Development Patterns in Multi-Sector Growth Models

Bernabé Edgar Cruz González

PhD in Economics

Development Patterns in Multi-Sector Growth Models

Bernabé Edgar Cruz González

Universitat de Barcelona

PhD in Economics | Bernabé Edgar Cruz González

Universitat
de Barcelona

PhD in Economics

Thesis title:

Development Patterns in Multi-Sector Growth Models

PhD student:

Bernabé Edgar Cruz González

Advisor:

Xavier Raurich Puigdevall

Date:

September 2015

Universitat de Barcelona

Acknowledgment

First, I would like to express my sincere gratitude to my advisor, Prof. Xavier Raurich, for the continuous support of my Ph.D study and research, for his patience, motivation, insightful comments and encouragement. His guidance helped me throughout my time researching and writing this thesis. My everlasting gratitude and acknowledgment for his invaluable time devoted to this research.

My sincere thanks also goes out to Fernando Sanchez-Losada, Monica Serrano, and Montserrat Vilalta, who supported me throughout the research process with comments and advice; sharing their experiences as Ph.D. students; and providing me a constant supply of coffee, chocolates and motley talks on topics that not only enriched my research, but also my spirit.

I thank my fellows from office 417 for the stimulating discussions and fellowship. I thank my friends Jose Luis and Manish for all the fun and discussions we have had in the last year. My special gratitude to Paola for her support and motivation she has provided since our paths crossed. I thank many other friends, who directly and indirectly supported me during my research project: Kristian, Kalo, Ferran, and Cristian.

Last, but not the least, I would like to thank my family. I thank my parents, Bernabé and Rosa, for their continued support both spiritual and material. Their life, an example of tenacity, perseverance and dedication to break through all the obstacles they faced, were and will always be my greatest motivation. I also thank my brothers, Juan and Nallely, for their unconditional moral support. Finally, and most importantly, a special thanks to Reyna for her support and encouragement at all pleasant and not so pleasant moments we shared during the researching and writing of this thesis. To all of them, my heartfelt love.

E.C

List of Figures

List of Tables

Contents

Introduction

Common patterns of structural change in the sectoral composition of production, consumption and labor force are observed across countries during the economic development process. These patterns of change consist mainly of a large shift of employment, production and consumption from agriculture to manufacturing, and then from manufacturing to the service sector. This process of structural transformation or structural change has been extensively documented by Clark (1940), Kuznets (1966), Chenery and Taylor (1968), and Herrendorf, Rogerson, and Valentinyi (2014). Empirical evidence shows that the decline in the employment share of agriculture and the increase in employment share of service is a systematic feature in both developed and developing countries. Figure 1 illustrates structural transformation in the case of developed countries.

Figure 1 plots the share of total employment in agriculture and service and the logarithm of gross domestic product per capita (GDP) in 1990 international dollars in the United States, 7 European and 2 Asian countries.[1] At the beginning of the development process, Figure 1 shows that almost 80% of the total labor force was employed in the agriculture sector, whereas the employment share in agriculture is less than 2% in the highest stage of development. In contrast, the employment share in the service sector increases from almost 10% to 75% during this development period. A similar pattern of structural change is observed in developing countries.[2] Figure 2 shows the time path of the employment shares in agriculture and services in developing countries.[3]

Beyond being a distinctiveness of the process of development, structural change is an important factor for economic growth. Kuznets (1966) presents evidence of high rates of product growth in developed countries while the

[1]European countries are Belgium, Spain, Finland, France, Netherlands, Sweden, and United Kingdom. Asian countries are Japan and Korea. Source: Herrendorf et al. (2014).

[2]Latin American countries are Argentina, Bolivia, Brazil, Chile, Colombia, Costa Rica, Mexico, Peru, and Venezuela. Asian countries are China, India, Indonesia, Malaysia, Philippines and Thailand. Sub-saharan Africa countires are Botswana, Eritrea & Ethiopia, Ghana, Kenya, Malawi, Mauritius, Nigeria, Senegal, South Africa, Tanzania and Zimbabwe. Sources: M.P. Timmer, G.J. de Vries, and K. de Vries (2014). "Patterns of Structural Change in Developing Countries." GGDC research memorandum 149, and The Maddison-Project, http://www.ggdc.net/maddison/maddison-project/home.htm, 2013 version.

[3]Chenery and Taylor (1968) pointed out that this dispersion in the patterns of structural change in developing countries is explained by cross-country specific factors, for instance, size of markets, initial factor endowments and differences in trade opportunities.

reallocation of labor takes place. This labor reallocation process also fosters economic growth in developing countries. In particular, Gollin, Parente, and Rogerson (2002) show that sectoral shifts explain 29 percent of the output growth in 69 developing countries during 1960-2000.

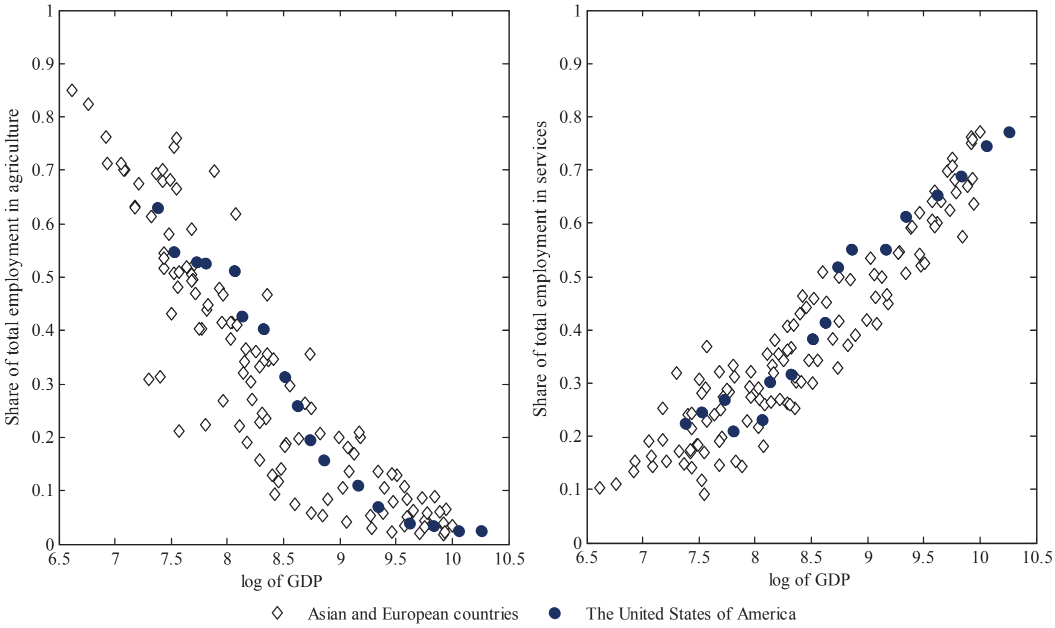

Figure 1: Structural change in developed countries.

Given the effect of structural change on economic growth, a renewed interest on the economic forces behind structural transformations had risen in the economic literature. In this regard, there is a growing literature that investigates the economic factors explaining both economic growth and structural change in a general equilibrium framework. Based on their assumptions on the structure of preferences and the sectoral production technologies, models of structural change are classified in two broad approaches: the demand and the supply explanations of structural change.[4]

The demand-based explanation emphasizes the role of changes in the composition of the demand on structural change. In this branch of the literature, demand changes are based on the assumption of cross-sector

[4]In the literature on structural change and growth, demand models are also known as models of income-effect, whereas models based on a supply explanation are also classified as models of price-effect.

differences in income-elasticity of the demand. Therefore, structural change is driven by the Engel law: as income rises, demand for agriculture goods decreases and less labor is demanded in the agriculture sector to produce goods. Thus, labor moves to those sectors that are facing an increasing demand for goods and services. Consequently, the shares of employment and value added in agriculture decrease as income increases, which is consistent with empirical evidence.

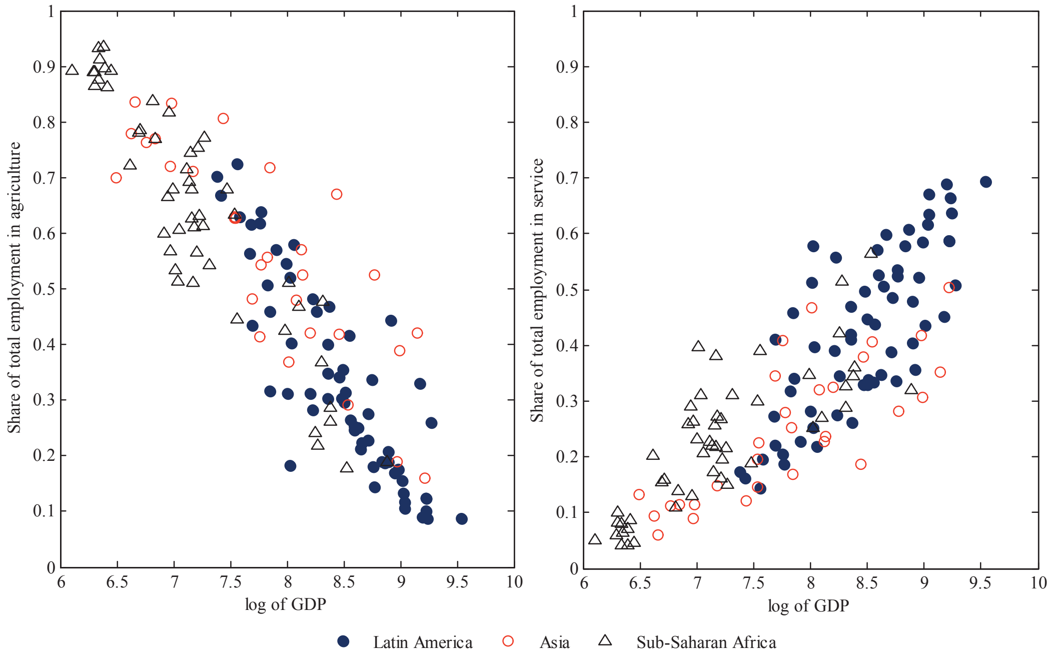

Figure 2: Structural change in developing countries.

Echeverría (1997) introduces this kind of preferences in a multisector growth model to analyze the effects of structural change on the growth rate. However, Echeverría's model is not consistent with the simultaneous existence of structural change and a constant capital-output ratio, which is one of the Kaldor's facts that characterizes the modern economic growth. By contrast, Kongsamut, Rebelo and Xie (2001) show that a model with non-homothetic preferences may encompass structural change and the features of an economy in balanced growth path. In this regard, Meckl (2002), Foellmi and Zweimüller (2002), and Boppart (2014) propose different specifications of non-homothetic preferences that, under conventional assumptions on the values of preference parameters, encompass the Kaldor's facts with changes in the sectoral composition of employment and production, the so-called Kuznets stylized facts.

The supply-based explanation emphasizes the role of technological differences across sectors to explain structural transformations. In this branch of the literature, sectoral differences in the growth rates of total factor productivity (TFP), on the one hand, and sectoral differences in physical capital intensity, on the other hand, drive structural change. In the first case, when there are only sectoral differences in the pace of technological progress, less labor is required to produce goods in the progressive sectors (those sectors with the highest TFP growth rates) and labor moves from the progressive to the stagnant sectors (those sectors with the lowest TFP growth rates). In the second case, as capital deepening takes place, less labor is demanded to produce goods in the capital-intensive sectors and labor moves from these sectors to the labor-intensive ones.

In order to be consistent with the observed time path of relative prices and labor reallocation, models based on these two approaches must satisfy conditions on the values of the elasticity of substitution across consumption goods and the elasticity of substitution between labor and capital. On the one hand, models that explain structural change based on the assumption of sectoral differences in technological progress assume that consumption goods are complementary. On the other hand, when cross-sector differences in capital intensity explain structural change, it is neccesary a low value of the elasticity of substitution between labor and capital.

Well-known examples of these two mechanisms are the seminal works of Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008). The former build a model where preferences are homothetic and structural change is driven by constant differences in sectoral growth rates of TFP. The latter build a model where differences in capital intensity between agriculture and non-agriculture sector. While Ngai and Pissarides' model explain structural change together with the features of an economy with a balanced growth, in Acemoglu and Guerrieri (2008) structural transformation and a balanced growth path only take place in the long run.[5]

This thesis contributes to this literature that considers the supply mechanisms of structural change. The three self-contained chapters of this

[5]There are several extension to study structural transformation and their relation with open economies (see Teignier, 2012; Herrendorf, Schmitz, Teixeira, 2012), income differences across countries (see Caselli,2005; Duarte and Restuccia, 2010; Herrendorf and Valentinyi, 2012; and Eberhardt and Vollrath, 2014), or missallocation (see Buera, Kaboski and Shin,2011; and Alonso-Carrera and Raurich, 2014).

thesis contribute to analyze the effects on structural change of non-constant technological progress, human capital accumulation, and changes in the uses of time.

In Chapter 1, I present a model based on Ngai and Pissarides (2007) that introduces a non-constant bias of sectoral technological progress. The aim of this chapter is to analyze the economic implications of non-constant technological bias for structural change. To this end, I assert that sectoral technological progress occurs via two channels. The first channel is a constantly increasing stock of knowledge generated exogenously, as in Ngai and Pissarides (2007). The second channel is technology adoption. I assume that the three broad sectors, namely agriculture, manufacturing and services, adopt new techniques for the production process that increases the stock of sectoral knowledge. This adoption process leads to an increase in sectoral technological progress at a non-constant growth rate. Under these assumptions, the model exhibits structural change, whereas aggregated GDP grows at a constant rate, which is consistent with empirical evidence.

This chapter contributes to the literature on structural change in two different directions. The first contribution is to show that a purely technological explanation could account for part of the sectoral transformations in the U.S. economy prior to the World War II. A model with constant biased technical change requires non-homothetic preferences to replicate the observed structural change prior to the World War II. In contrast, in the model developed in this chapter, the relative backwardness of the agricultural sector fosters the rate at which labor moves from this sector to the rest of the economy, which allows to explain actual data. The second contribution refers to the modeling approach of structural change. The model developed in this chapter shows that it is not necessary to assume implausible values of the elasticity of substitution across goods to model structural change. As Herrendorf et al. (2014) point out, in supply-explanation models, agriculture, manufacturing, and services need to be assumed as gross complementary goods in order to replicate the data. We conclude that by assuming non-constant sectoral TFP growth rates, the proposed model in this chapter can replicate the main trends in data and improve explanation power of standard models of structural change.

In Chapter 2, I present a continuous-time model of economic growth that encompasses the technological explanation of the Kuznets facts with endogenous technological progress. The model built in this chapter is a four-sector version of the endogenous growth model introduced by Uzawa (1965) and Lucas (1988). In this model, human capital accumulation drives structural change through two channels. In the first channel, differences in the rate of technical progress across the three broad sectors, namely agriculture, manufacturing and services, arise due to sector-specific strengths of an externality caused by the average stock of human capital. As occurs in Ngai and Pissarides (2007), these sectoral differences in productivity growth rates drive the changes in relative prices, which cause structural change. In the second channel, the initial stocks of human and physical capital cause structural change. In the model, individuals choose between investing in human and physical capital. When the initial ratio between these two stocks of capital differs from its long run value, individuals decide either to allocate more employment into the education sector (when the ratio between the two capital stocks is below its long run value) or allocate employment in the manufacturing sector in order to accumulate more physical capital. The imbalance between the two stocks of capital induces structural change.

This chapter contributes to the literature on structural change in two different directions. The first contribution is to propose an endogenous explanation of structural change and balanced growth. Previous structural change models assume that the sectoral technical progress is exogenous. To my knowledge, this is the first attempt in the literature to provide an endogenous explanation of the labor relocation across sectors compatible with balanced growth driven by human capital accumulation. The second contribution is to provide a growth model which can explain both the differences in income between countries and differences in the evolution of the sectoral composition. In standard models of structural change, the patterns of structural change are similar across countries, even when they differ in factor endowments. In the model developed in this chapter, I show that the differences in the stocks of physical and human capital induce differences across countries both in the initial sectoral composition as well as on structural change, which affects eventually the growth rate of aggregate economy.

In Chapter 3, coauthored with Xavier Raurich, we show that the development process of the U.S economy since the mid-twentieth century is characterized by two important patterns of sectoral change: a sustained increase in leisure time and an increasing expenditure on recreational services. We relate these two patterns of structural change by arguing that during leisure time we consume recreational services. The observed increase in leisure time then implies an increase in the consumption of these services, which introduces a new mechanism of structural change. In order to measure the relevance of this mechanism, we construct a multi-sector exogenous growth model with biased technological change. In our model, technological progress drives structural change through two different channels: a substitution channel and a wealth channel. On the one hand, the substitution channel is due to the assumption of biased technological progress. In this case, sectoral differences in productivity growth rates drive the changes in relative prices, which cause structural change as outlined by Ngai and Pissarides (2007). On the other hand, the wealth channel is the new mechanism of structural change introduced in this paper. Leisure rises with technological progress, which drives the increase in recreational services.

We use this model to study the effects of fiscal policy on both employment and GDP. In particular, we study the effect of increasing a labor income tax. This tax reduces the wage net of taxes and this causes the reduction of the labor supply when individuals can substitute leisure for consumption goods. In fact, the effects of labor taxes crucially depend on the substitution between leisure and the other consumption goods. As recreational activities crucially modify this substitution, the effects of taxes are modified when recreational activities are considered in our framework.

This chapter has two contributions. The first contribution is to consider the increasing demand of leisure to explain the rise of the services sectors. We show that the demand of leisure explains an increase in the share of recreational services in total service from 6% in 1947 to 14% in 2010. We show that this increase explains 26% of the observed increase in the service sector share of total value added. This result clearly shows that the effect of leisure on sectoral composition is sizable. The second contribution is to study the effects of fiscal policy in the framework of a multisectorial growth model. We show that the reduction in GDP due to an increase in the labor income tax is substantially larger when we consider that during leisure time we consume recreational services.

Chapter 1

Non-biased technical change and structural change

1.1 Introduction

Economic growth in developed economies is characterized by two prominent facts, namely, a constant capital-output ratio and changes in the relative sectoral shares in GDP as well as in the sectoral composition of the aggregate labor force. These are the so-called Kaldor-Kuznets stylized facts. The modern literature on structural change and economic growth encompasses these facts in the framework of multi-sectoral sectoral growth models (see Kongsamut, Rebelo and Xie, 2001; Meckl, 2002; Foellmi and Zweimuller, 2002; Ngai and Pissarides, 2007 and Acemoglu and Guerrieri, 2008).

A relevant explanation of structural change is sector biased technical change. According to this explanation, differences in rates of sectoral technical progress induce labor mobility from the progressive sectors (those with the highest productivity growth) to the stagnant sectors (those with the lowest productivity growth). This explanation goes back to Baumol (1967), while Ngai and Pissarides (2007) provide a modern formalization of the idea. They build a three-sector growth model where sectoral production functions differ only in total factor productivity (TFP) growth rates. Besides the assumptions of complementary goods and higher productivity growth in agriculture than in manufacturing and services, they assume that differences in TFP growth rates across sectors are constant. We refer to this assumption as constant biased technical change. As a consequence of these assumptions, this model replicates the main characteristic of the sectoral transformation: the fall of the agricultural sector and the rise of the service sector. However, the assumption of constant biased technical change is at odds with empirical evidence.

According to Dennis and Iscan (2007), and Alvarez-Cuadrado and Poshke (2011), technological progress at the sectoral level is not characterized by constant growth rates. Dennis and Iscan (2007) find that the differences in the respective rate of technological progress of the farm and non-farm sectors have been non-constant in the U.S. economy since the late 19th century. In particular, they show that sectoral technological progress is biased in favor of the non-farm sector at an early stage of development, which is followed by a shift in the bias of technological progress in favor of the farm sector. Alvarez-Cuadrado and Poshke (2012) find a similar pattern of sectoral technological progress across countries. They analyze available data for the relative prices of farm and non-farm goods for 11 advanced countries over the last two centuries. They find that changes in relative prices are related to changes in the bias of sectoral technological progress after controlling for the effects of international trade.

In this paper, we build a model based on Ngai and Pissarides's (2007) that introduces a non-constant bias of sectoral technological progress. Our aim is to analyze the economic implications of non-constant biased technical change for structural change. In order to address this aim, we assert that sectoral technological progress occurs via two channels. First, we assume that a constantly increasing stock of knowledge is generated exogenously, as in Ngai and Pissarides (2007). This channel captures the idea that technological progress can occur at the sector level based only on the available stock of knowledge in each sector. The second channel is technology adoption. We assume that a part of the sectoral technological progress is due to the adoption of new knowledge from the technological frontier. This frontier represents the maximum stock of new knowledge and ideas that is available in the economy and which can be adopted by the sectors. Adapting new techniques for the production process increases the stock of knowledge in each sector, which leads to an increase in sectoral technological progress. As in models of technology adoption, the distance or gap between the frontier and the sectoral technological level accounts for the stock of knowledge remaining to be adopted. This implies that relatively backward sectors, in the sense of having a higher gap relative to the frontier than others, tend to grow faster as long as there is a large stock of knowledge to be adopted.

To keep the model simple, we assert that the adoption rate of remaining knowledge occurs at an exogenous constant rate, which may differ between sectors. Under these assumptions, the growth rates of sectoral technological progress are not constant. We assume a functional form, in the spirit of the literature on technology adoption, for the sectoral productivity growth that is supported by data on agriculture, manufacturing and services TFP growth rates.

In the line with Ngai and Pissarides (2007), structural change in our model will be driven by differences in technological progress across sectors, whereas aggregated GDP, total expenditure consumption and capital grow at the same constant rate. Following Kongsamut et, al. (2001), we define this equilibrium path as a generalized balanced growth path (henceforth, GBGP). However, we show that the assumption of non-constant biased technical change introduces a relevant property in the patterns of structural change: sectoral composition can be degenerated or non-degenerated. The former characterizes an economy where the dominant sector is services and the weight of the remaining sectors is zero in the long run. The latter characterizes an economy where sectoral composition is asymptotically constant and with positive employment shares in all sectors.

We show that the nature of long-run sectoral composition depends on the sectoral ability to adopt knowledge. On the one hand, when all sectors can adopt technologies, sectoral composition is non-degenerated. This result arises because of our assumption regarding the adoption of knowledge from a common technological frontier. In this case, all sectoral TFP growth rates converge to the growth rate in the frontier and, consequently, asymptotic sectoral technology progress is unbiased. On the other hand, sectoral composition is degenerated if at least one sector producing only consumption goods cannot adopt new knowledge. We use this case to show that our model coincides with Ngai and Pissarides's model when no sector is able to adopt knowledge. Moreover, given the assumption of knowledge adoption, we show that the pace of structural change depends on technological backwardness in the agriculture sector. In particular, we show that a more marked degree of backwardness in agriculture causes labor to move rapidly from this sector to other sectors, thereby accelerating the pace of structural change.

In order to analyze how well a model based on non-constant biased technical change fits the features of the structural transformation, we conduct a numerical analysis of the model. To this end, we calibrate our model and a model based on the assumption of constant biased technical change to match the development process of the U.S. economy between 1870 and 2005. We use the second model as a benchmark for comparison. Based on these models, we simulate the time paths of the level of employment shares in agriculture, manufacturing and services. We compute the annual growth rates of the ratio between employment shares (RES, henceforth) in the agriculture and services sectors, and the RES between agriculture and manufacturing, where annual growth rates of these ratios represent the pace of industrialization in the economy. We then study the performance of both models in replicating these growth rates.

We evaluate the performance using two criteria. First, we examine the accuracy of both models when explaining the employment shares in agriculture, manufacturing and services over the period. To this end, we regress the actual values of employment shares on the simulated employment shares, and we then analyze how well these simulations fit the actual data on sectoral composition. As is standard in the literature, we report the root-mean-square error (RMSE), and the Akaike statistic for each regression, as measurements of accuracy. The second criterion is based on the value of the average annual growth rate of the RES obtained from numerical simulations. We compare the actual average annual growth rates of the RES with the growth rates obtained with our calibrated models. In particular, we compare the predicted with the actual average growth rates for three periods: 1870-1930, 1930-1950, and 1950-2005.

We focus on these periods because of the shifts in the sector biased technical change suggested by the data. According to Dennis and Iscan (2007), over these periods sectoral technical change shifts from being biased towards the non-farm sector to a bias in favor of the farm sector. These shifts in the bias of sectoral technical progress may accelerate the pace of industrialization in line with the technological explanation of structural change. By analyzing the performance of both models in predicting this change in the pace of industrialization over those periods, we can infer the importance of the shift in the bias of technical progress for explaining structural change.

Our numerical results show that a non-constant biased model fits the data better than a constant biased model. On the one hand, the numerical simulations based on our model fits the data better on the level of employment shares in agriculture, manufacturing and services than the benchmark model. This conclusion is robust to different values of the elasticity of substitution across goods.[1] In particular, we calibrate both

[1] The value of the elasticity of substitution has a significant role in the direction of structural change (see Ngai and Pissarides, 2007; and Herrendorf, et, al. 2014). In particular, a low value of the elasticity of substitution across goods is required in order

models by setting the value of the elasticity of substitution at 0.1, 0.5 and 0.9, and we obtain the simulated employment shares. Under these three scenarios, we highlight that the performance of the benchmark model increases as the utility function approaches a Leontief utility function. However, the accuracy is lower than that obtained with our model for the same values of the elasticity of substitution. That is, the model with non-constant technical bias provides a robust and better performance in replicating the structural change given the changes in the elasticity of substitution.

On the other hand, our model also provides a good fit with the data on the actual average growth growth rates of the RES. The numerical simulation based on our model replicates accurately the annual average growth rate of the RES before 1950. Indeed, our model explains 88 and 62 percent of the annual average growth of the RES between agriculture and manufacturing in the periods 1870-1930 and 1930-1950, respectively. In contrast, the benchmark model explains only 47 and 38 percent for the same periods. Interestingly, the accuracy of our model increases slightly when the elasticity of substitution increases to 0.90. In this case, our model explains 90 and 63 percent of the annual average growth rate of the RES for the same periods, whereas the accuracy of our benchmark model collapses to just 4 and 3 percent.

These numerical exercises show two interesting results. The first result is related to the discussion on how to model the process of sectoral transformation. Herrendorf et, al. (2014) show that if the aim is to obtain a good fit with the data using a consumption value-added specification, then the functional form of the utility that should be opted for is the Leontief utility function and assuming constant biased technical change. Our first result contributes to this discussion by showing that if the sectoral technological progress is modeled with exponential growth rates, then the Leontief utility function should be adopted in order to fit with the data, as Herrendorf et, al. (2014) point out. In contrast, if the utility function is assumed to differ from the Leontief specification, the performance of the model to fit the data will be poor under the assumption of constant biased technical change. Thus, our result suggests that a non-constant sectoral biased technological process is a necessary condition to model accurately structural change when a non-Leontief utility function is assumed.

The second result is related to the implications of non-exponential growth

to replicate the rise of the services sector (see Boppart, 2014).

rates of sectoral TFP for structural change. When differences in rates of technological progress are time-variant, we show that a model in which sectoral production functions differ only in TFP growth is able to provide a good fit to the data on structural change not only after World War II (WWII), but also prior to it. This crucially depends on assuming a technological backwardness of the agricultural sector. If the initial backwardness in agriculture is higher than in manufacturing and services, then the agriculture TFP growth rate increases inducing a non-constant decrease in the growth of relative prices of agricultural goods. This change in relative price implies that labor is rapidly pushed toward the stagnant sectors. In the case of constant technical biased change, growth rate of relative prices are constant, and therefore, it is also constant the pace at which labor moves from agriculture to other sectors. We show numerically that relaxing constant biased technical change, Baumol's effect can account for the process of industrialization in the early stage of development. In this regard, our results suggest that a purely technological approach to structural change is able to account for sectoral transformations in the U.S. economy prior to WWII.

The structure of the paper is as follows. In Section 2, we present empirical evidence of non-constant biased technological change. In Section 3, we build a model based on the assumption of non-constant biased technical change. In Section 4, we solve the model and characterize structural change. In Section 5, we present the main results of the numerical simulation. Finally, in Section 6, we present some concluding remarks and future lines of research, while the Appendix section contains the proofs of all the results of the paper.

1.2 The technology

We assert that sectoral technological progress occurs via two channels. The first channel is a stock of knowledge that increases at a constant rate and is generated exogenously. The second channel operates via the adoption of new knowledge from the technological frontier. This frontier encapsulates the maximum stock of new knowledge and ideas that are available for adoption by the sectors in the economy. These channels capture the idea that technological progress can occur both at the sector level, based on the available stock of knowledge in the sector, and based also on adoption from the technology frontier common to all sectors. In order to keep the model simple, we assume that adoption is costless. Furthermore, we assume that the stock of knowledge available in the technological frontier increases at an exogenous growth rate as follows

$$\frac{\dot{A}}{A} = \gamma, \tag{1.1}$$

where $\gamma > 0$ is the growth rate and A denotes the technology level in the frontier.[2] We then pose the law of motion of productivity in i sector as follows

$$\frac{\dot{A}_i}{A_i} = \phi_i + \omega_i \ln\left(\frac{A}{A_i}\right), \tag{1.2}$$

where $\omega_i > 0$ measures the rate of adoption, $\phi_i > 0$ measures the exogenous growth progress that takes place without adoption of knowledge, A_i is the level of TFP in the sector i and A/A_i measures the technological gap between sector i and the frontier. This gap accounts for the stock of knowledge remaining to be adopted.

To gain some intuition on the effect of the technological gap, let us suppose that $A > A_i$ and there is no exogenous technology growth in each sector, $\phi_i = 0$. In this case, sectoral technological progress depends only on the ability of each sector to adopt the remaining knowledge. Thus, differences in sectoral TFP growth rates or sectoral biased technical change will be determined by the magnitude of the technological gap and the rate at which technology is adopted across sectors. For simplicity, let us assume that all sectors can adopt knowledge from the frontier at the same rate, that is $\omega_i = \omega$ for all i. Therefore, sectoral biased technical change is due to differences in technological gaps. Those sectors with a lower stock of knowledge tend to grow faster than sectors that are closer to the frontier level. Although to the extent that backward sectors increase their TFP growth because of the adoption process, the growth rate decreases because fewer and fewer technologies from the frontier remain to be adopted. Eventually, both backward and advanced sectors converge to the frontier level, and this source of biased technological progress will vanish. On the contrary, the polar extreme case is when the source of biased technological progress lies on constant differences in exogenous growth rate, $\phi_i > 0$. If technology adoption is not possible, $\omega_i = 0$ for all i, sectoral biased technical change is due only to differences in ϕ_i across sectors, as in Ngai and Pissarides (2007). Hereinafter, we assume that $\omega_i \geq 0$, $\phi_i > 0$ for all i in order to analyze the implications of non-constant biased technical change for structural change. It is therefore convenient to derive the law of motion of technological gaps.

[2] In order to facilitate the notation we omit the time argument of all the variables.

Following Acemoglu (2008), we define the distance between sectors and the frontier as follows

$$v_i = \frac{A_i}{A}. \tag{1.3}$$

Taking the log-derivative of (1.3), and substituting (1.1) and (1.2), we obtain that the law of motion of technological gaps is

$$\frac{\dot{v}_i}{v_i} = \phi_i - \gamma - \omega_i \ln(v_i). \tag{1.4}$$

Once we solve (1.4),[3] it is easy to show that the long-run technological gap is

$$v_i^* = \begin{cases} 1 & if\ \phi_i = \gamma \\ \exp\left(\frac{\phi_i - \gamma}{\omega_i}\right) < 1, & if\ \phi_i < \gamma \\ \exp\left(\frac{\phi_i - \gamma}{\omega_i}\right) > 1, & if\ \phi_i > \gamma \end{cases}, \tag{1.5}$$

where v_i^* is the long-run gap in sector i. Note that there are three possible values that the technological gap can take in the long run. The first one, when the technological gap is equal to one, arises because exogenous sectoral technical progress is equal to the growth rate of the frontier, $\phi_i = \gamma$. This is the case when the sectoral TFP level catches up the frontier level in the long run. The second value is less than one. This occurs when the exogenous sectoral technical progress is lower than γ at the frontier, $\phi_i < \gamma$. The third value occurs when exogenous sectoral technical progress is higher than γ at the frontier, $\phi_i > \gamma$. In this case, the sectoral TFP level is larger than the frontier level. Next, we show that this case is not possible given our estimation of the technology in (1.2).

To analyze whether the technology proposed can explain the time path of sectoral TFP, we estimate the parameters in equation (1.2) using sectoral data on productivity for the U.S. economy provided by the EUKLEMS project.[4] In particular, we estimate equation (1.2) using the growth rates of three broad sectors, namely agriculture, manufacturing and services.[5] We choose these three broad sectors since the analysis of structural change is commonly performed at this level.

Figure 1 shows both the level and growth rates of the technological gaps

[3]See the Appendix B.

[4]This project has information about TFP growth across 74 sectors of the economy for the United States, Japan, and many countries in Europe for the period 1970-2005. For a summary overview of the methodology and construction of the EU KLEMS database, see O'Mahony, Mary and Marcel P. Timmer (2009).

[5]These broad sectors are defined as in Herrendorf et, al. (2014).

between the agriculture and services sectors, and between the agriculture and manufacturing sectors, as well as the trend in these series obtained with the Hodrick-Prescott filter. These gaps are measured by the ratios between the TFP in agriculture and services and the TFP in manufacturing. These ratios are defined as relative TFP. A superficial exploration of the plot shows that both relative TFPs have not been constant over the period 1970-2005 (see Figure 1; panel a, and b). Figure 1 points out that TFP in agriculture grew faster than TFP in manufacturing, meanwhile TFP in services grew at a lower rate than in manufacturing. These results are in line with those reported by Herrendorf et, al. (2014). Although a more careful inspection of data reveals that the trend of this relative sectoral TFP is not constant. In particular, both series show changes in trend around 1980 that have narrowed sectoral biased technical change. Despite variability in the growth rate of these series, shifts in the long-run trend of growth rates reveal the observed change in trend in relative TFP levels (see Figure 1, panels c and d).

[Insert Figure 1]

Table 1 shows the result of estimating the growth rates of TFP in equation (1.2) for the agriculture, manufacturing and services sectors. In order to estimate the parameters, we solve the differential equations in (1.2) under the assumption of exogenous growth of the technology frontier. Given the solution of (1.2) in Appendix B, we estimate the following system of equations

$$\ln A_i = \alpha_i + \beta_i t + e^{-\delta_i (t-n_i)}, \tag{1.6}$$

where

$$\alpha_i = \frac{\phi_i - \gamma}{\omega_i};\ \beta_i = \gamma;\ \text{and } \delta_i = \omega_i, \tag{1.7}$$

and n_i is the constant of integration. We estimate (1.6) imposing the constraints (1.7) by using non-linear squares.[6] The results are in Table 1. The point estimates for the rate of adoption, $\hat{\omega}_i$ and the exogenous growth progress $\hat{\phi}_i$ in agriculture and services are statistically different from zero. These results show that there exists a positive relation between the technological gap and the TFP growth in agriculture and services. In particular, the estimated rate of adoption $\hat{\omega}_i$ in agriculture is 0.026, whereas the estimated rate of adoption in services is, on average, 0.017. The point estimates for the rate of the exogenous growth progress $\hat{\phi}_i$ in agriculture and services are similar and statistically different from zero. Notably, the

[6] In Appendix B, we show the empirical strategy to estimate (1.4).

rate of adoption in the manufacturing sector is not statistically different from zero, and the estimated value of the exogenous growth progress $\hat{\phi}_m$ is close to the estimated growth rate of the technology frontier, $\hat{\gamma}$. Given the estimated parameters, we calculate the growth rate of the technological gap in agriculture and services.

[Insert Table 1]

These results suggest the existence of non-constant biased technical change in the U.S. economy across the agriculture, manufacturing and services sectors. We acknowledge that our results cover only a short period of time, nevertheless the reported results are in line with those reported by Dennis and Iscan (2009), who point out the existence of changes in relative TFP in farm and non-farm sectors over the period 1800-2000. Our results suggest that the bias in favor of technological progress in agriculture has declined over the period 1970-2005, converging to the growth rate of the manufacturing sector. This suggests again that biased technological progress is not constant.

In order to analyze the implications of non-constant biased technological progress on structural change, in the following section we build a three-sector growth model, based on the seminal work of Ngai and Pissarides (2007), which is characterized by non exponential sectoral TFP growth, as the empirical evidence indicates.

1.3 The model

We build a three-sector growth model in which the output in each sector is obtained from combining capital, K, and labor, L. We adopt the notation $a, s,$ and m to denote the agriculture, services, and manufacturing sectors, respectively. To facilitate the notation, we omit the time argument in all the variables. Following Ngai and Pissarides (2007), we assume that all sectors have the same capital intensity and produce an amount Y_i of commodity using the following production function:

$$Y_i = A_i \left(s_i K\right)^{\alpha} \left(u_i L\right)^{1-\alpha}, \text{ for } i = a, s, m, \tag{1.8}$$

where s_i and u_i are the shares of capital and labor allocated in sector i, A_i is the sectoral total factor productivity (TFP), and $\alpha \in (0, 1)$ is the intensity of capital in this sector. Obviously, both capital and labor shares satisfy

$$s_a + s_s + s_m = 1, \tag{1.9}$$

and

$$u_a + u_s + u_m = 1. \tag{1.10}$$

We also assume that population is constant and we normalize it to one. We refer to C_a and C_s as the amount of agricultural and service goods devoted to consumption, so that the following equation is satisfied

$$Y_i = C_i \text{ for } i = a, s. \tag{1.11}$$

We assume that the commodity Y_m, namely the manufacturing good, can be either consumed or added to the stock of aggregate capital. Thus, the law of motion of the capital stock is given by

$$\dot{K} = Y_m - \delta K - C_m, \tag{1.12}$$

where C_m is the amount of good Y_m devoted to consumption, and $\delta \in [0,1]$ is the depreciation rate of the capital stock.

The representative agent obtains utility from the consumption of agricultural, manufacturing and service commodities. In particular, we assume that the representative agent is characterized by the instantaneous utility function

$$U\left(\tilde{C}\right) = \ln\left(\tilde{C}\right), \tag{1.13}$$

where $\tilde{C}$ denotes a composite consumption good, which satisfies

$$\tilde{C} = \left(\eta_a C_a^{\frac{\epsilon-1}{\epsilon}} + \eta_s C_s^{\frac{\epsilon-1}{\epsilon}} + \eta_m C_m^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}, \tag{1.14}$$

where η_a, η_s, and η_m measure the relative preference for sectoral commodities, which are assumed to satisfy $\eta_a + \eta_s + \eta_m = 1$. The elasticity of substitution among commodities is denoted by the parameter $\epsilon > 0$.

1.4 The equilibrium

In this section, we obtain the system of differential equations characterizing the equilibrium. We use these equations to find the long-run equilibrium and to study how the introduction of technology adoption across sectors modifies the sectoral composition of the economy.

The representative agent maximizes the discounted sum of utilities

$$\int_0^\infty e^{-\rho t} U\left(\tilde{C}\right) dt, \tag{1.15}$$

subject to (1.9), (1.10), (1.11), and (2.2.5), where $\rho > 0$ is the subjective discount rate. In the Appendix, we obtain the following equations:

$$s_i = u_i, \text{ for } i = a, s, m; \tag{1.16}$$

and

$$p_a = \frac{A_m}{A_a}, \tag{1.17}$$

$$p_s = \frac{A_m}{A_s}, \tag{1.18}$$

where (1.16) is a set of static efficiency conditions for the allocation of factors, and (1.17) and (1.18) shows that relative prices, p_a and p_s, are functions of the ratio between the manufacturing sector productivity (the numeraire good) and agriculture and services, respectively.

To characterize the aggregate economy, we combine (1.11) and (1.17) to obtain the aggregate consumption expenditure, which is defined as $C = p_a C_a + p_s C_s + C_m$. As in Ngai and Pissarides (2007), we define the ratio of consumption expenditure on good i to consumption expenditure on the manufacturing good as

$$x_i \equiv \frac{p_i C_i}{C_m} = \left(\frac{\eta_i}{\eta_m}\right)^{\epsilon} p_i^{1-\epsilon}, \tag{1.19}$$

and using x_i, consumption expenditure can be rewritten as

$$C = C_m \left(1 + x_a + x_s\right). \tag{1.20}$$

Note that (1.19) only depends on relative prices. By combining (1.8), (1.16) and (1.17), we obtain the gross domestic product (GDP) as

$$Y = A_m K^{\alpha}. \tag{1.21}$$

Having obtained the equations that define the aggregate economy, we now characterize the sectoral employment share. In Appendix A, we show that substituting the market clearing condition (1.11) in (1.20) and taking into account the optimal capital shares and (1.23), the efficiency labor shares in the agriculture and services sector are

$$u_a = \left(\frac{x_a}{1 + x_a + x_s}\right)\frac{C}{Y}, \tag{1.22}$$

and

$$u_s = \left(\frac{x_s}{1 + x_a + x_s}\right)\frac{C}{Y}. \tag{1.23}$$

Equation (1.22) and (1.23) together with (1.10) define the sectoral composition of the economy.

