Nuclear reactions of heavy ions

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Abstract: In this work nuclear reactions of heavy ions are studied, focusing on elastic scattering. A classical and quantum description is made in order to obtain some important parameters, such as grazing angle, angular momentum or reaction cross sections, and to compare both descriptions.

I. INTRODUCTION. REACTION MECHANISMS

The aim of this work is to study nuclear reactions between heavy ions. We consider heavy ions those with mass number greater that carbon's (A=12), although this limit is not always considered to be the same [1]. The great interest of this reactions has many reasons; first, the complex nature of the particles allows the occurrence of many different reactions; besides, fusion reactions may produce nuclei with high excitation energy, which allows studying nuclear matter in some special conditions not commonly found in other ways.[2]

In these interactions different phenomena can take place depending on parameters like the projectile energy or the impact parameter. We can distinguish four interaction regions: fusion, incomplete fusion and deep inelastic collisions, peripheral and Coulomb regions.

When the two ions come very close to each other and the incident energy is high enough, the interaction leads to the formation of a compound nucleus; this is a fusion reaction. The ordered motion from the projectile and the target turns into a caothic thermal motion with a cascade of nucleon-nucleon interactions, a process that leads the nucleus to equilibrium (thermalization).

The interaction between both particles is described by a complex potential from which we will only consider the real part, this is:

$$V_{eff} = \frac{Z_1 Z_2 e^2}{r} + \frac{V_0}{1+e^{\frac{r-R_0}{a}}} + \frac{\hbar^2}{2\mu}\frac{L(L+1)}{r^2} \qquad (1)$$

where the first term is the Coulomb potential, the second one is the nuclear potential and the third one is the centrifugal term. We will see later that this potential has a well for certain values of the angular momentum below a critical value. If the energy is higher than the Coulomb barrier, the projectile can get trapped into the well, where is affected by nuclear interaction and may produce fusion. If the energy is lower, the projectile will only be affected by the Coulomb interaction.

When the two ions pass each other a bit further, but near enough to still allow a strong interaction, processes like deep inelastic and incomplete fusion reactions can take place. Due to this strong interaction, a considerable fraction of kinetic energy turns into internal excitation energy. In this region and for light projectiles binary fragmentation often occur, leading to processes like elastic or inelastic break-up (where both fragments come out) or incomplete fusion (where one of the fragments is emitted while the other is absorbed by the target, which can be excited). Deep inelastic collisions take place with heavier nuclei, with an incident energy of about $E_i \approx 10 MeV/nucleon$.

For larger distances between both ions (or lower energies) we come into the peripheral region, where nucleon transfer can occur and, for even larger impact parameters, elastic and inelastic scattering.

In inelastic scattering reactions, the projectile interacts with the target nucleus transferring some energy to it, so that the target can get excited. At low incident energies and for highly charged particles, the excitation of the target can be due to the Coulomb field (Coulomb excitation); at higher energies the excitation is due to both the nuclear interaction and the Coulomb field. In nucleon transfer reactions one or more nucleons are transferred from the projectile to the target or viceversa; these reactions can provide information about the structure of nuclei.

In this work I will focus on peripheral reactions and particularly on elastic scattering. Therefore, a more detailed explanation of the latter is given below.

II. ELASTIC SCATTERING. CLASSICAL DESCRIPTION

Elastic collisions are those that leave unaltered the state of the target. These will be described later through the so called optical potential. However, I will start giving a brief classical description in order to be able to compare with it the quantum optical potential model.[3] From the classical non-relativistic mechanics we can remember that the fact that the force is central ($\vec{F}(\vec{r}) = F(r)\hat{r}$) leads to i) the conservation of angular momentum($L = Mr^2\dot{\phi}$) and ii) the trajectory of the projectile is contained on a plane. r and ϕ are, respectively, the radial and angular coordinates of the projectile position.

The equations of motion can be written as

$$\dot{\phi} = L/(Mr^2), \tag{2}$$

$$M\ddot{r} = F(r) + \frac{L^2}{Mr^3} \tag{3}$$

The total energy can be expressed as:

$$E = T + V = \frac{1}{2}M\dot{r}^2 + \frac{L^2}{2Mr^2} + V(r)$$

Which, considering (2), can be rewritten as:

$$E - V = \frac{L^2}{2Mr^2} + \frac{L^2}{2Mr^4}(\frac{dr}{d\phi})^2$$

from where we can isolate $d\phi$ and obtain:

$$d\phi = \pm\frac{L/r^2}{\sqrt{2m(E-V) - L^2/r^2}}dr.$$

Using the condition of closest approach $|dr/d\phi|_{r_0} = 0$ (r_0 is the distance of closest approach) and considering that $2\alpha + \vartheta = \pi$ (where ϑ is the deflection angle and α is the angle between the position vector at r_0 and the asymptote of the outgoing trajectory), this gives the classical trajectory for a particle in a force field affected by the interaction potential V(r):

$$\vartheta(L) = \pi - 2\int_{r_0}^{\infty}\frac{L/r^2}{\sqrt{2M(E - V(r)) - L^2/r^2}}dr \tag{4}$$

