

The WKB Approximation: An application to the alpha decay

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Abstract: In this paper, the WKB approximation is presented in full detail, including a derivation of the connection formulas. A particular case is studied which applies the WKB approximation to alpha particle decay in various isotopes of Uranium nuclei, and is then used to find the alpha energy values using data from numerical computations in published literature. Finally a comparison between the WKB approximation to published studies is made, concluding a relationship between alpha decay energy and its parent nuclear radius may exist.

I. INTRODUCTION

The Wentzel-Kramers-Brillouin (WKB) Approximation was first introduced in quantum mechanics in 1926, although it had been developed earlier. This approximation is important since at the beginning of the development of quantum mechanics, physicists around the world were attempting to solve the Schrödinger equation. In 1928 Gamow used the Approximation to theoretically describe alpha decay for the first time.

In this project an overview of the WKB approximation is presented, including a detailed example of how the WKB method can be used to solve the one-dimensional Schrödinger problem. Problems in three-dimensions can often be reduced to one-dimensional (1D) problems by separation of angular and radial variables if the geometry of the problem allows, further expanding the usefulness of the WKB method. Alpha decay energy from various Uranium isotopes will be calculated using the WKB approximation and compared to values from literature.

II. THE WKB APPROXIMATION

We will consider the One-Dimensional Schrödinger equation, since the three-dimensional Schrödinger equation can often be reduced to the solution of a one-dimensional equation by separating the angular and radial variables. Thus, we have:

$$-\frac{\hbar^2}{2m}\psi''(x) + V(x)\psi(x) = E\psi(x). \quad (1)$$

We will consider a solution of the form

$$\psi(x) = N \exp\left[\frac{iS(x)}{\hbar}\right]. \quad (2)$$

By substituting equation (2) into (1) we find

$$-i\hbar S''(x) + [S'(x)]^2 = F(x), \quad (3)$$

where $F(x) = 2m[E - V(x)]$. The WKB method consists of expanding the function S as a power series of \hbar ,

$$S = S_0 + \hbar S_1 + \hbar^2 S_2 + \dots \quad (4)$$

Introducing equation (4) in (3) and equating terms of equal powers of \hbar :

$$\begin{aligned} [S'_0(x)]^2 &= F(x) \\ -iS''_0(x) + 2S'_0(x)S'_1(x) &= 0 \\ -iS''_1(x) + [S'_1(x)]^2 + 2S'_0(x)S'_2(x) &= 0 \\ &\vdots \end{aligned} \quad (5)$$

which can be easily solved recursively one by one just by integration and substitution. Thus:

$$S_0 = \pm \int^x \sqrt{F(x)} dx + C_0. \quad (6)$$

Rewriting the second expression in (5) yields

$$S'_1(x) = \frac{iS''_0(x)}{2S'_0(x)} = \frac{i}{2} [\ln |S'_0(x)|]'. \quad (7)$$

The approximation essentially consists of taking only the first two terms of the expansion (4) and neglecting the others. We will study to what extent this approximation is valid and consider the first neglected term:

$$S'_2(x) = \frac{iS''_1(x) - [S'_1(x)]^2}{2S'_0(x)}. \quad (8)$$

One may notice that the approximation will provide us with a good solution when (4) is a rapidly convergent series. Assuming that $V(x)$ is slowly varying with x (as well as S_0, S_1, \dots) it is reasonable to consider that $S_2 \sim 0$ when $|\hbar S'_1| \ll |S'_0|$. With the observation that if S'_1 is small its second derivative will also be small. Using the integral forms of S_1 and S_2 we have seen, we see that this condition translates into:

$$\hbar |F'(x)| \ll 2 [F(x)]^{\frac{3}{2}}. \quad (9)$$

Or equivalently,

$$\frac{1}{2} \left(\frac{\hbar}{\sqrt{2m[E - V(x)]}} \right) V'(x) \ll |E - V(x)|. \quad (10)$$

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By introducing a local wavelength, it becomes:

$$\lambda \equiv \frac{2\pi\hbar}{\sqrt{|F(x)|}} \Rightarrow \frac{\lambda}{4\pi} |V'(x)| \ll |E - V(x)|. \quad (11)$$

Therefore, when the relative difference of the potential energy and the wavelength is small the Approximation provides us with good results. This does not happen in what we will call the linear turning points (LTP), where $E - V = 0$. It is important to note that since the Approximation will only be acceptable when the potential is slowly and smoothly varying, as is the case when not near an LTP.

