Parity violation in Aharonov-Bohm systems: The spontaneous Hall effect

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We show how macroscopic manifestations of P (and T) symmetry breaking can arise in a simple system subject to Aharonov-Bohm interactions. Specifically, we study the conductivity of a gas of charged particles moving through a dilute array of flux tubes. The interaction of the electrons with the flux tubes is taken to be of a purely Aharonov-Bohm type. We find that the system exhibits a nonzero transverse conductivity, i.e., a spontaneous Hall effect. This is in contrast to the fact that the cross sections for both scattering and bremsstrahlung (soft photon emission) of a single electron from a flux tube are invariant under reflections. We argue that the asymmetry in the conductivity coefficients arises from many-body effects. On the other hand, the transverse conductivity has the same dependence on universal constants that appears in the quantum Hall effect, a result that we relate to the validity of the mean-field approximation.

I. INTRODUCTION

More than 30 years after it was first studied, the interaction of flux tubes and charged particles—the Aharonov-Bohm (AB) problem[1]—continues to show a tantalizing richness. In recent years it has received renewed attention due mainly to its intimate relation with Chern-Simons (CS) field theory and the occurrence of fractional statistics in planar physics. One can implement intermediate statistics on particles by attaching to them appropriate "statistical" (i.e., fictitious) electric charge and magnetic flux.[2] Two such particles are then subject to AB scattering, and acquire a statistical phase when moving around each other. Varying the values of the charge and flux one can continuously interpolate between bosons and fermions. Hence their generic name, anyons.

On the other hand, CS field theory provides a very convenient way to implement AB interactions between particles. The CS field strength vanishes outside isolated singularities; thus, there are no classical forces among particles. However, it is expected that a mean-field approximation, in which the particle is regarded as moving in an average uniform magnetic field, should be valid provided the flux per particle is small and that classical trajectories enclose a large number of flux tubes. This approach has been successfully employed to calculate various properties of the anyon gas.[3,4] In Ref. 4 a self-consistency argument is given to support the mean-field approximation.

An important issue in fractional statistics and AB systems is that of discrete (parity and time reversal) symmetry breaking. This problem becomes of notorious relevance when one considers the possible presence of anyonic excitations in high-T_c superconductors, which are believed to present time reversal noninvariance. Having these ideas in mind, March-Russell and Wilczek[5] sought for manifestations of P and T violation in CS theories. They found that the scattering cross section of identical anyons (which is directly related to the AB cross section) shows an asymmetry *provided an additional, nonstatistical interaction is included.* The parity-violating term arises from the interference between the amplitudes of AB scattering and additional interaction. Caenepeel and MacKenzie[6] have used these results to test further the validity of the mean-field approximation. They consider the motion of one anyon as the incoherent sum of two-particle scattering processes. They find that, whenever there is an additional interaction switched on, a typical trajectory is curved, its radius of curvature being large for anyons near either bosons or fermions. In this regime, they conclude, the mean-field approximation can be justified. However, the need for an additional interaction to show asymmetries remains somewhat obscure. The results of Kiers and Weiss[7] suggest that one can do without it and nevertheless find a handedness in the scattering: particles are deflected by a regular lattice of flux tubes if the full coherent scattering is taken into account. As we shall see, these are not the only ways to make parity breaking manifest.

As was already remarked in Ref. 5, broken T invariance should lead to asymmetries in transport coefficients. A thorough investigation of the experimentally observable consequences of T and P violation in high-T_c superconductors was carried out by Halperin, March-Russell, and Wilczek.[8] The standard Onsager reciprocity relations do not apply when P or T are broken, and anomalous transport phenomena are expected. Working in the framework of an effective London Lagrangian, they find that anyon systems should exhibit a Hall conductivity in the absence of an applied magnetic field.

In this paper we set out to study a simpler model which can be expected to capture some essential aspects of P (and T) violation in macroscopic magnitudes. Specifically, the model we analyze consists of a gas of charged particles moving through a dilute random array of flux tubes. We shall work with a pure AB interaction, not imposing *ab initio* an asymmetry in the scattering cross sections. We shall explicitly compute the electrical (dc) conductivity of the system, expecting to find a spontaneous Hall effect. From the viewpoint of mean-field theory the graininess of the array should be unimportant. In this case one would expect to find a transverse conductivity similar to that of the quantum Hall effect.

