



**WORKING PAPERS**

Col·lecció d'Economia E16/354

---

# Self-Financing Education, Borrowing Constraints, Government Policies, and Economic Growth

Hoang D. Duong

Fernando Sánchez-Losada



UNIVERSITAT DE  
BARCELONA

## Self-Financing Education, Borrowing Constraints, Government Policies, and Economic Growth

**Abstract:** We analyze how public policies for self-financing education, public fund for loans and deferred deductibility of education expenses, affect growth in an overlapping generations economy where individuals can be borrowing-constrained on human capital investment. We show that public loans positively affect growth in the unconstrained economy, while how tax deductibility affects growth depends on the magnitude of both public loans and tax deductibility. In the borrowing-constrained economy, public loans positively affect growth, while tax deductibility does not affect growth. Both government policies affect the borrowing-constraint tightness and, therefore, can shift the economy from being borrowing-constrained to unconstrained or vice versa.

JEL Codes: O40, H20, I22.

Keywords: Self-financing education, public education fund, tax deduction.

Hoang D. Duong  
Universitat Autònoma de Barcelona

Fernando Sánchez-Losada  
Universitat de Barcelona

**Acknowledgements:** We are very grateful to Jaime Alonso-Carreras, Marc Teignier and Montserrat Vilalta-Bufí. Fernando Sánchez-Losada acknowledges the financial support from the Ministerio de Economía y Competitividad and Fondo Europeo de Desarrollo Regional through grant ECO2015-66701-R (MINECO/FEDER, UE) and from the Government of Catalonia through grant 2014SGR493.

## 1. Introduction

As widely accepted in the literature, human capital accumulation is one of the main engines of growth (see Lucas, 1988). In any society, young individuals are characterized as not having accumulated assets in order to pay for education, an education that provides them with a human capital level and, then, will allow them to develop better careers and earn higher salaries. And, precisely, this level of education ultimately determines the welfare of the society. Financing human capital should therefore be attached a great importance to. Apart from altruistic parents and/or public education and/or public subsidies to education, young individuals can self-finance their education by getting loans from government and/or private financial markets and pay off their loans while working later on. This paper analyzes how government policies for self-financing education affect economic growth. Specifically, we stress the connection between these policies and the borrowing-constraint tightness of young individuals.

We consider an overlapping generations economy with endogenous human capital formation depending on investment in education and the level of human capital of the previous generation. When young, individuals borrow to invest in education, which endows them with a level of human capital. Individual loans come either from private credit markets or from public funds. However, due to the supply side of the financial market, individuals could be borrowing-constrained and, then, unable to finance their desired education. When adult, individuals work and use their incomes to consume, pay back the education loans, pay lump-sum taxes and save. When old, they consume their savings returns.

We analyze the importance of two public policies on the formation of human capital and, then, growth when human capital investment has no risk. In this way, having no risk and no altruistic parents, we highlight the pure effects of these policies on the financing of human capital and, then, growth without having any indirect financing effect. These policies are a public fund for education loans and deferred deductibility of education expenses.<sup>1</sup> Thus, and by not considering either public education or public subsidies, these two policies imply that the education of a generation will be ultimately paid by the same generation.<sup>2</sup> In this way, we concentrate on to what extent the government education policies affect economic growth when education is completely self-financing. We assume that both public policies are financed through lump-sum taxes. Therefore, they imply the same negative income effect for individuals and, then, aggregate savings will decrease. The difference between public loans for education and deferred deductibility of education expenses is two-fold. Firstly, while deductibility directly distorts the price of education, public loans indirectly distort the price via a higher supply of aggregate savings. Secondly, and perhaps most importantly, public loans can alleviate or break individuals' borrowing constraints because of this increase in aggregate savings, but deductibility of education expenses. Thus, the two policies have opposite effects on the borrowing-constraint tightness: while public loans lessen the pressure in the private credit market, tax deduction tightens the borrowing constraint.

Our results are categorized into three points: the effects of government policies on economic growth when young individuals are and are not borrowing-constrained, and the effects of government policies on the borrowing constraint tightness of young individuals. First, in the unconstrained economy, public loans always positively affect economic growth since the increase in public savings more than compensate the decrease in private savings as a consequence of the negative income effect for individuals due to the lump-sum tax. However, an increase in tax deductibility has two

---

<sup>1</sup>Although we will show that in U.S. this deduction is on the 100% of the interest rate, we consider the possibility to deduct also the principal, as the case of mortgage loans in some countries.

<sup>2</sup>Note that, although the public fund is built up by all the previous generations, individuals have to repay their loans.

opposite effects on the net price of education loans: a direct effect, since a higher tax deductibility implies a lower net price for education loans and, then, the loan demand increases; and an indirect effect, since this increase in the demand for loans leads to an increase in the equilibrium interest rate which, in turn, increases the net price and, then, the loans demand decreases. How this increase in tax deductibility affects economic growth depends on which effect is dominant. Thus, when the direct effect is dominant, an increase in tax deductibility positively affects economic growth since education investment increases, whereas when the indirect effect is dominant, it is the other way around. Overall, which effect is dominant depends on the magnitude of public loans, tax deductibility itself and the individual discount rate, since a higher discount rate means higher savings and, then, a lower net price for education loans. Specifically, when tax deductibility is sufficiently low, the direct effect is always dominant since an increase in tax deductibility implies a considerable reduction in the net price of loans. But when tax deductibility is sufficiently high, the effect of an increase in tax deductibility depends on the magnitude of public loans. Thus, when public loans are scarce, the indirect effect is dominant since an increase in tax deductibility will lead to a considerable increase in private loan demand. As a result, the interest rate will increase considerably. When public loans are sufficiently high, the increase in private loan demand will not be high enough and, then, the direct effect will dominate.

Second, in the borrowing-constrained economy, a numerical exercise suggests that public loans for education positively affects economic growth. An increase in public loans lessens the borrowing constraint since it allows more individuals to be able to access education loans and, hence, has a positive effect on education investment that, in turn, fosters economic growth. In contrast, an increase in tax deductibility does not affect economic growth. Individuals would increase the demand of loans as its net price becomes cheaper, but since the economy is borrowing-constrained, they cannot increase their loans.

Third, we show that both government policies determine if the economy is borrowing-constrained or not. Since private lenders worry about default, individuals can borrow at most a fraction of their life-cycle income. We define this fraction as the collateral rate. Then, there exists a particular value of this collateral rate, says the critical value, such that if the collateral rate is above it then individuals are not borrowing-constrained. We show that both government policies affect this critical collateral rate and, therefore, can shift the economy from being borrowing-constrained to unconstrained or vice versa. In particular, an increase in public loans has two effects on the critical value of the collateral rate. Firstly, there is a direct effect since the demand for private loans will decrease and, as a result, the economy will be more likely to be unconstrained. This, in turn, will positively affect economic growth. And, secondly, there is an indirect effect since a higher growth rate will consequently lead to a higher demand for loans and, as a result, the economy will be more likely to be constrained. Similarly, an increase in tax deductibility has also two effects on the critical collateral rate. Firstly, there is a direct effect since the demand for private loans will increase and, as a result, the economy will be more likely to be constrained. And secondly, there is an indirect effect via the growth rate which depends on the government policy values. A numerical exercise suggests that the critical collateral rate is decreasing in public loans whereas it is increasing in tax deductibility. In conclusion, alternative government policies affect in different ways the severity of the borrowing constraint and, then, growth.

