completely different from the previous ones as it unlinks the revaluation of the pensions from the Consumer Price Index.

The computation of the index of revaluation (IR) for the pensions of the year $t+1$ is articulated by a mathematical expression which includes various components that affect the financial balance of the system. The expression for the revaluation is:

$$IR_{t+1} = \bar{g}_{I,t+1} - \bar{g}_{p,t+1} - \bar{g}_{s,t+1} + \alpha \left[\frac{I^*_{t+1} - G^*_{t+1}}{G^*_{t+1}} \right]. \tag{1}$$

The first term involved in the calculation is $\bar{g}_{I,t+1}$, the rate of variation in per unit terms of the Social Security's revenues. The bar over the variable reflects that for its calculation the arithmetic mean will be used of eleven years centred on the year $t+1$. For the calculation, the data from year $t-5$ to year $t+6$ are used, as it is necessary to have twelve observations to obtain eleven variation rates. The second term ($\bar{g}_{p,t+1}$) is the variation in per unit terms of the number of contributory pensions to the system. The third term is $\bar{g}_{s,t+1}$. This measures the substitution effect expressed in per unit terms. The substitution effect is defined as the interannual variation of the mean pension in the system in one year in the absence of the revaluation of that year. Analogously to the rate of variation of the revenue, for the calculation of $\bar{g}_{p,t+1}$ and $\bar{g}_{s,t+1}$ the mean will be used of eleven variation rates centred on $t+1$. Other elements to be considered are I^*_{t+1} and G^*_{t+1}, the geometric means centred on $t+1$ of the revenue and expenditures of the system, respectively. According to the legislation, revenue will be taken to be the total of the system's aggregate expenditure and revenue corresponding to non-financial operations (chapters 1-7 in expenditure and 1-7 in revenue of the Social Security Budget) without regard for those corresponding to the National Institute of Health Management and the Institute for Older Persons and Social Services[1]. However, the part corresponding to revenue does not include the social contributions due to the cessation of activity of self-employed workers or the State's transfers for the financing of non-contributory benefits, except for the financing of the complements to the minimum pensions. The part corresponding to expenditure does not include the benefits paid for the cessation of activity of self-employed workers or non-contributory benefits, except for the complements to the minimum pensions. Also, regarding the liquidated accounts, the Social Security General Public Accounts Department will deduct from the above chapters those items that have no periodic character. Note that for the computation of the revenues and expenditures of the system, since the absolute amounts are used rather than rates of variation, it is only necessary to use eleven observations, not twelve.

Finally, the expression incorporates a parameter α which must take a value between 0.25 and 0.33, and will be revised every five years. The function of this parameter is to regulate the speed at which the system's budgetary imbalance is corrected. If α were to take the extreme value of 0, the imbalance will never be corrected, whereas if it were to take the extreme value of 1, the budgetary imbalance will be corrected in one year, the year of application.

In order to ensure the annual increase of the pensions, the new Law 23/2013 establishes bounds to their revaluation. The result obtained when applying Equation (1) can not result in an annual increase

[1]This definition of revenue and expenditure differs from the Draft's proposal, which only included the items of current operations (chapters 1 to 4 in expenditure and 1 to 5 in revenue). Thus, items would be excluded that relate to outlays for real property investments and capital transfers, and to revenues from the disposal of real property investments and capital transfers.

of neither less than 0.25% nor greater than the percentage variation of the CPI in the one-year period previous to the December of the year t plus 0.5%.

3 Basic considerations of the model

Having reviewed the recent legislative antecedents of the revaluation of the pensions of the Spanish Social Security system, in this section we shall analyse the economic model used for the derivation of the revaluation index. Despite the number of comments and reports that have emerged following the adoption of the IR, such as those mentioned in Section 1, only a few studies have carried out an in-depth analytical study of the model (see De las Heras et al. (2014) and AIReF (2014)).

The interest in and need for such an in-depth analysis of the technical aspects of the revaluation model arise from discussions with academics and consultants about its applicability. These meetings brought out evidence for the lack of transparency of certain aspects of the model, leading to mistakes in interpretation and to a disparity of opinions on its validity. We believe that the cause of these differing opinions lies in the omission of an important part of the model's derivation and an absence of any analysis in depth of its implications. The present study has the aim of filling in these gaps and of rigorously analysing the implications of the approach chosen to obtain the IR. We believe that a detailed study of the model will facilitate its better comprehension for subsequent further discussion.

From a strict reading of the Law 23/2013, it is difficult to acquire a correct understanding of the method used to revalue the pensions, since it limits itself to setting out Equation (1). The cause is twofold. On the one hand, the components of the expression are not defined mathematically, which can lead to mistakes in its application. And on the other, neither is it shown what the balance of the system is that is needed to derive the revaluation index. In the following sections, we shall break down the economic model behind the IR expression step by step to assess its applicability and limitations.

3.1 Formalization of the variables

To clarify the deduction of Equation (1), one must take as starting point the Committee of Experts Report on the sustainability factor of the state pension system. It reflects, among other aspects, the pensions' Annual Revaluation Factor (ARF) which was the origin of the IR that was finally set out in the Law 23/2013. Underlying the ARF is the idea of distributing the system's potential deficit or surplus among the existing pensioners, i.e., among those who already formed part of the system the year before. Thus the pensioners who have just joined the system are excluded from the benefits or charges that are to be distributed.

In Annex A3.1 the Committee of Experts Report, the derivation of ARF starts with the definition of the pension expenditure in the year $t+1$:

$$G_{t+1} = G_t(1+g_{t+1})(1+g_{p,t+1})(1+g_{s,t+1}), \tag{2}$$

where the factor $(1+g_{t+1})$ reflects the variation of the amount of the existing or surviving pensions (i.e., those that were already in the system the previous year t), the factor $(1+g_{p,t+1})$ reflects the variation of the number of pensions, and $(1+g_{s,t+1})$ (the substitution factor) reflects the variation of the mean pension produced by the different values of the amounts of the pensions that leave the system and of those which are newly incorporated in the year $t+1$. This expression decomposes the expenditure into three multiplicative factors. However, the report does not detail the process by which this expression is reached, and, similarly to the Law 23/2013, neither does it define any of the factors mathematically.