We next obtain the system of differential equations that characterizes the equilibrium. In Appendix A, we obtain that the growth rate of consumption expenditure is

$$\frac{\dot{C}}{C} = \alpha A_m K^{\alpha-1} - \left(\rho + \delta\right), \tag{1.24}$$

and using (2.2.5) and (1.21), we can express the law of motion of the capital stock in terms of total consumption expenditure as follows

$$\frac{\dot{K}}{K} = A_m K^{\alpha-1} - \frac{C}{K} - \delta. \tag{1.25}$$

Equation (1.24) tells us that the growth rate of total consumption expenditure is independent of relative prices effects. This result is attributed to our preferences being represented by a logarithmic utility function.

In order to characterize the equilibrium path, we rewrite (1.24) and (1.25) by using the following transformed variables $z = KA_m^{\frac{1}{\alpha-1}}$ and $c = CA_m^{\frac{1}{\alpha-1}}$, where z and c denote, respectively, capital and total consumption expenditure in efficiency units. By taking log-derivatives of z and c, and using (1.24), (1.25) and (1.4), we rewrite the dynamic system as follows

$$\frac{\dot{z}}{z} = z^{\alpha-1} - \frac{c}{z} - \delta - \frac{\dot{v}_m + \gamma v_m}{(1-\alpha)\, v_m}, \tag{1.26}$$

$$\frac{\dot{c}}{c} = \alpha z^{\alpha-1} - \rho - \delta - \frac{\dot{v}_m + \gamma v_m}{(1-\alpha)\, v_m}, \tag{1.27}$$

and

$$\frac{\dot{v}_m}{v_m} = \left(\phi_m - 1\right)\gamma - \omega_m \ln v_m. \tag{1.28}$$

Following Ngai and Pissarides (2007), we define structural change as the change in the employment shares. By taking log-derivatives of (1.17), (1.19), (1.22), (1.23); and taking into account (1.4), we obtain that the growth rate of the employment share are

$$\frac{\dot{u}_a}{u_a} = \frac{\dot{C/Y}}{C/Y} + (1-\epsilon)\left(\frac{\dot{v}_m}{v_m} - \frac{\dot{v}_a}{v_a}\right) - (1-\epsilon)\left[x_a\left(\frac{\dot{v}_m}{v_m} - \frac{\dot{v}_a}{v_a}\right) + x_s\left(\frac{\dot{v}_m}{v_m} - \frac{\dot{v}_s}{v_s}\right)\right], \quad (1.29)$$

and

$$\frac{\dot{u}_s}{u_s} = \frac{\dot{C/Y}}{C/Y} + (1-\epsilon)\left(\frac{\dot{v}_m}{v_m} - \frac{\dot{v}_s}{v_s}\right) - (1-\epsilon)\left[x_s\left(\frac{\dot{v}_m}{v_m} - \frac{\dot{v}_s}{v_s}\right) + x_a\left(\frac{\dot{v}_m}{v_m} - \frac{\dot{v}_a}{v_a}\right)\right]. \quad (1.30)$$

The dynamic equilibrium is thus characterized by a set of paths $\{z, c, v_m\}$ such that, given $z(0)$ and $v_m(0)$, solves equations $(1.26), (1.27)$ and (1.28), and satisfies the transversality condition

$$\lim_{t\to\infty} \mu_m \left(z v_m e^{\frac{\gamma}{1-\alpha}t}\right) = 0.$$

We define a balanced growth path (BGP, henceforth) equilibrium as an equilibrium path along which the efficiency units of capital, z, and total consumption expenditure, c, remain constant. The following result characterizes the steady-state equilibrium.

Proposition 1.1 *There exists an unique BGP, and the long-run values of the transformed variables are*

$$\begin{aligned} z^* &= \left(\frac{\alpha}{\gamma^* + \rho + \delta}\right)^{\frac{1}{1-\alpha}}, \\ c^* &= \left(\frac{(1-\alpha)\left(\delta + \gamma^*\right) + \rho}{\alpha}\right)\left(\frac{\alpha}{\gamma^* + \rho + \delta}\right)^{\frac{1}{1-\alpha}}, \\ v_m^* &= \exp\left(\frac{\phi_m - \gamma}{\omega_m}\right), \end{aligned}$$

where the long-run growth rate of GDP is

$$\gamma^* = \begin{cases} \frac{\gamma}{1-\alpha} & \text{if } \omega_m > 0 \\ \\ \frac{\phi_m}{1-\alpha} & \text{if } \omega_m = 0 \end{cases}.$$

Note that the GDP growth rate is higher if there is technology adoption in the manufacturing sector. Given the assumption of equal capital intensity across sectors, aggregate TFP is equal to the TFP in manufacturing. When technology adoption occurs in the manufacturing sector, the growth rate of technological progress in this sector increases and, consequently, so does the

GDP growth rate. On the other hand, when there is not technology adoption in the manufacturing sector, the GDP growth rate increases proportionally at the rate ϕ_m. The following propositions and definitions characterize the equilibrium path and the structural transformations in our economy.

Proposition 1.2 *The BGP is saddle-path stable.*

As Ngai and Pissarides (2007) pointed out, equations (1.26) and (1.27) are similar to the two differential equations in the one-sector Ramsey economy. Our model shows similar transitional dynamics to those of the Ramsey model if we assume that $\omega_m = 0$ or $A_m = A$ in the initial period. Obviously, in this case, the transitional dynamics are governed only by equation (1.26) and (1.27). In contrast, if $\omega_m > 0$ and $A_m \neq A$, the transitional dynamics are characterized by equations (1.26), (1.27), and (1.28); and the equilibrium dynamics are different from those obtained in Ngai and Pissarides model. And yet, in both cases, the patterns of structural change are not necessarily the same as those reported by Ngai and Pissarides (2007). For instance, when technology adoption occurs both in the agriculture and services sectors, the growth rates of sectoral TFP are not constant. This implies that the bias of sectoral technical progress is time varying and, therefore, the rate of reallocation of labor out of agriculture is not constant. This affects the pace and the patterns of structural transformation in the transitional and dynamics.

As is usual in this literature, we analyze the structural change that arises when aggregate variables are in the BGP i.e. the model satisfies the Kaldor facts. We, therefore, focus on characterizing the structural change that arises when the economy is in the BGP, and technology adoption occurs in manufacturing. At this point, we highlight that this assumption only has implications for the stationary solutions $\left(z^*, c^*, v_m^*\right)$ outlined in the previous propositions, and not for the structural changes that we characterize next. The following definition and propositions characterize the process of structural change of the economy along the equilibrium path and the sectoral composition in the long run.

Definition: Sectoral composition is degenerated if the asymptotic employment share of at least one sector is zero. Otherwise, sectoral composition is non-degenerated.

Proposition 1.3 *Necessary and sufficient conditions for the existence of a non-degenerated sectoral composition are $\omega_a \neq 0$ and $\omega_s \neq 0$. Otherwise, the sectoral composition is degenerated.*

Proposition 1.4 *In the BGP with non-degenerated sectoral composition, employment shares in the agriculture and services sectors are asymptotically:*

$$u_a^* = \frac{(1-\hat{\sigma})}{1+\left(\frac{\eta_m}{\eta_a}\right)^{\epsilon}\left(\frac{v_a^*}{v_m^*}\right)^{1-\epsilon}+\left(\frac{\eta_s}{\eta_m}\right)^{\epsilon}\left(\frac{v_m^*}{v_s^*}\right)^{1-\epsilon}},$$
$$u_s^* = \frac{(1-\hat{\sigma})}{1+\left(\frac{\eta_a}{\eta_m}\right)^{\epsilon}\left(\frac{v_m^*}{v_a^*}\right)^{1-\epsilon}+\left(\frac{\eta_m}{\eta_s}\right)^{\epsilon}\left(\frac{v_s^*}{v_m^*}\right)^{1-\epsilon}},$$

and

$$u_m^* = 1 - u_a^* - u_s^* > 0.$$

In the asymptotic BGP with degenerated sectoral composition, employment shares in the agriculture and services sectors are $u_m^* = \hat{\sigma}$, $u_s^* = 1-\hat{\sigma}$, *and* $u_a^* = 0$; *where*

$$\hat{\sigma} = \alpha\frac{\delta+\gamma^*}{\delta+\rho+\gamma^*}$$

is the savings rate along the aggregate balanced growth path, and v_i^*, *i=a,s,m are defined in* (2.5).

Proposition (1.3) shows the conditions under which the economy converges to a non-degenerated sectoral composition. These conditions require that sectors producing consumption goods adopt knowledge from the technological frontier. When these conditions are fulfilled, the rates of TFP growth in the agriculture and services sectors converge to the growth rate of the frontier. This implies that (1.29) and (1.30) are equal to zero in the long run, when both technology gaps in agricultural technology and services reach their stationary values. In contrast, a degenerated sectoral composition arises when one of the sectors that produces consumption goods (agriculture or services) is not able to adopt any knowledge from the frontier. It is important to note that these results are independent of whether technology adoption occurs in the manufacturing sector or not. That is, in our model, sectoral composition is determined by the technological characteristics of sectors producing only consumption goods. The extreme case of a degenerated economy occurs when no sector adopts knowledge. In that case, the implications of our model are the same as those in Ngai and Pissarides model (2007).

The results shown in Proposition (1.3) characterize the sectoral composition in the long run, whereas one of the main features of economic development is the structural transformations in the short run. To characterize this structural change, we focus on studying changes in the ratio between agriculture and services, and the ratio between agriculture and manufacturing employment shares (RES, henceforth), as a measure of the relative importance of agriculture in the economy. The annual relative variation in these ratios indicates the changes in the number of farm workers per worker engaged in non-agricultural activities. Thus, the growth rates of the RES show the pace of industrialization. Kuznets (1973) emphasized that a rapid decline in the RES (a higher growth rate) is one of the main features of structural transformations across countries. The following proposition characterizes the growth rate of the RES in our economy.

From using (1.29) and (1.30), the growth rate of the RES between agriculture and service is:

$$\frac{\dot{u}_a}{u_a} - \frac{\dot{u}_s}{u_s} = \underbrace{(1-\epsilon)\left(\phi_s - \phi_a\right)}_{\textit{Constant biased effect}} - \underbrace{(1-\epsilon)\left(\omega_s \ln v_s - \omega_a \ln v_a\right)}_{\textit{Backwardness effect}}.$$

This equation shows that the RES growth rates are functions of technological gaps between agriculture and other sectors, if technology adoption is possible in at least one of these sectors.[7] In this case, the RES growth rates depend on two components. The first component, the constant biased effect, is equal to the constant differences in the rate of exogenous technological progress between the service and agriculture sectors. The second component, the backwardness effect, depends on the difference in the distances of each sector to the technology frontier. To understand the effect of each component on the RES growth rate, let us assume that adoption of knowledge in agriculture and services is not possible. In this case, the constant biased effect determines the magnitude and direction of the RES growth rate. To replicate the observed structural change, a decreasing relative employment share in agriculture, a model with no adoption will require that $\phi_s < \phi_a$. In contrast, when knowledge adoption in both sectors is possible and we assume that there is no biased effect, the RES growth rate is determined by the backwardness effect. In this case, the RES growth rates are not constant because the technological gaps vary over time. To gain some intuition about the effect of the backwardness effect on RES growth rate, we assume that $\phi_s = \phi_a$. If the technological gap in agriculture is larger than the gap in the services, then the RES growth rate rises (the larger the magnitude

[7] For the sake of clarity, we present only the growth rate of the RES between agriculture and services. Despite the fact that the main feature of sectoral change is the polarization in the distribution of labor between agriculture and services, we also report numerically the growth rate of the RES between agriculture and manufacturing.

of the difference). Otherwise, the change in the RES growth rate is lower, if agriculture is closer than the services sector to the frontier (the lower the magnitude of the difference). That is, the second component measures the effect of backwardness in the agriculture sector on structural transformation. To analyze whether the effects of non-constant biased technical change is an important factor accounting for structural transformations, we conduct a numerical analysis of our model.

1.5 Numerical analysis

In this section we analyze the accuracy of the model for replicating the patterns of structural change observed in the United States during the period 1870-2005. Structural change is characterized by using both the employment shares in agriculture, manufacturing and services, and also by using the annual growth rates of the RES between agriculture and services, and between agriculture and manufacturing. To this end, we calibrate both the model based on non-constant biased technical change and the model built on the constant biased technical assumption to match the development process of the US economy in the period 1870-2005. We use both models to simulate the time path of the levels of sectoral employment shares, and we use them to calculate the growth rate of the RES. We then study the performance of both models in replicating these features of the structural change by taking into account two different criteria.

We first compare the accuracy of the non-constant biased model and the constant biased model's (our benchmark model) predictions on sectoral employment allocation by regressing actual employment shares in agriculture, and services on simulated data. We analyze how well these simulations fit actual data by reporting the root mean square error (RMSE), and the Akaike statistic for each regression. The second criterion is based on the value of the average annual growth rate of the RES obtained from numerical simulations. We compare the actual average annual growth rates of the RES with those growth rates obtained with our calibrated models. In particular, we compare the actual average growth rates of the RES for the periods 1870-1930; 1930-1950 and 1950-2005; and we then compare them to those predicted by our model and the benchmark. We focus on these periods because of the shifts in the sector biased technical change suggested by the data.

According to Dennis and Iscan (2007), over these periods sectoral technical change shifted from being biased towards the non-farm sector to being in favor of the farm sector. These changes affect the actual average growth rates of the RES and, therefore, comparing the performance of both models in predicting these changes provides a measure of the feasibility of the assumptions on which they are based for replicating the structural change in the U.S. economy. Based on these two criteria, we determine which model is more suitable for replicating the main patterns in the data. In what follows, we describe the strategy for calibrating both models and we present the main results.

1.5.1 Calibration strategy

To calibrate both non-constant and constant biased models, we first set the values of the parameters that are common in both frameworks. These parameters are $\alpha, \gamma, \rho, \delta, \eta_a, \eta_s, \epsilon$. From the The Economic Report of the President (2007), we set the value of $\alpha = 0.315$ to match the average labor income share for the period 1959-2005. We set the value of γ, ρ and δ so that they match the value of the average rate of GDP growth, the average capital-out ratio for the period of 1929-1998, and the interest rate. According to Ngai and Pissarides (2004), the average rate of GDP growth is around 2 percent, and the value of the capital-out ratio is 3. We set the interest rate equal to 5.2% in the steady-state as in Alonso-Carrera and Raurich (2010). Thus, we obtain that $\rho = 0.03$, $\delta = 0.05$ and the growth rate of the technology frontier $\gamma = 0.0137$. In the literature, there is not a specific estimation for the value of ϵ. Its value ranges from 0.002 to 0.89, depending on the calibration strategy and the estimation procedures applied (see Boppart, 2014). We perform three numerical simulations of both models by setting the value of ϵ equal to 0.1, 0.5, and 0.90 in order to cover the range of values reported in the literature. These values let us examine how our results change in response to shifts in the value of the elasticity of substitution. Obviously, these changes affect the values of η_a, η_s, v_a, and v_s. Therefore, in the case of the non-constant biased technical change, we set the value of the parameters η_a, and η_s, so that they match the expenditure consumption share in the agriculture and services sectors, at 2005 for given values of ϵ. Simultaneously, for each value of ϵ, we set initial values for technological gaps v_a, and v_s so that they match the employment labor shares in agriculture and services in 1870, respectively, and we normalize $v_m = 1$.[8] In the case of the constant biased

[8]This assumption implies that the manufacturing sector have reached the technological frontier. In this way, we do not need to impose a value for ω_m.

technical change model, we follow Ngai and Pissarides's procedure. We set the values of η_i and the initial values of the sectoral TFPs to match the values of sectoral employment shares in 1870 (see Ngai and Pissarides, 2004).

Finally, we set the values of $\omega_a, \omega_s, \omega_{,m}, \phi_a, \phi_s, \phi_m$ as follows. In the case of our benchmark model, constant biased technical change implies that $\omega_a = \omega_s = \omega_m = 0$, so that we then need to set the values of ϕ_a, ϕ_s, and ϕ_m so that they match TFP growth rates in agriculture, manufacturing and services according to equation (1.2). Ngai and Pissarides (2004) set the value for TFP growth in agriculture, manufacturing and services at 2.4%, 1.4% and 0.4% for the period 1870-2000. Accordingly, we set $\phi_a = 0.024$, $\phi_s = 0.04$, and $\phi_m = 0.014$. In the case of non-constant bias, both the rate of adoptions $(\omega_a, \omega_s, \omega_m)$ and the exogenous growth rates (ϕ_a, ϕ_s, ϕ_m) are obtained by using the growth rate of relative prices (1.17). As our aim is to analyze a long period of sectoral transformation, we estimate the value of these parameters using the information available on relative prices, rather than the estimated parameter values in Section 2. Limited availability of data for sectoral TFP growth would mean our having to use estimated values of these parameters for a short period (1970-2005). We exploit the fact that, given our assumptions, relative prices are linked to sectoral productivity, and hence to their growth rates by using equations (1.2) and (1.17). Thus, we overcome this data limitation by using time series of relative prices for the period 1929-2005.[9] The econometric procedure to estimate these parameters is shown in the Appendix B and results are in Table 2.[10] Table 3 summarizes the parameter values for the simulation of the two models.

[Insert Table 2 and Table 3]

1.5.2 Sectoral employment shares

Figure 2 shows the goodness of fit of our simulation based on both models. As can be seen with the naked eye, both models reproduce the main patterns of sectoral change: the decline of the agriculture sector and the rise of the

[9]Relative prices for 1929-1970 are from the Historical Statistics of the United States: Colonial Times to 1970, Part 1 and 2. The implicit price deflator for services in series E17, and the wholesale price index for industrial commodities and farm products in series E23-25, E42, E52-E53. Relative prices for 1970-2005 are from: Economic Report of the President, 2013. Price index for industrial commodities and farm products in table B-67. Price indexes for services, table B-62.

[10]Our simulated series are based on point estimated parameters $(\hat{\omega}_a, \hat{\omega}_s, \hat{\omega}_{,m}, \hat{\phi}_a, \hat{\phi}_s, \hat{\phi}_m)$. We also show the simulation series based on the confidence intervals that allow us to measure the model's robustness to a variation in the rates of adoption.

services sector. However, the accuracy of such predictions differs between the models. At first glance, it is evident that constant-biased model predictions change as the degree of elasticity varies. In particular, the predictions based on this model differ greatly from the actual values of labor shares in agriculture and services as elasticity increases. By contrast, the robustness of predictions based on a non-constant biased model is notorious to changes in this parameter. To analyze the degree of accuracy of the simulations further, we report three measurements of accuracy.

[Insert Figure 2]

Table 4 reports three measures that allow us to compare the accuracy of the models, and the robustness of these simulations to variations in the elasticity and the rate of adoption. Specifically, Table 4 reports the the root-mean-square error (RMSE), and the Akaike information criterion (AIC).[11] We calculate these accuracy measures by regressing actual labor shares in agriculture, manufacturing and services on those shares predicted by our non-constant biased model and the benchmark.[12]

[Insert Table 4]

Table 4 shows that both models are able to explain the dynamics of sectoral change. However, there are quantitative differences in their performance. On the one hand, reading from left to right, Table 4 shows the differences in accuracy across the models based on these statistics. The simulations based on the non-constant biased model provide a better fit than those based on the benchmark model. In the case of agriculture, for instance, changes in the RMSE are minimal in our model compared with those obtained by the benchmark model for three different values in the elasticity. In particular, the accuracy of the benchmark model decreases as the value of the elasticity increases, whereas the simulation based on the non-constant biased model is robust to replicate observed data. In the case of services, Table 4 shows similar results as in previous case, except in the

[11] RMSE is the standard deviation of the differences between observed and predicted values values. Finally, the AIC provides a measure for comparing models. AIC allows us to determine the probability that a model is the best model to replicate the data given the set of information and alternative models.

[12] Alternately, we use the filtered data of the labor shares in agriculture and services to make the regressions. Actual data were filtered by using the Hodrick-Prescott method to reduce fluctuations in actual data due to the business cycle. This approach does not change the results in the main text.

case of low elasticity ($\epsilon = 0.1$). In this case, the benchmark model reports a lower RMSE value than shown by our model. The results show that both models are able to explain qualitatively the structural change in the U.S. economy. However, they also show that the performance of the benchmark model decreases as the elasticity increases.

1.5.3 The growth rate of the RES

Tables 5 and 6 show the actual average growth rate of the RES between agriculture and services, and between agriculture and manufacturing, respectively, for three periods: 1870-1930; 1930-1950; and 1950-2005. Tables 5 and 6 also report the average growth rate of the RES that are calculated based on the simulation of non-constant biased and benchmark models for different values of the elasticity. Thus, Tables 5 and 6 allow us to compare the robustness of the models to replicate the structural change for variations of this parameter. From these tables, we can observe two interesting results.

[Insert Table 5 and Table 6]

First, we highlight the accuracy of the non-constant biased model for simulating the relative annual changes in the RES. In general, over the entire period considered, the non-constant biased model can account for most of the growth in the RES between agriculture and services, and between agriculture and manufacturing, whereas the benchmark model replicates poorly the observed growth rates. In particular, the non-constant biased model replicates the actual average annual growth rate in the RES in the early stage of development. For the periods 1870-1930 and 1930-1950, our model replicates 88 and 62 percent of the relative change in the RES between agriculture and services, and between agriculture and manufacturing, respectively. By contrast, the benchmark model replicates 42 and 25 percent, respectively. That is, the non-constant biased model improves the explanation for the relative change in the RES by around two times compared to the prediction from a model based on constant biased technical change.

Second, we highlight the robustness of the non-constant biased model respect to the benchmark model. Tables 5 and 6 also show the sensitivity of results to variations in the elasticity. On the one hand, our model can explain a large part of the labor reallocation in the post-war U.S. economy, regardless of the value of the elasticity. Herrendorf, Rogerson, and Valentinyi

(2009) calibrate utility function parameters to be consistent with the sectoral transformation and consumption data in the post-war U.S. economy under the assumption of constant biased technical change. They find that a Leontief utility specification ($\epsilon = 0$) is necessary to provide a good fit for both the value-added sectoral consumption and the sectoral labor shares data. Given our results, a non-constant sectoral technical progress can explain the sectoral transformation without imposing a Leontief utility function. We interpret these results as a measure of the importance of the technological explanation for the structural change in the United States during the post-war period.

Moreover, the literature points out that both technological and demand factors affect structural change throughout the development process in the United States (see Dennis and Iscan, 2007; Buera and Kaboski, 2009; and Herrendorf et, al., 2014). These papers highlight that a technological factor, such as the constant biased technical change, plays a major role in explaining the sectoral shift observed after WWII, whereas the income effect is the dominant factor in accounting for the structural transformation prior to 1950. Our findings show that if we move away from the assumption of constant biased technical change, a purely technological explanation could account for the sectoral transformations in the U.S. economy prior to WWII.

1.6 Concluding remarks

In this paper, we present a multi-sectoral sectoral growth model based on Ngai and Pissarides' model. In their model, sectoral technological progress is assumed to be a constant process. This implies that differences in TFP growth rates across sectors are constant over time. According to the literature, however, this assumption is at odds with empirical evidence.

We relax the constant biased technical assumption by asserting that sectoral TFP growth rates change due to technology adoption. We assert that a sector benefits from adopting new technologies or ideas (knowledge) available at the technological frontier. This process prompts their sectoral technological progress and induces non-constant sectoral TFP growth. Based on our proposed model, we analyze the implications for structural change when sectoral technological progress is not constant.

We find that predicted patterns of sectoral labor allocation across sectors are affected by the non-constant biased technical progress in two major ways. First, we find that labor allocation over time and sectoral composition in the long run are determined by a sector's ability to adopt knowledge. We show that if technology adoption occurs in every sector, then sectoral composition is constant in the long run, while the dynamic path of employment share is affected by the rate of technology adoption. Second, in our model, the pace of industrialization depends on the relative technology level in each sector. In contrast with a constant biased model, the growth rate of the RES depends on the technological gap between the agriculture, services and manufacturing sectors and to the frontier. We show that as long as the technological gap in the agriculture sector remains large, the pace of industrialization increases.

We analyze numerically the importance of non-constant biased technical change in explaining the structural change observed in the US economy in the period 1870-2005. We show that the patterns of sectoral labor allocation and the pace of industrialization are better explained by a model based on non-constant TFP growth than by a model based on constant biased technical change.

Our findings show that if we move away from the assumption of constant biased technical change, a purely technological explanation could account for part of the sectoral transformations in the U.S. economy prior to WWII. In our model, the relative backwardness of the agricultural sector at an early stage of development fosters the rate at which labor moves from this sector to the rest of the economy. We interpret this result as a suggestion for reconsidering the role of the technology in explaining structural transformation. In this regard, our results suggest that economic factors that promote technology adoption would foster the pace of industrialization and structural change. In this regard, a natural extension of our paper is to analyze in-depth those factors that promote sectoral technology progress, such as technological adoption, human capital, and R&D as possible future lines of research on the determinants of structural change.

Bibliography

[1] Acemoglu, D. and Guerrieri, V. (2008). “Capital Deepening and Nonbalanced Economic Growth,” Journal of Political Economy 116, 467-498.

[2] Alonso-Carrera, J., & Raurich, X. (2010). Growth, sectoral composition, and the evolution of income levels. Journal of Economic Dynamics and Control, 34(12), 2440-2460.

[3] Alvarez-Cuadrado, F., Monterio, G. and Turnovsky, S. (2004). “Habit Formation, Catching-up with the Joneses, and Economic Growth,” Journal of Economic Growth 9, 47-80.

[4] Baumol, W. J. (1967). "Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis." The American Economic Review 57(3): 415-426.

[5] Bond E., Wang, P. and Yip C. (1996). “A General Two-Sector Model of Endogenous Growth with Human and Physical Capital: Balanced Growth and Transitional Dynamics,” Journal of Economic Theory 68, 149-173.

[6] Boppart, T. (2014). Structural change and the kaldor facts in a growth model with relative price effects and Non-Gorman preferences. Econometrica, 82(6), 2167-2196.

[7] Buera, F., and Kaboski, J.P. (2009). “Can Traditional Theories of Structural Change Fit the Data?” Journal of the European Economic Association, 7 (2–3), 469–477.

[8] Dennis, B. N. and T. B. Iscan (2009). "Engel versus Baumol: Accounting for structural change using two centuries of U.S. data." Explorations in Economic History 46(2): 186-202.

[9] Echevarria, C. (1997). "Changes in Sectoral Composition Associated with Economic Growth," International Economic Review 38, 431-452.

[10] Foellmi, R. and Zweimüller, J. (2008). "Strucutral Change, Engel's Consumption Cycles and Kaldor's Facts of Economic Growth," Journal of Monetary Economics 55, 1317-1328.

[11] Gollin, D., Lagakos, D. and Waugh,M.E. (2014). "Agricultural Productivity Differences across Countries," American Economic Review, American Economic Association, vol. 104(5), pages 165-70, May.

[12] Herrendorf, B., et al. (2014). Chapter 6 - Growth and Structural Transformation. Handbook of Economic Growth. A. Philippe and N. D. Steven, Elsevier. Volume 2: 855-941.

[13] Joint Economic, C., et al. (2007). The economic report of the President: hearing before the Joint Economic Committee, Congress of the United States, One Hundred Ninth Congress, second session, February 16, 2006. Washington.

[14] Kongsamut, P., Rebelo, S. and Xie, D. (2001). "Beyond Balanced Growth," Review of Economic Studies 68, 869-882.

[15] Meckl, J. r. (2002). "Structural Change and Generalized Balanced Growth." Journal of Economics 77(3): 241.

[16] Ngai, R. and Pissarides, C. (2007). "Structural Change in a Multi-sector Model of Growth," American Economic Review 97, 429-443.

[17] O'Mahony, Mary and Marcel P. Timmer (2009), "Output, Input and Productivity Measures at the Industry Level: the EU KLEMS Database", Economic Journal, 119(538), pp. F374-F403.

[18] Steger, T.M., (2006). "Heterogeneous Consumption Goods, Sectoral Change and Economic Growth," Studies in Nonlinear Dynamics and Econometrics 10, No. 1, Article 2.

[19] United States. Bureau of the, C. (1975). Historical statistics of the United States, colonial times to 1970.

Appendix

1.A Equilibrium properties

Solution to the representative consumer optimization problem.

The Hamiltonian function associated with the maximization of (2.3.1) subject to (1.9), (1.10), (1.11), and (2.2.5) is

$$\mathscr{H} = \ln \tilde{C} + \sum_{i=a,s} \mu_i \left(Y_i - C_i\right) + \mu_m \left(Y_m - \delta K - C_m\right),$$

where μ_a, μ_s and μ_m are the co-state variables corresponding to the constraints (1.11) and (2.2.5), respectively. The first order conditions are

$$\eta_a \tilde{C}^{\frac{1-\epsilon}{\epsilon}} C_a^{-\frac{1}{\epsilon}} = \mu_a, \tag{1.A.1}$$

$$\eta_2 \tilde{C}^{\frac{1-\epsilon}{\epsilon}} C_s^{-\frac{1}{\epsilon}} = \mu_s, \tag{1.A.2}$$

$$\eta_3 \tilde{C}^{\frac{1-\epsilon}{\epsilon}} C_m^{-\frac{1}{\epsilon}} = \mu_m, \tag{1.A.3}$$

$$(1-\alpha)\,\mu_a \frac{Y_a}{l_a} = (1-\alpha)\,\mu_m \frac{Y_m}{l_m}, \tag{1.A.4}$$

$$(1-\alpha)\,\mu_s \frac{Y_s}{l_s} = (1-\alpha)\,\mu_m \frac{Y_m}{l_m}, \tag{A.5}$$

$$\alpha \mu_a \frac{Y_a}{v_a} = \alpha \mu_m \frac{Y_m}{v_m}, \tag{A.6}$$

$$\alpha \mu_s \frac{Y_s}{v_s} = \alpha \mu_m \frac{Y_m}{v_m}, \tag{A.7}$$

and

$$-\dot{\mu}_m + \mu_m \rho = \mu_a \alpha \frac{Y_a}{K} + \mu_s \alpha \frac{Y_s}{K} + \mu_m \left(\alpha \frac{Y_m}{K} - \delta\right). \tag{A.8}$$

The sectoral allocation of capital and relative prices

From combining $(1.A.4)$ and $(A.6)$, we obtain

$$v_a = v_m \frac{l_a}{l_m},$$

and combining $(A.5)$ and $(A.7)$, we obtain

$$v_s = v_m \frac{l_s}{l_m}.$$

We substitute v_a^* and v_s^* in (1.9), and taking into account (1.10), the optimal capital share in the manufacturing sector is

$$v_m = l_m, \tag{A.9}$$

which implies

$$v_a = l_a, \tag{A.10}$$

$$v_s = l_s. \tag{A.11}$$

By assuming that the manufacturing good is the numerarie and dividing equations $(1.A.4)$ by $(A.5)$, and combining (1.10), $(A.9)$, $(A.10)$ and $(A.11)$, we obtain the relative prices

$$p_a \equiv \frac{\mu_a}{\mu_m} = \frac{Y_m l_a}{Y_a l_m} = \frac{A_m}{A_a}, \tag{A.12}$$

$$p_s \equiv \frac{\mu_s}{\mu_m} = \frac{Y_m l_s}{Y_s l_m} = \frac{A_m}{A_s}. \tag{A.13}$$

Note that the relative prices are the ratio between the co-state variables.

The GDP is obtained by substitution of (1.16) and (1.17) in $(1.A.5)$. Firstly, we substitute (1.16) in (1.8) to obtain

$$Y_i = A_i K^{\alpha} l_i, \tag{A.14}$$

and GDP

$$Y = p_a Y_a + p_s Y_s + Y_m. \tag{1.A.5}$$

By combining with (1.17), $(A.14)$ and $(1.A.5)$, we obtain

$$Y = A_m \, K^{\alpha} \left(l_a + l_m + l_m\right),$$

and, given (1.10), we obtain that

$$Y = A_m \; K^{\alpha}.$$

The Euler equation

From (1.19), we obtain C_a and C_s as functions of C_m and the relative prices

$$C_i = \left(\frac{\eta_i}{\eta_m}\right)^{\epsilon} p_i^{-\epsilon} C_m \text{ for } i = a, s.$$

We then substitute these equation in (2.2.11) and combining with (2.*A*.3), we obtain

$$(1 + x_a + x_s)\, C_m = \mu_m^{-1}. \tag{1.A.6}$$

Substituting (1.*A*.6) in (1.20) we obtain

$$C = \mu_m^{-1},$$

where total expenditure is a function of the co-state variable corresponding to the constraint (2.2.5). By substituting (1.16) in (1.8), and combining with (*A*.8), we obtain the growth rate of the co-state variables μ_m as follows

$$-\hat{\mu}_m = \alpha A_m K^{\alpha-1} - \left(\rho + \delta\right). \tag{1.A.7}$$

We then log-differentiate $C = \mu_m^{-1}$ and combine with (1.*A*.7), we obtain (1.24).

Proof of Proposition 2.1. If $\omega_m > 0$, then the dynamic system is

$$\begin{aligned}
\hat{z} &= z^{\alpha-1} - \frac{c}{z} - \delta - \frac{\gamma\phi_m - \omega_m \ln u_m}{(1-\alpha)}, \\
\hat{c} &= \alpha z^{\alpha-1} - \left(\rho + \delta\right) - \frac{\gamma\phi_m - \omega_m \ln u_m}{(1-\alpha)}, \\
\hat{v}_m &= \left(\phi_m - 1\right)\gamma - \omega_m \ln v_m,
\end{aligned}$$

From equation $\hat{v}_m$, it follows that there is a unique steady value such that

$$v_m = \exp\left(-\frac{\left(1-\phi_m\right)\gamma}{\omega_m}\right),$$

and substituting in $\hat{z}$ and $\hat{c}$, we obtain

$$\begin{aligned}
\hat{z} &= z^{\alpha-1} - \frac{c}{z} - \delta - \frac{\gamma}{(1-\alpha)}, \\
\hat{c} &= \alpha z^{\alpha-1} - \left(\rho + \delta\right) - \frac{\gamma}{(1-\alpha)}.
\end{aligned}$$

The steady state, it must be satisfied that $\hat{z} = \hat{c} = 0$, implying that

$$0 = z^{\alpha-1} - \frac{c}{z} - \delta - \frac{\gamma}{(1-\alpha)},$$
$$0 = \alpha z^{\alpha-1} - \left(\rho + \delta\right) - \frac{\gamma}{(1-\alpha)}.$$

Solving the system for z and c, we obtain

$$z^* = \left(\alpha \frac{(1-\alpha)}{\gamma + (1-\alpha)\left(\rho+\delta\right)}\right)^{\frac{1}{1-\alpha}},$$
$$c^* = \frac{\left(\gamma + \rho + (1-\alpha)\delta\right)}{\alpha}\left(\frac{\alpha(1-\alpha)}{\gamma + (1-\alpha)\left(\delta+\rho\right)}\right)^{\frac{1}{1-\alpha}}.$$

If $\varpi_m = 0$, then the dynamic system at the steady state is

$$\hat{z} = z^{\alpha-1} - \frac{c}{z} - \delta - \frac{\phi_m \gamma}{(1-\alpha)},$$
$$\hat{c} = \alpha z^{\alpha-1} - \left(\rho + \delta\right) - \frac{\phi_m \gamma}{(1-\alpha)}.$$

At the steady state, we obtain

$$z^* = \left(\alpha \frac{(1-\alpha)}{\phi_m \gamma + (1-\alpha)\left(\rho+\delta\right)}\right)^{\frac{1}{1-\alpha}},$$
$$c^* = \frac{\left(\phi_m \gamma + \rho + (1-\alpha)\delta\right)}{\alpha}\left(\frac{\alpha(1-\alpha)}{\phi_m \gamma + (1-\alpha)\left(\delta+\rho\right)}\right)^{\frac{1}{1-\alpha}}.$$

Proof of Proposition 1.2. If $\omega_m > 0$, there are two state variables and one variable control. Using (1.26), (1.27), and (1.28), we obtain the following Jacobian matrix evaluated at the steady state

$$J = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix},$$

where

$$a_{11} \equiv \frac{\partial \hat{z}}{\partial z} = \rho, \qquad a_{12} \equiv \frac{\partial \hat{z}}{\partial c} = -1, \quad a_{13} \equiv \frac{\partial \hat{z}}{\partial v_m} = \frac{\omega_m}{1-\alpha}\frac{z^*}{v_m^*},$$
$$a_{21} \equiv \frac{\partial \hat{c}}{\partial z} = \alpha(\alpha-1)\, z^{*\alpha-2} c^*, \quad a_{23} \equiv \frac{\omega_m}{1-\alpha}\frac{c^*}{v_m^*}, \quad a_{33} \equiv \frac{\partial \hat{v}_m}{\partial v_m} = -\omega_m.$$

It is immediate to see that the eigenvalues are $\lambda_1 = -\omega_m$, and the two roots λ_2 and λ_3 are the solution of the following equation

$$Q(\lambda) = \lambda^2 - \lambda\left(\rho\right) + a_{21} = 0,$$

where the solutions are

$$\lambda_2, \lambda_3 = \frac{\rho \pm \sqrt{\rho^2 - 4a_{21}}}{2}.$$

Insofar as $a_{21} < 0$ and $\rho > 0$, it follows that one of the roots, for example λ_2 is always negative and the other one, λ_3, is positive. So, λ_1, $\lambda_2 < 0$ and $\lambda_3 > 0$. This result implies that there is a two-dimensional stable manifold in (z, c, v_3) space.

On the other hand, If $\omega_m = 0$, there is one state variable and one control variable. Using (1.26), (1.27), we obtain the following Jacobian matrix evaluated at the steady state

$$J = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix},$$

where

$$\begin{array}{ll} b_{11} \equiv \frac{\partial \hat{z}}{\partial z} = \rho, & b_{12} \equiv \frac{\partial \hat{z}}{\partial c} = -1, \\ b_{21} \equiv \frac{\partial \hat{c}}{\partial z} = \alpha\left(\alpha - 1\right) z^{*\alpha - 2} c^*, & b_{22} \equiv \frac{\partial \hat{c}}{\partial c} = 0. \end{array}$$

As

$$\det J = \lambda_1 \lambda_2 = -\left(b_{21}\right)\left(b_{12}\right) < 0,$$

the eigenvalues of the system are real numbers of opposite signs, and the steady state is saddle path stable.

