New neutrino physics effects on the universe expansion history

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Abstract: We investigate the impact of new neutrino physics on the expansion history of the universe, we analyzing first-order perturbations of neutrinos interacting with a very light and week interacting scalar field. Considering the interplay between neutrinos, scalar fields, and the cosmological environment, we explore the theoretical framework to uncover potential effects on the cosmic evolution.

I. INTRODUCTION

The extent of the universe and complexities of its development have long intrigued physicists. Among the cosmic trellis, normally thought of as elusive and spectral neutrinos have become a major player in shaping this large-scale structure. In the recent years cosmological observations get large attention to study properties of neutrinos complementing the ones from experiments.

Experimental examination of interactions characterized by small interaction coupling ($g \ll 1$) poses a challenge due to the proportionality of collision probability in scattering experiments to g^4. Consequently, such experiments are unreliable for studying these phenomena. Nevertheless, in the cosmological fluid, where the number of neutrinos is significantly large, we can effectively investigate the impact of these interactions when the inter-particle distance is much smaller than the inverse of the mediator mass ($1/M_\phi$).

In a cosmological context, the study of neutrinos has been rapidly growing in the recent decades. Given their abundance and weak interaction properties, neutrinos possess the potential to exert a significant influence on the evolution of the universe. An example of this is the study of the mass parameter, being cosmological data the one providing the best bound today. Our focus is directed towards the examination of long-range neutrino interactions, where effects are significant over substantial distances.

Several theoretical frameworks [1] have been developed to comprehend the interplay between neutrinos, scalar fields, and the overarching cosmological environment providing a theoretical foundation for exploring the nuanced effects that neutrinos can have on the large-scale structure of the universe.

In this context, this work seeks to extend this theoretical groundwork by examining first-order perturbations to the scalar field generated by neutrinos. Building upon the formalism developed in [1]-[2], we aim to uncover novel effects on the expansion history of the later-on universe. Through a meticulous analysis of the scalar and fermion field dynamics, we explore the consequences of new neutrino physics on cosmic evolution.

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Our investigation focuses into two limit scenarios: the behavior of ultra-relativistic neutrinos at the early universe, and those non-relativistic at the late universe evolution with low temperature. By analyzing these two regimes, we aim to provide a comprehensive understanding of how the scalar field generated by neutrinos shapes the evolution of the universe.

II. FORMALISM

The fundamental description of our system is encapsulated in the action denoted by

$$S = \int \sqrt{-\mathscr{G}} d^4x \left(-\frac{1}{2} D_\mu \hat{\phi} D^\mu \hat{\phi} - \frac{1}{2} M_\phi \hat{\phi}^2 \right. \\ \left. + i\psi \not{D} \overline{\psi} - m_0 \overline{\psi}\psi - g\hat{\phi}\overline{\psi}\psi \right) . \tag{1}$$

This action characterizes our system, involving the scalar field $\hat{\phi}$ and the fermion field $\overline{\psi}$. Here, $\mathscr{G}$ represents the determinant of the metric, and D_μ is the corresponding covariant derivative. The parameters M_ϕ and m_0 stand for the masses of the scalar and fermion fields, respectively, while g signifies the coupling strength. The Euler-Lagrange equations then yield the field equations

$$-D_\mu D^\mu \hat{\phi} + M_\phi^2 \hat{\phi} = -g\overline{\psi}\psi \, , \tag{2}$$

and

$$i\not{D}\psi - (m_0 + g\hat{\phi}\psi) = 0 \, . \tag{3}$$

In order to derive the classical scalar field equation produced by a collection of thermally distributed fermions, we take the expected value of the quantum field on the thermal state $\overline{\psi}\psi$, details can be found in [1],

$$\langle \overline{\psi}\psi \rangle = \int \frac{dP_1 dP_2 dP_3}{\sqrt{-\mathscr{G}}} \frac{\tilde{m}_0}{P^0} f(x^\mu, P_\mu) \, . \tag{4}$$