The outline of this paper is as follows: In Sec. II we set up our model, in which flux tubes are regarded as impurities distributed randomly, each acting independently on electrons. After presenting the formalism for calculating transport coefficients using field theory, we use it to compute the longitudinal conductivity of the system. This is essentially a calculation of vertex corrections expressed in the form of a ladder equation. In Sec. III, the parity-violating, transverse conductivity is computed along these lines. Although divergent terms appear in the calculations, all physical quantities are finite. Section IV contains a discussion of the results.

II. LONGITUDINAL CONDUCTIVITY THROUGH AN ARRAY OF FLUX TUBES

A. The model

We start from a Lagrangian describing a free electron gas interacting with an ensemble of flux tubes:

$$\mathcal{L} = \mathcal{L}_{\text{Fermi}} - \mathbf{j}\cdot\mathbf{A} - \rho\phi - \rho A^2/2m \;, \tag{2.1}$$

where $\mathcal{L}_{\text{Fermi}}$ is the free electron Lagrangian, and $\mathbf{j}$ and ρ its corresponding current and density.

The flux tubes are located at points $\mathbf{x}_a$, each being the source of a vector potential (κ is the flux per tube)

$$A_i(\mathbf{x}-\mathbf{x}_a) = \kappa\frac{\epsilon_{ij}(x-x_a)_j}{|\mathbf{x}-\mathbf{x}_a|^2} \tag{2.2}$$

and a scalar potential

$$\phi(\mathbf{x}-\mathbf{x}_a) = g\delta^{(2)}(\mathbf{x}-\mathbf{x}_a) \;. \tag{2.3}$$

Then $\mathbf{A}(\mathbf{x}) = \sum_{\mathbf{x}_a}\mathbf{A}(\mathbf{x}-\mathbf{x}_a)$, $\phi(\mathbf{x}) = \sum_{\mathbf{x}_a}\phi(\mathbf{x}-\mathbf{x}_a)$. The flux tube density will be denoted n_i, and the electron density n_e.

It is known that the presence of the contact interaction, Eq. (2.3), is needed whenever one treats the AB problem within the framework of CS theory.[9,10] When its strength g is properly adjusted, the AB scattering amplitude is reproduced.[9] We stress that this interaction has nothing to do with the additional interaction introduced in Ref. 5. In our model, the right value for g is

$$g = \pm\kappa/2m \;. \tag{2.4}$$

This is *half* the value found in Refs. 9 and 10. The difference is due to the fact that we are considering the flux tubes to be infinitely massive. Therefore the reduced mass of the electron/flux tube system is twice that of a pair of identical anyons. The choice of sign for g determines the "hand" of the interaction.

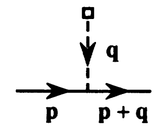

FIG. 1. Vertex for interaction between an electron (continuous line) and a flux tube (square). The amplitude is $u(\mathbf{p}+\mathbf{q},\mathbf{p}) = g + i\frac{\kappa}{m}\frac{\mathbf{p}\times\mathbf{q}}{q^2}$.

The vertex for interaction of flux tubes and electrons in momentum space is depicted in Fig. 1. The corresponding amplitude is

$$u(\mathbf{p}+\mathbf{q},\mathbf{p}) = g + i\frac{\kappa}{m}\frac{\mathbf{p}\times\mathbf{q}}{q^2}\;. \tag{2.5}$$

When g is given by Eq. (2.4), this corresponds (apart from a kinematical factor) to the AB scattering amplitude in the Born approximation. Flux tubes are assumed to be fixed objects with no internal excitations, so the electrons scatter from them elastically. This means that no frequency is carried by the interaction lines. As we shall show explicitly below, the 'seagull' interaction $\rho A^2/2m$ can be consistently neglected since it contributes higher orders in perturbation theory.

We expect that this simple model serves to approximate several many-body systems where some kind of AB interaction is present. Of course, more realistic interactions should be taken into account before it can be subject to experimental verification (for a detailed study see Ref. 8). In this paper, it will be used as a toy model for the study of the macroscopic effects that we expect to arise reflecting the presence of the underlying AB interaction. In particular, *can we make parity breaking show up anyhow in transport coefficients?* Our answer to this question will be in the affirmative.