This non-definition is a potential cause of misunderstandings and mistakes in its implementation. The reasons are the following.

The first multiplicative factor in Equation (2) is $(1 + g_{t+1})$. This factor reflects the nominal revaluation that can be applied in the year $t + 1$ to all the pensions already in the system in the year t. As this variable we shall consider to be unknown, no further formalization is needed. The formalization of the second factor $(1 + g_{p,t+1})$, relative to the rise in the number of pensions, does not seem to cause any doubt either. If N_t represents the number of pensions in the year t, the factor $(1 + g_{p,t+1})$ must be equal to the ratio N_{t+1}/N_t. Thus one has

$$(1 + g_{p,t+1}) = \frac{N_{t+1}}{N_t}. \tag{3}$$

The formalization of the substitution factor $(1 + g_{s,t+1})$ is more problematic based on its definition in the report. It is worth noting that in the wording of the Law 23/2013 there is a redefinition of the substitution effect, with it being defined as "the interannual variation of the system's mean pension in the absence of revaluation for that year". This definition differs from that used in the Committee of Experts Report. In any case, if one denotes the mean pension in the year t by $\bar{P}_t$, an equality coherent with both definitions is:

$$\bar{P}_{t+1}/\bar{P}_t = (1 + g_{t+1})(1 + g_{s,t+1}), \tag{4}$$

from which one deduces

$$(1 + g_{s,t+1}) = \frac{\bar{P}_{t+1}/(1 + g_{t+1})}{\bar{P}_t}. \tag{5}$$

Thus the substitution effect would be obtained by discounting from the mean pension in the year $t+1$ the revaluation of the pensions, and then dividing it by the mean pension of the year before. At this point it is important to note that the mean pension will be computed from all the pensions (also the existing ones), and not only those to which the revaluation is being applied.

It is easy to verify that the definition of the factors $(1 + g_{p,t+1})$ and $(1 + g_{s,t+1})$ in the previously described form is consistent with the decomposition of the expenditure described in Equation (2). Since the expenditure of any year can be expressed as the product of the number of pensions by the mean pension of that same year, one has $G_t = N_t\bar{P}_t$, and therefore it will also have to be satisfied that

$$\frac{G_{t+1}}{G_t} = \frac{N_{t+1}\bar{P}_{t+1}}{N_t\bar{P}_t}. \tag{6}$$

If one substitutes the expressions (3) and (4) into Equation (6), one recovers the expenditure formula in the year $t + 1$ reflected in (2).

An alternative way to reach the same decomposition of the expenditure of the year $t + 1$ is as follows. One knows that it is correct to multiply and divide simultaneously a variable by a non-negative variable without modifying the result. Therefore, one can consider the decomposition

$$G_{t+1} = G_{t+1}\frac{N_{t+1}}{N_{t+1}}\frac{N_t}{N_t}\frac{\bar{P}_{t+1}}{\bar{P}_{t+1}}\frac{\bar{P}_t}{\bar{P}_t}\frac{(1 + g_{t+1})}{(1 + g_{t+1})},$$

that will be correct as long as N_{t+1}, N_t, $\bar{P}_{t+1}$, $\bar{P}_t$, and $(1+g_{t+1})$ are non-negative variables. Due to the nature of the problem under study, it is reasonable to consider these variables to be strictly positive: it would not be logical to consider the sustainability of the pension system if, for example, the number of pensions or mean pension were null. It would be even less logical if the variables had negative values. Thus, considering that $G_{t+1} = \bar{P}_{t+1}N_{t+1}$, $G_t = \bar{P}_tN_t$, and Equations (3) and (4), simplifying one obtains the expression for the expenditure of the year $t + 1$ in Equation (2).

3.2 The basic model

The philosophy that underlies the basic model is to ensure a balanced budget. Therefore the expenditure of the year $t+1$ must be equal to revenue in the same year $t+1$. If one expresses the revenues of the year $t+1$ as $I_{t+1} = I_t(1+g_{I,t+1})$, where the factor $(1+g_{I,t+1})$ denotes the variation of the revenue of the year $t+1$ with respect to the year t, following the steps of Annex 3 in the Committee of Experts Report, one has

$$G_t(1+g_{t+1})(1+g_{p,t+1})(1+g_{s,t+1}) = I_t(1+g_{I,t+1}), \tag{7}$$

so that

$$(1+g_{t+1}) = \frac{(1+g_{I,t+1})}{(1+g_{p,t+1})(1+g_{s,t+1})}\frac{I_t}{G_t}. \tag{8}$$

Taking logarithms on both sides of the equation, and taking into account that $g \approx \ln(1+g)$ and that $\ln(I/G)) \approx (I-G)/G$, one finally obtains

$$g_{t+1} = g_{I,t+1} - g_{p,t+1} - g_{s,t+1} + \left(\frac{I_t - G_t}{G_t}\right), \tag{9}$$

where $g_{I,t+1} = (I_{t+1} - I_t)/I_t$, $g_{p,t+1} = (N_{t+1} - N_t)/N_t$, and the substitution effect is defined as

$$g_{s,t+1} = (\bar{P}_{t+1} - \bar{P}_t)/\bar{P}_t - g_{t+1}. \tag{10}$$

The goal of approximating Equation (8) is to obtain a simplified version of the result. For the exactness of calculation, however, the result of Equation (8) will always be preferable to its approximation.

