

# Characterization of a high stability audio band magnetic sensor

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**Abstract:** The LISA (Laser Interferometer Space Antenna) will be the first space-based gravitational wave observatory when it is launched around 2037. It will consist of 3 spacecraft separated by 2.5 million km in a triangular formation, each containing two test masses in free fall, following Earth in its orbit around the Sun. It will constitute an enormous interferometer, capable of detecting low frequency gravitational waves. To be able to do so, it is crucial to be able to keep the test masses in each spacecraft free from undesired accelerations that could be mistaken for gravitational waves. A relevant source of acceleration noise is the presence of magnetic field signals produced by equipment inside the spacecrafts. The aim of this work is to characterize a high stability audio band magnetic sensor capable of detecting these signals with the sensitivity required for the LISA mission.

## I. INTRODUCTION

Gravitational waves, in contrast to electromagnetic waves (which are created by the acceleration of electrical charges and propagate in the framework of space and time) are moving gravitational fields, ripples in the curvature of spacetime, generated by the acceleration of massive objects. They were conceptually first proposed by Oliver Heaviside in 1893 as the gravitational equivalent of electromagnetic waves [1], but it was Albert Einstein who predicted them in 1916 on the basis of his General Theory of Relativity and stated that they propagate at the maximum allowed speed in the universe: the speed of light.

It was not until almost a century later that Einstein's predictions were confirmed by experimental evidence, when the Laser Interferometer Gravitational-wave Observatory (LIGO), a ground-based gravitational wave observatory, detected the first gravitational waves from the merger of two stellar mass black holes [2]. Since then, LIGO and its European counterpart VIRGO, have announced the detection of several additional black hole systems as well as a neutron star merger.

LIGO is an interferometric detector, which uses laser interferometry to measure the motion induced by a gravitational wave between two separated test masses in free fall. The LIGO interferometer is constituted by two stations, separated by 3030 km: one in Hanford (Washington), and the other one in Livingston (Louisiana). Each station consists of two arms, which are 4 km long each, assembled in an L-shape. A gravitational wave that passes through the detector will slightly stretch one arm as it shortens the other; the strongest gravitational waves will change the distance between the ends of the arms by at most roughly  $10^{-18}$  m. LIGO's measuring system has been able to achieve a precision of  $S_h \approx 5 \cdot 10^{-22} \frac{1}{\sqrt{Hz}}$ , which allowed the first detections of gravitational waves. The two stations were built 3030 km apart in order to determine the source of the detected waves through triangulation, measuring the difference in arrival times of the waves to the two detectors.

Ground-based detectors such as LIGO are able to detect gravitational waves with frequencies in the range 10-1000 Hz. On Earth, noise sources such as seismic activity and the limited length of the interferometer arms make lower frequency regions of the spectrum inaccessible. Lower frequency gravitational waves have longer wavelength, and thus correspond to objects in wider orbits and potentially much heavier than those that ground-based detectors are able to search for, such as ultra-compact binaries within the Milky Way, supermassive black hole mergers, or extreme mass ratio inspirals.

That is why, in order to detect signals in the lower frequency range ( $10^{-4}$  Hz - 1 Hz), space-based detectors are required. LISA (Laser Interferometer Space Antenna) is planned to be the first space-based gravitational wave observatory, with its launch planned by 2037. The LISA mission is a collaboration between ESA (European Space Agency), NASA (National Aeronautics and Space Administration), and an international consortium of scientists.

The LISA mission will comprise three spacecraft flying in a triangular formation behind the Earth as our planet orbits the Sun. The spacecrafts will sit in a heliocentric orbit about 50 million km from Earth, with a distance of around 2.5 million km between each spacecraft. Each spacecraft will contain two test masses in free fall, and the distance between them will be constantly measured by laser interferometry in order to detect small variations produced by gravitational waves. The distance between the spacecrafts is going to act as the interferometer's arms, allowing the detection of lower frequency gravitational waves that is not feasible on Earth. The distance changes between the test masses caused by the gravitational waves are small (in the range of pm) compared to the variations caused by orbital dynamics (about 10000 km), but can be distinguished because the former are at mHz frequencies (1000 seconds timescale), whereas the latter have periods of many months and are quiet at mHz frequencies.