If we consider only Coulomb interaction, the potencial $V(r)$ on on (4) is

$$V(r) = \frac{Z_1Z_2e^2}{r} \tag{5}$$

and the integral can be solved analytically, giving:

$$\vartheta(L) = 2\arctan(\frac{MZ_1Z_2e^2}{L\sqrt{2ME}}) \tag{6}$$

However, for a more general potential that takes into account the nuclear interaction we cannot obtain an analytical solution and need to use numerical methods.

We can consider the scattering by a target nucleus of a beam of particles, all of them with the same mass M and energy E and each of them characterized by its angular momentum $L = bp$ (b is the impact parameter and $p = Mv$ the linear momentum). Then, the number of scattered particles with angle between θ and $\theta + d\theta$ (scattering angle respect to the polar axis) is the same that the number of particles that cross a plane perpendicular to the polar axis with impact parameter between b and $b + db$. Since the scattering cross-section is defined as the number of scattered particles per unit time for unit incident intensity, we can write:

$$J\frac{d\sigma}{d\Omega}2\pi\sin\theta d\theta = J2\pi b db$$

where J is the current density of the beam. From here we can obtain the classical differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}|\frac{db}{d\theta}| = \frac{L}{p^2\sin\theta}|\frac{dL}{d\theta}| \tag{7}$$

The scattering angle θ observed in experiments ($0 < \theta < \pi$) should be distinguished from the deflection angle ϑ, which is the polar angle of the asymptote of the outgoing trajectory, considering that the particle may plunge or orbit around the centre of force and therefore ϑ can be negative or greater than 2π. Both angles are related by $\theta = \pm\vartheta - 2\pi n$ with integer n such that $0 < \theta < \pi$. Eq. (7) is valid only in the case that $L(\vartheta)$ is a single-valued function. If it is multi-valued, the differential cross section takes the form:

$$\frac{d\sigma}{d\Omega} = \frac{1}{2p^2\sin\theta}\sum_i|\frac{dL^2}{d\vartheta}|_{L=L_i} \tag{8}$$

where we have considered the contributions form all the branches of $L(\vartheta)$.

We consider now a repulsive Coulombian interaction (5). Using the result from Eq. (8) and the expression $\vartheta(L)$ for the Coulomb potential (6) we obtain the differential cross section of Rutherford [4]:

$$\frac{d\sigma_R}{d\Omega} = (\frac{Z_1Z_2e^2}{4E})^2\frac{1}{\sin^4(\theta/2)} \tag{9}$$

We can see that does not depend on the signs of the charges. In the case of a Coulomb potential, the differential cross section obtained from non-relativistic quantum mechanics gives an identical result.
Integrating over all angles we obtain the total cross section (defined as the nombre of particles scattered in all directions per unit time for unit incident intensity):

$$\sigma_R = (\frac{Z_1Z_2e^2}{Mv^2E})^2[\frac{1}{1-\cos\theta}]_{-1}^{1}$$

which is infinite. This is because of the infinite range of the Coulomb interaction; actually σ will be infinite for any scattering field different from zero at any distance and will only be finite if the field has a cut. In quantum mechanics, however, potentials that tend to zero faster than $1/r^2$ have finite total cross-sections.

If we add an atractive field (nuclear interaction) to the Coulombian field, $\vartheta(L)$ gets modified and so the trajectories of the projectile [5]. As we reduce the impact parameter, the nuclear attractive field starts to modify the orbit; the limit case when this happens corresponds to the *grazing* trajectory. For lower b's it is possible to achieve a balance between nuclear and coulomb forces and have an *orbiting* trajectory. For even smaller b's, the nuclear interaction can be stronger than the repulsive force so that the orbit *plunges*, going out with negative deflection angle.

III. ELASTIC SCATTERING. DESCRIPTION BY THE OPTICAL MODEL POTENTIAL

The optical model consists on treating the scattering and absorption of nucleons by a nucleus in a similar way than scattering and absorption of light. As in optics, where a complex refraction index is used, for the nuclear reactions we can define a complex potential. This is done because a real potential would only explain scattering of the incident particles, but would not explain their removal from the elastic channel by inelastic processes, pre-equilibrium reactions and compound nucleus reactions. Therefore, the imaginary part of the potential is the one that takes away flux of particles from the elastic channel.

