# Progressions, Rays and Houses in Medieval Islamic Astrology: A Mathematical Classification[1]

Josep Casulleras and Jan P. Hogendijk

**Keywords**: Applied mathematics, medieval astrology, astrology in medieval Islam, planetary aspects, projection of rays, astrological houses, progressions, *tasyīr*, hour lines, position circles, medieval astronomical instruments.

**Abstract**
Medieval Islamic mathematicians and astronomers developed a variety of mathematical definitions and computations of the three astrological concepts of houses, rays (or aspects) and progressions. The medieval systems for the astrological houses have been classified by J.D. North and E.S. Kennedy, and the purpose of our paper is to attempt a similar classification for rays and progressions, on the basis of medieval Islamic astronomical handbooks and instruments. It turns out that there were at least six different systems for progressions, and no less than nine different systems for rays. We will investigate the historical relationships between these systems and we will also discuss the authors to whom the systems are attributed in the medieval Islamic sources.

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## 0. Preface

In November 1998, J.P. Hogendijk presented at a conference in the Dibner Institute a preliminary classification of medieval Islamic methods for defining the two astrological concepts of "progressions" and "projection of rays" (or "planetary aspects"). These concepts are associated with natal astrology but unconnected to the medieval practice of astronomy, just like the (related) division of the ecliptic into twelve astrological houses. Hogendijk's preliminary survey was inspired by the classification of the astrological houses by J.D. North (1986) and E.S. Kennedy (1996). The survey was not published at that time, but a photocopy of the lecture (Hogendijk 1998) was widely circulated among historians of Islamic science.

In October 2006, J. Casulleras published as his doctoral thesis an edition with translation and commentary of the entire Treatise on the Projection of Rays by the eleventh-century Andalusian mathematician and astronomer Ibn Muʿādh al-Jayyānī (died 1093). Only parts of this treatise had been studied before (Kennedy 1994, Casulleras 2004 and Hogendijk 2005). In the course of his research, Casulleras investigated numerous relevant sources on the three above-mentioned astrological concepts (cf. Casulleras 2010, pp. 49-170) which had not been covered in Hogendijk's preliminary classification. Many of these sources belong to the Western Islamic astrological tradition. It therefore seemed natural to us to produce a more definitive version of the previous classification as a joint work. This is what we intend to offer in the following pages. Although numerous changes and additions have been made to Hogendijk's lecture of 1998, and sub-sections have been added, we have maintained as much as possible the previous numbering of sections in order to be consistent with the citations in the recent research literature to the photocopied version of the lecture.[2] Only in one case we have changed the original structure: Section 4.8 in the 1998 lecture (Mathematical Properties of the Equatorial Methods) corresponds to our current Section 4.10.

## 1. Introduction

This paper concerns the history of applied mathematics in medieval Islamic civilization.[3] We will discuss the three astrological concepts of houses, rays and progressions and their mathematical interpretations. In Section 2 we introduce these concepts and explain their importance for the astrologers. The houses, rays and progressions could be defined according to mathematically different systems. The medieval systems for defining the houses were first classified by J.D. North (1986), and his classification was extended by E.S. Kennedy (1996). The purpose of this paper is to attempt a classification for rays and progressions similar to the classification which North and Kennedy gave for the houses. Our classification is based on the publications of many modern historians of Islamic science. A definition and reference for the use of progressions among the Arabs was given by O. Schirmer (1934), whereas a significant study on the rays in the Islamic area was published by E.S. Kennedy and H. Krikorian-Preisler (1972). An overview of the resolution of astrological questions by means of analogical instruments in al-Andalus was presented in 1990 by Calvo (published in 1998). Relevant medieval Arabic sources have been published or analyzed,[4] important older publications on Islamic astronomical instruments have been collected in Sezgin 1990-1991, and a comprehensive monograph on the same subject has been published by King (2005). It turns out that there were at least six different systems for

[2] Some of the publications that have used Hogendijk's original work, in chronological order, are: Samsó 1999, p. 117 (in the original) =19 (in the reprint); Samsó and Berrani 1999, pp. 302, 306, 310; Samsó 2004, p. 510; Samsó and Berrani 2005, pp. 188, 199, 202, 203, 204, 220; Casulleras 2007a, passim; Casulleras 2007b, pp. 48, 50, 52-55, 57-59, 61-63, 77; Díaz Fajardo 2008, pp. 24, 29, 30, 31; Casulleras 2008/2009, passim; Samsó 2009, pp. 31, 33; Casulleras 2009, passim; Díaz Fajardo 2010, pp. 302, 304, 305, 307, 311, 320, 322; Casulleras 2010, passim; Díaz Fajardo 2011, pp. 336, 340.

[3] The adjective "Islamic" in a term like "Islamic astrology" refers to culture and not to religion. Astrology was strongly condemned by some medieval Islamic theologians because they felt that it contradicted the omnipotence of God.

[4] We have not compiled a list of all the sources and studies that might be connected to the subject of the computations in medieval Arabic astrology, but an outline of it can be extracted from the references given at the end of this paper.

defining progressions, to be discussed in Section 3 of this paper, and no less than nine different systems for defining the rays, to be discussed in Section 4. We consider two systems as different if they have different (that is, mathematically inequivalent) geometric definitions. One geometric system can lead to different numerical computations.

For many medieval Islamic astrologers, the rays and progressions were not independent from each other and from the houses, and thus the history of one of these concepts may shed light on the history of the others. In Section 5 we introduce the systems of houses from the classification of Kennedy and North which are related to systems for progressions and rays. In Section 6 we discuss the usual attributions of the methods, we summarize some of their historical relationships, and we pose some questions deserving further research.

Sections 3 and 4 of our paper also involve Greek astrology, in particular the mathematical aspects of the *Tetrabiblos* of Ptolemy (ca. A.D. 150). We have added some references to medieval Hebrew and Latin texts but we have not really investigated the transmission and further development of progressions and rays in the medieval Hebrew and Latin scientific traditions. Our survey of the Islamic systems for progressions and rays is incomplete because there are many unpublished sources which we have not been able to consult. Nevertheless, our paper gives a general idea of the nature and the scope of these non-trivial applications of mathematics in medieval Islamic civilization. These applications are not only historically important, but to our mind they are also fascinating because they are so strange to modern scientific eyes. Future historians may well regard some present-day applications of mathematics with similar feelings of fascination.

## 2. Basic definitions

This section contains simple explanations of the astrological *houses*, *rays* and *progressions*. We assume that the reader has a rudimentary knowledge of spherical geometry and astronomical coordinate systems on the celestial sphere, as explained in any introduction to spherical astronomy (for example, Smart 1939).

### *2.1 Houses*

The ancient Greek astrologers introduced the division of the ecliptic into twelve “houses”.[5] This division depends on the horizon, and it is therefore not the same as the division of the ecliptic into the zodiacal signs Aries, Taurus, etc. In this paper we will only be concerned with systems based on the four cardinal points, that is to say the intersections of the ecliptic with the horizon and meridian planes.[6] In these systems the beginnings of the first, fourth, seventh and tenth houses are defined as the ascendent or rising point on the Eastern horizon (1), the intersection between ecliptic and the meridian below the horizon (4), the descendant or setting point on the Western horizon (7), and the midheaven, that is the intersection between ecliptic and the meridian above the horizon (10) (Figure 1).

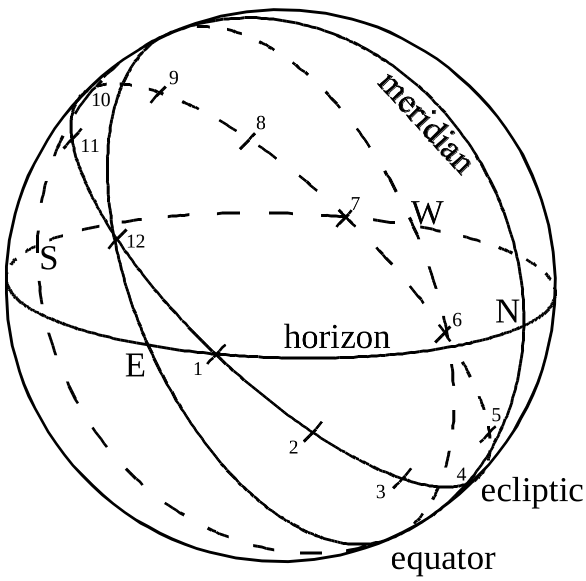


**Figure 1**

[5] See Bouché-Leclercq 1899, pp. 279.
[6] We will not be concerned here with systems where the beginning of the first house was 5 degrees away from the rising point. See for houses in general North 1986.

There were many different systems for the definition of the second, third, fifth, sixth, eighth, ninth, eleventh and twelfth houses in medieval Islamic astrology. In the simplest system, which was already used in ancient horoscopes (North 1986, p. 6, fn. 13), the remaining boundaries between the houses were obtained by trisecting each of the four arcs between two cardinal points. The Iranian astronomer al-Bīrūnī[7] (973-1046) called this system the "method of the ancients" (al-Bīrūnī 1954-1956, vol. 3, p. 1356; North 1986, p. 40), and it is called the "Dual Longitude Method" in the North-Kennedy classification.

