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A Coalitional Great Fish War Model with Quasi-Hyperbolic Discounting

Joakim Alderborn[1] · **Jesús Marín-Solano**[2]

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Abstract
This paper studies the coalitional great fish war model in a setting with $\beta\delta$-preferences (or quasi-hyperbolic discounting). We derive the equilibrium strategies and payoffs in this coalition game under non-cooperation, full cooperation, and partial cooperation, under simultaneous moves and with first-mover advantage for the coalition. We focus on the effects of the short-run discount factor β on the sustainability of cooperation. We find that the introduction of quasi-hyperbolic discounting can have a significant impact on the existence of stable coalitions. In particular, in contrast with the standard case of exponential discount functions, in which the coalitions of more than two players are always unstable under simultaneous moves, the grand coalition can be stable if the bias towards the present is sufficiently high (i.e., for small values of β). In addition, we show that, under simultaneous moves, there can be up to three sustainable coalitions, including the grand coalition, and larger coalitions are better from an ecological perspective, since total harvesting is lower. Thus, the consideration of quasi-hyperbolic discounting offers a promising avenue to address the challenge posed by small coalitions.

Keywords Fishery management · Coalitional game · Quasi-hyperbolic discounting · Sustainability of cooperation

1 Introduction

The great fish war model of Mirman (1979) and Levhari and Mirman (1980) is a parsimonious framework that has been extensively used to study the harvest of fish from a territory of water by a number of players under open access. It is a dynamic game formulated in a discrete time setting with an infinite planning horizon. The players are typically countries with

✉ Jesús Marín-Solano
jmarin@ub.edu

[1] Departament de Matemàtica Econòmica, Financera i Actuarial, Universitat de Barcelona, Barcelona, Spain

[2] Departament de Matemàtica Econòmica, Financera i Actuarial and BEAT, Universitat de Barcelona, Barcelona, Spain

access to international fisheries. In the standard case, there is a single state variable, which is the amount of fish in the sea at the beginning of a period, and each player has a single control or decision variable, which is their harvest over the period. Utility functions are logarithmic and discount functions are of the usual exponential form. The state dynamics is described by a power growth function of the fish stock, which, in the absence of any harvest, causes the fish stock to converge to a steady state called the saturation level. Most papers solve the model for both cooperative and non-cooperative solutions. In the former, the players form a coalition and jointly decide on the behavior of each coalition member. In the latter, the solution is usually the non-cooperative feedback Nash equilibrium of the game.

In the Levhari and Mirman (1980) model there were only two symmetric players. Okuguchi (1981) considered an n-country extension. An important result is that the fully non-cooperative setting yields a result similar to the prisoner's dilemma, and that cooperation is Pareto-improving, i.e., players are better off in the cooperative solution than in the non-cooperative solution. In addition, the steady state level to which the fish stock converges is higher under cooperation (harvest rates are lower), implying that ecological welfare suffers under non-cooperation. Fischer and Mirman (1992) extended the model by introducing a second fish species and allowing each player to harvest only one species each. In addition, the growth functions were constructed so that the amount of fish of one species affects the amount of the fish of the other species. This is justified, for example, if the species have a prey-predator relationship. This introduced a second externality, which the authors called a biological externality. In Fischer and Mirman (1996), the model was further extended by allowing each player to harvest both types of fish, meaning that there are two control variables per player. A somewhat similar scheme can be found in Datta and Mirman (1999), where each player can only fish one species, but players are allowed to trade their harvests with each other at market prices.

Several papers have analyzed cooperation in international fisheries by using a cooperative approach, comparing the full cooperation setting with the non-cooperative equilibrium. Cooperation instruments can be trigger strategies (Benhabib and Radner 1992; Cave 1987; Hamalainen et al. 1985), incentive strategies (Ehtamo and Hamalainen 1993) or transfers (Mazalov and Rettieva 2010; Pintassilgo et al. 2010).

Kwon (2006) introduced another variation of the model. In that paper, given an arbitrary number of players, a coalition of any size can be formed. This includes partial cooperation, in which some players are part of the coalition while the others are not (and they act as free riders). Utility transfers were not allowed, in the sense of the Levhari and Mirman model, where utilities were ordinal, not cardinal. It was proved that under simultaneous moves, independently of the number of players, the maximum size of a sustainable coalition is two, and that a coalition of size two can only be maintained for certain parameter values. Rettieva (2012) combined the idea of partial cooperation with the presence of two types of fish. In that model, a coalition is formed by the players who harvest the same type of fish, so that two separate coalitions can potentially form. Breton and Keoula (2012) investigated partial cooperation under a different type of condition for coalition stability. Instead of the standard Nash conjectures, they used rational conjectures, meaning that when players consider whether to leave or join a coalition, they take into account how other players will react to their decision.

Most of the papers mentioned above assume that the players are symmetric, which in the context of the great fish war model means that they have the same utility and exponential discount functions. This is also the case in Dahmouni et al. (2023), where a regulator imposes a moratorium on fishing when the stock reaches a predetermined critical low level. Exceptions are Levhari and Mirman (1980) and Fischer and Mirman (1992), where the two players are allowed to have different discount functions, but only in the non-cooperative solution. The first paper to more systematically study the effects of asymmetric discount factors in a fish war model was Breton and Keoula (2014). Due to the complexity of the problem, these authors studied in detail partial cooperation when there are two types of players, characterized by their discount functions. Under simultaneous moves, coalitions can be formed by players with the same or different discount functions, and it was proved that both events are possible, although, as in Kwon (2006), coalitions with more than two players cannot be maintained.

In the present study, we introduce quasi-hyperbolic discount functions. That is, while all of the above papers assume that agents (i.e. players) have constant discount rates, this is not the case here. Quasi-hyperbolic discount functions in a discrete time setting were first introduced by Phelps and Pollak (1968) and have been widely used in many contexts over the last three decades. Turan (2019) used them to study a simple fish war model with only two players, and only for the non-cooperative solution. We will do so in a more general way, with an arbitrary number of players, and with an emphasis of forming sustainable coalitions (partial and/or full cooperation). Quasi-hyperbolic discount functions represented by the so-called $\beta\delta$-preferences are very useful because they are probably the simplest type of discount function that deviates from standard geometric discounting. The introduction of a bias towards the present, represented by a value of β strictly lower than one, has interesting implications. While higher degrees of impatience in the long run, represented by lower values of the standard discount factor δ, make cooperation more difficult (see Kwon (2006)), a stronger bias towards the present in the short run, represented by a low β, can facilitate partial (and sometimes full) cooperation in an important way. Indeed, this is the main result of the present paper.

There is some empirical evidence that time preference biases may play a role in fisheries management. Hepburn et al. (2010) suggest that the collapse of the Canadian cod and the Peruvian anchovy in the 1970's and 1980's can both be explained in part by hyperbolic discounting by the agencies that regulate fishing industries. More generally, there are many studies of the effect of time preferences in fishing, although the results vary. For example, Fehr and Leibbrandt (2011) study shrimp fishermen in Brazil, and find that fishermen who are impatient use fishing instruments that are more likely to exploit the fishing grounds. On the other hand, Javaid et al. (2016) study fishermen in Zanzibar and find that higher time preferences lead to lower exploitation rates. They suggest that this counterintuitive result can be explained by the so-called disinvestment effect of time preferences, namely that impatient fishermen invest less resources in extraction technology, which means that they have smaller catches. For more studies, see the references in each of those papers.

The paper is organized as follows. The model is presented in Sect. 2. In the case of asymmetric players, represented by different degrees of impatience in both the short and the long run, the general solution is derived in Sect. 3, both under simultaneous moves and when the coalition has first-mover advantage. Section 4 reviews the notion of coalitional stability and discusses two different ways of measuring welfare in our problem with quasi-hyperbolic discounting. Section 5 analyzes in detail the stability of coalitions under partial cooperation for quasi-hyperbolic discounting agents. Section 6 concludes the paper.

2 The Model

There are M players, indexed by i. A coalition is formed by $m \in \{1, \ldots, M\}$ players. If $m=1$ we obtain the non-cooperative solution, because the coalition consists of only one player. If $m=M$ we obtain the fully cooperative solution because all players are part of the coalition. We will refer to the players who are part of the coalition as the insiders and to the players who are not part of the coalition (the singletons) as outsiders, and we define the sets

$$\mathbb{I} = \{i : i \text{ is an insider}\} \text{ and } \mathbb{O} = \{\, i : i \text{ is an outsider}\} \,.$$

There are n outsiders, so $m + n = M$. We also define for each insider their strategic weight $\lambda_i \in \{\lambda_1, \ldots, \lambda_m\}$. If a player is an insider, their strategic weight will determine their influence over the decisions of the coalition. Strategic weights are also present in the partial cooperation model of Breton and Keoula (2014), but not in that of Kwon (2006). In general, these weights can be exogenous (representing a higher bargaining power) or endogenous (e.g., by using a bargaining procedure). In this paper, in Sect. 5, we will assume that players are symmetric, so we will take $\lambda_i = 1/m$. Another limitation is that, as in Kwon (2006) and Breton and Keoula (2014), our model allows for only one coalition to form. All players who are not part of that coalition act individually.

The utility function of player $i \in \{1, \ldots, M\}$ at time $t \in \{0, 1, 2, \ldots\}$ is $u_{i,t} = \ln c_{i,t}$, where $c_{i,t}$ is the harvest of player i in period t. The fish stock at time t is x_t and its growth function is

$$x_{t+1} = f\left(x_t, c_{1,t}, ..., c_{M,t}\right) = \left(x_t - \sum_{i=1}^{M} c_{i,t}\right)^{\alpha}. \tag{1}$$

Without loss of generality, we have normalized the saturation level of the fish stock to be equal to one. That is, if there is no harvest, so that $c_{i,t} = 0$ for all i and t, the fish stock converges to one. The parameter $\alpha \in (0, 1)$ determines the capability of the fish stock to reproduce itself: the smaller is α (closer to 0), the greater is the fish stock's regeneration capacity. On the contrary, in the limit $\alpha \to 1^-$, the resource is nonrenewable and the growth function is linear.

The quasi-hyperbolic discount function for each player i is given by

$$\theta_t = \begin{cases} 1 & \text{if } t = 0 \\ \beta\delta^t & \text{if } t > 0. \end{cases}$$

This is a standard exponential discount function with the addition of the discount factor $\beta \in (0, 1]$, that affects all future utilities in the same way. The standard case is recovered by setting $\beta=1$. The lower is β, the higher is the bias towards the present in the player's short-run time preferences.[1] It follows that the intertemporal utility function (i.e. the sum of present and future utilities, discounted back to the present) of player i at time t is

[1] We can refer to $1/\beta$ as the player's *bias factor*. Turan (2019) refers to β as the player's altruism parameter.

$$J_{i,t}\left(x_t, c_i\right) = \ln c_{i,t} + \sum_{s=t+1}^{\infty} \beta\delta^{s-t} \ln c_{i,s}.$$

This is the function that the player aims to maximize. The coalition at time t has the intertemporal utility function

$$J_{c,t}\left(x_t, c_1, ..., c_M\right) = \sum_{i\in\mathbb{I}} \lambda_i \left\{ \ln c_{i,t} + \sum_{s=t+1}^{\infty} \beta\delta^{s-t} \ln c_{i,s} \right\}.$$

This is a sum of the intertemporal utility functions of the members of the coalition, weighted by the weights $\{\lambda_1, \ldots, \lambda_M\}$, with $\lambda_i > 0$ and $\sum_{j=1}^{M} \lambda_j = 1$.

