Phased array acousto-optic deflector

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Abstract: We study and model diffraction efficiency as function of frequency for a 2-element planar phased array TeO_2 acousto-optic deflector. This research aims to widen the bandwidth by means of acoustic beam steering and to find a phase delay that yields high efficiency throughout the working frequency range. The model is based on a geometric basis, which we then compare to experimental measures of efficiency with and without beam steering.

I. INTRODUCTION

The acousto-optic effect is based on the interaction between a light beam and an acoustic wave inside a medium. This interaction can be effectively studied through Bragg diffraction [1], a process by which an optical beam is diffracted by the periodic changes in refractive index caused by the acoustic wave, particularly when the angle between the optical ray and the acoustic wavefronts aligns with the Bragg angle θ_B, given by:

$$\sin\theta_B = \frac{1}{2}\frac{\lambda_0/n}{\Lambda} = \frac{\lambda_0}{2nV_a}f \tag{1}$$

Here, λ_0 is the light wavelength in vacuum, n is the refractive index of the medium, Λ is the wavelength of the acoustic beam, and V_a and f are the velocity and frequency of sound waves, respectively. In quantum theory, Bragg condition is interpreted as a case of conservation of momentum. A photon with wave vector $\mathbf{k_i}$ combines with an acoustic phonon with wave vector $\mathbf{K_a}$ to generate a new photon $\mathbf{k_d}$. This last photon meets the condition:

$$\mathbf{k_d} = \mathbf{k_i} + \mathbf{K_a} \tag{2}$$

as illustrated in FIG. 1.

Acousto-optic cells can be used in a variety of ways [1], such as optical modulation, filtering, switching, spectrum analysis, and the specific focus of this research, the acousto-optic deflector (AOD). In AODs, the deflection angle $2\theta_B$ between the incident wave vector ($\mathbf{k_i}$) and the diffracted wave vector ($\mathbf{k_d}$) is controlled by means of the sound frequency f. One of the key parameters of an acousto-optic deflector, known as the deflection angle range $\Delta\theta$ is directly proportional to the range of working frequencies, known as the deflector bandwidth Δf:

$$\Delta\theta \approx \frac{\lambda_0}{nV_a}\Delta f \tag{3}$$

By making a rough estimation using some typical parameters, such as $\lambda_0 = 600\,\text{nm}$, $n = 2$, $V_a = 1\,\text{km/s}$, and $\Delta f = 50\,\text{MHz}$, we observe that the scan angle is less than $1°$ inside the crystal ($\approx 1.5°$ in air), which is considerably low when compared to light deflectors that utilize alternative technologies [2].

According to Eq.(3), the way to increase the deflection angles is to increase the bandwidth while maintaining acceptable optical efficiency. Diffraction efficiency, alongside bandwidth, constitute the crucial parameters to take into consideration in the design of an AOD. Consequently, the primary quality criterion for an AOD is to maximize the bandwidth-efficiency product. It is important to note that for collimated incident light and sound beams, the Bragg condition (or perfect momentum matching) is only satisfied at a single frequency. If the frequency is varied within the bandwidth, it would be necessary for the incidence angle to change accordingly in order to maintain the strong coupling. This is unfeasible in practical experiments.

Fortunately, the acoustic wave exhibits angular divergence due to the finite and small size of the piezoelectric transducer which, under an applied electric field, vibrates to generate the sound wave in the medium. The acoustic divergence caused by diffraction means that for a single frequency f we have numerous sound waves with varying directions and amplitudes but with the same wave number $K_a = 2\pi f/v_a$. This allows the Bragg condition to be fulfilled for multiple frequencies without the need to modify the incident wave vector [3],[4], as illustrated in FIG. 1(b): if we fix the incident optical angle and increase the sound frequency, the longer acoustic wave vector propagating in the previous direction would not satisfy the Bragg condition (what is called "momentum mismatch"), but there still will be a diffracted sound component, travelling at a different direction, for which $\mathbf{k'_d} = \mathbf{k_i} + \mathbf{K'_a}$. The price to pay is a drop in the acousto-optic coupling efficiency due to a decrease in the amplitude of the interacting sound wave. This ultimately gives rise to a narrow range of frequencies that exhibit acceptable efficiency.