To be able to successfully detect such subtle distance variations, it is essential that the test masses are free from

spurious forces or accelerations, because those could be confused with the tidal deformations caused by the gravitational waves. Therefore, a requirement for the differential acceleration between the free falling test masses is set for the LISA mission [3]:

$$S_a^{1/2} \leq 3 \cdot 10^{-15} \frac{\text{ms}^{-2}}{\sqrt{\text{Hz}}} \cdot \sqrt{1 + \left(\frac{0.4\text{mHz}}{f}\right)^2} \cdot \sqrt{1 + \left(\frac{f}{8\text{mHz}}\right)^4} \quad (1)$$

where  $S_a^{1/2}$  is the maximum allowed test mass acceleration noise level. This value is given for a frequency region of  $10^{-4}$  Hz - 1 Hz.

In December 2015, a precursor mission called LISA Pathfinder was launched into space, with the intention of testing whether the LISA requirements could be achieved, and therefore whether the mission as a whole could be successful. LISA Pathfinder consisted of a single spacecraft containing two test masses separated by 37.5 cm (a reduced version of one of the LISA interferometer arms), the differential acceleration between which was being constantly measured. It arrived at the L1 Lagrange point (Sun-Earth reference system) 50 days after its launch, and for almost two years it stayed there testing all the critical technologies to be used in the future LISA mission, until it finally passivated in July 2017.

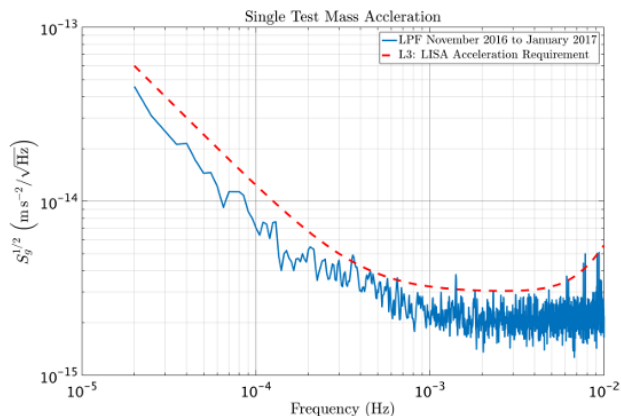


FIG. 1: Average test mass acceleration noise measured with LISA Pathfinder, compared against the LISA acceleration noise requirements for the target range of frequencies [4].

The LISA Pathfinder mission proved to be a success, since the measured acceleration noise was not only below its mission requirements (limited to  $S_a^{1/2} \leq 3 \cdot 10^{-14} \frac{\text{ms}^{-2}}{\sqrt{\text{Hz}}}$  at 1 mHz), but also below those of the future LISA mission, as can be seen in figure 1.

It is important, though, to identify the sources of this acceleration noise, in order to minimize it and prevent possible noise spikes in the future LISA mission. Some of those sources include residual gas pressure from imperfect vacuum conditions in the spacecraft, or the gravitational force from other elements of the satellite [5]. Another potentially relevant source of noise is the presence of mag-

netic field signals in the audio frequency band that could down-convert into the LISA detectable frequency region. Those undesired magnetic field signals, that can be generated by other on-board equipment within the spacecraft, can be detected by a magnetic sensor. The goal of this bachelor thesis is to characterize a high stability audio band magnetic sensor capable of detecting magnetic field signals in the audio range (15 Hz - 20 kHz), by studying its sensitivity and equivalent magnetic field noise and their respective frequency response.

## II. MAGNETIC FIELD SENSORS. SEARCH COILS

Magnetic fields can be detected and measured by a myriad of different devices, such as Hall effect sensors, anisotropic magnetoresistive (AMR) magnetometers, microelectromechanical systems (MEMS) magnetometers (including Lorentz force based and Electron Tunneling based sensors), magnetic tunnel junction (MTJ) magnetometers, superconducting quantum interference devices (SQUID), fluxgates and search coils, among many others. For the LISA mission, the most suitable ones are the search coils (1 nT - 1 T), because of the frequency range (the audio band) and magnitude of the magnetic field to be detected.

Search coils are also called induction magnetometers, because they are based on Faraday's law of induction. Their transfer function  $V = f(B)$  is

$$V = -n \cdot \frac{d\varphi}{dt} = -n \cdot A \cdot \frac{dB}{dt} = -\mu_o \cdot n \cdot A \cdot \frac{dH}{dt} \quad (2)$$

where  $\varphi$  is the magnetic flux going through the coil,  $A$  is the cross-area of the coil and  $n$  is its number of turns.