A. WKB Solution in the classically allowed regions

We will study how the solution of the WKB will behave in the classically allowed regions, as limited by the returning points x_a and x_b , but we may be able to consider these points in the limit. In these regions $E > V$, hence $F(x)$ is positive. If we define:

$$\kappa_a(x) \equiv \frac{1}{\hbar} \sqrt{F(x)} = \sqrt{\frac{2m}{\hbar} [E - V(x)]}. \quad (12)$$

The expressions for $S_0(x)$ and $S_1(x)$ will now be:

$$\begin{aligned} S_0(x) &= \pm \hbar \left(\int_X^x \kappa_a(x) dx + C_1 \right) \\ S_1(x) &= \frac{i}{2} \ln [\hbar \kappa_a(x)] + C_3, \end{aligned} \quad (13)$$

where $X \in (x_a, x_b)$. And the WKB solution will therefore be:

$$\tilde{\psi}(x) = \frac{A'}{\sqrt{\kappa_a(x)}} \exp \left[i \int_X^x \kappa_a(x) dx \right] + \frac{B'}{\sqrt{\kappa_a(x)}} \exp \left[-i \int_X^x \kappa_a(x) dx \right]. \quad (14)$$

Where A' and B' are arbitrary constants that will be found when applying the boundary conditions. Alternatively, the solution may be written as:

$$\tilde{\psi}(x) = \frac{A}{\sqrt{\kappa_a(x)}} \sin \left[\int_X^x \kappa_a(x) dx + C_1 \right] + \frac{B}{\sqrt{\kappa_a(x)}} \cos \left[- \int_X^x \kappa_a(x) dx + C_1 \right]. \quad (15)$$

B. WKB Solution for classically forbidden regions

Now we will study how the solution will behave in the classically forbidden region, which will be again limited by the returning points. In this region $E < V$, hence $F(x)$ is negative. Analogously we define $\kappa_f(x) \equiv \frac{1}{\hbar} \sqrt{-F(x)}$. The expressions for $S_0(x)$ and $S_1(x)$ will be the same as (13), but replacing $\kappa_a \rightarrow \kappa_f$ and constants. The WKB solution will then become:

$$\tilde{\psi}(x) = \frac{A'}{\sqrt{\kappa_f(x)}} \exp \left[\int_X^x \kappa_f(x) dx \right] + \frac{B'}{\sqrt{\kappa_f(x)}} \exp \left[- \int_X^x \kappa_f(x) dx \right]. \quad (16)$$

Again, A' and B' are arbitrary constants that will be found when applying the boundary conditions, and alternately written

$$\tilde{\psi}(x) = \frac{A}{\sqrt{\kappa_f(x)}} \sin \left[i \int_X^x \kappa_f(x) dx + C_1 \right] + \frac{B}{\sqrt{\kappa_f(x)}} \cos \left[-i \int_X^x \kappa_f(x) dx + C_1 \right]. \quad (17)$$

III. CONNECTION FORMULAE

To this point we have studied how the solution of Schrödinger's equation may be written in classically allowed regions and in the classically forbidden regions. However we know that the solution that the WKB approximation method provides it is not accurate near the LTPs, since λ becomes infinite at these points. It is no trivial matter finding a general solution valid for every point in the space. Furthermore, in order to find the arbitrary constants of the described expressions we will introduce now what it is commonly known as "Connection Formulae" that will enable us to enlarge the WKB solution along the linear turning points and construct an extended solution everywhere in the space.