Here, $\tilde{m}_0 = m_0 + g\hat{\phi}$ represents the effective fermion mass at zero-th order perturbation, and $f(x^\mu, P_\mu)$ stands for the statistical fermion distribution function. Since we are considering a distance scale $d \ll \frac{1}{H}$ we can assume a to be a constant and the metric can be written as

$$ds^2 = -d\tau^2 + \delta_{ij} dx^i dx^j \, , \tag{5}$$

Using the invariance of $\hat{\phi}$ equation (4) then simplifies to

$$-D_\mu D^\mu \hat{\phi} + M_\phi^2 \hat{\phi} = -g \int d^3\vec{p} \frac{\tilde{m}_0}{\sqrt{|\vec{p}|^2 + \tilde{m}_0^2}} f(x^\mu, P_\mu(\vec{p})) \,. \tag{6}$$

The evolution of the fermion distribution function $f(x^\mu, P_\mu)$ is governed by the Boltzmann equation

$$\frac{\partial f}{\partial x^0} + \frac{dx^i}{dx^0}\frac{\partial f}{\partial x^i} + \frac{dP_i}{dx^0}\frac{\partial f}{\partial P_i} = \left(\frac{\partial f}{\partial x^0}\right)_C \,. \tag{7}$$

In this context, the collision term on the right-hand side is represented by $\left(\frac{\partial f}{\partial x^0}\right)_C$. The coefficients $\frac{dx^i}{dx^0} = \frac{P^i}{P^0}$ and $\frac{dP_i}{dx^0}$ are determined by the geodesic equation

$$P^0 \frac{dP^\mu}{dx^0} + \Gamma^\mu_{\alpha\beta} P^\alpha P^\beta = -\tilde{m}_0 g \partial^\mu \hat{\phi} \,. \tag{8}$$

III. DEVELOPMENT

In this section, we will investigate the equation for the scalar field ϕ in the absence of perturbations, assuming a homogeneous and isotropic universe described by the FLRW metric

$$ds^2 = a(\tau)^2(-d\tau^2 + \delta_{ij}dx^i dx^j) \,. \tag{9}$$

Here, $\tau = \int \frac{dt}{a}$ denotes the conformal time, and t represents the cosmological time. The evolution of the scale factor is related to the total energy density of the Universe, ρ_{tot}, through the Friedman equation

$$\left(\frac{da}{d\tau}\right)^2 = \frac{8\pi G_N}{3} a(\tau)^4 \rho_{tot} \,. \tag{10}$$

In a homogeneous and isotropic Universe, the fermion distribution function depends solely on τ and the momentum modulus

$$f(x^\mu, P_\mu) = f_0(\tau, q) \,. \tag{11}$$

Here, $q = a|\vec{p}|$. In this scenario, the Boltzmann equation takes the form

$$\frac{\partial f_0}{\partial \tau} + \frac{dq}{d\tau}\frac{\partial f_0}{\partial q} = \left(\frac{\partial f_0}{\partial \tau}\right)_C \,. \tag{12}$$

To investigate long-range interaction effects, we omit the collision term in the Boltzmann equation. With some algebra, we deduce that $\frac{dq}{d\tau} = 0$ from component 0 of the geodesic equation. Using the definition

$$D_\mu D^\mu \hat{\phi} = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \hat{\phi}\right) \,. \tag{13}$$

the equation of motion (7) for a homogeneous scalar field $\phi_0(\tau)$ becomes

$$\frac{\ddot{\phi_0}}{a^2} + 2H\frac{\dot{\phi_0}}{a} + M_\phi^2 \phi_0 = -g \int d^3\vec{p} \frac{\tilde{m_0}}{\sqrt{|\vec{p}|^2 + \tilde{m_0}^2}} f(x^\mu, P_\mu(\vec{p})) \,. \tag{14}$$

being $H = \frac{\dot{a}}{a^2}$. Hence, we derive a Klein-Gordon equation with a field-dependent source term, leading to an effective correction to the scalar mass

$$M_T^2 = \frac{\partial}{\partial \phi_0}\left(g \int d^3\vec{p} \frac{\tilde{m}_0}{\sqrt{|\vec{p}|^2 + \tilde{m}_0^2}} f(x^\mu, P_\mu(\vec{p}))\right) \,. \tag{15}$$