In three dimensions we can solve the Schrödinger equation to find the elastic scattering differential cross section. This is done using the quantum scattering formalism of partial waves [6]. From the quantum scattering theory we obtain the wave function Ψ:

$$\Psi = -\frac{1}{2ikr}\sum_L (2L+1)P_L(\cos\theta)(e^{2i\delta_L}e^{ikr} - e^{-ikr})$$

(where δ_L are the phase-shifts) and we can find the angular distribution of the scattered nucleons, the total elastic cross section (integrated for all angles) and the reaction cross section.

The total optical potential can be expressed as:

$$V(r) = V_c(r) + U\cdot f_u(r) + iW\cdot f_w(r) + V_{so}(r) \quad (10)$$

The first term on (10) is just the Coulomb potential (11). The second term is the real part of the nuclear potential and third is the imaginary part; both of which (f_u and f_w) have a Saxon-Woods form. The fourth is the spin-orbit term, which allows calculating the polarization of the scattered beam, although we won't use this here.

$$V_C = \frac{Z_1Z_2e^2}{2R_C}(3 - \frac{r^2}{R_C^2})\ r \le R_C$$
$$V_C = \frac{Z_1Z_2e^2}{r}\ r \ge R_C \quad (11)$$

Typical parameters of the optical potential are: $U \approx 50MeV$, $W \approx 10MeV$, $r_0 \approx 1.2fm$, $a \approx 0.65fm$, $V_{so} \approx 4MeV$.

IV. RESULTS

In this section I will present the results obtained from some calculations using the fortran programs "traj_hi1.f", "traj_hi2.f" and "traj_hi3.f" [7], based on the classical theory, and the program "nvgopthi.f" [8], which uses the quantum theory of the optical potential model. Some magnitudes of interest in elastic scattering will be shown, such as the effective potential, the distance of closest approach in terms of the angular momentum, classical trajectories, the values of L and θ corresponding to the grazing trajectory and finally I will show some differential cross section values for different reactions in order to compare the classical with the optical model results.

A. Effective Potential and $r_0(L)$

As explained above, the effective potential between two heavy nuclei includes the Coulomb and the nuclear interaction. Figure (1) shows this potential as a funtion of the radial distance r between the two nuclei ^{16}O and ^{88}Sr for an incident energy of $E_{LAB} = 60MeV$, calculated using the program "traj_hi3.f". The red line corresponds to an angular momentum L=0 and the green one corresponds to the grazing trajectory (the blue horizontal line is the energy at the centre of mass frame of reference, $E_{CM} = \frac{A_2}{A_1+A_2}E_{LAB} = 50.769MeV$).

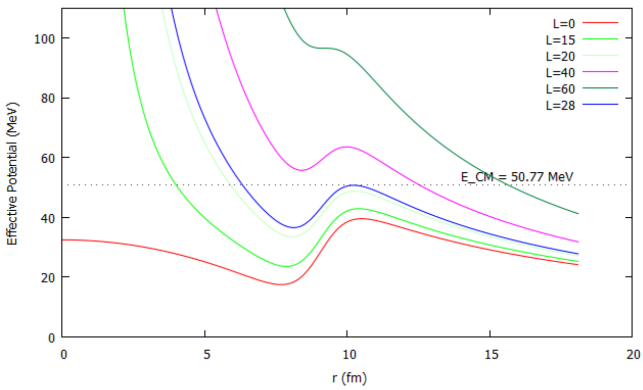

FIG. 1: Effective potential as a function of the radial distance

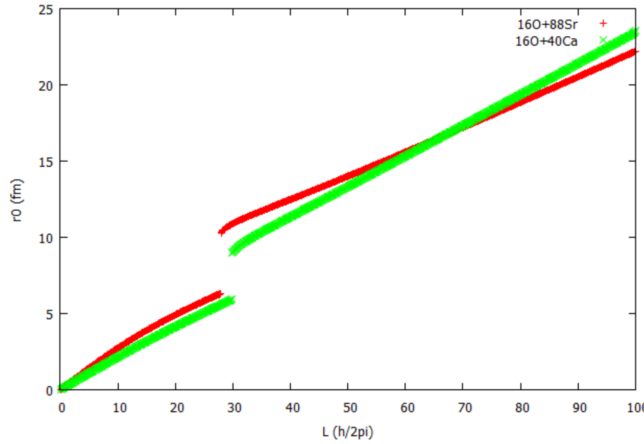

FIG. 2: Distance of closest approach in terms of the angular momentum for the reactions ${}^{16}O +{}^{88} Sr$ and ${}^{16}O +{}^{40} Ca$.