Search coils can have a ferromagnetic core or an empty core, in which case is called an air core coil; both have their pros and cons. Ferromagnetic-cored coils can achieve higher sensitivities with smaller coil dimensions, but their transfer function is not linear because it depends on frequency, temperature and flux density. They also present another issue, very relevant in the context of the LISA mission: the ferromagnetic core can alter the magnetic field and result in extra magnetic noise inside the spacecraft.

Air-cored coils, on the other hand, cannot achieve such high sensitivities, but in return they do not alter the magnetic field and therefore do not add any excess magnetic noise inside the spacecraft. This is the reason why they are more suited for the LISA mission requirements. The absolute sensitivity of an air-cored search coil is given by

$$S_a = \frac{V_{\text{out}}}{B} = 2\pi f \cdot A = 2\pi f \cdot A_o \cdot n = \frac{1}{8} \cdot \pi^2 \cdot (D + D_i)^2 \cdot n \cdot f \quad (3)$$

where  $A_o$  is the area of one turn of the wire, and  $D$  and  $D_i$  are the outer and inner diameter of the coil, respectively [6].

### III. SEARCH COIL PARAMETERS

The search coil characterized in this bachelor's thesis is a prototype of the one used in the LISA Pathfinder spacecraft, which successfully measured magnetic field signals below the required threshold. Its winding is made of copper wire.

Number of turns ( $n$ )	2400
Mean diametre of the coil ( $D_m$ )	113 mm
Area of one turn of the wire ( $A_0$ )	$10^4 \text{ mm}^2$
Total area of the coil ( $A$ )	$24 \text{ m}^2$
Diameter of the wire ( $d$ )	0.1007 mm
Length of one turn of wire ( $l_w$ )	0.3164 mm
Area of the wire ( $A_w$ )	$0.00797 \text{ mm}^2$
Resistance per length unit ( $\rho/A$ )	$2164 \text{ m}\Omega/\text{m}$

TABLE I: LISA Pathfinder search coil specifications.

In the ideal case, the output voltage signal is linear to the frequency of the measured magnetic field, as shown in equation (3). However, the coil presents an internal resistance  $R$  due to the copper wire winding, an inductance  $L$  because of its geometry, and a self-capacitance  $C$  because the multiple-layered arrangement of the wire winding acts as an energy-storing capacitor. Consequently, the search coil can be represented by an analogous RLC circuit, as shown in figure 3, where  $E$  is the electromotive force induced in the coil.

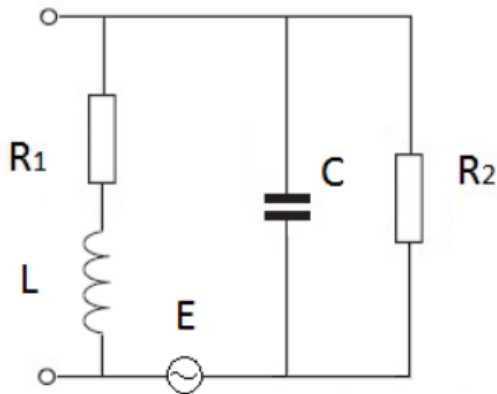


FIG. 2: Equivalent RLC circuit for the search coil.

The resistances  $R_1$  and  $R_2$ , the inductance  $L$ , and the capacitance  $C$  of the circuit needed to be determined experimentally.

$R_1$  was measured by applying a voltage to the search coil with a known intensity, using a multimeter. The obtained value was  $R_1 = 1821 \Omega$ .