B. Computation of the longitudinal conductivity

We shall regard flux tubes as impurities randomly distributed with density $n_i \ll 1$. The first effect one expects is the appearance of a finite longitudinal conductivity. This calculation will be done in this section, following the treatment of Ref. 11. Having checked the validity of the method for this model, we shall use it in the next section to find the transverse conductivity.

Consider then applying an external electric field $\mathbf{E}^{\text{ext}} = -\partial\mathbf{A}^{\text{ext}}/\partial t$ to the system. A longitudinal conductivity will be induced, the (linear) response being characterized by the conductivity tensor

$$J_i = \sigma_{ij}E_j^{\text{ext}} = i\omega\sigma_{ij}A_j^{\text{ext}} \;. \tag{2.6}$$

On the other hand, linear response theory yields the following relation for the current induced in an electron gas by an applied vector potential:

$$J_i = -\Big(\frac{n_e e^2}{m}\delta_{ij} + \Pi_{ij}(\mathbf{q},\omega)\Big)A_j^{\rm ext} \ . \tag{2.7}$$

From (2.6) we see that the dc conductivity can be obtained from (2.7) computing to $O(\omega)$ with $\mathbf{q} = \mathbf{0}$. $\Pi_{ij}(\mathbf{q},\omega)$ is the (retarded) current-current correlation function. It is best calculated in the imaginary time formalism. This means that frequencies will be discrete,

$$\omega_n = \begin{cases} 2\pi n/\beta & \text{for bosons,} \quad (2.8)\\ 2\pi(n+1)/\beta & \text{for fermions,} \quad (2.9)\end{cases}$$

while real frequency integrals $(2\pi)^{-1}\int d\omega$ are replaced by discrete sums $\beta^{-1}\sum_n$. In imaginary time,

$$\Pi_{ij}(\mathbf{q},\omega_n) = \int_0^\beta d\tau\ e^{i\omega_n\tau}\Pi_{ij}(\mathbf{q},\tau)\ , \tag{2.10}$$

$$\Pi_{ij}(\mathbf{q},\tau) = -\langle T\ j_i^\dagger(\mathbf{q},\tau)j_j(\mathbf{q},0)\rangle\ , \tag{2.11}$$

which can be calculated using standard diagrammatic techniques. After performing the frequency sums one can analytically continue to real retarded time by replacing $i\omega_n \to \omega + i0^+$.

The free electron propagator (no impurities present) is

$$G^0(\omega_n,\mathbf{p}) = (i\omega_n - \xi_\mathbf{p})^{-1}\ , \tag{2.12}$$

where $\xi_\mathbf{p} = \mathbf{p}^2/2m - \epsilon_F$ is the electron energy above the Fermi level.

The effect of impurities is that the electron propagator acquires an imaginary part corresponding to the finite lifetime of excitations above the Fermi level. The Dyson equation for the propagator leads to

$$G(\omega_n,\mathbf{p}) = \frac{1}{i\omega_n - \xi_\mathbf{p} - i\mathrm{Im}\Sigma(\omega_n,\mathbf{p})}\ . \tag{2.13}$$

The real part of the self-energy, $\mathrm{Re}\Sigma(\omega_n,\mathbf{p})$ has been absorbed in a renormalization of the Fermi level.

Since flux tubes are randomly distributed, we must average over the position of each tube. After averaging, the Green's function in the presence of impurities becomes translationally invariant: $\langle G(\mathbf{p},\mathbf{p}')\rangle = G(p)\delta^{(2)}(\mathbf{p}-\mathbf{p}')$. On the other hand, we shall be interested in excitations very near the Fermi surface. This will help us simplify many calculations, since all momenta will be strongly peaked around the Fermi value, p_F.