The presence of the parameter β has the effect that the players have time-inconsistent preferences. To see this, we note that $\theta_1/\theta_0 = \beta\delta$ whereas $\theta_{t+1}/\theta_t = \delta$ if $t \geq 1$. In words: for $\beta<1$, the discount factor is not constant and utility obtained two periods into the future is worth more relative to utility obtained one period into the future than is utility obtained one period into the future relative to utility obtained in the present. As a result, whereas $\delta<1$ represents the bias towards the present from a long-run perspective, $\beta<1$ is the bias towards the present in the short-run. We will show in this paper that the short-run and long-run biases can have opposite effects on the formation of stable coalitions.

Remark 1 *In the limit where beta goes to zero, players are myopic. There is some literature in the field of environmental economics suggesting that poor countries sometimes act as myopic agents for two possible reasons: because they are much more concerned about the present, or because, although they are aware of the dynamics, they do not have the instruments to monitor the future. Taking a value of beta lower than one, perhaps close to zero but not strictly zero, could represent this myopic behavior in a more realistic way than simply taking $\beta=0$.*

In the presence of time-inconsistent preferences, the standard method of defining a solution strategy as that which maximizes the intertemporal utility function becomes problematic. In particular, it is necessary to distinguish between a commitment solution and an intertemporal equilibrium solution. In a commitment solution, the allocation of utility (and of harvest) over all time periods follows what the player consider to be optimal from the perspective of period 0. That is, the players commit to a certain strategy and do not deviate from it even though deviation may be preferable from the perspective of a later period. The assumption of commitment looks unrealistic in intergenerational problems. In an intertemporal equilibrium, each player (being an individual or a coalition) at each point in time chooses the harvest that is the optimal response to their own future harvests and to the harvest of the other players. By construction, it is a Markovian (subgame perfect) Nash equilibrium in a sequential game. Hence, unlike the commitment solution, it can be implemented without any binding agreements that prevent players from acting in accordance with their preferences.

In this paper, we will solve the problem under no commitment, by assuming that players are completely rational, so they realize that their time preferences will change in the future, and decide according to the subgame intertemporal equilibria. This was also the approach followed in Breton and Keoula (2014), in which the source of the time-inconsistency was the asymmetry between players (if players have constant but different discount rates, any coalition formed among those players will have time-inconsistent preferences with a declining discount rate, see, e.g., Gollier and Zeckhauser 2005).

Remark 2 *The great fish war model of Mirman (1979) and Levhari and Mirman (1980) has several limiting assumptions. Concerning the dynamics (1), in the bioscience literature the growth function has an inverted U-shape, which is an important biological feature. This property is satisfied. In the absence of human activity, there is a unique steady state. However, the assumption of a logarithmic utility, unlike the more general power utility functions, imposes that the intertemporal elasticity of substitution is equal to one, and is in that sense rather restrictive. Its advantage is that when combined with the nonlinears dynamics (1), it is analytically tractable. In particular, linear strategies exist. But, unlike other models in resource economics, in the presence of harvesting there is still a unique positive steady state. Also, the symmetry of the agents imply, among others, that the marginal costs of extraction are the same. In addition, since* $u(0) = -\infty$*, under no fishing, the payment is also* $-\infty$*. Another consequence is that when solving a problem with time-inconsistent preferences, sophisticated agents (i.e., completely rational players following a subgame perfect equilibrium), as observed in Turan* (2019)*, will behave in the same way as naive agents, who are not aware of their changing time preferences. However, this result can have a positive effect in terms of interpretation of the results. Myopic players are expected to behave in a naive way. If we interpret the introduction of the short run discount factor* β *as a kind of myopic valuation, our results can be applied to this kind of agents with, maybe, quite low values of* β*. Another limitation of this model is that we assume that only one coalition forms. We remove the possibility of payment transfers because we work with utility functions, ranking the preferences of the agents, and not with profit functions.*

3 Equilibrium Solutions

In the present section we derive the equilibrium strategies under non-cooperation, full cooperation and partial cooperation. In the latter case, we will consider two possibilities: simultaneous moves and the case in which the coalition has first-mover advantage. As in Kwon (2006) and Breton and Keoula (2014), we assume that only one coalition forms. In all the cases, we will study the existence of stationary linear strategies, as is standard in the literature for this model. This implies that in each time period, players decide to extract a fixed fraction of the current fish stock. In the cooperative setting, we rule out a transfer mechanism as a way to sustain cooperation. This is in line with the spirit of the model, in which utilities represent just the ordering in the preferences. As a result, players in a coalition only negotiate a set of cooperative fishing strategies, and each player consumes (or takes profit) of just their own catches over time.

As in Breton and Keoula (2014), we define

$$\gamma \equiv \frac{\alpha\delta}{1-\alpha\delta}.$$

This object is referred to in the fish war literature as the bionomic multiplier. Its inverse γ^{-1} is called the bionomic discount rate, because it gives the discount rate that corresponds to the bionomic discount factor $\alpha\delta$. Essentially, $\alpha\delta$ is a modified discount factor that incorporates the nonlinear aspect of the growth function directly into the long-run discount function (see Breton and Keoula (2014) for more details).

3.1 The Non-Cooperative Case

First, let us study the existence of non-cooperative Nash equilibria. If $c_j^* = \phi_j^N(x)$ (along the paper, the superscript N will represent the non-cooperative setting) are the strategies of players $j \neq i$, for every $x_t = x$ and t, player i solves

$$V_i^N(x) = \max_{\{c_i\}} \left\{ \ln c_i + \beta\delta\bar{V}_i^N \left(\left(x - c_i - \sum_{j=1, j\neq i}^{M} \phi_j^N(x) \right)^{\alpha} \right) \right\}, \qquad \text{where} \tag{2}$$

$$\bar{V}_i^N(x) = \ln \phi_i^N(x) + \delta\bar{V}_i^N \left(\left(x - \sum_{j=1}^{M} \phi_j^N(x) \right)^{\alpha} \right). \tag{3}$$

This is the natural extension of the dynamic programming approach in the search of time-consistent solutions, followed by sophisticated agents in 1-person problems. In the above expressions, $V_i^N(x)$ is the value function. It is the valuation of player i, according to the way this player values the future at every moment, depending on the amount of the resource. The auxiliary value function $\bar{V}_i^N(x)$ collects the sum of all future discounted utilities. If $\beta=1$, for every $i \in \{1, \ldots, M\}$, $V_i^N(x) = \bar{V}_i^N(x)$. Otherwise, they are different. Depending on the setting, both functions have been used to measure welfare in the published literature. In Sect. 4 we discuss the rationale behind the use of these two functions as a measure of welfare in our problem. Along the paper, we will compute both the value functions and the auxiliary value functions.

In the Appendix we show that a solution to Eqs. (2), (3) exists, with

$$V_i^N(x) = A^N \ln x + B^N, \quad \bar{V}_i^N(x) = \bar{A}^N \ln x + \bar{B}^N, \tag{4}$$

and the corresponding linear strategies $\phi_i^N(x) = h^N x$, for $i \in \{1, \ldots, M\}$, where the harvest rate of player i is given by

$$h^N = \frac{1}{M + \beta\gamma}. \tag{5}$$

The total harvest rate is

$$H^N = Mh^N = \frac{M}{M + \beta\gamma} < 1. \tag{6}$$

Note that the harvest rates depend on the product $\beta\gamma$, not on each variable separately.

3.2 The Cooperative Case

Under cooperation, we have to solve

$$\max_{\{c_1,\ldots,c_M\}} \left\{ \sum_{i=1}^{M} \lambda_i \Big(\ln c_i + \beta\delta \bar{V}_i^c \Big(\Big(x - \sum_{j=1}^{M} c_j \Big)^{\alpha} \Big) \right\} , \tag{7}$$

where the superscript c stands for the cooperative solution. If $c_i^* = \phi_i^c(x)$ is the solution to (7), then the auxiliary value functions $\bar{V}_i^c$ verify, for $i \in \{1, \ldots, M\}$,

$$\bar{V}_i^c(x) = \ln \phi_i^c(x) + \delta \bar{V}_i^c \left(\left(x - \sum_{j=1}^{M} \phi_j^c(x) \right)^{\alpha} \right) . \tag{8}$$

As mentioned above, $\bar{V}_i^c(x)$ is the auxiliary value function, whereas the value function is given by

$$V_i^c(x) = \ln \phi_i^c(x) + \beta\delta \bar{V}_i^c \left(\left(x - \sum_{j=1}^{M} \phi_j^c(x) \right)^{\alpha} \right) . \tag{9}$$

A solution to Eqs. (7)–(9) exists (see the Appendix), with

$$V_i^c(x) = A_i^c \ln x + B_i^c, \quad \bar{V}_i^c(x) = \bar{A}_i^c \ln x + \bar{B}_i^c, \tag{10}$$

and

$$\phi_i^c(x) = h_i^c x = \frac{\lambda_i}{1+\beta\gamma} x, \tag{11}$$

for $i \in \{1, \ldots, M\}$. In particular, notice that $\dfrac{h_i^c}{h_j^c} = \dfrac{\lambda_j}{\lambda_i}$. The ratio of the harvest rates depends on the ratio of the strategic weights.

The total harvest is now

$$H^c = \sum_{i=1}^{M} h_i^c = \frac{1}{1+\beta\gamma} . \tag{12}$$

Comparing Eq. (12) with Eq. (6), we see that the fraction of the fish stock that is extracted is greater, and therefore that the steady state fish stock is smaller, in the non-cooperative solution than in the fully cooperative solution for every $M \geq 2$.

Finally note that, in the fully cooperative solution, in the same way as in the non-cooperative solution, what matters for the harvest rates is the product $\beta\gamma$, rather than each of variables separately. This means that, at first, it seems that the qualitative effects of the long-run

(δ) and the short-run (β) discount factors are exactly the same. However, this will not be the case when we analyze the effects of these two parameters on the benefits of cooperation, as we will see when we analyze the stability of (partial) cooperation.

3.3 Partial Cooperation: Simultaneous Moves

In the search of the solution under partial cooperation, we combine the ideas from the previous two cases. We have to search for the subgame perfect Nash equilibrium in a game with $M-m+1$ players. For $i \in \mathbb{I}$, let $\phi_i^{pc}(x)$ denote the equilibrium decision rule of the players in the coalition. If $\phi_k^{pn}(x)$, $k \in \mathbb{O}$, $k \neq j$, is the equilibrium decision rule of the remaining outsiders, outsider $j \in \mathbb{O}$ has to solve the dynamic programming equations

$$V_j^{pn}(x) = \max_{\{c_j\}} \left\{ \ln c_j + \beta\delta \bar{V}_j^{pn}\left(x - c_j - \sum_{i\in} \phi_i^{pc}(x) - \sum_{k\in\setminus\{j\}} \phi_k^{pn}(x)\right)^{\alpha}\right) \right\}, \quad \text{where} \tag{13}$$

$$\bar{V}_j^{pn}(x) = \ln \phi_j^{pn}(x) + \delta \bar{V}_j^{pn}\left(\left(x - \sum_{i\in} \phi_i^{pc}(x) - \sum_{k\in} \phi_k^{pn}(x)\right)^{\alpha}\right). \tag{14}$$

In a similar way, the insiders have to solve the problem

$$\max_{\{c_1,\ldots,c_m\}} \left\{ \sum_{i=1}^{m} \lambda_i \left(\ln c_i + \beta\delta \bar{V}_i^{pc}\left(\left(x - \sum_{j\in} c_j - \sum_{k\in} \phi_k^{pn}(x)\right)^{\alpha}\right) \right\}, \quad \text{where} \tag{15}$$

$$V_i^{pc}(x) = \ln \phi_i^{pc}(x) + \beta\delta \bar{V}_i^{pc}\left(\left(x - \sum_{j\in\mathbb{I}} \phi_j^{pc}(x) - \sum_{k\in\mathbb{O}} \phi_k^{pn}(x)\right)^{\alpha}\right). \tag{16}$$

The corresponding value functions are

$$V_i^{pc}(x) = \ln \phi_i^{pc}(x) + \beta\delta \bar{V}_i^{pc}\left(\left(x - \sum_{j\in\mathbb{I}} \phi_j^{pc}(x) - \sum_{k\in\mathbb{O}} \phi_k^{pn}(x)\right)^{\alpha}\right). \tag{17}$$

In the Appendix we show that a solution to Eqs. (13)–(17) exists, with

$$V_i^{pc}(x) = A_i^{pc} \ln x + B_i^{pc}, \quad V_j^{pn}(x) = A^{pn} \ln x + B^{pn}, \tag{18}$$

$$\bar{V}_i^{pc}(x) = \bar{A}_i^{pc} \ln x + \bar{B}_i^{pc}, \quad \bar{V}_j^{pn}(x) = \bar{A}^{pn} \ln x + \bar{B}^{pn} \tag{19}$$

and $\phi_i^{pc}(x) = h_i^{pc} x$, $\phi_j^{pn}(x) = h^{pn} x$, for $i \in \mathbb{I}$, $j \in \mathbb{O}$.