In order to calculate the inductance  $L$  of the search coil, it was needed to first calculate its impedance  $Z$ , which corresponds to the impedance relative to the

point where the phase is maximum. Therefore, an electronic impedance analyzer was used to measure the coil's impedance and its phase as a function of frequency. The maximum phase corresponded to an impedance value of  $Z_L = 15188 \Omega$ , at a frequency of  $f = 1767 \text{ Hz}$ . Then, inductance was obtained as

$$L = \frac{Z_L}{2\pi f} = \frac{15188 \Omega}{2\pi \cdot 1767 \text{ Hz}} \approx 1.37 \text{ H} \quad (4)$$

Correspondingly, the search coil's capacitance  $C$  corresponds to the impedance relative to the point where the phase is minimum. The previous data obtained with the impedance analyzer allowed to determine a minimum phase corresponding to an impedance value of  $Z_C = 16539 \Omega$ , at a frequency of  $f = 29086 \text{ Hz}$ . Then, capacitance was obtained as

$$C = \frac{1}{Z_C \cdot 2\pi f} = \frac{1}{16539 \Omega \cdot 2\pi \cdot 29086 \text{ Hz}} \approx 3.3 \cdot 10^{-10} \text{ F} \quad (5)$$

### IV. SENSITIVITY AND ITS FREQUENCY RESPONSE

In equation (3), the expression for the absolute sensitivity of an air-cored search coil was provided. However, that formula is an idealization, and in the real world the sensitivity of the coil depends on the transfer function of the equivalent RLC circuit in a complicated manner. Thereupon, taking as a starting point the circuit transfer function deduced by Ueda and Watanabe in [7], it can be inferred that the actual search coil sensitivity is

$$S = \frac{V_{\text{out}}}{B} = \frac{2\pi f \cdot A_o \cdot n}{\left[ (1 + \alpha)^2 + \gamma^2 \left( \frac{\alpha^2}{\beta^2} + \beta^2 - 2 \right) + \gamma^4 \right]^{\frac{1}{2}}} \quad (6)$$

where  $\alpha \equiv \frac{R_1}{R_2}$ ,  $\beta \equiv R_1 \sqrt{C/L}$ ,  $\gamma = \frac{\omega}{\omega_o} = 2\pi f \sqrt{LC}$ .

Having reached this expression, it is now possible to check the sensitivity's frequency response for different values of  $\alpha \equiv \frac{R_1}{R_2}$ .

In Figure 3 it can be seen how the sensitivity of the search coil is almost constant for  $R_2 = 6938.4 \Omega$ . Therefore, this is the  $R_2$  value we would require for the equivalent RLC circuit, since it allows a more stable sensitivity at a wide enough range of frequencies, while still keeping a high enough sensitivity value in the order of  $10^5 \text{ V/T}$ .

Since it was concluded that the RLC circuit model equivalence provided a satisfactory frequency response for the sensitivity, it was needed to test whether the behaviour of the real search coil was close enough to that of the model. In order to do so, an experiment was designed in which the search coil was exposed to a magnetic

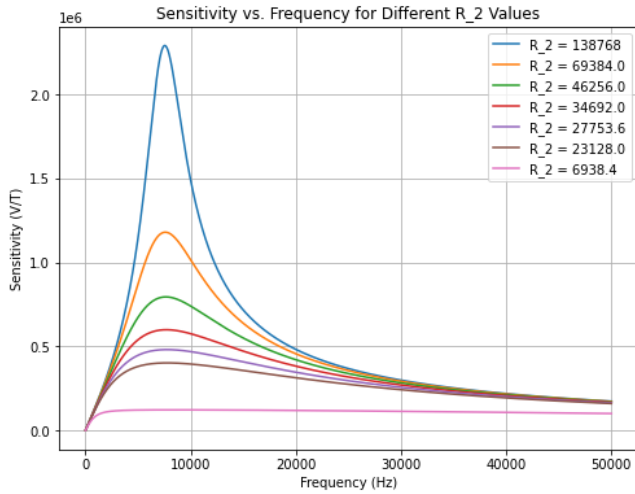


FIG. 3: Frequency response of the search coil sensitivity for different  $R_2$  values.

field created by a Helmholtz coil. The generated magnetic field could be calculated as  $B = V_{fg}/S_{fg}$ , where  $V_{fg}$  is the voltage measured by an auxiliary fluxgate magnetometer, and  $S_{fg}$  is the sensitivity of the fluxgate (a known value given by the fluxgate's manufacturer). Then, the experimental sensitivity value could be calculated as  $S_{sc} = V_{sc}/B$ , where  $V_{sc}$  is the voltage measured by the search coil and  $B$  is the applied magnetic field. 11 voltage values were measured for both the search coil and the fluxgate, at frequencies ranging from 40 Hz to 1 kHz.