In all systems, the first house is the part of the sphere below the Eastern horizon, and the houses are numbered counter-clockwise, for an observer in the temperate regions on the Northern hemisphere who looks to the south. The process of defining the houses is called in Arabic the "equalization of the houses",[8] and the beginning points are called the cusps[9] of the houses in the modern literature, and the centres[10] of the houses in Arabic.

During one daily revolution of the celestial sphere,[11] any celestial body will pass through all twelve houses. These houses were connected to many things, including life (1), possessions (2), brothers and sisters (3), parents (4), children (5), illness (6), marriage (7), death (8), travels (9), honours (10), friends (11), and enemies (12).[12] Lengthy descriptions of the qualities of the houses can be found in the work of the early tenth-century Christian astrologer Ibn Hibintā (1987, vol. 1, pp. 52-124). The qualities of the first, second, etc. houses were often connected to the qualities of the first, second, etc. signs of the zodiac.

Many modern astrologers define the houses according to the so-called systems of Regiomontanus, Campanus and Placidus. These three systems, all of medieval Islamic origin, will be mentioned in Sections 5.1-5.3.

[7] On al-Bīrūnī see, for example, Yano 2007 and the references given there.
[8] *taswiyat al-buyūt*.
[9] We do not know the origin of this term.
[10] *markaz*, pl. *marākiz*.
[11] This apparent revolution for a modern reader, who knows that the earth revolves around its axis, was a real revolution for a medieval astronomer.
[12] See Bouché-Leclercq 1899, pp. 280-285.

### *2.2 Rays*

The ancient Greek astrologers also introduced the doctrine that each planet[13] $P$ casts seven visual rays to other points $P_1 \ldots P_7$ of the ecliptic. The positions of these rays are defined by means of a regular hexagon, a square and an equilateral triangle.[14] In the simplest system, these polygons are inscribed in the ecliptic with an angular point at $P$, as in Figure 2, in which the arrow indicates the direction of increasing celestial longitude. The "rays" $PP_1$, $PP_2$, $PP_3$ ($PP_7$, $PP_6$, $PP_5$) are called the left (right) sextile, quartile and trine rays respectively, and $PP_4$ is the ray to the diametrically opposite point. If another celestial body $Q$ happens to be close to $P_i$, $Q$ is said to be "looked at" (Latin: *adspectus*) and the two bodies $P$ and $Q$ are said to make an aspect. This information is astrologically significant. For the interpretation, the theory of rays and aspects between the planets was often related to the theory of the aspects between the zodiacal signs.[15] The trine ray or aspect was regarded as beneficial, the sextile ray or aspect as less beneficial, the quartile ray or aspect as damaging, while the opinions diverged on the qualities of the opposite ray and aspect.

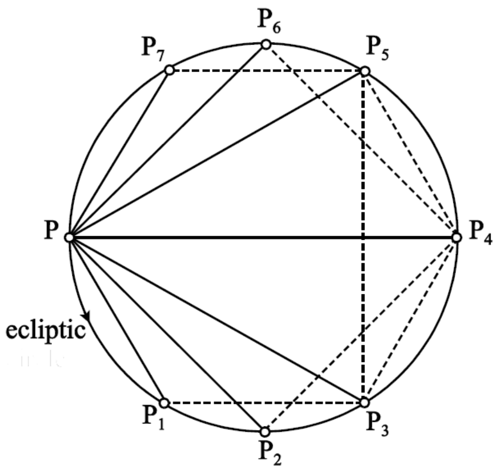


Figure 2

[13] In this connection, a planet is a celestial body which changes its position relative to the fixed stars. Thus the sun and moon are also considered to be planets.
[14] See Bouché-Leclercq 1899, pp. 177-178, 247-251 for the theory in antiquity.
[15] For this theory see Bouché-Leclercq 1899, pp. 165-177.

The medieval Islamic astrologers enthusiastically adopted the theory of the seven rays. Al-Bīrūnī points out that the planets cast light in all directions, not only in the direction of the astrological rays, but he argues that the points $P_i$ have some special influence, and that this effect can be compared to the theory of musical intervals. He also notes the connection between the tides and the quartile aspects between the moon and the sun (al-Bīrūnī 1954-1956, pp. 1379-1382).

The above-mentioned system for defining the rays (Figure 2) is still in use by modern astrologers, but it was not the most popular system in medieval Islamic astrology. In Section 4 we will list some examples of its use and we will discuss the more complicated systems which were preferred by most medieval Islamic astrologers, and which still await rediscovery by their modern successors.

### *2.3 Progressions*

The ancient Greek astrologers also introduced the doctrine that various events in the life of an individual could be predicted on the basis of the positions of the celestial bodies at the moment of birth. This doctrine was accepted and elaborated in Islamic astrology. We will explain the general idea in connection with the prediction of the moment of death of the individual (Figure 3). First the astrologer had to select two celestial bodies or other significant points (for example the ascendent, or one of the seven rays of a planet) in the celestial configuration at the moment of birth of the individual. The way in which these bodies or points were selected depended on astrological arguments which do not concern us here, although they were of course decisive for the resulting predictions.[16] One of these points was thought as emitting life-force, the other as a destructive point which destroys life.[17]

We now fix the emitting point $P$ in its initial position and we rotate the destructive point $F$ around the celestial axis (i.e. the line through the celestial North Pole $C$ and the centre of the sphere) until it reaches $P$. If this happens

[16] See Bouché-Leclercq 1899, pp. 411-422.
[17] The first point was called in Greek the *aphetic* body (Bouché-Leclercq 1899, pp. 415-416), in Arabic the *haylāj*, from Persian *hīlāj*, "lord of a building" (Nallino 1907, vol. 2, p. 355) and in Latin the *hyleg* or *significator*. The other body was called the *anairetic* body in Greek, *qāṭiʿ* in Arabic, and *promissor* in Latin (Schirmer 1934).

after rotation over *n* degrees,[18] the individual will live *n* solar years. Similarly, a rotation over one minute of arc corresponds to approximately six days of the life of the individual. The destructive point *F* has now "progressed" from its initial position, and the method is called "progression" or in Arabic *tasyīr*.[19] The emitting and the destructive points were always selected in such a way that the great circle distance between them is at most 90 degrees, so that $n \leq 90$.

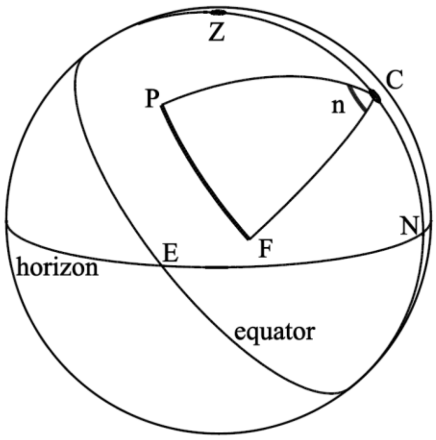


**Figure 3**

In Figure 3, the emitting point *P* "precedes"[20] *F* in the sense that *P* rises earlier than *F* on the Eastern horizon, and the rotation of *F* to *P* is in the direction of the daily motion of the celestial sphere. This situation was the most common one, and it will be assumed in the rest of this paper. Some astrologers were also willing to consider the destructive point "preceding" the emitting point, and rotation in the direction opposite to the daily motion of the sphere (see e.g. Ptolemy 1940, pp. 279-281, Ibn Hibintā 1987, vol. 1, p. 139).

[18] The ancient and medieval astrologers expressed this quantity not as an angle but as a rotation arc on the celestial equator.
[19] This technical term, literally: "making travel", is often used in modern articles on the history of the subject. Point *F* is also called *al-musayyar*, the point which is made to travel, and point *P* *al-musayyar ilay-hi*, the point toward which the travel is made.
[20] This terminology was used by al-Battānī (Nallino 1899, vol. 3, pp. 198-203) and al-Bīrūnī (1954-1956, p. 1393).

Figure 3 is drawn for the case where points *P* and *F* have the same declination, so the rotated point *F* can coincide with the initial position of *P*, and vice versa. Some astrologers believed that the progression could only have an effect in this case. The eleventh-century Maghribī astrologer Ibn Abī-l-Rijāl copies from Ibn Hibintā an approximate solution for checking whether the equatorial declinations of the two elements implied are the same, and mentions Dorotheus of Sidon[21] as his source[22] (Ibn Abī-l-Rijāl 1954, pp. 175-176; Ibn Hibintā 1987, vol. 1, p. 144; Díaz Fajardo 2008, pp. 95-103; Casulleras 2010, pp. 108-111). However, in almost all cases points *P* and *F* do not have the same declination, so the rotated point *F* can never coincide with *P*. In the next section we will see how the Greek and Islamic astrologers generalized the method for this case also.

Modern astrologers still predict the events in the life of an individual from the constellation at the moment of birth by letting the celestial bodies "progress" from their original positions, but they do not use the (apparent) daily rotation of the celestial sphere in this connection. Instead, they simply obtain the arc of the *tasyīr* on the ecliptic, a method which, as we shall see, is only one of a variety of alternatives existing in medieval sources.

In our presentation thus far, the concepts of houses, rays and progressions seem to be mathematically unrelated. In the next sections we will see that some systems for the mathematical definition of progressions were also adapted to define rays and houses, and that another system for houses was adapted to the rays. In another case of cross-fertilization, we have not been able to find out whether the system was first applied to the houses or to the rays. We will discuss the individual systems in Sections 3-5 and we will list the relationships in a chronological survey in Section 6.2.