1.B Estimation of the technology

Solution of differential equation

We then pose the law of motion of productivity in the i sector as follows

$$\frac{\dot{A}_i}{A_i} = \phi_i + \omega_i \ln\left(\frac{1}{v_i}\right),$$

where we define the inverse of the distance across sectors and the frontier as follows

$$v_i = \frac{A_i}{A}. \tag{1.B.1}$$

By taking the log-derivative of $(1.B.1)$, we obtain that the law of motion of technological gaps is

$$\dot{v}_i = \left(\phi_i - \gamma\right) v_i - \omega_i \ln\left(v_i\right) v_i. \tag{1.B.2}$$

We rewrite $(1.B.2)$ as follows

$$\frac{dv_i}{\left(\phi_i - \gamma\right) v_i - \omega_i \ln\left(v_i\right) v_i} = dt, \tag{1.B.3}$$

then $(1.B.3)$ can be integrated after a single substitution. Let

$$m = \phi_i - \gamma - \omega_i \ln\left(v_i\right),$$

where

$$\frac{dm}{dv_i} = \frac{-\omega_i}{v_i} \rightarrow \frac{dv_i}{v_i} = \frac{dm}{-\omega_i}.$$

Substituting in$(1.B.3)$, integrating and solving the integral equation,

$$\frac{1}{-\omega_i}\int \frac{1}{m_i} dm_i = \int dt,$$

we obtain

$$m_i = e^{-\omega_i(t+c)},$$

We substitute back into the m_i to obtain

$$v_i = \exp\left(\frac{\phi_i - \gamma}{\omega_i} + e^{\omega_i(c-t)}\right), \tag{1.B.4}$$

and substituting $(1.B.4)$ in $(1.B.1)$,we finally obtain

$$A_i = \exp\left(\frac{\phi_i - \gamma}{\omega_i} + e^{\omega_i(c-t)}\right) A. \tag{1.B.5}$$

Estimation Procedures: Using EUKLEMS (1970-2005)

From the definition of $(1.B.5)$, we can estimate ω_i and ϕ_i using the data on the TFP growth rates of the agriculture, manufacturing, and services sectors from the EUKLEMS database that covers 1970-2005. To this end, we estimate the following equation which derives from $(1.B.5)$ and our assumption that the technology frontier grows at a constant rate in equation (1.1). Taking logs in $(1.B.5)$ we obtain

$$\ln A_i = \frac{\phi_i - \gamma}{\omega_i} + \ln A_0 + e^{\omega_i(c-t)} + \gamma t,$$

and normalizing the initial stock in the frontier to one, $A_0 = 1$, we can estimate the following system of equations

$$\begin{aligned}
\ln A_a &= \alpha_a + \beta_a t + e^{-\delta_a(t-c_a)}, \\
\ln A_m &= \alpha_m + \beta_m t + e^{-\delta_m(t-c_m)}, \\
\ln A_s &= \alpha_s + \beta_s t + e^{-\delta_s(t-c_s)},
\end{aligned} \tag{1.B.6}$$

where

$$\alpha_i = \frac{\phi_i - \gamma}{\omega_i};\ \beta_i = \gamma;\ \text{and}\ \delta_i = \omega_i.$$

We estimate the parameters in $(1.B.6)$ constrained to $\beta_i = \gamma$ for all sector. We use non-linear squares to estimate $(1.B.6)$. We report the results in Table 1.

Estimation Procedures: Relative prices (1929-2005)

In order to have an estimation of ω_i and ϕ_i prior to 1970, we estimate the parameters in (1.2) by using relative prices as long as these are related to the dynamic of relative productivity in (1.17). The major problem in estimating adoption rates arises from the empirical specification of our law of motions, which depends on an unobservable factor (the technological frontier). In order to use relative prices as a proxies of sectoral TFP, here, we assume that the manufacturing sector is a proxy for the frontier.[13] Given this assumption, we know from Kruguer (2008) that the annual manufacturing TFP growth rate for the period 1870-2000 is around 0.014. Therefore,

$$\frac{\dot{A}_m}{A_m} \equiv \phi_m = 0.014.$$

From (1.17), we know that

$$\frac{\dot{p}_a}{p_a} = \frac{\dot{A}_m}{A_m} - \frac{\dot{A}_a}{A_a} = \phi_m - \phi_a - \omega_a \ln(p_a), \tag{1.B.7}$$

and

$$\frac{\dot{p}_s}{p_s} = \frac{\dot{A}_m}{A_m} - \frac{\dot{A}_s}{A_s} = \phi_m - \phi_s - \omega_s \ln(p_s). \tag{1.B.8}$$

We solve $(1.B.7)$ and $(1.B.8)$ to obtain

$$\ln p_a = \frac{\phi_m - \phi_a}{\omega_a} + e^{-\omega_a(C_a+t)}, \tag{1.B.9}$$

[13] In terms of our model, we can assume that the manufacturing sector is in the frontier. That is, TFP in manufacturing has reached the technology frontier.

and

$$\ln p_s = \frac{\phi_m - \phi_s}{\omega_s} + e^{-\omega_s(C_s+t)} \tag{1.B.10}$$

where C_a and C_s are constants of integration. We use nonlinear seemingly unrelated regression to estimate C_a and C_s; ϕ_a and ϕ_s; ω_a, and ω_s from (1.B.9) and (1.B.10) by fitting the following system of equations:

$$\ln p_i = \alpha_i + e^{\beta_i(C_i+t)} \text{ for } i = a, s,$$

where

$$\alpha_i = \frac{\phi_m - \phi_i}{\omega_i}; \text{ and } \beta_i = -\omega_i,$$

and the constant C_a and C_s subject to the constraint $\phi_m = 0.0140$. Tables 2 reports the estimated values of ω_a, and ω_s and the values of ϕ_a and ϕ_s, which are obtained by nonlinear combinations of the estimated parameters $\hat{\alpha}_i$ and $\hat{\beta}_i$ using nlcom (nonlinear combination) command in STATA.

1.C Figures and Tables

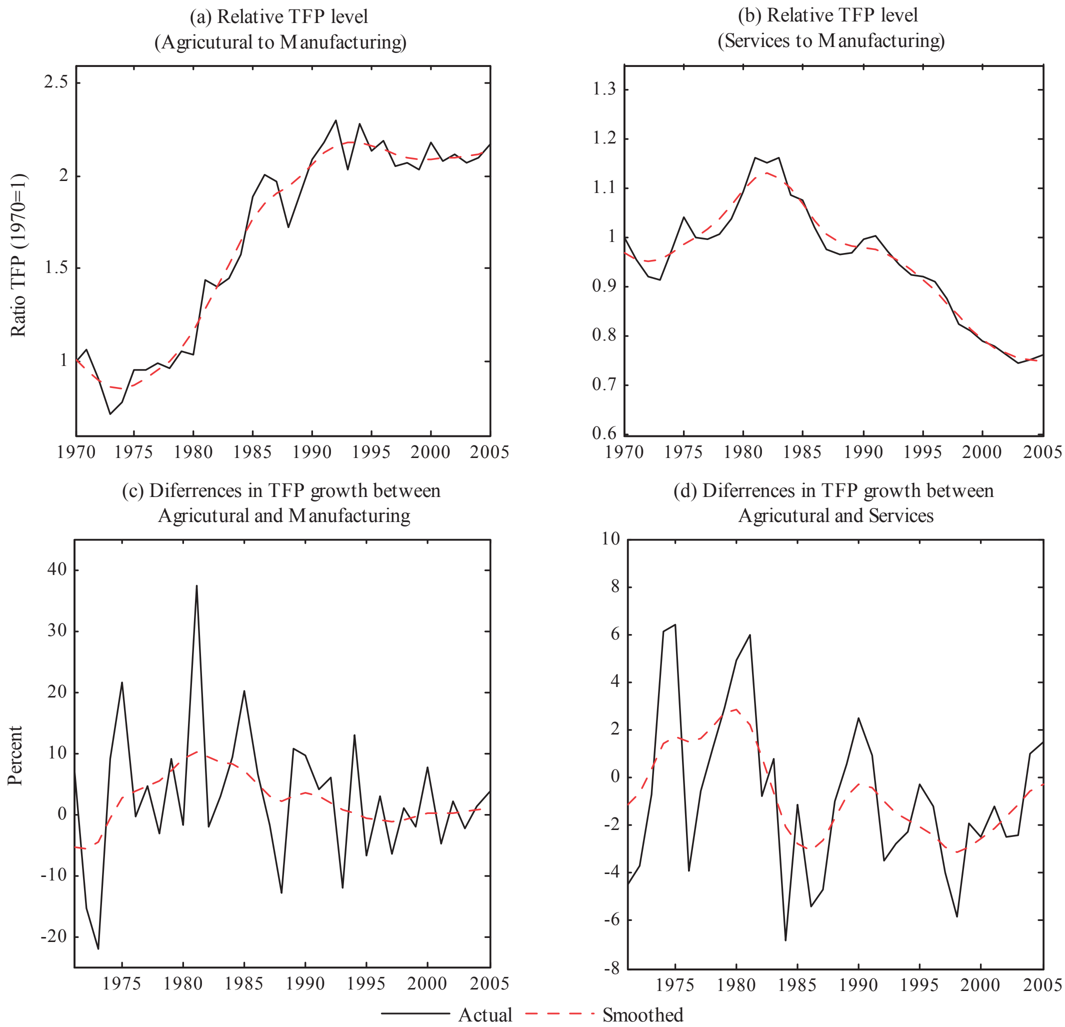

Figure 1.1: Relative Sectoral TFP (Levels and Growth). Panel (a) plots the ratio of TFP levels between agriculture and manufacturing sectors. Panel (b) plots the ratio of TFP levels between agriculture and services sectors. Panel (c) an (d) plot the actual and smoothed growth rates of these TFP ratios, respectively. We use the Hodrick-Prescott to filter actual data to obtain a trend component. We set the smooth parameter, lambda, equal to 6.25

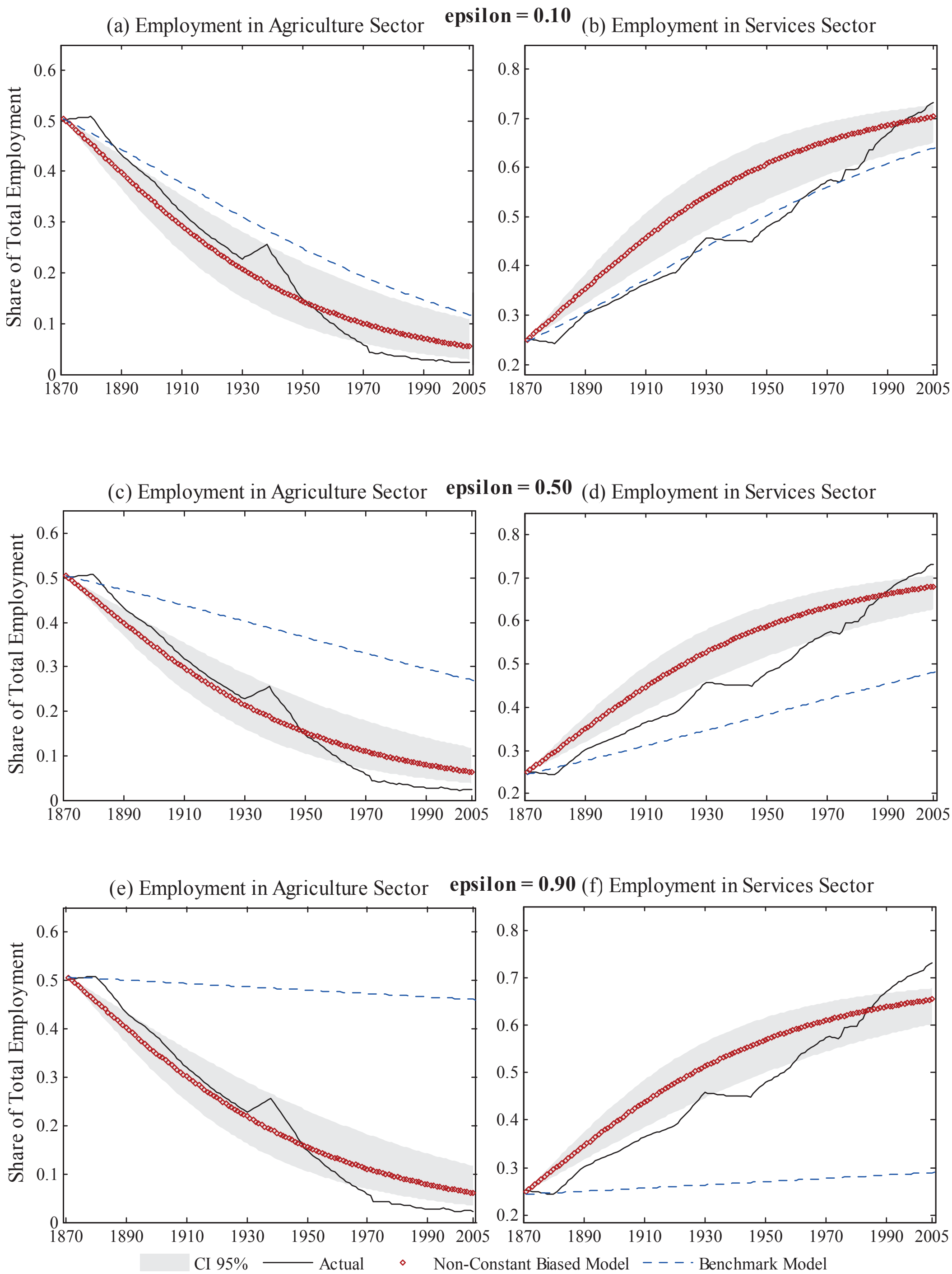

Figure 1.2: Patterns of Structural Change. Figure shows the simulated patters of labor shares obtained by assuming three different values of the elasticity of substitution. Thus, this figure shows the robustness of both models to changes in the elasticity. Here, Figure 2 also plots the predicted labor shares in the 95 percent confidence intervals values for the point estimates (shaded area).

Tables

Table 1.1: Estimation of adoption rates EUKLEMS 1970-2005

	Agriculture	Manufacturing	Services
ω	0.026***	-0.001	0.017***
	(-0.002)	(0.002)	(0.003)
φ	0.034***	0.011***	0.039***
	(0.002)	(0.002)	(0.007)
γ	0.012***	0.012***	0.012***
	(0.002)	(0.002)	(0.002)
R^2	0.93	0.63	0.76

Standard errors in parentheses
* $p<0.05$, ** $p<0.01$, *** $p<0.001$

Table 1.2: Estimation of adoption rates. Relative Agricultural and Services Prices

	Agriculture (mean)	Services (mean)	Agriculture (a)	Agriculture (b)	Services (a)	Services (b)
ω	0.005***	0.013***	0.004	0.006	0.005	0.021
	(0.001)	(0.004)				
φ	0.019***	0.025***	0.017	0.019	0.024	0.026
	(0.001)	(0.0005)				
R^2	0.42	0.84				

Standard errors in parentheses. (a) and (b) are the lower and upper values in the confidence interval.
* p<0.05, ** p<0.01, *** p<0.001

Table 1.3: Calibration

Parameters	Values	Targets	data
α	0.315	Labor income share	0.685
γ	0.0137	GDP growth rate (1870-2005)	0.02
ρ	0.03	Capital-output ratio	3
δ	0.05	Interest rate	0.052
ε	0.5	By assumption	
η_a	0.01	Consumption expenditure share in food	0.03
η_s	0.9	Consumption expenditure share in services	0.75
v_a	0.0013	Labor share in agriculture (1870)	50%
v_s	0.1803	Labor share in services (1870)	25%
v_m	1	Normalized	
ω_a	0.004	Estimation from relative agricultural price	
ω_s	0.01	Estimation from relative services price	
ω_m	0	By assumption	
φ_a	0.019	Estimation from relative agricultural price	
φ_s	0.025	Estimation from relative services price	
φ_m	1	Normalized	

Note: We perform three numerical simulations for three different values of the elasticity, and accordingly, the initial values of technological gap were set. Reported values in Table 3 are set by assuming ε=0.5. For the values of the elasticity equal to 0.1, and 0.9, we set the initial gaps in agriculture sector equal to 0.000005, and 0.001; and the initial gaps in the services sector are 0.053 and 0.000012, respectively. Finally, the values of ηa are 0.00001 and 0.0082 and the values of ηs are 0.9999 and 0.77 for ε=0.1 and ε=0.9, respectively.

Table 1.4: Accuracy Measures of Simulated Structural Change

Case	Accuracy measure	Agriculture		Manufacturing		Services	
		(a)	(b)	(a)	(b)	(a)	(b)
$\varepsilon = 0.10$	RMSE	**0.032**	0.071	**0.044**	0.044	0.046	**0.030**
	AIC	**-546**	-328	**-459**	-457	-446	**-559**
$\varepsilon = 0.50$	RMSE	**0.032**	0.123	**0.030**	0.039	**0.048**	0.054
	AIC	**-546**	-183	**-566**	-494	**-438**	-406
$\varepsilon = 0.90$	RMSE	**0.035**	0.157	**0.037**	0.039	**0.050**	0.119
	AIC	**-518**	-116	**-504**	-493	**-426**	-191
Observations		135		135		135	

We calculate these accuracy-measures by regressing actual labor shares in agriculture, manufacturing and services on those predicted shares by our non-constant biased model and the benchmark for the period 1870-2005. Here, we report the results for three values of the elasticity of substitution. Column (a) reports the statistical measures for the non-constant biased model. Column (b) reports the statistical measures for the benchmark model.

Table 1.5: Average growth rate: RES between Agriculture and Services

		(a)	(b)	(c)	(d)	(e)
$\varepsilon = 0.10$	1870-1930	-2.32	-2.78	-1.78	1.20	0.77
	1930-1950	-2.30	-2.35	-1.78	1.02	0.77
	1950-2005	-3.98	-2.03	-1.78	0.51	0.45
$\varepsilon = 0.50$	1870-1930	-2.32	-2.67	-0.96	1.15	0.41
	1930-1950	-2.30	-2.21	-0.96	0.96	0.42
	1950-2005	-3.98	-1.86	-0.96	0.47	0.24
$\varepsilon = 0.90$	1870-1930	-2.32	-2.60	-0.19	1.12	0.08
	1930-1950	-2.30	-2.18	-0.19	0.95	0.08
	1950-2005	-3.98	-1.97	-0.19	0.49	0.05

Column (a) reports the actual average annual growth rates. Columns (b) and (c) report the predicted average growth rates based on the non-constant and the benchmark model, respectively. Columns (d) and (e) report the fraction of actual growth rate that is replicated by the model (b) and (c), respectively.

Table 1.6: Average growth rate: RES between Agriculture and Manufacturing

		(a)	(b)	(c)	(d)	(e)
$\varepsilon = 0.10$	1870-1930	-1.74	-1.53	-0.82	0.88	0.47
	1930-1950	-2.87	-1.77	-1.09	0.62	0.38
	1950-2005	-2.44	-1.72	-1.31	0.71	0.54
$\varepsilon = 0.50$	1870-1930	-1.74	-1.52	-0.39	0.88	0.22
	1930-1950	-2.87	-1.70	-0.47	0.59	0.16
	1950-2005	-2.44	-1.59	-0.54	0.65	0.22
$\varepsilon = 0.90$	1870-1930	-1.74	-1.57	-0.07	0.90	0.04
	1930-1950	-2.87	-1.78	-0.07	0.62	0.03
	1950-2005	-2.44	-1.79	-0.08	0.73	0.03

Column (a) reports the actual average annual growth rates. Columns (b) and (c) report the predicted average growth rates based on the non-constant and the benchmark model, respectively. Columns (d) and (e) report the fraction of actual growth rate that is replicated by the model (b) and (c), respectively.

Chapter 2

Kuznets meets Lucas: Structural Change and Human Capital

2.1 Introduction

In the current literature on structural change and economic growth, there are two explanations for the shift from agriculture to non-agriculture activities, the so-called Kuznets facts. The first explanation emphasizes the role of changes in the composition of the demand on structural change. These changes are based on the Engel law: as income rises, demand for agriculture products decreases as well as the share of agriculture in GDP (see Kongsamut, Rebelo and Xie, 2001; Meckl, 2002; Foellmi and Zweimuller, 2002). The second explanation relies on technological differences across sectors. In this branch of the literature, sectoral differences in rates of technological progress or in capital intensity explain structural change. In Ngai and Pissarides (2007), for instance, labor moves from the progressive sectors (those with the highest productivity growth) to the stagnant sectors (those with the lowest productivity growth), whereas, in Acemoglu and Guerrieri (2008), labor moves from the more capital-intensive sectors to the less capital-intensive sectors. Interestingly, while these demand and supply based explains the Kuznets facts in the framework of multisectoral growth models, both explanations rely on growth models, where technology progress is exogenous.[1] We contribute to this literature by explaining structural change in a model where technological progress is endogenous.

In this paper, we build a continuous-time model of economic growth

[1] See Herrendorf (2013) for a thorough review of the empirical evidence on the dynamics of the sectoral composition in developed countries.

that encompasses the technological explanation of the Kuznets facts with endogenous technological progress. To this end, we consider a four-sector version of the endogenous growth model introduced by Uzawa (1965) and Lucas (1988). Firms in three sectors produce goods that are devoted to consumption or investment. We identify these sector as the agriculture, manufacturing and services sectors. Firms in the fourth sector, that we define as the educational sector, produce a good that is devoted only to increase the stock of human capital. Whereas firms in the educational sector produce using only the time that individuals devote to education, we assume that firms in the non-educational sector produce consumption goods by combining physical and human capital. As in the Lucas' model, we also assume that the economy-wide average human capital causes an externality in the non-educational sectors.[2] Specifically, we assume sector specific strengths of the externality.

In this model, human capital accumulation drives structural change through two channels. In the first channel, human capital accumulation causes endogenous sectoral technical progress. The differences in the rate of technical progress across sectors arise due to sector-specific strengths of the externality. As occurs in Ngai and Pissarides (2007), these differences in productivity growth rates across sectors induce changes in relative prices, which cause structural change. We show that the empirical relevant case occurs when external effects are higher in the agriculture sector than in manufacturing and services sectors. In this case, the relative price of agriculture goods decreases and induces labor to move toward the service sector. We refer to this channel as the endogenous Baumol effect on structural change.

In the second channel, the initial stocks of human and physical capital cause structural change. In the model, individuals choose between investing in human and physical capital. When the initial ratio between this two stocks of capital differs from its long run value, individuals decide either to allocate more employment into the education sector (when the ratio between the two capital stock is below its long run value) or allocate employment in the manufacturing sector in order to accumulate more physical capital (when the ratio between the two capital stock is above its long run value). We refer to this channel as the endogenous investment effect on structural change. We show that the latter channel affects structural change only during the transition to the equilibrium, whereas the former continues driving structural

[2]See Tamura (2006) for an analysis of the effects of the externality on the accumulation of human capital.

change in the long run.

We use this model to analyze numerically the effects of these two channels on structural change and investigate their implications for development. To this end, we propose two numerical exercises. In the first one, we analyze the accuracy of this model in explaining observed patterns of structural change in the United States along the 20th century, a period of time characterized by a fast accumulation of human capital. In the second exercise, we extend the numerical analysis to outline the role of imbalances between physical and human capital in explaining structural change as well as income differences across countries.

Our numerical exercises show that the model is capable to explain i) structural change; ii) the differences between employment and added value shares across sectors; iii) the time path of human accumulation in the U.S economy. Furthermore, we found that the imbalance between physical and human capital can provide an explanation for the heterogeneity across countries in both the sectoral composition and income differences. We conclude that human capital is an important factor explaining not only economic growth, but also cross-country differences in sectoral composition.

The paper is organized as follows. Section 2 presents the basic model. Section 3 characterizes the equilibrium dynamics. Section 4 develops the numerical analysis. Section 5 presents some concluding remarks, while the appendices contains the proofs of all the results of the paper.

2.2 The model

We consider a four-sector model. The first sector produces a good that can be consumed or invested in physical capital. We refer to this sector as the manufacturing sector and we denote it by using the sub-index m. The second and third sectors produce only goods devoted to consumption. We refer to these sectors as the agricultural and the services sectors, and we denote them by the sub-indexes a and s, respectively. Finally, the fourth sector produces a good that is exclusively devoted to increase the stock of human capital. We refer to this sector as the education sector and we denote it by using the sub-index h.

In our economy, firms in agriculture, manufacturing, and services sectors produce consumption goods by combining physical and human capital, that we denote as k and h, respectively. Following Ngai and Pissarides (2007), we assume that the capital-output elasticity is equal across sectors and they produce an amount of commodity y_i by using the following production function:

$$y_i = A_i \bar{h}^{\psi_i} (s_i k)^{\alpha} (u_i h)^{1-\alpha}, \text{ for } i = a, m, s, \tag{2.2.1}$$

where s_i and u_i are the shares of physical and human capital allocated in the sector i; A_i is a parameter which stands for a sector-specific productivity level; and $\alpha \in (0,1)$ is the capital-output elasticity.[3] As in Lucas (1988), we introduce externalities in production, but not in the human capital accumulation process. these externalities are generated by economy-wide average human capital $\bar{h}$. In equation (2.2.1), the parameter $\psi_i \geq 0$ measures the elasticity of output with respect to the aggregate external effect of human capital. Given the assumptions on the production function, total factor productivity (TFP) across sectors is defined by $A_i \bar{h}^{\psi_i}$. This implies that the differences in rates of sectoral technical progress across sectors are driven only by differences in the strength of productive externalities across sectors. Thus, if $\psi_m = \psi_{a.}$ and $\psi_m = \psi_s$, sector technical change would be unbiased. In order to replicate the actual time paths of the relative prices of agriculture and services, we make the assumption that $\psi_a > \psi_m$ and $\psi_m > \psi_s$ holds in order to investigate how structural change is affected by the strength of these externalities.[4]

In our model, human capital is accumulated through a commodity y_h, which is produced in the education sector. Firms produce y_h from human capital. For the sake of simplicity, we assume that the production function is a linear function of the form

$$y_h = B u_h h, \tag{2.2.2}$$

where $B > 0$ is a constant which stands for a productivity level, and u_h is the share of labor devoted to increase human capital. Obviously, both physical capital and employment shares in (2.2.1) and (2.2.2) must satisfy the following

[3] Despite the fact that capital intensive varies across sector (Echevarria, 1997; Valentinyi and Herrendorf, 2008), we assume that there are not sectoral differences in capital intensive in order to focus on the effects of human capital on sectoral productivities. As a result, we exclude from the model capital deepening (see Acemoglu and Guerrieri, 2008). Dennis and Iscan (2008) show that the effect of capital deepening on structural change in U.S. economy is marginal and complementary to the biased technical change. Moreover, Herrendorf, Herrington and Valentinyi (2013) found that a Cobb-Douglas sectoral production functions that differ only in technical progress capture main trend in the postwar US structural change.

[4] A particular case of our model $\psi_m = \psi_s = \psi_s = 0$ is Alonso-Carrera, Caballé and Raurich (2015). They build a model with human capital accumulation, heterogenous consumption goods, homothetic preferences and sectoral production functions with differences in physical capital intensity.

equations

$$s_a + s_m + s_s = 1, \tag{2.2.3}$$

and

$$u_a + u_m + u_s + u_h = 1. \tag{2.2.4}$$

We assume the commodity y_m is the only commodity that can be either consumed or added to the stock of aggregate capital. This implies that $y_a = c_a$, $y_s = c_s$ and

$$\dot{k} = y_m - c_m - \delta k, \tag{2.2.5}$$

where c_a, c_m and c_s are the amounts of agricultural, manufacturing and service goods devoted to consumption, and $\delta \in [0,1]$ is the depreciation rate of the physical capital stock. As we assume that y_h is devoted exclusively to increase the stock of human capital, the law of motion of this stock is given by

$$\dot{h} = B u_h h. \tag{2.2.6}$$

Note that we assume that there is no depreciation of human capital, whereas physical capital depreciates at a constant rate.[5]

We assume that the economy is populated by a single infinitely lived representative agent endowed with k units of physical capital and h units of human capital. Let w be the wage per unit of human capital and r the real interest rate. We assume perfect sectoral mobility so that the wage and interest rate are identical across sectors. Thus, the budget constraint of the consumer is given by

$$wh + rk = c_m + p_s c_s + p_a c_a + I_k + p_h I_h, \tag{2.2.7}$$

where w, and r are the wage and interest rate; p_s, p_a, and p_h are the relative prices of services, agriculture goods and human capital in terms of the manufacturing good; I_h and I_k are the gross investment in human and physical capital, respectively, and thus

$$I_k = \dot{k} + \delta k, \tag{2.2.8}$$

$$I_h = \dot{h}. \tag{2.2.9}$$

The representative agent obtains utility from the consumption of services, agricultural and manufacturing goods. In particular, we assume that the

[5]This assumption does not change the main results of the paper.

instantaneous utility function is

$$U(\tilde{c}) = \ln \tilde{c}, \tag{2.2.10}$$

where $\tilde{c}$ denotes the composite consumption good. This good satisfies

$$\tilde{c} = \left(\eta_a c_a^{\frac{\epsilon-1}{\epsilon}} + \eta_m c_m^{\frac{\epsilon-1}{\epsilon}} + \eta_s c_s^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{2.2.11}$$

where η_m, η_s, and η_a measure the weights of sectoral consumptions in utility, and ϵ is the elasticity of substitution across consumption goods. Following Ngai and Pissarides (2007), we assume that $\eta_m + \eta_s + \eta_a = 1$ and $\epsilon < 1$. The latter assumption implies that goods are complements. Ngai and Pissarides (2007) show that this is a necessary condition to explain the observed patterns of structural change in a model with homothetic preferences.

2.3 The equilibrium

In this section, we obtain the system of differential equations that characterizes the equilibrium. We use these equations to find the long-run equilibrium and we study how human capital accumulation modifies the sectoral composition during the transition and along the steady state.

The representative agent maximizes the discounted sum of utilities

$$\int_0^\infty e^{-\rho t} U(\tilde{c})\, dt, \tag{2.3.1}$$

subject to (2.2.7), (2.2.8), (2.2.9) and $y_a = c_a$, $y_s = c_s$, where $\rho > 0$ is the subjective discount rate. In the appendix, we show that the solution to this optimization problem is given by the transversality conditions are $\lim_{t\to\infty} \mu_2 k = 0$, and $\lim_{t\to\infty} \mu_h h = 0$, where μ_2 and μ_h are the shadow prices of physical and human capital, respectively, and the following equations:

$$\frac{\dot{c}_m}{c_m} = r - \delta - \rho - (1-\epsilon) \sum_{i=a,s} \sigma_i \frac{\dot{p}_i}{p_i}, \tag{2.3.2}$$

where

$$\sigma_i = \frac{\bar{\eta}_i p_i^{1-\epsilon}}{1 + \bar{\eta}_i p_i^{1-\epsilon}}, \tag{2.3.3}$$

$\bar{\eta}_i = (\eta_i/\eta_m)^\epsilon > 0$ for $i = a, s$. It is shown in the appendix that σ_i is the ratio of consumption expenditures between consumption goods and the manufacturing good. Equation (2.3.2) is the Euler equation. This equation equals the rate of return on investment in physical capital (r) and the increase of marginal utility from consuming an additional unit of the manufacturing good. Given the structure of preferences, the marginal utility from consuming manufacturing goods depends on marginal changes in consumption of agricultural goods and services, which are captured by the summation term in equation (2.3.2). As discussed by Alonso-Carrera, Caballe and Raurich (2015), this term depends on the growth rate of relative prices. Later, we show that the aggregate consumption expenditure is independent of relative prices effects, given the utility function in (2.2.10). Finally, as there is a representative household, the average stock of human capital obviously coincides with the economy-wide stock of human capital, i.e. $\bar{h} = h$.

The supply side in our model is characterized by perfectly competitive firms. Thus, firms maximize profits in each sector without taking into account the effect of average human capital. In the appendix, we solve the firm's problem. From this solution, we obtain that the capital shares are

$$s_i = \frac{u_i}{u_a + u_m + u_s}, \; i = a, m, s; \tag{2.3.4}$$

the relative prices of services and agricultural goods are

$$p_i = \left(\frac{A_m}{A_i}\right) h^{\psi_m - \psi_i}, \; i = a, s; \tag{2.3.5}$$

and the relative price of human capital is

$$p_h = (1-\alpha)\left(\frac{A_m}{B}\right)\left[\frac{k}{(u_a + u_m + u_s)\, h}\right]^{\alpha} h^{\psi_m}. \tag{2.3.6}$$

From (2.3.5), we obtain the growth rate of relative prices depends on the growth rate of human capital according to the following equation:

$$\frac{\dot{p}_i}{p_i} = \left(\psi_m - \psi_i\right)\frac{\dot{h}}{h}, \; i = a, s. \tag{2.3.7}$$

According to (2.3.7), the growth rate of relative prices of agriculture and services may be positive or negative. This result depends on the difference between the intensity of the externalities in these sectors and the manufacturing sector, and the sign of the growth rate of human capital. Empirical evidence shows that the price of agriculture goods decreases, whereas the price of services increases. Thus, we assume that $\psi_a > \psi_m$ and $\psi_m > \psi_s$ so that the time path of relative prices are consistent with the empirical evidence given a positive growth rate of human capital. Finally, the relative price of human capital is a function of the stock of human capital.

2.3.1 Aggregated economy and sectoral composition

In this section, we characterize the aggregate economy. To this end, we first define the aggregate output and the total consumption expenditure in our economy. Q stands for the gross output of our economy (henceforth, GDP). Given the relative prices (2.3.5) and (2.3.6), GDP is

$$Q = y + p_h y_h,$$

where $y = y_m + p_s y_s + p_a y_a$ is the sum of the value of output in the non-educational sectors, and $p_h y_h$ is the value, in units of manufacturing good, of the gross investment in human capital. By using (2.2.1), (2.3.4), and (2.3.5), we obtain

$$y = A_m k^{\alpha} \left(u_m + u_a + u_s\right)^{1-\alpha} h^{1-\alpha+\psi_m}. \tag{2.3.8}$$

Let c denote the total consumption expenditure, which is defined as follows

$$c = c_m + p_s c_s + p_a c_a. \tag{2.3.9}$$

Using (2.3.4) and the definitions in (2.3.8) and (2.3.9), in the Appendix A, we obtain the share of physical capital in agriculture, manufacturing, and services sectors are given by

$$s_m = 1 - \sigma \frac{c}{y}, \tag{2.3.10}$$

$$s_i = \sigma_i \frac{c}{y}, \tag{2.3.11}$$

where σ_i for $i = a, s$ are defined in (2.3.3), and the auxiliary variable σ is

$$\sigma = \sigma_a + \sigma_s.$$

In Appendix A, we show that substituting the marker clearing conditions, $y_a = c_a$ and $y_s = c_s$, in (2.3.9) and (2.3.11), the employment shares in agriculture and services sector are:

$$
\begin{aligned}
u_i &= \sigma_i \frac{c}{y} \chi, \; i = a, s; && (2.3.12)\\
u_m &= \chi \left(1 - \sigma \frac{c}{y} \right), && (2.3.13)
\end{aligned}
$$

where

$$\chi = \left(\frac{1-\alpha}{\alpha} \right) \frac{CIS}{LIS}. \tag{2.3.14}$$

where CIS and LIS stand for the capital and the labor income shares in the aggregate economy. In Appendix A, we show that χ depends on the ratio between human and physical capital and, therefore, it captures the effect of investment in both capitals on sectoral composition. In the Appendix A, we show that total consumption expenditure is

$$c = c_m \left(1 + \bar{\eta}_a p_a^{1-\epsilon} + \bar{\eta}_s p_s^{1-\epsilon} \right), \tag{2.3.15}$$

and taking log-derivatives, the growth rate of consumption expenditure is

$$\frac{\dot{c}}{c} = r - \delta - \rho. \tag{2.3.16}$$

(2.3.16) is the Euler condition that depends only on the interest rate. As explained in Ngai and Pissarides (2007), this result arises because we assume that the utility function is logarithmic in the consumption composite $\tilde{c}$, which implies an intertemporal elasticity of substitution equal to one. Thus, aggregate consumption expenditure is independent of changes in relative prices of agriculture and services. From the firm first-order conditions, we show, in the Appendix A, that $r = \alpha A_m k^{\alpha-1} h^{1-\alpha+\psi_m} (u_m + u_s + u_a)^{1-\alpha}$. The Euler equation can then be rewritten as

$$\frac{\dot{c}}{c} = \alpha A_m k^{\alpha-1} h^{1-\alpha+\psi_m} (u_m + u_s + u_a)^{1-\alpha} - \delta - \rho. \tag{2.3.17}$$

By using (2.2.1) and the shares of physical capital (2.3.10) and (2.3.12), we write the law of motion (2.2.5) as follows

$$\frac{\dot{k}}{k} = A_m k^{\alpha-1} h^{1-\alpha+\psi_m} \frac{u_m}{(u_m + u_s + u_a)^{\alpha}} - \frac{c_m}{k} - \delta. \tag{2.3.18}$$

In order to obtain a transformed system, we use the variable z, that is defined as

$$z = \frac{k}{u_m + u_s + u_a} h^{\frac{1-\alpha+\psi_m}{\alpha-1}}, \tag{2.3.19}$$

and we introduce one new transformed variable $q \equiv \frac{c}{k}$, which stands for the ratio between total consumption expenditure to aggregate physical capital. In the Appendix, we use the relation between total consumption expenditure (c) and consumption expenditure in manufacturing good (c_m) to obtain

$$c = \frac{c_m}{1-\sigma}. \tag{2.3.20}$$

In Appendix A, we show that (2.3.21) can be rewritten as a function of transformed variables z and q by using (2.3.19), (2.3.12) and $q = c/k$. Thus, the growth rate of physical capital is

$$\frac{\dot{k}}{k} = A_m z^{\alpha-1} - q - \delta. \tag{2.3.21}$$

Given the employment shares (2.3.12), we write the law of motion of human capital as follows

$$\frac{\dot{h}}{h} = B\left(1-\chi\right). \tag{2.3.22}$$

and using (2.3.19), Euler equation can be rewritten as

$$\frac{\dot{c}}{c} = \alpha A_m z^{\alpha-1} r - \delta - \rho. \tag{2.3.23}$$

Finally, we obtain the growth rate of the transformed variables z, χ, and q. By taking log-derivatives in $q = c/k$, and using (2.3.21) and (2.3.23), the growth rate of q is

$$\frac{\dot{q}}{q} = (\alpha-1)\, A_m z^{\alpha-1} + q - \rho. \tag{2.3.24}$$

In Appendix A, we show that the growth rate of z is given by

$$\frac{\dot{z}}{z} = A_m z^{\alpha-1} - \varpi\left(1-\chi\right) - \delta_z, \tag{2.3.25}$$

where $\varpi = \frac{B}{\alpha}\left(\frac{\psi_m}{1-\alpha}\right)$ and $\delta_z = \frac{B}{\alpha}$, and the growth rate of χ is given by

$$\frac{\dot{\chi}}{\chi} = \theta\left(1-\chi\right) - q + \delta_\chi, \tag{2.3.26}$$

where $\theta = \left(\frac{\psi_m-\alpha}{\alpha}\right) B$ and $\delta_\chi = \frac{B}{\alpha} - \delta$.

The dynamic equilibrium is a path $\{q, z, \chi\}$ such that, given the initial value of z_0 and χ_0, solve equations (2.3.24), (2.3.25), and (2.3.26) and satisfies the transversality conditions.

2.3.2 Structural change

Following Ngai and Pissarides (2007), we define structural change as the change in the allocation of employment shares. By taking-log-derivative of (2.3.12) and using (2.3.7), the growth rate of employment shares is

$$\frac{\dot{u}_s}{u_s} = (1-\epsilon)\frac{\dot{h}}{h}\left[(\psi_m-\psi_s)(1-\sigma_s)-(\psi_m-\psi_a)\sigma_a\right]+\frac{\dot{c/y}}{c/y}+\frac{\dot{\chi}}{\chi}, \quad (2.3.27)$$

$$\frac{\dot{u}_a}{u_a} = (1-\epsilon)\frac{\dot{h}}{h}\left[(\psi_m-\psi_a)(1-\sigma_a)-(\psi_m-\psi_s)\sigma_s\right]+\frac{\dot{c/y}}{c/y}+\frac{\dot{\chi}}{\chi}. \quad (2.3.28)$$

From equations (2.3.27) and (2.3.28), note that structural change arises from two channels.[6] The first channel, that corresponds to the first term in (2.3.27) and (2.3.28), shows the change in employment shares due to the bias of sectoral technical progress. In this case, changes in employment shares are induced by the changes in relative prices, which are captured by the variables σ_a and σ_s. Given the assumption on the strength of sectoral externality, that is $\psi_a > \psi_m$ and $\psi_m > \psi_s$, and assuming $\dot{c/y} = \dot{\chi} = 0$ and $\dot{h} > 0$, in the Appendix A, we show that, in the limit, the values of σ_a and σ_s are zero and one, respectively. This implies the growth rate of employment share in services converge to zero, on the one hand, and the growth rate of agriculture converge to a negative value in the long run, on the other hand. In this case, the employment share in agriculture is continuously decreasing, and labor is moving from this sector to manufacturing, services and educational sector. We call this effect as endogenous Baumol effect.

The second channel, the second and the third terms in (2.3.27) and (2.3.28), shows the change in employment shares due to the imbalance between physical and human capital. If the initial ratio between these two stocks of capital differs from its steady state value, individuals decide either to allocate more labor into education sector or allocate labor in the manufacturing sector in order to accumulate more physical capital. The first case takes place when human capital is relatively scarce in comparison to physical capital. In this case, the marginal return of human capital increases and more labor is allocated to the educational sector instead of the sector

[6] The growth rate of the employment share in manufacturing is easy computable by taking log-derivatives in (2.3.12).

which produces physical capital. On the other hand, when physical capital is relatively scarce in comparison to human capital, the second case arises. In this case (net) marginal return of physical capital increases and more labor is allocated to the manufacturing sector. These changes in marginal returns of physical and human capital induces changes labor and capital income shares. Furthermore, These changes in investment in physical and human capital change also affect endogenously the term c/y, which will decrease (increase) in the first (second) case. We refer to this channel as the imbalance effect on structural change.