There are several authors that approach this problem differently, yet nearly all of them consider the potential to have a linear and slowly varying behavior near the LTP. Then, the Schrödinger's equation can be solved exactly at this points by introducing a new solution involving the Bessel function of order $\pm \frac{1}{3}$. This new solution will asymptotically approach the WKB solution found previously on both sides. It is in this last step that the solutions are matched with arbitrary constants of the WKB approximation, generating a continuous solution across any LTP boundary.

Thus lets us consider that $V(x)$ has a linear behaviour in the LTPs:

$$V(x) \simeq V(x_r) + V'(x_r)(x - x_r) = E + V'(x_r)(x - x_r), \quad (18)$$

where x_r is a Linear Turning Point (LTP), and where we have used $F(x) \simeq -2mV'(x_r)(x - x_r)$. The Schrödinger's equation around a LTP will then be:

$$-\frac{\hbar^2}{2m} \psi''(x) + V'(x_r)(x - x_r) \psi(x) = 0. \quad (19)$$

Doing the following change of variables will simplify drastically our equation and its solution.

$$s = \frac{x - x_r}{\alpha} \quad \alpha = \left[\frac{2m}{\hbar^2} |V'(x)| \right]^{1/3} \rightarrow \left[\frac{d^2}{ds^2} - \frac{\alpha^3 2m V'(x_r) s}{\hbar^2} \right] \psi(s) = 0. \quad (20)$$

Our equation will now look like:

$$\begin{cases} \psi''(s) - s\psi(s) = 0 & \text{if } V'(x_r) > 0 \\ \psi''(s) + s\psi(s) = 0 & \text{if } V'(x_r) < 0 \end{cases} \quad (21)$$

One may notice that if $\psi(s)$ is the solution for the first equation in the system, $\psi(-s)$ will then be the solution for the second one.

A. Airy's Functions and the solution

The general solution for the $\psi(s) - s\psi'(s) = 0$ is $\psi(s) = aAi(s) + bBi(s)$ being $Ai(s)$ and $Bi(s)$ the Airy's functions, which are particular solutions.

$$\begin{aligned} Ai(s) &= \sqrt{\frac{s}{9}} \left[I_{-1/3} \left(\frac{2s^{3/2}}{3} \right) - I_{1/3} \left(\frac{2s^{3/2}}{3} \right) \right] \\ Bi(s) &= \sqrt{\frac{s}{3}} \left[I_{-1/3} \left(\frac{2s^{3/2}}{3} \right) + I_{1/3} \left(\frac{2s^{3/2}}{3} \right) \right], \end{aligned} \quad (22)$$

where $I_\nu(x)$ are the Bessel functions. One may recall that when $x \rightarrow \infty$, which happens when $\hbar \rightarrow 0$, the Airy's functions have an asymptotic behavior and may be written in terms of sinus and cosinus. Therefore, the general solution of $\psi''(s) - s\psi(s) = 0$ that we want to consider is

$$\begin{aligned} \psi(s) &= \frac{a}{\sqrt{\pi}(-s)^{1/4}} \sin \left(\frac{2}{3}(-s)^{3/2} + \frac{\pi}{4} \right) \\ &+ \frac{b}{\sqrt{\pi}(-s)^{1/4}} \cos \left(\frac{2}{3}(-s)^{3/2} + \frac{\pi}{4} \right) \quad \text{if } s < 0 \end{aligned} \quad (23)$$

$$\begin{aligned} \psi(s) &= \frac{a}{2\sqrt{\pi}s^{1/4}} \exp \left(\frac{-2}{3}s^{3/2} \right) + \\ &\frac{b}{\sqrt{\pi}s^{1/4}} \exp \left(\frac{2}{3}s^{3/2} \right) \quad \text{if } s > 0. \end{aligned}$$

B. Solution at the right of a classically allowed regions

When the LTP is at the right of a classically allowed region $V'(x_r) > 0$ and the exact solution of $\psi(s) - s\psi'(s) = 0$ is (24). We have already seen how the WKB solution will be at both sides of the LTP, by using equations (15) and (16) and by changing the integration limits properly. Furthermore, near the LTPs we have that

$$\kappa_a(x) = \sqrt{\frac{2m}{\hbar^2} V'(x_r)(x_r - x)} \quad \kappa_f(x) = \sqrt{\frac{2m}{\hbar^2} V'(x_r)(x - x_r)}. \quad (24)$$