## 3. Systems for Progressions

In this and the following sections we use the following notation for any celestial body *P* or any point *P* on the (moving) celestial sphere:

- $\alpha_\varphi(P)$ is the oblique ascension of *P* in a locality with Northern geographical latitude $\varphi$. This is the length of the arc of the celestial equator which

[21] Dorotheus of Sidon was a Greek astrologer who lived in the first century A.D. On the Arabic traces of his works see (Sezgin 1974-1984, vol. 7, pp. 32-38; Pingree 1978).

[22] Díaz Fajardo (2008, p. 98) notes that the quotation is not found in the preserved works of Dorotheus (1976).

rises on the Eastern horizon in the time interval which begins with the rising of the vernal point $V$ and ends with the next rising of $P$ (Figure 4).

- $\alpha_0(P)$ is the right ascension of $P$.
- $\alpha_{-\varphi}(P)$ is the oblique descension of $P$, i.e. the length of the arc of the celestial equator which sets on the Western horizon in the time interval which begins with the setting of the vernal point and ends with the next setting of $P$, at a locality with Northern geographical latitude $\varphi$. This quantity is equal to the oblique ascension in a locality with southern latitude $-\varphi$; such latitudes were sometimes considered by the Islamic astronomers.[23]

Oblique ascensions and descensions are usually defined only for points $P$ on the ecliptic (see Figure 4, in which $\bar{\varphi} = 90^\circ - \varphi$, $\varepsilon \approx 23^\circ\ 30'$ the obliquity of the ecliptic, and Pedersen 1974, pp. 99-113, for all details, including computation and tables). We use this notation for arbitrary points $P$ for sake of brevity.

All angles and arcs will be reckoned modulo 360. Thus, $40^\circ - 320^\circ = 80^\circ$.

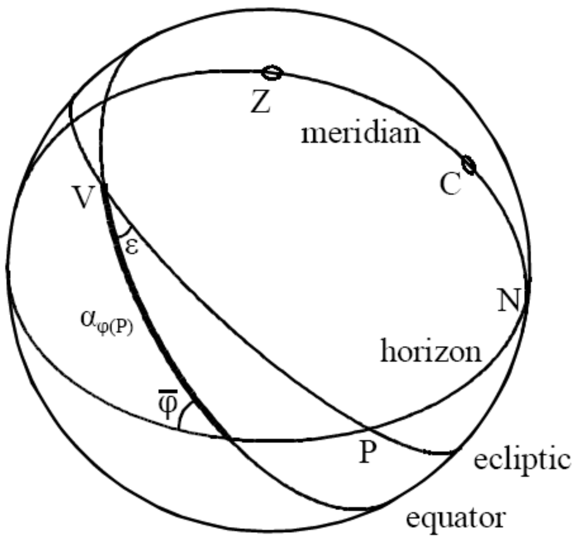


Figure 4

[23] Cf. Hogendijk 1989, pp. 176-178.

### *3.1 Progressions for predicting the moment of death*

We now assume an emitting point $P$ and a destructive point $F$ as in Figure 3. Required to compute the number of degrees $n$ over which point $F$ has to be rotated (in the direction of the daily motion of the universe) around the celestial axis in such a way that the image of $F$ is in a similar position as $P$. The number $n$ in degrees is the life of the individual in solar years. The problem is, of course, how being in a similar position has to be defined if $F$ and $P$ do not have the same declination. At least six methods were used in medieval Islamic astrology.

#### *3.1.0 Direct Method*

As the name may suggest, in this method for progressions the number $n$ in degrees is simply assumed to be the difference between the ecliptic longitudes of $F$ and $P$. In the eleventh century, al-Bīrūnī alludes implicitly to the use of this method for progressions in his *Kitāb al-Tafhīm*, stating that the *tasyīr* of the nativities must not be calculated using ecliptic degrees (al-Bīrūnī 1934, p. 326). Thus, the method was known in the East but it seems that it was not commonly applied. In contrast, in al-Andalus, the eleventh-century astrologer al-Istijī strongly defends the use of the simple procedure for the *tasyīr* and for the projection of rays (cf. Section 4.1), and even considers it incorrect to use the equator in the computation (cf. Samsó and Berrani 1999, pp. 303, 305-306, and 2005, pp. 187-188, 230). The method is also documented in the works of later authors of the Maghrib, such as Ibn ᶜAzzūz (died 1354) and Ibn Qunfudh (died 1407), both from Constantine (Algeria), and Abū ʿAbd Allāh al-Baqqār, who was working in Fez in 1418 (cf. Samsó and Berrani 2005, p. 188; Samsó 1999, p. 117=19; Casulleras 2008/2009, pp. 258-259, and 2010, pp. 105-106). It may also be worth noting here that this is the system currently used by modern astrologers.

#### *3.1.1 Right Ascension Method*

Two points are in a similar position if they are on a great semicircle whose endpoints are the celestial poles (Figure 5). This is to say:

$$n = \alpha_0(F) - \alpha_0(P)\ .$$

This system is mentioned in a treatise on the astrolabe by the Jewish mathematician Abraham ibn ʿEzra (ca. 1090-1167), who received his mathematical training in Islamic Spain (Viladrich and Martí 1983, p. 90). The Greek astronomer Ptolemy says that this system is correct only if *P* is in the meridian plane (1940, p. 290).[24] The same principle is stated by al-Bīrūnī (1934, p. 326-327), Ibn Abī-l-Rijāl (1954, p. 174a) and, according to Ibn Qunfudh, by the philosopher al-Kindī[25], who died around 870 (Díaz Fajardo 2008, pp. 125-126). Al-Istijī relates the method to "a group of Persians"[26] that make their projection of rays and their progressions using right ascensions (Samsó and Berrani 1999, p. 304, and 2005, pp. 201, 234).

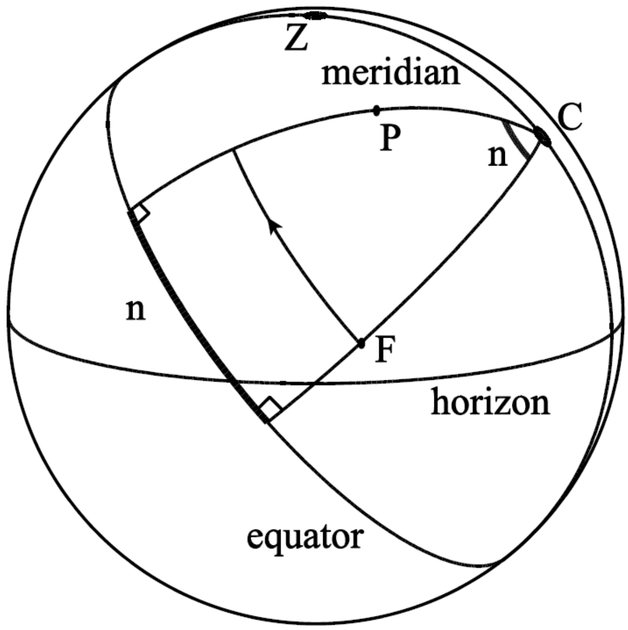


Figure 5

[24] Bouché-Leclercq 1899, p. 411 indicates that the system was used in all cases, but he misunderstood Ptolemy's *Tetrabiblos*.
[25] On al-Kindī see, for example, Cooper 2007.
[26] *Ṭā'ifa min al-furs*.

### *3.1.2 Oblique Ascension Method*

Two points are in a similar position if they are on the Eastern horizon, or a half circle obtained by rotating the Eastern horizon on the celestial sphere around the celestial poles (Figure 6). This is to say:

$$n = \alpha_\varphi(F) - \alpha_\varphi(P) .$$

The system is mentioned by Ptolemy in his astrological work *Tetrabiblos* (1940, p. 290). He says that this is the usual system but that it is correct only if $P$ is on the Eastern horizon. As happens with the Right Ascension Method, the same reasoning is transmitted by al-Bīrūnī and Ibn Abī-l-Rijāl, and related to al-Kindī by Ibn Qunfudh.