The paper is organized as follows. Following a literature review, in the next section, we present the model and define the fundamental concepts. In Section 3 and Section 4, we study the effects of the public fund and tax deduction on economic growth when the borrowing constraint is not binding and binding, respectively. In Section 5, we derive the critical value of the collateral rate which determines if the economy is constrained or unconstrained and analyze the interactions of both government policies and the borrowing constraint tightness via this critical value. Section 6

concludes the paper. All proofs are in the Appendix.

**Literature review.** In contrast to our paper, considerable attention of economists has focused on studying the formation of human capital, education policies and their effects on the economy in the presence of altruism. For example, Glomm and Ravikumar (1992) and Eckstein and Zilcha (1994) discuss the distinction between economies with public education and those with private education. Milesi-Ferretti and Roubini (1998) and Brauning and Vidal (2000) study the effect of a public subsidy on private education. And Zhang (1996) and Blankenau (2005) analyze the effects of both, public education and public subsidies. But little attention has been devoted if parents are not altruistic. In this case, why then to publicly finance education if parents are not altruistic? While Soares (2003) shows that agents that get a large fraction of their income from the return on their physical capital are interested in a higher level of human capital of future workers and, therefore, support for public funding of education, Boldrin and Montes (2005) propose public education as a borrowing-lending scheme: working individuals want to pay public education to young because they will pay back a public pension when old.

In the recent years, a large body of literature document the connection between individual abilities, borrowing constraints, public policies and schooling decisions. Thus, while Abbott et al. (2016) find that the educational financial aid system in the U.S. improves welfare, and removing it would reduce GDP by 4-5 percentage points in the long run, Garriga and Keightley (2016) find that the impact of borrowing constraints on schooling enrollment are significant when the constraints are severely tightened and the option to work while in school is removed. Closely related to our work, Lochner and Monge-Naranjo (2011, 2012) examine the effects of borrowing constraints, government public loans and subsidies to education on schooling attainment in the presence of innate abilities. They suggest that endogenous borrowing constraints make human capital investment more sensitive to government education subsidies and private lending markets play an important role in how human capital accumulation responds to changes in policies. Nevertheless, our focus is rather on the interaction between borrowing constraints, self-financing education and growth. A complementary analysis is Findeisen and Sachs (2016), who show that an education public loan system coupled with income-contingent repayment can always be designed in a Pareto optimal way. To our knowledge, only Stancheva (2016) introduces deferred tax deductibility of human capital expenses. However, different from us, she uses tax deductibility as one of the fiscal instruments in the design of a second-best optimal tax scheme for human capital accumulation over the life-cycle.

As apposed to our work where we consider no risks of human capital investment, a series of other papers study the role of government policies in education, such as taxes and subsidies, in the presence of idiosyncratic labor income risk (see Krebs, 2003, Kass and Zink, 2011, or Krueger and Ludwig, 2016) or risk during the human capital accumulation process (see Tsiddon, 1992, Kalemli-Ozcan et al., 2000, Gottardi et al., 2015, Lochner and Monge-Naranjo, 2016). Specifically, Krebs (2003) studies the connection between human capital risk and growth and conclude that a reduction in uninsurable idiosyncratic labor income risk decreases physical capital investment, but increases human capital investment, growth and welfare. Krueger and Ludwig (2016) find that progressive taxes provide social insurance against idiosyncratic wage risk but distort the education decision of households such that optimally chosen tertiary education subsidies mitigate these distortions. And Gottardi et al. (2015), in an environment with uninsurable risk to human capital accumulation, conclude that it is beneficial to tax both labor and capital income.

## 2. The Economy

### 2.1. Households

Consider an overlapping generations economy in which individuals live for three periods: in the first period they study, in the second period they work, and in the third period they retire. Working population at time  $t$  is  $N_t$  and grows at the rate  $n$ . An individual born at time  $t - 1$  has to borrow  $l_{t-1}$  to invest in education, which endows her with a number of efficiency units of labor, measured by the human capital level  $h_t$ . She is endowed with one unit of labor time that will be supplied inelastically in the second period. Human capital depends on the investment in education and the level of human capital in the previous period. In particular, we assume

$$h_t = \delta l_{t-1}^\gamma h_{t-1}^{1-\gamma}, \quad (2.1)$$

where  $\gamma \in (0, 1)$ . The educational loan can be public or private. Thus,  $l_{t-1} = l_{t-1}^{pr} + l_{t-1}^{pu}$ , where  $l_{t-1}^{pr}$  is the private loan and  $l_{t-1}^{pu}$  is the public loan. In the second period, the individual works and gets an income  $w_t h_t$ , where  $w_t$  is the wage per efficiency unit of labor. She consumes  $c_{1t}$ , saves  $s_t$ , pays two lump-sum taxes  $v_t$  and  $m_t$ ,<sup>3</sup> and repays the loan of the previous period  $R_t (1 - g_t) l_{t-1}$ , where  $R_t = 1 + r_t$  is the interest factor,  $r_t$  is the interest rate, and  $g_t$  is a proportional tax-deductible amount on the education expenses. Note that we consider the possibility to deduct both the interest rate and the principal of the loan. The budget constraint in the second period of an individual born at time  $t - 1$  is

$$w_t h_t - v_t - m_t = R_t (1 - g_t) l_{t-1} + c_{1t} + s_t. \quad (2.2)$$

In the third period, the individual uses the return from savings  $R_{t+1} s_t$  to consume  $c_{2t+1}$ . Thus,

$$c_{2t+1} = R_{t+1} s_t. \quad (2.3)$$

Moreover, since private lenders worry about default, individuals face the following borrowing constraint when asking for private loans in the first period:

$$l_{t-1}^{pr} \leq \phi w_t h_t, \quad (2.4)$$

where  $\phi \in (0, 1)$  states the maximum quantity individuals can borrow from the private capital market given their expected future income. We define this fraction as the collateral rate. Note that individuals want  $l_{t-1}^{pu}$  as big as possible, since the lower  $l_{t-1}^{pr} = l_{t-1} - l_{t-1}^{pu}$ , the more likely the restriction is not binding. Thus, public loans can alleviate or break individuals' borrowing constraints, but deductibility of education expenses. Combining (2.1) and (2.4), the restriction can be written as

$$\phi w_t \delta l_{t-1}^\gamma h_{t-1}^{1-\gamma} - l_{t-1}^{pr} \geq 0. \quad (2.5)$$

The individual maximizes  $\ln c_{1t} + \beta \ln c_{2t+1}$  subject to (2.1), (2.2), (2.3) and (2.4). The optimal condition regardless of the borrowing constraint is

$$c_{2t+1} = c_{1t} \beta R_{t+1}, \quad (2.6)$$

which equates the marginal rate of substitution to the relative price. When the borrowing constraint is not binding, the optimal condition with respect to the loan is

$$R_t (1 - g_t) - \gamma w_t \delta l_{t-1}^{\gamma-1} h_{t-1}^{1-\gamma} = 0, \quad (2.7)$$