Our task now is to compute $\mathrm{Im}\Sigma$. Diagrams with only one impurity line give (using translation invariance) a constant which represents a shift of the Fermi energy, irrelevant for our purposes. Now, under the assumptions that (a) the impurity density n_i is low enough, and (b) scattering by impurities is weak (small κ, Born approximation), then the main contribution to the imaginary part of the self-energy is the one shown in Fig. 2. For low densities the interference between scattering from different impurities is negligible, i.e., the scattering is incoherent. Summing over the position of the impurities gives

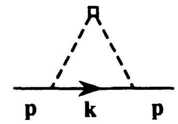

FIG. 2. Main contribution to the imaginary part of the self-energy. The impurity lines give a contribution $|u(\mathbf{k},\mathbf{p})|^2$.

then a factor n_i. On the other hand, the seagull vertex, although of the same order as the diagrams in Fig. 2, does not contribute to the imaginary part.

These considerations lead to

$$\mathrm{Im}\Sigma(\omega_n,\mathbf{p}) = n_i\mathrm{Im}\int\frac{d^2k}{(2\pi)^2}G^0(\omega_n,\mathbf{k})|u(\mathbf{k},\mathbf{p})|^2$$
$$= n_i m\int\frac{d\xi_\mathbf{k}}{2\pi}\frac{\omega_n}{\omega_n^2+\xi_\mathbf{k}^2}\int\frac{d\varphi}{2\pi}|u(\varphi)|^2\ . \tag{2.14}$$

Here we have used $|\mathbf{p}|,|\mathbf{k}| \sim p_F$ and assumed that, for $\mathbf{k}$ near the Fermi surface, $|u(\mathbf{k},\mathbf{p})|^2$ is a slowly varying function. This term is

$$|u(\mathbf{k},\mathbf{p})|^2 = |u(\varphi)|^2 = \frac{\kappa^2}{2m^2}\frac{1}{1-\cos\varphi}\ . \tag{2.15}$$

This is (up to a factor $m/2\pi v_F$) the differential cross section $d\sigma/d\varphi$ for AB scattering[1] in the Born approximation. It is even in the scattering angle.

Values of $\xi_\mathbf{k}$ far from the Fermi surface give negligible contributions to the integral over energies. Therefore we can extend the range of integration from $(-\epsilon_F,+\infty)$ to $(-\infty,+\infty)$. One finds

$$\mathrm{Im}\ \Sigma(\omega_n,\mathbf{p}) = -\frac{\mathrm{sgn}\ \omega_n}{2\tau}\ , \tag{2.16}$$

where

$$\tau^{-1} = n_i m\int\frac{d\varphi}{2\pi}|u(\varphi)|^2\ . \tag{2.17}$$

The integrated cross section is $\sigma = (n_i v_F \tau)^{-1}$. According to this, τ is the mean time between collisions in the Born approximation (2τ is the lifetime for an excitation near the Fermi surface). Observe that τ^{-1} is divergent, reflecting the long range of the AB interaction. The integral in Eq. (2.17) has to be considered as regularized with a cutoff for small φ, which eventually has to be sent to zero. We do not write it explicitly since we shall find that it disappears from physical macroscopic magnitudes.

Substitution of (2.16) into (2.13) yields,

$$G(\omega_n,\mathbf{p}) = \frac{1}{i\omega_n - \xi_\mathbf{p} + i\mathrm{sgn}\ \omega_n/2\tau}\ . \tag{2.18}$$

The computation of the polarization tensor must take into account the fact that the average (over positions of flux tubes) of two propagators does not equal the product of the separate averages. The resulting terms can be conveniently included in the form of a vertex term, as in Fig. 3. The vector vertex $\mathbf{\Gamma}$ takes account of diagrams where an impurity interacts with both the upper and lower electron lines in Fig. 3. Although it may seem at first that these terms contain higher powers in the flux tube concentration n_i, this is not the case since they also contain higher powers of $\mathrm{Im}\Sigma$. The seagull vertex can be easily seen to yield a contribution of higher order in κ, and therefore will be neglected. Under the same assumptions made above in the computation of $\mathrm{Im}\Sigma$, the main contribution comes from the ladder diagrams (see Fig. 4), and can be found by solving the following integral equation:

$$\mathbf{\Gamma}(\mathbf{p},\mathbf{p}+\mathbf{q}) = 2\mathbf{p}+\mathbf{q}+n_i\int\frac{d^2p'}{(2\pi)^2}\,G(\omega_p,\mathbf{p}')\left\{|u(\mathbf{p},\mathbf{p}')|^2+ig\frac{\kappa}{m}\frac{\mathbf{q}\times(\mathbf{p}-\mathbf{p}')}{|\mathbf{p}-\mathbf{p}'|^2}\right.$$
$$\left.+\frac{\kappa^2}{m^2}\frac{(\mathbf{p}'\times\mathbf{p})\;\mathbf{q}\times(\mathbf{p}-\mathbf{p}')}{|\mathbf{p}-\mathbf{p}'|^4}\right\}\mathbf{\Gamma}(\mathbf{p}',\mathbf{p}'+\mathbf{q})G(\omega_p+\omega,\mathbf{p}'+\mathbf{q})\ . \qquad (2.19)$$

(We are not writing the explicit frequency dependence of $\mathbf{\Gamma}$.) In general, this equation is very difficult to solve. However, we know that to compute the longitudinal conductivity we must take $\mathbf{q}=\mathbf{0}$. The resulting equation is

$$\mathbf{\Gamma}^0(\mathbf{p}) = 2\mathbf{p}+n_i\int\frac{d^2p'}{(2\pi)^2}\,G(\omega_p,\mathbf{p}')|u(\mathbf{p},\mathbf{p}')|^2$$
$$\times\mathbf{\Gamma}^0(\mathbf{p}')G(\omega_p+\omega,\mathbf{p}')\ . \qquad (2.20)$$

Now, it is clear that $\mathbf{\Gamma}^0(\mathbf{p})\propto\mathbf{p}$. Write then $\mathbf{\Gamma}^0(\mathbf{p})=(2+\Lambda)\mathbf{p}$, where, for $|\mathbf{p}|\sim p_F$, Λ can be taken as independent of $|\mathbf{p}|$. Equation (2.20) gives then the following equation for Λ:

$$\Lambda = n_i m(2+\Lambda)\int\frac{d\xi_{\mathbf{p}'}}{2\pi}G(\omega_p,\mathbf{p}')G(\omega_p+\omega,\mathbf{p}')$$
$$\times\int\frac{d\varphi}{2\pi}|u(\varphi)|^2\frac{\mathbf{p}\cdot\mathbf{p}'}{p_F^2}\ . \qquad (2.21)$$

The integral over energies can be easily performed extending the limits to $(-\infty,+\infty)$, and using contour integration. We quote the result since it will be used repeatedly:

$$\int\frac{d\xi_{\mathbf{p}'}}{2\pi}G(\omega_p,\mathbf{p}')G(\omega_p+\omega,\mathbf{p}') = \frac{\theta(-\omega_p)\theta(\omega_p+\omega)}{\omega+1/\tau}\ , \qquad (2.22)$$

for $\omega>0$ (considering $\omega<0$ does not affect our final results). Then

$$\Lambda = \frac{2\theta(-\omega_p)\theta(\omega_p+\omega)}{\tau_1(\omega+1/\tau_{\mathrm{tr}})} \qquad (\omega>0)\ , \qquad (2.23)$$

with

$$\tau_1^{-1} = n_i m\int\frac{d\varphi}{2\pi}|u(\varphi)|^2\cos\varphi\ , \qquad (2.24)$$

$$\tau_{\mathrm{tr}}^{-1} = \tau^{-1}-\tau_1^{-1} = \frac{n_i\kappa^2}{2m}\ . \qquad (2.25)$$

τ_{tr} is the "transport time" between collisions. The divergence in τ_1^{-1} is of the same kind as the one in τ^{-1}, so that their difference is finite. Therefore, the dependence on the regularization of integrals disappears from τ_{tr}.

Now we are ready to compute $\Pi_{ij}(\omega)\equiv\Pi_{ij}(\mathbf{q}=0,\omega)$. From Fig. 3(a) we read

$$\Pi_{ij}(\omega) = \frac{e^2}{4m^2\beta}\sum_{\omega_p}\int\frac{d^2p}{(2\pi)^2}$$
$$\times 2p_i\ G(\omega_p,\mathbf{p})\Gamma_j^0(\mathbf{p})G(\omega_p+\omega,\mathbf{p})\ . \qquad (2.26)$$

Equation (2.26) can now be solved. Calculations are quite standard (see Ref. 11). The longitudinal conductivity is eventually found to be

$$\sigma_L = \frac{n_e e^2\tau_{\mathrm{tr}}}{m} = 2\frac{n_e e^2}{n_i\kappa^2}\ . \qquad (2.27)$$

Remarkably, this is a nonzero, finite quantity, in spite of the long-range interaction. In contrast, if the interaction with impurities were Coulombian, τ_{tr}^{-1} would be divergent, and therefore $\sigma_L\to 0$. However, the transverse nature of the AB interaction makes the divergence in $|u(\varphi)|^2$ milder.