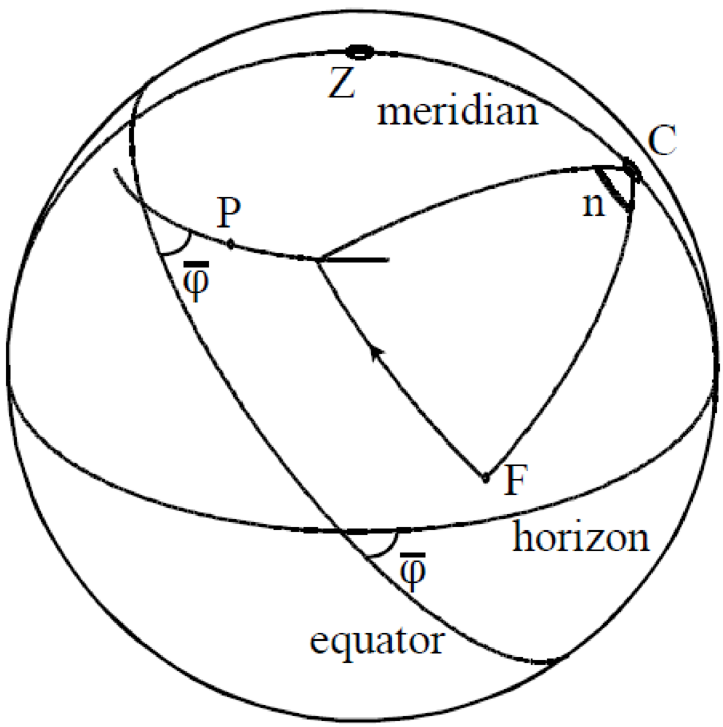


Figure 6

### *3.1.3 Position Semicircle Method*

We call a position semicircle a semicircle on the celestial sphere whose end-points are the North point *N* and the South point *S* of the horizon (Figure 7).[27] The principle of this method is stated by Ptolemy in the *Tetrabiblos*:[28] Two points are in a similar position if they are on the same position semicircle. Al-Bīrūnī (1934, p. 326-327), Ibn Abī-l-Rijāl (1954, p. 174a) and al-Kindī (according to Ibn Qunfudh) are probably thinking of the same principle when refer to the use of "mixed ascensions" for planets between the horizon and the meridian (Díaz Fajardo 2008, pp. 125-126). Thus *F* has to be rotated over an angle *n* in such a way that its image is on the position circle *NPS*.

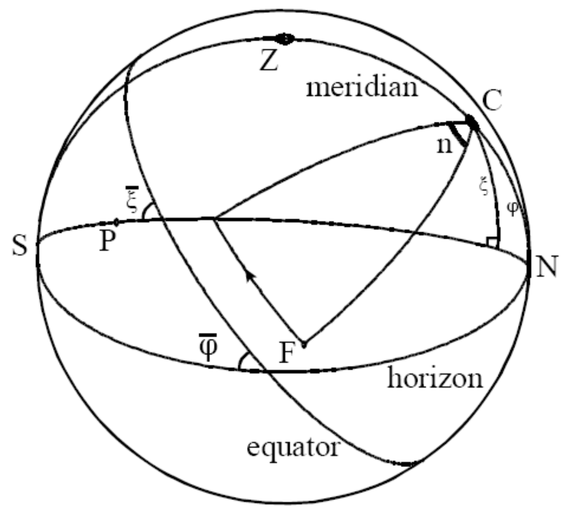


Figure 7

[27] An early definition of the position (semi)circles is found in the treatise on the construction of the astrolabe by the ninth-century astronomer al-Farghānī (2005, pp. 5, 10, 60-63). On this author see, for example, De Young 2007.

[28] "For a place is similar and the same if it has the same position in the same direction with reference both to the horizon and to the meridian. This is most nearly true of those which lie upon one of those semicircles which are described through the [inter]sections of the meridian and the horizon ..." (Ptolemy 1940, p. 291).

If $P$ is on the meridian, we obtain the right ascension method, and if $P$ is on the Eastern horizon, the oblique ascension method. For $P$ neither on the meridian nor on the horizon, Ptolemy uses in his computation an approximate method which we call the Hour Line Method, and which will be described in Section 3.1.4 below because it is based on a different geometrical principle.

Al-Bīrūnī presents a computation according to the Position Circle Method as "the method which I prefer in progressions" (al-Bīrūnī 1954-1956, pp. 1397-1399). A similar computation is found in the works of later Islamic astronomers, including the Andalusian astronomer Ibn al-Raqqām[29] (died 1315), in chapter 63 of his *Mustawfī Zīj*[30] (p. 221), and the king of Samarkand Ulugh Beg[31] in the first half of the fifteenth century (Sédillot 1853, pp. 208-209, 211-212). The basic idea is as follows.

First assume that point $P$ in the Eastern hemisphere. Let $\xi$ be the minimum distance (on a great circle) from the celestial North Pole ($C$ in Figure 7) to the position circle through $P$. We have $0 \leq \xi \leq \varphi$, with $\xi = 0$ for $P$ in the meridian plane and $\xi = \varphi$ for $P$ in the horizon plane.

We can now consider the position semicircle through $P$ as the Eastern horizon for a geographical locality with latitude $\xi$. Thus $n$ can be found if a table of oblique ascensions for the latitude $\xi$ is available, using the formula

$$n = \alpha_{\xi}(F) - \alpha_{\xi}(P)\ .$$

For points $P$ is in the Western hemisphere, $NPS$ can be considered in the same way as a Western horizon for a locality with latitude $\xi$ with $0 \leq \xi \leq \varphi$, and we find $n$ as the difference between two oblique descensions:

$$n = \alpha_{-\xi}(F) - \alpha_{-\xi}(P)\ .$$

Ulugh Beg explains (Sédillot 1853, p. 206) that $NPS$ can be considered as an Eastern horizon for a locality with southern latitude $-\xi$.

Al-Bīrūnī gives the computation of $\xi$ from the azimuth and altitude of $P$, and Ulugh Beg (Sédillot 1853, pp. 205-208) discusses various methods for the computation of $\xi$ for a given point $P$ (Sédillot 1853, pp. 205-208).[32]

[29] On Ibn al-Raqqām see Casulleras 2007c. On his *Mustawfī Zīj*, Samsó 2011, 522, 566-529 and the references given there.

[30] We use the unpublished notes on this text kindly provided by Julio Samsó.

[31] On this author, see van Dalen 2007b.

[32] Some Islamic astronomers called the position (semi)circle through $P$ the "incident horizon" (*ufuq ḥādith*), and $\xi$ the "latitude of the incident horizon". See Kennedy 1996, pp. 555-556. In Sédillot's translation of the *Zīj* of Ulugh Beg (Sédillot 1853, p. 205), these quantities are called

Some medieval Islamic astrologers describe the determination of $n$ by means of a special astrolabe plate, called a “progression” plate or a plate for the projection of rays. This plate displays the stereographical projections of the position circles which intersect the equator at equal intervals of, for example, five degrees (see Figure 8). The plate is used as follows. Set the astrolabe at the celestial configuration at birth, find the positions of the pointer on the rim and find $P$ on the spider.[33] Fix on the plate the position semicircle on which $P$ is located. Turn the spider until $F$ arrives at this position semicircle, and note the new position of the pointer on the rim. The difference between the two positions is the angle of rotation $n$. This easy method did not have the accuracy necessary for serious astrological predictions.

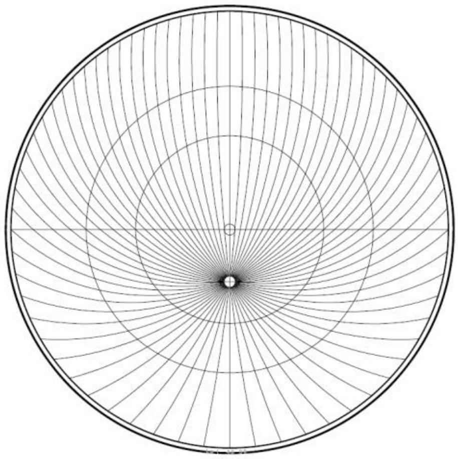

**Figure 8**

“(latitude de) l’horizon du cas de fortune”, which corresponds to “*(ʿarḍ-i) ufuq-i ḥādith*” in the Persian original (Sédillot 1847, p. 437).

[33] The principle of the astrolabe is as follows. The spider contains the stereographic projections of the ecliptic and some bright fixed stars, projected from the celestial South Pole. Only the part of the heavens between the North Pole and the tropic of Capricorn is projected. The plate contains stereographic projections of circles in a horizontal coordinate system, projected from the celestial South Pole. The spider is placed on the plate in such a way that it can be rotated around an axis which corresponds to the projection of the celestial North Pole. This rotation corresponds to the apparent daily motion of the celestial sphere. The plate is fixed to the rim, which is concentric with the projection of the North Pole, and which is divided into 360 degrees. A pointer attached to the spider is used to measure angles of rotation $n$ on the rim (see, for example, Michel 1947).

This plate for progressions and rays is described by various authors, including al-Bīrūnī (Samsó 1996, p. 592) and al-Marrākushī (1984, vol. 2, pp. 54-55), and in the thirteenth-century *Libro dell Ataçir* written for the Castilian King Alphonso X by the astronomer Rabiçag, that is, Rabbī Isḥāq ibn Sīd (see Viladrich and Martí 1983 and Rico 1863-1867, vol. 2, pp. 295-309).[34]

Several existing astrolabes contain plates with lines that match the geometrical approach here explained. Consequently, they can be used for the progressions and the projection of rays. However, we have to bear in mind that, when lacking the descriptions on the use of a plate, it is not always possible to know the methods and practices on which its design was based, since the application of two of the methods for the projection of rays (cf. Sections 4.4 and 4.6) and the method for the progressions which we are dealing with here produce the same pattern on an astrolabe plate. We list here some examples of this kind of plate that have been published:

- The astrolabe constructed in A.D. 984-985 by the Iranian astronomer al-Khujandī includes a plate entitled "projection of rays and progressions (*tasyīrāt*)", with position circles which intersect the equator in the end-points of five-degree intervals. The geographical latitude is not stated but turns out to be 33º (King 1991, p. 162; King 1995, p. 87; King 2005, #111, cf. pp. 50, 508: plate *f*, 514, 940). The position circles are numbered from 5º to 90º in each quadrant.[35]

- An Andalusian plate from the eleventh-century, which is surprisingly preserved in an ottoman astrolabe,[36] has an inscription stating that it is devised for "the division of houses and the projection of rays" at the "latitude of 35º", and projections of position circles arranged on the equator on divi-

[34] Curiously, Rabiçag calls the position circles "circulos de los tiempos", i.e. hour lines, so he considered the exact method he was using as equivalent to the approximate Hour Line Method (see the next section). In the Castilian text (Rico 1863-1867, vol. 2, p. 303), the North and South points of the horizon are incorrectly called *polos de la villa*. In the summary of the *Libro dell Ataçir* in (Viladrich and Martí 1983, p. 81), the position circles (through the North and South points of the horizon) have therefore been confused with the azimuthal circles (through the two poles of the locality, namely the zenith and the nadir).