In equation (15), there exist two characteristic timescales. Firstly, there is the Hubble time scale, denoted as H^{-1}, which governs both the Hubble friction term $\frac{2H}{a}\dot{\phi}_0$ and the rate at which the right-hand side changes. Secondly, we have the inverse of the scalar field mass, denoted as $M_{eff}^{-1} = \left(M_\phi^2 + M_T^2\right)^{-1/2}$, which controls the characteristic oscillation time of the scalar field.
The relative values of M_{eff} and H determine different scenarios. We are particularly interested in the case where $M_{eff} \gg H$, as this situation corresponds to the adiabatic approximation. In this scenario, we can express the field as $\phi_0(\tau) = \overline{\phi_0(\tau)} + \varphi(\tau)$, where $\overline{\phi_0}$ satisfies

$$M_\phi^2 \overline{\phi_0} = -g \int d^3\vec{p} \frac{\tilde{m}_0}{\sqrt{|\vec{p}|^2 + \tilde{m}_0^2}} f(x^\mu, P_\mu(\vec{p})) \,. \tag{16}$$

The evolution of $\varphi(\tau)$ can then be described by the equation

$$\frac{\ddot{\varphi}}{a^2} + 2H\frac{\dot{\varphi}}{a} + \left(M_\phi^2 + \overline{M_T^2}\right)\varphi + \mathscr{O}(\varphi^2) = \mathscr{O}(H^2\overline{\phi_0}) \,. \tag{17}$$

Here, $\overline{M_T^2}$ is determined by equation (16) evaluated at $\phi_0 = \overline{\phi_0}$. The scalar field can be separated into a component sourced by the fermions (equation 19) and a rapidly oscillating component (satisfying equation 20). The latter corresponds to a background of ϕ particles at rest and is non-zero only if set by the initial condition, with small corrections of order $\mathscr{O}\left(\frac{H^2}{M_T^2 + M^2}\right)$. As it does not affect the scalar field sourced by the fermions $\overline{\phi_0}$, we will not study it.

A. Solution for a Thermal Fermion Relic

To compute the scalar field ϕ_0 and determine the macroscopic properties of the system, we need to specify the fermion distribution function f_0. We assume that the fermions were in thermal equilibrium in the past and that they thermally decoupled while still relativistic, before long-range effects became relevant. In this case, we can use a Fermi-Dirac distribution for the fermion distribution function

$$f_0(\tau, a|\vec{p}|) = \frac{\mathbf{g}}{(2\pi)^3} \frac{1}{e^{|\vec{p}|/T(a)} + 1} \,. \tag{18}$$

Here, $\mathbf{g}$ represents the number of internal degrees of freedom of the fermion (including particles, antiparticles, and any internal quantum numbers), and T is the temperature. The Boltzmann equation implies that $T \propto \frac{1}{a}$. This distribution is applicable to neutrinos and other hot thermal relics,

with exceptions being particles that never reached thermal equilibrium (e.g., those produced through freeze-in) or particles with non-negligible chemical potential.
Given the explicit expression of $f_0(\tau, a|\vec{p}|)$, we can solve equation (17). To simplify notation, we use $\overline{\phi_0} = \phi_0$. The equation can be rewritten as

$$M_\phi^2 \phi_0 = \frac{-g\mathbf{g}\tilde{m}_0}{2\pi^2} T^2 I_{1/2}(\alpha)\,. \tag{19}$$

Here, we integrated over the (θ, ϕ) components of $\vec{p}$, introduced the variable $p = xT$, defined $\alpha = \frac{\tilde{m}}{T}$, and introduced the integral

$$I_n(\alpha) = \int_0^\infty \frac{x^2}{1+e^x} \frac{dx}{(x^2+\alpha^2)^n}\,. \tag{20}$$

Now we can self-consistently solve the former equation to obtain the results shown in FIG.1.