The harvest rates for the outsiders are, for $j \in \mathbb{O}$,

$$h_j^{pn} = h^{pn} = \frac{1}{M-m+1+\beta\gamma}. \tag{20}$$

For the insiders, for $i \in \mathbb{I}$, the harvest rates are

$$h_i^{pc} = \frac{\lambda_i}{M - m + 1 + \beta\gamma}. \tag{21}$$

Next, we observe from (20) and (21) that when a coalition of size *m* forms, the total harvesting rate $H(M, m)$ is

$$H\left(M,m\right) = \sum_{i\in\mathbb{I}} \frac{\lambda_i}{M-m+1+\beta\gamma} + \left(M-m\right)\frac{1}{M-m+1+\beta\gamma} = \frac{M-m+1}{M-m+1+\beta\gamma}.$$

Then

$$\frac{\partial H\left(M,m\right)}{\partial m} = -\frac{\beta\gamma}{\left(M-m+1+\beta\gamma\right)^2} < 0.$$

As a result, as the size of the coalition increases, the total harvest decreases, and thus the long-run fish stock increases. Therefore, the steady state is always higher under partial cooperation than that under full non-cooperation.

3.4 Partial Cooperation: First-Mover Advantage

Suppose that the coalition can act as the leader. In that case, the outsiders, after having observed the harvest rates of the members of the coalition, c_i^L, for $i \in \mathbb{I}$, will solve, for every $j \in \mathbb{O}$, the dynamic programming equation

$$V_j^F(x) = \max_{\{c_j\}} \left\{ \ln c_j + \beta\delta\bar{V}_j^F\left(x - c_j - \sum_{i\in\mathbb{I}} c_i^L - \sum_{k\in\mathbb{O}\setminus\{j\}} \phi_k^F(x)\right)^\alpha \right\}, \tag{22}$$

where $\phi_k^F(x)$ denote the strategies of the remaining (noncooperating) followers. The auxiliary value functions $\bar{V}_j^F$ verify

$$\bar{V}_j^F(x) = \ln\phi_j^F(x) + \delta\bar{V}_j^F\left(\left(x - \sum_{i\in\mathbb{I}}\phi_i^L(x) - \sum_{k\in\mathbb{O}}\phi_k^F(x)\right)^\alpha\right). \tag{23}$$

Here, $c_i^L = \phi_i^L(x)$ denotes the strategy of the members of the leading coalition.

By solving (22), (23) we will obtain the strategies of the followers as a function of the actions of the leaders, i.e., $c_j^F = \phi_j^F(x, \{c_i\}_{i\in\mathbb{I}})$. Then, the problem of the cooperating players is

$$\max_{\{c_l, l\in\mathbb{I}\}} \left\{ \sum_{i\in\mathbb{I}} \lambda_i \left(\ln c_i + \beta\delta\bar{V}_i^L\left(\left(x - \sum_{j\in\mathbb{I}} c_j - \sum_{k\in\mathbb{O}} \phi_j^F(x, \{c_i\}_{i\in\mathbb{I}})\right)^\alpha\right)\right)\right\}, \text{ where} \tag{24}$$

$$\bar{V}_i^L(x) = \ln\phi_i^L(x) + \delta\bar{V}_i^L\left(\left(x - \sum_{j\in\mathbb{I}}\phi_j^L(x) - \sum_{k\in\mathbb{O}}\phi_k^F(x)\right)^\alpha\right). \tag{25}$$

The corresponding value functions are, for $i \in \mathbb{I}$,

$$V_i^L(x) = \ln \phi_i^L(x) + \beta\delta\bar{V}_i^L\left(\left(x - \sum_{j\in\mathbb{I}} \phi_j^L(x) - \sum_{k\in\mathbb{O}} \phi_k^F(x)\right)^\alpha\right). \tag{26}$$

In the Appendix we show that a solution to Eqs. (22)–(26) exists, with

$$V_i^L(x) = A_i^L \ln x + B_i^L, V_j^F(x) = A^F \ln x + B^F, \tag{27}$$

$$\bar{V}_i^L(x) = \bar{A}_i^L \ln x + \bar{B}_i^L, \bar{V}_j^F(x) = \bar{A}^F \ln x + \bar{B}^F \tag{28}$$

and $\phi_i^L(x) = h_i^L x$, $\phi_j^F(x) = h^F x$, for $i =\in \mathbb{I}$ and $j \in \mathbb{O}$.

The harvest rates of the insiders are, for $i \in \mathbb{I}$,

$$h_i^L = \frac{\lambda_i}{1 + \beta\gamma}. \tag{29}$$

For the outsiders, for $i \in \mathbb{O}$, we obtain

$$h^F = \frac{\beta\gamma}{(M - m + \beta\gamma)(1 + \beta\gamma)}. \tag{30}$$

4 Coalitional Stability and Welfare Measures

Kwon (2006) was the first to study the partial cooperation fish war game. Stability of partial cooperation was analyzed in the sense of d'Aspremont et al. (1983):

- A coalition is *internally stable* if there are no incentives for an insider to withdraw from the coalition.
- A coalition is *externally stable* if there are no incentives for an outsider to join the coalition.
- A coalition is *stable* if it is both internally and externally stable.

In the case with geometric discounting and simultaneous moves, Kwon (2006) proved that, if the cooperating coalition has size m, then the harvest rate of a free rider is m times the harvest rate of a player participating in the coalition. Also, the maximum size of a sustainable coalition is two, and a coalition of size two is stable only if $\delta\alpha$ is sufficiently high (i.e., near to one). Partial cooperation is facilitated if the coalition is the dominant player (it has first-mover advantage). In that framework there always exist at least one and no more than two sustainable coalitions (in which case coalitions differ in size by one country).

Breton and Keoula (2014) extended the results of Kwon (2006) to a situation in which the players have different discount factors. For the analysis of coalitional stability, due to the complexity of the problem, the authors centered their attention in the setting in which there are two types of players, characterized by their discount factors. In that case, under simultaneous moves, the maximum size of stable coalitions is still two, as in Kwon (2006). Therefore, introducing asymmetries in the discount rates does not solve the puzzle of small coalitions.

In contrast with Kwon (2006) and Breton and Keoula (2014), in the present paper we will show that the introduction of quasi-hyperbolic discounting can facilitate the formation of coalitions of higher sizes. But first, we will briefly discuss how to measure welfare and then the benefits of cooperation.

The natural way to measure the welfare of the different players is with the value function V_i^l, where $l \in \{N, c, pn, pc, F, L\}$, depending on the setting. The value function represents the way in which players taking a decision at period *t* measure their welfare at the same period *t*, and is used in most previous papers with one decision maker in the literature. Unfortunately, its expression is quite cumbersome and it is difficult to derive theoretical results.

There is an alternative way to measure the welfare, by using the auxiliary value functions $\bar{V}_i^l$, where $l \in \{N, c, pn, pc, F, L\}$. This function is related to the long-run utility criterion used in O'Donoghue and Rabin (1999). Its expression is simpler, and we can derive more analytical results. Its rationale is the following: if there are T periods (T can be infinite, as in our model), indexed by $t \in \{1, \ldots, T\}$, the auxiliary value functions correspond to the way decision makers would value their future welfare in a previous period $t=0$. Since in the evaluation of all the future periods, both under (partial) cooperation and under non-cooperation, the welfare is multiplied by the same constant number $\beta\delta$, we can ignore this multiplicative factor when we analyze the benefits of (partial) cooperation. An advantage of the long-run utility criterion is that it measures the welfare with a unique number, independently on the period considered (we refer to O'Donoghue and Rabin (1999) for a more detailed discussion). In the setting of our particular problem, we could interpret this criterion in different ways:

- First, it can correspond to a situation in which the players precommit themselves, before the problem starts, to maintain their coalitions, after an analysis of their future actions. However, this is in contradiction with the spirit of our setting, in which we assume that there is no commitment when the harvest rates are computed.
- Alternatively, it would be the way in which an external agent (e.g., a social planner) with no short-term bias toward the present (so that $\beta=1$) would evaluate the welfare of the different agents. But this is unrealistic if there is no social planner making the decisions about whether to cooperate or not. However, the long-run utility criterion seems to be a reasonable way to measure welfare once the coalitions (whose members make their decisions according to the value function) are formed.
- Turan (2019) used this criterion in a two-player problem. One player has time-consistent preferences, but ignores which are the time preferences of the second player. Then, the second player, who has time-consistent preferences, can take advantage of acting as if that player had time-inconsistent preferences ($\beta<1$). Hence, both players measure their welfare according to the auxiliary value function, although the first player thinks that the second player uses the value function.
- Finally, if the short-run discount factor β is sufficiently high (i.e., close to 1), the auxiliary value function provides a good approximation to the value function. From the previous section, the auxiliary value function is the sum of the present utility and all future

discounted utilities, where the discount factor is δ. In turn, the value function is the sum of present utility and the product of β times the sum of all future discounted utilities (where the discount factor is δ). Thus, if the short-run discount factor β is sufficiently close to 1, one can expect small qualitative differences.

In the present paper we will study the problem under both criteria, although we pay more attention to the first one, with the (more realistic) value function, in the analysis of stable coalitions. As expected, there will be some differences, but they will not be large.

If we use the value function as the measure of welfare, in the case of simultaneous moves, a coalition is stable if, and only if, the following two conditions hold:

- For all $i \in \mathbb{I}$, $V_i^{pc}(x; \{\mathbb{I}, \mathbb{O}\}) \geq V_i^{pn}(x; \{\mathbb{I} \setminus \{i\}, \mathbb{O} \cup \{i\}\})$ (internal stability).
- For all $j \in \mathbb{O}$, $V_j^{pn}(x; \{\mathbb{I}, \mathbb{O}\}) \geq V_j^{pc}(x; \{\mathbb{I} \cup \{j\}, \mathbb{O} \setminus \{j\}\})$ (external stability).

In a similar way, under the long-run utility criterion, also in the case of simultaneous moves, a coalition $\mathbb{I}$ is stable if, and only if,

- For all $i \in \mathbb{I}$, $\bar{V}_i^{pc}(x; \{\mathbb{I}, \mathbb{O}\}) \geq \bar{V}_i^{pn}(x; \{\mathbb{I} \setminus \{i\}, \mathbb{O} \cup \{i\}\})$ (internal stability);
- And if, for all $j \in \mathbb{O}$, $\bar{V}_j^{pn}(x; \{\mathbb{I}, \mathbb{O}\}) \geq \bar{V}_j^{pc}(x; \{\mathbb{I} \cup \{j\}, \mathbb{O} \setminus \{j\}\})$ (external stability).

For the characterization of the stability under first-mover advantage, we have simply to substitute in the above expressions the upper indices $\{pc\}$ and $\{pn\}$ by L and F, respectively.