[35] In King 2005, p. 940, is mentioned that this plate has markings for the equalization of the houses, but we do not see these markings on the published plate.

[36] This plate is going to be published soon in King 2012 (?). It belongs to a group of plates of unknown author preserved in a private collection in Belgium, mentioned in King 2005, #4040, cf. pp. 940, 945, 962: where *Sotheby's London 30.5.1991 Catalogue*, 136, lot 391, is quoted. Thanks to the kindness of the author, we could examine some photographs of this instrument before being published.

sions at each six-degree interval around the horizon. It also bears thicker marks for the limits of the houses according to what North (1986, pp. 35-38) called the Equatorial (fixed boundaries) Method (see our Section 5.1, below), and the numbering of the position circles is repeated within each of the houses.

- A Moroccan astrolabe constructed in A.D. 1208 contains four plates entitled "the projection of rays for latitude 33º 40'" (31º, 37º 30', 38º 30'), with position circles intersecting the equator at the endpoints of ten-degree intervals. In the published plate there are also thicker marks for the limits of the houses according to the Equatorial Method and the numbering of the position circles repeated within each of the houses (Sarrus 1853, p. 17, Planche 5).

- An astrolabe constructed in A.D. 1304-1305 by Aḥmad ibn Ḥusayn ibn Bāṣo[37] includes one plate with position circles intersecting the equator at the endpoints of ten-degree intervals, entitled "The method of al-Ghāfiqī for latitude 37º 30'" (North 1986, p. 64). The plate also has ordinal numbers for the astrological houses according to the Equatorial Method. We will discuss the attribution of this method to "al-Ghāfiqī" in Section 5.1.

- In the context of the Latin West, a Spanish astrolabe, which Moreno, van Cleempoel and King (2002, pp. 346, 348; Casulleras 2010, pp. 135-136) date in the sixteenth century, contains a two-sided plate for the latitudes of 41º 30' and 40º, probably for the cities of Valladolid and Toledo, respectively. Both faces contain, below the horizon, the projection of position circles crossing the equator at the endpoints of three-degree intervals, Roman numerals for the (first to seventh) astrological houses (Equatorial Method) and the inscription "*Circvli positionvm*".

A solution by means of a celestial globe is to be found in an appendix, written by the thirteenth-century Jewish astronomer Don Moshé for King Alphonso X, and appended to the *Libro del Alcora* (Rico 1863-1867, vol. 1, p. 206; Viladrich and Martí 1983, pp. 89-90; Samsó 1997, pp. 201, 208-209). A solution by means of an armillary sphere is in another Alfonsine book: the *Libro Segundo de las Armellas* (Rico 1863-1867, vol. 2, p. 67; Nolte 1922, p. 46). The Andalusian astronomers al-Zarqālluh (died 1100) and Ḥusayn Ibn Bāṣo (died 1316) explain how the progressions can be computed accor-

[37] He was the son of Ḥusayn Ibn Bāṣo who authored a treatise on a universal instrument (edition, Spanish translation and study in Ibn Bāṣo 1993).

ding to this method by means of their universal astrolabe plates (Rico 1863-1867, vol. 3, pp. 207, 211; Puig 1987, pp. 82, 85-86; Ibn Bāṣo 1993, 202: Spanish, 179-181: Arabic, Ch. 156).[38]

Although the principle of the method is stated in Ptolemy's *Tetrabiblos*, Islamic astronomers do not attribute this method to Ptolemy. Al-Zarqālluh and Ibn Bāṣo attribute this method to the legendary Hermes, and they both mention the *Zīj* (astronomical handbook with tables) of the Andalusian astronomer Abū'l-Qāsim Ibn al-Samḥ[39] (979-1035) as the source of this attribution. Don Moshé and the *Libro Segundo de las Armellas* also attribute the method to Hermes. We will discuss the attributions to Hermes in Section 6.1 below.

### *3.1.4 Hour Line Method*

In the *Tetrabiblos*, Ptolemy presents the method of this section as an approximation to the Position Semicircle Method of the previous section. The *Tetrabiblos* was translated into Arabic. The work is mentioned around the year 880, by the Eastern astronomer al-Battānī[40] (Nallino 1899, vol. 3 p. 203, line 1) as the source for this method. The attribution to Ptolemy is also found in al-Bīrūnī's *Qānūn* (1954-1956, p. 1396), and this method was very likely seen as the practical resolution for the "mixed ascensions" that we have mentioned in the previous section.

In this method, two points are supposed to be in a similar position if they are on the same hour line. The hour lines are defined as follows. On any given day, the period between sunrise and sunset is divided into 12 equal periods called seasonal (day) hours. The length of the seasonal hours varies with the season, but the time interval between sunrise and noon is always 6 seasonal hours. On different days, one can consider the positions of the (centre of the) sun at a fixed number (less than 6) of seasonal hours before (or after) noon. These points define an hour line above the horizon. Similarly,

[38] In her useful survey of methods for progressions, (Calvo 1998, p. 37, Fig. 7), Calvo distinguishes between an "equatorial method" found in the *Libro de Ataçir*, and a "methode du premier vertical", attributed to Hermes, and described in the treatises by al-Zarqālluh and Ibn Bāṣo, the *Libro de las Armellas*, and in Don Moshé's appendix to the *El Libro del Alcora*. Inspection of the texts shows that these two methods are the same.

[39] On Ibn al-Samḥ see Rius 2007a.

[40] On al-Battānī, who wrote a commentary on the *Tetrabiblos*, see, for example, van Dalen 2007a.

the period between sunset and the next sunrise is divided into 12 equal seasonal night hours, and by means of these hours one can define the hour lines under the horizon (cf. Hogendijk 2001, pp. 4-6).

Thus, our problem is now as follows: For two given points $F$ and $P$ on the celestial sphere, to compute the number $n$ such that if $F$ is rotated around the celestial axis over an angle $n$, its image is on the same hour line as $P$ (Figure 9).

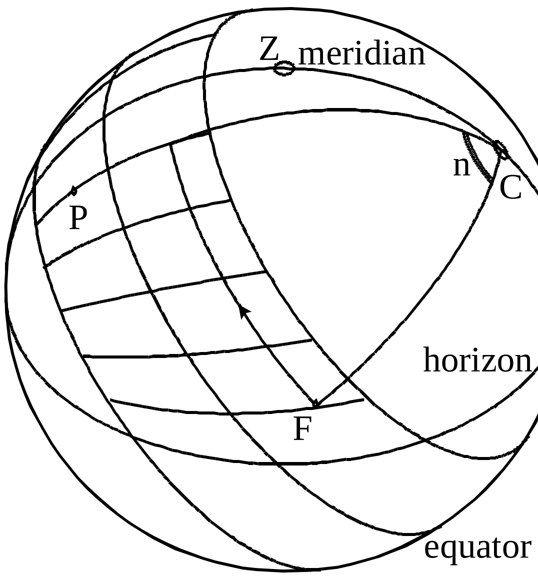


Figure 9

Ḥusayn Ibn Bāṣo describes in Chapter 152 of his treatise on the general plate (Ibn Bāṣo 1993, pp. 167-169: Arabic, 195-196: Spanish) the determination of $n$ by means of a special astrolabe plate "that some people make for the projection of rays […] for a particular latitude […], and that can be also used for the *tasyīr*". This plate is "mentioned in the previous chapter"[41], and it is prescribed to have hour lines distributed along 180 divisions on each

[41] Chapter 151, devoted to the projection of rays according to the Method of Ptolemy, cf. our Section 4.5.

side of the horizon.[42] He calls this the "method of Ptolemy".[43] The astrolabe of A.D. 1304-1305 by his son Aḥmad, mentioned above, has two plates with hour lines for all multiples of $\frac{2}{3}$ seasonal hours before or after noon and midnight (North 1986, pp. 62-63). One of them is entitled "the method of Ptolemy for latitude 37º 30'", whereas the other only has the indication "for latitude 33º". The hour lines on the plates can be used in the method for the *tasyīr* of this Section, and in the methods for the rays that we explain in Sections 4.5 and 4.7. Again, inspection of these plates is not sufficient to determine the methods and practices for which they were conceived. However, these plates also have ordinal numbers for the limits of the astrological houses and, therefore, we know for sure that the constructor had in mind the method for the division of houses of our Section 5.3.