---

<sup>3</sup>Although we could have only one lump-sum tax, for ease of exposition we consider two different ones.

which equates the marginal income to the marginal cost of the loan. When the borrowing constraint is binding, then (2.5) holds with strict equality.<sup>4</sup>

## 2.2. Firms

Firms maximize profits,  $(K_t)^\alpha(N_t h_t)^{1-\alpha} - w_t N_t h_t - R_t K_t$ , where  $K_t$  is capital and  $\alpha \in (0, 1)$ . The optimal conditions are

$$R_t = \alpha \left( \frac{K_t}{N_t h_t} \right)^{\alpha-1} = \alpha \left( \frac{k_t}{h_t} \right)^{\alpha-1} \quad (2.8)$$

and

$$w_t = (1 - \alpha) \left( \frac{K_t}{N_t h_t} \right)^\alpha = (1 - \alpha) \left( \frac{k_t}{h_t} \right)^\alpha, \quad (2.9)$$

where  $k_t \equiv K_t/N_t$  is capital per capita. Dividing (2.8) by (2.9), we have

$$\frac{R_t}{w_t} = \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{h_t}{k_t} \right). \quad (2.10)$$

## 2.3. Government

The government levies workers two types of lump-sum taxes: a tax  $v_t$  to finance the tax deduction of the education loans,

$$v_t = g_t R_t l_{t-1}, \quad (2.11)$$

and a tax  $m_t$  to build a public fund for education loans. Defining  $F_t$  as the public fund, and noting that (2.11) implies that the interest rate paid for the public loan becomes a net income for the government, the fund's accumulation law is

$$F_t - F_{t-1} = N_t m_t + N_t (R_t - 1) l_{t-1}^{pu}, \quad (2.12)$$

which means that the increase in the public fund consists of the lump-sum tax and the interest rate of the public loan. Rewriting this equation in per capita terms, we have

$$f_t - \frac{f_{t-1}}{1+n} = m_t + (R_t - 1) l_{t-1}^{pu}. \quad (2.13)$$

Government loans are

$$N_{t+1} l_t^{pu} = F_t \leq N_{t+1} l_t$$

and, then,

$$(1+n) l_t^{pu} = f_t. \quad (2.14)$$

Combining (2.13) and (2.14), we have

$$(1+n) l_t^{pu} = m_t + R_t l_{t-1}^{pu}. \quad (2.15)$$

We assume the government fixes both  $g_t$  and  $l_t^{pu}$ . Then  $v_t$  and  $m_t$  will be endogenous.

---

<sup>4</sup>In this case,  $R_t(1-g_t) - \gamma w_t \delta l_{t-1}^{\gamma-1} h_{t-1}^{1-\gamma} < 0$ , which means that the individual wants to increase the loan, but she cannot, since it is given by (2.5).

## 2.4. Capital Market Clearing Condition

Savings  $N_t s_t$  are lent to firms or to young individuals. Therefore,

$$s_t = (1 + n)(k_{t+1} + l_t^{pr}). \quad (2.16)$$

Next, we derive the balanced growth path depending on the existence of financial frictions, that is, if the borrowing constraint is binding or not.

## 3. Non-Financial Frictions

### 3.1. Balanced Growth Path

Since the economy grows, we define  $l_{t-1}^{pu} = \rho_{t-1} l_{t-1}$  and, as  $l_{t-1} = l_{t-1}^{pr} + l_{t-1}^{pu}$ , then  $l_t^{pr} = (1 - \rho_t) l_t$ , where  $\rho_t \in (0, 1)$  is the proportion of the public loan over the total loan at time  $t$ . Since the borrowing constraint is not binding, combining (2.1) and (2.7) we obtain

$$w_t h_t = \frac{R_t (1 - g_t)}{\gamma} l_{t-1}. \quad (3.1)$$

Combining (2.2), (2.3), (2.6), (2.11) and (2.15) yields

$$\left(\frac{1 + \beta}{\beta}\right) s_t = w_t h_t - (1 - \rho_{t-1}) R_t l_{t-1} - (1 + n) \rho_t l_t. \quad (3.2)$$

From (2.10) and (3.1), we have

$$k_t = \left(\frac{\alpha}{1 - \alpha}\right) \left(\frac{1 - g_t}{\gamma}\right) l_{t-1}. \quad (3.3)$$

Substituting (2.16) and (3.1) into (3.2), and after using (3.3), we obtain

$$\begin{aligned} (1 + n) \left( \left[ \frac{1 + \beta}{\beta} \right] \left[ \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{1 - g_{t+1}}{\gamma} \right) + (1 - \rho_t) \right] + \rho_t \right) \varphi_{t-1} \\ = \left[ \left( \frac{1 - g_t}{\gamma} \right) - (1 - \rho_{t-1}) \right] R_t, \end{aligned} \quad (3.4)$$

where  $\varphi_{t-1} = l_t / l_{t-1}$  is the loan's growth factor. From (2.7), (2.8) and (2.9), we have

$$R_t = \alpha^\alpha (1 - \alpha)^{(1-\alpha)} \left( \frac{\gamma \delta}{1 - g_t} \right)^{(1-\alpha)} \left( \frac{h_{t-1}}{l_{t-1}} \right)^{(1-\gamma)(1-\alpha)}. \quad (3.5)$$

And combining this equation with (2.1) yields

$$R_t = \alpha^\alpha (1 - \alpha)^{(1-\alpha)} \left( \frac{\gamma \delta}{1 - g_t} \right)^{(1-\alpha)} \left( \frac{\delta}{\varphi_{t-1}} \right)^{\frac{(1-\gamma)(1-\alpha)}{\gamma}}. \quad (3.6)$$

Finally, substituting (3.6) into (3.4) and evaluating at the balanced growth path, we obtain

$$\varphi_u = \left\{ \frac{\left[ \left( \frac{1-g}{\gamma} \right) - (1 - \rho) \right] \left[ \frac{\alpha \left( \frac{\alpha}{1-\alpha} \right)^{(1-\alpha)} (1-\alpha) \gamma \delta^{\frac{1}{\gamma}}}{(1-g)} \right]^{1-\alpha}}{(1 + n) \left( \left[ \frac{1+\beta}{\beta} \right] \left[ \left( \frac{1-g}{\gamma} \right) \left( \frac{\alpha}{1-\alpha} \right) + (1 - \rho) \right] + \rho \right)} \right\}^{\frac{1}{1 + \frac{(1-\gamma)(1-\alpha)}{\gamma}}}, \quad (3.7)$$

where the subscript  $u$  denotes the unconstrained economy. Next propositions summarize the consequences on economic growth of a change in the public policy.



**Proposition 3.1.** *When individuals are not borrowing-constrained, public loans for education have always a positive impact on growth. That is, the higher the value of  $\rho$ , the higher the value of  $\varphi_u$ .*

Public loans always positively affect economic growth since the increase in public savings more than compensate the decrease in private savings as a consequence of the negative income effect for individuals due to the tax.