A smart solution to further expand the bandwidth while simultaneously maintaining maximum efficiency is by beam steering the acoustic wave[5]. This can be accomplished by utilizing phased-array transducers, which enable the manipulation of the acoustic wave angle θ_a by introducing a phase shift between the signals of each transducer [6],[7]. The constructive and destructive interferences resulting from the multiple acoustic waves generated by the transducers allow steering of the acoustic wave: by selecting the appropriate phase shifts, it is possible to radiate the acoustic wave in the direction that

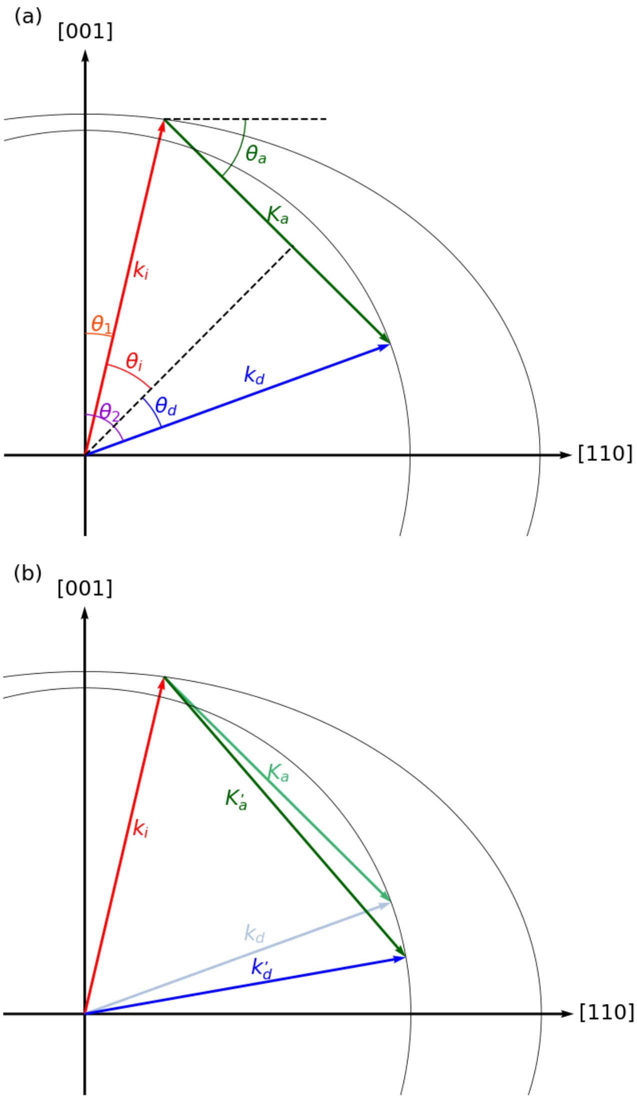

FIG. 1: (a) Wave vector diagram of birefringent acousto-optic interaction when Bragg condition is perfectly satisfied. The wave vectors $\mathbf{k_i}$ and $\mathbf{k_d}$ must lie on the index surfaces of the crystal (thin gray lines). For birefrigent acousto-optic interaction, the incident optical wave commonly propagates with the extraordinary index of refraction while the diffracted one is an ordinary wave. Note that in this case, the incident and diffracted angles satisfying the Bragg condition are different, $\theta_\mathbf{i} \neq \theta_d$. $\mathbf{K_a}$ is the sound wave vector, propagating at an angle θ_a from the [110] direction. The incident and diffracted beams close angles θ_1 and θ_2 with the [001] direction, and θ_i and θ_d with the normal to $\mathbf{K_a}$. (b) If the same incident optical beam is maintained and the frequency f increased, the new, longer wave vector $\mathbf{K_a'}$ would need to have a different direction in order to satisfy Bragg condition.

best satisfies the Bragg condition for each frequency, resulting in a maximized diffraction efficiency across the bandwidth.

For AODs, often a TeO_2 (paratellurite) crystal is utilized, which is renowned for its elasto-optic properties[8]: its birefringence, optical activity and a high figure of merit when the acoustic beam travels along the [110] direction (slow-shear mode), result in a good performance in terms of the bandwidth-efficiency product.