And by using the change of variables described before in (20) we have $\kappa_a = \alpha^{-1}(-s)^{1/2}$ and $\kappa_f = \alpha^{-1}s^{1/2}$. Therefore,

$$\int_x^{x_r} \kappa_a(x) dx = \frac{2}{3}(-s)^{3/2} \quad \int_{x_r}^x \kappa_f(x) dx = \frac{2}{3}s^{3/2}. \quad (25)$$

By substitution, we find

$$\begin{aligned} \tilde{\psi}_1(s) &\simeq \frac{A_1}{(-s)^{1/4}} \sin \left(\frac{2}{3}(-s)^{3/2} + C_1 \right) + \frac{B_1}{(-s)^{1/4}} \cos \left(\frac{2}{3}(-s)^{3/2} + C_1 \right) \\ \tilde{\psi}_2(s) &\simeq \frac{A_2}{s^{1/4}} e^{-\frac{2}{3}s^{3/2}} + \frac{B_2}{s^{1/4}} e^{\frac{2}{3}s^{3/2}}. \end{aligned} \quad (26)$$

Finally by comparison with (24) it is obvious that the WKB solution for a linear potential matches the dominant asymptotic terms of the exact solution. And provided that $\tilde{\psi}_1(x)$ and $\tilde{\psi}_2(x)$ have to match the solution in each side of the LTP, we can therefore obtain the relationship between the unknown coefficients $A_2 = \frac{A_1}{2}$, $B_2 = B_1$ and $C_1 = \frac{\pi}{4}$ obtaining:

$$\begin{aligned} \tilde{\psi}_1(x) &= \frac{A_1}{\sqrt{\kappa_a(x)}} \sin \left[\int_{x_r}^x \kappa_a(x) dx + \frac{\pi}{4} \right] + \frac{B_1}{\sqrt{\kappa_a(x)}} \cos \left[\int_{x_r}^x \kappa_a(x) dx + \frac{\pi}{4} \right] \\ \tilde{\psi}_2(x) &= \frac{A_1}{2\sqrt{\kappa_f(x)}} \exp \left[- \int_{x_r}^x \kappa_f(x) dx \right] + \frac{B_1}{\sqrt{\kappa_f(x)}} \exp \left[\int_{x_r}^x \kappa_f(x) dx \right]. \end{aligned} \quad (27)$$

As we have been mentioning all along this will be valid as long as the potential in the LTP is linear, thus if $\exists(a, b) \ni x_r$ narrow enough that satisfies $V(x) \simeq V(x_r) + V'(x_r)(x - x_r)$ and wide enough that the WKB solution is acceptable in the limiting points a and b .

C. Solution at the left of a classically allowed region

When the LTP is at the left of a classically allowed region, $V'(x_r) < 0$. The exact solution satisfies $\psi(s) + s\psi'(s) = 0$ and can be written as $\psi(s) = aAi(-s) + bBi(-s)$. Likewise what we have previously done,

$$\begin{aligned} \psi(s) &\simeq \frac{a}{\sqrt{\pi}s^{1/4}} \sin \left(\frac{2}{3}s^{3/2} + \frac{\pi}{4} \right) + \frac{b}{\sqrt{\pi}s^{1/4}} \cos \left(\frac{2}{3}s^{3/2} + \frac{\pi}{4} \right) \quad \text{if } s > 0 \\ \psi(s) &\simeq \frac{1}{\sqrt{\pi}(-s)^{1/4}} \left[\frac{a}{2} \exp \left(\frac{-2}{3}(-s)^{3/2} \right) + b \exp \left(\frac{2}{3}(-s)^{3/2} \right) \right] \quad \text{if } s < 0. \end{aligned} \quad (28)$$