The implications in the great fish war model of a different concept of stability, the farsighted coalition stability, based on rational conjectures, were explored in Breton and Keoula (2012). Under the farsighted assumption, players in a coalitional game take into account that a deviation of a single player will lead to the formation of a new coalition structure as the result of possibly successive moves of their rivals in order to improve their payoffs. Breton and Keoula (2012) proved that the farsighted assumption predicts a wide scope for cooperation of coalitions, sustained by credible threats of successive deviations. We will not use this stability concept in our paper.

In the following section we will study the stability of cooperation in the case of M symmetric quasi-hyperbolic players.

5 Results

5.1 Simultaneous Moves

The harvest rates under non-cooperation are given by (5). In the cooperative case, if the players are symmetric, their weights are given by $\lambda_i = 1/M$ and, from (11), the harvest rates are

$$h_i^c = \frac{1}{M(1+\beta\gamma)}.$$

First, we show that, for $M \geq 2$, it is always profitable for the players to cooperate. All the proofs can be found in the Appendix.

Lemma 1 *If $M \geq 2$, $\bar{V}_i^c(x) > \bar{V}_i^N(x)$, for every $i \in \{1, \ldots, M\}$.*

Lemma 2 *If $M \geq 2$, $V_i^c(x) > V_i^N(x)$, for every $i \in \{1, \ldots, M\}$.*

Therefore, if we compare the welfares, both under the long-run utility criterion (that makes use of the auxiliary value functions $\bar{V}_i^c(x)$ and $\bar{V}_i^N(x)$) and with the value functions ($V_i^c(x)$, $V_i^N(x)$), players are better off if they cooperate in comparison with no cooperation at all. This reasonable result, that is obvious under time-consistent preferences ($\beta=1$), is not straightforward if $\beta<1$, for the two following reasons. First, we are using two different ways of measuring the welfare. Second, when solving the problems over the set of time-consistent strategies, with the introduction of time-inconsistent preferences, unlike the standard case, the set of non-cooperative time-consistent strategies is not included, in general, into the set of time-consistent cooperative strategies. In the case with $\beta=1$ and asymmetric players, a related result was proved in Breton and Keoula (2014).

Next, under partial cooperation, if the players in the coalition are symmetric, their weights are given by $\lambda_i = 1/m$ for all $i \in \mathbb{I}$. From (21) and (20) we obtain

$$h_j^{pc} = h^{pc} = \frac{1}{m(1+M-m)+m\beta\gamma}, \quad h_k^{pn} = h^{pn} = mh^{pc}, \tag{31}$$

for $j \in \mathbb{I}$, $k \in \mathbb{O}$. Then $\dfrac{\partial h^{pc}}{\partial \alpha} < 0$, $\dfrac{\partial h^{pc}}{\partial \delta} < 0$ and $\dfrac{\partial h^{pc}}{\partial \beta} < 0$. Obviously, the same inequalities are satisfied for the harvest rates of non-cooperating countries. Also, $\dfrac{\partial h^{pn}}{\partial m} > 0$, and

$$\frac{\partial h^{pc}}{\partial m} > (<) 0 \Leftrightarrow m < (>) \frac{1}{2}\left[M + \frac{1-\alpha\delta(1-\beta)}{1-\alpha\delta}\right].$$

Let us study the stability of partial cooperation. As a preliminary step, we will consider first the (less realistic) stability under the long-run utility criterion, represented by the auxiliary value function. Then, we will study in more detail the problem with the value function.

5.1.1 Coalitional Stability: Long-Run Utility Criterion

If we define $\bar{\Lambda}(M,m) = (1-\delta)(1-\delta\alpha)\left[\bar{V}^{pn}(M,m) - \bar{V}^{pc}(M,m+1)\right]$, a coalition of size *m* is stable under the long-run utility criterion if, and only if, $\bar{\Lambda}(M,m) \geq 0$ (external stability) and $\bar{\Lambda}(M,m-1) \leq 0$ (internal stability). In our problem,

$$\bar{\Lambda}(M,m) = (1-\delta\alpha)\ln(m+1) + \ln\frac{(1-\delta\alpha)(M-m)+\beta\delta\alpha}{(1-\delta\alpha)(M-m+1)+\beta\delta\alpha}. \tag{32}$$

Note that the stability conditions do not depend, in this model, on the initial conditions, described by the initial resource stock. Therefore, a stable coalition at the initial time will be remain stable throughout the entire planning horizon.

It is easy to check that

$$\frac{\partial \bar{\Lambda}\,(M,m)}{\partial \beta} > 0,\; \frac{\partial \bar{\Lambda}\,(M,m)}{\partial M} > 0.$$

Let us center our attention on the stability of the grand coalition. In that case, we have:

Proposition 1 *Under the long-run utility criterion, the grand coalition is stable if, and only if,*

$$(1-\delta\alpha)\ln M - \ln \frac{2\,(1-\delta\alpha)+\beta\delta\alpha}{1-\delta\alpha+\beta\delta\alpha} \leq 0.$$

Therefore, unlike Kwon (2006) (and also Breton and Keoula (2014)), the maximum size of stable coalitions can be higher than two. Observe also that, from Lemma 1, if the grand coalition is stable, then it is also profitable, in the sense that the payments for the cooperating players are not lower than their payments in the fully non-cooperative setting.

Since $\dfrac{\partial \bar{\Lambda}(M,M-1)}{\partial \beta} > 0$, note that the short and long term discount factors, β and δ, have the opposite influence on the stability of the grand coalition. As shown in Kwon (2006) in the case when β=1, stable coalitions (of maximum size two) can exist only if $\delta\alpha$ is sufficiently high. Therefore, it is convenient to have high values of the discount factor δ, i.e., low discount rates. Note, indeed, that when β=1, $\dfrac{\partial \bar{\Lambda}(M,M-1)}{\partial(\delta\alpha)} = -\ln M < 0$. On the contrary, high discount rates in the short-term, i.e., low values of β, facilitate the stability of the grand coalition. From Proposition 1 we have:

Corollary 1 *Given M, if* $\delta\alpha > 1 - \dfrac{\ln 2}{\ln M}$, *then there exists a value* $\bar{\beta} \in (0,1)$ *such that, for every* $\beta < \bar{\beta}$, *the grand coalition is stable. If* $\delta\alpha < 1 - \dfrac{\ln 2}{\ln M}$, *the grand coalition is always unstable.*

For example, assume that $\delta\alpha = 0.9$. If there are three players, M=3, the grand coalition is stable if β is lower than 0.8456. If the number of players increases, lower values of the parameter β are needed. If M=4, the grand coalition is stable if $\beta \leq 0.6361$; and if M=5, then $\beta \leq 0.521$ is needed. As in Kwon (2006) and in Breton and Keoula (2014)), when the number of players increase, the conditions for the stability of coalitions become more demanding. This is a consequence of the fact that, from (32),

$$\frac{\partial \bar{\Lambda}\,(M,M-1)}{\partial M} > 0.$$

5.1.2 Coalitional Stability: Value Function

The previous results suggest that we can expect the existence of stable coalitions of larger size under quasi-hyperbolic discounting. In the present section we show that this is, indeed, the case when we analyze the stability of coalitions when players measure their welfare with their value functions, and we characterize the number of stable coalitions.

Let us define $\Lambda(M,m) = (1-\delta)\left[V^{pn}(M,m) - V^{pc}(M,m+1)\right]$. A coalition of size *m* is stable if, and only if, $\Lambda(M,m) \geq 0$ (external stability) and $\Lambda(M,m-1) \leq 0$ (internal stability). By substituting (21) and (20) into (51) and (52), after some calculations we obtain

$$\Lambda(M,m) = [1-\delta(1-\beta)]\ln(m+1) - [1-\delta(1-\beta)+\beta\gamma]\ln\left(\frac{M-m+\beta\gamma+1}{M-m+\beta\gamma}\right). \quad (33)$$

Again, the stability conditions do not depend on the resource stock, so a stable coalition at the initial time will be remain stable (sustainable) throughout the entire planning horizon.

Also, as in the previous case, $\dfrac{\partial\Lambda(M,m)}{\partial M} > 0$ but, on the contrary, $\dfrac{\partial\Lambda(M,m)}{\partial\beta}$ can be positive or negative. Hence, the way the short-run discount rate is affecting cooperation is ambiguous. However, after some calculations we can easily check the two following useful properties of function $\Lambda(M,m)$:

$$\frac{\partial^2\Lambda\left(M,m\right)}{\partial\beta^2} > 0 \quad \left(\Lambda\left(M,m\right) \text{is strictly convex in} \beta\right), \text{ and} \quad (34)$$

$$\frac{\partial^2\Lambda\left(M,m\right)}{\partial m^2} < 0 \quad \left(\Lambda\left(M,m\right) \text{is strictly concave in} m\right). \quad (35)$$

The convexity of the stability function with respect to β can lead to situations where the set of values of β for which a coalition of a given size is stable can be the union of two disjoint sets. We discuss this issue in detail in Sect. 5.1.3 and, mainly, in Sect. 5.1.4. The concavity of $\Lambda(M,m)$ with respect to the members of a coalition plays a crucial role in our characterization of the number of stable coalitions (or, more precisely, on the number of stable coalitions with different sizes), as we show in the proof of Proposition 3.

First, as in Sect. 5.1.1, we will focus our attention on the stability of the grand coalition, that from Lemma 2 is always profitable. By taking $m = M-1$ in (33) we obtain:

Proposition 2 *Under the value function criterion, the grand coalition is stable if, and only if,*

$$[1-\delta(1-\beta)]\ln M - [1-\delta(1-\beta)+\beta\gamma]\ln\left(\frac{2+\beta\gamma}{1+\beta\gamma}\right) \leq 0.$$

Since

$$\frac{\partial\Lambda(M,M-1,\beta)}{\partial M} > 0,$$

as with the long-run utility criterion, the stability of the grand coalition becomes more problematic when the number of players increases. Indeed, we have:

Corollary 2 *Given α, β and δ, according to the value function criterion, the grand coalition is stable if, and only if,*

$$M \leq \left(\frac{2+\beta\gamma}{1+\beta\gamma} \right)^{1+\frac{\beta\gamma}{1-\delta(1-\beta)}}.$$

In contrast to Kwon (2006) and Breton and Keoula (2014), when $\beta < 1$, the grand coalition can become stable. For example, if $\alpha = \delta = 0.95$, so that $\delta\alpha = 0.925$ (these are approximately the values taken in the previous subsection), in the case of three players ($M=3$), the grand coalition is stable if $\beta \in (0.0036, 0.8388)$. For $M=4$, the grand coalition is stable if $\beta \in (0.0066, 0.6167)$; and for $M=5$, if $\beta \in (0.0093, 0.4980)$. If there are many players, for example if $M=20$, the grand coalition will only be stable if $\beta \in (0.0502, 0.1242)$. If M is sufficiently large, the grand coalition eventually becomes unstable; however, stable coalitions of smaller sizes may still exist.

It remains to study the stability of partial coalitions. Unlike previous results in the literature, in the following proposition we prove that there can be stable coalitions of different sizes (see the Appendix for the proof). More precisely, we have:

Proposition 3 *In the Great Fish War model with quasi-hyperbolic discounting and symmetric players, under simultaneous moves, there can be stable coalitions of up to three different sizes.*

(1) *If there is only one stable coalition, it can be of any size.*
(2) *If there are two stable coalitions, there are two possibilities:*

- *The two coalitions differ in size by one player.*
- *One of the coalitions is the grand coalition.*

(3) *If there are three stable coalitions, one is the grand coalition and the other two differ in size by one player.*

Figure 1 illustrates the different possibilities. We take $M=8$, $\delta=0.97$ and $\alpha = 0.96$). We discuss this example in more detail in Sect. 5.1.3.