The aim of this research is to study and characterize a TeO_2 phased-array AOD that incorporates two transducers, with the view in finding the configuration that maximizes the efficiency-bandwidth product.

II. THEORETICAL MODEL

The study of the deflection of light by periodic sound waves is complicated and requires the solution of coupled-wave equation between the incident electric field, the acoustic field locally modifying the index of refraction of the medium, and the diffracted field. In acousto-optic deflectors, most of the light is either transmitted in the zeroth order (undiffracted) or deflected in the first order of diffraction. The efficiency η is then defined as the ratio between the intensity of light in the first order and the intensity of light in the zeroth order. It is well known that maximum efficiency occurs for perfect momentum matching and can be even close to 100% depending on the strength of acousto-optic interaction, the interaction length and the power of the sound wave. Things become harsh when the sound frequency is varied and there is a momentum mismatch that inevitably causes a decrease in the amount of light being effectively deflected.

Our goal is to compute the efficiency curves (efficiency as a function of sound frequency) by using a simplified geometric model in which the drop in efficiency is solely due to the reduction in the intensity of the acoustic angular component that best suits Bragg condition, according to the far-field or Fraunhofer diffraction distribution of the sound transducers. For a single transducer, we have compared our results with the exact solution given in the literature, and the results differ by less than 2%. We will study the efficiency curves when two transducers are used powered by the same periodic signals, and then how these curves can be improved by properly tuning the phase shift between the two channels to steer the acoustic beam at each frequency.

Applying Eq.(2) for a anisotropic medium we obtain the fundamental equations of anisotropic AO diffraction[9]:

$$\sin\theta_i = \\ = \frac{\lambda_0}{2n_i(\theta_1)V_a(\theta_a)}\left[f + \frac{V_a^2(\theta_a)}{\lambda_0^2 f}\left(n_i^2(\theta_1) - n_2^2(\theta_2)\right)\right] \tag{4a}$$

$$\sin\theta_d = \\ = \frac{\lambda_0}{2n_d(\theta_2)V_a(\theta_a)}\left[f - \frac{V_a^2(\theta_a)}{\lambda_0^2 f}\left(n_i^2(\theta_1) - n_2^2(\theta_2)\right)\right] \tag{4b}$$

Here, n_i and n_d are the refractive indexes of the incident and diffracted light waves, respectively. Angles θ_a, θ_i, θ_d, θ_1 and θ_2 are illustrated in FIG. 1.

Due to the anisotropy and rotatory power δ of the material, the refractive indexes for incident and diffracted rays depend on θ_1 and θ_2 [3].

$$n_i(\theta_1) = \frac{n_e n_o(1+\delta)}{\sqrt{n_e^2 \cos^2\theta_1 + n_o^2(1+\delta)^2 \sin^2\theta_1}} \quad (5a)$$

$$n_d(\theta_2) = \frac{n_o(1-\delta)}{\sqrt{\cos^2\theta_2 + \sin^2\theta_1}} \quad (5b)$$

We have assumed that the incident light corresponds to the extraordinary mode while the diffracted light is ordinary light, which is the usual scenario[9]. Sound velocity is also dependent on the direction of the acoustic wave. It is at its slowest when it propagates along [110], $V_t = 616\,\mathrm{m/s}$, and it reaches maximum velocity along [001], $V_z = 2104\,\mathrm{m/s}$. [10]

$$V_a^2 = V_t^2 \cos^2\theta_a + V_z^2 \sin^2\theta_a \quad (6)$$

To solve Eqs.(4) and find at each frequency the required directions of the optical and acoustic wave vectors for momentum matching, it is necessary to fix one of them: either θ_1, θ_2 or θ_a. We start by fixing the direction θ_a, which we call $\bar{\theta}_a$, supposing that we have a collimated sound wave, and determining the directions of the optical beams. In this case, the angle θ_1 satisfying Bragg condition varies with frequency, and should be selected by rotating the AOD. Solutions $\theta_1(f)$ and $\theta_2(f)$ are obtained from Eqs.(4), which can then be converted to $\theta_i(f)$ and $\theta_d(f)$. Note that all the angles here are referring to the deflection occurring inside the crystal. To obtain the deflection outside the crystal, which are the ones that can be measured, we simply use Snell's law.