One way to interpret Equation (7) is to consider the expenditure of the year $t+1$ as a three variable function: the number of current pensions in $t+1$, the mean pension in $t+1$, and the revaluation of the existing pensions in $t+1$. That is, if the goal is to modify the expenditure of the year $t+1$, there are three paths to take. One is to modify the number of pensions in $t+1$. This option means tightening the conditions to obtain a pension (e.g., delay the retirement age). A second option consists of modifying the mean pension of the year $t+1$. This option means, for example, altering the computation of the base pension or the coefficients of reduction of the retirement pensions. Finally, there is the option of varying the pensions' revaluation. This is precisely the path the model considers to balance revenues with expenditures.

Instead of considering as endogenous variables the number of pensions of the year $t+1$ and the mean pension of $t+1$, one can consider in their place the number of new pensions claimed in $t+1$ and the expenditure that these new pensions mean. This simplification would be made by adding two reasonable hypotheses. On the one hand, one assumes that no legislative reform can affect the drop-outs from the system. I.e., a tightening of the law cannot result in an increase in the number of drop-outs from the system, by excluding the pension benefits which met the former but not the new requirements. On the other hand, one also assumes that no legislative reform will affect the amounts of former pensions other than by way of annual pension revaluations.

Just as the information for the year $t+1$ is included by way of endogenous variables, the information for the year t is transmitted through exogenous variables. In particular, the information for the year t is reflected by the variables $\bar{P}_t$ and N_t. This information is considered to be exogenous since it is not possible to modify it in order to alter the expenditure in year $t+1$.

From the above interpretation, equation $G_{t+1} = I_{t+1}$ can be characterized as a contour I_{t+1} of the function of expenditure in $t+1$. Thus, the three exogenous expenditure function variables can be modified while the constraint is met. The proposal of both the Committee of Experts Report and the Law 23/2013 is to allow the variables N_{t+1} and $\bar{P}_{t+1}$ to evolve so that the remaining variable, the pension revaluation, is determined dependently to meet the budgetary target set in the contour curve.

Once budgetary balance has been expressed through Equation (7), one simply has to recast that expression to give the revaluation factor $(1 + g_{t+1})$. Therefore, from the formula (9), it may appear that the revaluation to apply to the existing pensions to balance the accounts is explicitly decomposed into various elements. Quantifying each of these elements separately and adding (or subtracting) them, one can calculate the revaluation to apply to the existing pensions corresponding to the year $t + 1$.

However, the interpretation of the revaluation in this sense would not be correct. The definition of the substitution effect (10) means that it is a function of the revaluation. Hence the revaluation can neither be decomposed nor explicitly expressed in the elements of the expression (9). I.e., the calculation of the revaluation requires the datum of the substitution effect and, in turn, the calculation of the substitution effect requires the revaluation to have been calculated.

One solution to this problem is to calculate the revaluation and the substitution effect simultaneously. Considering the implicit relationship

$$g_{t+1} - g_{I,t+1} + g_{p,t+1} + g_{s,t+1} - \left(\frac{I_t - G_t}{G_t}\right) = 0 \tag{11}$$

one can evaluate the revaluation g_{t+1} numerically. Below, we shall show that this procedure is unnecessary since this basic model can be solved analytically. An analytical solution is more satisfactory, and has major advantages over the numerical solution, both computationally and interpretatively.

We have detected the mistake of interpreting the revaluation as an explicit decomposition of various elements among our professional colleagues. As noted above, it originates from the lack of formalization of the elements involved in the expression (9), especially the substitution effect. From the above results, it follows that the only correct way to calculate the substitution effect is by Equation (10). The use of any other expression will lead to inaccuracies. This misinterpretation is not, however, easy to detect. This is because, although the substitution effect may have been incorrectly calculated (without taking into account that it depends on the revaluation), budgetary equilibrium will continue to be met, but always at the cost of restricting either the new pension expenditure in year $t + 1$ or the number of pensions in year $t + 1$, or both together, two variables on which it is not intended to act.

The explanation is the following. Having estimated independently the substitution factor for the year $t + 1$, with result x, one would be adding a constraint to the model. Specifically, the constraint added would be

$$(\bar{P}_{t+1} - \bar{P}_t)/\bar{P}_t - g_{t+1} = x. \tag{12}$$

To analyse the implications of the inclusion of this constraint, we first redefine components involved in it in terms of the number of new pensions and the expenditure on them. This redefinition will also be very useful in the following sections.

The mean pension in year $t+1$ is the ratio of the expenditures produced by all pensions in the year $t+1$ to the number of pensions in the same year. In addition, the mean pension in year $t+1$ incorporates both the existing or surviving pensions and the new pensions. Let m be the number of surviving pensions in the year $t + 1$, i.e., the number of pensions already in the system in the year t and remaining in the year $t + 1$. Thus, the number of new pensions, i.e., the number of people entering the pension system in the year $t + 1$ is $N_{t+1} - m$. Let us denote by P^i_{t+1} the amount of the i-th pension in year $t + 1$. Then one has

$$\bar{P}_{t+1} = \frac{\sum_{i=1}^{N_{t+1}} P^i_{t+1}}{N_{t+1}} = \frac{\sum_{i=1}^{m} P^i_{t+1} + \sum_{i=m+1}^{N_{t+1}} P^i_{t+1}}{N_{t+1}}.$$

The amount of the existing pensions in the year $t+1$, i.e., of those already in the system in the year t and surviving in $t+1$, will be the same as in the previous year multiplied by the year $t+1$'s revaluation factor, so that the above expression can be rewritten as

$$\bar{P}_{t+1} = \frac{\sum_{i=1}^{m} P_t^i(1+g_{t+1}) + \sum_{i=m+1}^{N_{t+1}} P_{t+1}^i}{N_{t+1}}.$$

In order to simplify notation, one can express the expenditure produced in the year t by the pensions surviving from year t to year $t+1$ by G_t^s, and G_{t+1}^a for the expenditure produced in the year $t+1$ by the new pensions. Thus, the above expression will become

$$\bar{P}_{t+1} = \frac{G_t^s(1+g_{t+1}) + G_{t+1}^a}{N_{t+1}}.$$

In turn, the number of pensions in $t+1$ can also be broken down into the number of pensions that have survived from the year t to the year $t+1$ plus the number of pensions that have joined the system in year $t+1$. I.e., $N_{t+1} = N_{t+1}^s + N_{t+1}^a$. Taking these decompositions into account, the constraint (12) can be rewritten as

$$\frac{G_t^s(1+g_{t+1}) + G_{t+1}^a}{\bar{P}_t(N_{t+1}^s + N_{t+1}^a)} - (1+g_{t+1}) = x, \tag{13}$$

which makes explicit the relationship between the three variables N_{t+1}^a, G_{t+1}^a, and g_{t+1}.