Fig. (2) shows, for two different reactions, the distance of closest approach as a function of the angular momentum L. We can see a jump in r_0, that corresponds to the angular momentum of the grazing trajectory, L_{gr}. Below, the nuclear interaction starts to make effect. The distance of closest approach corresponding to the discontinuity in fig.(2) can be compared (table I) to the strong interaction radius R_{int}, which is the distance where nuclear interaction starts to be relevant:

$$R_{int} = R_1 + R_2 + 3.2fm$$

($R_i = 1.12A_i^{1/3} - 0.94A_i^{-1/3}$ are the half-density radii).

Reaction	$r_0^{gr} \pm 0.15$(fm)	R_{int}(fm)	discrepancy
${}^{16}O + {}^{88}Sr$	10.2	10.4	0.2
${}^{16}O + {}^{40}Ca$	8.9	9.2	0.3

TABLE I: Comparison between the distance of closest approach corresponding to L_{gr} and the strong interaction distance R_{int}. We can see that both values are quite similar, with a discrepancy within two times the error.

B. Trajectories

For the reaction ${}^{16}O +{}^{88} Sr$ I have chosen a set of L values around the approximate value of L_{gr} obtained from fig.(2) (from 27 to 28.5 with intervals of 0.1, in units of $\hbar$), and drawn some of their trajectories using the data obtained by running "traj_hi1.f". In fig.(3) we can see, in black colour, the grazing trajectory corresponding to the angular momentum

$$L_{gr}^{classical} = (28.0 \pm 0.1)\hbar$$

The trajectories above suffer basically only the Coulomb interaction and the ones beneath are affected by the nuclear interaction.

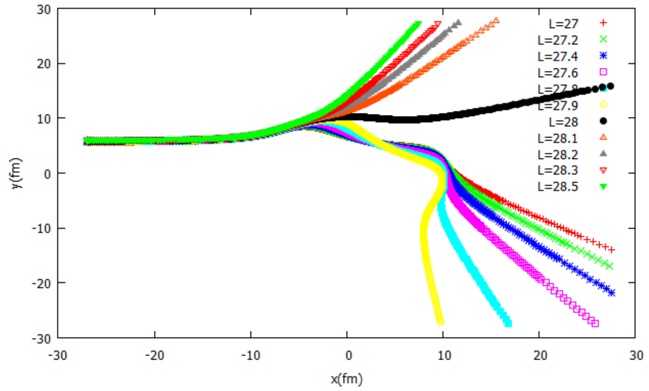

FIG. 3: Trajectories of the projectile ${}^{16}O$ when interacting with a nucleus of ${}^{88}Sr$. Each line corresponds to a different vaule of the angular momentum L.

C. Optical model

Now we intend to obtain another estimation of L_{gr} using the optical potential model. To do so we have used the program "nvgopthi.f", which gives the differential cross section as ratio to the Rutherford value in terms of the scattering angle. The result of this is shown in fig.(4).

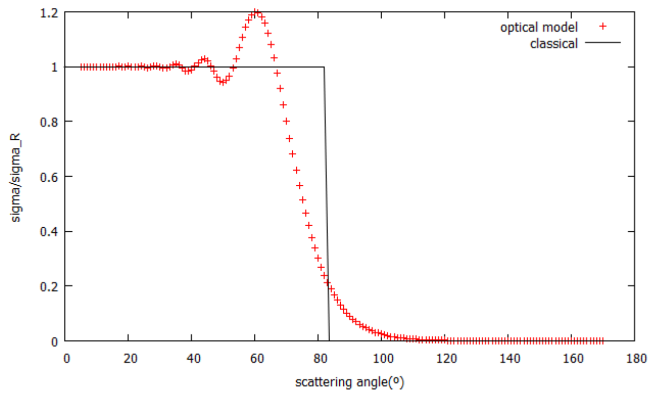

FIG. 4: σ/σ_R as a function of θ. The black line shows a simple classical approach. For $\theta > \theta_{gr}$, σ for elastic scattering goes to zero (the nuclei interact so strongly that an elastic collision cannot take place), while for $\theta < \theta_{gr}$ it takes the σ_R value. In red is shown the optical model result, that presents some oscillations and falls approximately exponentially.