As mentioned in section I, rotating the AOD is inadequate due to the requirement of additional mechanical components. It is worth commenting here that acousto-optic deflectors are competitive devices due to their extremely fast response, in the order of hundreds of kHz, and the need of mechanical elements would inadequately slow them down. So we will now fix the angle of incidence to the value that achieves perfect momentum match for a certain frequency f_B and solve Eqs.(4) in order to obtain the diffracted angle $\theta_2(f)$ and, more importantly, the angle of the sound wave $\theta_a(f)$ by solving Eqs.(4).

Subsequently, we move on to calculate the diffraction efficiency. Given the acoustic wave and transducers dimension parameters, maximum efficiency in the diffracted beam occurs when Bragg condition is satisfied:

$$\eta_0 = \sin^2\left(\frac{\pi}{\lambda_0}\sqrt{\frac{M_2 L P_a}{2H\cos\theta_i\cos\theta_d}}\right) \quad (7)$$

Here, P_a is the acoustic wave power, L and H are the length and the height of the transducer, respectively, and M_2 is the acousto-optic figure of merit, given by the following expression:

$$M_2 = \frac{n_i^3(\theta_1) n_d^3(\theta_2) p^2}{\rho V_a^3(\theta_a)} \quad (8)$$

p is the effective photo-elastic coefficient for our configuration, and $\rho = 5.99\,\mathrm{g/m^3}$ is the density of paratellurite. The divergence of the acoustic waves due to the finite shape of the transducers allow momentum matching at the cost of reducing the efficiency compared to the maximum one given by Eq. (7). These effects are described by the following expressions:

$$H_1 = \mathrm{sinc}^2\, x = \frac{\sin^2(\pi x)}{(\pi x)^2} \quad (9)$$

$$H_2 = \cos^2\left(\pi\frac{xD}{L} - \phi\right) \quad (10)$$

with x being:

$$x = \frac{L(\theta_a - \bar{\theta}_a)}{V_a} f$$

H_1 is the angular spectrum of the acoustic field intensity stimulated by a single rectangular transducer of length L, H_2 is the Young interference pattern produced by 2 transducers separated a distance D and $\theta_a - \bar{\theta}_a$ represents the angular deviation of the sound component with respect to the original one. Angle ϕ is the phase shift between the transducers, which allows steering the sound wave by an angle $\phi f/(V_a D)$ and minimize the total angular deviation. The final efficiency is then calculated as the product of Bragg diffraction efficiency η_0 and the diffraction pattern of the transducers:

$$\eta = \eta_0 \times H_1 \times H_2 \quad (11)$$

We will next describe the experimental set-up in which the AOD was used to deflect light and efficiency curves were obtained and optimized. The results will be compared with the predictions of our model.

III. EXPERIMENTAL SET-UP

The light source being used is a laser with a wavelength of 532 nm, linearly polarized and adjustable power output. The light beam is initially passed through a high-quality polarizer to increase the extinction ratio of linear polarization, a rotating half-wave plate and a rotating quarter-wave plate to adjust the polarization to the extraordinary mode for TeO_2, which is almost circular close to the optical axis. Then the beam is expanded and collimated before entering the deflector. The AOD is mounted on an automatized platform to control the incidence angle with precision.

The AOD possesses two input channels for injecting radio frequency (RF) signals. These are synthesized using MATLAB and generated with a high speed DAC (PX-DAC4800), the two signals then go through an amplifier before entering the AOD. The signals generated for this experiment are single frequency sinusoidal waves, with varying phase delays to maximize the efficiency