And the WKB solution being:

$$\begin{aligned} \tilde{\psi}_1(x) &= \frac{A_1}{\sqrt{\kappa_f(x)}} \exp \left[- \int_{x_r}^x \kappa_f(x) dx \right] + \frac{B_1}{\sqrt{\kappa_f(x)}} \exp \left[\int_{x_r}^x \kappa_f(x) dx \right] \\ \tilde{\psi}_2(x) &= \frac{A_2}{\sqrt{\kappa_a(x)}} \sin \left[\int_{x_r}^x \kappa_a(x) dx + C_2 \right] + \frac{B_2}{\sqrt{\kappa_a(x)}} \cos \left[\int_{x_r}^x \kappa_a(x) dx + C_2 \right]. \end{aligned} \quad (29)$$

Again by using the change described in (20) we find now that $\kappa_f = \alpha^{-1}(-s)^{1/2}$ and $\kappa_a = \alpha^{-1}s^{1/2}$. By substitution in the WKB solution (29) we have that

$$\begin{aligned} \tilde{\psi}_1(s) &\simeq A_1(-s)^{-1/4} \exp \left(\frac{-2}{3}(-s)^{3/2} \right) + B_1(-s)^{-1/4} \exp \left(\frac{2}{3}(-s)^{3/2} \right) \\ \tilde{\psi}_2(s) &\simeq A_2s^{-1/4} \sin \left(\frac{2}{3}s^{3/2} + C_2 \right) + B_2s^{-1/4} \cos \left(\frac{2}{3}s^{3/2} + C_2 \right). \end{aligned} \quad (30)$$

Finally, we find the relationship between the constants comparing it with (28), $A_2 = 2A_1$, $B_2 = B_1$ and $C_2 = \frac{\pi}{4}$. We have described a complete method to find the solution of the Schrödinger's equation everywhere in the space.

IV. WKB APPLICATIONS AND THE ALPHA DECAY

The WKB approximation can be used to calculate the bound state energy levels of a one-dimensional potential well. To study the penetration of a potential barrier, potential wells with several turning points. We will study with deep detail the alpha decay, we will find the energy values of the alpha particle in different Uranium isotopes. We will assume that the alpha particle is confined by a Coulomb barrier in a potential well of radius R as in [[1]]. We will represent the potential of the strong nuclear binding as a simple potential well.

$$V(r) = \begin{cases} -V_0, & \text{if } r < R \\ \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{r}, & \text{if } r > R. \end{cases} \quad (31)$$

For simplicity we assume we have s-waves and that we can write $\Psi(r) = \frac{1}{r}u(r)$, where the wavefunction is:

$$u(r) = N \begin{cases} \sin Kr, & \text{if } r < R \\ \frac{1}{\sqrt{\kappa_f(r)}} \left[A e^{\int_R^r \kappa_f(x) dx} + B e^{-\int_R^r \kappa_f(x) dx} \right], & \text{if } R \leq r \leq b \\ \frac{1}{\sqrt{\kappa_a}} C \exp \left[i \int_b^r \kappa_a(x) dx \right] & \text{if } r > b, \end{cases} \quad (32)$$

where $K = \sqrt{2m(E + V_0)}$.

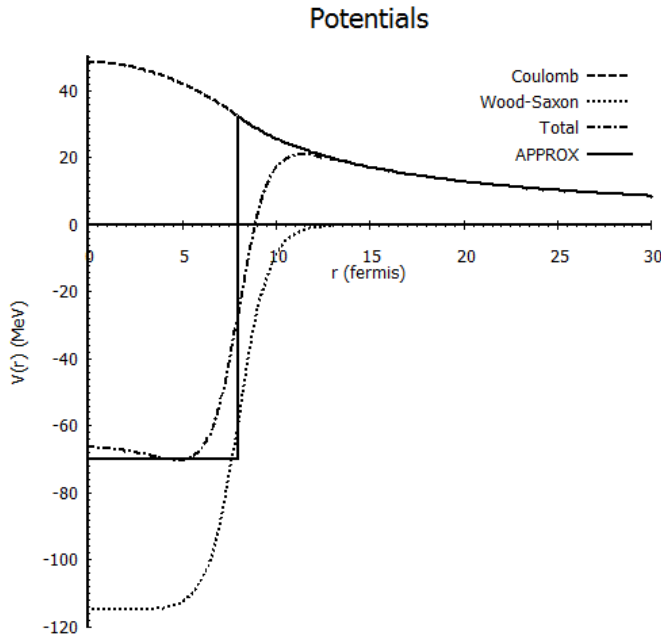


FIG. 1: Four energy potentials are plotted versus the Uranium nuclear radius: Coulombic, Wood-Saxon, total potential, and WKB approximation as in eq.(31)