We now explain the computation of $n$ in modern notation, in a way close to the medieval procedures. Let the declination of $P$ be $\delta(P)$. We first compute the ascensional difference $\Delta(P) = \alpha_0(P) - \alpha_\varphi(P)$ from $\sin \Delta(P) = \tan \varphi \tan \delta(P)$. Here $\varphi$ is the geographical latitude, $|\Delta(P)| < 90°$, and $\Delta(P)$ and $\delta(P)$ have the same sign. Then, if $P$ is above the horizon, the length of a seasonal day hour (if the centre of the sun coincides with $P$) is $\sigma(P) = 15 + \frac{1}{6}\Delta(P)$ time-degrees.[44] For $P$ under the horizon, the length of a seasonal night hour (if the centre of the sun coincides with $P$) is $\sigma = 15 - \frac{1}{6}\Delta(P)$ time-degrees.

Now consider the case where $P$ and $F$ are above the horizon in the Eastern hemisphere. We first find the so-called "distance"[45] $d(P)$ of $P$ to the meridian in seasonal hours. If the hour angle of $P$ is $u(P)$, we have $d(P) = \frac{|u(P)|}{\sigma(P)}$. Similarly we find $d(F)$.

[42] Ibn Bāṣo correctly explains how to plot these lines using equal divisions on the equator and the two tropics, but he then incorrectly states that the hour lines pass through the North point of the horizon.

[43] In her survey of progression methods, Calvo (1998, p. 37, Fig. 7), Calvo calls this method the "equatorial method", but this is not strictly correct because the special plate considered by Ḥusayn ibn Bāṣo contains hour lines, as he explains in his description of the plate in the preceding chapter (Ibn Bāṣo 1993, Ch. 151, pp. 163-164 Arabic, 193-194 Spanish).

[44] One time-degree is $\frac{1}{360}$ of one complete revolution of the celestial sphere (a revolution is close to 24 equinoctial hours). Thus a time-degree is the time-interval in which the celestial sphere rotates over one degree, and which corresponds to approximately four minutes.

[45] Arabic: *buʿd*.

Then we have (compare Figure 9, in which $d(P) = 1$, $d(F) = 5$, $\sigma(F) \approx 18^\circ$):

$$n = (d(F) - d(P)) \times \sigma(F). \tag{1}$$

This method is given by Ibn Hibintā with worked examples (Ibn Hibintā 1987, vol. 1, pp. 134-143).[46] Using $\Delta(P) = \alpha_0(P) - \alpha_\varphi(P)$ and $|u(F) - u(P)| = \alpha_0(F) - \alpha_0(P)$, (1) can be expressed as

$$n = \alpha_0(F) - \alpha_0(P) + \frac{d(P)}{6}\,((\alpha_\varphi(F) - \alpha_\varphi(P)) - (\alpha_0(F) - \alpha_0(P))). \tag{2}$$

Formula (2) is valid for all positions of $P$ in the Eastern hemisphere and all positions of $F$; for points $P$ in the Western hemisphere of the celestial sphere, change $\varphi$ to $-\varphi$. This method is presented by al-Battānī (Nallino 1903, vol. 1, pp. 131-134, vol. 3, pp. 200-202), Kūshyār ibn Labbān (tenth-century Iran) (Yano and Viladrich 1991, pp. 4-7; Ibn Labbān 1997, pp. 160-167), al-Bīrūnī (1954-1956, pp. 1394-1397),[47] in the treatise by al-Zarqālluh on his universal plate (Puig 1987, pp. 81-82), (Rico 1863-1867, vol. 3, pp. 205-206), and in the Alphonsine *Libro de las Armellas* (Rico 1863-1867, vol. 2, p. 67), and by Ibn ʿEzra (Viladrich and Martí, p. 91).[48]

Other equivalent forms are also possible. Al-Battānī (Nallino 1903, vol. 1, pp. 134, 317; Nallino 1899, vol. 3, pp. 202-203; Yano and Viladrich 1991, p. 7) presents another method, which is as follows for points $P$ and $F$ in the Eastern hemisphere:[49] If $M$ is the intersection between the ecliptic and the meridian above the horizon,

$$n = \alpha_0(F) - \alpha_0(M) - \mathrm{d}(P) \times \sigma(F). \tag{3}$$

[46] For $F$ under the horizon and $P$ above the horizon, Ibn Hibintā finds $n$ as sum of two arcs $n_1$ and $n_2$, where $n_1$ represents the travel of $F$ from its initial position to a position on the horizon and $n_2$ the travel from this position of the horizon to the hour line through $P$.

[47] Al-Bīrūnī calls the right term of (2) a "mixed ascension" (*maṭāliʿ muzawwaja*, al-Bīrūnī 1954-1956, p. 1394, line 2).

[48] Ibn ʿEzra says that this was the method of Hermes, Donoreus (i.e. Dorotheus?), Ptolomeus, Messella (Māshā'allāh), Andruzagar, Abū Maʿshar, and all other early astrologers with the exception of Avennouasth (Ibn Nawbakht) the Christian and Anurizi (al-Nayrīzī), see the commentary of Viladrich and Martí 1983, p. 91).

[49] This formula, which is a variation of (1), can be derived algebraically from (2) using $\sigma(F) = \frac{1}{6}(90^\circ + \alpha_0(F) - \alpha_\varphi(F))$ and $\frac{d(P)}{6} = \frac{\alpha_0(P) - \alpha_0(M)}{90^\circ + \alpha_0(P) - \alpha_\varphi(P)}$. The formula is therefore mathematically equivalent to (2). We do not understand the statement in Yano and Viladrich 1991, p. 7 to the effect that (3) is less accurate than (2).

For other positions of $F$, analogous formulas are valid.

Ḥusayn ibn Bāṣo gives a solution by means of his universal astrolabe plate (Ibn Bāṣo 1993, Ch. 152, pp. 169-170 Arabic, 196-197 Spanish), in which he follows the above-mentioned method of computation (2) step by step.

In the eleventh century, al-Istijī criticized al-Battānī for using this method. Nevertheless, the same method occurs in works by later authors in the Maghrib, such as the twelfth-century astronomer Ibn al-Kammād[50] (chapter 29 of the *Muqtabis Zīj*, fols. 17*v*-18*r*), Ibn al-Raqqām (chapter 63 of the *Mustawfī Zīj*, p. 222), Ibn ʿAzzūz, who says that his tables for the projection of rays in the *Muwāfiq Zīj* (cf. Section 4.5), which are based on the use of hour lines, have their fruit in the *tasyīr* (Casulleras 2007b, p. 62), and al-Baqqār (fl. Fes, 1411-1418), among others (cf. Díaz Fajardo 2010).

For points $P$ on the horizon or meridian, the Hour Line Method gives the same results as the Position Semicircle Method. We now investigate the difference between the two methods for other positions of $P$, mathematically and historically.

In the *Tetrabiblos*, Ptolemy introduces hour lines as approximations to position semicircles in the following passage: "... one of those semicircles (i.e. position semicircles) which are described through the [inter]sections of the meridian and the horizon, each of which at the same position makes nearly the same temporal (i.e. seasonal) hour" (Ptolemy 1940, pp. 290-291).[51] Thus Ptolemy says that a position semicircle is nearly an hour line.

We now check if this is the case in his own worked example for the latitude of Lower Egypt ($\varphi = 30^\circ\ 22'$),[52] where point $P$, the vernal point, is in the Western hemisphere of the celestial sphere with $d(P) = 3$, and $F$ is the beginning point of Gemini on the ecliptic (60°). He finds $n = 64^\circ$ by the Hour Line Method (Ptolemy 1940, pp. 300-301).

The latitude of the position circle through $P$ is[53] $\xi \approx 22^\circ\ 30'$ and hence by the Position Semicircle Method $n = \alpha_{-\xi}(60^\circ) - \alpha_{-\xi}(0^\circ) \approx 66^\circ\ 54'$. The difference of 2° 54' corresponds to 2 years and 11 months in the life of the individual in question, so the result is disastrous for a serious astrologer. For $1 \leq$

[50] On this author, see Comes 2007.

[51] The passage is incorrectly explained in footnotes 1 and 2 in Ptolemy 1940, pp. 290-291.

[52] This locality is defined by the assumption of a longest day of exactly 14 equinoctial hours, see Pedersen 1974, p. 108.

[53] If $\theta$ is the distance along the equator between (a) the intersection of the position circle through $P$ and (b) the meridian, then $\tan \xi = \tan \varphi \sin \theta$.

$d(P) \leq 5$, the error is always at least one degree for localities on the latitude of Lower Egypt.[54]

The difference between the Hour Line Method and the Position Circle Method can also be illustrated geometrically. According to a celebrated theorem in the theory of sundials, the (seasonal) hour lines are very nearly great circles on the celestial sphere, so their images on a horizontal sundial are very nearly straight lines. However, if the hour lines were (almost) position circles, their images would be almost parallel to each other and to the North-South axis. This is evidently not the case (Figure 10 represents a sundial with markings for the seasonal hours at a latitude of 40º), and many surviving sundials from antiquity show that their makers were aware of this fact (Gibbs 1976, pp. 323-338). Thus Ptolemy could have been aware of the significant difference between hour lines and position semicircles.

