

# Beam steering with spatial light modulators: Quantisation effects

Author: Astrid Ribot Thunnissen

Facultat de Física, Universitat de Barcelona, Diagonal 645, 08028 Barcelona, Spain\*.

**Abstract:** Beam steering can be achieved by sending holograms to a phase-only spatial light modulator (SLM). The use of holograms, defined by linear phases, induce a one dimensional transverse shift of the beam spot at the focal plane of a lens. An SLM is formed by  $N$  pixels and uses a limited number of grey levels,  $M$ . These limitations produce the quantisation of the phase values introduced on the incoming beam and decrease the accuracy on spot positioning.

We have built a system in which a laser beam is focused by a lens after being reflected by an SLM. Different positions of the focal spot have been captured by a camera and accurately determined by image processing. Measurements were done with  $N=100$  and  $M=16$ . We proved that quantisation effects are not relevant in most of the positions, except for certain locations, which produce such errors on spot positioning that need to be avoided.

## I. INTRODUCTION

Optical tweezers (or optical traps) are strongly focused laser beams capable of trapping and moving microscopic particles [1]. In the last years, the design of optical tweezers has been improved by means of phase-only spatial light modulators (SLM) [2]. In particular, the position of the trap position can be controlled by encoding holograms on the SLM, what spatially modifies the laser's wave front before the focusing step.

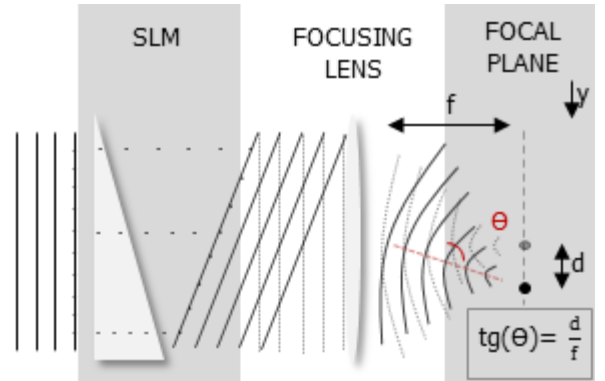
The accuracy on trap positioning is decreased due to the limited pixel's number and limited number of possible phase values in the SLM modulation. These effects have been studied in detail in [3].

Our work has consisted in designing an optical tweezers experimental setup to control and study the localisation of the traps. To do that, we have first computed the holograms sent to the SLM, taking into account the specifications and constraints of the setup. We have then designed a system to measure the traps' centre accurately, which has allowed us to study the effects on trap positioning due to the limitation of the number of pixels and the phase's values.

## II. THEORETICAL BACKGROUND

### A. Beam steering with spatial light modulators

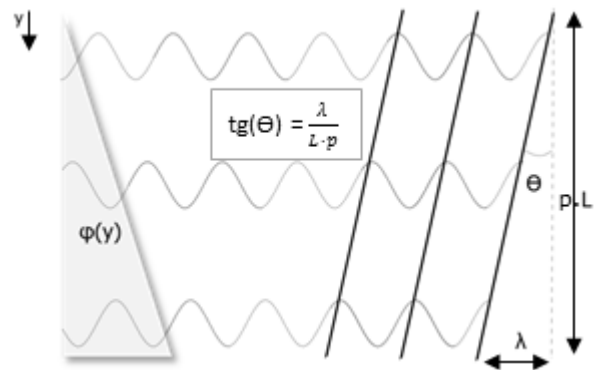
Phase-only spatial light modulators are used in beam steering. In order to do that, holograms are encoded to the SLM, producing different beam's directions. Holograms are pixel images defined by grey levels. The electronics of the SLM converts the value of each pixel's grey level to applied voltages. If the grey levels have different values, a change of the incoming beam's direction is achieved. In fact, the different grey levels correspond to different  $\varphi$  phases –values from 0 to  $2\pi$ – as they are linearly related. When they are introduced on the incoming plane wave front, the direction of the beam is deviated. Particularly, when phases change linearly along the  $y$ -axis, as defined in Equation (1), this deviation corresponds to a  $\theta$  tilt at the plane wave front. In consequence, a one dimension  $d$  transverse shift of the spot is induced at the lens' focal plane. This effect is represented in Fig. 1.