In Fig.(1) a constant energy of $0 < E < V_m a x$ will intersect the approximation potential twice, thus creating three regions. Now we study the matching relations at $r = b$: So if we are in region II, $R \leq r \leq b$ we will use the formulas described for when we are at the left of a classically allowed region and s will be negative. Notice that

$$\int_R^b \kappa_f(x) dx + \int_b^r \kappa_r(x) dx = \int_R^r \kappa_f(x) dx. \quad (33)$$

If we do the following changes of variables

$$\gamma = \int_R^b \kappa_f(x) dx \quad A' = B e^{-\gamma} \quad B' = A e^{\gamma}, \quad (34)$$

the WKB will then be

$$\frac{1}{\sqrt{\kappa_f(r)}} \left[A' e^{\int_r^b \kappa_f(x) dx} + B' e^{-\int_r^b \kappa_f(x) dx} \right] \quad (35)$$

And with the change of variables defined in (20), and being the solution of $u_p(s) = a A i(-s) + b B i(-s)$, we have:

$$\begin{aligned} u_{WKB}(s) &\simeq A'(-s)^{-1/4} \exp\left(\frac{2}{3}(-s)^{3/2}\right) + B'(-s)^{-1/4} \exp\left(\frac{-2}{3}(-s)^{3/2}\right) \\ u_p(s) &\simeq \frac{1}{\sqrt{\pi}(-s)^{1/4}} \left[\frac{a}{2} \exp\left(\frac{-2}{3}(-s)^{3/2}\right) + b \exp\left(\frac{2}{3}(-s)^{3/2}\right) \right]. \end{aligned} \quad (36)$$

Thus $a = 2\sqrt{\pi}B'$ and $b = \sqrt{\pi}A'$. Now on the other side of point b, in region III, thus $s > 0$ and using equation (28):

$$\begin{aligned} u_p(s) &\simeq \frac{1}{2\sqrt{\pi}s^{1/4}} \left[(-ia + b) e^{i\pi/4} e^{i2/3s^{3/2}} + (ia + b) e^{-i\pi/4} e^{-i2/3s^{3/2}} \right] \\ u_{WKB} &= \frac{C}{s^{1/4}} \exp\left[i\frac{2}{3}s^{3/2}\right]. \end{aligned} \quad (37)$$

Thus we have that $ia + b = 0$ and $b = C\sqrt{\pi}e^{-i\pi/4}$, $a = iC\sqrt{\pi}e^{-i\pi/4}$. Therefore,

$$\begin{aligned} A' &= C e^{-i\pi/4} \\ 2B' &= i e^{-i\pi/4} \Rightarrow A e^{\gamma} = \frac{iC}{2} e^{-i\pi/4} \Rightarrow A = \frac{iB}{2} e^{-2\gamma}. \\ B e^{-\gamma} &= C e^{-i\pi/4} \end{aligned} \quad (38)$$

By continuity of the wavefunction and its derivative at $r = R$:

$$\begin{aligned} \sin KR &= \frac{A + B}{\sqrt{\kappa_f}} \\ \cos KR &= \frac{\sqrt{\kappa_a}}{K} (A - B) \end{aligned} \Rightarrow \tan KR = \frac{A + B}{A - B} \frac{K}{\kappa_a}. \quad (39)$$

By substitution in (38) we have

$$\frac{K}{\kappa_a} + \tan KR = \frac{i e^{-2\gamma}}{2} \left(\tan KR - \frac{K}{\kappa_a} \right). \quad (40)$$

The right-hand side of the last equation is extremely small. Furthermore, the first term of the left-hand side

can be neglected since for high Coulomb barriers and low-lying states $K \ll \kappa_\alpha(R)^7$. The real part of the eigenvalue will almost be the same as in the case of an infinite square well: $\tan KR \simeq 0 \Rightarrow KR = n\pi \quad n \in \mathbb{N}$. And by the definition of K we find:

$$E_\alpha = \left(\frac{\hbar n\pi}{R}\right)^2 \frac{1}{2m} - V_0. \quad (41)$$

Notice that the constant well depth V_0 has an electrostatic component and a strong force component.