Thus, calculating the revaluation by means of the expression

$$g_{t+1} = g_{I,t+1} - g_{p,t+1} - x + \frac{I_t - G_t}{G_t},$$

one obtains a revaluation of pensions that would meet the budgetary balance condition. Implicitly, however, it would be restricting the new pension expenditure in the year $t+1$, or the number of pensions in the year $t+1$, or both simultaneously.

For example, let us suppose that, with the number of pensions for the year $t+1$ estimated, we proceed to calculate the variation in the number of pension properly but the substitution factor is estimated incorrectly (i.e., differently from the result obtained by using Equation (10)) as x. Then, solving the constraint (13) for G_{t+1}^a, one has that the expenditure produced by the new pensions would be restricted to

$$G_{t+1}^a = (x+1+g_{t+1})\bar{P}_t(N_{t+1}^s + N_{t+1}^a) - G_t^s(1+g_{t+1}).$$

On the contrary, the budgetary balance criterion would not be satisfied.

In summary, the above example illustrates the mistake committed by incorrectly interpreting the formula (9) since there is only one consistent way to proceed to calculate the substitution effect, which is to use Equation (10). Furthermore, the implicit basic model defined in the terms of Law 23/2013 requires the use of numerical methods to calculate the revaluation. In the next subsection, we shall show how it is possible to more successfully perform the same calculation algebraically.

3.3 Alternative derivation of the basic model

Starting from the aforementioned problems, in this subsection we shall derive the revaluation factor in an alternative form in the basic model to avoid those problems of formalization. The result of this new method of calculation is equivalent to that obtained by the formula (8). Its application is simpler, however, since its solution does not require the use of numerical methods.

Recalling that expenditure in year $t+1$ may be decomposed into expenditure produced by the surviving pensions and the expenditure produced by the new pensions, one has that

$$G_{t+1} = G_t^s(1+g_{t+1}) + G_{t+1}^a. \tag{14}$$

Now, in accordance with the philosophy of budgetary balance, expenditure in the year $t+1$ will have to be equal to revenue in $t+1$, i.e.,

$$G_t^s(1+g_{t+1}) + G_{t+1}^a = I_{t+1}.$$

The revaluation factor will thus be

$$(1+g_{t+1}) = \frac{I_{t+1} - G_{t+1}^a}{G_t^s}. \tag{15}$$

This proposal gives a more direct interpretation of formula (8). If the goal is to distribute the possible surplus or deficit that the system might incur in the year $t+1$ among the existing pensions, then it is as simple as just subtracting from the expected revenue the expenditure associated with the new pensions since this adjustment will not affect them, and then dividing the result among the expenditure produced by the surviving pensions.

To the extent that the expenditure caused by the new pensions is small in proportion to the expenditure produced by the surviving pensions, the revaluation factor may easily be approximated by

$$(1+g_{t+1}) \approx \frac{I_{t+1}}{G_{t+1}}.$$

This result is not inconsistent with the model proposed in the Committee of Experts Report. The relationship between the two is as follows. From Equation (8), replacing the substitution factor by expression (5), one obtains

$$(1+g_{t+1}) = \frac{(1+g_{I,t+1})}{(1+g_{p,t+1})\frac{\bar{P}_{t+1}/(1+g_{t+1})}{\bar{P}_t}} \frac{I_t}{G_t}.$$

Substituting $\bar{P}_{t+1} = \left(G_t^s(1+g_{t+1}) + G_{t+1}^a\right)/N_{t+1}$ and $\bar{P}_t = G_t/N_t$ in the above expression and simplifying, one again obtains the expression for the revaluation factor (15).

Thus one has an alternative form of calculating in the basic model the revaluation factor of Spanish pensions. Instead of having to calculate numerically as is necessary with Equation (8), the alternative Equation (15) can provide an equivalent result analytically, and therefore accurately and with less computational cost.

3.4 Extensions to the basic model

The formula for the pension revaluation contained in Law 23/2013 incorporates modifications to the basic model described in this section. None of these extensions were considered as initial assumptions of the model, but are instead added to the final expression. That is why each of these extensions requires a detailed analysis.

Firstly, the Law 23/2013 sees as expenditures of the system the total expenditures for non-financial transactions. This definition is inconsistent with the definition of expenditure in (2) since this latter only includes expenditure directly deriving from the amount of the pensions, as one deduces from the definition of the substitution factor. To be consistent with the definition of Law 23/2013, Equation (2) would have to refer to mean expenditure in non-financial transactions per pension instead of mean pension. This discrepancy between mean pension and mean expenditure is crucial for the results. Another alternative to correct this inconsistency would be to add to the definition of expenditures those costs not directly deriving from the payment of the amount of the pensions. This alternative would imply imputing to costs certain expenditures that function as fixed costs of the system. This would have a leverage effect on the revaluation. In addition, the Law considers budgeted expenditures (and revenues). These may differ from those finally settled, and thus not give a correct picture of the actual situation of the system's accounts.

Secondly, the model incorporates a parameter α which multiplies the last term of Equation (9). It controls the speed at which the system's imbalances are corrected. Throughout this section, and for the sole purpose of simplicity, we have considered the particular case in which $\alpha = 1$, so that any deficit or surplus in the system is compensated in one year. In Section 4, we will relax this assumption to carefully analyse the implications of the introduction of this parameter.