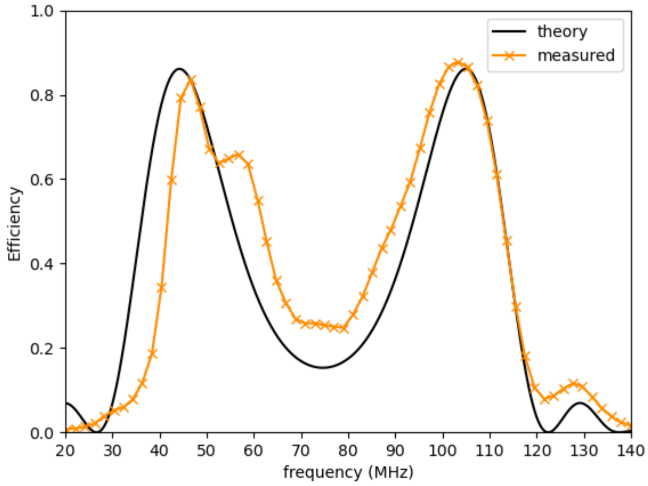

FIG. 2: Efficiency curves of acousto-optic interaction when the angle of incidence is set to match Bragg condition for $f_B = 105\,\mathrm{MHz}$ and $\phi = 0$. We compare the results of our model with the measurements.

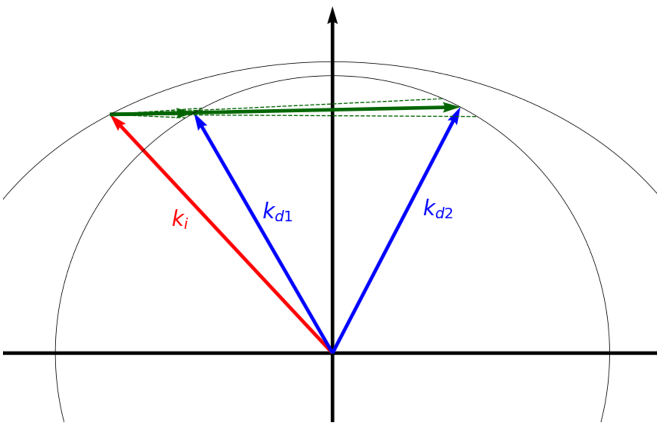

FIG. 3: Representation in k-vector diagram of the two frequencies with perfect momentum matching and the divergence of the acoustic waves.

at each frequency. Frequency range of the output signals varies from 20-150 MHz with the maximum recommended $P_a = 0.8\,\mathrm{W}$.

The light coming out of the AOD is composed of undiffracted light, the first order and some weaker higher orders. The different rays pass through a lens following a beam splitter. One part is directed and focused towards a CMOS camera to capture the position of each diffracted beam, while the other part is directed towards a power meter for measures of efficiency. To discard the unwanted orders, a slit is placed to block the zeroth order and a circular analyzer used to only let through the ordinary light. This way, rediffracted light from the first order[11] (see Section IV. and V.) can also be blocked, allowing to isolate the first order diffracted beam for efficiency measurements.