Note that:

- When $\beta=0.5$, the only stable coalition is when $m=2$. This is because $\Lambda(8,1)$ is negative and $\Lambda(8,2)$ is positive. Since $\Lambda(8,m) > 0$ for $m \geq 2$, any coalition of size greater or equal than three is internally unstable.
- When $\beta=0.01$, the stability function is positive for $m \in [1,6]$, but is negative for $m=7$. Hence, only the grand coalition is stable.
- When $\beta=0.35$, one stable coalition is when $m=3$. This is because $\Lambda(8,3)$ is positive and $\Lambda(8,2)$ is negative. The global coalition is also stable because $\Lambda(8,7)$ is negative.

- For $\beta \approx 0.3269$ the coalitions with three and four members are both stable. This is because $\Lambda(8,2) < 0$, $\Lambda(8,3) = 0$ and $\Lambda(8,4) > 0$. The grand coalition is also stable ($\Lambda(8,7) < 0$), so in this case there are three stable coalitions.

5.1.3 Numerical Solutions

In order to appreciate better the effects of the different parameters of the model on the stability of the different coalitions, in this section we present a more detailed numerical solutions.

In Tables 1 and 2 we analyze the stability of the grand coalition for some values of the parameters. For some fixed values of δ and β, we write the values of the parameter α such that the grand coalition is stable, for several values of M. Table 1 illustrates the results for the value function representing the welfare perceived by the agents. Table 2 shows the results according to the long-run utility criterion.

First, we can observe that the results with the two criteria used to measure the welfare are quite similar. We have the same qualitative patterns, and the threshold values of the parameter α are relatively similar. Hence, we can expect that the long-run utility criterion is a good proxy of the results on the sustainability of cooperation obtained with the use of the value function. The long-run utility criterion, which is less realistic in our problem, favours slightly the stability of cooperation of the grand coalition. A similar result is expected in the case of partial cooperation. Recall that the value function is obtained by adding the utility of the present period plus the auxiliary value function (representing the long-run utility) discounted by $\beta\delta$. As a result, if $\beta < 1$, it assigns a higher value to the utility derived from present harvests in comparison with the long-run utility. But this quantity is clearly higher under non-cooperation. However, we are changing the relative weight of the utility in just one period, whereas the number of periods is infinite. Therefore, unless β is sufficiently small, the results will be very similar. Observe that the differences in the threshold values of the parameter α increase when we move from $\beta = 0.66$ to $\beta = 0.5$.

Next, let us center our attention on the stability of cooperation when we measure the welfare with the value function. When analysing the effects of the different parameters, we see that the result advanced above remains: the perspectives of cooperation of the grand coalition increase when δ and α increase, and when β decreases. This result looks quite surpris-

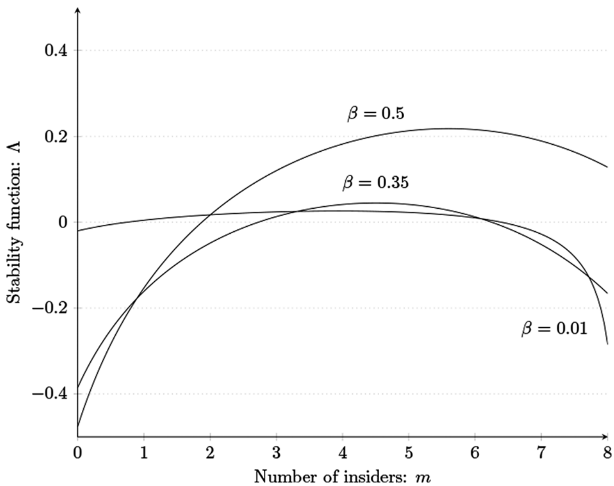

Fig. 1 Plotting the stability function against the number of insiders

Table 1 Stability of the grand coalition

β	δ	$M=3$	$M=4$	$M=5$	$M=6$	$M=7$
0.66	0.99	$\alpha \in [0.7088, 0.9473)$	$\alpha \in [0.9473, 1]$	$\nexists$	$\nexists$	$\nexists$
0.5	0.99	$\alpha \in [0.5845, 0.7863)$	$\alpha \in [0.7863, 0.8911]$	$\alpha \in [0.8911, 0.9562]$	$\alpha \in [0.9562, 1]$	$\nexists$
0.66	0.95	$\alpha \in [0.7557, 1]$	$\nexists$	$\nexists$	$\nexists$	$\nexists$
0.5	0.95	$\alpha \in [0.6324, 0.8436)$	$\alpha \in [0.8436, 0.9517]$	$\alpha \in [0.9517, 1]$	$\nexists$	$\nexists$

Value function criterion

Table 2 Stability of the grand coalition

β	δ	$M=3$	$M=4$	$M=5$	$M=6$	$M=7$
0.66	0.99	$\alpha \in [0.7047, 0.9432)$	$\alpha \in [0.9432, 1]$	$\nexists$	$\nexists$	$\nexists$
0.5	0.99	$\alpha \in [0.5789, 0.7804)$	$\alpha \in [0.7804, 0.8854]$	$\alpha \in [0.8854, 0.9508]$	$\alpha \in [0.9508, 0.9960]$	$\alpha \in [0.9960, 1]$
0.66	0.95	$\alpha \in [0.7344, 0.9829]$	$\alpha \in [0.9829, 1]$	$\nexists$	$\nexists$	$\nexists$
0.5	0.95	$\alpha \in [0.6633, 0.8133)$	$\alpha \in [0.8133, 0.9227]$	$\alpha \in [0.9227, 0.9909]$	$\alpha \in [0.9909, 1]$	$\nexists$

Long-run utility criterion

ing. Recall that the harvest rates, both under cooperation and under non-cooperation, move in the same direction with respect to small changes in the two discount parameters δ and β. Hence, one could expect that the impact on the total welfare should be similar, reducing it in both cases in comparison with the case when $\beta=1$. However, when we analyze the difference of the welfares, when $\beta<1$, we observe that the reduction of this difference is higher if the agent decides not to cooperate, facilitating in this way the possibility of cooperation.

If we compare our numerical solutions with those in Table 1 in Kwon (2006), we observe that when $\delta=0.99$, we need $\alpha \geq 0.7085$, so that $\alpha\delta = 0.7014$, to obtain a size-two sustainable coalition if $M=3$ (conditions are much more demanding when M increases). If $\beta=0.66$, the grand coalition is stable if $M=3$ and $M=4$; and if $\beta=0.5$, there are values of α in that interval such that the grand coalition can be stable if there are three to six players. If $\delta=0.85$, we need $\alpha=0.7384$ in order to obtain stable coalitions of size two when $M=3$. But for these values of the parameters, the grand coalition can be stable if $M = 3, 4, 5$, for some specific values of β in the interval $[0.5, 0.66]$.

Finally, Table 3 shows, for some given values of the remaining parameters, which are the values of β guaranteeing the formation of coalitions if there are eight players ($M=8$). In the symmetric setting with $\beta=1$, Kwon (2006) showed that it is necessary that $\alpha\delta \geq 0.9318$ for the formation of a size-two coalition, whereas coalitions of higher sizes are always unstable. In our solution, we take $\delta=0.97$ and $\alpha=0.96$, so that their product is approximately the above threshold level.

The first important result that we observe is that there is not a monotonic behaviour in terms of the size of the coalition. The grand coalition becomes stable for low values of β, while coalitions of sizes 5 to 7 are always unstable. Coalitions of sizes 2, 3 and 4 can be stable for appropriate values of the short run discount factor. In these cases, coalitions of a lower number of players are more willing to form, as expected.

The second, and more surprising result, is that there is also not a monotonic behaviour in terms of β. First of all, for the values of β for which a coalition of size 4 is stable, the grand coalition is also stable. For $m=2$, 3 and 4, the length of the interval, as well as the maximum value of β, increase when the size of the stable coalitions decrease.[2] Next, for sufficiently low values of β, the coalitions become unstable, but for extremely low values of this parameter the coalitions are stable. These very low values of β could represent agents who are very (but not completely) myopic in the short-run.

To get a better idea of what is happening in Table 3, consider Fig. 2, where we have plotted $\Lambda(M, m)$ against β for some of the values of m reported in Table 3. If the graph is positive for a particular m, the coalition of size m is externally stable. If it is negative, the coalition of size m +1 is internally stable. Recall that, from (34), (35), the stability function is strictly concave in β and convex in m. In our example, there seems to be a saddle point for some $(m, \beta) \in \{(0, 8), (0, 1)\}$.

The first thing to note is that all the graphs have a minimum for some $\beta \in (0, 1)$. This is why, for $m=2$ and $m=3$, stability is obtained within two separate intervals of β. As m grows, the graphs initially shift upwards, so that they are negative for fewer β. This makes it less likely that a coalition is internally stable. Hence, stability is obtained for fewer β and for $m=5$ not at all. For higher m, the graphs start shifting downwards for small β, making it more likely that internal stability is obtained. However, we still do not have external stabil-

[2] In the case of $m=2$, we do not obtain a stable coalition if $\beta=1$ because $\alpha\delta$ is slightly under the threshold level (0.9318) in Kwon (2006).

ity for those m until we get to $m=8$, in which case external stability is satisfied automatically because there is no player who can join the coalition. Hence, in this case, stability is obtained for every β such that the graph is negative.

5.1.4 Discussion

The most surprising take-away from Table 3 and Fig. 2 is that, when $m=2$ and $m=3$, coalitional stability is obtained for two intervals of β. Clearly, this is a result of the convexity of the stability function $\Lambda(M,m)$. What then explains the convexity? Why is a coalition externally stable for small and large β and externally unstable in between?

First, we should note that analyzing the effect of the short-run discount factor on stability is, in a sense, easier than analyzing the impact of the long-run discount factor on stability. This is because while δ influences the trade-off between utility in every period with utility in every other period, β influences only the trade-off between the present and "the future", where the future is every future period taken as a whole. Indeed, from a mathematical point of view, the maximization problems in Eqs. (22) and (26) can be interpreted as having only two time periods.

Imagine now that player i is an outsider considering whether to join an already existing coalition. If she decides to join, she will reduce the fraction of the fish stock that she extracts. This lowers her utility in both the present and the future. On the other hand, if she joins, since both she and the already existing insiders will reduce their extraction in the present, there will be more fish available in the future. This increases her future utility. Hence, the costs of joining the coalition are in the present and in the future, while the benefits are only in the future. This suggests that she is more likely to join the coalition if she has a high short-run discount factor. That is, a coalition is less likely to be externally stable for small β, and the functions in Fig. 2 should be decreasing.

However, there is a also a second mechanism at work. The main reason for why player i may want to join the coalition is that it prompts the already existing members to reduce their present extraction. This is because agent i herself doesn't bare the cost of this reduction but shares in the benefit. Now, it is clear from Eq. (31) that an increase in β leads to a large decrease in present extraction for all players, especially if γ is large. (In Table 3, $\gamma\approx 13$). This means that in the event that player i joins the coalition, the decrease in present extraction by the already existing members will also be smaller. This reduces the benefit of joining the coalition when the short-run discount factor is large. That is, a coalition is less likely to be externally stable for large β, and the functions in Fig. 1 should be increasing.

For small β, the first of these mechanisms dominate because even a small increase in β means that a lot of extraction should be moved from the present to the future. (Note that, when $\beta=0$, the whole resource is extracted in the present, and future utility is minus infinity.) For large β, the second mechanism becomes more important.