Thirdly, in the formula to apply in Law 23/2013, moving averages are considered for revenue and expenditures. This last change has important implications from the point of view of application of the model. It modifies considerably the technical aspects. Its analysis will be studied in Section 5.

And fourthly, there are set a lower bound of 0.25% and an upper bound of $CPI + 0.5\%$. One notes that this would lead to a contradiction for CPI values strictly less than -0.25%.

4 The parameter α

In a refinement of the model, the Committee of Experts Report introduces a coefficient α in the expression for calculating the revaluation factor:

$$(1 + g_{t+1}) = \frac{(1 + g_{I,t+1})}{(1 + g_{p,t+1})(1 + g_{s,t+1})} \left(\frac{I_t}{G_t} \right)^{\alpha}.$$

The values of this parameter are in the range $[0, 1]$. The objective with it is to introduce the possibility of regulating the speed of adjustment of the imbalance between revenues and expenditures. A value of α close to 0 means that such adjustment will be slow and gradual. Instead, values close to 1 will mean that a deficit will be significantly reduced in a small number of periods. In the extreme case with the coefficient taking the value 1, the deficit would be canceled in the first period. Law 23/2013 fixes the allowed range of variation of α to be between 0.25 and 0.33, and its value will be revised every five years. The Law also sets the value at 0.25 for the first five years of implementation.

Because of the definition of substitution factor, the revaluation factor in the expression above must be calculated numerically. Nevertheless, this problem can be dealt with by extending the simplified model presented in the previous section to allow modulation of the speed at which the system's deficit is absorbed in a form that yields results algebraically. To take this possibility into account, the expenditure in the year t has to be introduced, so that, from the alternative expression for the revaluation factor (15) one obtains

$$(1 + g_{t+1}) = \frac{I_{t+1} - G^a_{t+1}}{G^s_t} \frac{G_t}{G_t}. \tag{16}$$

Grouping terms and given that $I_{t+1} = I_t(1 + g_{I,t+1})$, the addition of the parameter α means that

$$(1 + g_{t+1}) = G_t \frac{(1 + g_{I,t+1})}{G^s_t} \left(\frac{I_t}{G_t} \right)^{\alpha} - \frac{G^a_{t+1}}{G^s_t}. \tag{17}$$

In this way, one can obtain the same result as in the extended model of the Expert Report, but analytically.

The implications of introducing α as a parameter in the model are little detailed in the Committee of Experts Report. The purpose of introducing this parameter is to permit the delay in the possible cuts in the amount of the pensions, cuts which are associated with a negative value of the revaluation (or a moderation of its positive value) during the first years after the reform.

It is important to note that the introduction of the parameter α directly into Equation (8) has the effect of modifying the model's equilibrium condition. This will no longer be equality between revenue and expenditures, but be given by the equality

$$\left(\frac{I_t}{G_t}\right)^{1-\alpha} = \frac{I_{t+1}}{G_{t+1}}. \tag{18}$$

From the definition of expenditures in the year $t+1$ as $G_{t+1} = G_t^s(1+g_{t+1}) + G_{t+1}^a$ and the above equilibrium condition, it is easy to see that one obtains the expression (17) to calculate the revaluation factor.

Therefore, an analysis of the revaluation factor shows that the coefficient α does not adjust the annual imbalances, but instead is their cause. Also, one must keep in mind that any possible debt that might exist at the time of implantation of the revaluation factor will never be absorbed by future revaluations. The same would be the case with any possible deficits that might be generated which would not be reabsorbed, and would accumulate in the form of a debt that can not be liquidated.

To demonstrate the above statements, let us compute the financial result generated by the model over the course of years. Let H_t be the deficit (or surplus) generated by the system in the year t. From Equation (18), it is easy to see that if in the initial period $t = 0$ the system is in equilibrium it remains so for any future period t. If not, the deficit (or surplus) will be $H_t = I_t - G_t$, or, which is the same,

$$H_t = I_{t-1}(1+g_{I,t}) - \left(G_{t-1}^s(1+g_t) + G_t^a\right).$$

Substituting the revaluation factor (17) in the above expression and simplifying, one has

$$H_t = I_{t-1}(1+g_{I,t}) - \left(G_{t-1}(1+g_{I,t})\left(\frac{I_{t-1}}{G_{t-1}}\right)^{\alpha}\right).$$

The above equality holds if $I_t = I_{t-1}(1+g_{I,t})$ and $G_t = \left(G_{t-1}(1+g_{I,t})\left(\frac{I_{t-1}}{G_{t-1}}\right)^{\alpha}\right)$, simultaneously, so that it must also be satisfied that

$$\frac{I_t}{G_t} = \frac{I_{t-1}}{G_{t-1}\left(\frac{I_{t-1}}{G_{t-1}}\right)^{\alpha}}.$$

Simplifying, one obtains the difference equation $\frac{I_t}{G_t} = \left(\frac{I_{t-1}}{G_{t-1}}\right)^{1-\alpha}$, precisely the same equilibrium equation of the model. Solving this equation, one finds that the expression for the ratio between revenue and expenditure for any period t starting from initial revenue and expenditure I_0, G_0 is

$$\frac{I_t}{G_t} = \left(\frac{I_0}{G_0}\right)^{(1-\alpha)^t}. \tag{19}$$

From this expression it is easy to see that if $\alpha \in (0,1)$ then $\lim_{t\to\infty}\frac{I_t}{G_t} = 1$, from which it follows that revenues and expenditures tend to equalize in the long run, and therefore the system's deficit will tend to 0.