**FIG. 1:** Introducing  $\varphi(y)$  linear phases with the SLM on the incoming plane wave causes a tilt on its direction. Consequently, it produces a 1D transverse shift of the spot at the focal plane of the lens. We have illustrated this effect with a prism, as it is equivalent.

$$\varphi(y) = \frac{y}{L} \cdot 2\pi \quad (1)$$

As represented in Fig. 2, the introduction of a  $\varphi$  linear phase as defined in Equation (1) is directly related to the  $\theta$  deviation of the plane wave front.



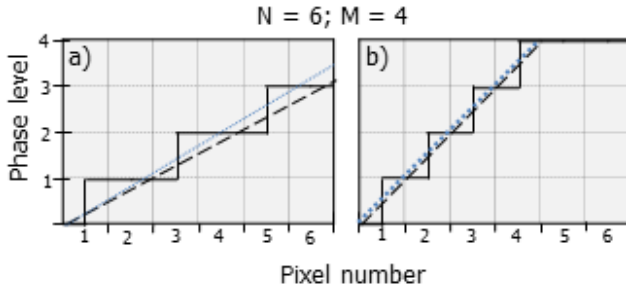
**FIG. 2:** The linear phase shift produces a tilt on the optical wave front by an angle  $\theta$ .  $L$  is the period of the phase, producing a difference of  $\lambda$  on the optical path.  $p$  is the pixel width.

\* Electronic address: aribotth7.alumnes@ub.edu

## B. Quantisation Effects

As explained, beam steering can be achieved by sending holograms to an SLM. Its accuracy is related to the  $N$  pixels that conform the SLM (it is not an analogue equipment). In addition, the limited  $M$  number of grey levels quantises the phase values that can be introduced to the incoming wave front. The effect that these limitations have on spot localisation is detailed below.

To this point, we have explained that the phases introduced by the SLM to the incoming plane wave front induce a  $\theta$  tilt on it. In addition, limiting  $N$  and  $M$  produces a staircase on the out-coming wave front as shown in Fig. 3 for two different spot positions. A plane wave front is associated to the staircase wave fronts. Fig. 3 illustrates the variation of the tilt caused by an ideal plane wave front –not limiting  $M$  and  $N$  values- and a real grated one. This difference induces a shift on the spot position at the focal plane.



**FIG. 3:** Ideal phase (blue dotted), realized staircase phase modulation (black solid) and the mean phase tilt of the realized modulation (black dashed). Number of pixels  $N=6$ , number of phase levels  $M=4$ . (a) and (b) correspond to different spot positions. The tilt in (b) is perfectly reproduced as it gives an exact number of phase values per pixel.

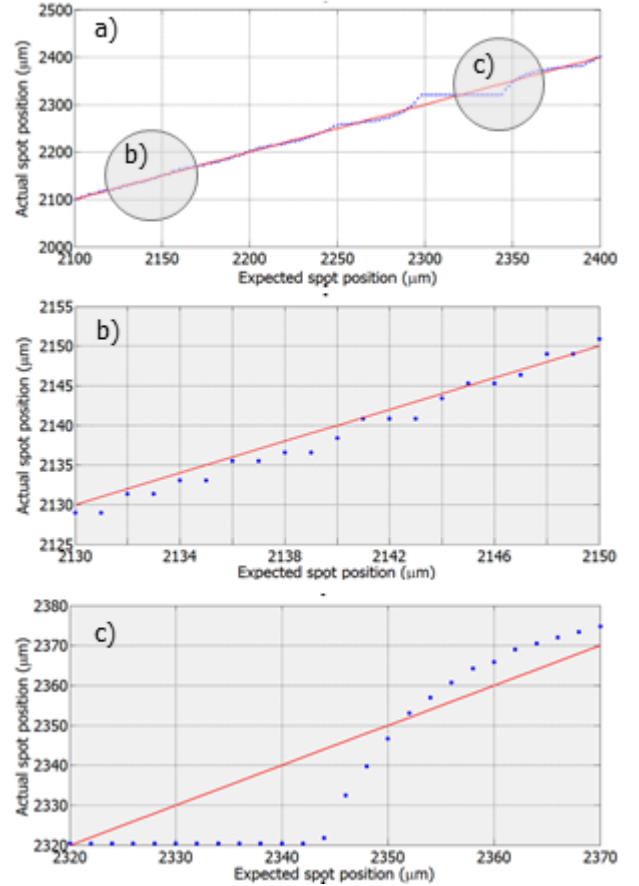
It is clear that these quantisation effects decrease steering's or spot localisation accuracy. Curiously and as proved in [3], these effects induce different errors depending on the spot's position.