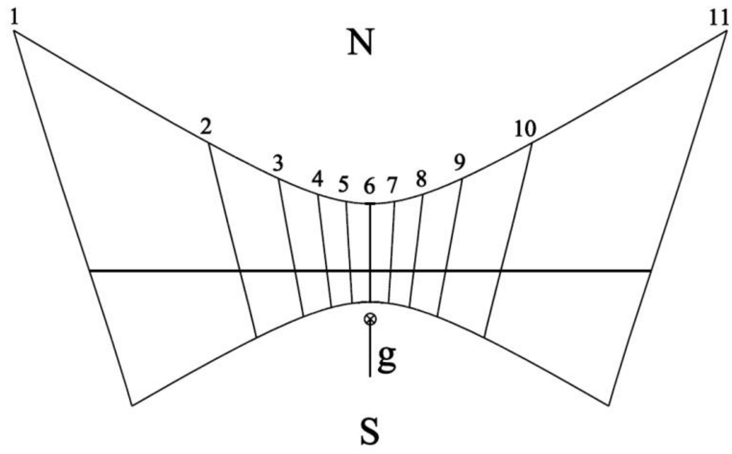


Figure 10

This situation becomes even more surprising if one realizes that Ptolemy wrote a work *Analemma* on the theory of sundials. In this work he introduced six solar coordinates, including the *verticalis*, that is the distance between the zenith and the position semicircle through the (center) of the sun.

[54] The fact that this error went unnoticed for such a long time can be used as an argument against the validity of this type of prediction.

He then tabulated these coordinates for various latitudes and various times of the year, for the beginning of every seasonal hour. Thus he must have noticed the change of the *verticalis* (and hence the position semicircle) at the beginning of each seasonal hour as function of the time of the year.[55] All but one of the tables are lost, and the *Analemma* seems not to have been transmitted into Arabic. It is probably a coincidence that the tenth-century Iranian mathematician Kūshyār ibn Labbān believed that the Ptolemy who authored the *Tetrabiblos* was not the same as the Ptolemy who wrote the *Almagest* (Ibn Labbān 1997, pp. 160-161).

Most Islamic astrologers show no awareness of the fact that the Hour Line Method is not mathematically equivalent to the Position Semicircle Method. However, al-Bīrūnī was more critical. He treats the notion of "distance" $d(P)$ in seasonal hours between point $P$ and the meridian with misgivings (al-Bīrūnī 1954-1956, p. 1376), and he regards the Hour Line Method as a "theoretically insufficient"[56] approximation and simplification of the Position Semicircle Method (1954-1956, p. 1398, lines 1, 16-17).

According to Ibn Hibintā, the astrologer Māshā'allāh (died ca. 815) simplified the method of this section in a way which boils down to the following. In formula (2) take $d(P) = 0$ for $P$ in the tenth and fourth house, $d(P) = 2$ for $P$ in the eleventh and third house, $d(P) = 4$ for $P$ in the twelfth and second house, and $d(P) = 6$ for $P$ in the first house. For points $F$ and $P$ on the ecliptic, Māshā'allāh called $\alpha_0(F) - \alpha_0(P) + \frac{1}{3}((\alpha_\varphi(F) - \alpha_\varphi(P)) - (\alpha_0(F) - \alpha_0(P)))$ the ascension of arc $FP$ in the eleventh (and third) house, and so on (Ibn Hibintā 1987, vol. 1, pp. 131-134). The passage by Ibn Hibintā was copied by Ibn Abī-l-Rijāl (cf. Díaz Fajardo 2008, pp. 125-136, and 2011, pp. 348-352).

### *3.1.5 Distance Method*

In this method $n$ is the great circle distance between points $P$ and $F$. We have found this method in the treatise by Ḥusayn ibn Bāṣo on his universal astrolabe plate, in Chapter 153 on the projection of rays according to al-Battānī (see Section 4.1) (Ibn Bāṣo 1993, pp. 174 Arabic, 199 Spanish), and in Chapter 63 of Ibn al-Raqqām's *Mustawfī Zīj* (ca. 1280-1290, pp. 220-221).

[55] See for a summary of the *Analemma* and a description of the tables Nallino 1907, vol. 2, pp. 848-856.
[56] Arabic: *ghayr murḍī fī ṭarīq al-naẓar.*

### *3.1.6 Hypothetical use of the Prime Vertical*

As we shall see (in Sections 4.9 and 5.2), there are astrolabe plates containing position circles arranged on equal divisions of the great circle through the East and West points of the horizon and the zenith, which is called the prime vertical circle. The division of houses according to this geometrical approach is illustrated on these plates in a way that corresponds to a well known procedure (cf. Section 5.2). The "projection of rays" is mentioned explicitly on two of the published plates of this kind that we know (cf. Section 4.9). These plates could also be used for performing operations for the *tasyīr*, but this is not indicated on any of them by a self-explanatory, unambiguous way. Moreover, the texts do not mention the use of this geometrical approach for the progressions. Therefore, there is no definite evidence of a method in which the arcs of the progressions are to be measured along the prime vertical circle instead of the equator or the ecliptic.

### *3.2 Progressions for predictions for a given date*

Thus far we have discussed the problem to find the rotation arc $n$ if the emitting point $P$ and the destructive point $F$ are given. The astrologers were frequently confronted with the need to make a prediction for a given number $n$ of years after the birth of an individual. In this case, $P$ and $n$ are given and $F$ has to be determined on the ecliptic in such a way that rotation of $F$ around the celestial axis over an angle $n$ produces a point in a similar position as point $P$. The astrologer then interpreted the position of point $F$ in a zodiacal sign of the ecliptic and the celestial bodies, rays, etc. close to it, in order to make the desired prediction.

The mathematical problem of finding $F$ is easy to solve for the Oblique Ascension Method and the Right Ascension Method, and also for the Positon Semicircle Method, if the latitude $\xi$ of the position circle through $P$ can be computed, and oblique ascension tables for sufficiently many latitudes between 0 and $\varphi$ are available.

For the Hour Line Method, one has to determine $F$ in such a way that formula (2) is satisfied. For points $P$ not in the horizon and meridian planes, Kūshyār ibn Labbān, (Yano and Viladrich 1991, p. 6) and al-Bīrūnī (1954-1956, p. 1400) solved this problem in the following way by linear interpolation:

First assume that $P$ is in the Eastern hemisphere. Find points $F_0$ and $F_\varphi$ on the ecliptic such that

$$n = \alpha_0(F_0) - \alpha_0(P)\ ,$$

$$n = \alpha_\varphi(F_\varphi) - \alpha_\varphi(P)\ .$$

If $P$ is on the Eastern horizon, $F = F_\varphi$. If $P$ is in the meridian plane, $F = F_0$.

For $P$ between the Eastern horizon and the meridian, point $F$ on the ecliptic is defined by

$$\lambda(F) = \lambda(F_0) + \frac{d(P)}{6} \times (\lambda(F_\varphi) - \lambda(F_0)). \tag{4}$$

Here $\lambda$ denotes ecliptic longitude and $d(P)$ is the distance between $P$ and the meridian in seasonal day hours if $P$ is above the horizon or in seasonal night hours if $P$ is under the horizon.

Al-Bīrūnī calls the arc between the vernal point and $F_0$ “the first arc” and the arc between the vernal point and $F_\varphi$ “the second arc”. For $P$ in the Western hemisphere, change $\varphi$ to $-\varphi$.

Al-Bīrūnī and Kūshyār do not mention the fact that this computation is only an approximation, and hence the question arises whether they related this computation to the Hour Line Method. The following evidence shows that they must have conceived (4) as a solution to (2) for given $n$ and $P$. Kūshyār only presents (2) and no other methods for the computation of $n$ from $P$ and $F$. Al-Bīrūnī also mentions the Position Semicircle Method, but he goes on to compute the latitude $\xi$ of this position circle, and he then hints that the ascensions and descensions for this latitude have to be used to find $F$ from $n$ and $P$.[57] Since the formula (4) is unnecessary for the Position Semicircle Method, it has to belong to the only other method which he presents, that is the Hour Line Method. See Section 4.5 for further evidence.

[57] This is shown by the somewhat confusing terminology which al-Bīrūnī uses here: “the latitude of the circle of the progression (i.e. $\xi$), and that is the horizon such that the preceding (body) travels by its ascensions and descensions” (al-Bīrūnī 1954-1956, p. 1399, lines 7-8). This seems to contradict our explanations, in which $F$ (and not $P$) is made to travel, but al-Bīrūnī explains the terminology a little later: “... the position at which the preceding (body) arrives by means of the progression, that is to say, the position of the ecliptic which arrives by the first motion (i.e. the daily rotation) at the circle of it (the preceding body)” (1954-1956, p. 1400, lines 11-12). The notion that $P$ (not $F$) travels is more natural for the astrologer who has to compute a series $F_1$, $F_2$ for a successive number of values $n = 1, 2...$ The $F_i$ can be seen as images of $P$. The terminological confusion between $P$ travelling to $F$ and $F$ travelling to $P$ occurs in many sources, for example: Ibn al-Samḥ 1986.