We highlight that these channels affect structural change simultaneously. For instance, when human capital is relative scarce and χ is not on this equilibrium path, the imbalance effect fosters reallocation of labor across sector, but also, it fosters the growth rate of human capital, and, therefore, the Baumol effect on structural change. However, these possible chained effects are only effective along the transition toward the equilibrium path.

2.3.3 The equilibrium path

We next characterize the equilibrium path. We define a balanced growth path (BGP, henceforth) as an equilibrium path along which the efficiency units of capital z, q, and χ remain constant, and, therefore, the aggregate physical capital, total expenditure consumption and human capital grow at the same constant rate. In the numerical simulation there is a unique equilibrium path converging towards this BGP. The following propositions characterizes the BGP equilibrium.[7]

Proposition 2.1 *There is a unique BGP along which the BGP values of q, z, and χ are, respectively,*

$$\begin{aligned} q^* &= \frac{B\left(1-\alpha+\psi_m\right)-\psi_m\left(\delta+\rho\right)}{\alpha}+\rho, \\ z^* &= \left[\frac{\alpha\left(1-\alpha\right)A_m}{B\left(1-\alpha+\psi_m\right)-\left(\delta+\rho\right)\psi_m}\right]^{\frac{1}{1-\alpha}}, \end{aligned} \tag{2.3.29}$$

and

$$\chi^* = \frac{\delta+\rho}{B}.$$

Propositions (2.1) establish that the economy is along an equilibrium path where aggregate physical capital and the total consumption expenditure

[7] We provide all the proofs in the Appendix B.

grow at the same constant growth rate. In the Appendix B, we show that in this equilibrium path, the growth rate of aggregate physical capital (γ_k), and total expenditure consumption (γ_c) are given by

$$\gamma = \gamma_k = \gamma_c = \varphi\gamma_h = B - \delta - \rho, \quad (2.3.30)$$

where γ is the growth rate of GDP, γ_h is the growth rate of human capital, and φ is a factor of proportionality equal to $(1-\alpha)/(1-\alpha+\psi_m)$. These results imply that along this equilibrium path the interest rate in this economy is also constant.

Economic growth in developed economies is characterized by two prominent facts, namely, a constant capital-output ratio and changes in the relative sectoral shares in GDP as well as the sectoral composition of the aggregate labor force. These are the so-called Kaldor-Kuznets facts. Kongsamut et al (2001) define this equilibrium path as a Generalized Balanced Growth Path (henceforth, GBGP). In the following proposition, we define the necessary conditions for a GBGP to arise in our model.

Proposition 2.2 *If $\epsilon \neq 1$ and $\psi_m \neq \psi_a$ or $\psi_m \neq \psi_s$, then the equilibrium path is a GBGP, along which the interest rate remains constant and there is structural change.*

In the GBGP equilibrium, employment shares in agriculture, services and manufacturing are not constant, as follows from (2.3.27) and (2.3.28). When the transformed variables are in their BGP values, physical capital and total expenditure grow at the same rate as well as GDP and human capital grow at a constant rate. That implies that c/y and χ are constant. Thus, in the GBGP the imbalance effect on structural change is not affecting the sectoral composition. The only driving force behind structural change is the bias of sectoral technical progress. This result shows that in the framework of the Uzawa-Lucas model, structural change compatible with a GBGP arise if there exists externalities with sector-specific strength.

2.4 Numerical analysis

In this section, we analyze numerically the effects of human capital accumulation on structural change and their implications for development. To this end, we propose two numerical exercises. First, we use this model to analyze the accuracy of this endogenous mechanism in explaining observed patterns of structural change in the United States along the 20th century, a period of time characterized by a fast accumulation of human capital. That is, we analyze how important is the human capital accumulation to predict the levels of employment shares in agriculture, manufacturing and services. Second, we extend the numerical analysis to focus on the role of human capital accumulation in explaining cross-country income differences through differences in the structural change.

To these ends, we propose the following strategy. In a first step, we calibrate the model to explain the structural change of the U.S. economy in the period 1947-2005. We focus on this period because of the fraction of the non-working time that individuals allocate to education is robustly constant, namely around a 0.11, which is consistent with an economy on a BGP.[8] We then obtain the simulated time path of employment shares and calculate the accuracy of these predictions. In a second step, we use our calibrated model to simulate transitional dynamics by varying the initial values of physical and human capital. We choose the initial values of these state variables so that the employment share in the non-educational sector approaches its actual value at the beginning of the 20th century. Then, we analyze the accuracy of our model to encompass the process of human capital accumulation and the pace of structural change along this century. To this end, we use the simulated employment shares to fit actual data and obtain a set of accuracy measures of our simulations.

Finally, we use the model to analyze the role of human capital accumulation in explaining cross country differences in structural change. In particular, we investigate how differences in the initial ratio between physical and human capital can account for the differences in both the initial sectoral composition and in income. To answer this question, we simulated two economies that starts with the same level of GDP and they only differ in the initial stock of physical and human capital. In what follows, we present the calibration strategy.

2.4.1 Calibration

To calibrate the model, we assume that the post-war U.S. economy is in the BGP at 1947. Then, we set α to match the average income labor share for the

[8]Francis and Ramey (2009) shows that the fraction of available time devotes to school has been increasing since the early 20 century. However, after World War II, schooling time has been roughly constant.

period. According with data reported by the *Economic President Report* (2009 and 2012) the labor income share is about 0.65. The values of ρ, δ, ψ_m, and B are calculated to match the following targets: a 2% the growth rate of GDP, a 5.4% interest rate as reported by Prescott (2001), a 2.5 capital-output ratio as reported by Maddison (1995), and the fraction of labor allocated in the educational sector in 1947 is 0.11 (from *Historical Statistics of the United States, millennium edition)*.[9] The parameters η_s and η_a are set to match values of two ratios in 1947: the first one is the ratio between agriculture and manufacturing consumption expenditures, and the second, the ratio between services and manufacturing consumption expenditures. Both ratios are reported by Herrendorf (2013). We normalize the technology level in manufacturing, $A_m = 1$, and we set the values of A_s, and A_a to match the relative price level in 1947, according to the prices reported in NIPA. Finally, we set the values of the sector externalities ψ_s, and ψ_a to match the average growth rate of relative prices for the period 1947-2005, and we let ϵ take value of 0.5 as in Ngai and Pissarides (2008). Table 1 reports the values of the parameters.

[Insert Table 1]

2.4.2 GBGP: U.S. economy (1947-2005)

In this section, we show the main results of our numerical simulation. We simulate the U.S. economy by assuming that it is on a BGP since 1947. Figures 1 to 4 plot the results of our numerical simulation for the U.S economy in the period 1947 to 2005. Figure 1 plots the time path of the relative prices. Panel (a) plots the fall in the relative price of agriculture. The model captures the main trend in data despite the variation in the price of agriculture during the period. Panel (b) plots the price of services. Once again, the model replicates the main trend of data.

[Insert Figure 1]

By using these simulated relative prices, we simulate the rest of variables in our economy. In particular, we use these relative prices to simulated shares in total employment, shares in GDP, and share in total consumption expenditures of agriculture, manufacturing and services. Figure 2 shows the simulated employment shares. A first visual exploration of our results

[9] Share in total employment of educational sector account for all workers that are related to formation of human capital. This is professors, trainers, scientists.

shows that the model is able to replicate qualitatively the main trends in data and it almost matches initial values for the data series. In particular, the model explains the fall and the rise of agriculture and services, respectively. Nonetheless, the model is unable to replicate the characteristic hump-shape in employment share in manufacturing. This result shows the limitations of the relative price mechanism. Herrendorf (2013) notes that this price-mechanism proposed by Ngai and Pissarides (2008) may account for the hump-shape in both employment and nominal shares of manufacturing under certain parameter values. In this regard, we explore for which range of parameters the model can fit better to the data. As we show below, improvements in accuracy will depend on the value of the elasticity parameter ϵ.

[Insert Figure 2]

Table 2 reports two measures of accuracy to give us a notion of the quantitative fit of the model to the data. The first measure is the statistic R^2. By regressing actual data on simulated data and using OLS regression, we report the simple coefficient of correlation (Pearson coefficient) which exceeds 0.9. Because two-time series could be highly correlated even when both series are distant to each other, we report a complementary statistic that measures the accuracy of predicted to real data. The second statistic Theil's U-index indicates the degree of accuracy of prediction. The Theil coefficient is scale invariant and it lies between zero and one. If the Theil coefficient equals zero then we have a perfect fit. Based on these two indicators, we can say that calibrated externalities across the industries can account for the structural change in the U.S. economy.

[Insert Table 2]

Figure 3 plots both the time path for actual and simulated consumption and value added shares. Figure 3 notes that the model simulates the main trends in data and, particularly, the simulated share of GDP in services. Furthermore, the model accounts for the observed differences between shares of GDP in agriculture, manufacturing and services and the employment share in those sectors. By comparing Figure 2 and Figure 3, we note that the model explains the initial level difference between of the share of GDP. This value is around 52%. In the case of employment share in services, the simulated data is around 48%. This is a lower value than the simulated share of GDP in services, but it overestimated the actual employment share in 1947. The difference in both shares in GDP and employment of services is explained by the educational sector in the model.

[Insert Figure 3]

Finally, Figure 3 also shows the shares of total expenditure in agriculture, manufacturing and services. In this case, the model fails to match the initial values of the shares of agriculture and services in the total consumption expenditure, but match the initial value of manufacturing. Although the model does not quantitatively replicate the trend of the series, it is able to qualitatively replicate the differences among the shares of total consumption expenditure,GDP and employment in agriculture, manufacturing and services. These are important results from our model. Buera and Kaboski (2009) highlight that traditional theories of structural change are not available to replicate the behavior of these differences.

2.4.3 Transitional dynamics: U.S economy (1900-1947)

In this section, we use our calibrated model to analyze numerically the effect of the imbalance in the ratio between physical and human capital on the pace of structural change. With this purpose, we induce a transition process in our model by varying the initial values of physical and human capital from their steady-state values in the previous exercise. In particular, we chose the initial values of k and h so that the initial employment shares match their observed values in 1900. These values are 0.4767 and 0.1901, respectively. This implies that the initial value of the ratio between human and physical capital is below its steady-state values in 1947.

[Insert Figure 4]

Figure 4 plots the results for this simulation. In the first panel, Figure 4 plots the structural change along the transitional path. Simulated paths capture the main trend in data. For instance, given the initial values of physical and human capital, the model replicates the hump-shape in the share of employment in manufacturing.[10] Moreover, the simulated transition of employment shares in agriculture and services is near to actual value since 1900. In the second panel, Figure 4 plots the actual and simulated paths

[10]Obviuously, the model is not able to account for decline in manufacturing in 1920-1921, when the industrial production falls around 30% (O'Brien, 1997).

of time available to schooling. In this case, simulated time for schooling increases at higher rate than actual data, but it captures the increasing trend before 1947. We then conclude that human capital accumulation may account for a part of structural change along the twenty century in the U.S. economy.

2.4.4 Development patterns

In this numerical exercise, we analyze the role of imbalances in the ratio between human and physical capital in explaining the differences in the structural change. Differences in human capital have been stressed as important factors to explain income divergence across countries in the literature of economic growth. Here, we propose to extend our framework to analyze how initial relative human capital endowment affects the pace of structural change.

To this end, we simulated two economies, economies A and B. These economies differ only in the initial values of physical and human capital and, therefore, that are identical in all fundamentals. We assume that, in Country A, the initial stocks of physical and human capital are arbitrarily set equal to 0.85 and 0.378, respectively. Meanwhile, we assume that in Country B the initial stocks are set equal to 0.93785 and 0.36, respectively. These values have been chosen so that both countries have the same GDP level at $t = 0$. Then, we calculate the ratio of physical to human capital across both countries to determinate the distance from their steady-state values. Thus, Country A has a ratio of 2.2487, which is a 26% lower than its steady-state value, whereas Country B's ratio is -2.605, which is a 14% lower than its steady-state value. Comparing the ratios between countries, we note that, in Country B, physical capital is more abundant than in Country A. Given these initial values, Figure 5 plots the simulated time paths of GDP (levels and growth rates), and the structural change that arises in both economies.

[Insert Figure 5]

Many interesting results emerge from this numerical exercise. Firstly, along the development process, there is divergence in GDP levels across countries (see Figure 5, panel a). Note that Country A grows faster than Country B. The reason for this result is that, in Country A, the imbalance between physical and human capital is lower than its stationary values. In both countries, more resources (labor) are devoted to the education sector, the factor relative scarce. This implies an increase of the growth rate of human capital and, consequently, the growth rate of GDP. However, the magnitude of the increase in growth rate of GDP differs between countries. This cross-country difference in GDP growth lies in the size of the imbalance across countries. These differential in growth rates are plotted by Figure 5, panel b. As the economy accumulates human capital, imbalances converge to its steady-state values, and their effects on GDP growth rate will tend to vanish. Although our model also predicts that countries will converge in growth rates in the long-run but with permanent differences in GDP levels as in Lucas (1988).

Secondly, there is cross-country difference in the pace of industrialization process. Figure 5 panels c and d plots the structural change in both countries along the transitional dynamics. It shows the employment shares in agriculture, manufacturing and services (where we include the educational sector). As can be seen with the naked eye, both countries show a different sectoral composition at the beginning of the development process. In particular, Country A starts with a larger agriculture sector compared to the share in agriculture in Country B. In this case, we say that Country B is relative more industrialized that Country A. However, the pace of structural change in Country A is higher than in Country B. Because the imbalance in Country A is larger, the large is the amount of labor allocated to accumulate human capital. The higher amount of labor allocates in the educational sector yields an increase in the rate of human capital accumulation. This effect fosters the change in relative prices and, consequently, it fosters structural change. Thus, Country A shows a more remarkable structural change than Country B along the transition toward their respectively steady states. Furthermore, in the hypothetical case that both countries begin on a BGP, both economies would show structural change due to the accumulation of human capital, but in the long run both converge to the same sectoral composition, where the services sector is the dominant sector. These results show that human capital is an important factor explaining not only observed structural along the development process in a country, but also cross-country differences in sectoral composition.

2.5 Concluding remarks

In this paper, we present a multi-sectorial growth model which allows for changes in sectoral composition and human capital accumulation along the balanced growth path. As occurs in multi-sector growth models based on the assumption of biased technical change, the structural change along the equilibrium path is driven by the changes of relative prices. A novelty in our proposal is that the growth of relative prices is endogenous and we show that the imbalance in the ratio between physical and human capital is a mechanism for structural change. By assuming that sectoral technical progress depends on the stock of human capital, the model is capable to replicate not only the Kaldor-Kuznets facts, but also replicates the differences among the employment and sectoral value added shares, as well as share in consumption expenditure, which other models are not able to explain. Buera and Kaboski (2009) highlight that traditional theories of structural change are not available to replicate the behavior of these differences. They argue that models that incorporate, for instance, sector-specific factor distortions can contribute to amend the standard models. We contribute to the literature on structural change by showing that these differences can be explained without sector specific factor distortions.

Moreover, we analyze the effect of human capital accumulation on structural change in the transition path. We found that the imbalance between physical and human capital can account for the observed structural change in the U.S. economy along the 20th century. In this regard, we explore how this imbalance may provide an explanation for the heterogeneity in the sectoral composition across countries. Based on our numerical examples, we conjecture that differences in sectoral composition may arise due to differences in endowments of human capital across countries.

We interpret this result as a suggestion to reconsider the role of human capital in explaining structural transformation in the spirit of the literature of development. According to this literature, human capital accumulation is a significant factor behind the industrialization. It fosters sectoral technological progress through technology adoption or development of new technologies that accelerate the process of reallocation of labor across sectors (see Nelson and Phelps, 1966; Temple and Voth, 1998; Quamrul and Galor, 2011). In this regard, a natural extension of our paper is to investigate if under these conditions both sectoral technological adoption, R&D and human capital can explain the Kaldor-Kuznets facts.

Bibliography

[1] Acemoglu, Daron and Guerrieri, Veronica. "Capital Deepening and Nonbalanced Economic Growth." Journal of Political Economy, 2008, 116(3), pp. 467-98.

[2] Alonso-Carrera, Jaime; Caballé, Jordi and Raurich, Xavier. "Sectoral Composition and Macroeconomic Dynamics." 2011.

[3] Alvarez-Cuadrado, Francisco and Poschke, Markus. "Structural Change out of Agriculture: Labor Push Versus Labor Pull." American Economic Journal: Macroeconomics, 2011, 3(3), pp. 127-58.

[4] Baumol, William J. "Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis." The American Economic Review, 1967, 57(3), pp. 415-26.

[5] Bond, Eric W. "A General Two-Sector Model of Endogenous Growth with Human and Physical Capital: Balanced Growth and Transitional Dynamics." Journal of economic theory, 1996, 68(1), pp. 149-73.

[6] Boppart, Timo. "Structural Change and the Kaldor Facts in a Growth Model with Relative Price Effects and Non-Gorman Preferences," Society for Economic Dynamics, 2013.

[7] Buera, Francisco J. and Kaboski, Joseph P. "Can Traditional Theories of Structural Change Fit the Data?" Journal of the European Economic Association, 2009, 7(2/3), pp. 469-77.

[8] Caballe, Jordi and Santos, Manuel S. "On Endogenous Growth with Physical and Human Capital." Journal of Political Economy, 1993, 101(6), pp. 1042-67.

[9] Carter, Susan B. and Cambridge University, Press. "Historical Statistics of the United States," Cambridge [England]: Cambridge University Press, 2006.

[10] Dennis, Benjamin N. and Ä°ÅŸcan, Talan B. "Engel Versus Baumol: Accounting for Structural Change Using Two Centuries of U.S. Data." Explorations in Economic History, 2009, 46(2), pp. 186-202.

[11] Dolores Guilló, María; Papageorgiou, Chris and Perez-Sebastian, Fidel. "A Unified Theory of Structural Change." Journal of Economic Dynamics and Control, 2011, 35(9), pp. 1393-404.

[12] Echevarria, Cristina. "Changes in Sectoral Composition Associated with Economic Growth." International economic review, 1997, 38(2), pp. 431-52.

[13] Foellmi, Reto and Zweimüller, Josef. "Structural Change, Engel's Consumption Cycles and Kaldor's Facts of Economic Growth." Journal of Monetary Economics, 2008, 55(7), pp. 1317-28.

[14] Francis, Neville and Ramey, Valerie A. "A Century of Work and Leisure." American economic journal, 2009, 1(2), pp. 189-224.

[15] Galor, Oded. "Inequality, Human Capital Formation and the Process of Development," Institute for the Study of Labor (IZA), 2012.

[16] Galor, Oded and Moav, Omer. "Inequality in Landownership, the Emergence of Human-Capital Promoting Institutions, and the Great Divergence." The review of economic studies, 2009, 76(1), pp. 143-79.

[17] Galor, Oded and Weil, David N. "Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond." The American Economic Review, 2000, 90(4), pp. 806-28.

[18] Gupta, Manash Ranjan and Chakraborty, Bidisha. "Uniqueness and Indeterminacy of the Equilibrium Growth Path in the Uzawa-Lucas Model with Sector Specific Externalities." Japanese Economic Review, 2007, 58(3), pp. 400-06.

[19] Herrendorf, Berthold and Rogerson, Richard. "Two Perspectives on Preferences and Structural Transformation." The American Economic Review, 2013, 103, pp. 2752-89.

[20] Herrendorf, Berthold and Valentinyi, Ãkos. "Measuring Factor Income Shares at the Sectoral Level." Review of economic dynamics, 2008, 11, pp. 820-35.

[21] Herrendorf, Berthold; Rogerson, Richard; and Valentinyi, Ákos. "Growth and Structural Transformation," P. Aghion, Durlauf, Steven, Handbook of Economic Growth. Amsterdam and NewYork: North Holland,, 2014, 855-941.

[22] Kongsamut, Piyabha; Rebelo, Sergio and Xie, Danyang. "Beyond Balanced Growth." The review of economic studies, 2001, 68(4), pp. 869-82.

[23] Kuznets, Simon. "Modern Economic Growth: Findings and Reflections." The American Economic Review, 1973, 63(3), pp. 247-58.

[24] Lucas, Robert E. "On the Mechanics of Economic Development." Journal of Monetary Economics, 1988, 22(1), pp. 3-42.

[25] Maddison, Angus. Monitoring the World Economy, 1820-1992. Washington, D.C: Development Centre of the Organisation for Economic Co-operation and Development, 1995.

[26] Mulligan, Casey B. "Transitional Dynamics in Two-Sector Models of Endogenous Growth." The Quarterly Journal of Economics, 1993, 108(3), pp. 739-73.

[27] Nelson, Richard R. and Phelps, Edmund S. "Investment in Humans, Technological Diffusion, and Economic Growth." The American Economic Review, 1966, 56(1/2), pp. 69-75.

[28] O'Brien, Anthony Patrick. "Depression of 1920-1921," T. F. C. David Glasner, Business Cycles and Depressions: An Encyclopedia. New York: Garland Publishing, 1997, 151-53.

[29] Pissarides, Christopher A. and Ngai, L. Rachel. "Structural Change in a Multisector Model of Growth." The American Economic Review, 2007, 97, pp. 429-43.

[30] Ramey, Valerie A., and Neville Francis. 2009. "A Century of Work and Leisure." American Economic Journal: Macroeconomics, 1(2): 189-224.

[31] Robert, E. Lucas, Jr. "The Industrial Revolution: Past and Future." The Region, 2004, 18, pp. 4.

[32] Romer, Paul M. and Jones, Charles I. "The New Kaldor Facts: Ideas, Institutions, Population, and Human Capital." American Economic Journal: Macroeconomics, 2010, 2(1), pp. 224-45.

[33] Tamura, Robert. "Human Capital and Economic Development." Journal of development economics, 2006, 79, pp. 26-72.

[34] ____. "Human Capital and the Switch from Agriculture to Industry." Journal of economic dynamics & control, 2002, 27(2), pp. 207-42.

[35] Temple, Jonathan and Voth, Hans-Joachim. "Human Capital, Equipment Investment, and Industrialization." European Economic Review, 1998, 42(7), pp. 1343-62.

[36] United, States; President and Council of Economic, Advisers. "Economic Report of the President: Transmitted to the Congress February 2011, Together with the Annual Report of the Council of Economic Advisers," Washington[, D.C.]: U.S. G.P.O, 2011.

[37] Vandenbroucke, Guillaume and Restuccia, Diego. "A Century of Human Capital and Hours." Economic inquiry, 2013, 51(3), pp. 1849-66.

Appendix

2.A Derivation of the main equations

Solution to the representative consumer optimization problem.
The Hamiltonian function associated with the maximization of (2.2.10) subject to (2.2.3), $y_a = c_a,\ y_s = c_s$, and (2.2.5) is

$$\mathscr{H} = \ln \tilde{c} + \mu_1 \left(wh + rk - c_m - p_s c_s - p_a c_a - I_k - p_h I_h \right) + \mu_2 \left(I_k - \delta k \right) + \mu_h \left(I_h - \delta h \right)$$

where μ_1, μ_2 and μ_h are the co-state variables corresponding to the constraints (2.2.7), (2.2.8), and (2.2.9), respectively. The first order conditions are

$$\begin{aligned}
\eta_a \tilde{c}^{\frac{1-\epsilon}{\epsilon}} c_a^{-\frac{1}{\epsilon}} &= p_a \mu_1, & (2.A.1)\\
\eta_s \tilde{c}^{\frac{1-\epsilon}{\epsilon}} c_s^{-\frac{1}{\epsilon}} &= p_s \mu_1, & (2.A.2)\\
\eta_m \tilde{c}^{\frac{1-\epsilon}{\epsilon}} c_m^{-\frac{1}{\epsilon}} &= \mu_1, & (2.A.3)\\
\mu_1 &= \mu_2, & (2.A.4)\\
p_h \mu_1 &= \mu_h, & (2.A.5)\\
r\mu_1 - \delta \mu_2 &= -\dot{\mu}_2 + \rho \mu_2, & (2.A.6)\\
w\mu_1 - \delta \mu_h &= -\dot{\mu}_h + \rho \mu_h. & (2.A.7)
\end{aligned}$$

Euler's equation.
To obtain the Euler equation in the main text, we first combine (2.A.1), (2.A.3), (2.A.2) and (2.A.3) to obtain

$$\begin{aligned}
c_a &= \left(\frac{\eta_a}{\eta_m}\right)^{\epsilon} \frac{c_m}{p_a^{\epsilon}}, & (2.A.8)\\
c_s &= \left(\frac{\eta_s}{\eta_m}\right)^{\epsilon} \frac{c_m}{p_s^{\epsilon}}.
\end{aligned}$$

By substituting c_a and c_s in (2.2.11) we obtain that

$$\tilde{c} = \left[p_a^{1-\epsilon} \left(\frac{\eta_a}{\eta_m} \right)^{\epsilon} + 1 + p_s^{1-\epsilon} \left(\frac{\eta_s}{\eta_m} \right)^{\epsilon} \right]^{\frac{\epsilon}{\epsilon-1}} \eta_m^{\frac{\epsilon}{\epsilon-1}} c_m, \tag{2.A.9}$$

and substituting (2.A.9) in (2.A.3), we obtain

$$\frac{1}{c_m} = \mu_1 \left[1 + p_a^{1-\epsilon} \left(\frac{\eta_a}{\eta_m} \right)^{\epsilon} + p_s^{1-\epsilon} \left(\frac{\eta_s}{\eta_m} \right)^{\epsilon} \right]. \tag{2.A.10}$$

and taking log-derivatives in (2.A.10), we obtain that

$$\frac{\dot{c}_m}{c_m} = -\frac{\dot{\mu}_1}{\mu_1} - (1-\epsilon) \left(\sigma_a \frac{\dot{p}_a}{p_a} + \sigma_s \frac{\dot{p}_s}{p_s} \right).$$

where $\bar{\eta}_a = \left(\frac{\eta_a}{\eta_m} \right)^{\epsilon}$, $\bar{\eta}_s = \left(\frac{\eta_s}{\eta_m} \right)^{\epsilon}$,

$$\begin{aligned} \sigma_a &= \frac{\bar{\eta}_a p_a^{1-\epsilon}}{1 + \bar{\eta}_a p_a^{1-\epsilon} + \bar{\eta}_s p_s^{1-\epsilon}}, \\ \sigma_s &= \frac{\bar{\eta}_s p_s^{1-\epsilon}}{1 + \bar{\eta}_a p_a^{1-\epsilon} + \bar{\eta}_s p_s^{1-\epsilon}}. \end{aligned} \tag{2.A.11}$$

Finally, we substitute (2.A.4) in (2.A.6)

$$r - \rho - \delta = -\frac{\dot{\mu}_2}{\mu_2},$$

and from (2.A.4) implies that

$$-\frac{\dot{\mu}_1}{\mu_1} = -\frac{\dot{\mu}_2}{\mu_2} = r - \rho - \delta. \tag{2.A.12}$$

By substituting (2.A.12) in (2.A.10), we obtain

$$\frac{\dot{c}_m}{c_m} = r - \rho - \delta - (1-\epsilon) \left(\sigma_a \frac{\dot{p}_a}{p_a} + \sigma_s \frac{\dot{p}_s}{p_s} \right) \tag{2.A.13}$$

which is the equation (2.3.2) in the main text.

The sectoral allocation of capital and relative prices
Competitive firms in the production sector, namely agriculture, manufacturing and services, maximize profits by choosing labor and capital. The maximization problem is

$$\max \pi_i = p_i y_i - r(s_i k) - w(u_i h)$$

where

$$y_i = A_i (s_i k)^{\alpha} (u_i h)^{1-\alpha} \bar{h}^{\psi_i}.$$

Then the first order conditions are

$$\begin{aligned} p_a (1-\alpha) \frac{A_a (s_a k)^{\alpha} (u_a h)^{1-\alpha} \bar{h}^{\psi_a}}{u_a h} &= w, \quad (2.A.14) \\ p_a \alpha \frac{A_a (s_a k)^{\alpha} (u_a h)^{1-\alpha} \bar{h}^{\psi_a}}{s_a k} &= r, \quad (2.A.15) \end{aligned}$$

$$\begin{aligned} p_s (1-\alpha) \frac{A_s (s_s k)^{\alpha} (u_s h)^{1-\alpha} \bar{h}^{\psi_s}}{u_s h} &= w, \quad (2.A.16) \\ p_s \alpha \frac{A_s (s_s k)^{\alpha} (u_s h)^{1-\alpha} \bar{h}^{\psi_s}}{s_s k} &= r, \quad (2.A.17) \end{aligned}$$

$$\begin{aligned} (1-\alpha) \frac{A_m (s_m k)^{\alpha} (u_m h)^{1-\alpha} \bar{h}^{\psi_m}}{u_m h} &= w, \quad (2.A.18) \\ \alpha \frac{A_m (s_m k)^{\alpha} (u_m h)^{1-\alpha} \bar{h}^{\psi_m}}{s_m k} &= r, \quad (2.A.19) \end{aligned}$$

where p_a and p_s are the relative prices of agriculture and services sectors. Finally, firms in the educational sector, choose labor until

$$p_h B = w \quad (2.A.20)$$

where p_h is the relative price of human capital in units of manufacturing good. From combining $(2.A.14)$, $(2.A.16)$ and $(2.A.18)$, we obtain

$$\begin{aligned} p_a &= \frac{A_m}{A_a} \left(\frac{s_m u_a}{s_a u_m} \right)^{\alpha} \bar{h}^{\psi_m - \psi_a}, \quad (2.A.21) \\ p_s &= \frac{A_m}{A_s} \left(\frac{s_m u_s}{s_s u_m} \right)^{\alpha} \bar{h}^{\psi_m - \psi_s}, \quad (2.A.22) \end{aligned}$$

and from equations $(2.A.15)$, $(2.A.17)$ and $(2.A.19)$ we obtain that

$$p_a = \frac{A_m}{A_a} \frac{(s_a u_m)^{1-\alpha}}{(s_m u_a)^{1-\alpha}} \bar{h}^{\psi_m - \psi_a}, \quad (2.A.23)$$

$$p_s = \frac{A_m}{A_s}\frac{(s_s u_m)^{1-\alpha}}{(s_m u_s)^{1-\alpha}}\bar{h}^{\psi_m-\psi_s}. \tag{2.A.24}$$

From (2.*A*.21), (2.*A*.22), (2.*A*.23), and (2.*A*.24) we obtain that labor and capital shares must satisfy that

$$s_a u_m = s_m u_a, \tag{2.A.25}$$
$$s_s u_m = s_m u_s. \tag{2.A.26}$$

From (2.2.3) we obtain that capital shares are given by

$$\begin{aligned} s_s &= \frac{u_s}{u_m+u_s+u_a}, \\ s_a &= \frac{u_a}{u_m+u_s+u_a}, \\ s_m &= \frac{u_m}{u_m+u_s+u_a}. \end{aligned} \tag{2.A.27}$$

which is the equation (2.3.4) in the main text. The relative prices of services and agriculture good is obtained by substituting (2.A.27) in (2.A.23) and (2.A.24) and taking into account (2.2.3). Thus, relative prices are

$$p_a = \frac{A_m}{A_a}\bar{h}^{\psi_m-\psi_a},$$

and

$$p_s = \frac{A_m}{A_s}\bar{h}^{\psi_m-\psi_s},$$

which is the equation (2.3.5) in the main text. We obtain the relative price of human capital by susbtituting equation (2.*A*.14) in (2.*A*.20), and taking into account the capital shares in (2.*A*.27) to obtain that

$$p_h = (1-\alpha)\frac{A_m}{B}\left(\frac{1}{u_m+u_s+u_a}\frac{k}{h}\right)^{\alpha}\bar{h}^{\psi_m},$$

which is the equation (2.3.6) in the main text.

Aggregate economy

To characterize the aggregate economy, we first obtain aggregate output and the total consumption expenditure in our economy. Let Q stands for the gross output of our economy (henceforth, GDP). Then, GDP is

$$Q = y + p_h y_h.$$

where $y = y_m + p_s y_s + p_a y_a$ is the sum of the value of output in the non-educational sectors. Given the relative prices (2.3.5) and sectoral production function (2.2.1), together the efficient capital allocation (2.A.27), and the fact that all workers in the economy are identical, $\bar{h}$ is just h, we obtain the value of output in the non-educational sectors is

$$y = A_m k^{\alpha} \left(u_m + u_a + u_s\right)^{1-\alpha} h^{1-\alpha+\psi_m}. \tag{2.A.28}$$

Total consumption expenditure is defined as follows

$$c = c_m \left(1 + \bar{\eta}_a p_a^{1-\epsilon} + \bar{\eta}_s p_s^{1-\epsilon}\right). \tag{2.A.29}$$

The sectoral allocation of labor

We use the ratio between (2.A.28) and (2.A.29) to characterize the employment shares in this economy. This ratio is equal to

$$\frac{c}{y} = \frac{c_m \left(1 + \bar{\eta}_a p_a^{1-\epsilon} + \bar{\eta}_s p_s^{1-\epsilon}\right)}{A_m k^{\alpha} \left(u_m + u_s + u_a\right)^{1-\alpha} h^{1-\alpha+\psi_m}}, \tag{2.A.30}$$

we take into account that the constraints $p_a c_a = p_a y_a$ and $p_s c_s = p_s y_s$. Using (2.A.8) relative prices (2.3.5), sectoral production function in (2.2.1), together with the efficient capital allocation (2.A.27), we obtain

$$\frac{\bar{\eta}_a p_a^{1-\epsilon} c_m}{A_m k^{\alpha} h^{1-\alpha+\psi_m} \left(u_m + u_s + u_a\right)^{1-\alpha}} = \frac{u_a}{u_m + u_s + u_a}. \tag{2.A.31}$$

$$\frac{\bar{\eta}_s p_s^{1-\epsilon} c_m}{A_m k^{\alpha} h^{1-\alpha+\psi_m} \left(u_m + u_s + u_a\right)^{1-\alpha}} = \frac{u_s}{u_m + u_s + u_a}. \tag{2.A.32}$$

Now, we use (2.A.30) to solve for c_m as follows

$$c_m = \frac{A_m k^{\alpha} \left(u_m + u_s + u_a\right)^{1-\alpha} h^{1-\alpha+\psi_m}}{\left(1 + \bar{\eta}_a p_a^{1-\epsilon} + \bar{\eta}_s p_s^{1-\epsilon}\right)} \frac{c}{y},$$

and substituting in (2.A.31) and (2.A.32), we obtain

$$\frac{u_a}{u_m + u_s + u_a} = \sigma_a \frac{c}{y}, \tag{2.A.33}$$

$$\frac{u_s}{u_m + u_s + u_a} = \sigma_s \frac{c}{y}. \tag{2.A.34}$$

By substituting (2.A.33) and (2.A.34) in (2.A.27), we find the physical capital share in agriculture as a function of relative prices,

$$s_a = \sigma_a \frac{c}{y}, \text{ and } s_s = \sigma_s \frac{c}{y},$$

which are equations (2.3.11) in the main text. To find the share of physical capital in manufacturing sector, equation (2.3.10) in the main text, we substitute the share in agriculture and service in the constraint $s_m = 1 - s_a - s_s$. Thus,

$$s_m = 1 - \sigma \frac{c}{y},$$

where $\sigma = \sigma_a + \sigma_s$. Finally, we find the employment shares in agriculture, manufacturing and services. We use equation (2.3.6) to find manufacturing employment share as a function of the employment share in agriculture and services as follows. First, we obtain the following expression from (2.3.6):

$$(u_a + u_m + u_s)^{\alpha} = (1-\alpha) \frac{A_m}{B} \frac{1}{p_h} (k)^{\alpha-1} h^{1-\alpha+\psi_m} \left(\frac{k}{h} \right), \tag{2.A.35}$$

Second, we substitute the stationary variable z, defined in (2.3.19), in (2.A.35) to obtain the following expression

$$(u_a + u_m + u_s) = (1-\alpha) \frac{A_m}{B} \frac{z^{\alpha-1}}{p_h} \left(\frac{k}{h} \right) \equiv \chi. \tag{2.A.36}$$

We substitute (2.A.36) in (2.A.33), (2.A.34) and (2.A.36) to obtain the employment shares

$$\begin{aligned} u_a &= \sigma_a \frac{c}{y} \chi, \\ u_s &= \sigma_s \frac{c}{y} \chi, \\ u_m &= \chi \left(1 - \sigma \frac{c}{y} \right), \end{aligned}$$

where

$$\chi \equiv (1-\alpha) \frac{A_m}{B} \frac{z^{\alpha-1}}{p_h} \left(\frac{k}{h} \right) = \frac{(1-\alpha)}{\alpha} \frac{rk}{wh}.$$

Substituting (2.3.5) and (2.A.27) in (2.A.15), we obtain that

$$r = \alpha A_m \left(\frac{k}{u_m + u_s + u_a} \right)^{\alpha-1} h^{1-\alpha+\psi_m}$$

and using (2.3.19), we obtain

$$r = \alpha A_m z^{\alpha - 1}, \tag{2.A.37}$$

and from (2.A.20), we obtain that

$$w = p_h B. \tag{2.A.38}$$

Differential equations

Total consumption expenditure is obtained by substituting (2.A.8) in (2.3.9). Thus, we obtain

$$c = c_m \left(1 + \bar{\eta}_a p_a^{1-\epsilon} + \bar{\eta}_s p_s^{1-\epsilon}\right),$$

which is equation (2.3.15) in the main text. Using (2.A.11), we can write total consumption expenditure as follows

$$c = \frac{c_m}{\frac{1}{\left(1+\bar{\eta}_a p_a^{1-\epsilon}+\bar{\eta}_s p_s^{1-\epsilon}\right)}} = \frac{c_m}{1-(\sigma_a+\sigma_s)},$$

and, using $\sigma_a + \sigma_s = \sigma$, we obtain

$$c = \frac{c_m}{1-\sigma}.$$

By taking log-derivatives in (2.A.29), we obtain

$$\frac{\dot{c}}{c} = \frac{\dot{c}_m}{c_m} + \frac{1}{\left(1+\bar{\eta}_a p_a^{1-\epsilon}+\bar{\eta}_s p_s^{1-\epsilon}\right)} \left[(1-\epsilon)\bar{\eta}_a p_a^{1-\epsilon}\frac{\dot{p}_a}{p_a} + (1-\epsilon)\bar{\eta}_s p_s^{1-\epsilon}\right]$$

and using (2.A.13) and definitions in (2.A.11), we obtain

$$\begin{aligned}
\frac{\dot{c}}{c} &= r-\rho-\delta-(1-\epsilon)\left(\sigma_a\frac{\dot{p}_a}{p_a}+\sigma_s\frac{\dot{p}_s}{p_s}\right)+(1-\epsilon)\left(\sigma_a\frac{\dot{p}_a}{p_a}+\sigma_s\frac{\dot{p}_s}{p_s}\right)\\
\frac{\dot{c}}{c} &= r-\delta-\rho.\\
\frac{\dot{c}}{c} &= \alpha A_m z^{\alpha-1}-\delta-\rho.
\end{aligned}$$

The growth rate of physical capital is obtained as follows. From definition of z, we obtain that $k^{\alpha-1} = \frac{z^{\alpha-1}(u_m+u_s+u_a)^{\alpha-1}}{h^{1-\alpha+\psi_m}}$. By substituting this results in $\dot{k}/k$, we obtain

$$\frac{\dot{k}}{k} = A_m z^{\alpha-1}\frac{u_m}{(u_m+u_s+u_a)} - \frac{c_m}{k} - \delta.$$

By substituting $u_m = \chi\left(1-\sigma\frac{c}{y}\right)$, $(u_a+u_m+u_s) = \chi$, and $c(1-\sigma) = c_m$, and $y/k = A_m z^{\alpha-1}$equation above can be rewritten as

$$\begin{aligned}
\frac{\dot{k}}{k} &= A_m z^{\alpha-1}\left(1-\sigma\frac{c}{y}\right)-(1-\sigma)\frac{c}{y}\frac{y}{k}-\delta,\\
&= A_m z^{\alpha-1}-[\sigma+(1-\sigma)]\frac{c}{y}A_m z^{\alpha-1}-\delta,\\
&= A_m z^{\alpha-1}-\frac{c}{k}-\delta,
\end{aligned}$$

and using $q = c/k$, we obtain equation in (2.3.21).