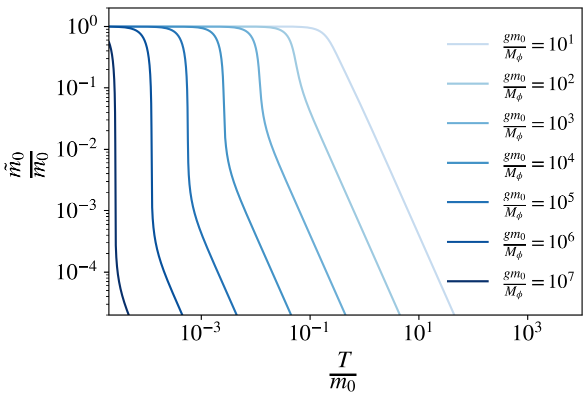

FIG. 1: Effective fermion mass in terms of the temperature

We can compute the contribution to the energy density as

$$\rho = \frac{1}{2} M_\phi^2 \phi_0^2 + \int d^3\vec{p} \sqrt{|\vec{p}|^2 + \tilde{m}^2} f_0(\tau, a|\vec{p}|)\,. \tag{21}$$

shown in FIG. 2.

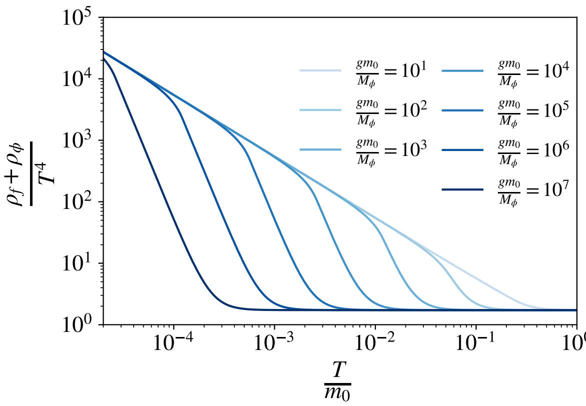

FIG. 2: Energy density in terms of the temperature

B. Perturbation to the Metric and the Fermion Distribution

We will investigate a first-order perturbation described as follows:

$$ds^2 = a^2(\tau)\left(-(1+2\varphi)d\tau^2 + (1-2\psi)\delta_{ij}dx^i dx^j\right)\,. \tag{22}$$

These perturbations can vary with respect to both $\vec{x}$ and τ. Furthermore, we will introduce perturbations in the scalar field, denoted as $\tilde{\phi}_0 = \phi_0(1+\delta\phi_0)$, where ϕ_0 represents the zeroth-order scalar field solution, and in the particle statistical distribution $f = f_0(1+\delta f)$, where f_0 represents the original Fermi-Dirac distribution. We will consider small and soft perturbations, implying $\delta\phi_0 \ll 1$, $\delta\dot{\phi}_0 \ll 1$, and the same for other perturbations. We will proceed to linearize our equations to the first order.
Following some algebraic manipulations and linearization, we derive from equation (7):

$$(1-2\varphi)\frac{\delta\ddot{\phi}_0}{a^2} + \frac{2H}{a}\delta\dot{\phi}_0 - \frac{1+2\psi}{a^2}\nabla^2\delta\phi_0 + \tilde{M}_\phi^2\delta\phi_0 = \frac{M_\phi^2}{I_{1/2}(\alpha_0)}\int_0^\infty du \frac{u^2}{1+e^u}\frac{\delta f(\tau, u = |\vec{p}|/T, |\vec{x}|)}{\sqrt{u^2+\alpha_0^2}} \tag{23}$$

where $\tilde{M}_\phi^2 =$

$$M_\phi^2\left[1 + \frac{\mathbf{g}}{2\pi^2}\left(\frac{g\tilde{m}_0}{M_\phi}\right)^2\left(\alpha_0^{-2} I_{1/2}(\alpha_0) - I_{3/2}(\alpha_0)\right)\right]$$

being $\tilde{m}_0$ the zero-th order fermion effective mass. In the following figure we observe the behaviour of the effective field mass $\tilde{M}_\phi$.