Fig. 4 is a simulation of actual –staircase wave front- versus ideal –continuous wave front- spot positions. In particular, in Fig. 4 (a) it is proved that the maximum error is found close to positions where the phase periodicity ( $L$ ) is defined by Equation (2):

$$L_{match} = \frac{M}{n} \quad (n = 1, 2, 3 \dots) \quad (2)$$

By means of the relationships between  $d$ ,  $L$  and  $\theta$  found in Fig. 1 and Fig. 2, the expression of Equation (3) is deduced. This permits to know the exact positions on the focal plane where the quantisation effects are more relevant.

$$d_{match} = \frac{\lambda \cdot f}{L_{match} \cdot p} \quad (3)$$

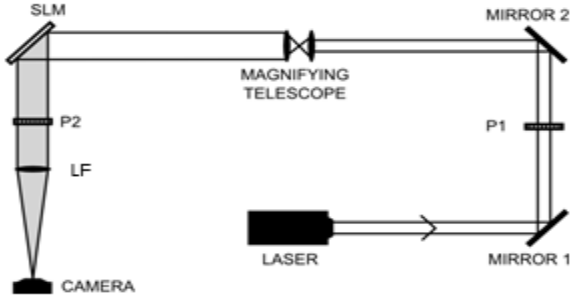


**FIG. 4:** Simulation of actual (blue dotted) versus ideal (red solid) spot positions for  $N=100$ ,  $M=16$  (parameters of our experiment). (a): Spot positions from  $2100\mu\text{m}$  to  $2400\mu\text{m}$  by intervals of  $2\mu\text{m}$ . (b): Spot positions from  $2130\mu\text{m}$  to  $2150\mu\text{m}$  by intervals of  $1\mu\text{m}$ . The quantisation effects of the grated phase in these positions are schematised in Fig. 3 (a). (c): Spot positions from  $2320\mu\text{m}$  ( $d_{match}$ ) to  $2370\mu\text{m}$  by intervals of  $2\mu\text{m}$ . While at  $d_{match}$  the aimed position is perfectly achieved (Fig. 3 (b)), curiously around this position the errors become very big.

## III. PROCEDURE

### A. Experimental Setup

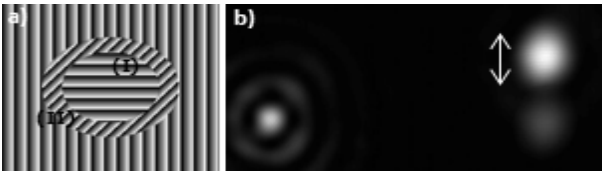
Fig. 5 schematizes the experimental setup used in our study. A  $\text{TEM}_{00}$  laser continuous-wave ( $\lambda = 532\text{nm}$ ) with a Gaussian beam profile is expanded by a magnifying telescope. It is sent then to an SLM, Holoeye LC-R 2500 which has an active area of  $19.5\text{mm} \times 14.6\text{mm}$  and supports images with a resolution of  $1024 \times 768$  pixels (pixel pitch  $p = 19\mu\text{m}$ ). The out-coming beam is focused by lens LF ( $f \sim 1000\text{mm}$ ), and this produces a spot on its focal plane where the camera (pixel size =  $6\mu\text{m}$ ) is placed. Using the frames taken with the camera, we analyse the shift of the spot's position, produced by the different holograms sent to the SLM.