The growth rate of relative price of human capital is obtained as follows. We combine (2.A.4), (2.A.5), (2.A.6), and (2.A.7) to obtain:

$$\frac{\dot{p}_h}{p_h} = r - \frac{w}{p_h},$$

and using (2.A.37) and (2.A.38), we obtain that

$$\frac{\dot{p}_h}{p_h} = \alpha A_m z^{\alpha-1} - B. \tag{2.A.39}$$

Then, we obtain the growth rate of z by substituting (2.A.36) in (2.3.19), we obtain that

$$z^{\alpha} = \frac{B}{(1-\alpha)\, A_m} \frac{p_h}{h^{\frac{\psi_m}{1-\alpha}}},$$

and taking log-derivatives, we obtain that

$$\frac{\dot{z}}{z} = \frac{1}{\alpha}\left(\frac{\dot{p}_h}{p_h} - \frac{\psi_m}{1-\alpha}\frac{\dot{h}}{h}\right),$$

and using (2.3.22) and (2.A.39), we can rewrite $\dot{z}/z$ as follows

$$\frac{\dot{z}}{z} = A_m z^{\alpha-1} - \varpi\left(1-\chi\right) - \delta_z,$$

where $\varpi = \left(\frac{B}{\alpha}\right)\frac{\psi_m}{1-\alpha}$ and $\delta_z = \frac{B}{\alpha}$.

We obtain the growth rate of χ by taking log-derivatives in (2.A.36), to obtain

$$\frac{\dot{\chi}}{\chi} = (\alpha - 1)\frac{\dot{z}}{z} - \frac{\dot{p}_h}{p_h} + \frac{\dot{k}}{k} - \frac{\dot{h}}{h},$$

and substituting (2.3.21), (2.3.22), (2.3.25) and (2.A.39), and after some algebra, we obtain

$$\frac{\dot{\chi}}{\chi} = \theta\left(1-\chi\right) - q + \delta_{\chi},$$

where $\theta = \left(\frac{\psi_m - \alpha}{\alpha}\right) B$ and $\delta_{\chi} = \frac{B}{\alpha} - \delta$.

Finally, the growth rate of employment shares is obtain by taking log-derivatives in (2.3.12)

$$\frac{\dot{u}_a}{u_a} = \frac{\dot{\sigma}_a}{\sigma_a} + \frac{\dot{\overline{c/y}}}{c/y} + \frac{\dot{\chi}}{\chi},$$

and

$$\frac{\dot{u}_s}{u_s} = \frac{\dot{\sigma}_s}{\sigma_s} + \frac{\dot{\overline{c/y}}}{c/y} + \frac{\dot{\chi}}{\chi}.$$

By taking log-derivatives in (2.3.3), for the agriculture sector, we obtain

$$\frac{\dot{\sigma}_a}{\sigma_a} = (1-\epsilon)\frac{\dot{p}_a}{p_a} - \frac{(1-\epsilon)}{\left(1+\bar{\eta}_a p_a^{1-\epsilon}+\bar{\eta}_s p_s^{1-\epsilon}\right)}\left[\bar{\eta}_a p_a^{1-\epsilon}\frac{\dot{p}_a}{p_a}+\bar{\eta}_s p_s^{1-\epsilon}\frac{\dot{p}_s}{p_s}\right],$$

which can be rewritten using (2.3.7) and (2.3.3) to obtain

$$\frac{\dot{\sigma}_a}{\sigma_a} = \left[\left(\psi_m-\psi_a\right)(1-\sigma_a)-\left(\psi_m-\psi_s\right)\sigma_s\right](1-\epsilon)\frac{\dot{h}}{h},$$

and substituting in $\dot{u}_a/u_a$, we obtain the growth rate of employment share in agriculture reported in main text. By taking log-derivatives in (2.3.3), for the agriculture sector, we obtain

$$\frac{\dot{\sigma}_s}{\sigma_s} = (1-\epsilon)\frac{\dot{p}_s}{p_s} - \frac{(1-\epsilon)}{\left(1+\bar{\eta}_a p_a^{1-\epsilon}+\bar{\eta}_s p_s^{1-\epsilon}\right)}\left[\bar{\eta}_a p_a^{1-\epsilon}\frac{\dot{p}_a}{p_a}+\bar{\eta}_s p_s^{1-\epsilon}\frac{\dot{p}_s}{p_s}\right],$$

which can be rewritten using (2.3.7) and (2.3.3) to obtain

$$\frac{\dot{\sigma}_s}{\sigma_s} = \left[\left(\psi_m-\psi_s\right)(1-\sigma_s)-\left(\psi_m-\psi_a\right)\sigma_a\right](1-\epsilon)\frac{\dot{h}}{h},$$

and substituting in $\dot{u}_s/u_s$, we obtain the growth rate of employment share in services reported in main text.

Note that $\psi_a > \psi_m$ and $\psi_m > \psi_s$, and $\epsilon < 1$, so that the time path of relative prices are consistent with the empirical evidence given a positive growth rate of human capital, implies that

$$\begin{aligned} \lim_{t\to\infty} p_a &\rightarrow 0, \\ \lim_{t\to\infty} p_s &\rightarrow \infty, \end{aligned}$$

and

$$\begin{aligned} \lim_{t\to\infty} \sigma_a &\rightarrow 0, \\ \lim_{t\to\infty} \sigma_s &\rightarrow 1. \end{aligned}$$

Therefore,

$$\begin{aligned} \lim_{t\to\infty}\frac{\dot{\sigma}_a}{\sigma_a} &= \left(\psi_s-\psi_a\right)(1-\epsilon)\frac{\dot{h}}{h} < 0, \\ \lim_{t\to\infty}\frac{\dot{\sigma}_s}{\sigma_s} &= 0. \end{aligned}$$

2.B Proof of Propositions

Proof of Proposition 2.1. The steady state, the growth rates of transformed variables z, q and χ are equal to zero by definition, then we obtain the following system of equations,

$$0 = (\alpha - 1) A_m z^{\alpha-1} + q - \rho, \tag{2.B.1}$$

$$0 = A_m z^{\alpha-1} - \varpi\left(1-\chi\right) - \delta_z, \tag{2.B.2}$$

$$0 = \theta\left(1-\chi\right) - q + \delta_\chi. \tag{2.B.3}$$

It is straightforward to show that there exists a unique values for z, q and χ that solve the system. From $(2.B.3)$ there exist a unique value for χ so that satisfied.

$$\chi^* = \frac{\delta + \rho}{B},$$

We substitute this value into $(2.B.1)$ and $(2.B.2)$ to obtain

$$z^* = \left(\alpha \frac{A_m (1-\alpha)}{B\left(1-\alpha+\psi_m\right) - \left(\delta+\rho\right)\psi_m}\right)^{\frac{1}{1-\alpha}},$$

$$q^* = \frac{B\left(1-\alpha+\psi_m\right)}{\alpha} + \rho - \frac{\psi_m}{\alpha}\left(\delta+\rho\right). \blacksquare$$

Proof of Proposition-2.2. By substituting z^*, q^* and χ^* in

$$\frac{\dot{k}}{k} = A_m z^{\alpha-1} - q - \delta,$$
$$\frac{\dot{c}}{c} = \alpha A_m k^{\alpha-1} h^{1-\alpha+\psi_m} \chi^{1-\alpha} - \delta - \rho,$$

we obtain

$$\frac{\dot{k}}{k} = \frac{\dot{c}}{c} = \frac{\left[B - \left(\delta+\rho\right)\right]\left(1-\alpha+\psi_3\right)}{1-\alpha},$$

and given χ^*, the growth rate of human capital is

$$\frac{\dot{h}}{h} = B - \left(\rho + \delta\right).$$

where we can define the parameter $\phi = \frac{1-\alpha}{(1-\alpha+\psi_3)}$ and substitute it in k and c taking into account $\frac{\dot{h}}{h}$.■

2.C Figures and Tables

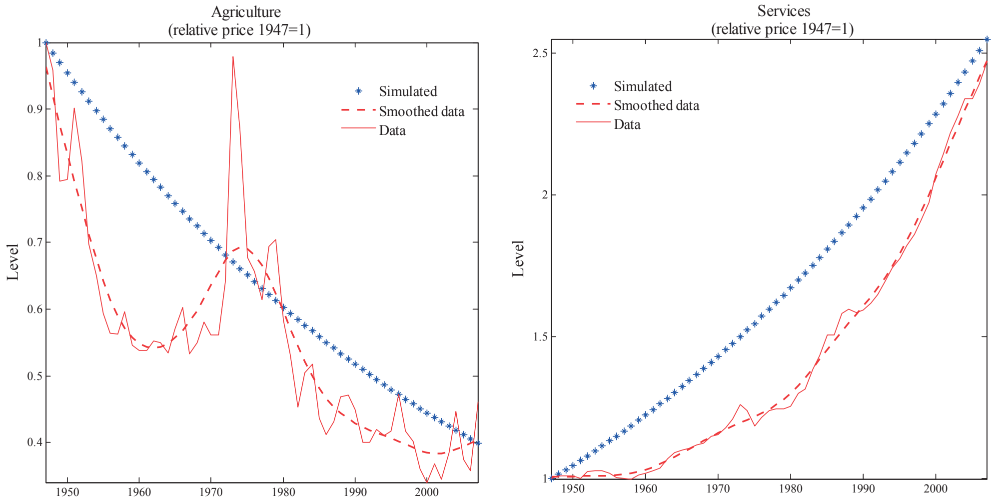

Figure 2.1: Time path of relative prices. Figures plot the simulated path of relative prices. Actual data (solid line) normalized to unity in 1947. Smoothed actual data was obtained by applying the Hodrick-Prescott filter. We choose a value of 6.25 for the smoothed parameter in order to pick up the trend in data. Source: Producer Prices Index from NIPA

.

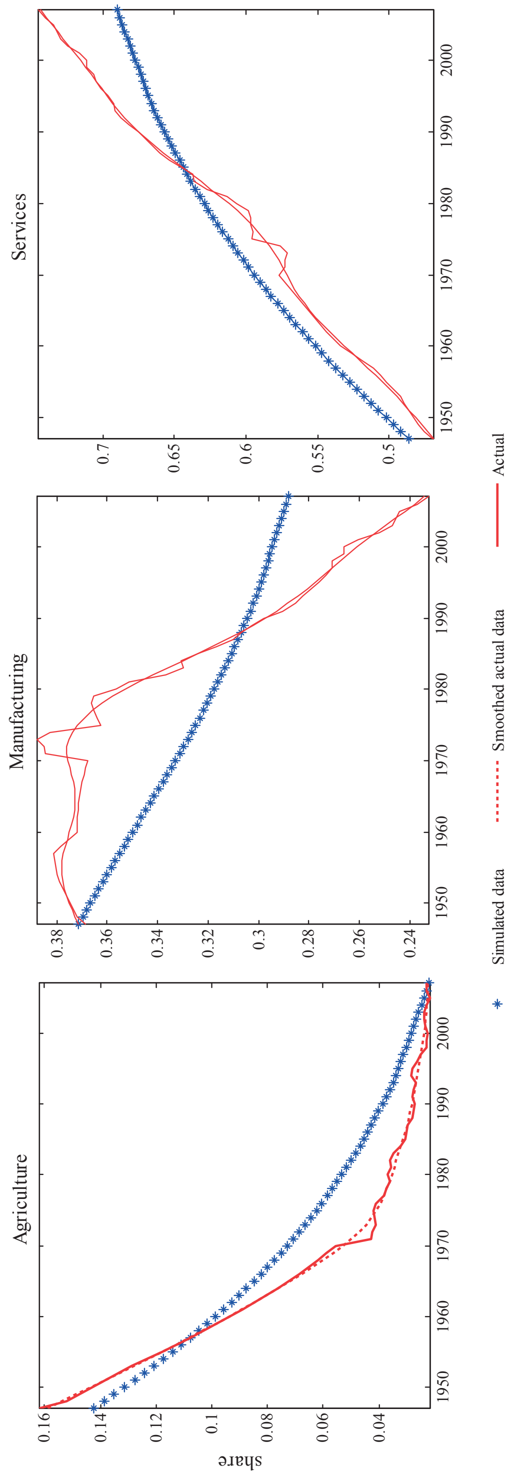

Figure 2.2: Employment shares 1947-2005, actual and simulated time paths. This figure plots the time path of actual employment share in agriculture, manufacturing and service, and THE simulated time path. In the first panel, simulated path matches the actual value of agriculture employment share in 2005, but it fails to match initial value. Source: de Vries and Timmer (2007)," Groningen Growth and Development Centre 10-sector database". http://www.ggdc.net/

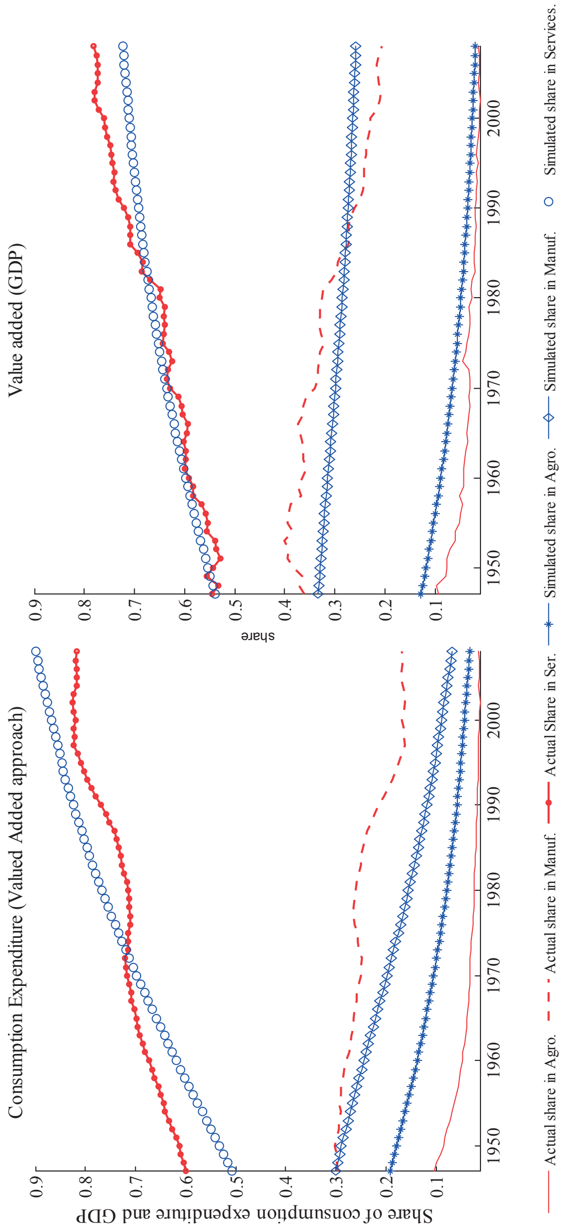

Figure 2.3: Consumption and GDP shares, actual and simulated data. This figure plots both actual and simulated shares of consumption expenditure and GDP in agriculture, manufacturing and service. We use the available data reported by Herrendorf, Rogerson, and Valentinyi (2014) for consumption expenditure based on the value added approach. Note that the model predicts the differences in levels between actual consumption expenditure and actual value shares across sectors.

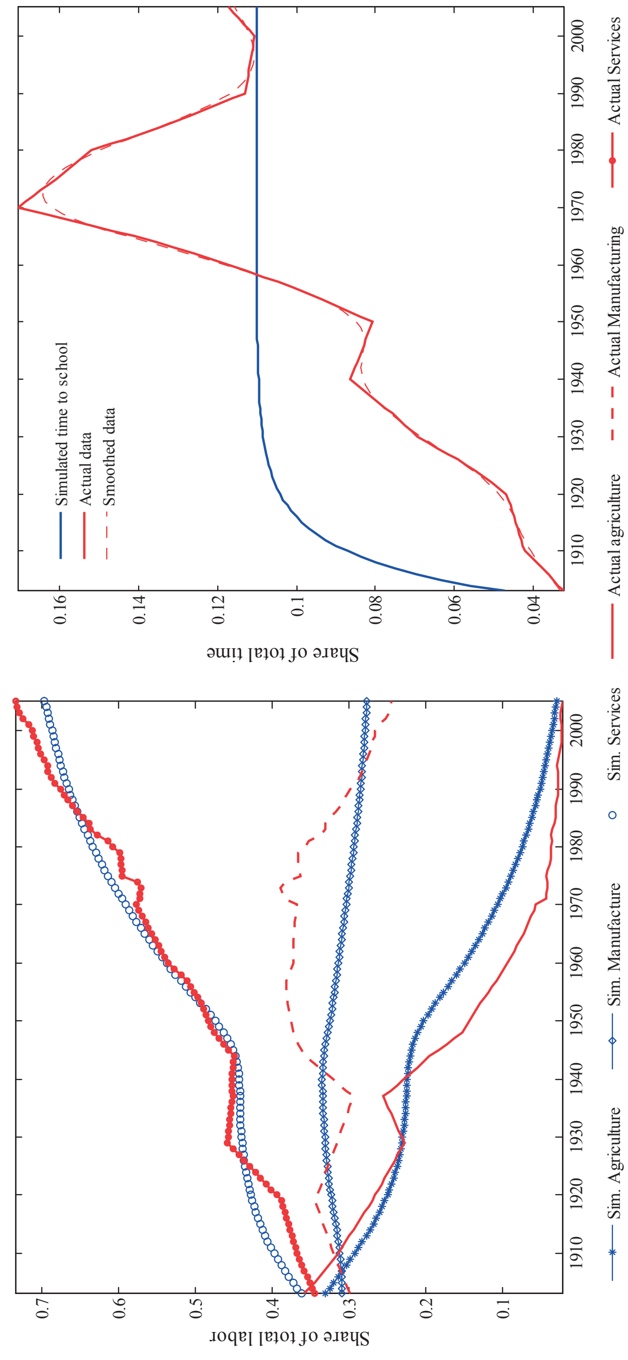

Figure 2.4: Structural change and time to shooling, 1900-2005.

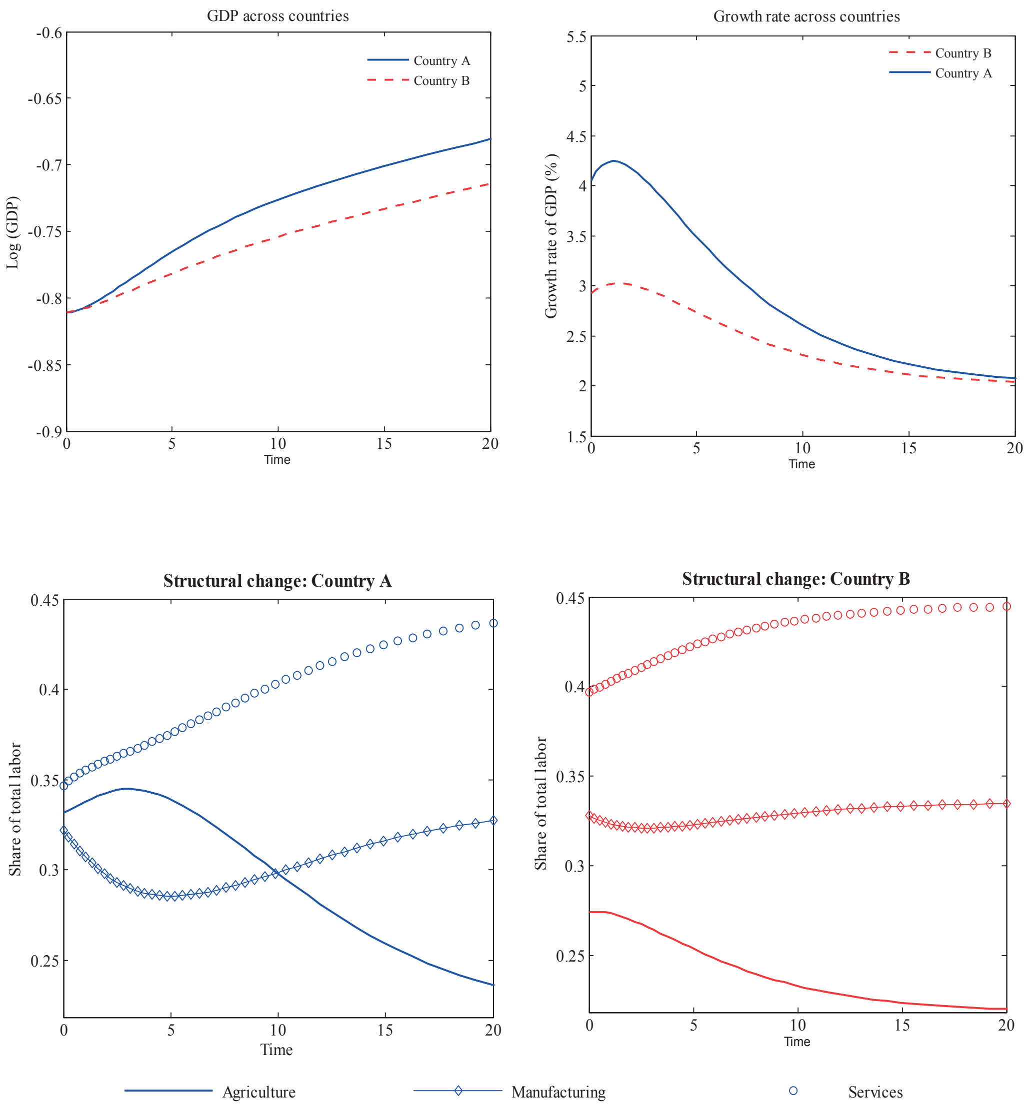

Figure 2.5: Development patterns between two simulated countries.

Table 2.1: Calibration: parameters and targets

Parameter		Value	Target	U.S.
Consumption Preferences				
Discount factor	ρ	0.0320	Growth rate of GDP	0.02
Manufacturing	η_m	0.1246	-	–
Services	η_s	0.8498	R. expenditure serv.(1947)	0.41
Agriculture	η_a	0.0256	R. expenditure farm.(1947)	2.13
Elasticity substitution	ϵ	0.50	Ngai and Pissarides (2008)	–
Sectoral technologies				
Capital share	α	0.374	Labor income share	0.65
Annual depreciation	δ	0.141	Interest rate (%)	5.4
Technology level				
Manufacturing	A_m	1	Normalized	–
Services	A_s	3.324	R. price of farm (1947)	0.27
Agriculture	A_a	0.962	R. price of services (1947)	1.05
Education	B	0.147	Labor share in educational sector	0.11
Externalities				
Manufacturing	ψ_m	0.148	Capita-output ratio	2.5
Services	ψ_s	0.050	Growth rate of price (ser)	.014
Agriculture	ψ_a	0.157	Growth rate of price (agro)	−.014

Table 2.2: Accuracy of simulated time paths: labor shares

Case	**Sector**	**Theil U**	**R^2**
	Agriculture	0.0580	0.952
$\epsilon = 0.30$	Manufacturing	0.0363	0.756
	Services	0.0134	0.959
	Agriculture	0.0464	0.932
$\epsilon = 0.15$	Manufacturing	0.0372	0.761
	Services	0.0153	0.9739
	Agriculture	0.0923	0.957
$\epsilon = 0.50$	Manufacturing	0.0350	0.765
	Services	0.0108	0.948

Chapter 3

Leisure Time and the Sectoral Composition of Employment

3.1 Introduction

The second half of the twenty century has been characterized by two important patterns of structural change. First, the process of structural change in the sectoral composition of employment. This process consists of a large shift of employment and production from the agriculture and manufacturing sectors to the service sector. Figures 1 and 2 illustrate this process in the case of the US economy, during the period 1947-2010. As follows from Figure 1, in the mid of the twenty century, almost 20% of employment was employed in the agriculture sector, whereas by the end of the century only 2% is employed in this sector. In contrast, employment in the service sector increases from almost 50% to 75% of total employment during this period. Finally, employment in the manufacturing sector declines during the second half of the twenty century. Figure 2 shows a similar pattern for the shares of value added in the three sectors. The recent multisector growth literature has explained these patterns of structural change as the result of non-homothetic preferences (Kongsamunt, Rebelo and Xie, 2001) or changes in relative prices (Acemoglu and Guerrieri, 2008; Ngai and Pissariades, 2007). More recently, this literature has shown that the rise of the service sector can only be explained from combining both non-homothetic preferences and changes in relative prices (Boppart, 2014; Dennis and Iscan, 2008; Foellmi and Zweimuller, 2008). Herrendorf, Rogerson and Valentinyi (2014) offer an exhaustive review of this literature.

[Insert Figures 1 and 2]

Second, the change in the uses of time is another relevant process of structural change that has occurred during the second half of the last century. Using survey data, Aguiar and Hurst (2013) and Francis and Ramey (2009) show the evolution of the uses of time in the US economy during the second half of the last century. Figures 3 and 4 show these changes in the uses of time. As follows from these figures, during the second half of the twenty century there has been a clear increase in the time devoted to leisure. According to these figures, the time devoted to leisure increases during the period from 45% of the total time to 54%. Obviously, this implies a reduction in the amount of time devoted to work.

[Insert Figures 3 and 4]

The reduction in the time devoted to work is mainly explained as the consequence of a wealth effect: as wealth increases, agents want to consume a larger amount of leisure and, therefore, they reduce the time devoted to work. Note that this explanation is completely independent of the multisectoral structure of the economy. There are few papers relating the patterns of structural change in the sectoral composition of employment with changes in the uses of time (See Buera and Kaboski, 2012; Gollin, Parente and Rogerson, 2004; and Ngai and Pissariades, 2008). In these papers, the relationship is based on home production and its different substitutability with the market production of the different sectors. In contrast, in this paper, we propose an explanation based on the recreational nature of leisure. We consider that during the leisure time we consume recreational services. The mechanism explaining the two patterns of structural change is then as follows. As the economy develops, households devote a larger amount of time to leisure activities, which consume recreational services. It follows that part of the increase in the service sector can be explained by the increase in leisure.

In this paper, we quantify the impact on structural change of the proposed mechanism. To this end, we measure the fraction of the value added of the service sector explained by recreational services. The details of the procedure followed to obtain this fraction are explained in Appendix D and the results are displayed in Figure 5. As explained in Appendix D, data availability limits the period analyzed to be between 1947 and 2010. Figure 5 displays the time path of the fraction of the value added in the service sector directly explained by recreational activities. This fraction increases from 6% in 1947 to 14% in 2010. This increase is large and explains 26% of the observed increase in the service sector share of total value added. This clearly shows that the effect of leisure on sectoral composition is sizeable.

[Insert Figure 5]

In order to study the effects on structural change of recreational activities, we construct a multi-sector exogenous growth model. In the supply side, we distinguish among three sector specific technologies that are used to produce agriculture goods, manufacturing goods and services. These technologies are differentiated only by the exogenous growth rate of total factor productivity (TFP). In the demand side, we assume that households obtain utility from consuming agriculture and manufacturing goods, services and recreational activities. Following Ngai and Pissariades (2007), we assume a constant elasticity of substitution (CES) utility function. Therefore, the only new feature of this model is the introduction of recreational activities. These activities are defined as a CES function of both the amount of time devoted to leisure and of the consumption of recreational services. We assume that the elasticity of substitution of recreational activities (the elasticity between leisure time and recreational services) is different from the elasticity of substitution of consumption goods (between recreational activities and the consumption of goods or services produced in the three sectors). In fact, the utility function considered in this paper is a non-homothetic version of the nested CES function introduced by Sato (1967).

Technological progress drives structural change through two different channels: a wealth channel and a substitution channel. On the one hand, the substitution channel is due to the assumption of biased technological progress. Consistent with empirical evidence, we will assume that the sector experiencing the largest TFP growth is the agriculture sector and the one experiencing the smallest TFP growth is the service sector. This biased technological progress causes the reduction of the relative price of agriculture goods in units of manufactured goods and the increase in the relative price of services in units of manufactured goods. As outlined by Ngai and Pissariades (2007), the effect on structural change of relative price changes will depend on the value of the elasticity of substitution of consumption goods.

On the other hand, the wealth channel is the new mechanism of structural change introduced in this paper. Leisure rises with technological progress, which drives the increase in recreational services. Obviously, the effect of this mechanism on the sectoral composition will depend on the value of the elasticity of substitution of recreational activities.

The interaction between the two channels explains the process of structural change in this economy. This process drives the economy to different asymptotic long run equilibria, depending on the value of the two elasticities of substitution. These asymptotic equilibria will be differentiated by the long run values of five variables that measure structural change: leisure, the shares of employment devoted to the three sectors and the share of added value in the service sector devoted to recreational services. Section 3 provides a complete characterization of these long run equilibria. We show that these long run asymptotic equilibria consist of corner solutions implying that the value of these variables converges to its minimum or maximum possible values. These corner solutions arise because technological progress is permanently biased towards a given sector and, therefore, they must be interpreted as the long run equilibrium that an economy would attain if the bias in technological progress were permanent. Interestingly, they inform about the direction of structural change implied by the model. We use this asymptotic equilibria to conclude that the observed patterns of structural change can only be explained if both elasticities of substitution are smaller than one and the elasticity of substitution of recreational activities is larger than the elasticity of substitution of consumption goods.

In Section 4, we calibrate and numerically simulate the equilibrium. We consider three different economies. The first one is our benchmark economy where individuals obtain utility from recreational activities. In the second economy, we do not consider these activities and we instead assume that individuals obtain utility directly from leisure. Finally, the third economy is a standard multisector growth model without leisure. From the comparison among these economies, we show that the performance of the simulated economies in explaining the observed patterns of structural change is enhanced by the introduction of recreational activities. Moreover, the benchmark economy explains the observed increase in leisure time and almost all the increase in the share of recreational services. We then conclude that recreational activities are an important feature of structural change.

In Section 5, we use the model to study the effects of fiscal policy on employment and gross domestic product (GDP). Following the analysis of Prescott (2004) and Rogerson (2008), we also study the effects of an increase in the labor income tax rate. We show that increasing this tax reduces employment and GDP, both in the benchmark economy with recreational activities and in the economy with leisure. However, the effect of this policy is substantially larger in the benchmark economy. This is due to the fact that the introduction of recreational activities increases the substitutability between leisure and consumption goods. An increase in the labor income tax reduces the wage after taxes, which causes both a substitution and an income effect. The net effect of a tax increase on GDP will depend on the strength of these two effects. The introduction of recreational services, by increasing the substitutability between leisure and consumption goods, reduces the strength of the wealth effect. This explains that the reduction in GDP due to a tax increase is substantially larger when we consider that individuals derive utility from leisure through recreational activities.

3.2 The model

We build a three-sector exogenous growth model. We distinguish between the agriculture, service and the manufacturing sectors. The agriculture and service sectors only produce a consumption good, whereas the manufacturing sector produces both a consumption and an investment good. We assume that the consumption good produced in the service sector can be devoted to either recreational or non-recreational activities. Finally, we also assume that the manufacturing sector is the numeraire of the economy.

3.2.1 Firms

Each sector i produces by using the following constant returns to scale Cobb-Douglas technology:

$$Y_i = A_i \left(s_i K\right)^{\alpha} \left(u_i L\right)^{1-\alpha}, \; i = a, s, m, \tag{3.2.1}$$

where Y_i is the amount produced in sector i, $\alpha \in (0,1)$ is the capital output elasticity, s_i is the share of total capital K devoted to sector i, u_i is the share of total employment L employed in sector i, A_i measures total factor productivity (TFP) in sector i, and the subindexes a, s and m amount for the agriculture, services and manufacturing sectors, respectively. Obviously, the capital shares and the employment shares satisfy

$$s_a + s_m + s_s = 1,$$

and

$$u_a + u_s + u_m = 1.$$

We assume that TFP grows in each sector at a constant growth rate γ_i. Consistent with empirical evidence, we assume that $\gamma_a > \gamma_m > \gamma_s$.

Each individual has a time endowment of measure one that can devote to either leisure activities or labor. Let l be the amount of time an individual devotes to work and N the constant number of individuals. Then, total employment in the economy satisfies $L = lN$. It follows that (3.2.1) can be rewritten in per capita terms as

$$y_i = A_i\,(s_i k)^{\alpha}\,(u_i l)^{1-\alpha}\,,\ i = a, s, m, \tag{3.2.2}$$

where $y_i = Y_i/N$ and $k = K/N$.

Perfect competition and perfect factors' mobility imply that each factor is paid according to its marginal productivity and that marginal productivities equalize across sectors, implying that

$$r = \alpha p_i A_i\,(s_i k)^{\alpha-1}\,(u_i l)^{1-\alpha} - \delta, \tag{3.2.3}$$

and

$$w = (1-\alpha)\,p_i A_i\,(s_i k)^{\alpha}\,(u_i l)^{-\alpha}\,, \tag{3.2.4}$$

where r is the rental price of capital, w is the wage per unit of employment, p_i is the relative price and $\delta \in (0,1)$ is the depreciation rate of capital. From using (3.2.3) and (3.2.4), we obtain $s_i = u_i$ and

$$p_i = \frac{A_m}{A_i}. \tag{3.2.5}$$

Given the assumed ranking of TFP growth rates, the relative price of agriculture, p_a, decreases and the relative price of services, p_s, increases.

3.2.2 Consumers

Individuals are infinitely lived consumers. Each individual has a time endowment of measure one. As l is the amount of time an individual devotes to work, $1-l$ is the amount of time devoted to leisure activities. Consumers obtain income from capital and labor and use it to investment and consume. Therefore, the consumers' budget constraint is

$$wl + rk = E + \dot{k}, \tag{3.2.6}$$

where $E = c_m + p_s c_s + p_a c_a$ is total consumption expenditures.

The consumers' utility is

$$u = \int_0^{\infty} e^{-\rho t} \ln C dt, \tag{3.2.7}$$

where $\rho > 0$ is the subjective discount rate and C is the following composite consumption good:

$$C = \left[\eta_a c_a^{\frac{\varepsilon-1}{\varepsilon}} + \eta_m c_m^{\frac{\varepsilon-1}{\varepsilon}} + \eta_s (x c_s)^{\frac{\varepsilon-1}{\varepsilon}} + \eta_l c_l^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where c_a is the amount consumed of agriculture goods, c_m is the amount consumed of manufactured goods, c_s is the amount consumed of service goods, c_l is the amount consumed of recreational activities, $x \in [0,1]$ is the fraction of services devoted to non-recreational consumption, $\varepsilon > 0$ is elasticity of substitution between the different consumption goods, and $\eta_i > 0$ measures the weight of the different consumption goods in the utility function. We assume that $\eta_a + \eta_s + \eta_l + \eta_m = 1$. We also assume that recreational activities depend on both leisure and the amount consumed of services, according to the following function:

$$c_l = \left\{ \beta \left[(1-x) c_s \right]^{\frac{\sigma-1}{\sigma}} + \left(1-\beta\right) \left(1 - l - \bar{l}\right)^{\psi\left(\frac{\sigma-1}{\sigma}\right)} \right\}^{\frac{\sigma}{\sigma-1}}, \tag{3.2.8}$$

where $\sigma > 0$ is the elasticity of substitution between recreational services and leisure, $\bar{l}$ is a minimum requirement of leisure, $\psi \in (0,1)$ determines the wage elasticity of the labor supply and $\beta \in [0,1]$ measures the weight of recreational services in recreational activities.[1] On the one hand, $\bar{l}$ is introduced to guarantee a minimum amount of leisure. On the other hand, the preference parameters ψ disentangles σ from the elasticity of substitution of the labor supply with respect to the wage. This is necessary in order to explain the observed increases in both leisure and in the fraction of recreational services.

Consumers decide on leisure, the value of consumption expenditures, the sectoral composition of these expenditures and the fraction of services devoted to recreational activities, in order to maximize (3.2.7) subject to (3.2.6). The solution of this maximization problem is characterized by the following equations:

$$\frac{c_m}{E} = \frac{1}{\kappa_1}, \tag{3.2.9}$$

[1]In our analysis we do not consider $\sigma = 1$ or $\varepsilon = 1$.

$$\frac{p_s c_s}{E} = \left(p_s \frac{\eta_m}{\eta_s}\right)^{-\varepsilon} \left(\frac{1}{x}\right) \left(\frac{p_s}{\kappa_1}\right), \tag{3.2.10}$$

$$\frac{p_a c_a}{E} = \left(p_a \frac{\eta_m}{\eta_a}\right)^{-\varepsilon} \left(\frac{p_a}{\kappa_1}\right), \tag{3.2.11}$$

$$x = \frac{\kappa_4}{1+\kappa_4}, \tag{3.2.12}$$

$$1 - l = \bar{l} + \left(\frac{w\eta_m}{\eta_l (1-\beta) \psi \kappa_2^{\frac{\varepsilon-\sigma}{\varepsilon\sigma}}}\right)^{-\frac{\varepsilon}{(1-\varepsilon)\psi+\varepsilon}} \left(\frac{E}{\kappa_1}\right)^{\frac{1}{(1-\varepsilon)\psi+\varepsilon}}, \tag{3.2.13}$$

and

$$\frac{\dot{E}}{E} = r - \rho - \frac{\dot{\kappa}_7}{\kappa_7}, \tag{3.2.14}$$

where $\{\kappa_i\}_{i=1}^{7}$ are complex functions of both the prices and the wage that are obtained in Appendix A. Equations (3.2.9), (3.2.10) and (3.2.11) characterize the sectoral composition of consumption expenditures, while (3.2.12) determines the fraction of services devoted to non-recreational activities. Equation (3.2.13) determines the amount of leisure and it then implicitly characterizes the labor supply. Finally, (3.2.14) is the Euler condition driving the intertemporal trade-off between consuming today and in the future.[2]

3.3 Equilibrium

Let $z = k/lA_m^{\frac{1}{1-\alpha}}$ be the capital stock per efficiency unit of employment in the economy. Using this definition and (3.2.3), we obtain the interest rate as

$$r = \alpha z^{\alpha-1} - \delta, \tag{3.3.1}$$

and using (3.2.4) we obtain

$$w = (1-\alpha) A_m^{\frac{1}{1-\alpha}} z^{\alpha}. \tag{3.3.2}$$

We define per capita GDP as $Q = p_a y_a + p_s y_s + y_m$ and using (3.2.2) and (3.2.4) we obtain

$$Q = A_m^{\frac{1}{1-\alpha}} z^{\alpha} l. \tag{3.3.3}$$

[2]As follows from (3.2.14), the growth rate of consumption expenditures depends on the growth rate of κ_7 and therefore it depends on the growth rate of prices. Alonso-Carrera, Caballé and Raurich (2015) discuss why the growth of prices affects the Euler condition in multisector growth models.

Note that per capita GDP depends on the time devoted to work, l.

Let $q = E/Q$ be consumption expenditure per unit of GDP. Using this variable, the resource constraint of this economy can be written as

$$\dot{k} = Q\left(1-q\right) - \delta k. \tag{3.3.4}$$

The agriculture and service sectors only produce a consumption good and, thus, the market clearing condition in these sectors is $y_i = c_i$, $i = a, s$. From using this market clearing conditions and (3.2.11), (3.2.10), (3.3.2) and (3.3.3), we obtain the employment shares in the service sector,

$$u_s = p_s\left(p_s\frac{\eta_m}{\eta_s}\right)^{-\varepsilon}\left(\frac{1}{x_s}\right)\left(\frac{q}{\kappa_1}\right), \tag{3.3.5}$$

and in the agriculture sector,

$$u_a = p_a\left(p_a\frac{\eta_m}{\eta_a}\right)^{-\varepsilon}\left(\frac{q}{\kappa_1}\right). \tag{3.3.6}$$

The employment share in the manufacturing sector is

$$u_m = 1 - u_s - u_a. \tag{3.3.7}$$

Finally, from using (3.2.13) and (3.3.3), we obtain the amount of time devoted to work

$$l = 1 - \bar{l} - \left(\frac{w\eta_m}{\eta_l\left(1-\beta\right)\psi\kappa_2^{\frac{\varepsilon-\sigma}{\varepsilon\sigma}}}\right)^{-\frac{\varepsilon}{(1-\varepsilon)\psi+\varepsilon}}\left(\frac{qA_m^{\frac{1}{1-\alpha}}z^{\alpha}l}{\kappa_1}\right)^{\frac{1}{(1-\varepsilon)\psi+\varepsilon}}. \tag{3.3.8}$$

Equations (3.2.12), (3.3.5), (3.3.6), (3.3.7) and (3.3.8) show that the sectoral composition of the economy and the amount of time devoted to work depend on relative prices the wage and the time path of z and q. In Appendix B, we obtain the following system of differential equations governing the time path of these two variables:

$$\frac{\dot{z}}{z} = \frac{\left[\left(1-q\right)z^{\alpha-1} - \delta - \frac{\gamma_m}{1-\alpha}\right]l + \left(1-l-\bar{l}\right)\omega_{10}}{l - \left(1-l-\bar{l}\right)\omega_{11}}, \tag{3.3.9}$$

and

$$\frac{\dot{q}}{q} = (\omega_9+\alpha)\left(\delta+\frac{\gamma_m}{1-\alpha}\right)-\delta-\rho-\omega_8-\frac{\gamma_m}{1-\alpha}+\left[\alpha-(\omega_9+\alpha)\left(1-q\right)\right]z^{\alpha-1}$$
$$-\left[\omega_9-(1-\alpha)\right]\left(1-l-\bar{l}\right)\left(\frac{\omega_{10}+\left[\left(1-q\right)z^{\alpha-1}-\delta-\frac{\gamma_m}{1-\alpha}\right]\omega_{11}}{l-\left(1-l-\bar{l}\right)\omega_{11}}\right),\quad(3.3.10)$$

where $\{\omega_i\}_{i=1}^{11}$ are functions of the relative prices and of the wage defined in Appendix B.