5.2 First-Mover Advantage

In the case when the coalition has first-mover advantage, under partial cooperation, from (29) and (30) we obtain

Table 3 Partial cooperation

Size of the coalition	Stability of partial cooperation. Value function criterion
$m=8$	$\beta \in [0.0056, 0.3844]$
$m=7$	Unstable for every β
$m=6$	Unstable for every β
$m=5$	Unstable for every β
$m=4$	$\beta \in [0.2936, 0.3269]$
$m=3$	$\beta \in [0.0272, 0.0332] \cup [0.3269, 0.4676]$
$m=2$	$\beta \in [0.0138, 0.0272] \cup [0.4676, 0.9952]$

$M=8$, $\delta=0.97$, $\alpha=0.96$

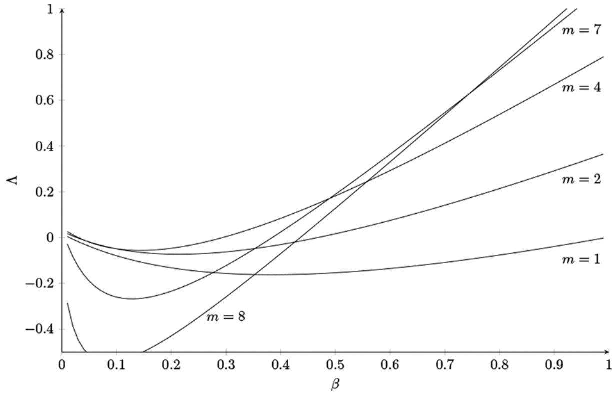

Fig. 2 Plotting Λ against β for different values of m

$$h_j^L = h^L = \frac{1}{m(1+\beta\gamma)}, \quad h_k^F = h^F = \frac{\beta\gamma}{(M-m+\beta\gamma)(1+\beta\gamma)}, \tag{36}$$

for $j \in \mathbb{I}$, $k \in \mathbb{O}$. Notice that $mh^L > h^F$, i.e., as in the standard case, under first-mover advantage, the catch of the whole coalition is strictly greater than the catch of a non-cooperating player. In addition, we observe that, concerning the catches of the cooperating countries, $\frac{\partial h^L}{\partial \alpha} < 0$, $\frac{\partial h^L}{\partial \delta} < 0$ and $\frac{\partial h^L}{\partial \beta} < 0$. The effects of β and γ on the catches of the noncooperating countries are ambiguous, and depends on the sign of $M - m - \beta^2\gamma^2$. The total number of players M does not affect h^L, whereas h^F increases with m. Finally, $\frac{\partial h^L}{\partial m} < 0$ and $\frac{\partial h^F}{\partial m} < 0$. From all these properties one can expect that the results concerning the stability of partial cooperation will differ from those obtained if moves are simultaneous.

Next we study the stability of partial cooperation under both criteria.

5.2.1 Coalitional Stability: Long-Run Utility Criterion

If we define $\bar{\Lambda}(M,m) = (1-\delta)(1-\delta\alpha)\left[\bar{V}^F(M,m) - \bar{V}^L(M,m+1)\right]$, recall that a coalition of size m is stable under the long-run utility criterion if, and only if, $\bar{\Lambda}(M,m) \geq 0$ (external stability) and $\bar{\Lambda}(M,m-1) \leq 0$ (internal stability). In our problem, substituting

(29) and (30) into the expressions of the auxiliary value functions (see Appendix 4), after some algebra we obtain

$$\begin{aligned}\bar{\Lambda}(M,m) = {} & (1-\delta\alpha)\ln(m+1) + (1-\delta\alpha)\ln(\beta\delta\alpha) \\ & + \delta\alpha \ln((1-\delta\alpha)(M-m-1)+\beta\delta\alpha) - \ln((1-\delta\alpha)(M-m)+\beta\delta\alpha)\,. \end{aligned} \tag{37}$$

Lemma 3 *The function* $\bar{\Lambda}(M,m)$ *verifies:*

1. *For every* $m \leq M-1$, $\dfrac{\partial \bar{\Lambda}(M,m)}{\partial m} \geq 0$.
2. *For every* $m \leq M-1$, $\dfrac{\partial \bar{\Lambda}(M,m)}{\partial \beta} > 0$.
3. *For every m, there exists a value* $\beta^* \in (0,1]$ *such that* $\bar{\Lambda}(M,m) < 0$*, for every* $\beta < \beta^*$.

If $\beta = 1$, the first property was already obtained in Kwon (2006), and plays a crucial role in the description of stable coalitions under first-mover advantage. The monotonicity of the dependence of $\bar{\Lambda}(M,m)$ on β is the another ingredient that we need to characterize the number of stable coalitions under quasi-hyperbolic time preferences. The following proposition extends Proposition 2 in Kwon (2006) to our setting. It also analyzes the stability of the grand coalition.

Proposition 4 *If, in the analysis of the stability of cooperation, the welfare is measured by the auxiliary value function (long-run utility criterion), then we have:*

1. *There always exists at least one and no more than two sustainable coalitions for any given* $M \geq 3$*, α, β and δ.*
2. *If there are two sustainable coalitions, they differ in size by one player.*
3. *There exists a value* β^* *such that, if* $\beta < \beta^*$*, the grand coalition is the unique stable coalition.*

It remains to check if stable coalitions are also profitable. The following proposition states that this is the case when we measure the welfare with the long-run utility criterion.

Proposition 5 *In the first-mover advantage case, under the long-run utility criterion, if a partial coalition is stable, then it is also profitable.*

5.2.2 Coalitional Stability: Value Function

In this case, let us define $\Lambda(M,m) = (1-\delta)(1-\delta\alpha)\left[V^F(M,m) - V^L(M,m+1)\right]$. A coalition of size $m<M$ is stable under the value function criterion if, and only if, $\Lambda(M,m) \geq 0$ (external stability) and $\Lambda(M,m-1) \leq 0$ (internal stability). If $m=M$, the grand coalition is stable if $\Lambda(M,M-1) \leq 0$. As in the previous subsection, substituting (29) and (30) into the expressions of the value functions (see Appendix 4), after some algebra we obtain

$$\begin{aligned}\Lambda(M,m) = {} & (1-\delta\alpha)(1-\delta(1-\beta))\ln(m+1) + (1-\delta\alpha)(1-\delta(1-\beta))\ln(\beta\delta\alpha) \\ & + \beta\delta\alpha\ln((1-\delta\alpha)(M-m-1)+\beta\delta\alpha) - ((1-\delta\alpha)(1-\delta(1-\beta))+\beta\delta\alpha)\ln((1-\delta\alpha)(M-m)+\beta\delta\alpha)\,. \end{aligned} \tag{38}$$

The expression (38) is more cumbersome than that in the previous section (Eq. 37) and it is unclear which is the effect of the parameters m and β on $\Lambda(M,m)$. In any case, a close observation of (38) suggests that, if β is sufficiently low, $\Lambda(M,m)$ can become negative (as in the third Statement in Lemma 3). This suggests that the grand coalition will be stable (recall Statement 3 of Proposition 4). More interestingly, recall that, from the construction of the value function, if β is sufficiently high (close to 1), the auxiliary value function provides a good approximation of the value function. Hence, one can expect that the Statements 1 and 2 of Lemma 3 and Proposition 4 will be preserved for those values of β.

With respect to the profitability of stable coalitions according to the value function criterion, from Kwon (2006) we know that this is the case if $\beta=1$. Therefore, by continuity, if β is sufficiently close to 1, stable coalitions will be profitable. In the limit when $\beta \to 0^+$, the difference $V^L(x) - V^N(x)$ goes to zero. However, it is not clear whether the condition of stability of a coalition suffices to guarantee the profitability of the coalition. Indeed, it is easy to find values of the parameters such that $V^L(x) - V^N(x)$ is negative. In our numerical solutions we will check if stable coalitions are also profitable.

5.2.3 Numerical Solutions

In this section we present the numerical results for the same parameter values of δ and β as in the case with simultaneous moves. Tables 4 and 5 show the results concerning the stability of the grand coalition. As above, for some fixed values of δ and β, we write the values of the parameter α such that the grand coalition is stable and profitable, for several values of M. We start, in Table 4, with the study of the stability and profitability when we measure the welfare with the value functions. It is noteworthy that, in all these cases, for all the values for which a coalition is stable, it is also profitable. Next, Table 5 shows the results according to the long-run utility criterion.

Note that, in both cases, as in Kwon (2006) and Breton and Keoula (2014), it is much easier to maintain a coalition if it can act as a leader. Note also that the results with the two criteria are very similar. Thus, the long-run utility criterion, with the (more tractable) auxiliary value function, provides a very good approximation to the threshold values for which the full coalition is stable and profitable under first-mover advantage. As in the case of simultaneous moves, the long-run utility criterion is less demanding with respect to the value of the parameter α for the formation of the grand coalition, but here the difference is very small. Therefore, if the analysis of coalition formation is made with agents measuring their individual welfare with the value function, but the profitability is measured with the long-run utility criterion, stable coalitions are profitable in all cases.

When we center our attention on the sensitivity with respect to the parameters, for equal values of the long-run discount factor δ, more short-biased players (with lower values of β) are more willing to collaborate in the formation of the grand coalition. Indeed, according to

Table 4 Stability and profitability of the grand coalition

β	δ	$M=3$	$M=4$	$M=5$	$M=6$	$M=7$
0.66	0.99	$\alpha \in [0,1]$	$\alpha \in [0,1]$	$\alpha \in [0,0.7679]$	$\alpha \in [0,0.5386]$	$\alpha \in [0,0.4137]$
0.5	0.99	$\alpha \in [0,1]$	$\alpha \in [0,1]$	$\alpha \in [0,1]$	$\alpha \in [0,1]$	$\alpha \in [0,1]$
0.66	0.95	$\alpha \in [0,1]$	$\alpha \in [0,1]$	$\alpha \in [0,0.7623]$	$\alpha \in [0,0.5433]$	$\alpha \in [0,0.4206]$
0.5	0.95	$\alpha \in [0,1]$	$\alpha \in [0,1]$	$\alpha \in [0,1]$	$\alpha \in [0,1]$	$\alpha \in [0,0.8544]$

Value function criterion

Table 5 Stability and profitability of the grand coalition

β	δ	$M=3$	$M=4$	$M=5$	$M=6$	$M=7$
0.66	0.99	$\alpha \in [0, 1]$	$\alpha \in [0, 1]$	$\alpha \in [0, 0.7781]$	$\alpha \in [0, 0.5432]$	$\alpha \in [0, 0.4164]$
0.5	0.99	$\alpha \in [0, 1]$	$\alpha \in [0, 1]$	$\alpha \in [0, 1]$	$\alpha \in [0, 1]$	$\alpha \in [0, 1]$
0.66	0.95	$\alpha \in [0, 1]$	$\alpha \in [0, 1]$	$\alpha \in [0, 0.8108]$	$\alpha \in [0, 0.5661]$	$\alpha \in [0, 0.4340]$
0.5	0.95	$\alpha \in [0, 1]$	$\alpha \in [0, 1]$	$\alpha \in [0, 1]$	$\alpha \in [0, 1]$	$\alpha \in [0, 1]$

Long-run utility criterion

Table II in Kwon (2006), for $M=7$ (and $\beta=1$), the grand coalition is stable (and profitable) for $\alpha\delta = 0.1$ (and lower), but it is no longer stable for values of $\alpha\delta \geq 0.3$. In our model, for $\beta=0.66$, higher (but not too high) values of $\alpha\delta$ allow the stability of the grand coalition. And when $\beta=0.5$, we get stability for many values of these parameters.

6 Concluding Remarks

To the best of our knowledge, this paper is the first to study and derive analytical results for the great fish war model under partial cooperation (i.e., coalition versus fringe) when players have quasi-hyperbolic discount functions. We believe that the consideration of short-run biases, as described in the model with $\beta\delta$-preferences introduced by Phelps and Pollak (1968), is relevant in fishery agreement negotiations. Such preferences can be justified in several ways. In general, there is strong evidence that many individuals have a bias toward the present resulting in time-inconsistent preferences. In our setting, where agents may be countries negotiating fishery agreements, regimes in democratic countries may exhibit a bias toward the present because they want to be re-elected. Authoritarian regimes may act in a similar way to prevent public disorder. It also seems natural to think that developing countries, whose fishermen have lower rents, may also have a stronger biases toward the present.