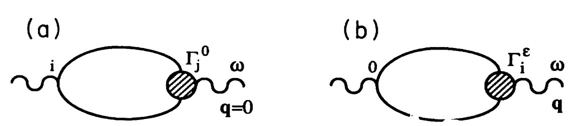

FIG. 3. (a) Current-current correlation function Π_{ij}. (b) Parity-violating density-current correlation function Π_{0i}^{ϵ}. The electron lines correspond to the propagator $G(\omega_n,\mathbf{p})$ dressed by the interaction with impurities (Fig. 2).

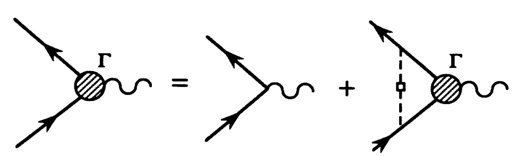

FIG. 4. Ladder equation for the vector vertex $\mathbf{\Gamma}$.

III. THE TRANSVERSE CONDUCTIVITY

Here we shall use the techniques developed in the previous section to make the presence of a transverse conductivity manifest. A current perpendicular to the applied field must satisfy a relation of the form

$$J_i = \sigma_\perp \epsilon_{ij} E_j^{\text{ext}} \ . \tag{3.1}$$

Here, the presence of a transverse conductivity $\sigma_\perp$ is a signal of P and T spontaneous symmetry breaking. The relation $J_i = \sigma_L E_i^{\text{ext}}$ already breaks T symmetry, but this is naturally expected since dissipative effects are present. However, T breaking is qualitatively different in (3.1) because the transverse current does not dissipate energy ($\mathbf{J}\cdot\mathbf{E}^{\text{ext}}=0$).

We find it convenient to choose a gauge in which the external field takes the form $\mathbf{E}^{\text{ext}} = -\nabla\phi^{\text{ext}}$, or, in Fourier components, $\mathbf{E}^{\text{ext}} = -i\mathbf{q}\phi^{\text{ext}}$. Therefore

$$J_i = -i\sigma_\perp \epsilon_{ij} q_j \phi^{\text{ext}} \ . \tag{3.2}$$

This means that, since $J_i = -\Pi_{0i}\phi^{\text{ext}}$, it will suffice to compute the density-current correlation function Π_{0i} to $O(q)$, and then take the limit $\omega \to 0$ [Fig. 3(b)].

The vertex $\mathbf{\Gamma}(\mathbf{p},\mathbf{p}+\mathbf{q})$ can be expanded in powers of $\mathbf{q}$ as follows:

$$\mathbf{\Gamma}(\mathbf{p},\mathbf{p}+\mathbf{q}) = \mathbf{\Gamma}^0(\mathbf{p},\mathbf{p}) + \mathbf{\Gamma}^\epsilon(\mathbf{p},\mathbf{p}+\mathbf{q}) + \mathbf{\Gamma}^{(\text{no }\epsilon)}(\mathbf{p},\mathbf{p}+\mathbf{q}) + O(q^2) \ . \tag{3.3}$$