Given initial conditions $z(0)$ and $A_i(0)$ $i=a,s,m$, a dynamic equilibrium is a path of $\{z,q,u_a,u_s,u_m,l,x,p_a,p_s,w\}_{t=0}^{\infty}$ that solves the system of differential equations (3.3.9) and (3.3.10) and satisfies (3.2.12), (3.2.5), (3.3.2), (3.3.5), (3.3.6), (3.3.7), (3.3.8), and $A_i=A_i(0)\,e^{-\gamma_i t}$, $i=a,s,m$.

A balanced growth path (BGP) equilibrium is an equilibrium path along which the interest rate and the ratio of capital to GDP remain constant.

Appendix C proves the following four propositions regarding the BGP equilibrium.

Proposition 3.1 *There is a unique asymptotic BGP along which variables characterizing the sectoral composition remain constant.*

In the numerical simulation there is a unique equilibrium path converging towards this BGP. The assumption of permanent bias in technological progress implies that the BGP equilibrium can only be attained asymptotically when the employment shares and employment converge to a corner solution. The following propositions obtain the long run values of these variables.

Proposition 3.2 *The long run values of employment, the ratio of capital to efficiency units and the ratio of consumption expenditures to GDP are:*

$$l^*=\begin{cases}0 & if\ \ \sigma<1,\ \varepsilon<1\\ 1-\bar{l} & otherwise.\end{cases},$$

$$z^*=\left(\frac{(1-\alpha)\left(\delta+\rho\right)+\gamma_m}{\alpha\left(1-\alpha\right)}\right)^{\frac{1}{\alpha-1}},$$

and

$$q^* = \begin{cases} 1-\alpha\left(\frac{\delta+\frac{\gamma m}{1-\alpha}-(1-\sigma)\left(\frac{\alpha\gamma m}{1-\alpha}+\gamma_s\right)}{\delta+\rho+\frac{\gamma m}{1-\alpha}}\right) & if \quad \sigma<1,\ \varepsilon<1 \text{ and } \sigma>\varepsilon \\ 1-\alpha\left(\frac{\delta+\frac{\gamma m}{1-\alpha}-(1-\varepsilon)\left(\frac{\alpha\gamma m}{1-\alpha}+\gamma_s\right)}{\delta+\rho+\frac{\gamma m}{1-\alpha}}\right) & if \quad \sigma<1,\ \varepsilon<1 \text{ and } \sigma<\varepsilon \\ 1-\alpha\left(\frac{\delta+\frac{\gamma m}{1-\alpha}}{\delta+\rho+\frac{\gamma m}{1-\alpha}}\right) & otherwise \end{cases}.$$

The wage increases as the economy develops. This implies that if individuals can substitute leisure for other consumption goods employment will increase and converge to its maximum value. This happens when either $\sigma > 1$ or $\varepsilon > 1$. It follows that employment decreases as the economy grows only when leisure is complementary with respect to the other consumption goods, which requires that both $\varepsilon < 1$ and $\sigma < 1$. As Figures 3 and 4 clearly show that employment decreases as the economy develops, we should consider the case in which $\varepsilon < 1$ and $\sigma < 1$. In the following two propositions we show that these values of the elasticities of substitution are also consistent with the observed patterns of structural change.

Proposition 3.3 *The long run values of the sectoral composition of employment are:*

1. $u_a^* = q^*$, $u_s^* = 0$ *and* $u_m^* = 1-q^*$ *if either* $i)$ $\varepsilon > 1$, $\sigma > 1$ *and* $\varepsilon < \sigma$; $ii)$ $\varepsilon > 1$ *and* $\sigma < 1$; *or* $iii)$ $\varepsilon = \sigma > 1$.

2. $u_a^* = 0$, $u_s^* = q^*$ *and* $u_m^* = 1-q^*$ *if either* $i)$ $\varepsilon > 1$, $\sigma > 1$ *and* $\varepsilon > \sigma$; *or* $ii)$ $\varepsilon < 1$.

Proposition 3.3 shows that the employment share in agriculture declines to zero if either the different consumption goods are complementary goods ($\varepsilon < 1$) or they are substitutes ($\varepsilon > 1$ and $\sigma > 1$) but services are more complementary than the other consumption goods ($\varepsilon > \sigma$). The first case corresponds exactly to the case considered in Ngai and Pissariades (2007). As these authors explain, when the price of agriculture declines the employment share in this sector decreases only if the consumption goods are complements. The second case arises in this paper because recreational activities allow to disentangle the substitution of agriculture goods from the substitution of services.

Proposition 3.4 *The long run value of the fraction of services devoted to non-recreational activities is*

$$x^* = \begin{cases} \frac{1}{1+\left(\frac{\eta_l}{\eta_s}\beta\right)^{\sigma}} & if \quad \sigma = \varepsilon \\ 1 & if \quad either\ \sigma > 1\ and\ \varepsilon < \sigma\ or\ \sigma < 1\ and\ \sigma < \varepsilon \\ 0 & if \quad either\ \sigma > 1\ and\ \varepsilon > \sigma\ or\ \sigma < 1\ and\ \sigma > \varepsilon \end{cases}.$$

Figure 5 shows that the fraction of services devoted to recreational activities has increased. This implies that the variable x should decline. Proposition 3.4 shows that this happens when either $\sigma > 1$ or $\varepsilon < \sigma < 1$. As the economy develops, both leisure and consumption of services increases. However, the increase in the consumption of services is substantially larger and faster than the increase in leisure. As a consequence, if leisure and recreational services were strong complements, then the fraction of services devoted to recreational activities would decline. It follows that this fraction increases only when leisure and recreational services are substitutes $(\sigma > 1)$ or not strong complements $(1 > \sigma > \varepsilon)$.

We conclude that the equilibrium path implied by this model is compatible with the observed patterns of structural change when i) the utility function exhibits complementaries both among the different consumption goods $(\varepsilon < 1)$ and between leisure and recreational services $(\sigma < 1)$ and ii) when the complementarity between leisure and services is smaller than the complementarity among the different consumption goods $(\sigma > \varepsilon)$. The first condition is already obtained in Ngai and Pissariades (2007). The second condition is a contribution of this paper and it is related to the capacity of the model to explain the process of structural change between recreational and non-recreational activities. These constraints on the value of the elasticities of substitution are considered in the numerical analysis of the following section.

3.4 Structural change

In this section we simulate the economy in order to analyze if the mechanism proposed in this paper contributes to explain the observed patterns of structural change. To this end, we calibrate the parameters of the economy as follows: $\gamma_m = 1.37\%$ in order to have a long run GDP growth rate equal to 2%; $\gamma_s = 0.64\%$ and $\gamma_a = 3.57\%$ in order to match the growth rate of prices obtained by Herrendorf, et al. (2013); we set $\alpha = 0.35$ in order to match the average value of the LIS in the US during this period; $\rho = 0.032$ so that the long run interest rate equals 5.2%; $\delta = 5.6\%$ in order to obtain a long run ratio of investment to capital equal to 7.6%; we normalize $A_m(0) = 1$ and we set $A_s(0) = 1.4633$ and $A_a(0) = 0.2327$ in order to obtain the initial relative prices of services and agriculture in units of the manufacturing goods obtained by Herrendorf, et al. (2014). The values of the two elasticities, $\varepsilon = 0.25$ and $\sigma = 0.98$, are set to obtain the best fit in explaining the time path of both x and l. We also assume that $z_0 = z^*$, which implies that the equilibrium exhibits almost balanced growth. This is consistent with the observed time path of the interest rate and of the ratio between capital to GDP in the US economy. The rest of parameters, β, ψ, $\bar{l}$, η_a, η_s and η_l, are set to distinguish between three different economies. In Economy 1, these six parameters are set to match the value in 1947 of the following variables l, x, u_a, u_s and the value in 2010 of l and x. Obviously, Economy 1 corresponds to our benchmark economy. In Economy 2, we assume that $\beta = 0$ implying that $x = 0$ and the rest of parameters are set as in Economy 1. In Economy 3, we assume that $\beta = 0$ and $\eta_l = 0$, which implies that $x = 0$ and $l = 1 - \bar{l}$. The rest of parameters are set to match the value in 1947 of u_a, u_s and u_m. The parameters in the three economies are summarized in Table 1.

[Insert Table 1 and Table 2]

Figure 6 illustrates the patterns of structural change in Economy 1. As follows from this figure, this economy explains all the observed reduction in the amount of employment and it also explains almost all the increase in the share of recreational services (90% of the increase observed in the period 1947-2010). Moreover, the model captures the observed trends in the process of structural change in the sectoral composition of employment. More precisely, the model explains part of the reduction in the employment shares of the agriculture and manufacturing sectors (78% and 22%, respectively) and it also explains part of the increase in the employment share in the service sector (49%). We conclude that the model provides a reasonable explanation of the patterns of structural change. This explanation is based on the interaction between two mechanisms: the substitution and the wealth effect. The first one is the classical effect associated to biased technological change, whereas the second one is associated to the introduction of recreational activities. In order to disentangle between the two mechanisms, we proceed to study the performance of Economies 2 and 3.

[Insert Figure 6]

Figure 7 displays the patterns of structural change in Economy 2. In this economy, we assume that there are no recreational services and, therefore, we do not consider the mechanism proposed in this paper. This simulated economy captures the main trends of structural change in the sectoral composition of employment, but the performance is worse than in Economy 1. From the comparison between Economies 1 and 2, we conclude that the introduction of recreational activities contributes to explain both the reduction in the labor supply and the process of structural change in the sectoral composition of employment.

[Insert Figure 7]

Figure 8 displays the patterns of structural change in Economy 3. In this economy, we assume that there is no leisure. The model then coincides with the model in Ngai and Pissariades (2007). In this case, the mechanism driving structural change is simply biased technological change and the performance of this economy is worse.

[Insert Figure 8]

Table 3 provides two different measures of performance: the percentage of total variation explained by the simulated economies and the sum of the square of the residuals. The first measure captures the capacity of the simulated economies to explain the observed long run trends in the variables. The second one is a standard measure of performance during all the period. From the comparison between these different measures we conclude that introduction of recreational activities improves the performance of the simulated economies in explaining both the reduction in the amount of labor and the changes in the sectoral composition of employment. Leisure increases with economic development, which contributes to explain the observed changes in the sectoral composition of employment when recreational activities are introduced. These activities introduce a complementarity between leisure and services. The increase in leisure then increases both the fraction of services devoted to recreational services and the share of employment in the service sector.

[Insert Table 3]

3.5 Fiscal policy

Growth models with leisure introduce an adequate framework to study the effects of fiscal policy on both employment and GDP. Prescott (2004) and Rogerson (2008) among many others have studied the effects of increasing the labor income tax. This tax reduces the wage net of taxes and this causes the reduction of the labor supply when individuals can substitute leisure for consumption goods. In fact, the effects of labor taxes crucially depend on the substitution between leisure and the other consumption goods. As recreational activities crucially modify this substitution, the effects of taxes are modified when recreational activities are considered. To study this differential impact of labor income taxes, in this section we compare the effect of a tax increase in Economies 1 and 2. We follow Prescott (2004) and we consider the consequence of increasing the effective labor income tax from the US average level, 40%, to the French average level, 59%. For the sake of simplicity, we assume that government revenues are returned to the individuals as a lump-sum subsidy. It follows that labor income taxes only modify the labor supply implicitly obtained in (3.2.13) and (3.3.8). The wage in these two equations should be replaced by the wage net of taxes.

We calibrate Economies 1 and 2 so that they explain employment and the sectoral composition when taxes are at the US level. Table 4 provides the new values of the parameters of these two economies.

[Insert Table 4]

Figure 9 shows the effects of this tax increase in Economy 2, where individuals directly derive utility from leisure. The tax increase reduces employment both initially and during the transition. The initial reduction of 1.5% is explained by the reduction in the wage net of taxes. This implies that GDP also decreases initially. This lower GDP reduces capital accumulation, which, in turn, reduces even further employment and GDP during the transition. As an example, In 2010, the employment and GDP loss due to the increase in taxes is around 3%.

[Insert Figure 9]

Figure 10 studies the effects of the tax increase in Economy 1, where individuals derive utility from recreational activities and not from the direct consumption of leisure. As follows from this figure, the effects on employment and GDP are substantially larger. Initially employment and

GDP decrease about 3%. This larger initial reduction of GDP causes a larger reduction in capital accumulation, which, in turn, implies a larger GDP loss during the transition. In 2010, the employment and GDP loss is about 6%. It follows that the effect of taxes on both employment and GDP is twice larger when we take into account that individuals derive utility from leisure through the consumption of recreational activities. These activities introduce the possibility that individuals can substitute leisure time for services. This increases the substitutability of leisure for other consumption goods. As a consequence, after the tax increase, agents can substitute to a larger extend leisure for consumption goods. This explains the larger impact that a tax increase has when we consider recreational activities.

[Insert Figure 10]

3.6 Concluding remarks

The purpose of this paper is to explain two important patterns of structural change observed during the second half of the last century; first, the large shift of employment and production from the agriculture and manufacturing sectors to the service sector, and, second, the sustained increase in leisure time. We contribute to existing literature on structural change by introducing a mechanism that relates these two patterns of structural change. We argue that during leisure time we consume recreational services. The observed increase in leisure time then implies an increase in the consumption of these services, which introduces a mechanism explaining structural change in the sectoral composition of employment. We measure the relevance of this mechanism and we make two contributions. First, we measure the fraction of the value added of the service sector explained by recreational services. We show that this fraction has increased from 6% to 14% during the period 1947-2010. Obviously, this substantial increase provides support to our mechanism, which has a sizeable effect on sectoral composition. Indeed, we show that 26% of the observed increase in the added value share of the service sector is explained by the increase in recreational services. Second, we construct a multi-sector exogenous growth model with biased technological change. The new feature of the model is the introduction of recreational activities, which depend on both leisure time and on the consumption of recreational services. We introduce these activities by assuming a nested CES utility function. We show that biased technological progress drives structural change through two different channels: a wealth channel and a substitution channel. We calibrate the model and we show that the model explains the reduction in the time devoted to work, the increase in the fraction of recreational services and the main changes in the sectoral composition of employment. Moreover, we compare the performance of our economy with recreational activities with the performance of other economies without these activities. We show that the introduction of recreational activities improves the performance of the simulated economies in explaining both the reduction in the amount of labor and the process of structural change. We then conclude that recreational activities are an important feature of structural change.

There are large differences in the amount of time devoted to work between the US and European economies. Prescott (2004) has convincingly argued that large part of these differences can be explained by the differences between the labor income taxes in the US and Europe. These taxes reduce employment and, as a consequence, they also reduce GDP. In this paper, we show that the effect of a tax increase on the amount of employment and on the level of GDP is substantially larger when we assume that individuals derive utility from recreational activities. These activities increase the substitution of leisure by other consumption goods, which explains the larger impact of fiscal policy.

Bibliography

[1] Abraham, K., Spletzer, J. and Stewart, J. (1998). “Divergent Trends in Alternative Wage Series.” In John Haltiwanger, Marilyn E. Manser, and Robert Topel, editors. Labor Statistics Measurement Issues. University of Chicago Press.

[2] Acemoglu, D. , Guerrieri, V. (2008). Capital deepening and non-balanced economic growth. Journal of Political Economy, 116, 467-498.

[3] Aguiar, M. and Hurst, E. (2007). Measuring trends in leisure: the allocation of time over five decades, Quarterly Journal of Economics, 122, 969-1006.

[4] Aguiar, M., Hurst, E. and Karabarbounis, L. (2013). Time Use during the Great Recession. American Economic Review, 103, 1664-96.

[5] Alonso-Carrera, J., Caballé, J. and Raurich, X. (2015). Sectoral Composition and Macroeconomic Dynamics. BE J. Macroecon. Advances, 15, 1-42,

[6] Boppart, T. (2014). Structural change and the Kaldor facts in a growth model with relative price effects and non-Gorman preferences, Econometrica, 82, 2167-2196.

[7] Buera, F. and Kaboski, J. (2012). Scale and the origins of structural change, Journal of Economic Theory, 147, 684-712.

[8] Dennis, B. and Iscan, T. (2009). Engel versus Baumol: accounting for sectoral change using two centuries of US data. Explorations in Economic History, 46, 186-202.

[9] Francis, N. and Ramey, V. (2009). A century of work and leisure, American Economic Journal: Macroeconomics, 1, 189-224.

[10] Foellmi, R. and Zweimüller, J. (2008). Structural change, Engel's consumption cycles and Kaldor's facts of economic growth. Journal of Monetary Economics, 55,1317-1328.

[11] Gollin, D., Parente, S. and Rogerson, R. (2004). Homework, farmwork, and international differences in productivity, Review of Economic Dynamics, 7, 827-850.

[12] Herrendorf, B. , Rogerson, R. and Valentinyi, Á. (2013). Two Perspectives on Preferences and Structural Transformation. American Economic Review, 103, 2752-2789.

[13] Herrendorf, B. ., Rogerson, R. and Valentinyi, Á. (2014). Growth and Structural Transformation, Handbook of Economic Growth, in: Handbook of Economic Growth, edition 1, volume 2, chapter 6, pages 855-941 Elsevier.

[14] Kongsamunt, P., Rebelo, S., Xie, D. (2001). Beyond balanced growth. Review of Economic Studies, 68, 869-882.

[15] Ngai, L. R., Pissariades, C. (2007). Structural change in a multisector model of growth. American Economic Review, 97, 429-443.

[16] Ngai, L. R., Pissariades, C. (2008). Trends in hours and economic growth, Review of Economic Dynamics, 11, 239-256.

[17] Prescott, E.C. (2004). Why do Americans work so much more than Europeans? Federal Reserve Bank of Minneapolis Quarterly Review, 2-13, 28.

[18] Rogerson, R. (2008). Structural transformation and the deterioration of European labor market outcomes, Journal of Political Economy, 116, 235-259.

[19] Sato, K. (1967). A Two-Level Constant-Elasticity-of-Substitution Production Function, Review of Economic Studies, 43, 201-218.

[20] M.P. Timmer, G.J .de Vries, and K.deVries (2014). Patterns of Structural Change in Developing Countries. GGDC research memorandum 149.

Appendix

3.A Solution of the consumers' problem

Consumers maximize the utility function subject to the budget constraint (3.2.6). The Hamiltonian present value associated to this maximization problem is

$$\mathcal{H} = \ln C + \lambda \left(wl + rk - c_m - p_s c_s - p_a c_a \right).$$

The first order conditions with respect to x, c_a, c_m, c_s, l and k are, respectively,

$$\frac{x^{-\frac{1}{\varepsilon}}}{(1-x)^{-\frac{1}{\sigma}}} c_s^{\frac{\varepsilon-\sigma}{\varepsilon\sigma}} = \left(\frac{\eta_l \beta}{\eta_s} \right) c_l^{\frac{\varepsilon-\sigma}{\varepsilon\sigma}}, \tag{3.A.1}$$

$$C^{\frac{1-\varepsilon}{\varepsilon}} \eta_a c_a^{-\frac{1}{\varepsilon}} = \lambda p_a, \tag{3.A.2}$$

$$C^{\frac{1-\varepsilon}{\varepsilon}} \eta_m c_m^{-\frac{1}{\varepsilon}} = \lambda, \tag{3.A.3}$$

$$C^{\frac{1-\varepsilon}{\varepsilon}} \eta_s (x c_s)^{-\frac{1}{\varepsilon}} = \lambda p_s, \tag{3.A.4}$$

$$C^{\frac{1-\varepsilon}{\varepsilon}} \eta_l \psi c_l^{\frac{\varepsilon-\sigma}{\varepsilon\sigma}} \left(1-\beta\right) \left(1 - \bar{l} - l\right)^{\frac{(\sigma-1)\psi-\sigma}{\sigma}} = \lambda w, \tag{3.A.5}$$

and

$$\dot{\lambda} = -\left(r - \rho\right)\lambda. \tag{3.A.6}$$

We proceed to obtain c_l, c_s, c_m, c_a, l, and x as functions of prices, wages and total consumption expenditures, E, where $E = c_m + p_a c_a + p_s c_s$. To this end, we combine (3.A.3), (3.A.2) and (3.A.4) to obtain (3.2.9), (3.2.10) and (3.2.11) in the main text, where the function κ_1 in this equations is

$$\kappa_1 = 1 + p_a \left(p_a \frac{\eta_m}{\eta_a} \right)^{-\varepsilon} + p_s \left(p_s \frac{\eta_m}{\eta_s} \right)^{-\varepsilon} \frac{1}{x}. \tag{3.A.7}$$

We next use (3.A.4), (3.A.5) and (3.A.1) to obtain

$$(1-x)\,c_s = \left(\frac{\psi\left(1-\beta\right)p_s}{w\beta}\right)^{-\sigma}\left(1-\bar{l}-l\right)^{(1-\sigma)\psi+\sigma}. \tag{3.A.8}$$

We substitute (3.A.8) in (3.2.8) to obtain

$$c_l = \kappa_2\,(1-l)^{\psi}, \tag{3.A.9}$$

where

$$\kappa_2 = \left[\beta\left(\frac{\psi\left(1-\beta\right)p_s}{w\beta}\right)^{1-\sigma}\left(1-\bar{l}-l\right)^{(\sigma-1)(1-\psi)} + 1-\beta\right]^{\frac{\sigma}{\sigma-1}}.$$

κ_2 can be rewritten as

$$\kappa_2 = \kappa_3\left(\frac{w}{1-\beta}\right)^{\sigma}, \tag{3.A.10}$$

where

$$\kappa_3 = \left(\beta^{\sigma}\left(\psi p_s\right)^{1-\sigma}\left(1-\bar{l}-l\right)^{(\sigma-1)(1-\psi)} + \left(1-\beta^{\sigma}\right)w^{1-\sigma}\right)^{\frac{\sigma}{\sigma-1}}. \tag{3.A.11}$$

From combining (3.A.3), (3.A.5) and (3.A.9), we obtain (3.2.13) in the main text. We combine (3.A.9), (3.A.1), (3.2.13) and (3.2.10) to obtain (3.2.12) in the main text. The expression of the function κ_4 in equation (3.2.12) is

$$\kappa_4 = \left(\frac{\eta_l\beta}{\eta_s}\right)^{-\sigma}\left(\frac{\eta_s}{p_s\eta_m}\right)^{\varepsilon-\sigma}\left(\frac{w\eta_m}{\eta_l\left(1-\beta\right)\psi}\right)^{\frac{\psi(\varepsilon-\sigma)}{(1-\varepsilon)\psi+\varepsilon}} c_m^{\frac{(\sigma-\varepsilon)(\psi-1)}{(1-\varepsilon)\psi+\varepsilon}}\,\kappa_2^{\left(\frac{(1-\sigma)\psi+\sigma}{(1-\varepsilon)\psi+\varepsilon}\right)\left(\frac{\sigma-\varepsilon}{\sigma}\right)}. \tag{3.A.12}$$

In what follows, we obtain the expression of the Euler condition. To this end, we first use (3.A.9) and (3.2.13) to obtain

$$c_l = \kappa_2^{\left(\frac{\sigma+\psi(1-\sigma)}{(1-\varepsilon)\psi+\varepsilon}\right)\frac{\varepsilon}{\sigma}}\left(\frac{w\eta_m}{\eta_l\left(1-\beta\right)\psi}\right)^{-\frac{\varepsilon\psi}{(1-\varepsilon)\psi+\varepsilon}} c_m^{\frac{\psi}{(1-\varepsilon)\psi+\varepsilon}}. \tag{3.A.13}$$

We next substitute (3.2.11), (3.2.10) and (3.A.13) in the definition of C to obtain

$$\left(\frac{C}{c_m}\right)^{\frac{\varepsilon-1}{\varepsilon}}\frac{1}{\eta_m} = \kappa_6,$$

where

$$\kappa_6 = \overline{\eta}_a p_a^{1-\varepsilon} + 1 + \overline{\eta}_s p_s^{1-\varepsilon} + \overline{\eta}_l {\kappa_5}^{1-\varepsilon}, \tag{3.A.14}$$

$$\kappa_5 = \kappa_2^{-\left(\frac{\sigma+\psi(1-\sigma)}{(1-\varepsilon)\psi+\varepsilon}\right)\frac{1}{\sigma}} \left(\frac{w\eta_m}{\eta_l(1-\beta)\psi}\right)^{\frac{\psi}{(1-\varepsilon)\psi+\varepsilon}} c_m^{\frac{-(\psi-1)}{(1-\varepsilon)\psi+\varepsilon}}, \tag{3.A.15}$$

$\overline{\eta}_a = (\eta_a/\eta_m)^{\varepsilon}$, $\overline{\eta}_s = (\eta_s/\eta_m)^{\varepsilon}$, and $\overline{\eta}_l = (\eta_l/\eta_m)^{\varepsilon}$. We rewrite (3.A.3) and substitute the previous relations to obtain

$$\lambda = \frac{1}{\kappa_6 c_m}. \tag{3.A.16}$$

From using (3.2.9), we obtain

$$\frac{1}{\lambda} = \kappa_7 E, \tag{3.A.17}$$

where

$$\kappa_7 = \frac{\kappa_6}{1 + \overline{\eta}_a p_a^{1-\varepsilon} + \overline{\eta}_s p_s^{1-\varepsilon}\frac{1}{x}}. \tag{3.A.18}$$

Finally, from combining (3.A.6) and (3.A.16), the Euler condition (3.2.14) in the main text is obtained.

Note that equations (3.2.9)-(3.2.14) in the main text depend on $\{\kappa_i\}_{i=1}^{7}$. From using (3.A.7), (3.A.10), (3.A.11), (3.A.12), (3.A.15), (3.A.14) and (3.A.18) it follows that $\{\kappa_i\}_{i=1}^{7}$ are functions only of the relative price and the wage. This implies that x only depends on prices and the wage.

3.B System of Differential equations

In this appendix we obtain the system of differential equations governing the time path of the variables z and q. The fist step is to obtain the expression of $\dot{\kappa}_7/\kappa_7$. We first combine (3.A.18) and (3.A.12) to obtain

$$\kappa_7 = \frac{\kappa_6}{1 + \overline{\eta}_a p_a^{1-\varepsilon} + \overline{\eta}_s p_s^{1-\varepsilon} + \beta^{\sigma} p_s^{1-\sigma}\omega_0}, \tag{3.B.1}$$

where

$$\omega_0 = \frac{\overline{\eta}_s}{\beta^{\sigma}}\frac{p_s^{\sigma-\varepsilon}}{\kappa_4},$$

and from using (3.A.12) it follows that

$$\omega_0 = \overline{\eta}_l^{\frac{\sigma+(1-\sigma)\psi}{(1-\varepsilon)\psi+\varepsilon}} \left[\left(\frac{1}{\psi}\right)^{\psi}\left(\frac{w}{1-\beta}\right)^{\sigma(\psi-1)}\right]^{\frac{\sigma-\varepsilon}{(1-\varepsilon)\psi+\varepsilon}} c_m^{-\frac{(\sigma-\varepsilon)(\psi-1)}{(1-\varepsilon)\psi+\varepsilon}} \kappa_3^{-\left(\frac{(1-\sigma)\psi+\sigma}{(1-\varepsilon)\psi+\varepsilon}\right)\left(\frac{\sigma-\varepsilon}{\sigma}\right)}, \tag{3.B.2}$$

We log-differentiate with respect to time the previous equation to obtain

$$\begin{aligned}\frac{\dot{\kappa}_7}{\kappa_7} &= \frac{\dot{\kappa}_6}{\kappa_6} - \frac{(1-\varepsilon)\overline{\eta}_a p_a^{(1-\varepsilon)}\left(\gamma_m-\gamma_a\right)}{1+\overline{\eta}_a p_a^{1-\varepsilon}+\overline{\eta}_s p_s^{1-\varepsilon}+\beta^\sigma p_s^{1-\sigma}\omega_0} \\ &\quad - \frac{\left[(1-\varepsilon)\overline{\eta}_s p_s^{(1-\varepsilon)}+(1-\sigma)\,p_s^{1-\sigma}\beta^\sigma\omega_0\right]\left(\gamma_m-\gamma_s\right)}{1+\overline{\eta}_a p_a^{1-\varepsilon}+\overline{\eta}_s p_s^{1-\varepsilon}+\beta^\sigma p_s^{1-\sigma}\omega_0} \\ &\quad - \left(\frac{\beta^\sigma p_s^{1-\sigma}\omega_0}{1+\overline{\eta}_a p_a^{1-\varepsilon}+\overline{\eta}_s p_s^{1-\varepsilon}+\beta^\sigma p_s^{1-\sigma}\omega_0}\right)\frac{\dot{\omega}_0}{\omega_0}.\end{aligned} \tag{3.B.3}$$

From using (3.B.2), we get

$$\frac{\dot{\omega}_0}{\omega_0} = \frac{\sigma\left(\psi-1\right)(\sigma-\varepsilon)}{(1-\varepsilon)\,\psi+\varepsilon}\frac{\dot{w}}{w} - \frac{(\sigma-\varepsilon)\left(\psi-1\right)}{(1-\varepsilon)\,\psi+\varepsilon}\frac{\dot{c}_m}{c_m} - \left(\frac{(1-\sigma)\,\psi+\sigma}{(1-\varepsilon)\,\psi+\varepsilon}\right)\left(\frac{\sigma-\varepsilon}{\sigma}\right)\frac{\dot{\kappa}_3}{\kappa_3}.$$

We next use (3.A.16) and (3.A.6) to obtain

$$\frac{\dot{c}_m}{c_m} = r-\rho-\frac{\dot{\kappa}_6}{\kappa_6}.$$

The growth rate of wages is obtained from (3.3.2) and it is

$$\frac{\dot{w}}{w} = \frac{\gamma_m}{1-\alpha}+\alpha\frac{\dot{z}}{z},$$

and we use (3.A.11) and (3.2.4) to obtain

$$\frac{\dot{\kappa}_3}{\kappa_3} = \omega_1+\omega_2\frac{\dot{z}}{z}+\omega_3\frac{\dot{l}}{1-l-\overline{l}}, \tag{3.B.4}$$

where

$$\omega_1 = -\sigma\left(\frac{\beta^\sigma\left(\psi p_s\right)^{1-\sigma}\left(1-l-\overline{l}\right)^{(\sigma-1)(1-\psi)}\left(\gamma_m-\gamma_s\right)+\left(1-\beta\right)^\sigma w^{1-\sigma}\left(\frac{\gamma_m}{1-\alpha}\right)}{\beta^\sigma\left(\psi p_s\right)^{1-\sigma}\left(1-l-\overline{l}\right)^{(\sigma-1)(1-\psi)}+\left(1-\beta\right)^\sigma w^{1-\sigma}}\right), \tag{3.B.5}$$

$$\omega_2 = -\frac{\sigma\left(1-\beta\right)^\sigma w^{1-\sigma}\alpha}{\beta^\sigma\left(\psi p_s\right)^{1-\sigma}\left(1-l-\overline{l}\right)^{(\sigma-1)(1-\psi)}+\left(1-\beta\right)^\sigma w^{1-\sigma}}, \tag{3.B.6}$$

and

$$\omega_3 = -\sigma\frac{\left(1-\psi\right)\beta^\sigma\left(\psi p_s\right)^{1-\sigma}\left(1-l-\overline{l}\right)^{(\sigma-1)(1-\psi)}}{\beta^\sigma\left(\psi p_s\right)^{1-\sigma}\left(1-l-\overline{l}\right)^{(\sigma-1)(1-\psi)}+\left(1-\beta\right)^\sigma w^{1-\sigma}}. \tag{3.B.7}$$

From using (3.A.15), we get

$$\frac{\dot{\kappa}_5}{\kappa_5} = \zeta_1 + \zeta_2\frac{\dot{z}}{z} + \zeta_3\frac{\dot{l}}{1-l-\bar{l}} + \zeta_4\frac{\dot{\kappa}_6}{\kappa_6},$$

where

$$\begin{aligned}
\zeta_1 &= -\left(\frac{\sigma+\psi(1-\sigma)}{(1-\varepsilon)\psi+\varepsilon}\right)\frac{\omega_1}{\sigma} + \left(\frac{\sigma\left(\psi-1\right)}{(1-\varepsilon)\psi+\varepsilon}\right)\frac{\gamma_m}{1-\alpha} - \zeta_4\left(r-\rho\right),\\
\zeta_2 &= -\left(\frac{\sigma+\psi(1-\sigma)}{(1-\varepsilon)\psi+\varepsilon}\right)\frac{\omega_2}{\sigma} + \left(\frac{\sigma\left(\psi-1\right)}{(1-\varepsilon)\psi+\varepsilon}\right)\alpha,\\
\zeta_3 &= -\left(\frac{\sigma+\psi(1-\sigma)}{(1-\varepsilon)\psi+\varepsilon}\right)\frac{\omega_3}{\sigma},\\
\zeta_4 &= -\frac{1-\psi}{(1-\varepsilon)\psi+\varepsilon}.
\end{aligned}$$

From using (3.A.14), we obtain

$$\frac{\dot{\kappa}_6}{\kappa_6} = \frac{(1-\varepsilon)\overline{\eta}_a p_a^{1-\varepsilon}\left(\gamma_m-\gamma_a\right) + (1-\varepsilon)\overline{\eta}_s p_s^{1-\varepsilon}\left(\gamma_m-\gamma_s\right) + (1-\varepsilon)\overline{\eta}_l {\kappa_5}^{1-\varepsilon}\frac{\dot{\kappa}_5}{\kappa_5}}{\kappa_6},$$

and we use (3.B.4) to obtain

$$\frac{\dot{\kappa}_6}{\kappa_6} = \omega_4 + \omega_5\frac{\dot{z}}{z} + \omega_6\frac{\dot{l}}{1-l-\bar{l}}, \tag{3.B.8}$$

where

$$\omega_4 = \frac{(1-\varepsilon)\overline{\eta}_a p_a^{1-\varepsilon}\left(\gamma_m-\gamma_a\right) + (1-\varepsilon)\overline{\eta}_s p_s^{1-\varepsilon}\left(\gamma_m-\gamma_s\right) + (1-\varepsilon)\overline{\eta}_l {\kappa_5}^{1-\varepsilon}\zeta_1}{\kappa_6 - (1-\varepsilon)\overline{\eta}_l {\kappa_5}^{1-\varepsilon}\zeta_4}, \tag{3.B.9}$$

$$\omega_5 = \frac{(1-\varepsilon)\overline{\eta}_l {\kappa_5}^{1-\varepsilon}\zeta_2}{\kappa_6 - (1-\varepsilon)\overline{\eta}_l {\kappa_5}^{1-\varepsilon}\zeta_4}, \tag{3.B.10}$$

and

$$\omega_6 = \frac{(1-\varepsilon)\overline{\eta}_l \kappa_5^{1-\varepsilon}\zeta_3}{\kappa_6 - (1-\varepsilon)\overline{\eta}_l {\kappa_5}^{1-\varepsilon}\zeta_4}.$$

We next rewrite

$$\frac{\dot{\omega}_0}{\omega_0} = \theta_1 + \theta_2\frac{\dot{z}}{z} + \theta_3\frac{\dot{l}}{1-l-\bar{l}},$$

where

$$\theta_1 = \frac{\sigma\left(\psi-1\right)(\sigma-\varepsilon)\gamma_m}{\left[(1-\varepsilon)\psi+\varepsilon\right](1-\alpha)} - \frac{(\sigma-\varepsilon)\left(\psi-1\right)\left(r-\rho-\omega_4\right)}{(1-\varepsilon)\psi+\varepsilon} - \frac{\left[(1-\sigma)\psi+\sigma\right]\omega_1(\sigma-\varepsilon)}{\left[(1-\varepsilon)\psi+\varepsilon\right]\sigma}, \tag{3.B.11}$$

$$\theta_2 = \frac{\sigma\left(\psi-1\right)(\sigma-\varepsilon)\alpha}{(1-\varepsilon)\psi+\varepsilon} + \frac{(\sigma-\varepsilon)\left(\psi-1\right)}{(1-\varepsilon)\psi+\varepsilon}\omega_5 - \left(\frac{(1-\sigma)\psi+\sigma}{(1-\varepsilon)\psi+\varepsilon}\right)\left(\frac{\sigma-\varepsilon}{\sigma}\right)\omega_2, \tag{3.B.12}$$

and

$$\theta_3 = \frac{(\sigma-\varepsilon)\left(\psi-1\right)}{(1-\varepsilon)\psi+\varepsilon}\omega_6 - \left(\frac{(1-\sigma)\psi+\sigma}{(1-\varepsilon)\psi+\varepsilon}\right)\left(\frac{\sigma-\varepsilon}{\sigma}\right)\omega_3. \tag{3.B.13}$$

We substitute (3.B.4) and (3.B.8) in (3.B.3) to obtain

$$\frac{\dot{\kappa}_7}{\kappa_7} = \omega_7 + \omega_8\frac{\dot{z}}{z} + \omega_9\frac{\dot{l}}{1-l-\bar{l}}, \tag{3.B.14}$$

where

$$\omega_7 = \omega_4 - \frac{(1-\varepsilon)\overline{\eta}_a p_a^{1-\varepsilon}\left(\gamma_m-\gamma_a\right) + \left[(1-\varepsilon)\overline{\eta}_s p_s^{1-\varepsilon} + (1-\sigma)\beta^\sigma p_s^{1-\sigma}\omega_0\right]\left(\gamma_m-\gamma_s\right)}{1+\overline{\eta}_a p_a^{1-\varepsilon}+\overline{\eta}_s p_s^{1-\varepsilon}+\beta^\sigma p_s^{1-\sigma}\omega_0} - \frac{\beta^\sigma p_s^{1-\sigma}\omega_0\theta_1}{1+\overline{\eta}_a p_a^{1-\varepsilon}+\overline{\eta}_s p_s^{1-\varepsilon}+\beta^\sigma p_s^{1-\sigma}\omega_0}, \tag{3.B.15}$$

$$\omega_8 = \omega_5 - \frac{\beta^\sigma p_s^{1-\sigma}\omega_0\theta_2}{1+\overline{\eta}_a p_a^{1-\varepsilon}+\overline{\eta}_s p_s^{1-\varepsilon}+\beta^\sigma p_s^{1-\sigma}\omega_0},$$

and

$$\omega_9 = \omega_6 - \frac{\beta^\sigma p_s^{1-\sigma}\omega_0\theta_3}{1+\overline{\eta}_a p_a^{1-\varepsilon}+\overline{\eta}_s p_s^{1-\varepsilon}+\beta^\sigma p_s^{1-\sigma}\omega_0}.$$

The second step is to obtain the growth rate of employment. We first combine (3.2.13) and (3.A.10) to obtain

$$1-l-\bar{l} = \left(\frac{w\eta_m}{\eta_l\left(1-\beta\right)\psi\kappa_2^{\frac{\varepsilon-\sigma}{\varepsilon\sigma}}}\right)^{-\frac{\varepsilon}{(1-\varepsilon)\psi+\varepsilon}} c_m^{\frac{1}{(1-\varepsilon)\psi+\varepsilon}}. \tag{3.B.16}$$

We use (3.A.16) and (3.A.17) to get

$$c_m = \frac{\kappa_7}{\kappa_6}E. \tag{3.B.17}$$

We combine (3.B.16) and (3.B.17) to obtain

$$1-l-\bar{l}=(1-\beta)^{\frac{\sigma}{(1-\varepsilon)\psi+\varepsilon}}\left(\psi^{\varepsilon}\bar{\eta}_{l}\right)^{\frac{1}{(1-\varepsilon)\psi+\varepsilon}}\kappa_{3}^{\frac{\varepsilon-\sigma}{((1-\varepsilon)\psi+\varepsilon)\sigma}}w^{\frac{-\sigma}{(1-\varepsilon)\psi+\varepsilon}}\left(\frac{\kappa_{7}}{\kappa_{6}}E\right)^{\frac{1}{(1-\varepsilon)\psi+\varepsilon}}, \quad (3.B.18)$$

and we log-differentiate this equation

$$\begin{aligned}-\frac{\dot{l}}{1-l-\bar{l}} &= \left(\frac{\varepsilon-\sigma}{((1-\varepsilon)\psi+\varepsilon)\sigma}\right)\frac{\dot{\kappa}_{3}}{\kappa_{3}}- \\ &\quad \frac{\sigma}{(1-\varepsilon)\psi+\varepsilon}\frac{\dot{w}}{w}+\frac{1}{(1-\varepsilon)\psi+\varepsilon}\left(\frac{\dot{E}}{E}+\frac{\dot{\kappa}_{7}}{\kappa_{7}}-\frac{\dot{\kappa}_{6}}{\kappa_{6}}\right).\end{aligned} \quad (3.B.19)$$

We substitute the growth rate of wages, (3.B.4), (3.B.8), (3.2.14) and (3.3.1) to rewrite (3.B.19) as follows

$$\frac{\dot{l}}{l}=-\left(\frac{1-l-\bar{l}}{l}\right)\left(\omega_{10}+\omega_{11}\frac{\dot{z}}{z}\right), \quad (3.B.20)$$

where

$$\omega_{10}=\frac{\left(\frac{\varepsilon-\sigma}{\sigma}\right)\omega_{1}-\sigma\left(\frac{\gamma_{m}}{1-\alpha}\right)+\alpha z^{\alpha-1}-\delta-\rho-\omega_{4}}{(1-\varepsilon)\psi+\varepsilon+\left(\frac{\varepsilon-\sigma}{\sigma}\right)\omega_{3}-\omega_{6}}, \quad (3.B.21)$$

and

$$\omega_{11}=\frac{\left(\frac{\varepsilon-\sigma}{\sigma}\right)\omega_{2}-\sigma\alpha-\omega_{5}}{(1-\varepsilon)\psi+\varepsilon+\left(\frac{\varepsilon-\sigma}{\sigma}\right)\omega_{3}-\omega_{6}}. \quad (3.B.22)$$

Finally, we proceed to obtain the system of differential equations governing the time path of z and q. We first use (3.3.3) and (3.3.4) to obtain

$$\frac{\dot{k}}{k}=(1-q)z^{\alpha-1}-\delta. \quad (3.B.23)$$

We combine (3.2.14) and (3.3.1) to obtain

$$\frac{\dot{E}}{E}=\alpha z^{\alpha-1}-\delta-\rho-\frac{\dot{\kappa}_{7}}{\kappa_{7}}. \quad (3.B.24)$$

From log-differentiating the definition of z and using (3.B.23) we obtain the dynamic equation for z

$$\frac{\dot{z}}{z}=(1-q)z^{\alpha-1}-\delta-\frac{\gamma_{m}}{1-\alpha}-\frac{\dot{l}}{l}, \quad (3.B.25)$$

which can be rewritten as

$$\frac{\dot{z}}{z}=\frac{l\left[\left(1-q\right)z^{\alpha-1}-\delta-\frac{\gamma_m}{1-\alpha}\right]+\left(1-l-\bar{l}\right)\omega_{10}}{l-\left(1-l-\bar{l}\right)\omega_{11}}.$$

We use (3.B.25) and (3.B.20) to obtain (3.3.9) in the main text and

$$\dot{l}=-l\left(1-l-\bar{l}\right)\left(\frac{\omega_{10}+\left[\left(1-q\right)z^{\alpha-1}-\delta-\frac{\gamma_m}{1-\alpha}\right]\omega_{11}}{l-\left(1-l-\bar{l}\right)\omega_{11}}\right). \tag{3.B.26}$$

From log-differentiating the definition of q and using (3.B.24), (3.B.14), (3.B.25) and (3.B.26) we obtain (3.3.10) in the main text.

$$\begin{aligned}\frac{\dot{q}}{q} &= \left(\omega_8+\alpha\right)\left(\delta+\frac{\gamma_m}{1-\alpha}\right)-\delta-\rho-\omega_7-\frac{\gamma_m}{1-\alpha}+\left[\alpha-\left(\omega_8+\alpha\right)\left(1-q\right)\right]z^{\alpha-1}\\ &\quad+\left\{\omega_9 l-\left(1-l-\bar{l}\right)\left[\omega_8-\left(1-\alpha\right)\right]\right\}\left(\frac{\omega_{10}+\left[\left(1-q\right)z^{\alpha-1}-\delta-\frac{\gamma_m}{1-\alpha}\right]\omega_{11}}{l-\left(1-l-\bar{l}\right)\omega_{11}}\right).\end{aligned}$$

3.C Balanced Growth Path

In order to obtain the BGP of this economy we will follow a four steps procedure. First, we will compute the long run values of prices. Second, we will obtain the long run values of the auxiliary variables κ_3, κ_6, and $\{\omega_i\}_{i=1}^{11}$. Third, we will compute the long run values of employment and of the transformed variables, z and q, and, finally, we will obtain the long run sectoral composition of the economy.