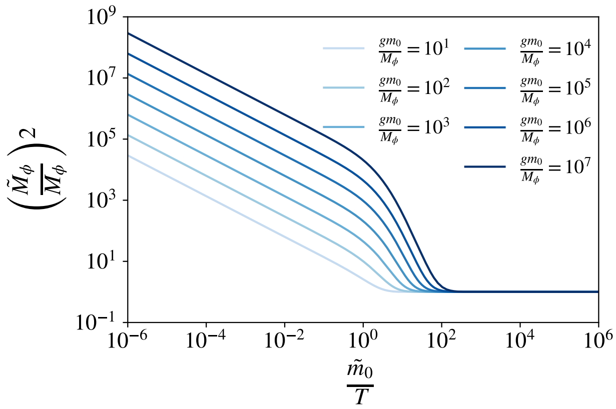

FIG. 3: Effective field mass in terms of the ratio $\frac{\tilde{m}_0}{T}$

For the FLRW metric, the geodesic equation, from which we obtain $\frac{dP^i}{dx^0}$, takes the form:

$$P^0\frac{dP^i}{dx^0} + 2HP^0P^i = -\tilde{m}g\partial_i\tilde{\phi}_0 \tag{24}$$

By substituting this into the Boltzmann equation and linearizing the equations, we obtain a second equation:

$$\delta\dot{f} + \frac{\vec{q}\cdot\vec{\nabla}}{\epsilon}\delta f + A\hat{n}\cdot\vec{\nabla}\delta\phi_0 = 2H\vec{q}\cdot\vec{\nabla}_q\delta f \tag{25}$$

where $q = ap$ $\epsilon = \sqrt{q^2+\tilde{m}_0^2a^2}$ is the energy of the particles and $A = \frac{\tilde{m}_0 g\phi_0 a^2}{\epsilon T}\frac{e^{q/T}}{1+e^{q/T}} = -\frac{g\tilde{m}_0\phi_0 a^2}{q\epsilon}\frac{d\log f_0}{d\log q}$ acts as an effective coupling constant between the fermions and the scalar field. Considering a typical momentum of the particles $q = T$, we obtain the plot shown in FIG. 4.

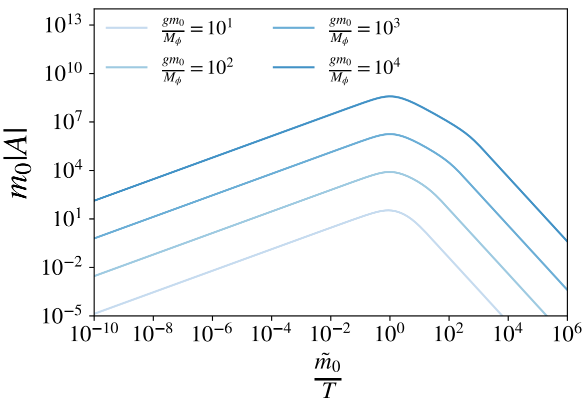

FIG. 4: Value of the coupling constant A in terms of the ratio $\frac{\tilde{m}_0}{T}$. Higher coupling values $\frac{gm_0}{M_\phi}$ generate numerical instability.