We have computed the zeroth-order term $\mathbf{\Gamma}^0$ in the previous section. The first-order contribution has been split into a part $\mathbf{\Gamma}^\epsilon$ containing the vector $\mathbf{q}_\perp$ (defined to have components $\epsilon_{ij}q_j$), and another $\mathbf{\Gamma}^{(\text{no }\epsilon)}$, containing $\mathbf{q}$. We shall be interested only in $\mathbf{\Gamma}^\epsilon$. The integral equation for it can be obtained from Eq. (2.19) [observe that the last term inside the brackets in Eq. (2.19) does not contribute to $\mathbf{\Gamma}^\epsilon$, but to $\mathbf{\Gamma}^{(\text{no }\epsilon)}$, since $(\mathbf{q}\times\mathbf{p})(\mathbf{p}'\times\mathbf{p}) = p^2\mathbf{q}\cdot\mathbf{p}' - (\mathbf{q}\cdot\mathbf{p})(\mathbf{p}'\cdot\mathbf{p})$]:

$$\mathbf{\Gamma}^\epsilon(\mathbf{p},\mathbf{p}+\mathbf{q}) = n_i \int \frac{d^2p'}{(2\pi)^2}\, G(\omega_p,\mathbf{p}')\left\{ -ig\frac{\kappa}{m}\frac{(\mathbf{p}-\mathbf{p}')\cdot\mathbf{q}_\perp}{|\mathbf{p}-\mathbf{p}'|^2}\,\mathbf{\Gamma}^0(\mathbf{p}',\mathbf{p}') + |u(\mathbf{p},\mathbf{p}')|^2\mathbf{\Gamma}^\epsilon(\mathbf{p}',\mathbf{p}'+\mathbf{q})\right\} G(\omega_p+\omega,\mathbf{p}') \ . \tag{3.4}$$

Changing the sign of g [Eq. (2.4)] would reverse the flow of the transverse current.

The ansatz that allows us to solve (3.4) is not very hard to guess. After some examination, one is led to write

$$\mathbf{\Gamma}^\epsilon(\mathbf{p},\mathbf{p}+\mathbf{q}) = \frac{1}{p_F^2}\mathbf{p}(\mathbf{p}\cdot\mathbf{q}_\perp)\Lambda_1 + \mathbf{q}_\perp\Lambda_2 \ , \tag{3.5}$$

where, again, Λ_1 and Λ_2 are independent of $|\mathbf{p}|$.

The following integrals are needed to solve Eq. (3.4) (φ is the angle between $\mathbf{p}$ and $\mathbf{p}'$):

$$n_i m \int \frac{d\varphi}{2\pi}|u(\varphi)|^2\, \mathbf{p}'(\mathbf{p}'\cdot\mathbf{q}_\perp) = \left(\frac{1}{\tau_1} - \frac{1}{\tau_{\text{tr}}}\right)\mathbf{p}(\mathbf{p}\cdot\mathbf{q}_\perp) + \frac{p_F^2}{\tau_{\text{tr}}}\mathbf{q}_\perp \ , \tag{3.6}$$

$$\frac{n_i\kappa^2}{m}\int\frac{d\varphi}{2\pi}\,\frac{(\mathbf{p}-\mathbf{p}')\cdot\mathbf{q}_\perp}{|\mathbf{p}-\mathbf{p}'|^2}\mathbf{p}' = \frac{1}{\tau_{\text{tr}}}\left(\frac{1}{p_F^2}\mathbf{p}(\mathbf{p}\cdot\mathbf{q}_\perp) - \mathbf{q}_\perp\right) . \tag{3.7}$$

Calculations are now straightforward, and lead (again for $\omega > 0$) to

$$\Lambda_1 = -i\frac{(\Lambda+2)}{2}\,\frac{\theta(-\omega_p)\theta(\omega_p+\omega)}{\omega\tau_{\text{tr}}+2} \ , \tag{3.8}$$

$$\Lambda_2 = i\frac{(\Lambda+2)}{2}\,\frac{\theta(-\omega_p)\theta(\omega_p+\omega)}{\omega\tau_{\text{tr}}}\left(1 - \frac{1}{\omega\tau_{\text{tr}}+2}\right) . \tag{3.9}$$

Now we can use the vertex $\mathbf{\Gamma}^\epsilon$ to find the part of $\mathbf{\Pi} \equiv (\Pi_{0i})$ that yields parity violation. This is [see Fig. 3(b)],

$$\mathbf{\Pi}^\epsilon = \frac{e^2}{2m\beta}\sum_{\omega_p}\int\frac{d^2p}{(2\pi)^2} \times G(\omega_p,\mathbf{p})\mathbf{\Gamma}^\epsilon(\mathbf{p},\mathbf{p}+\mathbf{q})G(\omega_p+\omega,\mathbf{p}+\mathbf{q}) \ . \tag{3.10}$$