Now, considering that $I_t = I_0 \prod_{i=1}^{t}(1+g_{I,i})$ and $G_t = I_t \left(\frac{G_0}{I_0}\right)^{(1-\alpha)^t}$, one can finally calculate the financial result for any time t from the initial conditions of the system:

$$H_t = I_0 \prod_{i=1}^{t}(1+g_{I,i}) \left(1 - \left(\frac{G_0}{I_0}\right)^{(1-\alpha)^t}\right).$$

There are two very important aspects to note in this result. Firstly, the deficit (or surplus) system does not depend on expenditure on new pensions. It only depends on the initial state of the system, the increases in revenues, and the choice of the parameter α. Secondly, the potential surplus or deficit of the system depends on the sign of the term $(1-(G_0/I_0))^{(1-\alpha)^t}$. When this term is positive, the system will produce a surplus in its accounts. On the contrary, when it is negative, the system will generate a deficit. Taking into account again that $\alpha \in (0,1)$, one has that the financial result of the system will depend for all t on its initial value, since if the ratio G_0/I_0 is less than unity (and therefore revenues initially exceed expenditures), the value of H_t will always be positive. Similarly, if in the initial state expenditures exceed revenue then the financial result will always be negative, so that the system will incur deficits indefinitely. Therefore, it would be impossible for the Social Security Reserve Fund to accumulate any possible surpluses because there would never be any.

Recall that the way the model is set out does not allow the system to absorb the accumulated debt. So the only way to avoid a permanent deficit is to impose an upper bound on the revaluations. Then financial surpluses would not impact in their full value on the surviving pensions, being instead capable of covering potential shortfalls.

Regarding debt, its expression will depend on the sum of the financial results. Denoting by D_t the debt in year t, one has for $t = 1, 2, \ldots$

$$D_t = D_0 + \sum_{j=1}^{t} I_0 \prod_{i=1}^{j}(1+g_{I,i}) \left(1 - \frac{G_0}{I_0}\right)^{(1-\alpha)^j}.$$

A positive value of debt will mean an excess of resources, while a negative value will mean that there are outstanding accumulated deficits. Since we demonstrated above the convergence of the deficit and its monotonic character with respect to the initial conditions, the debt will also be bounded. However, as was commented on above, if there existed some initial debt, it will never be settled, but will increase over time until reaching its upper bound.

An alternative to imposing an upper bound to the revaluation of existing pensions is to force a budgetary equilibrium that includes the gradual absorption of the debt. For example, if the equilibrium equation were of the form

$$\frac{I_{t+1}+D_t}{G_{t+1}} = \left(\frac{I_t + D_{t-1}}{G_t}\right)^{1-\alpha},$$

since the results above prove that $\lim_{t\to\infty} \frac{I_{t+1}+D_t}{G_{t+1}} = 1$, as the periods pass the revenue plus the debt will converge to the expenditures. In this case, the revaluation factor would take the form

$$(1+g_{t+1}) = \frac{I_{t+1}+D_t}{G_t^s} \left(\frac{I_t + D_{t-1}}{G_t}\right)^{\alpha-1} - \frac{G_{t+1}^a}{G_t^s}.$$

For the particular case of $\alpha = 1$, all the debt is transferred to the existing pensions in a single period, and is therefore settled in that one period.

5 The moving averages

The most complex extension of the model presented by the Expert Committee is to consider moving arithmetic and geometric means for the computation of the elements involved in determining the revaluation index. According to their Report, the aim of using means is to smooth out the impact of the economic cycle. It recommends considering a window of eleven or thirteen years for the calculation. Law 23/2013 finally fixed at eleven the number of years to use, centred on $t+1$, the year for which the revaluation is calculated.

Perhaps the main criticism of the use of means is the need to estimate the expenditures and revenues of the system for the future six years, i.e., from year $t+1$ to year $t+6$. According to Law 23/2013, the Ministry of Economy and Finance shall provide the necessary forecasts of macroeconomic variables needed for this estimation. That the Government itself will estimate these revenues and expenditures raises doubts about the arbitrariness to which the calculation of the revaluation index may be subject. With it being in the hands of the government to establish those projections, and therefore the values of the means involved in the computation, the result of Equation (1) can be fixed arbitrarily. This point is crucial, since one of the key advantages of implementing an automatic balancing mechanism is to avoid, insofar as possible, government intervention.

In this section, we wish to reflect on other methodologically problematic issues which arise from the introduction of moving averages to compute the revaluation index using Equation (1). In particular, we shall discuss two major drawbacks. The first is that Equation (1) has no analytical solution. And the second is that the model presents problems of dynamic consistency, so that there is no guarantee that revenues and expenditures will compensate each other in either the short or the long terms.

To demonstrate the problem of the lack of an analytical solution, let us recall that pension expenditure in year $t+1$ is given by $G_{t+1} = G_t^s(1+g_{t+1}) + G_{t+1}^a$. If one denotes by G_t^b the expenditure produced by exits from the system, i.e., the expenditure produced by those pensions that do not survive to $t+1$, then one has that $G_t = G_t^s + G_t^b$. Thus, substituting, one obtains

$$G_{t+1} = (G_t - G_t^b)(1+g_{t+1}) + G_{t+1}^a. \tag{20}$$

In this way, one year's expenditure can be related to the expenditure of the previous year. The expenditure of period $t+2$ will then be

$$\begin{aligned}
G_{t+2} &= \left(G_{t+1} - G_{t+1}^b\right)(1+g_{t+2}) + G_{t+2}^a \\
&= \left[\left(\left(G_t - G_t^b\right)(1+g_{t+1}) + G_{t+1}^a\right) - G_{t+1}^b\right](1+g_{t+2}) + G_{t+2}^a \\
&= G_t^s(1+g_{t+1})(1+g_{t+2}) + \left(G_{t+1}^a - Gb_{t+1}\right)(1+g_{t+2}) + G_{t+2}^a.
\end{aligned}$$

Continuing with the process, one can obtain the expression of expenditure for the period $t+n$:

$$G_{t+n} = G_t^s \prod_{i=1}^{n}(1+g_{t+i}) + \sum_{i=1}^{n-1}\left(G_{t+i}^a - G_{t+i}^b\right)\prod_{j=i+1}^{n}(1+g_{t+j}) + G_{t+n}^a. \tag{21}$$

Note that the decision to modify the pension revaluation in year $t+1$ will explicitly affect the expenditure in all future periods.