We have four unknowns, namely $(\psi, \varphi, \delta\phi_0, \delta f)$, and only two equations. The remaining two equations for gravitational perturbations are provided in [2]. The equations we derived are complex due to the presence of momentum, time, and space derivatives, making them a system of partial integro-differential equations. Nevertheless, we can make some approximations to extract the underlying physics more clearly in specific limits.
We are studying long distances compared to the typical inter-particle distance of neutrinos but short when compared to the cosmological distances, i.e., $\frac{1}{M_\phi} \ll H^{-1}$. Hence, even when considering long-distance neutrino interactions, we can safely neglect the effect of gravity. Hence, we can consider $\psi \approx \varphi \approx 0$ and we will consider the factor scale $a(\tau)$ as a constant, since its effect will be negligible. Neutrinos have very small masses, are electrically neutral, and do not readily clump together or form structures on small scales like dark matter or baryonic matter. We can assume that the perturbation in the distribution does not depend on momentum. Thus, we can rewrite the equations as follows

$$\begin{cases} \frac{\delta\ddot{\phi}_0}{a^2} - \frac{1}{a^2}\nabla^2\delta\phi_0 + \tilde{M}_\phi^2\delta\phi_0 - M_\phi^2\delta f = 0 \\ \delta\dot{f} + \frac{\vec{q}\cdot\vec{\nabla}}{\epsilon}\delta f + \hat{n}\cdot\vec{\nabla}A\delta\phi_0 = 0 \end{cases} \tag{26}$$

We will solve the equations in two limits: particles at rest, i.e. $T \ll \tilde{m}_0$, and for ultra relativistic particles, i.e. $T \gg \tilde{m}_0$. In order to solve the equations we will use the typical mathematical methods following the example given in [3]

1. *Early universe (relativistic particles)*

In our equations, both $\tilde{M}_\phi^2$ and A act as coupling constant between the fermion scalar field and the particle distribution perturbations. As expected, in the ultra-relativistic limit the equations decouple, since $\tilde{M}_\phi^2 \gg M_\phi^2$ we can neglect δf from the first equations, and $A \ll 1$ if the coupling constant g is not too large (i.e. $\frac{gm_0}{M_\phi} \lesssim 10^7$) and we can neglect it in front of δf in the second equation. We can rewrite

$$\begin{cases} \frac{\delta\ddot{\phi}_0}{a^2} + \left(\frac{k^2}{a^2} + \tilde{M}_\phi^2\right)\delta\phi_0 = 0 \\ \delta\dot{f} + ik\mu\delta f = 0 \end{cases} \tag{27}$$

where $\mu = \hat{k}\cdot\hat{n}$. In this scenario, the perturbations in the energy density will have the kinetic contribution and the particle distribution contribution, being

$$\delta\rho_f = \int q^2 dq d\Omega q f_0(q)\delta f = \frac{7\pi^2}{240}\mathbf{g}\frac{\sin(kt)}{kt}T^4\delta f(0) \tag{28}$$

where we used $\delta f = \delta f(0)e^{-ik\mu t}$ and $\epsilon = q$ in the ultra-relativistic limit. Comparing this result to the zero-th order contribution,

$$\rho_f = \int q^2 dq d\Omega q f_0(q) = \frac{7\pi^2}{240}\mathbf{g}T^4 \tag{29}$$

we observe $\frac{\delta\rho_f}{\rho_f} = \frac{sin(kt)}{kt}\delta f(0)$. On the other hand, the perturbation in the scalar field generates a contribution to the energy density of

$$\delta\rho_\phi = \phi_0^2\tilde{M}_\phi^2\delta\phi_0(0)\cos\left(\omega(k)t\right) \tag{30}$$

being $\omega(k) = \sqrt{k^2+a^2\tilde{M}_\phi^2}$. To obtain this result we only considered long range contribution to the energy density, i.e., $k^2\delta\phi_0 \ll \tilde{M}_\phi^2$. Comparing to the zero-th order kinetic contribution,