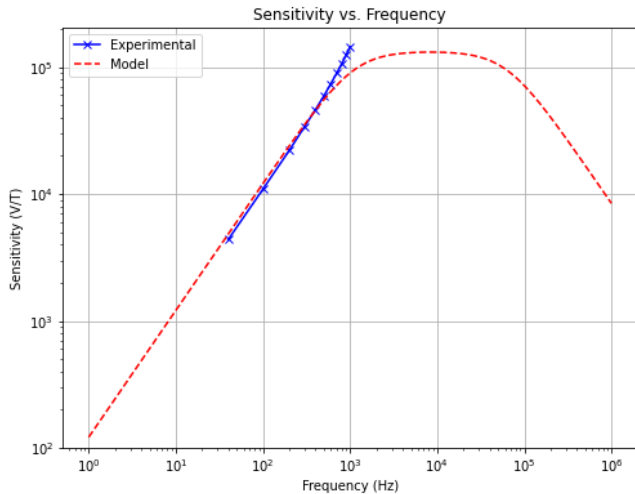


FIG. 4: Comparison between the sensitivity predicted by the RLC circuit model and the experimental sensitivity measured with the search coil.

As can be seen in Figure 4, the experimental sensitivity matches the one predicted by the model remarkably well in the 50-500 Hz range. This is the spectrum where it is more important to have a good sensitivity, since it is

the expected frequency range of the magnetic field noise that we aim to detect. For frequencies higher than 500 Hz, the calculated sensitivity begins to diverge from the model one; the reason for this behaviour is unknown, but it could be due to unreliable fluxgate measurements in higher frequencies.

## V. EQUIVALENT MAGNETIC FIELD NOISE

The equivalent magnetic field noise  $e_B$  of the search coil is given by

$$e_B = \frac{e_{total}}{S} \quad (7)$$

where  $S$  is the search coil sensitivity and  $e_{total}$  is the sum of all contributions to the total noise, such as thermal noise density from resistances, or voltage and current noise from an amplifier. Some degree of amplifier noise is unavoidable, since the signal provided by an induction search coil is generally weak and needs to be amplified with an additional device. A theoretical prediction of  $e_{total}$  was also performed, but it could not be fit in this paper due to lack of space.

In order to measure the equivalent magnetic field noise, another experiment was planned, in which the search coil was going to be connected in a circuit with a data acquisition board. Such device is capable of detecting small random noises for different input voltages. The circuit's power supply was to be placed inside a Faraday cage, for the sake of shielding the search coil from external electromagnetic interferences. Both the coil and the Faraday cage containing the power supply were going to be placed inside a mu-metal shielding case, so as to reduce as much as possible the exterior magnetic field. Then the measurements with the data acquisition board would have been performed.

However, due to time constraints, it was not possible to carry out this experiment. Instead, a Python script was used to generate data which simulates the one that would have been obtained in the experiment.

The script generates magnetic noise data that imitates the one emitted by the search coil, and then adds two spikes (at 1 kHz and 10 kHz respectively) with an amplitude of 1 nT. Then it calculates and plots the power spectrum of the magnetic field noise (its distribution into frequency components). It does so for four different amounts of data points at a constant 40 kHz sampling frequency, up until  $10^5$  points for an integration time of 2.5 s.

We can see in figure 5 that, as long as we have enough data points (so specially for  $N = 10^4$  and  $N = 10^5$ ) we can obtain a good frequency resolution for the magnetic field noise power spectrum. This is relevant, because the search coil will be producing a high amount of data that can not be downloaded to Earth due to a limited telemetry budget in LISA. Therefore, an in-flight processing is

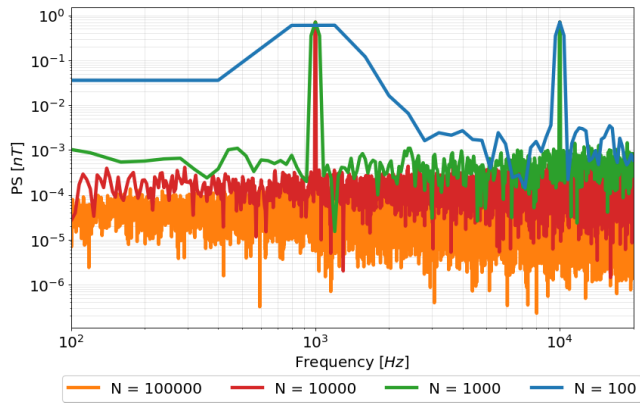


FIG. 5: Equivalent magnetic field noise simulation.

necessary, and according to this first raw estimation, retrieving data samples every 2.5 s could be enough.