In our model, we find that, as in the case of geometric discounting, there exist equilibrium fishing rules that are linear functions of the current fish stock. The main new result is that the introduction of quasi-hyperbolic discounting can facilitate the formation of larger stable coalitions. It is important to note that the stability conditions in this model are independent of the resource stock, so that stable coalitions formed at the initial time remain stable over the entire planning horizon. Also, players who are not in a coalition harvest much more than coalition members. As a result, we show that the total harvest is lower (and the long-run fish stock higher) when coalitions of larger size are formed. In our problem, we prove that there can be up to three stable coalitions in the case of simultaneous moves. In particular, the grand coalition can become stable under both simultaneous moves and first-mover advantage for the coalition. This was not the case in the setting with symmetric or asymmetric players with geometric discounting. Specifically, as shown in Kwon (2006) and Breton and Keoula (2014), in their cases there are no stable coalitions of size greater than two under simultaneous moves, and the parameter conditions for the stability of size two coalitions can be very stringent.

Appendix

1 Subgame Perfect Non-Cooperative Equilibrium

From the first order condition in Eq. (2), using (4) we easily obtain

$$c_i^* = \frac{1}{\alpha\beta\delta\bar{A}_i}\Big(x - \sum_{j\neq i}\phi_j^N(x) - c_i^*\Big),$$

hence

$$\phi_i^N(x) = \frac{1}{\alpha\beta\delta\bar{A}_i}\Big(x - \sum_{j=1}^{M}\phi_j^N(x)\Big). \tag{39}$$

Taking the sum for $i \in \{1, \ldots, M\}$ and simplifying,

$$\sum_{i=1}^{M}\phi_i^N(x) = \frac{\sum_{j=1}^{M}\left(\alpha\beta\delta\bar{A}_i\right)^{-1}}{1+\sum_{j=1}^{M}\left(\alpha\beta\delta\bar{A}_i\right)^{-1}}.$$

Substituting in (39), $\phi_i^N(x) = h_i^N x$, where

$$h_i^N = \frac{\left(\alpha\beta\delta\bar{A}_i\right)^{-1}}{1+\sum_{j=1}^{M}\left(\alpha\beta\delta\bar{A}_i\right)^{-1}}. \tag{40}$$

Next, let us compute the coefficients $\bar{A}_i^N$ and $\bar{B}_i^N$ in the auxiliary value function $\bar{V}_i^N(x)$. From (2) and (4), we obtain

$$\bar{A}_i^N = \frac{1}{1-\delta\alpha},$$

$$\bar{B}_i^N = \frac{1}{1-\delta}\left[\ln h_i^N + \frac{\delta\alpha}{1-\delta\alpha}\ln(1-H^N)\right],$$

where $H^N = \sum_{i=1}^{M} h_i^N$ denotes the total harvest rate. By substituting the value of $\bar{A}_i^N$ in (40), (5) follows. The auxiliary value function is

$$\bar{V}_i^N(x) = \frac{1}{1-\delta\alpha}\ln x - \frac{1}{1-\delta}\left[\ln(\beta\gamma) + \frac{1}{1-\delta\alpha}\ln(1+\frac{M}{\beta\gamma})\right]. \tag{41}$$

It remains to compute the value functions $V_i^N(x)$. By substituting in (2) we obtain

$$V_i^N(x) = \ln h_i^N + \ln x + \beta\delta[\bar{A}_i^N \alpha \ln x + \bar{A}_i^N \alpha \ln(1 - H^N) + \bar{B}_i^N]$$
$$= \frac{1 - \delta\alpha(1-\beta)}{1 - \delta\alpha} \ln x + \frac{1}{1-\delta}[(1 - \delta(1-\beta)) \ln h_i^N + \frac{\beta\delta\alpha}{1 - \delta\alpha} \ln(1 - H^N)].$$

Alternatively,

$$V_i^N(x) = \frac{1 - \delta\alpha(1-\beta)}{1 - \delta\alpha} \ln x -$$
$$\frac{1}{1-\delta}\left[(1 - \delta(1-\beta)) \ln(\beta\gamma) + \frac{1 - \delta(1 + \alpha - \delta\alpha)(1-\beta)}{1 - \alpha\delta} \ln\left(1 + \frac{M}{\beta\gamma}\right)\right]. \tag{42}$$

2 Cooperative Solution

From the first order condition in Eq. (7), using (10), we obtain

$$c_i^* = \frac{\lambda_i}{\alpha \sum_{j=1}^M \lambda_j \beta\delta \bar{A}_j^c}\left(x - \sum_{k=1}^M c_k^*\right). \tag{43}$$

By taking the sum for $i \in \{1, \ldots, M\}$ in (43) and taking into account that $\sum_{j=1}^M \lambda_j = 1$, we get

$$\sum_{i=1}^M c_i^*(x) = \frac{1}{1 + \alpha \sum_{j=1}^M \lambda_j \beta\delta \bar{A}_j^c} x.$$

Substituting in (43) we obtain the linear strategies $c_i^* = \phi_i^c(x) = h_i^c x$, where

$$h_i^c = \frac{\lambda_i}{1 + \alpha \sum_{j=1}^M \lambda_j \beta\delta \bar{A}_j^c}. \tag{44}$$

For the calculation of the coefficients $\bar{A}_i^c$ and $\bar{B}_i^c$ of the auxiliary value function $\bar{V}_i^c(x)$, from (7) and (10) we obtain

$$\bar{A}_i^c = \frac{1}{1 - \alpha\delta}, \tag{45}$$

$$\bar{B}_i^c = \frac{1}{1-\delta}\left[\ln \lambda_i + \gamma_i \ln\left(\sum_{j=1}^M \lambda_j \beta\gamma\right) - \frac{1}{1 - \alpha\delta} \ln\left(1 + \beta\gamma\right)\right]. \tag{46}$$

By substituting (45) in (44), Eq. 11 follows.

Finally, we compute the value functions $V_i^c(x)$, for $i \in \{1, \ldots, M\}$, by substituting (11), (45) and (46) in (9) to obtain

$$V_i^c(x) = \frac{1-\alpha\delta(1-\beta)}{1-\alpha\delta}\ln x + \frac{1}{1-\delta}\left[\Big(1-\delta(1-\beta)\Big)\ln\lambda_i + \beta\gamma\ln(\beta\gamma)\right.$$
$$\left. - \frac{1}{1-\alpha\delta}\Big(1-\delta(1-\beta)(1+\alpha-\alpha\delta)\Big)\ln\Big(1+\beta\gamma\Big)\right].$$

3 Partial Cooperation: Simultaneous Moves

For the members of the coalition, from the first order condition in Eq. (15), using (19), we obtain

$$c_i^{pc} = \frac{\lambda_i}{\alpha\sum\limits_{j\in\mathbb{I}}\lambda_j\beta\delta\bar{A}_j^{pc}}\left(x - \sum_{k\in\mathbb{I}} c_k^{pc} - \sum_{l\in\mathbb{O}}\phi_l^{pn}(x)\right). \tag{47}$$

In a similar way, for the outsiders, the combination of the first order optimality condition in (13) with (19) gives rise to

$$c_j^{pn} = \left(\alpha\beta\delta\bar{A}_j^{pn}\right)^{-1}\left(x - c_i^{pn} - \sum_{k\in\mathbb{I}}\phi_k^{pc} - \sum_{l\in\mathbb{O}\setminus\{j\}}\phi_l^{pn}(x)\right). \tag{48}$$

By identifying $c_i^{pc} = \phi_i^{pc}(x)$, $c_j^{pn} = \phi_j^{pn}(x)$, and taking the sum in $i\in\mathbb{I}$ in (47) and the sum in $j\in\mathbb{O}$ in (48), we can easily solve

$$\sum_{i\in\mathbb{I}}\phi_i^{pc}(x) = \frac{1}{1+\left(\sum_{j\in\mathbb{I}}\alpha\lambda_j\beta\delta\bar{A}_j^{pc}\right)\left(1+\sum_{k\in\mathbb{O}}\left(\alpha\beta\delta\bar{A}_k^{pn}\right)^{-1}\right)}x,$$

$$\sum_{i\in\mathbb{O}}\phi_i^{pn}(x) = \frac{\left(\sum_{j\in\mathbb{I}}\alpha\lambda_j\beta\delta\bar{A}_j^{pc}\right)\left(\sum_{k\in\mathbb{O}}\left(\alpha\beta\delta\bar{A}_k^{pn}\right)^{-1}\right)}{1+\left(\sum_{j\in\mathbb{I}}\alpha\lambda_j\beta\delta\bar{A}_j^{pc}\right)\left(1+\sum_{k\in\mathbb{O}}\left(\alpha\beta\delta\bar{A}_k^{pn}\right)^{-1}\right)}x.$$

By substituting in (47), (48) we obtain $\phi_i^{pc}(x) = h_i^{pc}x$ and $\phi_i^{pn}(x) = h_i^{pn}x$, where

$$h_i^{pc} = \frac{\lambda_i}{1+\left(\sum_{j\in\mathbb{I}}\alpha\lambda_j\beta\delta\bar{A}_j^{pc}\right)\left(1+\sum_{k\in\mathbb{O}}\left(\alpha\beta\delta\bar{A}_k^{pn}\right)^{-1}\right)} \tag{49}$$

and

$$h_i^{pn} = \frac{\left(\beta\delta\bar{A}_i^{pn}\right)^{-1}\left(\sum_{j\in\mathbb{I}}\alpha\lambda_j\beta\delta\bar{A}_j^{pc}\right)}{1+\left(\sum_{j\in\mathbb{I}}\alpha\lambda_j\beta\delta\bar{A}_j^{pc}\right)\left(1+\sum_{k\in\mathbb{O}}\left(\alpha\beta\delta\bar{A}_k^{pn}\right)^{-1}\right)}. \tag{50}$$

Next, from the auxiliary value functions (14) and (16), together with (19), we easily obtain that

$$\bar{A}_i^{pc} = \frac{1}{1-\alpha\delta} \text{ and } \bar{A}_j^{pn} = \frac{1}{1-\alpha\delta},$$

and substituting in (49), (50), the harvest rates (20), (21) follow.

The auxiliary value functions are

$$\bar{V}_i^{pc} = \frac{1}{1-\alpha\delta}\ln x + \frac{1}{1-\delta}\left[\ln h_i^{pc} + \gamma_i \ln(1 - H(M,m))\right],$$

$$\bar{V}_i^{pn} = \frac{1}{1-\alpha\delta}\ln x + \frac{1}{1-\delta}\left[\ln h_i^{pn} + \gamma_i \ln(1 - H(M,m))\right],$$

where $H(M,m)$ denotes the sum of all the harvest rates, hence

$$1 - H(M,m) = 1 - \sum_{i\in\mathbb{I}} h_i^{pc} - \sum_{j\in\mathbb{O}} h_j^{nc},$$

where h_i^{pc} and h_j^{nc} are given by (21) and (20), respectively.