$$\rho_\phi = \frac{1}{2}\phi^2 M_\phi^2 \tag{31}$$

we obtain $\frac{\delta\rho_\phi}{\rho_\phi} = \frac{\mathbf{g}}{\pi^2}\left(\frac{gT}{M_\phi}\right)^2\delta\phi_0(0)\cos(\omega(k)t)$, where we used $\tilde{M}_\phi^2 \approx \frac{\mathbf{g}}{2\pi^2}(gT)^2$ in the ultra-relativistic limit.
The results obtained can be interpreted as follows: When the particles are close together, i.e., closer than the characteristic distance $a\tilde{M}_\phi$, given a certain temperature, the kinetic contribution dominates and we observe an oscillation in the energy density with a constant amplitude $\phi_0^2\tilde{M}_\phi^2\delta\phi_0$, while for longer distances we observe an oscillation with an increasing amplitude with distance and decreasing with time, $\frac{7\pi^2}{240}\mathbf{g}\frac{\sin(kt)}{kt}T^4\delta f(0)$, since the larger the distance the larger the number of particles that

contribute to the field and hence larger perturbations. It is important to note that for longer distances, since the amplitude grows, the linear approximation we performed breaks, we would need to solve the full non-approximated equation in order to rigorously obtain the correct result.

2. *Late universe evolution*

As observed in FIG. 3, in this case we have $\tilde{M}_\phi \approx M_\phi$. In the Fourier space we can write

$$\begin{cases} \frac{\delta\ddot{\phi_0}}{a^2} + \left(\frac{k^2}{a^2} + M_\phi^2\right)\delta\phi_0 - M_\phi^2 \delta f = 0 \\ \delta\dot{f} + i\frac{qk}{\tilde{m}_0}\mu\delta f - i\frac{(\tilde{m}_0 - m_0)a^2 k}{q}\mu\frac{d\log f_0}{d\log q}\delta\phi_0 = 0 \end{cases} \tag{32}$$

We will work in the regime where $\tilde{m}_0$ has a plateau $\tilde{m}_0 = m_0$ as shown in FIG. 1. We observe that the equations decouple since when T is low, the particles are very diluted due to the universe expansion and do not interact with each other.
Multiplying both sides of the second equation by $\int q^2 dq f_0(q)$ and performing the integration considering that δf and $\delta\phi_0$ do not depend on q, as previously justified, we obtain

$$\delta\dot{f} + i\frac{k\mu}{\tilde{m}_0}\frac{7\pi^4}{180\zeta(3)}T\delta f = 0 \tag{33}$$

where the linear term in T is rapidly suppressed when the particles are at rest and we can rewrite

$$\delta\dot{f} = 0 \tag{34}$$

Hence, the perturbation in the fermion distribution remains a constant $\delta f = \delta f(0)$. On the other hand, $\delta\rho_\phi$ will depend on $\phi_0^2 M_\phi^2$, which we argued to be zero in this regime. Since we considered very low T, i.e., late part of the cosmological evolution, the cooling of the universe will be slow and we can take T to be a constant. Hence, the perturbations in the total energy density will be a constant, $\delta\rho = \delta\rho_f = \frac{3g\zeta(3)}{2\pi}\tilde{m}_0 T^3 \delta f(0)$. Notice that in this case gravity should not be neglected and is precisely the effect that dominates the dynamical evolution of the perturbations.

IV. CONCLUSIONS

We studied the zero-th order field equations for neutrinos, following the formalism given in [1]. Considering small inhomogeneities, we separately analyzed the ultra-relativistic and non-relativistic scenarios of the scalar-mediated field generated by massive neutrinos, assuming scales much smaller that cosmological distance scales and therefore expected to be dominated by the new long range physics driven by the scalar field.
The perturbations on the energy density exhibit oscillatory behavior with a constant amplitude for short distances and an increasing amplitude for long distances in the ultra-relativistic scenario, while remaining constant in the non-relativistic case.
Our investigation focuses on the early stages of the universe and the present times. However, overlooking the transient state where $T \sim \tilde{m}_0$ may alter results in the non-relativistic limit. Further study in this topic is needed.

V. ACKNOWLEDGMENTS

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