**Proposition 3.2.** *When individuals are not borrowing-constrained, there exist  $\bar{\beta}(\rho)$  and  $\bar{g}(\rho)$  such that when  $\beta > \bar{\beta}(\rho)$ , if  $g < \bar{g}(\rho)$  then  $\partial\varphi_u/\partial g > 0$ , and if  $g \geq \bar{g}(\rho)$  then  $\partial\varphi_u/\partial g \leq 0$ ; and when  $\beta \leq \bar{\beta}(\rho)$  then  $\partial\varphi_u/\partial g \leq 0$ .*

An increase in tax deductibility has two opposite effects on the net price of education loans: a direct effect, since a higher tax deductibility implies a lower net price for education loans and, then, the loan demand increases; and an indirect effect, since this increase in the demand for loans leads to an increase in the equilibrium interest rate which, in turn, increases the net price and, then, the loans demand decreases. How this increase in tax deductibility affects economic growth depends on which effect is dominant. Thus, when the direct effect is dominant, an increase in tax deductibility positively affects economic growth since education investment increases, whereas when the indirect effect is dominant, it is the other way around. Overall, which effect is dominant depends on the magnitude of public loans, tax deductibility itself and the individual discount rate, since a higher discount rate means higher savings and, then, a lower net price for education loans. Specifically, when tax deductibility is sufficiently low, the direct effect is always dominant since an increase in tax deductibility implies a considerable reduction in the net price of loans. But when tax deductibility is sufficiently high, the effect of an increase in tax deductibility depends on the magnitude of public loans. Thus, when public loans are scarce, the indirect effect is dominant since an increase in tax deductibility will lead to a considerable increase in private loan demand. As a result, the interest rate will increase considerably. When public loans are sufficiently high, the increase in private loan demand will not be high enough and, then, the direct effect will dominate.

### 3.2. Numerical exercise

Next, we illustrate the previous proposition through a numerical exercise.<sup>5</sup> The strategy is as follows: firstly, we calibrate for the values of  $\gamma$  and  $\delta$  using U.S. economy statistics; and secondly, using these calibrated parameters, we show how the combination of the values of  $g$  and  $\rho$  decides their effects on the growth rate  $\varphi_u$ . The below table resumes the parameter values that we use in the calibration exercise (a detailed explanation is in the Appendix). With these parameter values, from equations (3.4) and (3.6) we obtain  $\gamma = 0.1040049078$  and  $\delta = 2.293560488$ .

$\alpha$	$\beta$	$g$	$\rho$	$n$	$\varphi$	$R$
0.33	0.739	0.1897	0.3	0.24458	1.48595	2.86294

In order to show how tax deduction affects the economy, we check the sign of the derivative of the growth rate  $\varphi_u$  with respect to  $g$ . Since the sign of the derivative depends on the value of  $\rho$  (see the Appendix), for each value of  $g$  there exists a threshold value of  $\bar{\rho}$  such that if  $\rho < \bar{\rho}$  then  $\partial\varphi_u/\partial g < 0$ , if  $\rho > \bar{\rho}$  then  $\partial\varphi_u/\partial g > 0$ , and if  $\rho = \bar{\rho}$  then  $\partial\varphi_u/\partial g = 0$ . Figure 3.1. shows the associated values of  $\bar{\rho}$  for each value of  $g$ . For a sufficiently low value of  $g$ ,  $\partial\varphi_u/\partial g > 0$  no matter

<sup>5</sup>According to Cameron and Taber (2004), there is no evidence of borrowing constraints in education in the U.S. Therefore, we calibrate the parameters for the unconstrained economy.

the value of  $\rho$ . But for a sufficiently high value of  $g$ , the magnitude of  $\rho$  decides the sign of  $\partial\varphi_u/\partial g$ . In particular,  $\partial\varphi_u/\partial g < 0$  and  $\partial\varphi_u/\partial g > 0$  when the combinations of values of  $\rho$  and  $g$  lie on the left side and the right side of the continuous line, respectively.

There are two opposite effects of a change in tax deductibility on the net price of education loans  $R_t(1 - g_t)$ : a direct effect via  $(1 - g_t)$  and an indirect effect via  $R_t$ . An increase in  $g$  directly implies a lower net price for education loans but, as a consequence, the demand for loans will increase and, then, leads to an increase in the interest rate  $R_t$ . Therefore, how a change in tax deductibility  $g$  affects the demand for education loans depends on which effect dominates. Figure 3.1. shows that when  $g < 0.584$ , the direct effect always dominates and an increase in  $g$  leads to an increase in education loans which, in turn, has a positive impact on economic growth. For  $0.584 < g < 0.9558$ , the dominating effect depends on the magnitude of public loans for education. When the proportion of public loans over total loans is sufficiently high, the increase in private loans due to an increase in tax deductibility will not be high enough to make the indirect effect via  $R_t$  be the dominating effect. In the U.S. economy, where there is only a 100% of tax-deduction on the interest rate and, then,  $g = 0.1897$ , we should deduct a considerable part of the principal of the loan in order that  $g$  decreases  $\varphi_u$ .

Figure 3.1. also illustrates how the results change if we use a proportional tax on income instead of a lump-sum tax to finance for tax deduction. In this case, we can define a price wedge  $R_t(1 - g_t)/(1 - \tau_t)$  instead of the net price of loans. Now, it is more likely that tax deductibility has a positive impact on growth.

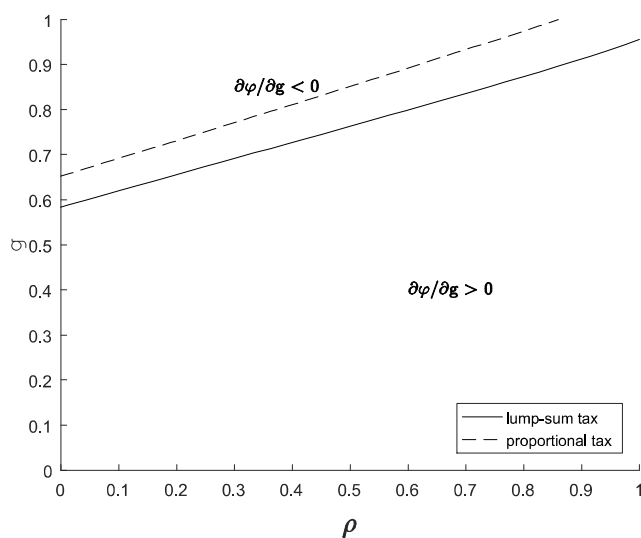


Figure 3.1. The sign of  $\partial\varphi_u/\partial g$  as a function of  $\rho$  and  $g$  in the cases of a lump-sum tax and a proportional tax.

Figure 3.2 illustrates how tax deductibility affects  $\varphi_u$  when we set  $\rho = 0.3$ , as in the U.S. economy statistics. It shows that an increase in  $g$  decreases the growth rate only when a considerable part of the principal of the loan is deducted. Moreover, according to Proposition 3.2, it is the case that  $\beta = 0.739 > \bar{\beta}(0.3)$ .