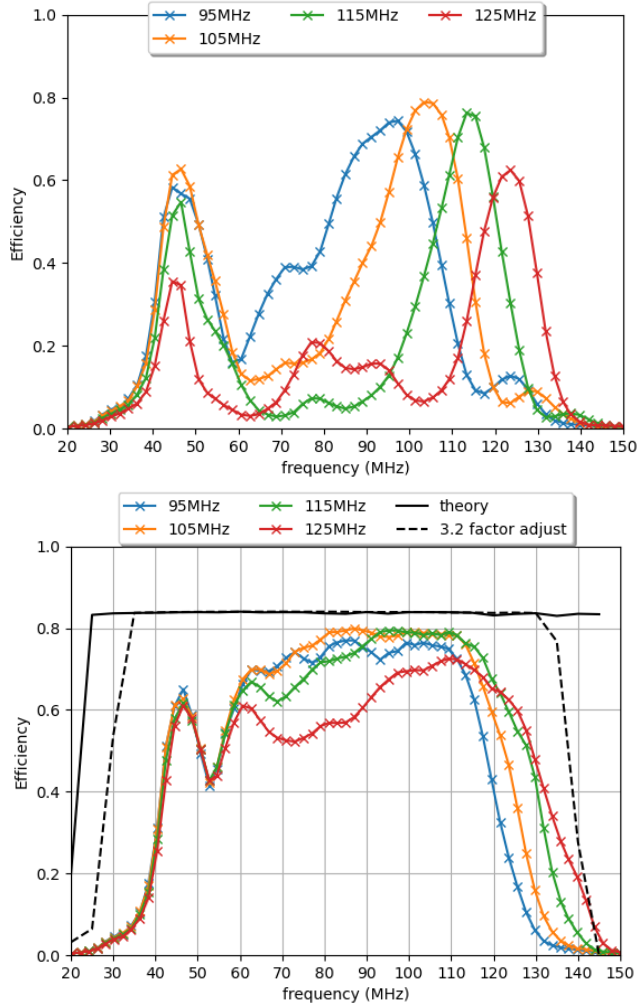

FIG. 4: Measured efficiency as a function of acoustic frequency of the first order diffracted beam when maximizing frequencies f_B 95, 105, 115, and 125 MHz, with (a) phase delay $\phi = 0$ and (b) optimal phase delay for each frequency, includes the model prediction and a adjusted model with a 3.2 factor correction in the optimal phase shift calculation.

IV. RESULTS

For the modelization, $\bar{\theta}_a = -1.1°$ is used as the angle between the acoustic wave and the [110] direction as it produces the best fit to measured data. FIG. 2 shows the calculated and measured efficiency for the case of maximum efficiency at 105 MHz without phase shift using the AOD model presented here. These two peaks at 45 MHz and 105 MHz correspond to the two frequencies that satisfy Bragg diffraction, the k-vector diagram in FIG.3 illustrates this case. Here we can clearly see that due to the divergence of the acoustic wave, the efficiency does not immediately drop to zero for frequencies around the peaks.

In FIG.4, four overlayed measurements of efficiency for four different f_B are shown. One (a) with no phase delay and the other one (b) with optimal phase shift, so that the acoustic wave deviation always produces the maximum efficiency possible. From the measurements shown in FIG.4(b) we can verify that acoustic beam steering does indeed increase the bandwidth while being able to maintaing acceptable efficiency. However, as f_B increases, a gradual loss in maximum efficiency is observed. Efficiency for $f_B = 125\,\text{MHz}$ is around 15% lower than efficiency for $f_B = 105\,\text{MHz}$. This can be explained due to secondary acoustic beams originating from the interference of the transducers, as phase shift is increased[12]. The principal acoustic beam loses energy to the side lobes resulting in a lower AO diffraction efficiency.

Making a comparison between the two efficiencies measured for $f_B = 105\,\text{MHz}$, we can notice that without beam steering we have a 3 dB bandwidth of 18 MHz, from about 94 to 112 MHz. The maximum diffraction efficiency is around 78% at its peak. In contrast, when beam steering is utilized and the optimal phase shift is applied, we have a 3 dB bandwidth of 65 MHz, from 57 to 122 MHz, while maintaining around 78% efficiency. It is interesting to note that for every f_B measured drom 95 to 125 MHz they present the same dip in efficiency at around 53 MHz. This energy loss can be attributed to the rediffraction of the first order, which will be explained in the next section.

V. CONCLUSION

This approximate model of anisotropic AO interaction presented here is able to describe with enough precision the peaks of efficiency and bandwidth of a 2-transducer AOD with no phase shift. Position of the peaks in FIG.4 align with the predictions of the model. We show FIG.2 as an example. Additionally, the angle of the acoustic wave $\bar{\theta}_a = -1.1^\circ$ used for the calculations can be considered as valid for purposes of crystal orientation determination.

One of the problems encountered during the investigation is the rediffraction of the first order beam, that occurs when the Bragg condition is satisfied again for the diffracted beam, this time the rediffracted beam has an extraordinary polarized state, so it can be filtered away. We can observe this effect in FIG.4(b) where a significant dip in efficiency occurs around 55 MHz.

Future models will also include more precise simulations of the acoustic wave generated by the transducers. We think our model overestimates both the angular sound diffraction pattern and the steering produced by ϕ. Still, the conclusions about the use of phased array transducers to increase the bandwidth-efficiency product applicable. With optimal phase shift, the deflector is able to achieve a considerable efficiency ($> 55\%$) with a bandwidth of more than 60 MHz for most configurations. If rediffraction is not taken into account, the bandwidth could reach 80 MHz.

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