First, as $\gamma_a>\gamma_m>\gamma_s$, equations (3.2.5) and (3.3.2) imply that $\lim_{t\to\infty} w=\infty$, $\lim_{t\to\infty} p_s=\infty$ and $\lim_{t\to\infty} p_a=0$. Taking this into account, we obtain the long run values of the different auxiliary variables. We first use (3.A.15) and (3.2.13) to obtain

$$\kappa_5=\kappa_2^{-\frac{1}{\sigma}}\left(\frac{w\eta_m}{\eta_l\left(1-\beta\right)\psi}\right)\left(1-l-\bar{l}\right)^{1-\psi},$$

and from using (3.A.10) we obtain

$$\begin{aligned}\kappa_5 &= \kappa_3^{-\frac{1}{\sigma}}\left(\frac{\eta_m}{\eta_l\psi}\right)\left(1-l-\bar{l}\right)^{1-\psi} = \\ &= \left[\left(\frac{p_s}{w}\right)^{1-\sigma}\beta^{\sigma}+(1-\beta)^{\sigma}\left(\frac{\left(1-l-\bar{l}\right)^{(1-\psi)}}{\psi}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}\frac{\eta_m}{\eta_l}w.\end{aligned}$$

Thus,

$$\kappa_5^* = \begin{cases} \beta^{\frac{\sigma}{1-\sigma}}\frac{\eta_m}{\eta_l}p_s = \infty \text{ when } l^* = 1-\bar{l} \\ (1-\beta)^{\frac{\sigma}{1-\sigma}}\frac{\eta_m}{\psi\eta_l}w = \infty \text{ when } l^* = 0 \end{cases},$$

We next use (3.A.14) to obtain

$$\kappa_6^* = \begin{cases} \bar{\eta}_a p_a^{1-\varepsilon}+1+\left(\bar{\eta}_s+\bar{\eta}_l\left(\beta^{\frac{\sigma}{1-\sigma}}\frac{\eta_m}{\eta_l}\right)^{1-\varepsilon}\right)p_s^{1-\varepsilon} = \infty \text{ when } l^* = 1-\bar{l} \\ \bar{\eta}_a p_a^{1-\varepsilon}+1+\bar{\eta}_s p_s^{1-\varepsilon}+\bar{\eta}_l\left((1-\beta)^{\frac{\sigma}{1-\sigma}}\frac{\eta_m}{\psi\eta_l}\right)^{1-\varepsilon}w^{1-\varepsilon} = \infty \text{ when } l^* = 0 \end{cases},$$

and we use (3.A.11) to obtain $\kappa_3^* = 0$. From using (3.A.7) we obtain that $\kappa_1^* = \infty$ and from using (3.A.10) we obtain

$$\kappa_2^* = \begin{cases} \beta_l^{\frac{\sigma}{\sigma-1}} \text{ when } \sigma < 1 \\ \infty \text{ when } \sigma > 1 \end{cases}.$$

In order to obtain the long run value of κ_4, we rewrite (3.A.12) as

$$\kappa_4 = \left(\frac{\eta_l\beta}{\eta_s}\right)^{-\sigma}\left(\frac{\eta_s}{\eta_l(1-\beta)\psi}\right)^{\varepsilon-\sigma}\left[\beta\left(\frac{\psi(1-\beta)}{\beta}\right)^{1-\sigma}+(1-\beta)\left(\frac{\left(1-l-\bar{l}\right)^{1-\psi}w}{p_s}\right)^{1-\sigma}\right]^{\frac{\sigma-\varepsilon}{\sigma-1}}.$$

Note first that $\frac{\left(1-l-\bar{l}\right)^{1-\psi}w}{p_s}$ diverges to infinite. To see this, note that the growth rate of this term in the long run is

$$(1-\psi)\left(\frac{-\dot{l}}{1-l-\bar{l}}\right)+\frac{\dot{w}}{w}-\frac{\dot{p}_s}{p_s} = \frac{\gamma_m}{1-\alpha}-(\gamma_m-\gamma_s) > 0.$$

Then, we obtain that

$$\kappa_4^* = \begin{cases} \infty \text{ if } \sigma < 1 \text{ and } \varepsilon > \sigma \\ 0 \text{ if } \sigma < 1 \text{ and } \varepsilon < \sigma \\ \left(\frac{\eta_s}{\beta\eta_l}\right)^{\varepsilon}\beta^{\left(\frac{\sigma-\varepsilon}{\sigma-1}\right)} \text{ if } \sigma > 1 \end{cases}.$$

From (3.B.5) and (3.B.6) , we obtain

$$\omega_1^* = \begin{cases} -\frac{\sigma\gamma_m}{1-\alpha} & if \quad \sigma < 1 \\ -\sigma\left(\gamma_m - \gamma_s\right) & if \quad \sigma > 1 \end{cases},$$

$$\omega_2^* = \begin{cases} -\alpha\sigma & if \quad \sigma < 1 \\ 0 & if \quad \sigma > 1 \end{cases},$$

and

$$\omega_3^* = \begin{cases} 0 & if \quad \sigma < 1 \\ -\sigma\left(1-\psi\right) & if \quad \sigma > 1 \end{cases}.$$

Next, we obtain

$$\begin{aligned}
\zeta_1^* &= -\left(\frac{\sigma + \psi(1-\sigma)}{(1-\varepsilon)\psi + \varepsilon}\right)\frac{\omega_1^*}{\sigma} + \left(\frac{\sigma\left(\psi - 1\right)}{(1-\varepsilon)\psi + \varepsilon}\right)\frac{\gamma_m}{1-\alpha} - \zeta_4^*\left(r-\rho\right), \\
\zeta_2 &= -\left(\frac{\sigma + \psi(1-\sigma)}{(1-\varepsilon)\psi + \varepsilon}\right)\frac{\omega_2^*}{\sigma} + \left(\frac{\sigma\left(\psi - 1\right)}{(1-\varepsilon)\psi + \varepsilon}\right)\alpha, \\
\zeta_3 &= -\left(\frac{\sigma + \psi(1-\sigma)}{(1-\varepsilon)\psi + \varepsilon}\right)\frac{\omega_3^*}{\sigma}, \\
\zeta_4^* &= -\frac{1-\psi}{(1-\varepsilon)\psi + \varepsilon}.
\end{aligned}$$

We use the long run values of κ_3 and κ_6 and equations (3.B.9) and (3.B.10) to obtain the long run values of ω_4

$$w_4^* = \begin{cases} (1-\varepsilon)\frac{\gamma_m}{1-\alpha} \quad if \ \sigma < 1 \text{ and } \varepsilon < 1 \\ (1-\varepsilon)\left[\left(\sigma + \psi(1-\sigma)\right)\left(\gamma_m - \gamma_s\right) + \frac{(1-\psi)(1-\sigma)\gamma_m}{1-\alpha}\right] \quad if \quad \sigma > 1 \text{ and } \varepsilon < 1 \\ 0 \quad if \quad \varepsilon > 1 \end{cases},$$

and of ω_5

$$\omega_5^* = \begin{cases} \psi(1-\varepsilon)\alpha \quad if \quad \sigma < 1 \text{and } \varepsilon < 1 \\ -\alpha\sigma(1-\varepsilon)\left(1-\psi\right) \quad if \quad \sigma > 1 \text{and } \varepsilon < 1 \\ 0 \ if \quad \varepsilon > 1 \end{cases}.$$

We next use (3.A.14) to obtain the long run value of ω_6

$$\omega_6^* = \begin{cases} 0 \text{ if } \varepsilon > 1 \text{ or } \varepsilon < 1 \text{ and } \sigma < 1 \\ \left(1-\psi\right)(1-\varepsilon)\left[\sigma + \psi(1-\sigma)\right] \text{ if } \varepsilon < 1 \text{ and } \sigma > 1 \end{cases}.$$

We also obtain the following long run values:

$$\theta_1^* = \frac{\sigma\left(\psi - 1\right)(\sigma - \varepsilon)}{(1-\varepsilon)\psi + \varepsilon}\left(\frac{\gamma_m}{1-\alpha}\right) - \frac{(\sigma - \varepsilon)\left(\psi - 1\right)}{(1-\varepsilon)\psi + \varepsilon}\left(r - \rho - \omega_4^*\right) - \left(\frac{(1-\sigma)\psi + \sigma}{(1-\varepsilon)\psi + \varepsilon}\right)\left(\frac{\sigma - \varepsilon}{\sigma}\right)\omega_1^*,$$

$$\theta_2^* = \frac{\sigma\left(\psi-1\right)(\sigma-\varepsilon)}{(1-\varepsilon)\psi+\varepsilon}\alpha + \frac{(\sigma-\varepsilon)\left(\psi-1\right)}{(1-\varepsilon)\psi+\varepsilon}\omega_5^* - \left(\frac{(1-\sigma)\psi+\sigma}{(1-\varepsilon)\psi+\varepsilon}\right)\left(\frac{\sigma-\varepsilon}{\sigma}\right)\omega_2^*,$$

and

$$\theta_3^* = \frac{(\sigma-\varepsilon)\left(\psi-1\right)}{(1-\varepsilon)\psi+\varepsilon}\omega_6^* - \left(\frac{(1-\sigma)\psi+\sigma}{(1-\varepsilon)\psi+\varepsilon}\right)\left(\frac{\sigma-\varepsilon}{\sigma}\right)\omega_3^*.$$

From using (3.B.2), (3.B.16) and (3.A.10) we obtain

$$\omega_0^* = \begin{cases} \infty \text{ when } \sigma > \varepsilon \\ 0 \text{ when } \sigma < \varepsilon \end{cases}, \tag{3.C.1}$$

$$w_7^* = \begin{cases} \omega_4^* - (1-\sigma)\left(\gamma_m-\gamma_s\right) - \theta_1 \text{ if } \varepsilon < 1, \sigma < 1 \text{ and } \varepsilon < \sigma \\ \omega_4^* - (1-\varepsilon)\left(\gamma_m-\gamma_s\right) \text{ if } \varepsilon < 1, \sigma < 1 \text{ and } \varepsilon > \sigma \\ \omega_4^* - \frac{(1-\varepsilon)\overline{\eta}_s + (1-\sigma)\overline{\eta}_l \beta^{\left(\frac{1-\varepsilon}{1-\sigma}\right)\sigma}}{\overline{\eta}_l \beta^{\left(\frac{1-\varepsilon}{1-\sigma}\right)\sigma}}\left(\gamma_m-\gamma_s\right) - \theta_1 \text{if } \varepsilon < 1 \text{ and } \sigma > 1 \\ \omega_4^* - (1-\varepsilon)\left(\gamma_m-\gamma_a\right) \text{ if } \varepsilon > 1 \end{cases},$$

$$\omega_8^* = \begin{cases} \omega_5^* - \theta_2 \text{ if either } \varepsilon < 1 \text{ and } \sigma > 1 \text{ or } \varepsilon < 1, \sigma < 1 \text{ and } \varepsilon < \sigma \\ \omega_5^* \text{ if } \varepsilon > 1 \text{ or } \varepsilon < 1, \sigma < 1 \text{ and } \varepsilon > \sigma \end{cases},$$

and

$$\omega_9^* = \begin{cases} \omega_6^* - \theta_3 \text{ if either } \varepsilon < 1 \text{ and } \sigma > 1 \text{ or } \varepsilon < 1, \sigma < 1 \text{ and } \varepsilon < \sigma \\ \omega_6^* \text{ if either } \varepsilon > 1 \text{ or } \varepsilon < 1, \sigma < 1 \text{ and } \varepsilon > \sigma \end{cases}.$$

Using the previous results and (3.B.21) we obtain

$$\omega_{10}^* = \begin{cases} \frac{\alpha z^{\alpha-1} - \delta - \rho - \frac{\gamma m}{1-\alpha}}{(1-\varepsilon)\psi+\varepsilon} \text{ if } \varepsilon < 1 \text{ and } \sigma < 1 \\ \frac{\alpha z^{\alpha-1} - \delta - \rho - \left[(1-\varepsilon)\left(\psi-1\right)+1\right](1-\sigma)\left(\gamma_m-\gamma_s\right) - \left[(1-\varepsilon)\left(1-\psi\right)(1-\sigma)+\sigma\right]\left(\frac{\gamma m}{1-\alpha}\right)}{(1-\varepsilon)\psi+\varepsilon-\left[(\varepsilon-\sigma)+(1-\varepsilon)\left(\sigma+\psi(1-\sigma)\right)\right]\left(1-\psi\right)} \text{ if } \varepsilon < 1 \text{ and } \sigma > 1 \\ \frac{-\varepsilon\left(\frac{\gamma m}{1-\alpha}\right) + \alpha z^{\alpha-1} - \delta - \rho - (1-\varepsilon)\left(\gamma_m-\gamma_a\right)}{(1-\varepsilon)\psi+\varepsilon} \text{ if } \varepsilon > 1 \text{ and } \sigma < 1 \\ \frac{-(\varepsilon-\sigma)\left(\gamma_m-\gamma_s\right) - \sigma\left(\frac{\gamma m}{1-\alpha}\right) + \alpha z^{\alpha-1} - \delta - \rho - (1-\varepsilon)\left(\gamma_m-\gamma_a\right)}{(1-\varepsilon)\psi+\varepsilon-(\varepsilon-\sigma)\left(1-\psi\right)} \text{ if } \varepsilon > 1 \text{ and } \sigma > 1 \end{cases},$$

and using (3.B.22) we obtain

$$\omega_{11}^* = \begin{cases} -\alpha \text{ if } \varepsilon < 1 \text{ and } \sigma < 1 \\ -\sigma\alpha\left(\frac{1-(1-\varepsilon)\left(1-\psi\right)}{(1-\varepsilon)\psi+\varepsilon-(\varepsilon-\sigma)\left(1-\psi\right)-(1-\varepsilon)\left(\sigma+\psi(1-\sigma)\right)\left(1-\psi\right)}\right) \text{ if } \varepsilon < 1 \text{ and } \sigma > 1 \\ \frac{-\alpha\varepsilon-\omega_5}{(1-\varepsilon)\psi+\varepsilon} \text{ if } \varepsilon > 1 \text{ and } \sigma < 1 \\ -\sigma\alpha\left(\frac{1-(1-\varepsilon)\left(1-\psi\right)}{(1-\varepsilon)\psi+\varepsilon-(\varepsilon-\sigma)\left(1-\psi\right)}\right) \text{ if } \varepsilon > 1 \text{ and } \sigma > 1 \end{cases}.$$

We proceed to obtain the long run values of labor and of the transformed variables. We first use (3.B.18) and the definition of q to obtain

$$1-l-\bar{l}=\kappa_3^{\frac{\varepsilon-\sigma}{\sigma}\frac{1}{(1-\varepsilon)\psi+\varepsilon}}\left(\frac{w^{1-\sigma}}{\kappa_1}\right)^{\frac{1}{(1-\varepsilon)\psi+\varepsilon}}\left(\frac{\bar{\eta}_l\left(1-\beta\right)^{\sigma}\psi^{\varepsilon}ql}{1-\alpha}\right)^{\frac{1}{(1-\varepsilon)\psi+\varepsilon}}.$$

and from using (3.A.7) we obtain

$$1-l-\bar{l}=\kappa_3^{\frac{\varepsilon-\sigma}{\sigma}\frac{1}{(1-\varepsilon)\psi+\varepsilon}}\left(\frac{w^{1-\sigma}}{1+\overline{\eta}_a p_a^{1-\varepsilon}+\overline{\eta}_s p_s^{1-\varepsilon}+\beta^{\sigma}p_s^{1-\sigma}\omega_0}\right)^{\frac{1}{(1-\varepsilon)\psi+\varepsilon}}\left(\frac{\bar{\eta}_l\left(1-\beta\right)^{\sigma}ql\psi^{\varepsilon}}{1-\alpha}\right)^{\frac{1}{(1-\varepsilon)\psi+\varepsilon}}.$$

From using (3.C.1), we obtain

$$\left(1-l-\bar{l}\right)^{(1-\varepsilon)\psi+\varepsilon}$$
$$=\frac{\bar{\eta}_l(1-\beta)^{\sigma}q\psi^{\varepsilon}l}{(1-\alpha)\left(\frac{1}{w^{1-\varepsilon}}+\overline{\eta}_a\frac{p_a^{1-\varepsilon}}{w^{1-\varepsilon}}+\overline{\eta}_s\frac{p_s^{1-\varepsilon}}{w^{1-\varepsilon}}\right)\left[\beta^{\sigma}\left(\frac{\psi p_s}{w(1-l)^{(1-\psi)}}\right)^{1-\sigma}+(1-\beta)^{\sigma}\right]^{\frac{\sigma-\varepsilon}{\sigma-1}}+(1-\alpha)\beta^{\sigma}\frac{p_s^{1-\sigma}}{w^{1-\sigma}}\left(\frac{1}{\psi}\right)^{\sigma-\varepsilon}\overline{\eta}_l(1-l)^{(1-\psi)(\sigma-\varepsilon)}}.$$

From this expression, it can be shown that $l^* = 0$ if $\sigma < 1$ and $\varepsilon < 1$. Otherwise, $l^* = 1-\bar{l}$. These are the long run values obtained in Proposition 2.4. In what follows we assume that $\sigma < 1$ and $\varepsilon < 1$ so that $l^* = 0$. We assume that $\dot{z} = 0$ and $\dot{q} = 0$ in equations (3.3.9) and (3.3.10) to obtain the long run values z^* and q^* in Proposition 2.4.

In the last step, we obtain the long run sectoral composition. To this end, we first use (3.A.12) to obtain

$$\kappa_4^* = \begin{cases} \infty \text{ if } \sigma < 1 \text{ and } \varepsilon > \sigma \\ 0 \text{ if } \sigma < 1 \text{ and } \varepsilon < \sigma \end{cases},$$

and using (3.2.12) we obtain the long value of x^* shown in Proposition 2.5. We next use (3.A.7) and (3.3.5) and (3.3.6) to obtain

$$u_s = \frac{p_s^{1-\varepsilon}\overline{\eta}_s}{x+p_a^{1-\varepsilon}\overline{\eta}_a x+p_s^{1-\varepsilon}\overline{\eta}_s}q,$$

and

$$u_a = \frac{p_a^{1-\varepsilon}\overline{\eta}_a x}{x+p_a^{1-\varepsilon}\overline{\eta}_a x+p_s^{1-\varepsilon}\overline{\eta}_s}q.$$

We use these expressions to obtain the long run values of the employment shares that are displayed in Proposition 2.6.

3.D Working and Leisure hours and Recreational Services

In this section we discuss data sources and we explain the procedures that we follow to obtain the shares of total time devoted to work and leisure and the share of value added in the service sector explained by recreational services that we report in the main text.

3.D.1 Labor supply and leisure

Market working hours

We report the share of working hours on total available time in Figure 1. To obtain this share, we first need to compute average weekly working hours per employed person in the period 1947-2010. We use two data sets. The first one is the Census Data Population Survey (CPS). This survey provides information on average weekly working hours reported by the population above 16 years for the period 1947-1998 in an annual basis.[3] The second data set is provided by Aguiar and Hurst (2007) and Aguiar, Hurst, and Karabarbounis (2013). We will refer to them as AH. These authors link five major time-use surveys that all together provide the trends in the allocation of time in the period 1965 to 2010. Based on these surveys, AH provides information on average weekly working (market and non-market) hours, leisure, home production, personal care, and child care for the years 1965, 1975, 1985, 1993, 2003, and 2010. The following table shows the average market working hours per week obtained by AH.

Table D.1: Average market working hours per week

	1965	1975	1985	1993	2003	2010
Hours	35.98	33.79	32.67	33.22	31.71	30.54

Source: Aguiar and Hurts (2007) and Aguiar, Hurst, and Karabarbounis (2013).

[3]Sundstrom, William A. , "Average weekly hours worked in nonagricultural industries, by age, sex, and race: 1948–1999." Table Ba4597-4607 in Historical Statistics of the United States, Earliest Times to the Present: Millennial Edition, edited by Susan B. Carter, Scott Sigmund Gartner, Michael R. Haines, Alan L. Olmstead, Richard Sutch, and Gavin Wright. New York: Cambridge University Press, 2006.

To obtain the market working hours from 1947 to 2010 reported in Figure 1, we first interpolate linearly working hours between the initial and final years reported in the previous table in order to obtain the amount of hours worked in the market in the period 1965-2010. We then chain the CPS to AH data to obtain market working hours for the period prior to 1965. We joint together both data sets by rescaling the reported working hours in CPS to make its value in 1965 equal to the reported hours in AH. The following figure plots the trend in market working hours reported in CPS, AH and the chained time series.[4]

[Insert Figure 11]

Leisure

We compute the amount of hours devoted to leisure by using the weekly leisure hours reported by AH. Aguiar and Hurts (2007) show four different measures of leisure based on the type of activities related with non-working time. The first measure of leisure, denoted Leisure 1, is composed of average weekly hours devoted to sports, exercise, socialize, travel, reading, hobbies, tv, radio, entertainment, and other leisure activities. The second measure, denoted Leisure 2, is composed of Leisure 1 plus the average weekly hours devoted to sleep, eating and personal care. The third measure, Leisure 3, is composed of Leisure 2 and the average weekly hours devoted to child care. Finally, Leisure 4 is computed as the residual between the total available weekly time (168 hrs) and working hours (market and non-market).

In our model, individuals enjoy their leisure time together with the consumption of recreational services. As a consequence, the appropriate measure of leisure in our analysis is Leisure 1. The table below reports the data on this measure of leisure. We interpolate linearly these data to obtain a complete time series for the period 1965-2010.

[4]From visual inspections, it is clear that CPS overestimates the number of working hours compared with those reported by AH. According to Abraham et al. (1998), this discrepancy in working hours is due to the period in which the surveys are carried out. In the case of the CPS data, by design, the weeks in which they carried out the data collection avoids major holiday periods, periods in which average working hours tend to be lower. This implies an overestimation of working hours. By controlling for differences in the weeks that surveys are carried out, reported working hours tend to be similar between the time-use surveys and CPS. Due to CPS overestimate working hours and diary data is presumed to provide a higher reliable measure of how individuals allocate time, we choose AH dataset as a basis for our calculations.

Table D.2: Average hours devoted to leisure

	1965	1975	1985	1993	2003	2010
Hours	30.77	33.24	34.78	37.47	35.33	36.92

Source: Aguiar and Hurts (2007) and Aguiar, Hurst, and Karabarbounis (2013).

In order to obtain a measure of hours devoted to leisure prior to 1965 consistent with Leisure 1, we assume that any increase (or decrease) in market working hours from 1947 to 1964 is equivalent to an increase (or decrease) in time devoted to leisure. That is, we assume that there is a mirror effect between working and leisure hours. This assumption has the important implication that time devoted to personal care, child care, and home production remains constant. This is consistent with available evidence for the period after 1965, which shows that weekly hours devoted to these tasks have remained robustly constant. The following table shows the average weekly hours devoted to these activities reported in AH.[5] Beyond the decrease in the average hours in 1993, the figures for these 4 decades is roughly the same, which allows us to assume that this pattern remains prior to 1965. Based on this assumption, we compute the time series of weekly leisure hours using the average working hours for the period 1947-1964 to extend the time series reported in AH.

Table D.3: Average hours devoted to personal and child care, and home production

	1965	1975	1985	1993	2003	2010
Hours	101.25	100.97	100.55	97.31	100.96	100.54

Source: Based on Aguiar and Hurts (2007).

Note that both market working hours and leisure hours are not reported directly in the main text. Instead, these time series are reported as shares of total available time. In what follows, we show the procedure to obtain these shares.

Rescaling total available time

We first rescale the total time available as follows

$$T_s = T - (P_c + H_m + C_c) \equiv l_h + w_h,$$

[5] To obtain data in Table 6, we calculate the difference between total time in a week (168 hrs) and total time devote to market working hours, reported in Table 1, and hours devoted to Leisure 1.

where T_s is the re-scaled available time, T is the total available time in a week (168hrs), P_c is time spend on personal care, H_m is time devoted to home production, C_c is time spend on child care, l_h are the hours devoted to leisure and w_h are the hours devoted to work. Then, the shares of leisure and working hours on total available time are, respectively,

$$s_{l_h} = \frac{l_h}{T_s}, \quad s_{w_h} = \frac{l_h}{T_s}.$$

By using the computed time series of weekly hours of leisure and work, we can obtain the time series of s_{l_h} and s_{w_h}. Finally, we use the Hodrick-Prescott filter to remove business cycle changes. In this way, we obtain the smooth time series that capture the long run trends that we report in Figures 1 and 2.

3.D.2 Recreational Services

In this appendix we compute the share of value added in the service sector generated in recreational activities. To this end, we use information from input-output tables (IO) available for the period 1947-2010.[6] Using IO tables allows us to compute the added value of those industries that provide services that are consumed during leisure time.

In an ideal world, for the calculation of added value of recreational services, we would link every leisure activity to a commodity in the IO tables and, in turn, each commodity with a particular industry. In that hypothetical case, the sum of added values generated by industries corresponds to the added value of the recreational services sector. However, changes in the industrial classification during the period and lack for information in higher detailed industrial level for some years do not allow us to apply directly this strategy. To deal with these issues, we use the Bridge Tables published by the Bureau of Economic Analysis (BEA). The Bridge Tables provide the IO commodity codes that connect personal consumption estimates from the input-output accounts to the consumption categories (PCE) used by the National Income and Product Accounts (NIPA). We use these codes to identify recreational industries in the IO tables.[7] This procedure allows us to

[6]Bureau of Economic Analysis (BEA) publishes IO tables for the years 1947, 1958, 1963, 1967, 1972 1977, 1982, 1987, 1992, 1998. After 1998, IO tables are published annually from 1999 to 2010 . To download IO table prior to 1977, see http://www.bea.gov/industry/io_benchmark.htm . For the years after 1977, IO tables are available in http://www.bea.gov/industry/io_histsic.htm

[7]Those that provide services consumed during leisure time.

identify the industrial classification for industries that provide recreational services even in periods when reclassification of industries take place. Given the IO codes, we use the transaction matrix of the IO table to compute the added value of recreational industries.

Table D.4: SIC classification and IO code for Recreational Services (1967)

Leisure activities	PCE code	IO codes	SIC
Civic	9918	770500	84, 86, 8921
Sports & exercise	9830, 9900	760200	84
Entertainment, and hobbies	9820, 9910	760200	79
Socializing	9918	720300	723, 724
Travel	9400	720100	70, 81
Tv,radio, movies	9500, 9600, 9810	760100	78

The previous table shows an example of this procedure. Using PCE category codes, we assign a code to each of the activities that characterize our measure of leisure. For instance, sports and exercise activities, which cover all activities related to attend sport events and travel related to sports, are coded by 9830 and 9900 in the PCE classification in 1967. Based on the Bridge Table, the sub-industries that provide this kind of services are coded 760200 and classified into the sector 84 according to the Standard Industrial Classification (SIC) at that time. Using the IO table published by BEA in 1967, we can compute the added value of this sub-industry as well as the share of this sub-industry in the aggregate sector (84) that it belongs to. Following the same procedure for the others activities, we can compute the added value for all sub-industries. We then aggregate all added values from these industries to obtain the added value for the recreational services sector in 1967. We repeat the procedure described here for all the IO tables with high detail disaggregate industry levels.[8]

When the IO tables only provide data for more aggregate industrial level, i.e. two digits level, we compute the added value of recreational industries by using the computed weights of each sub-industry for years with detailed information. In order to reduce substantially any potential over or subestimation of real weights of sub-industries in the recreational

[8] IO tables with two-digit detail are those tables published in 1947 and 1958.

sector, we use the computed weights of the closest years from which we have available detailed IO tables.[9] Once we have computed the added value of the recreational sector, we then linearly interpolate between the years for which IO tables are available.

To obtain the share of value added in the service sector explained by the recreational sector that we report in Figure 5, we use the time series of added value personal consumption expenditure calculated by Herrendorf, Valentiny and Rogerson (2013). They calculate the added value of agricultural, industrial goods and services generated by personal consumption, which excludes government and transportation costs. Thus, we calculate the ratio between the added value of recreational services and the added value of the service sector generated by personal consumption to obtain the share reported in the paper. As before, we use the Hodrick-Prescott filter to eliminate any variation in data due to business cycles.

[9] Overestimation may be due to changes in the composition of demand for new leisure activities. For example, the introduction of the VCR in the 80's substantially changes the demand for cinema tickets. Using the computed weight of the cinema industry in added valued of recreational sector in the 60s to compute added value of cinema industry in the 90s overestimates the weight of this sector in the recreational sector.

3.E Figures and Tables

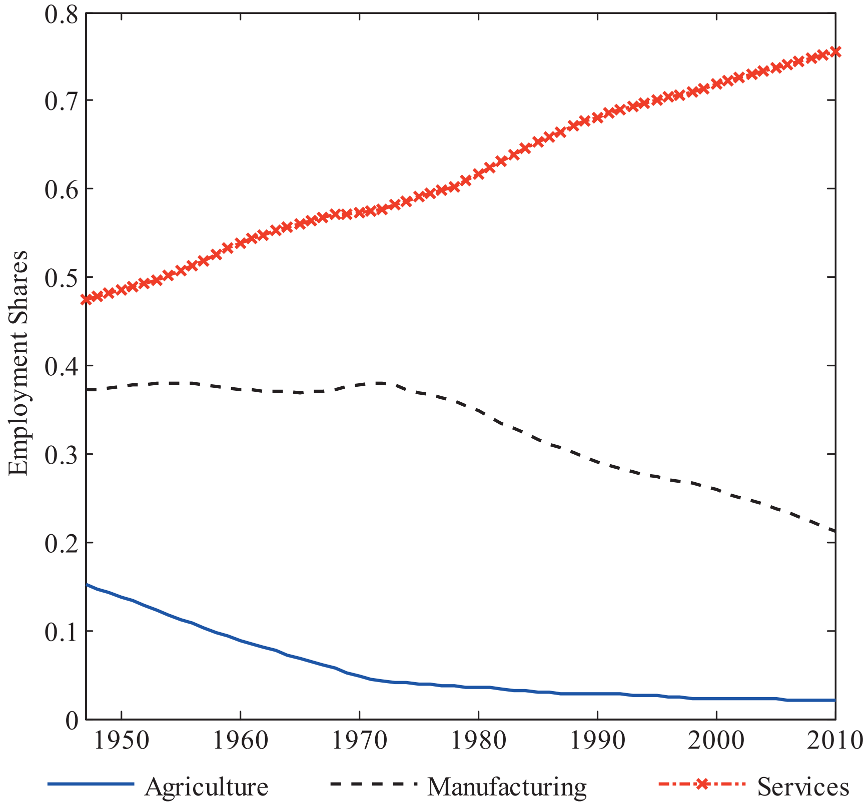

Figure 3.1: Sectoral composition of employment. Source. M.P. Timmer, G.J .de Vries, and K.deVries (2014).

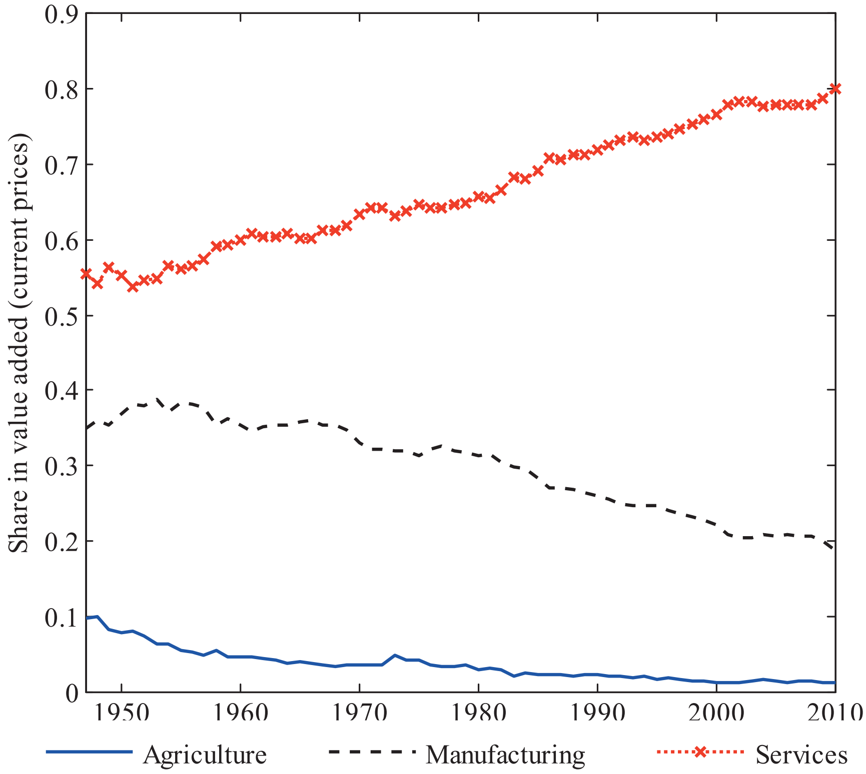

Figure 3.2: Sectoral composition of GDP. Source. M.P. Timmer, G.J .de Vries, and K.deVries (2014).