V. RESULTS

Formula (41) has been used to find the energy of the emitted alpha particles for different Uranium isotopes with even atomic number A and orbital quantum number $l = 0$. The values have been compared with the ones obtained by numerical computation [[2]]. In Segiel, et. al., the strong interaction is approximated by a Wood-Saxon potential where the nuclear radius is approximated by $R = A^{1/3}1.25\text{fm}$ using $A - 4$ instead of the atomic number, the skin parameter is taken to be 0.8fm , the values of the Wood-Saxon potential have been numerically adjusted to obtain values of the ejected alpha particle consistent with experimental values. For equation (41), we use the numerically-adjusted Wood-Saxon potentials in [[2]], which were obtained with a radial quantum number $n = 10$. With these values we find the first positive alpha particle energies at $n = 10$ as well. Negative energy values are incompatible with the described system since we have only considered $E > 0$. For $n < 10$ the Wood-Saxon potential is weaker [[2]] and smaller values must be used. In Table I, the energy values of the alpha

Isotope	$E_\alpha(A-4)(\text{MeV})$	$E_\alpha(A-2)(\text{MeV})$	$E_\alpha(A)(\text{MeV})$	$E_{\alpha exp}(\text{MeV})$
U^{238}	4.87	4.29	3.71	4.27
U^{236}	5.34	4.75	4.16	4.57
U^{234}	5.79	5.19	4.59	4.86
U^{232}	6.63	6.01	5.41	5.41
U^{230}	7.50	6.88	6.27	5.99
U^{228}	8.68	8.05	7.43	6.80
U^{226}	10.05	9.41	8.78	7.72
U^{224}	11.41	10.76	10.12	8.62

TABLE I: Energy of the alpha particle of different Uranium Isotopes and different radii.

particle are shown with different values for A in calculating R as $(A-4)$, $(A-2)$, and A , in Columns I, II, and III, respectively, and then compared with known values from [[2]] in Column IV. The experimental values in [[3]] match the the values in [[2]] because the Wood-Saxon potential was a free parameter that could be adjusted. Rearranging 41, we find what the Wood-Saxon should be in our case, which is shown in Table II versus our adjusted parameters for R .

Isotope	$V_0[A-4](\text{MeV})$	$V_0[A-2](\text{MeV})$	$V_0[A](\text{MeV})$	[[2]] values (MeV)
U^{238}	-116.33	-115.74	-115.16	-115.72
U^{236}	-116.62	-116.02	-115.43	-115.85
U^{234}	-116.94	-116.33	-115.74	-116.00
U^{232}	-116.99	-116.38	-115.78	-115.78
U^{230}	-117.04	-116.41	-115.80	-115.53
U^{228}	-116.85	-116.22	-115.60	-114.98
U^{226}	-116.60	-115.96	-115.33	-114.25
U^{224}	-116.33	-115.68	-115.04	-113.54

TABLE II: Wood-Saxon potential energy for different Uranium Isotopes and different radii.

VI. CONCLUSIONS

The WKB Approximation has been studied and applied to alpha decay since it has not been reviewed in any of the courses taken during the Physics Bachelor program. Although [[2]] is a quite rough numerical calculation and WKB is just an approximation, all the results are in the same range and agree well with experimental results. This analysis is limited by the initial Wood-Saxon values used from source [[2]]. Additionally, the effect on the nuclear radius should be studied to a more precise value to better understand the interaction between the four nucleons which become the alpha particle as it escapes and the parent nucleus.

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