It can also be seen that the equivalent B noise stays constant with frequency (if we exclude the two added spikes). This is not too good of an approximation to reality, since the magnetic field noise actually has a minimum plateau between  $10^3$  and  $10^5$  Hz and then it increases for lower and higher frequencies.

The magnetic field value, on the other hand, is a better approximation to reality, albeit a bit optimistic. The simulation provides values in the vicinity of  $10^{-5}$  nT/ $\sqrt{\text{Hz}}$ , while previous works detected a magnetic noise of  $8.5 \cdot 10^{-5}$  nT/ $\sqrt{\text{Hz}}$  for a frequency of  $10^4$  Hz. The values previously obtained for the equivalent magnetic noise of the search coil were already lower than the

LISA mission requirements, so we can assume that if the values given by the simulation were real, the experiment would be considered a success.

## VI. CONCLUSIONS

- The search coil sensitivity measured experimentally matches the one predicted by the model in the target frequency range (50-500 Hz), and it is below the LISA mission requirements.
- Assuming the equivalent magnetic field noise simulation is a realistic enough estimation of reality, the search coil also fulfills LISA magnetic field noise requirements.
- As already hinted by its success in LISA Pathfinder, the search coil can be a very suitable option to fulfill the magnetometer role in the future LISA mission.

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- [1] De Mees, T. "The Prediction of Gravitational Waves in 1893". *The General Science Journal* (2014).
  - [2] Abbott, B. P. et al. (LIGO Scientific Collaboration and Virgo Collaboration). "Observation of Gravitational Waves from a Binary Black Hole Merger". *Physical Review Letters* **116**, 061102 (2016).
  - [3] Mateos, I. et al. "Temperature coefficient improvement for low noise magnetic measurements in LISA". *Journal of Physics: Conference Series* **363**, 012051 (2012).
  - [4] Danzmann, K. et al. "Laser Interferometer Space Antenna. A proposal in response to the ESA call for L3 mission concepts". arXiv preprint. 1702.00786 (2017).
  - [5] Armano, M. et al. "Beyond the required LISA free-fall performance: new LISA Pathfinder results down to 20  $\mu\text{Hz}$ ". *Physical Review Letters* **120**, 061101 (2018).
  - [6] Ho Zhang, J. J. "Design of an antenna for the detection of kHz electromagnetic disturbances on-board LISA". Master Thesis, UPC (2019).
  - [7] Ueda, H. and Watanabe, T. "Several Problems about Sensitivity and Frequency Response of an Induction Magnetometer". *Science reports of the Tohoku University, Ser. 5, Geophysics* (1975).
  - [8] Pudney, M. et al. "ADAM - Advanced DC and AC magnetic verification for the LISA mission and beyond" ESA Workshop on Aerospace EMC (2022).
  - [9] Armano, M. et al. "Disentangling the magnetic force noise contribution in LISA Pathfinder". *Journal of Physics: Conference Series* **610**, 012024 (2015).
  - [10] Boileau, G. et al. "Figures of merit for a stochastic gravitational-wave background measurement by LISA: Implications of LISA Pathfinder noise correlations". *Physical Review D* **106**, 063025 (2022).
  - [11] Castelli, E. "LISA Pathfinder noise performance results: disturbances in the sub-mHz frequency band and projection to LISA". Doctoral Thesis, Univ. of Trento (2020).
  - [12] Joffre, E. et al. "LISA: Heliocentric formation design for the laser interferometer space antenna mission". *Advances in Space Research* **67**, 11, 3868-3879 (2021).
  - [13] Bendat, J. S. and Piersol, A. G., *Random Data: Analysis and Measurement Procedures*, (John-Wiley & Sons, New York 2010, 4th. ed.).
  - [14] Analog Devices, Inc. "Voltage feedback op amp gain and bandwidth". MT-033 Tutorial (2009).
  - [15] Analog Devices, Inc. "4 MHz, 7 nV/ $\sqrt{\text{Hz}}$ , low offset and drift, high precision amplifiers". ADA4077-1/ADA4077-2/ADA4077-4 Data Sheet (2022).