The ansatz (3.5) gives the following expression for $\mathbf{\Pi}^\epsilon$ after integrating the angles:

$$\mathbf{\Pi}^\epsilon = \mathbf{q}_\perp\frac{e^2}{2\beta}\sum_{\omega_p}\left(\frac{\Lambda_1}{2}+\Lambda_2\right) \times \int\frac{d\xi_{\mathbf{p}}}{2\pi}G(\omega_p,\mathbf{p})G(\omega_p+\omega,\mathbf{p}) \ . \tag{3.11}$$

Substituting the results above, one finds

$$\mathbf{\Pi}^\epsilon = i\frac{e^2/8\pi}{1-i\omega\tau_{\text{tr}}}\mathbf{q}_\perp \ . \tag{3.12}$$

We have made the continuation $\omega \to -i\omega$. Highly nontrivial cancellations of τ^{-1}, τ_1^{-1} have concurred again to yield a finite result. Taking $\omega \to 0$ we find the transverse conductivity

$$\sigma_\perp = e^2/8\pi \ , \tag{3.13}$$

which is independent of n_e, n_i, and κ. At first sight, this seems to lead to the nonsensical result that taking either n_i or κ to be zero, one still finds a finite transverse conductivity. Of course, this is wrong: these limits must be taken in Eq. (3.12) before $\omega \to 0$, and then one obtains

$\sigma_\perp = 0$. Also, one can check that choosing $g = -\kappa/2m$ reverses the sign of the conductivity.

IV. DISCUSSION

There are two aspects of this result that deserve some explanation: first, the appearance of the transverse conductivity; second, its dependence on universal constants.

It may seem somewhat surprising to find a nonzero transverse conductivity. Consider the motion of a single electron in the presence of a flux tube. The differential scattering cross section (2.15) is invariant under $\varphi \rightarrow -\varphi$, i.e., a typical electron trajectory is not deflected. In order to make asymmetries appear, March-Russell and Wilczek[5] were forced to introduce an additional interaction (in the form of a generic phase shift). Then parity is broken through interference terms. This is explicitly illustrated by Suzuki *et al.* in Ref. 12, where the additional interaction is taken to be a hard-disk repulsion. The situation here is different, since the interaction with the flux tubes is purely AB. Of course, one could argue that we actually have another interaction, namely, that with the external electromagnetic field. But we have reasons to believe that this by itself is not the origin of parity breaking. We have analyzed a closely related problem: the bremsstrahlung for emission of soft photons in the presence of a flux tube. Vertex corrections have been taken into account. It turns out that, after summing over the scattering angle of the emitted photon, the corresponding cross section for electrons is invariant under parity. It seems that an electron interacting with a flux tube emits photons in a left-right symmetric way.

There is however another difference between our problem and that of the scattering of a single electron from a single flux tube: it is essential in our calculation to take into account the many-body effects. Apart from yielding a damping of excited electrons, many-body effects are present in the ladder resummation that we perform to find the vertices. One can check that if only the first term in the series is included, the transverse conductivity turns out to be zero. Of course, it is not consistent to do so, as we have argued before. We need to sum all the terms and it is then that the asymmetry appears. Therefore parity breaking does arise at the macroscopic level, although its microscopic origin is hidden.

The universal dependence of the transverse conductivity is less surprising once it is noticed that it is of the kind familiar from the quantum Hall effect. This seems to provide support to the mean-field approximation, since we find a behavior analogous to that of an electron gas in the presence of a magnetic field. However, we are dealing with a perturbative approximation, so we do not expect to find the Landau level structure responsible for the integer spacing in the Hall conductivity. Nor, evidently, can our model take into account the null longitudinal resistivity that appears simultaneously with it. Nevertheless, it is remarkable to find in this simple model such a semi quantitative agreement with mean-field theory.

The procedure we have described above can be modified to compute other transport coefficients, such as thermal conductivities; these will also show parity violation. However, it seems more interesting to take one step further and consider the conductivity of a gas of anyons with the full CS interaction taken into account. Calculations with this model are much more difficult, but the outcome would be a full description of transport phenomena in anyonic systems.

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