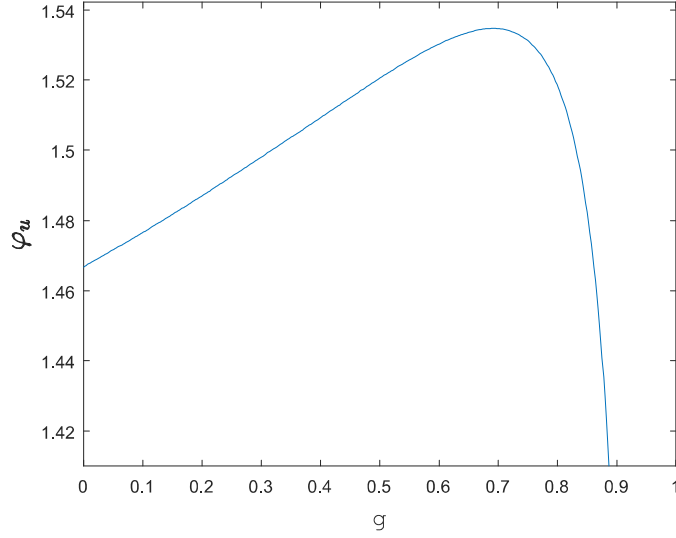


Figure 3.2. The effects of  $g$  on  $\varphi_u$  when  $\rho = 0.3$ .

## 4. Financial Frictions

### 4.1. Balanced Growth Path

When individuals are borrowing-constrained, then (2.4) is binding. Thus,

$$w_t h_t = \left( \frac{1 - \rho_{t-1}}{\phi} \right) l_{t-1}. \quad (4.1)$$

From (2.16), (3.2) and (4.1), we obtain

$$\left( \frac{1 + \beta}{\beta} \right) (1 + n) [k_{t+1} + (1 - \rho_t) l_t] = \left( \frac{1 - \rho_{t-1}}{\phi} \right) l_{t-1} - (1 - \rho_{t-1}) R_t l_{t-1} - (1 + n) \rho_t l_t. \quad (4.2)$$

From (2.10) and (4.1) we have

$$k_t = \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{1 - \rho_{t-1}}{\phi} \right) \left( \frac{l_{t-1}}{R_t} \right). \quad (4.3)$$

Substituting this equation into (4.2) yields

$$\begin{aligned} (1 + n) \left( \left[ \frac{1 + \beta}{\beta} \right] \left[ \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{1 - \rho_t}{\phi} \right) \frac{1}{R_{t+1}} + (1 - \rho_t) \right] + \rho_t \right) \varphi_{t-1} \\ = \left( \frac{1 - \rho_{t-1}}{\phi} \right) - (1 - \rho_{t-1}) R_t. \end{aligned} \quad (4.4)$$

Combining (2.1) and (4.1) gives

$$w_t = \left( \frac{1 - \rho_{t-1}}{\phi} \right) \left( \frac{1}{\delta} \right)^{\frac{1}{\gamma}} \varphi_{t-1}^{\left( \frac{1-\gamma}{\gamma} \right)}. \quad (4.5)$$

Substituting this equation into (2.9) and using (2.8) yields

$$R_t = \alpha \left[ \phi \left( \frac{1 - \alpha}{1 - \rho_{t-1}} \right) \right]^{\left(\frac{1-\alpha}{\alpha}\right)} \delta^{\left(\frac{1-\alpha}{\alpha\gamma}\right)} \varphi_{t-1}^{-\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1-\alpha}{\alpha}\right)}. \quad (4.6)$$

And finally, substituting (4.6) into (4.4) and evaluating at the balanced growth path, we obtain

$$\begin{aligned} (1+n) \left( \left[ \frac{1+\beta}{\beta} \right] \left[ \left( \frac{1-\rho}{(1-\alpha)\phi} \right)^{\left(2-\frac{1}{\alpha}\right)} \delta^{\left(\frac{\alpha-1}{\alpha\gamma}\right)} \varphi_c^{\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1-\alpha}{\alpha}\right)} + (1-\rho) \right] + \rho \right) \varphi_c \\ = \left( \frac{1-\rho}{\phi} \right) - (1-\rho)\alpha \left[ \phi \left( \frac{1-\alpha}{1-\rho} \right) \right]^{\left(\frac{1-\alpha}{\alpha}\right)} \delta^{\left(\frac{1-\alpha}{\alpha\gamma}\right)} \varphi_c^{-\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1-\alpha}{\alpha}\right)}, \end{aligned} \quad (4.7)$$

where the subscript  $c$  denotes the constrained economy.

Although we cannot generalize, our numerical exercise suggests that public loans for education positively affects economic growth. An increase in public loans lessens the borrowing constraint since it allows more individuals to be able to access education loans and, hence, has a positive effect on education investment that, in turn, fosters economic growth. In contrast, and as we can see from (4.7),  $\varphi_c$  does not depend on  $g$ . An increase of tax deductibility increases the demand of loans as its net price becomes cheaper, but since the economy is borrowing-constrained, individuals cannot increase their loans. However, it could be the case that a decrease in tax deductibility leads to a decrease in the demand of loans which, in turn, might shift the economy from being borrowing-constrained to unconstrained. Moreover, a change in public loans could also have similar effects on the economy, since it might break the borrowing constraint via affecting the demand of private loans. We analyze these effects in details in the next section.

## 4.2. Numerical Exercise

Using the same parameter values as in the previous section, Figure 4.1. shows that when the economy is constrained, the growth rate is strictly increasing and concave in  $\phi$ . The higher the value of  $\phi$ , the less the economy is constrained, the higher the investment in human capital and, hence, the higher the economic growth.

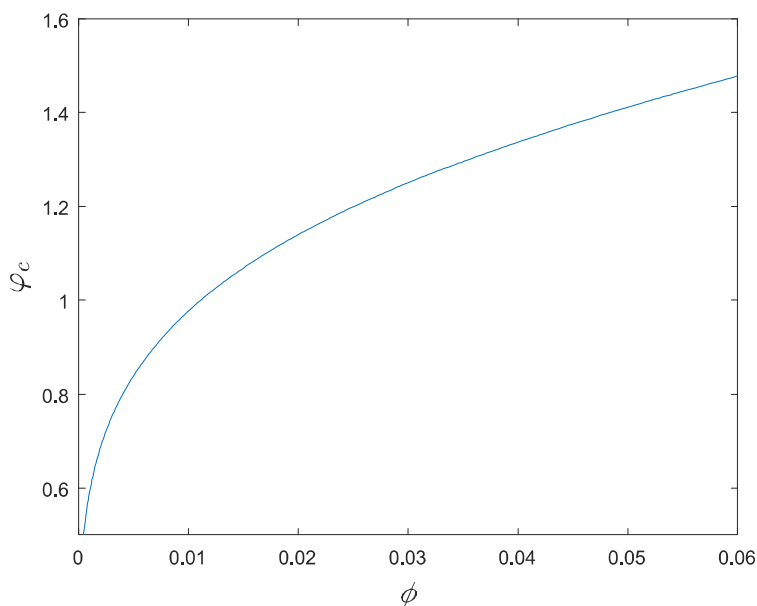


Figure 4.1. The effects of  $\phi$  on  $\varphi_c$ .