**FIG. 5:** Experimental setup for beam steering in holographic optical tweezers. The SLM is tilt  $45^\circ$  (in the horizontal direction) with respect to the optical axis and sandwiched between two linear polarizers, P1 and P2, orientated at  $74^\circ$  and  $108^\circ$ , respectively, to achieve phase-only modulation. The mirrors are used to provide a good alignment control. The magnifying telescope expands the laser beam. LF is the focusing lens which forms a spot of light at its focal plane.

### B. Hologram design

So far, we have described the theoretical background for hologram design. From now on, we will develop the experimental details that need to be taken into consideration in hologram computation. For this purpose, we created a Matlab program. Due to the SLM non-flatness [4], a modification to the described holograms has to be made: the full area of the SLM is not being used since the shape of the resulting spot became too deteriorated. We only selected central pixels for our experiments; the rest were set to a grating that deflects the light to positions outside the measurement area. Also, in order to avoid the effect that drift (due to mechanical vibrations, air currents, etc.) has on localising the spot centre, the following technique is used: we created 2 spots instead of one. Therefore, the hologram (see Fig. 6 (a)) contains two concentric regions with the same area -we are looking for 2 spots with the same intensity-. The hologram from the smallest annular region (Fig. 6 (a) I) produces the spot to which the transversal shift will be applied (Fig. 6 (b) I). The reference spot (Fig. 6 (b) II) is produced by the bigger central region (Fig. 6 (a) II).

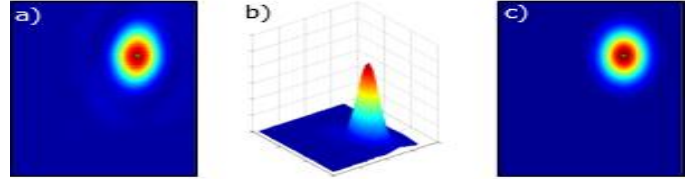


**FIG. 6** (a): Example of a hologram sent to the SLM. The region of the central pixels is selected to achieve a circular spot on the focal plane as a common optical trap is. We created an elliptic (and not circular) pixel selection in order to compensate the factor introduced in the horizontal direction by the  $45^\circ$  SLM tilt with respect to the optical axis. In (I) the ellipse's vertical axis corresponds to  $N=100$  pixels. In (II) the ellipse's vertical axis corresponds to  $N=141$  pixels (b): Example of a frame taken on the focal plane. The different holograms produce a vertical transverse shift on (I), while (II) remains in the same position. Experiment results are achieved by measuring the relative displacement between (I) and (II), to avoid drift.

### C. Spot centre localisation

For each different beam position, we have captured two spots by means of the camera: One spot has been used as the reference point, while the transversal shift produced by the hologram has been applied to the second one. We measured the relative displacement between the 2 spots for each captured frame, what suppresses the error caused by the drift.

In order to do that, we needed to locate the spot centres: we used a Matlab code [5] which adjusts the intensity's spots to a 2D Gaussian function. The spot centres correspond to the maximum intensity value of the Gaussian function.



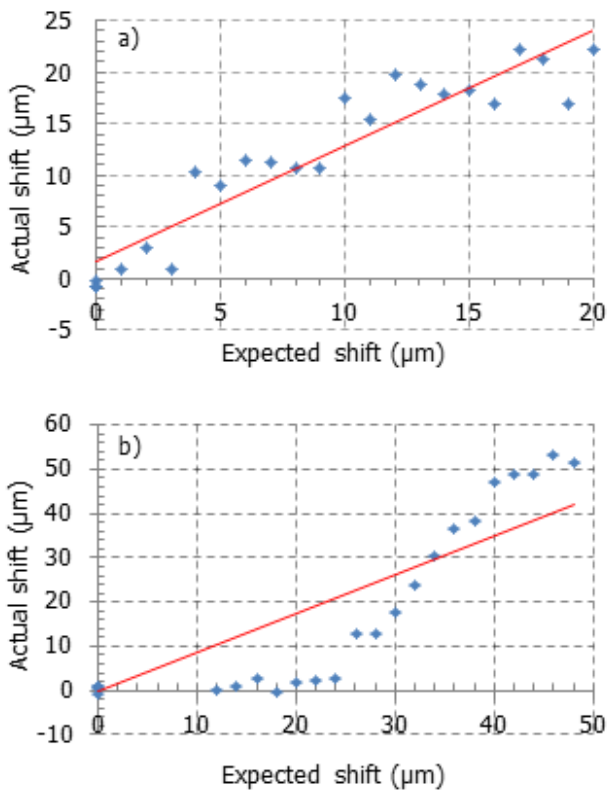
**FIG. 7:** (a): Image of an experimental spot using a Matlab program. (b): 2D Gaussian function that corresponds to Fig. 7(a) intensity's adjustment. (c): Image of the adjusted spot found applying the adjustment of (b) on (a).