Substituting in (17) we finally obtain the expressions of the value functions of cooperating and non-cooperating players,

$$V_i^{pc} = (1+\beta\gamma)\ln x + \frac{1}{1-\delta}\left[(1-\delta(1-\beta))\ln h_i^{pc} + \beta\gamma\ln(1-H(M,m))\right], \quad (51)$$

$$V_i^{pn} = (1+\beta\gamma)\ln x + \frac{1}{1-\delta}\left[(1-\delta(1-\beta))\ln h_i^{pn} + \beta\gamma\ln(1-H(M,m))\right]. \quad (52)$$

4 Partial Cooperating: First-Mover Advantage

Given c_i^L, $i \in \mathbb{I}$, we study first the problem of the followers, by solving (22). From the first order optimality condition, we obtain

$$c_j^F = \phi_i^F = (\beta\gamma)^{-1}\left(x - \sum_{i\in\mathbb{I}} c_i^L - \sum_{k\in\mathbb{O}} \phi_k^F\right). \quad (53)$$

In Eq. (53) we have made use of the fact that, combining (23) and (28), we derive $\bar{A}_j^F = 1/(1-\alpha\delta)$. By taking the sum for all $j \in \mathbb{O}$ and solving, we obtain

$$\sum_{j\in\mathbb{O}} \phi_j^F = \frac{(M-m)(\beta\gamma)^{-1}}{1+(M-m)(\beta\gamma)^{-1}}\left(x - \sum_{i\in\mathbb{I}} c_i^L\right),$$

and substituting in (53) we have

$$\phi_j^F = \frac{(\beta\gamma)^{-1}}{1 + (M - m)(\beta\gamma)^{-1}} \left(x - \sum_{i \in \mathbb{I}} c_i^L \right). \tag{54}$$

Next, let us solve the problem for the leaders. By substituting (54) in (24), applying the first order optimality condition in (24), and using (25) and (28), we derive

$$c_i^L = \phi_i^L(x) = \frac{\lambda_i}{\beta\gamma} \left(x - \sum_{k \in \mathbb{I}} \phi_k^L(x) \right). \tag{55}$$

In Eq. (55), by taking the sum for all $i \in \mathbb{I}$, we get

$$\sum_{i \in \mathbb{I}} \phi_i^L(x) = \frac{x}{1 + \beta\gamma},$$

and substituting in Eqs. (54) and (55), the harvest rates (29) and (30) follow.

The auxiliary value functions are

$$\bar{V}_i^L = \frac{1}{1 - \alpha\delta} \ln x + \frac{1}{1 - \delta} \left[\ln h_i^L + \gamma \ln(1 - H(M, m)) \right],$$

$$\bar{V}_j^F = \frac{1}{1 - \alpha\delta} \ln x + \frac{1}{1 - \delta} \left[\ln h_i^F + \gamma \ln(1 - H(M, m)) \right],$$

with h_i^L and h_j^F given by (29) and (30), respectively, and H is the sum of all the harvest rates.

Finally, substituting in (27), the expressions of the value functions of cooperating and non-cooperating players are

$$V_i^L = (1 + \beta\gamma) \ln x + \frac{1}{1 - \delta} \left[(1 - \delta(1 - \beta)) \ln h_i^L + \beta\gamma \ln(1 - H(M, m)) \right],$$

$$V_j^F = (1 + \beta\gamma) \ln x + \frac{1}{1 - \delta} \left[(1 - \delta(1 - \beta)) \ln h_j^F + \beta\gamma \ln(1 - H(M, m)) \right].$$

5 Proofs

Proof of Lemma 1: We can write

$$\bar{V}_i^c(x) - \bar{V}_i^N(x) = \frac{f(M)}{1 - \delta},$$

where

$$f(M) = \frac{1}{1-\delta\alpha}\ln\left(\frac{M+\beta\gamma}{1+\beta\gamma}\right) - \ln M.$$

The result follows by observing that $f(1) = 0$ and $f'(M) > 0$, for $M>1$. □

Proof of Lemma 2: In this case, after some algebra we obtain that

$$V_i^c(x) - V_i^N(x) = \frac{g(M)}{1-\delta},$$

where

$$g(M) = \frac{(1-(1-\delta)(\beta+\delta\alpha(1-\beta))\ln(M(1-\delta\alpha)-\beta\delta\alpha) + (1-\delta\alpha)(1-\delta+\beta\delta)\ln M}{(1-\delta)(1-\delta\alpha)} \\ - \frac{(1-\delta(1-\beta)(1+\alpha-\delta\alpha)\ln(1-\delta\alpha(1-\beta))}{(1-\delta)(1-\delta\alpha)}.$$

Since $g(1) = 0$ and $g'(M) > 0$, for $M>1$, the result follows. □

Proof of Proposition 1: Since we are studying the stability of the grand coalition, we have to check the internal stability of the coalition of size *M*, i.e., if $\bar{\Lambda}(M, M-1) \leq 0$. By substituting $m = M-1$ in (32), we obtain

$$\bar{\Lambda}(M, M-1) = (1-\delta\alpha)\ln M - \ln\frac{2(1-\delta\alpha)+\beta\delta\alpha}{1-\delta\alpha+\beta\delta\alpha}$$

and the result follows. □

Proof of Corollary 1: Since $\dfrac{\partial\bar{\Lambda}(M, M-1)}{\partial\beta} > 0$, then $\bar{\Lambda}(M, M-1)$ increases monotonically when β decreases. We analyze what happens for low values of β.

Since $\lim_{\beta\to 0^+} \bar{\Lambda}(M, M-1) = (1-\delta\alpha)\ln M - \ln 2$, if $\delta\alpha < 1 - \dfrac{\ln 2}{\ln M}$, the grand coalition can not be stable. But if $\delta\alpha > 1 - \dfrac{\ln 2}{\ln M}$, the grand coalition will be stable if β is sufficiently low. □

Proof of Proposition 3: The results follows from the strict convexity of function $\Lambda(M, m)$ with respect to *m* (see (35)). Depending on the location of the roots of $\Lambda(M, m)$ as a function of *m* in the interval $m \in [1, M]$, we have the following scenarios:

(1) There are no roots in the interval $[1, M]$. There are two possibilities:

- If $\Lambda(M, m)$ is strictly positive in that interval, all the coalitions are internally unstable, so there are no stable coalitions.

- If $\Lambda(M, m)$ is strictly negative in that interval, all the coalitions are internally stable but externally unstable. Then the grand coalition is the unique stable coalition.

(2) There is only one root m^* in the interval $[1, M]$. Then we have the following possibilities:

- If $m^* = 1$ and $\Lambda(M, m) > 0$ for $m > m^*$, then coalitions of size two are the unique stable coalitions.
- If $m^* \in \{2, 3, \ldots, M-1\}$ and $\Lambda(M, m) > 0$ for $m > m^*$, then coalitions of size m^* and coalitions of size $m^* + 1$ are stable.
- If $m^* \notin \{2, 3, \ldots, M-1\}$ and $\Lambda(M, m) > 0$ for $m > m^*$, then coalitions of size $[m^*]$ are stable.
- If $m^* \leq M-1$ and $\Lambda(M, m) < 0$ for $m > m^*$, then the grand coalition is the unique stable coalition. On the contrary, if $m^* > M-1$ with $\Lambda(M, m) < 0$ for $m > m^*$, then there are no stable coalitions.

(3) There are two roots in the interval $[1, M-1]$, given by $m_1 < m_2$. In this case, by combining the scenarios with one root, if $M \geq 4$, there always exists at least one stable coalition, and we can have situations with one, two or three stable coalitions. For M=3 there always exist stable coalitions of size two, three or both. □

Proof of Lemma 3: For the first part, after rearrangement terms we obtain

$$\frac{\partial \bar{\Lambda}(M, m)}{\partial m} = \frac{(1-\delta\alpha)\left[(1-\delta\alpha)(M-m-1)+\beta\delta\alpha\right]}{\left[(1-\delta\alpha)(M-m)+\beta\delta\alpha\right]\left[(1-\delta\alpha)(M-m-1)+\beta\delta\alpha\right]} \geq 0$$

if $m \leq M-1$.

As for the second part, after several calculations we obtain

$$\frac{\partial \bar{\Lambda}(M, m)}{\partial \beta} = \frac{(M-m)\left[(1-\delta\alpha)^3(M-m-1)+\left(1-\delta\alpha\right)^2\beta\delta\alpha\right]-(1-\delta\alpha)^2\beta\delta\alpha+\beta^2+\delta^3+\alpha^3-\beta^2\delta^2\alpha^2+(1-\delta\alpha)\beta\delta\alpha}{\beta\left[(1-\delta\alpha)(M-m)+\beta\delta\alpha\right]\left[(1-\delta\alpha)(M-m-1)+\beta\delta\alpha\right]}. \tag{56}$$

For the case in which $m = M-1$, that is the highest possible value of *m*, the expression above simplifies to

$$\frac{\partial \bar{\Lambda}(M, M-1)}{\partial \beta} = \frac{(1-\delta\alpha)(1-\beta\delta\alpha)\beta\delta\alpha}{\beta\left[(1-\delta\alpha)(M-m)+\beta\delta\alpha\right]\left[(1-\delta\alpha)(M-m-1)+\beta\delta\alpha\right]} > 0.$$

Since $\dfrac{\partial \bar{\Lambda}(M, m)}{\partial \beta}$ in (56) is a decreasing function in *m*, then $\dfrac{\partial \bar{\Lambda}(M, m)}{\partial \beta} > 0$ for every $m \leq M-1$.

The final part follows from the previous result and the fact that $\lim\limits_{\beta \to 0^+} \bar{\Lambda}(M, m) = -\infty$. □

Proof of Proposition 4: First, let us assume that there exists a stable coalition of size $\bar{m}$. Then, necessarily, $\bar{\Lambda}(M, \bar{m}) \geq 0$ and $\bar{\Lambda}(M, \bar{m}-1) \leq 0$. There are three possibilities:

- $\bar{\Lambda}(M, \bar{m}) > 0$ and $\bar{\Lambda}(M, \bar{m}-1) < 0$. In this case, since from Lemma 3 the function $\bar{\Lambda}(M, m)$ is strictly decreasing in *m*, then $\bar{\Lambda}(M, m) < 0$ if $m \leq \bar{m}-1$ and $\bar{\Lambda}(M, m) > 0$ if $m \geq \bar{m}$. As a result, there are no more stable coalitions.
- $\bar{\Lambda}(M, \bar{m}) = 0$ and $\bar{\Lambda}(M, \bar{m}-1) < 0$. As $\bar{\Lambda}(M, m)$ is strictly decreasing in *m*, then $\bar{\Lambda}(M, \bar{m}+1) > 0$ and $\bar{\Lambda}(M, \bar{m}) = 0$, so a coalition of size $\bar{m}+1$ is stable, and there are no more stable coalitions.
- $\bar{\Lambda}(M, \bar{m}) > 0$ and $\bar{\Lambda}(M, \bar{m}-1) = 0$. By using the above reasoning, $\bar{\Lambda}(M, \bar{m}-1) = 0$ and $\bar{\Lambda}(M, \bar{m}-2) < 0$ and a coalition of size $\bar{m}-1$ is stable (and there are no more stable coalitions).

Next, recall that, from Lemma 3, $\bar{\Lambda}(M, m)$ is strictly increasing in β, i.e., m and β affect in opposite directions the value of $\bar{\Lambda}(M, m)$. From Lemma 3 we know that there exists β^* such that, for $\beta < \beta^*$, $\bar{\Lambda}(M, M-1) < 0$. This means that, for β sufficiently small, the grand coalition is stable. In principle, higher values of β could prevent the formation of stable coalitions. However, in the worst of cases, which happens when β = 1, from Proposition 2 in Kwon (2006), we know that there exists at least one stable coalition. As a result, for every $\beta \in (0, 1]$ there exists always at least one stable coalition, and if β is sufficiently low the grand coalition is the unique stable coalition .□

Proof of Proposition 5: After some algebra, we obtain that $\bar{V}^L(x) \geq \bar{V}^N(x)$ if, and only if, $f(M, m\gamma, \beta) = (1+\gamma)\ln\frac{M+\beta\gamma}{1+\beta\gamma} - \ln m \geq 0$. If $\beta = 1$, from Proposition 3 in Kwon (2006), we already know that a sustainable coalition is also profitable. In addition, it is straighforward to check that $\dfrac{\partial f(M, m\gamma, \beta)}{\partial \beta} < 0$, and the result follows. □

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Declarations

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