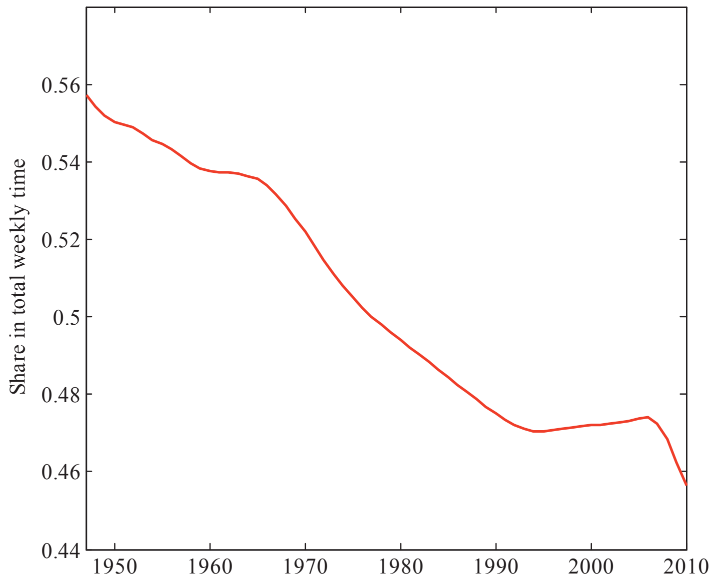

Figure 3.3: Time devoted to work. Own elaboration. See Appendix D

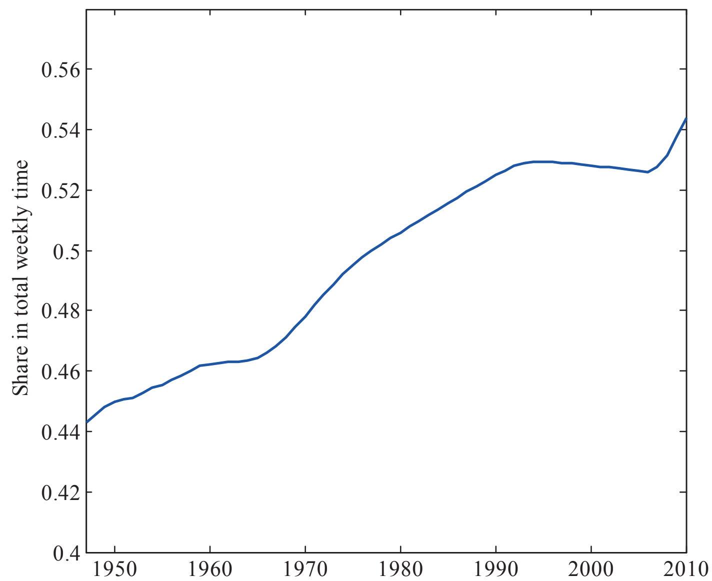

Figure 3.4: Time devoted to leisure. Own elaboration. See Appendix D

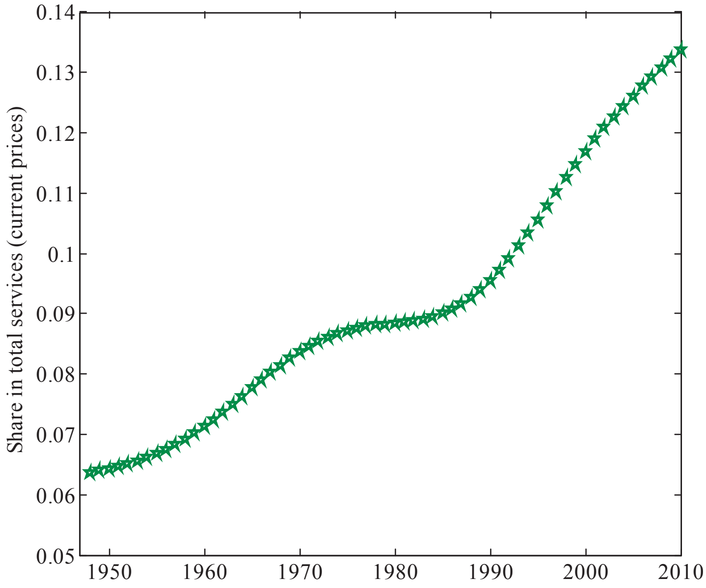

Figure 3.5: Recreational services. Own elaboration. See Appendix D

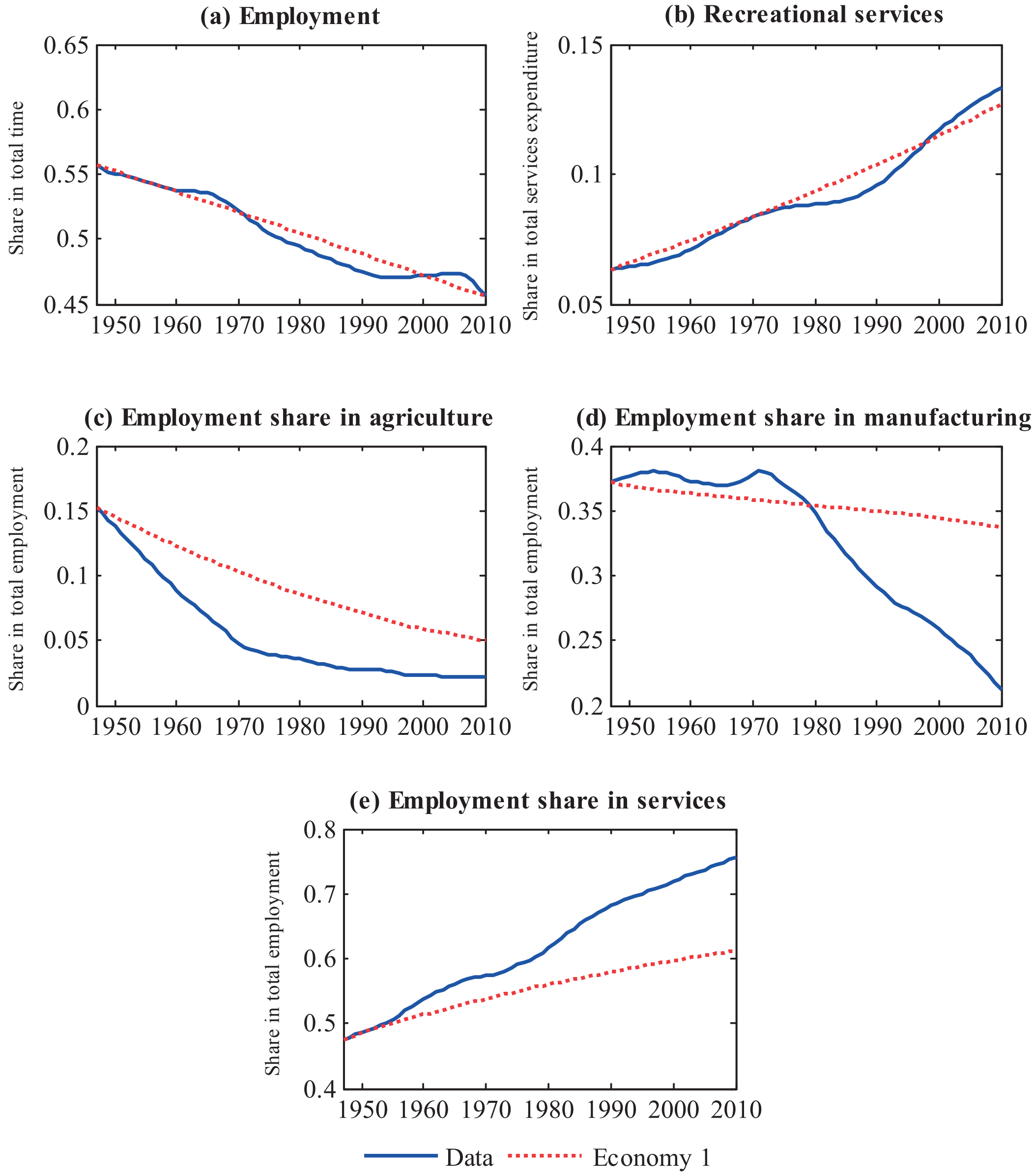

Figure 3.6: Numerical simulations of Economy 1.

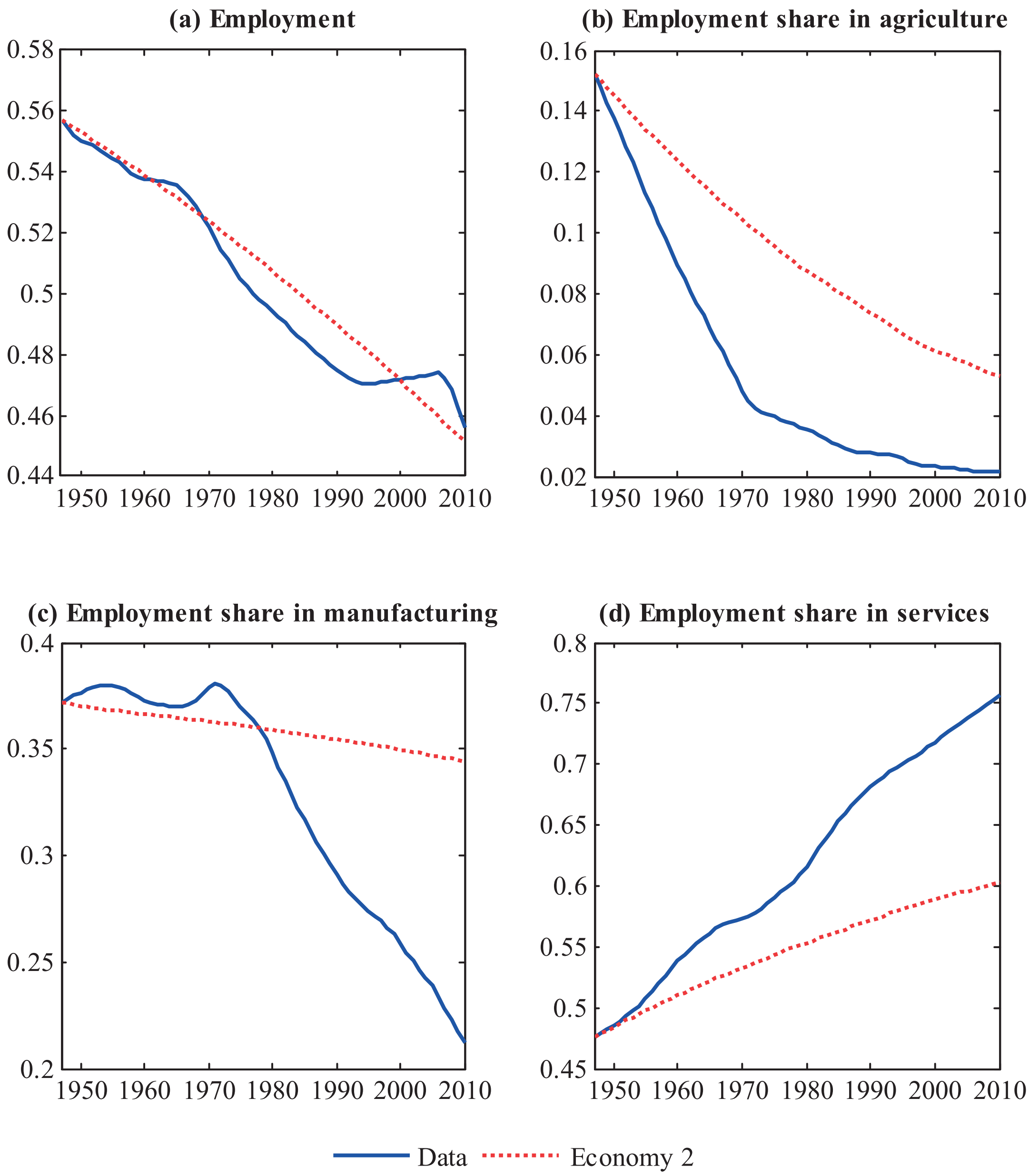

Figure 3.7: Numerical simulations of Economy 2.

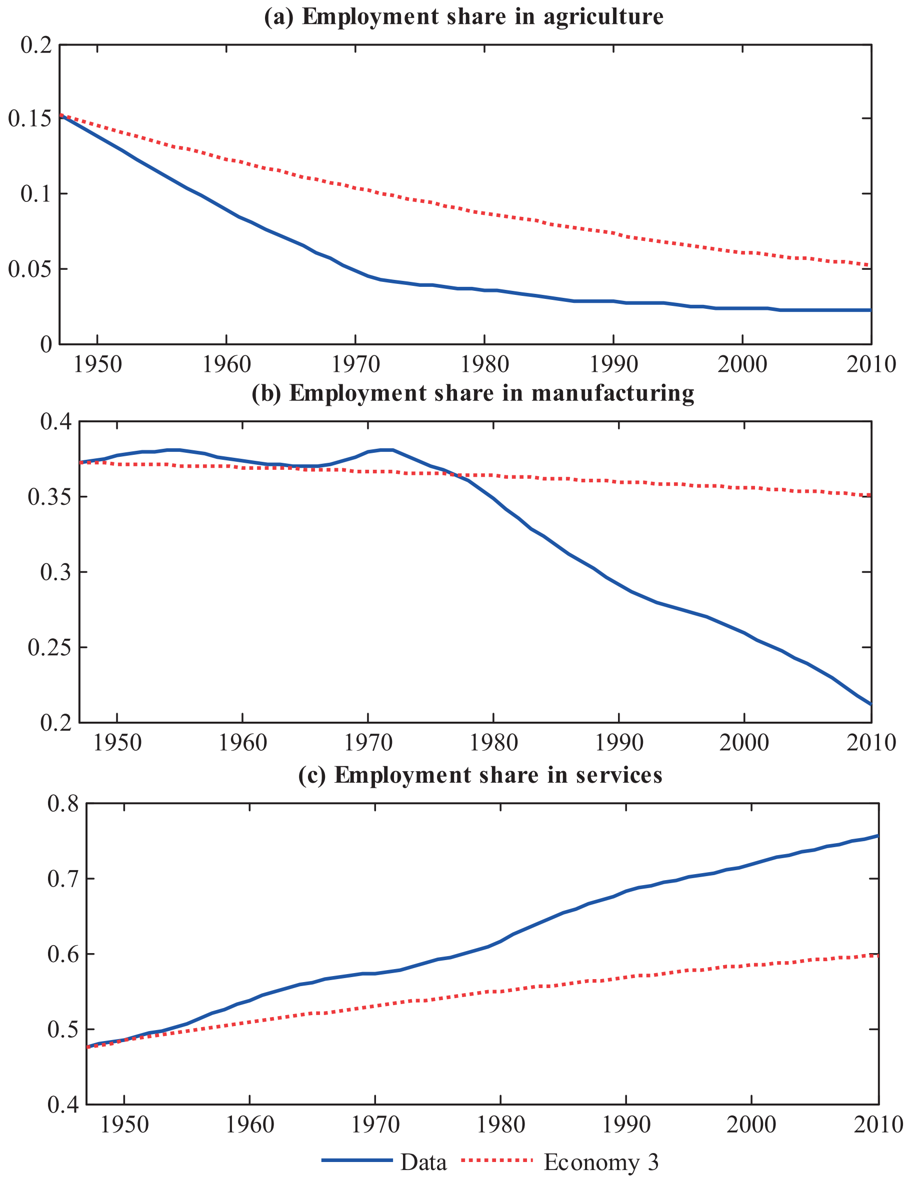

Figure 3.8: Numerical simulations of Economy 3.

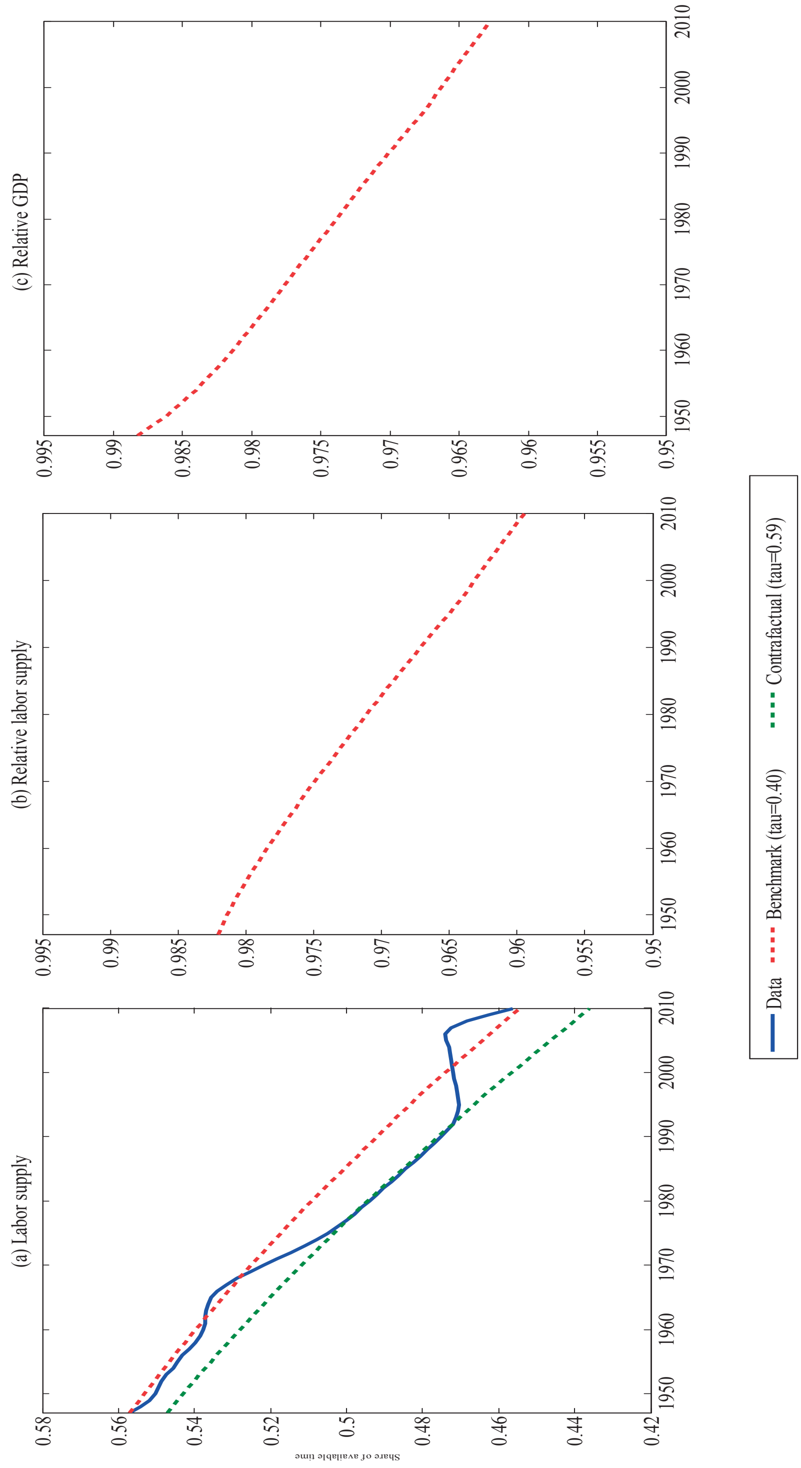

Figure 3.9: Tax increase from 40 to 59 percent in Economy 2.

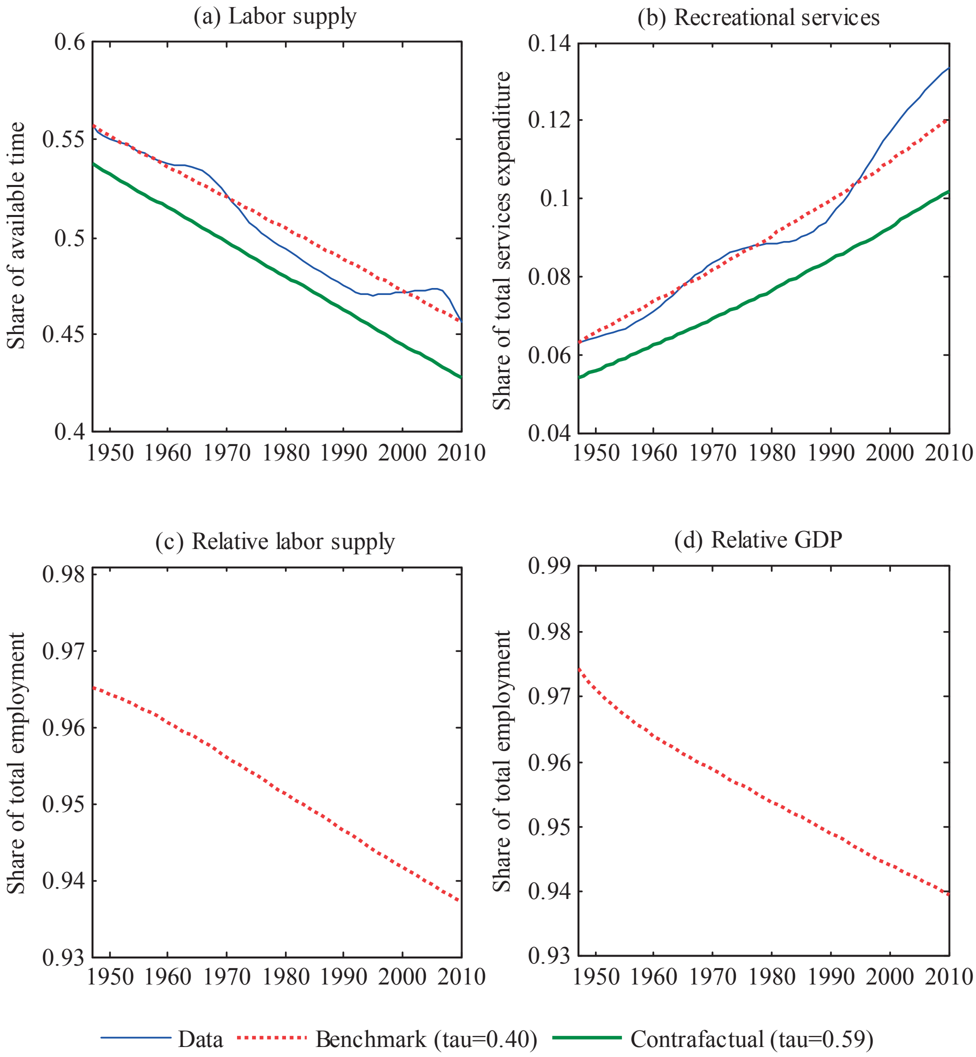

Figure 3.10: Tax increase from 40 to 59 percent in Economy 1.

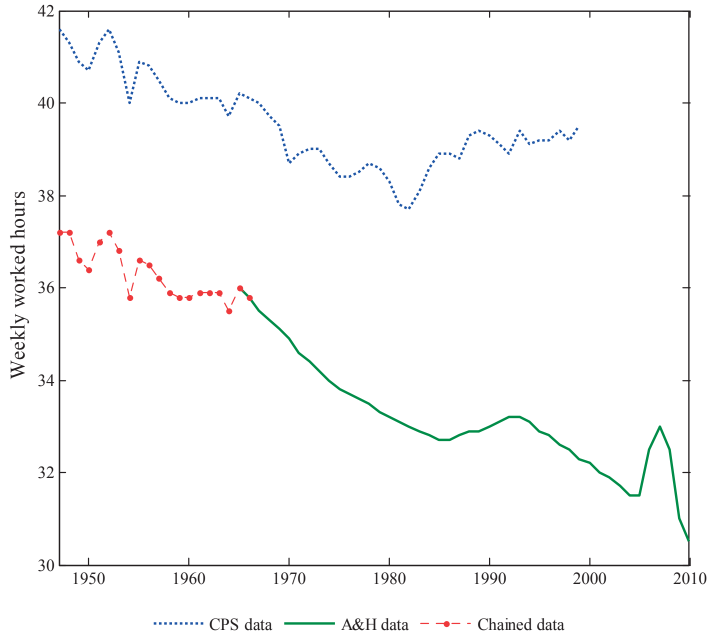

Figure 3.11: Average market working hours.

Table 3.1: Calibration: parameters and targets

Parameters	**Targets**
$\gamma_m = 0.014$	Long run growth rate of GDP is 2%
$\gamma_s = 0.0061$	Growth rate of p_s^*
$\gamma_a = 0.035$	Growth rate of p_a^*
$\rho = 0.032$	Long run interest rate is 5.2%
$\delta = 0.056$	Long run ratio of capital to GDP is 7.6%
$\alpha = 0.35$	Average value of the labor income share
$A_m(0) = 1$	Normalization
$A_s(0) = 1.4633$	$p_s(0) = 0.6833$ value in 1947*
$A_a(0) = 0.2327$	$p_a(0) = 4.2975$ value in 1947*
$\sigma = 0.98$	Best fit in the time path of x and l
$\varepsilon = 0.25$	Best fit in the time path of x and l
$z_0 = z^*$	Consistent with almost BGP.

Note: *- Herrendorf et al.(2013)

Table 3.2:
Calibration: parameters and targets. Three economies

Parameters	Targets
Economy 1	
$\beta = 0.105$	$x(1947) = 0.0632$
$\overline{\eta}_l = 1.95$	$l(1947) = 0.5572$
$\overline{\eta}_a = 0.33$	$u_a(1947) = 0.1522$
$\overline{\eta}_s = 3.86$	$u_m(1947) = 0.3722$
$\overline{l} = 0.387$	$l(2010) = 0.47$
$\psi = 0.5$	$x(2010) = 0.135$
Economy 2	
$\beta = 0$	$x(1947) = 0$
$\overline{\eta}_l = 1.53$	$l(1947) = 0.557$
$\overline{\eta}_a = 0.3$	$u_a(1947) = 0.152$
$\overline{\eta}_s = 3.96$	$u_m(1947) = 0.372$
$\overline{l} = 0.618$	$l(2010) = 0.47$
$\psi = 0.5$	$x(2010) = 0$
Economy 3	
$\beta = 0$	$x(1947) = 0$
$\overline{\eta}_l = 0$	$l(1947) = 1 - \overline{l}$
$\overline{\eta}_a = 0.32$	$u_a(1947) = 0.1522$
$\overline{\eta}_s = 3.98$	$u_m(1947) = 0.3722$
$---$	$l(2010) = 1 - \overline{l}$
$---$	$x(2010) = 0$

Table 3.3: Performance of the simulations

	Economy 1		**Economy 2**		**Economy 3**	
	Variation*	SSR**	Variation	SSR	Variation	SSR
u_a	78%	0.32	76%	0.33	76%	0.33
u_m	22%	0.43	17%	0.46	13%	0.48
u_s	49%	0.64	45%	0.68	43%	0.71
l	100%	0.06	104%	0.07	.....	.....
x_s	90%	0.03	.....	.....	.....	.....

* Variation is the part of the total change explained by the simulation.

** SSR is the sum of the square of the residuals.

Table 3.4: Parameters and Targets. Labor Tax rate equal to 40 percent

Economy 1	
Parameters	**Targets**
$\beta = 0.165$	$x\,(1947) = 0.0632$
$\overline{\eta}_l = 1.482$	$l\,(1947) = 0.5572$
$\overline{\eta}_a = 0.328$	$u_a\,(1947) = 0.1522$
$\overline{\eta}_s = 3.810$	$u_m\,(1947) = 0.3722$
$\overline{l} = 0.6534$	$l\,(2010) = 0.47$
$\psi = 0.5$	$x\,(2010) = 0.135$
Economy 2	
$\beta = 0$	$x\,(1947) = 0$
$\overline{\eta}_l = 1.312$	$l\,(1947) = 0.557$
$\overline{\eta}_a = 0.3197$	$u_a\,(1947) = 0.152$
$\overline{\eta}_s = 3.965$	$u_m\,(1947) = 0.372$
$\overline{l} = 0.61525$	$l\,(2010) = 0.47$
$\psi = 0.5$	$x\,(2010) = 0$
Notes: *. Herrendorf, et al. (2013)	

Conclusions

This thesis investigates the implications of technology adoption, human capital accumulation, and changes in the use of time on economic development. In this section, I present a summary of each chapter that compose the thesis, and I mention directions of future research.

In Chapter 1, I present a multi-sectoral growth model where sectoral technological differences drive structural change. Empirical evidence suggests that the differences in rates of technical progress across sectors are time-variant, implying that the bias in technological change is not constant. I analyze the implications of this non-constant sectoral biased technical change for structural change and assess whether this is an important factor behind structural transformations. To this end, I calibrate the model to match the development of the U.S. economy during the twentieth century. The main findings show that a purely technological approach is able to replicate the sectoral transformations in the U.S. economy not only after but also prior to the World War II. Moreover, I find that this result is robust for higher values of the elasticity of substitution across goods larger than those used in simulations of the standard models of structural change based on constant biased technical change. I conclude that non-constant sectoral biased technical change improves the explanatory power of the model.

In Chapter 2, I develop a multi-sector growth model with human capital accumulation. In this model, human capital induces structural change through two channels: changes in relative prices due to differences in the strength of sectoral externalities, and changes in the investment rate of physical and human capital. This model allows to provide an endogenous explanation for the growth of relative prices and, therefore, of structural change. To investigate the effects of this endogenous mechanism on structural change, I calibrate the model to match the development patterns of U.S. economy during the twentieth century. Based on the numerical simulations, I find that the model replicates the qualitative patterns of structural change during the transitional towards the equilibrium path. I also find that the model replicates qualitatively the differences between employment and value added shares across sectors, as well as the path of human capital accumulation. Moreover, I outline that imbalances between physical and human capital contribute to explain structural change as well as income differences across countries. Based on numerical examples, I conclude that the imbalance between these two capital stocks can provide an explanation for the observed heterogeneity in the sectoral composition across countries.

Finally, in Chapter 3, we aim to explain two important patterns of structural change. First, a sustained increase in the amount of time devoted to leisure activities and, second, an increase in the consumption of recreational services. In order to measure the relevance of this mechanism, we construct a multi-sector exogenous growth model with biased technological change. The new feature of the model is the introduction of recreational activities, which depend on both leisure time and on the consumption of recreational services. We introduce these activities by assuming a nested CES utility function. We show that the model explains the two patterns of structural change. Our findings show that the change in the use of time, mainly driven by increasing leisure time, has important effect on structural change. In particular, our numerical simulation shows that leisure is an important factor to explain the rise of services.

The results derived from these chapters outline that the mechanisms explaining structural change are complex. For example, the results of Chapters 1 and 2 reveal the importance of economic factors, as the investment in human capital or R & D, that foster rates of sectoral technological progress and, hence, the pattern of development. Chapter 3 shows how individual decisions on the uses of time have important implications for structural change.

The purpose of future research is to continue studying the determinants of structural change. A natural line of research is an in-depth analysis of the factors that promote technological adoption and its relationship with the sectoral composition. In particular, an extension of Chapter 1 is to consider the effects of sector-specific equipment investment on promoting an endogenous technical progress. In this framework, endogenous biased of sectoral productivity growth could arise because of sectoral differences in returns of equipment investment. Furthermore, as pointed out in the literature of economic development, a key factor in the process of industrialization is the accumulation of human capital (see Nelson and Phelps, 1966; Temple and Voth, 1998 and Quamrul and Galor, 2011). An extension of Chapter 2 is to analyze the role of human capital in the adoption of new technologies to explain structural change in developing countries. In this case, human capital would be the engine of this technology adoption identified in Chapter 1. Moreover, in the context of a multisectorial model based on the features previously pointed out, we could analyze the effects of educational policies and infer the implications for economic growth.

Finally, in Chapter 3, we show that changes in time use have important implications for structural change and GDP. A possible line of research is to analyze the relationship between changes in the use of time and its links with the sectoral composition of the economy. In particular, we can analyze the effects of changes in the time devoted to childcare on the sectoral composition. To the extent that income increases, it is observed that parents with higher incomes spend more time in child care (see Kimmel & Connelly, 2007; and Guryan, Hurst, Kearney, 2008). Therefore, economic development increases child care and may have a large impact on sectoral composition.

Bibliography

[1] Abraham, K., Spletzer, J. and Stewart, J,. 1998. "Divergent Trends in Alternative Wage Series." In John Haltiwanger, Marilyn E. Manser, and Robert Topel, editors. Labor Statistics Measurement Issues. University of Chicago Press.

[2] Acemoglu, D. , Guerrieri, V., 2008. “Capital deepening and non-balanced economic growth.” Journal of Political Economy, 116, 467-498.

[3] Aguiar, M. and Hurst, E., 2007. “Measuring trends in leisure: the allocation of time over five decades.” Quarterly Journal of Economics, 122, 969-1006.

[4] Aguiar, M., Hurst, E. and Karabarbounis, L., 2013. “Time Use during the Great Recession.” American Economic Review, 103, 1664-96.

[5] Alonso-Carrera, J., & Raurich, X., 2010. “Growth, sectoral composition, and the evolution of income levels.” Journal of Economic Dynamics and Control, 34(12), 2440-2460.

[6] Alonso-Carrera, J., Caballé, J. and Raurich, X., 2015. “Sectoral Composition and Macroeconomic Dynamics.” BE J. Macroeconomics. Advances, 15, 1-42,

[7] Alvarez-Cuadrado, F., Monterio, G. and Turnovsky, S., 2004. "Habit Formation, Catching-up with the Joneses, and Economic Growth." Journal of Economic Growth 9, 47-80.

[8] Alvarez-Cuadrado, Francisco and Poschke, Markus., 2011. "Structural Change out of Agriculture: Labor Push Versus Labor Pull." American Economic Journal: Macroeconomics, 3(3), pp. 127-58.

[9] Baumol, W. J., 1967. "Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis." The American Economic Review 57(3): 415-426.

[10] Bond E., Wang, P. and Yip C., 1996. "A General Two-Sector Model of Endogenous Growth with Human and Physical Capital: Balanced Growth and Transitional Dynamics." Journal of Economic Theory 68, 149-173.

[11] Boppart, T., 2014. “Structural change and the kaldor facts in a growth model with relative price effects and Non-Gorman preferences.” Econometrica, 82(6), 2167-2196.

[12] Buera, F. and Kaboski, J., 2012. “Scale and the origins of structural change.” Journal of Economic Theory, 147, 684-712.

[13] Buera, F., and Kaboski, J., 2009. "Can Traditional Theories of Structural Change Fit the Data?" Journal of the European Economic Association, 7 (2–3), 469–477.

[14] Buera, Francisco J., Joseph P. Kaboski, and Yongseok Shin. 2011. "Finance and Development: A Tale of Two Sectors." American Economic Review, 101(5): 1964-2002.

[15] Caballé, J. and Santos, M., 1993. "On Endogenous Growth with Physical and Human Capital." Journal of Political Economy, 101(6), pp. 1042-67.

[16] Carter, Susan B. and Cambridge University, Press. "Historical Statistics of the United States." Cambridge [England]: Cambridge University Press, 2006.

[17] Caselli, F. (2005). Chapter 9 Accounting for Cross-Country Income Differences. Handbook of Economic Growth. A. Philippe and N. D. Steven, Elsevier. Volume 1, Part A: 679-741.

[18] Dennis, B. and Iscan, T., 2009. “Engel versus Baumol: accounting for sectoral change using two centuries of US data.” Explorations in Economic History, 46, 186-202.

[19] Dolores, Papageorgiou, and Perez-Sebastian, 2011. "A Unified Theory of Structural Change." Journal of Economic Dynamics and Control, 35(9), pp. 1393-404.

[20] Duarte and Restuccia, 2010. “The role of structural transformation in aggregate productivity.” Quarterly Journal of Economics, 125, 129-173.

[21] Eberhardt and Vollrath, 2014. "Agricultural Technology and Structural Change." CSAE Working Paper Series 2014-21, Centre for the Study of African Economies, University of Oxford.

[22] Echevarria, C., 1997. "Changes in Sectoral Composition Associated with Economic Growth." International Economic Review 38, 431-452.

[23] Foellmi, R. and Zweimüller, J., 2008. “Structural change, Engel’s consumption cycles and Kaldor’s facts of economic growth.” Journal of Monetary Economics, 55,1317-1328.

[24] Francis, N. and Ramey, V., 2009. “A century of work and leisure.” American Economic Journal: Macroeconomics, 1, 189-224.

[25] Galor, O and Moav, O., 2009. "Inequality in Landownership, the Emergence of Human-Capital Promoting Institutions, and the Great Divergence." The review of economic studies, 76(1), pp. 143-79.

[26] Galor, O and Weil, D., 2000. "Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond." The American Economic Review, 90(4), pp. 806-28.

[27] Galor, Oded. "Inequality, Human Capital Formation and the Process of Development," Institute for the Study of Labor (IZA), 2012.

[28] Gollin, D., Lagakos, D. and Waugh,M.E. , 2014. "Agricultural Productivity Differences across Countries." American Economic Review, American Economic Association, vol. 104(5), pages 165-70, May.

[29] Gollin, D., Parente, S. and Rogerson, R., 2004. “Homework, farmwork, and international differences in productivity.” Review of Economic Dynamics, 7, 827-850.

[30] Gupta, Manash Ranjan and Chakraborty, Bidisha., 2007. "Uniqueness and Indeterminacy of the Equilibrium Growth Path in the Uzawa-Lucas Model with Sector Specific Externalities." Japanese Economic Review, 58(3), pp. 400-06.

[31] Guryan, Jonathan, Erik Hurst, and Melissa Kearney. 2008. "Parental Education and Parental Time with Children." Journal of Economic Perspectives, 22(3): 23-46.

[32] Herrendorf, Schmitz, and Teixeira, 2012. "The Role Of Transportation In U.S. Economic Development: 1840–1860." International Economic Review, vol. 53(3), pages 693-716, 08.

[33] Herrendorf, B. , Rogerson, R. and Valentinyi, Á., 2013. “Two Perspectives on Preferences and Structural Transformation.” American Economic Review, 103, 2752-2789.

[34] Herrendorf, B. ., Rogerson, R. and Valentinyi, Á., 2014. “Growth and Structural Transformation.” Handbook of Economic Growth, in: Handbook of Economic Growth, edition 1, volume 2, chapter 6, pages 855-941 Elsevier.

[35] Herrendorf, Berthold and Valentinyi, Ãkos., 2008. "Measuring Factor Income Shares at the Sectoral Level." Review of economic dynamics, 11, pp. 820-35.

[36] Joint Economic, C., et al. 2007, The economic report of the President: hearing before the Joint Economic Committee, Congress of the United States, One Hundred Ninth Congress, second session, February 16, 2006. Washington.

[37] Kimmel, Jean and Rachel Connelly, 2007. “Mothers’ Time Choices: Caregiving, Leisure, Home Production, and Paid Work” Journal of Human Resources, 42(3), 643-61.

[38] Kongsamut, P., Rebelo, S. and Xie, D., 2001. "Beyond Balanced Growth." Review of Economic Studies 68, 869-882.

[39] Kuznets, Simon, 1973. "Modern Economic Growth: Findings and Reflections." The American Economic Review, 63(3), pp. 247-58.

[40] Lucas, Robert E., 1998. "On the Mechanics of Economic Development." Journal of Monetary Economics, 1988, 22(1), pp. 3-42.

[41] M.P. Timmer, G.J .de Vries, and K.deVries, 2014. “Patterns of Structural Change in Developing Countries.” GGDC research memorandum 149.

[42] Maddison, Angus., “Monitoring the World Economy, 1820-1992.” Washington, D.C: Development Centre of the Organisation for Economic Co-operation and Development, 1995.

[43] Meckl, J. r., 2002. "Structural Change and Generalized Balanced Growth." Journal of Economics 77(3): 241.

[44] Mulligan, Casey B.1 1993. "Transitional Dynamics in Two-Sector Models of Endogenous Growth." The Quarterly Journal of Economics, 108(3), pp. 739-73.

[45] Nelson, Richard R. and Phelps, Edmund S., 1966. "Investment in Humans, Technological Diffusion, and Economic Growth." The American Economic Review, 56(1/2), pp. 69-75.

[46] Ngai, L. R., Pissariades, C., 2007. “Structural Change in a Multi-sector Model of Growth.” American Economic Review, 97, 429-443.

[47] Ngai, L. R., Pissariades, C., 2008. “Trends in hours and economic growth.” Review of Economic Dynamics, 11, 239-256.

[48] O’Brien, Anthony Patrick. "Depression of 1920-1921," T. F. C. David Glasner, Business Cycles and Depressions: An Encyclopedia. New York: Garland Publishing, 1997, 151-53.

[49] O’Mahony, Mary and Marcel P. Timmer, 2009. "Output, Input and Productivity Measures at the Industry Level: the EU KLEMS Database." Economic Journal, 119(538), pp. F374-F403.

[50] Prescott, E.C., 2004. “Why do Americans work so much more than Europeans?” Federal Reserve Bank of Minneapolis Quarterly Review, 2-13, 28.

[51] Ramey, Valerie A., and Neville Francis, 2009. "A Century of Work and Leisure." American Economic Journal: Macroeconomics, 1(2): 189-224.

[52] Robert, E. Lucas, Jr., 2004. "The Industrial Revolution: Past and Future." The Region, 18, pp. 4.

[53] Rogerson, R., 2008. “Structural transformation and the deterioration of European labor market outcomes.” Journal of Political Economy, 116, 235-259.

[54] Romer, Paul M. and Jones, Charles I., 2010. "The New Kaldor Facts: Ideas, Institutions, Population, and Human Capital." American Economic Journal: Macroeconomics, 2(1), pp. 224-45.

[55] Sato, K., 1967. “A Two-Level Constant-Elasticity-of-Substitution Production Function.” Review of Economic Studies, 43, 201-218.

[56] Steger, T.M., 2006. "Heterogeneous Consumption Goods, Sectoral Change and Economic Growth." Studies in Nonlinear Dynamics and Econometrics 10, No. 1, Article 2.

[57] Tamura, Robert, 2002. "Human Capital and the Switch from Agriculture to Industry." Journal of economic dynamics & control, 27(2), pp. 207-42.

[58] Tamura, Robert, 2006. "Human Capital and Economic Development." Journal of development economics, 79, pp. 26-72.

[59] Teignier,Marc, 2012. “The Role of Trade in Structural Transformation.” Manuscript. Universidad de Alicante

[60] Temple, Jonathan and Voth, Hans-Joachim, 1998. "Human Capital, Equipment Investment, and Industrialization." European Economic Review, 42(7), pp. 1343-62.

[61] United States. Bureau of the, C. 1975. Historical statistics of the United States, colonial times to 1970.

[62] United, States; President and Council of Economic, Advisers. "Economic Report of the President: Transmitted to the Congress February 2011, Together with the Annual Report of the Council of Economic Advisers," Washington[, D.C.]: U.S. G.P.O, 2011.

[63] Vandenbroucke, Guillaume and Restuccia, Diego., 2013. "A Century of Human Capital and Hours." Economic inquiry, 51(3), pp. 1849-66.