In Equation (1), the term G_{t+1}^* represents the geometric mean of expenditures centred on $t+1$. This requires the expenditure data from period $t-4$ to period $t+7$. The expenditure data for $t-3$ to t derive from historical data. In contrast, the expenditure data for $t+1$ to $t+6$ must be estimated. Here the problem lies in the computation of $G^*_{t+1} = \prod_{i=0}^{10} G_{t+i-3}$. The product of these terms, defined above in (21), will lead to a sum of terms in $(1 + g_{t+1})$ raised to different powers, the largest of then being the eleventh power. For this reason, since it will be impossible to isolate the factor $(1 + g_{t+1})$ from the term G^*_{t+1}, Equation (1) will have no analytical solution. The revaluation may, however, be computed numerically, as we shall show in the next section.

The second problem to consider with the use of moving averages is the model's lack of dynamic consistency. In calculating the revaluation index each year, one is choosing the index which satisfies the corresponding condition of equilibrium. One of its components, the geometric moving average of expenditures, includes both historical expenditures and future projections. To calculate the revaluation index in period $t+1$ it is therefore necessary to make assumptions about the value of future revaluation indices, in particular, for the period $t+2$. However, once the period $t+1$ has passed and one is preparing to calculate the revaluation factor for the following period $t+2$, one will find that this does not necessarily coincide with the value projected in the previous period to calculate the revaluation index for $t+1$. Hence, at $t+2$, one will find that the equilibrium condition at $t+1$ is no longer satisfied. This process will be repeated time after time. This inconsistency would continue to hold even if the forecast mean pension and numbers of new and of leaving pensions were fulfilled perfectly. As a result, there will be no guarantee of the long-term convergence of the ratio of the mean values of expenditure and revenue.

A possible alternative solution to this problem of dynamic inconsistency would be to calculate the revaluation indices for all future periods simultaneously. This approach would, however, also entail many difficulties. Given that the problem would have to be solved numerically, and that (as was shown above) the revaluation index for each period would also have to be calculated numerically, this approach would yield results with serious stability problems.

6 Numerical results

In this section, we shall present a numerical example of the applicability of the revaluation index of Spanish pensions, illustrating results obtained in the foregoing sections. We shall take 2014 as the reference year t. Hence, we will calculate the revaluation index for the year $t+1 = 2015$.

Taking Equation (1) into account, one needs data for the system's revenue and expenditure for the 12 years from $t-5 = 2009$ to $t+6 = 2020$. In the case of the geometric mean of revenues and expenditures, it will be sufficient to employ data from the period 2010-2020. To calculate the arithmetic means of the rates of change of revenue, number of pensions, and substitution effect, we will use data for the period 2009-2020 (i.e., to calculate the mean variation of eleven periods, twelve years' data are needed).

Since it is impossible to obtain from publicly available information the revenue and expenditure amounts for 2009-2013 as defined in Law 23/2013, we will take the data used in AIReF (2014) provided by the Ministry of Employment and Social Security. We took the values of the number of pensions and of the mean pension from the Statistical Report published by the National Institute of Social Security for the respective years.

For practical purposes, except for the value of IR_t, the information for the year t is unavailable at the time of calculating the revaluation index for the year $t+1$. The data will there have to be estimated from the information available as was done for the values corresponding to the period 2014-2020. Since the purpose of this section is not calculate with precision the revaluation index for 2015, but simply to illustrate some of the issues that have been discussed in the preceding sections, we shall make simple assumptions for the estimates (for an in-depth methodological discussion of the revenue and expenditure forecasts, see AIReF (2014)).

In the case of revenues, from the AIReF (2014) data we shall assume that the annual variation is 4.092%. For the estimation of the expenditures, the Committee of Experts Report, Annex A3.2, proposes calculating the expenditure for the year $t+1$ by the expression

$$G_{t+1} = N_{t+1}\bar{P}_{t+1}$$

with $\bar{P}_{t+1} = \bar{P}_t(1+g_t)(1+g_{s,t})$. I.e., the mean pension of year $t+1$ would increase in accordance with the revaluation of the pensions and the substitution effect of year t. We found this proposal to be unsatisfactory for two reasons. Firstly, the results of Section 3 show that this expression does not model a direct link between the expenditure of year $t+1$ and pension revaluations. Correct in this sense would be to use Equation (20) for the expenditure. Secondly, the proposal presents the problem of describing an expenditure that is only affected by the number of pensions and the mean pension. However, the amount of Social Security expenditure is greater than the product of these two factors. It includes many other concepts, as can be seen in the data of Table 1 which details the difference between mean budgeted expenditure and mean pension (considering fourteen monthly payments) as well as the percentage difference between the two for the years 2009-2013.

Year	**Num. pensions**	**Mean pension**	**Mean expenditure**	**% difference**
2009	8 614 876	760.68	902.10	18.59%
2010	8 749 054	786.51	927.82	17.97%
2011	8 871 435	811.46	937.32	15.51%
2012	9 008 348	837.02	943.81	12.76%
2013	9 154 617	862.74	962.35	11.55%

Table 1: Percentage difference between mean expenditure and mean pension, 2009-2013.

As one sees, the differences, although diminishing, remain significant.

One can resolve the problems caused by the incorrect definition of expenditure by modifying Equation (20) to incorporate costs that do not derive from the amount of the pensions. Let GF_t be the system's expenditure generated for reasons other than payment of the amount of pensions in the year t. Then,

$$G_{t+1} = (G_t - G_t^b)(1+g_{t+1}) + G_{t+1}^a + GF_{t+1}. \tag{22}$$

Taking as referents the data in Table 1, let us assume for simplicity that the expenditures not directly deriving from the payment of pensions for the years 2014-2020 stay constant in absolute value and equal to that of year 2013. This approach considers the expenditures GF_{t+1} to be fixed costs of the system, so that the pension revaluations will not affect their amount. The drawback of this approach is the leverage effect on the revaluation.