We cannot plot  $\varphi_c$  against  $\rho$  since we have no value of  $\phi$  in the real economy. However, Figure 4.2. shows the effects of public loans on the growth rate for different values of  $\phi$ . For the same value of  $\rho$ , an increase in  $\phi$  lessens the borrowing constraint and allows to increase education loans via private loans which, in turn, increases growth.

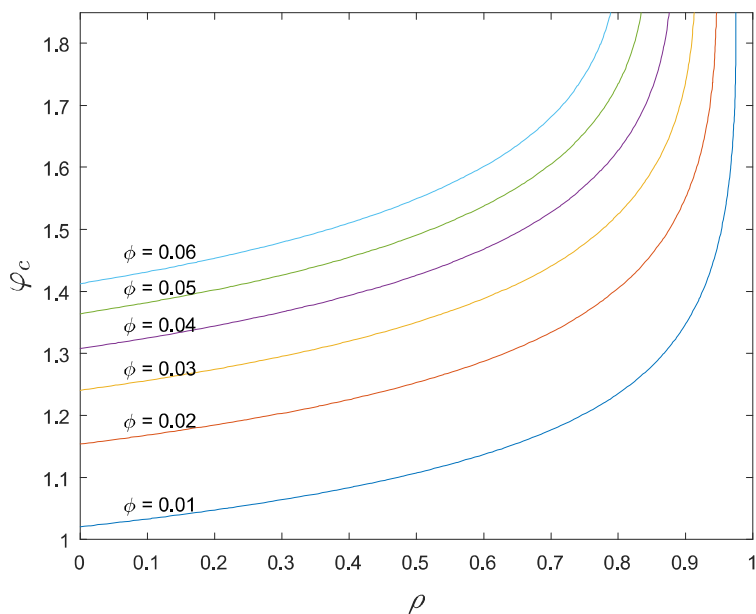


Figure 4.2. The effects of  $\rho$  on  $\varphi_c$  for different values of  $\phi$ .

Note that figures 4.1. and 4.2. assume that individuals are borrowing-constrained for all values of  $\phi$ , although this is not the case if  $\phi$  is sufficiently high.<sup>6</sup> In the next section, we analyze how public policies determine if the economy is borrowing-constrained or not.

<sup>6</sup>Given that we have assumed a 2% yearly growth rate, this value is 0.06.

## 5. Critical Value of Collateral Rate

Since we have defined the collateral rate as the fraction of the life-cycle income that individuals can borrow at most, there exists a particular value of this collateral rate, says the critical value, such that if the collateral rate is above it then individuals are not borrowing-constrained. Define  $\bar{\phi}$  as the level of the collateral rate that makes the borrowing constraint just binding. In other words, (2.7) is satisfied at the same time that (2.4) is binding.<sup>7</sup> Then, using (2.5) and (2.7), we have

$$\bar{\phi} = \frac{\gamma(1-\rho)}{R}. \quad (5.1)$$

Note that  $\bar{\phi} = 0$  when  $\rho = 1$ , that is, when all the loans come from public funds, individuals have no need to ask for private loan and, therefore, they are financially unconstrained. Combining this equation with (3.6) gives

$$\bar{\phi} = \left[ \frac{\gamma(1-\rho)}{\alpha^\alpha(1-\alpha)^{1-\alpha}} \right] \left( \frac{1-g}{\gamma\delta} \right)^{(1-\alpha)} \left( \frac{\varphi_u}{\delta} \right)^{\frac{(1-\gamma)(1-\alpha)}{\gamma}}. \quad (5.2)$$

Next proposition states when individuals are borrowing-constrained or are not.

**Proposition 5.1.** (a) *If  $\phi \leq \bar{\phi}$  then the economy is financially constrained, i.e. the borrowing constraint holds with equality.* (b) *If  $\phi > \bar{\phi}$  then the economy is financially unconstrained, i.e. the borrowing constraint does not hold.*

An increase in public loans has two effects on the critical value of the collateral rate. Firstly, there is a direct effect since the demand for private loans will decrease and, as a result, the economy will be more likely to be unconstrained. This, in turn, will positively affect economic growth. And, secondly, there is an indirect effect since a higher growth rate will consequently lead to a higher demand for loans and, as a result, the economy will be more likely to be constrained. Similarly, an increase in tax deductibility has also two effects on the critical collateral rate. Firstly, there is a direct effect since the demand for private loans will increase and, as a result, the economy will be more likely to be constrained. And secondly, there is an indirect effect via the growth rate which, as stated in Proposition 3.2., depends on the government policy values.

### 5.1. Numerical Exercise

Using the same parameter values as in the previous sections, Figure 5.1. shows that the critical value  $\bar{\phi}$  is decreasing in  $\rho$ . Therefore, the direct effect dominates the indirect effect and, thus, a public fund for education loans could shift the economy from being financially constrained to financially unconstrained.

---

<sup>7</sup>We follow Caballé (1998), where he finds a critical value for the individual altruistic level.

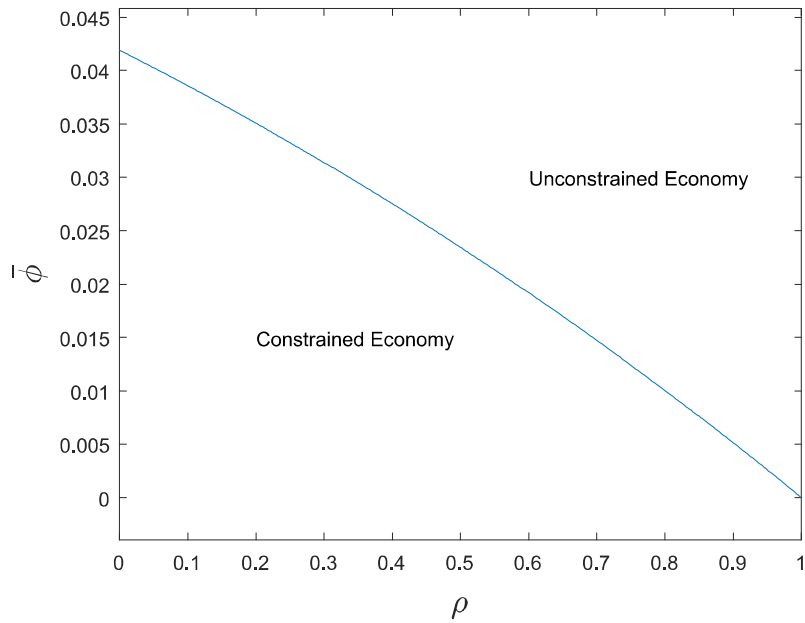


Figure 5.1. The effects of  $\rho$  on  $\bar{\phi}$  when  $g = 0.1897$ .