The angle θ_{gr} of the grazing trajectory can be found considering that $\theta_{gr} = \theta_{1/4}$, where the "quarter-point angle" $\theta_{1/4}$ is the one that corresponds to $\sigma/\sigma_R = 0.25$ [9].

From the data obtained from numerical calculations (file "nvgopthi.dat") we get:

$$\theta_{gr} = (82 \pm 1)^{\text{o}}$$

that corresponds to $\sigma/\sigma_R = 0.2399$. We can obtain the grazing angular momentum using:

$$L_{gr}^{opt} = n \cdot \cot(\frac{\theta_{1/4}}{2})$$

where $n = \frac{Z_1 Z_2 e^2}{\hbar v_\infty}$ is the Sommerfeld parameter (v_∞ is the projectile initial velocity). For the reaction of ^{16}O+^{88}Sr, with initial kinetic energy $E_{LAB} = 60MeV$, and considering the units $\frac{e^2}{\hbar c} = \frac{1}{137}$, we obtain:

$$L_{gr}^{opt} = (28.6 \pm 1.2)\hbar$$

Comparing this result with that obtained on section B (trajectories of the classical model) we can see that the results are compatible, since $d < 2 \cdot \delta(L)$, and the relative discrepancy is

$$d = \frac{L_{classical} - L_{optical}}{L_{classical}} = 0.0196 \approx 2\%$$

D. Reaction cross section

Reaction cross sections have been calculated for the classical and optical methods for some reactions of ^{16}O with different targets [10]. The results obtained are shown in Table (II).

Target	$\sigma_r^{classical}(fm^2)$	$\sigma_r^{optical}(fm^2)$	relative discrepancy
^{92}Mo	50.90	60.32	0.12
^{96}Zr	70.88	80.91	0.12
^{92}Zr	65.77	75.53	0.16
^{88}Sr	75.33	85.14	0.12
^{86}Sr	72.60	82.26	0.13
^{54}Fe	108.96	117.18	0.07
^{52}Cr	119.95	128.07	0.06
^{50}Ti	130.88	138.88	0.06
^{48}Ca	141.77	149.60	0.05
^{40}Ca	122.85	129.87	0.05
^{64}Ni	112.81	121.84	0.07
^{62}Ni	109.25	118.04	0.07
^{60}Ni	105.59	114.25	0.08
^{58}Ni	101.82	110.28	0.08

TABLE II: Reaction cross section values (integrated over all angles) for the interaction of ^{16}O with the different targets in the first column.

We can see that the discrepancy between the two methods is not relevant; in other words, the classical description gives a very good aproximation for the cross section with the considered reactions.

V. CONCLUSIONS

In fig.(2) we can see that for the ^{40}Ca target ($Z = 20$) the L_{gr} value is greater than for the ^{88}Sr target ($Z = 38$). This is because for smaller atomic number, the Coulomb interaction is weaker (see (5)) and therefore the limit value for angular momentum in which nuclear interaction starts to be relevant increases.

Considering the results obtained either for the grazing angular momentum and for the reaction cross sections, we can conclude that the difference between the classical description and the optical model is not very significant. Even though the nuclear interactions treated occur in a scale in which it would seem that quantum effects are dominant, the heavy ions we consider have short wavelenghts and often large angular momentum, which implies that the interactions can be described as classical particles that move along a localized trajectory (fig.(3)). When quantum effects (like interference or diffraction) become relevant, the classical theory can be modified to take them into account (this is called semi-classical approximation).

Acknowledgments

I would like to thank first of all my advisor Xavier Vinyes, for his useful help and kindness; and also my dear friends Raimon Luna and Carles Rivero, for their infinite patience and support.

[1] In some cases the α particle is considered the borderline for heavy ions
[2] Hodgson, Gadioli. *Introductory nuclear physics*, Oxford science publications, 1997. Chapter 23.
[3] Salvat. Notes *Elastic collision of charged particles with atoms*
[4] Deduced in 1911 for scattering of α particles by atomic nuclei.It describes suitably the angular distribution of scattered particles in many cases, except when the two ions are equal
[5] See: G.R.Satchler. *Introduction to Nuclear Reactions.* MacMillan, London 1990. pages 132-137
[6] Hodgson, Gadioli. Op. cit. Chapters 13, 20
[7] F. Salvat, "traj_hi.f", Unpublished
[8] P.E.Hodgson, "nvgopthi.f", Oxford University Report.
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[10] F.D. Becchetti. *Elastic and inelastic scattering of* ^{16}O *and* ^{12}C *from* $A = 40-96$*nuclei.* Nuclear physics, North-Holland Publishing Co., Amsterdam