With respect to the expenditure caused directly by the payment of pensions, this also depends on the numbers of new entries into and of exits from the system, and the amount of their mean pensions. We extrapolated the annual mean of the last five years' data of the 2013 Statistical Report of the National Social Security Institute to the period 2014-2020. The annual percentage variations were: in the number of newly entering pensions 6.12%, in that of exiting pensions 4.60%, in the mean pension of the newly entering pensions 3.66%, and in the mean pension of the exiting pensions 4.14%.

Another set of data required was the revaluation of the pensions for the years 2009-2014. These values are listed in Table 2. Note that, for the 2013 pension revaluation, the law Real Decreto-Ley 29/2012 establishes a different type of revaluation which depends on the amount of the pension. Although the datum for the exact percentage of the overall revaluation is not available, we estimated it to be 1.55% from the distribution of pensions by intervals of amount from the Statistical Report of the National Social Security Institute for the year 2012.

Year	2009	2010	2011	2012	2013	2014
Revaluation	2.0%	2.3%	0.00%	1.00%	1.55%	0.25%

Table 2: Percentage revaluation of Spain's pensions for the years $2009 - 2014$.

Once the data had been acquired, we proceeded to calculate the revaluation index for 2015. Recall that the first component of Equation (1) is $\bar{g}_{I,2015}$, i.e., the arithmetic mean centred on $t+1 = 2015$ of the eleven values of the fractional (i.e., not percentage) rate of change of the Social Security system's revenue. The second component, $\bar{g}_{p,2015}$, is the arithmetic mean centred on $t+1 = 2015$ of the eleven values of the fractional rate of change of the number of contributory pensions in the Social Security system. The third component of the formula is $\bar{g}_{s,2015}$, the arithmetic mean centred on $t+1 = 2015$ of eleven substitution effect values expressed as fractions. As detailed in Section 3, we shall use Equation (10) for each year from $t - 4 = 2010$ to $t + 6 = 2020$. For each of these years, one needs to know the corresponding revaluation index. Here we make the assumption that the revaluation index for the years that need to be estimated is constant and equal to that of the year $t + 1 = 2015$, provided that this value lies within the upper and lower bounds. Since one needs the revaluation index for the year 2015 in particular, and this datum is unknown since it is precisely what one intends to calculate, we shall consider the implicit expression

$$g_{2015} - \bar{g}_{I,2015} + \bar{g}_{p,2015} + \bar{g}_{s,2015} - \alpha \left(\frac{I^*_{2015} - G^*_{2015}}{G^*_{2015}} \right) = 0 \tag{23}$$

to calculate numerically the 2015 revaluation index, the substitution effects, and the expenditure for each year, which also depends explicitly on its corresponding revaluation index through Equation (22).

The last component consists of three elements: the parameter α, and the geometric means centred on $t + 1 = 2015$ of the eleven values of the amounts of the revenues and of the expenditures of the system, denoted by I^*_{2015} and G^*_{2015}, respectively. We use 0.25 for the value of the parameter α, as is required by Law 23/2013 for the first five years of implementation. The values of I^*_{2015} and G^*_{2015} are straightforward to calculate from the acquired and the estimated data. Note, however, that the results of previous sections have shown that it would have been more consistent to centre the means on the period t, and not on $t + 1$ as is required by the Law.

With this information, the revaluation to apply to all the pensions in the year 2015 can be calculated numerically. The result is $g_{2015} = -1.73\%$. In this calculation, we ignored the lower bound of 0.25% for the revaluation at $t + 1$ as is established legally. The reason was that this would have given a distorted view of the real result, and because in general applying Equation (23) for each year might not have a solution.

Table 3 lists the results of the calculations of the system's revenues, expenditures, number of pensions, and mean pension. The revenue and expenditure values are in millions of euros.

From the above results, one can disaggregate the revaluation index value into its components: $g_{I,2014} = 2.30\%$, $\bar{g}_{p,2014} = 1.52\%$, $g_{s,2014} = 1.79\%$, $I^*_{2015} = 126\,216.38$, and $G^*_{2014} = 129\,938.37$, with these last two values being in millions of euros.

	Year	Revenue	Expenditure	Pensions	Mean pension
t-5	2009	117 397.00	108 800.30	8 614 876	760.68
t-4	2010	116 458.20	113 646.10	8 749 054	786.51
t-3	2011	116 119.00	116 415.80	8 871 435	811.46
t-2	2012	113 081.30	119 029.80	9 008 348	837.02
t-1	2013	113 505.10	123 339.50	9 154 617	862.74
t	2014	118 149.73	127 001.07	9 293 576	877.98
t+1	2015	122 984.42	128 671.77	9 434 645	877.50
t+2	2016	128 016.94	132 658.82	9 577 855	894.12
t+3	2017	133 255.39	136 822.07	9 723 238	911.33
t+4	2018	138 708.20	141 169.10	9 870 829	929.16
t+5	2019	144 384.14	145 707.80	10 020 659	947.62
t+6	2020	150 292.34	150 446.36	10 172 764	966.72

Table 3: Statistical data for the years 2009-2020.

7 Conclusions

In this paper, we have analysed the technical aspects of the computation of the revaluation index of Spain's pensions. Based on a simplified model, we analysed step by step the elements that comprise it, starting with the correct definition of the factors involved in its calculation. We then examined the implications of adding directly the coefficient α to the final expression in order to regulate the rate at which the deficit is reduced, and of the use of moving averages to calculate the magnitudes involved. The results allowed us to show how, due to the convergence of revenue and expenditure, the introduction of the parameter α into the final expression does not significantly condition the model. The introduction of the moving averages into the final expression, however, does have a strong impact because it prevents the revaluation index being computed analytically and no longer ensures the dynamic consistency of the model. Finally, we analysed how redefining the expenditure variable leads to severe inconsistency in the model, as was illustrated in a numerical example.

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