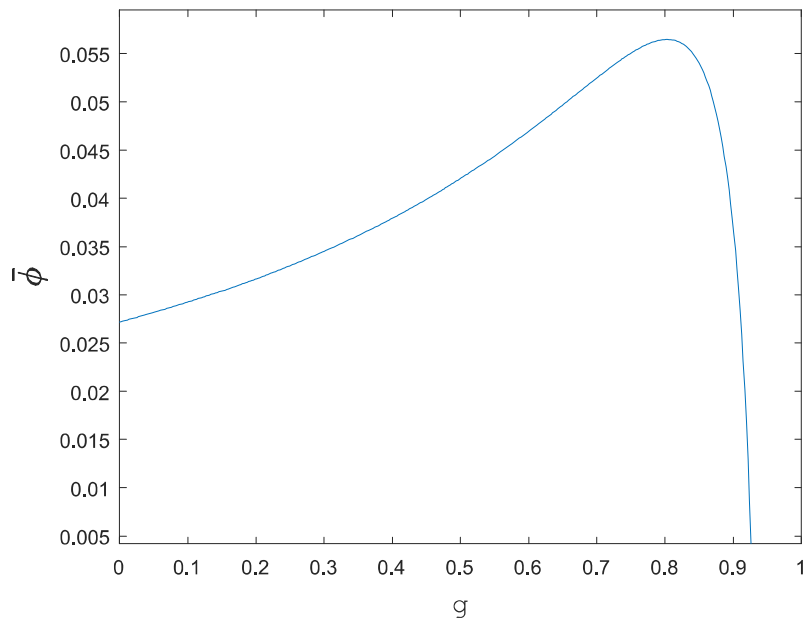


Figure 5.2. The effects of  $g$  on  $\bar{\phi}$  when  $\rho = 0.3$ .

Figure 5.2. shows that there is a value of tax deductibility  $g$ , says  $\hat{g}$ , such that if  $g < \hat{g}$  then an increase in tax deductibility will increase the critical value of collateral rate, whereas if  $g > \hat{g}$  then an increase in tax deductibility will lead to a decrease in the critical collateral value. Recall that for the U.S. case there is only a 100% of tax-deduction on the interest rate, so that  $g = 0.1897 < \hat{g}$ . When tax deductibility increases, both the public and private demand for education loans increase, and this increase in private loan demand worsens the borrowing constraint.

## 6. Conclusions

In this paper, we develop a three period overlapping generations economy to analyze to what extent a public fund for education loans and deferred deductibility of education expenses affect economic growth. These two policies imply that the education of a generation is completely self-financed by the same generation. Since private lenders worry about default, individuals can borrow at most a fraction of their life-cycle income. We define this fraction as the collateral rate. Thus, individuals could be borrowing-constrained and, then, unable to finance their desired education. We show that there exists a particular value of the collateral rate, says the critical value, such that if the collateral rate is above it then individuals are not borrowing-constrained. Moreover, government policies could affect this critical collateral rate and, then, determine if the economy is borrowing-constrained or not.

We show that when young individuals are not borrowing-constrained, public loans always positively affect economic growth since the increase in public savings more than compensate the decrease in private savings as a consequence of the negative income effect for individuals due to lump-sum taxes. A numerical exercise suggests the same positive effect when young individuals are borrowing-constrained. This numerical exercise also suggests that the critical collateral rate is decreasing in public loans.

When young individuals are not borrowing-constrained, an increase in tax deductibility has two opposite effects on the net price of education loans: a direct effect, since a higher tax deductibility implies a lower net price for education loans and, then, the loan demand increases; and an indirect effect, since this increase in the demand for loans leads to an increase in the equilibrium interest rate which, in turn, increases the net price and, then, the loans demand decreases. How an increase in tax deductibility affects economic growth depends on which effect is dominant. In contrast, an increase in tax deductibility does not affect economic growth when young individuals are borrowing-constrained. A numerical exercise suggests that the critical collateral rate is increasing in tax deductibility.

In conclusion, alternative government policies affect in different ways both economic growth and the severity of the borrowing constraint. Future work should study how the endogeneization of labor when young, as in Garriga and Keightley (2016) and Abbott et al. (2016), affects the relationship between both education policies and growth. While working when young reduces the demand of education loans and, hence, lessens the borrowing constraint, individuals have less time to attend classes. Thus, the final effects of both education policies on the acquisition of human capital could change.



# Appendix

## A. Proof of Proposition 3.1.

It is straightforward to show that  $\partial\varphi_u/\partial g > 0$ .

## B. Proof of Proposition 3.2.

Calculating  $\partial\varphi_u/\partial g$  we obtain that

$$\text{sign} (d\varphi_u/dg) = \text{sign} (f(g)),$$

where

$$\begin{aligned} f(g) = & \left\{ -\left(\frac{\alpha}{\gamma}\right) (1-g) - (1-\rho)(1-\alpha) \right\} \left\{ (1+\beta) \left[ \left(\frac{1-g}{\gamma}\right) \left(\frac{\alpha}{1-\alpha}\right) + 1 \right] - \rho \right\} \\ & + \left[ \left(\frac{1-g}{\gamma}\right) - (1-\rho) \right] \left[ \frac{(1-g)(1+\beta)}{\gamma} \right] \left(\frac{\alpha}{1-\alpha}\right). \end{aligned}$$

Since  $f(g)$  is a quadratic function of  $g$  with a positive coefficient of  $g^2$  and  $f(g) < 0$  when  $g = 1$ , we can conclude that  $f(g)$  has two roots,  $\bar{g}$  and  $\hat{g}$ , such that  $\bar{g} < 1 < \hat{g}$ . If  $f(0) > 0$  then  $\bar{g} > 0$ ,  $f(g) > 0$  for  $0 \leq g < \bar{g} < 1$  and  $f(g) < 0$  for  $\bar{g} < g < 1$ . Then,

$$\begin{aligned} f(0) = & \left\{ -\frac{\alpha}{\gamma} - (1-\rho)(1-\alpha) \right\} \left\{ (1+\beta) \left[ \left(\frac{1}{\gamma}\right) \left(\frac{\alpha}{1-\alpha}\right) + 1 \right] - \rho \right\} \\ & + \left[ \frac{1}{\gamma} - (1-\rho) \right] \frac{(1+\beta)}{\gamma} \frac{\alpha}{1-\alpha} \end{aligned}$$

is positive whenever

$$\beta > \bar{\beta} \equiv 1 - \frac{\rho \left\{ \frac{\alpha}{\gamma} + (1-\rho)(1-\alpha) \right\}}{\left[ (2-\alpha)(1-\rho) - \left(\frac{1-\alpha}{\gamma}\right) \right] \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{1}{\gamma}\right) + \frac{\alpha}{\gamma} + (1-\rho)(1-\alpha)}.$$

## C. Numerical Exercise Values

We consider the U.S. economy is financially unconstrained, and that one period in our economy is equivalent to 20 years in the real economy. Therefore, we fix  $\alpha = 0.33$ ,  $\beta = 0.739$  so that the individual time discount value for one year is 0.985,  $n = 0.24458$  so that population growth per year is 1.1%,  $\varphi = 1.48595$  so that the economic growth rate is 2% per year, and  $R = 2.86294$  so that the interest rate per year is 5.4%. Moreover, following Li (2013), we set  $\rho = 0.3$ . According to <https://studentaid.ed.gov/sa/types/loans/subsidized-unsubsidized>, the maximum undergraduate public student loan amount is 57,500 USD, and according to <https://www.irs.gov/publications/p970/ch04.html>, for individuals with income less than 60,000 USD, tax deduction on student loans is only on the interest rate and with a maximum of 2,500 USD. Taking into account that the public interest rate for student loans is 4.29%, we could assume that tax deduction covers all the student loan interest rate. Therefore, we set  $g = 0.1897$  to comply with the definition of tax-deductible amount in our model.<sup>8</sup>

<sup>8</sup>Instead, we use  $R = 1.234$  as the interest factor to calculate this value of  $g$  since we consider an accumulative interest rate in a period of 4 years as the typical duration of undergraduate studies.