## IV. RESULTS AND DISCUSSION

Measurements were done for two different spot positions: far from  $d_{\text{match}}$  and close to it. We took 20 measures around each position by intervals of  $2\mu\text{m}$ , reaching  $< 1\mu\text{m}$  error measurements. We limited M and N to low values ( $M=16$ ,  $N=100$ ) to ensure finding the quantisation effects.

Actual versus expected positions are shown in Fig. 8, with their respective simulations. We also represented the relation that would have been found using an ideal plane wave front instead of the grated one.

A maximum linear fit residual of  $\sim 6\mu\text{m}$  was achieved for positions far from  $d_{\text{match}}$ , (Fig. 8 (a)) while a  $\sim 20\mu\text{m}$  value was found close to it (Fig. 8 (b)). These results entail to conclude that close to  $d_{\text{match}}$  positions, quantisation effects produce big inaccuracies on spot localisation, which should be avoided if precision in the experiments is important. Far from  $d_{\text{match}}$ , quantisation effects do not damage location accuracy. Experimental results were very similar to the predicted by the simulations in both cases (see Fig. 4).



**FIG. 8:** Experimental measurements (blue dots) and ideal results (solid red) of the spot transverse shifts. The shifts are referred to the reference spot's position. (a): Spot's shifts from  $0\mu\text{m}$  up to  $20\mu\text{m}$  by intervals of  $1\mu\text{m}$ . The reference spot position was located at  $2130\mu\text{m}$  from the centre (when no holograms is sent). (b): Spot's shifts from  $0\mu\text{m}$  up to  $50\mu\text{m}$  by intervals of  $2\mu\text{m}$ . The reference spot position was located at  $2320\mu\text{m}$  ( $d_{\text{match}}$ ) from the centre.

Using our experimental setup, big spots were needed to study the quantisation effects. This big spots were achieved

by limiting  $M$  and  $N$  to low values and using a lens with a big focal length. Nevertheless, it is crucial to understand that in this case, our experiment does not really function as an optical tweezers, that is, no particle would get trapped at the focal spot. Even though, by means of a focusing lens with a shorter focal length and, therefore, a higher numerical aperture (NA), the tweezers would work.

Not limiting  $M$  and  $N$  ( $M=256$ ,  $N=768$ ) would permit to achieve 100 times lower quantisation effects ( $\sim 0.2\mu\text{m}$ ). In this case, we would not be able to measure them (we found measurement errors  $\sim 1\mu\text{m}$ ), so the experimental set up would need to be improved. In order to do that, we could replace the focusing lens by a teleobjective system with an even higher focal length. This variation on the experimental setup would permit to obtain bigger spots and measure quantisation effects even if no limitation on  $N$  and  $M$  was applied. However, this lens' change could produce extra aberrations that would be necessary to take into account on the hologram design to get the required precision on the results.

Errors of  $\sim 0.2\mu\text{m}$  due to quantisation effects should not be passed over using common optical traps, where (in some cases) nanometrical precision is needed. We propose the use of Equation (3) to localise  $d_{\text{match}}$ , around which it has been proved that quantisation effects produce important inaccuracies on spot localisation. Measurements should be done far from these positions.

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