## D. Proof of Proposition 5.1.

(a) We proceed by contradiction. Assume that the loan  $l$  is freely chosen with  $\phi \leq \bar{\phi}$  and the borrowing restriction does not hold. Then, defining  $\bar{l}$  as the loan associated to  $\bar{\phi}$ ,  $l \leq \bar{l}$  cannot be, since then the borrowing restriction would hold. Therefore, it must be that  $l \geq \bar{l}$ . Then, defining  $\bar{\varphi}$  as the growth rate associated to  $\bar{\phi}$ , in a balanced growth path it must be true that  $\varphi \geq \bar{\varphi}$ ,<sup>9</sup> otherwise it would exist a  $T$  such that  $l_{t-1+T} < \bar{l}_{t-1+T}$ . From (5.1), the  $\bar{R}$  associated to  $\bar{\phi}$  is

$$\bar{R} = \frac{1}{\bar{\phi}} \frac{\gamma(1-\rho)}{(1-g)}.$$

From (2.5) and (2.7), we have

$$R > \frac{1}{\phi} \frac{\gamma(1-\rho)}{(1-g)},$$

since the borrowing restriction does not hold. Then,  $\phi \leq \bar{\phi}$  implies that  $R > \bar{R}$ , where the strict inequality comes from the fact that the borrowing restriction does not hold. From (3.6) and  $R > \bar{R}$  we have  $\varphi < \bar{\varphi}$ , which cannot be.

(b) We proceed by contradiction. Assume that the borrowing restriction holds with equality with  $\phi > \bar{\phi}$  so that  $l \leq \bar{l}$ . Then, in a balanced growth path it must be true that  $\bar{\varphi} \geq \varphi$ . Since the collateral restriction holds, from (2.7) it must be true that

$$R(1-g) - \gamma w \delta \left( \frac{l}{h} \right)^{\gamma-1} < 0,$$

which combined with (2.5) gives

$$R < \frac{1}{\phi} \frac{\gamma(1-\rho)}{(1-g)}.$$

Then,  $\phi > \bar{\phi}$  implies that  $R < \bar{R}$ . From (4.6),  $\phi > \bar{\phi}$  and  $R < \bar{R}$  we have that  $\bar{\varphi} < \varphi$ , which cannot be.

---

<sup>9</sup>In fact,  $\varphi = \bar{\varphi}$  because  $\bar{\varphi}$  is the growth rate associated to the unconstrained economy.

## References

- [1] Abbott, B., Gallipoli, G., Meghir, C. and G. L. Violante, 2006, Education Policy and Intergenerational Transfers in Equilibrium, IFS Working Paper W16/04.
- [2] Blankenau, W., 2005, Public Schooling, College Subsidies and Growth, *Journal of Economic Dynamics and Control* 29, 487-507.
- [3] Boldrin, M. and A. Montes, 2005, The Intergenerational State Education and Pensions, *Review of Economic Studies* 00, 1-14.
- [4] Brauning, M. and J. Vidal, 2000, Private versus Public Financing of Education and Endogenous Growth, *Journal of Population Economics* 13, 387-401.
- [5] Caballé, J., 1998, Growth Effects of Taxation under Altruism and Low Elasticity of Intertemporal Substitution, *Economic Journal* 108, 92-104.
- [6] Cameron, S. V. and C. Taber, 2004, Estimation of Educational Borrowing Constraints Using Returns to Schooling, *Journal of Political Economy* 112, 132-82.
- [7] Eckstein, Z. and I. Zilcha, 1994, The Effects of Compulsory Schooling on Growth, Income Distribution and Welfare, *Journal of Public Economics* 55, 339-59.
- [8] Findeisen, S. and D. Sachs, 2016, Education and Optimal Dynamic Taxation: The Role of Income-Contingent Student Loans, *Journal of Public Economics* 138, 1-21.
- [9] Garriga, C. and M. P. Keightley, 2016, A General Equilibrium Theory of College with Education Subsidies, In-School labor Supply, and Borrowing Constraints, mimeo.
- [10] Glomm, G. and B. Ravikumar, 1992, Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality, *Journal of Political Economy* 4, 818-34.
- [11] Gottardi, P., Kajii, A. and T. Nakajima, 2015, Optimal Taxation and Debt with Uninsurable Risks to Human Capital Accumulation, *American Economic Review* 105, 3443-70.
- [12] Kaas, K. and S. Zink, 2011, Human Capital Investment with Competitive Labor Search, *European Economic Review* 55, 520-34.
- [13] Kalemli-Ozcan, S., Ryder, H. and D. N. Weil, 2000, Mortality Decline, Human Capital Investment and Economic Growth, *Journal of Development Economics* 62, 1-23.
- [14] Krebs, T., 2003, Human Capital Risk and Economic Growth, *Quarterly Journal of Economics* 118, 709-44.
- [15] Krueger, D. and A. Ludwig, 2016, On the Optimal Provision of Social Insurance: Progressive Taxation versus Education Subsidies in General Equilibrium, *Journal of Monetary Economics* 77, 72-98.
- [16] Li, W., 2013, The Economics of Student Loan Borrowing and Repayment, *Federal Reserve Bank of Philadelphia Business Review*, Third Quarter 2013, 1-10.
- [17] Lochner, L. J. and A. Monge-Naranjo, 2011, The Nature of Credit Constraints and Human Capital, *American Economic Review* 101, 2487-2529.

- [18] Lochner, L. J. and A. Monge-Naranjo, 2012, Credit Constraints in Education, *Annual Review of Economics* 4, 225-56.
- [19] Lochner, L. J. and A. Monge-Naranjo, 2016, Student Loans and Repayment: Theory, Evidence and Policy, *Handbook of Education Economics*, forthcoming.
- [20] Lucas, R.E., 1988, On the Mechanics of Economic Development, *Journal of Monetary Economics* 22, 3-42.
- [21] Milesi-Ferretti, G. and N. Roubini, 1998, On the Taxation of Human and Physical Capital in Models of Endogenous Growth, *Journal of Public Economics* 70, 237-54.
- [22] Soares, J., 2003, Self-Interest and Public Funding of Education, *Journal of Public Economics* 87, 703-27.
- [23] Stantcheva, S., 2016, Optimal Taxation and Human Capital Policies Over the Life Cycle, *Journal of Political Economy*, forthcoming.
- [24] Tsiddon, D., 1992, A Moral Hazard Trap to Growth, *International Economic Review* 33, 299-321.
- [25] Zhang, J., 1996, Optimal Public Investment in Education and Endogenous Growth, *Scandinavian Journal of Economics* 98, 387-404.