## 4. Systems for Casting the Rays

In this section we classify all methods for the construction of the astrological rays of which we have found traces in medieval Islamic sources. The names for these methods are our own inventions. The simple ecliptic method has already been defined in the introduction. It is based on the idea that the intervals of 60º, 90º, etc. which determine the rays have to be measured on the ecliptic. In Section 4.1 we report some examples of authors using this method and we add the definitions of two alternatives which involve in the computations the ecliptic latitude of the object that casts its rays. The seven methods of Sections 4.2 to 4.8 are based on the idea that the intervals have to be measured on the celestial equator. The extant source material suggests that these seven equatorial methods were at least as popular as the simple ecliptic method.[58] An interesting enquiry is to try and guess the practical significance of the method (or methods) presented in Section 4.9, only attested in preserved astrolabes and based on the use of the prime vertical circle (defined in Section 3.1.6 above).

We have described the non-ecliptic methods of Sections 4.2 to 4.9 in such a way that the case of planets having non-zero latitude is automatically included and, in fact, most sources ignore the latitude of the planet. Nevertheless, some authors prescribe modifications of their methods to account for the case of non-zero latitude.[59] Fortunately, there is no need to mention all these variations here.

Unless indicated otherwise, in the following we consider a planet $P$ on the ecliptic, and we suppose that the planet casts its left and right sextile rays to points $L$ and $R$ on the ecliptic. The definitions of $L$ and $R$ according to the different methods will be presented below. To obtain the definitions of the corresponding quartile and trine rays, change 60º to 90º and 120º in the definitions, and consider that the ray to the opposite point is cast to the point on the ecliptic diametrically opposite $P$.

Some mathematical properties of the different methods will be compared

[58] Ulugh Beg (Sédillot 1853, p. 209) says that are many methods for the projection of rays, but that the (equatorial) methods of Sections 4.4 and 4.7 are much more used than the others.

[59] One example of this is in the work by Ibn Hibintā copied by Ibn Abī-l-Rijāl, which we have mentioned in Section 2.3 above. The approximate procedure for determining if the two objects involved in the *tasyīr* have the same declination is extended to the projection of rays, and used for projecting the object that casts its rays onto the ecliptic by means of a circle parallel to the equator (see Casulleras 2007a, pp. 44-45; Ibn Abī-l-Rijāl 1954, p. 176b).

in Section 4.10.

### *4.1 Ecliptic Methods*

Suppose that $P$ is in the ecliptic (Figure 2). Inscribe a regular hexagon, a square and an equilateral triangle in the ecliptic with angular points at $P$. Then $P$ casts its rays to points $L = P_1$, $P_2$, $P_3$, $P_4$, $P_5$, $P_6$ and $R = P_7$ in Figure 2. If $P$ is not on the ecliptic, most astrologers ignored its latitude and took instead of $P$ the point with the same ecliptic longitude. Thus the celestial longitudes of $L$ and $R$ can be obtained by adding or subtracting 60º from the celestial longitude of $P$.

This method for the projection of rays could have been used by Ptolemy, since he mentions the astrological rays without indicating how they have to be computed. In chapter 54 of his *Ṣābi' Zīj*, al-Battānī approves the application of this method only for the projection of rays of planets without ecliptic latitude (Nallino 1903, vol. 1, p. 129, and 1899, vol. 3, p. 194; cf. Samsó and Berrani 1999, p. 306). Although he describes far more sophisticated methods in his *Qānūn*, al-Bīrūnī states in his *Tafhīm* that the different aspects are determined by the ecliptic signs (al-Bīrūnī 1934, pp. 225, 259-260). We have seen in Section 3.1.0 that, in the Maghrib, al-Istijī trusts in the use of this procedure for the *tasyīr* and the projection of rays, and Ibn ʿAzzūz believes that the computation of the aspects must be performed on the ecliptic following this simple method (Casulleras 2007b, pp. 47, 62, 72, 75, and 2008/2009, pp. 258-259). We also recall that it is the system used by modern astrologers.

In order to take account of non-zero ecliptic latitude of $P$, the following two generalized methods were designed:

According to al-Battānī, points $L$ and $R$ are located on the ecliptic at great circle distances of 60º from $P$, even if $P$ is not on the ecliptic, as happens in Figure 11 (Nallino 1903, vol. 1, pp. 131, 307-308; and 1899, vol. 3, pp. 196-197, Chapter 54). His laborious computation of the celestial longitudes of $L$ and $R$ was simplified by the tenth-century Iranian astronomer Ibn al-Ḥusayn al-Ṣūfī, whose method is described by al-Bīrūnī with tables (Kennedy and Krikorian 1972, pp. 5-6; al-Bīrūnī 1954-1956, pp. 1385-1388). The quartile and trine rays were also cast to the ecliptic to points at great circle distances of 90º and 120º from $P$, and the opposite ray was cast to the point on the celestial sphere diametrically opposite $P$.

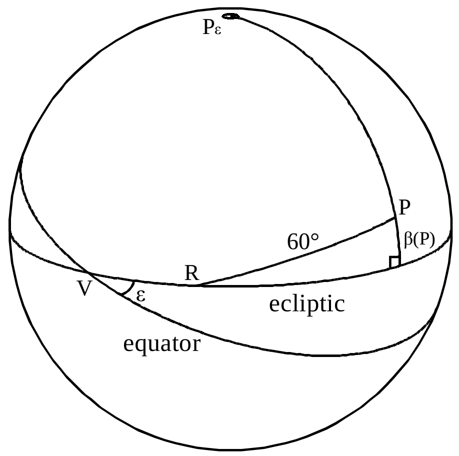


Figure 11

A passage with a similar title and concerned with similar topics as al-Battānī's text is found in an earlier work by Yaḥyā ibn Abī Manṣūr[60] (died 830) and, therefore, the method may not have been created by al-Battānī (cf. van Dalen 2004, p. 30). However, in al-Andalus this procedure is generally attributed to al-Battānī. It appears in the eleventh century (Calvo 1998, p. 43) in the treatises on the universal plates by Azarquiel (Puig 1987, pp. 32, 80; Rico y Sinobas 1863-1867, vol. 3, p. 206) and ʿAlī b. Khalaf (Rico y Sinobas 1863-1867, vol. 3, p. 123), and in the *Epistle on Tasyīr and the Projection of Rays* by Abū Marwān al-Istijī (Samsó and Berrani 1999, pp. 305-306, and 2005, pp. 204, 235). It is also found, subsequently, in the alfonsine *Libro Segundo de las Armellas*, in the *Treatise on the General Plate* by Ḥusayn b. Bāṣo (1993, 197-199: Spanish, 171-174: Arabic) and in two *Zīj*es by Ibn al-Raqqām: the *Shāmil* (Abdurrahman 1996, pp. 158-159) and the *Mustawfī* (pages 216-217, chapter 61). We have seen in Section 3.1.5 that these two last authors also extended the method to the progressions.

For $P$ not on the ecliptic al-Bīrūnī defines points $P_2$, $P_6$ on the ecliptic at a great circle distance of 90 degrees from $P$ and he draws the great circle through $P$, $P_2$ and $P_6$. The regular polygons have to be inscribed into this great circle with $P$ as angular point. Thus the two quartile rays are cast to $P_2$ and $P_6$ on the ecliptic, but the remaining rays are cast to points not on the

[60] On this author, see van Dalen 2007c.

ecliptic. Al-Bīrūnī computed tables for the longitudes and latitudes of the rays (1954-1956, pp. 1389-1392). Kennedy and Krikorian (1972, p. 7) pointed out that this method seems to have been originated by al-Bīrūnī himself. Since it is not found in other sources, we may suspect that it did not have much success among the practitioners of astrology.

### *4.2 Right Ascension Method*

Points $L$ and $R$ are on the ecliptic such that

$$60^\circ = \alpha_0(L) - \alpha_0(P) ,$$

$$60^\circ = \alpha_0(P) - \alpha_0(R) .$$

Geometric interpretation: Find the intersection of the celestial equator and the semicircle through $P$ and the celestial North and South Poles. Let this intersection be an angular point of a regular hexagon and a square inscribed in the celestial equator. Draw the semicircles through the other angular points and the celestial North and South Poles. The planet $P$ casts its rays to the intersections of these semicircles with the ecliptic. See Figure 12, in which only $P$ and $L$ are shown.

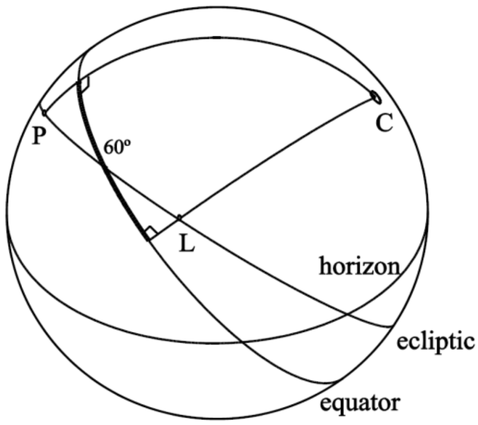


Figure 12

This method is related to "Drīnūsh" (probable Dorotheus of Sidon) by Yaḥyā ibn Abī Manṣūr (van Dalen 2004, p. 30). As we have seen in Section 3.1.1, al-Istijī relates the method to the Persians. Al-Zarqālluh and Ḥusayn ibn Bāṣo say: "It has been said that the (pre-Islamic) Persians produce the projection of rays in the right sphere (i.e. using right ascensions) only, but this is for people who live on the equator, so we have not mentioned it" (Ibn Bāṣo 1993, p. 174 Arabic, 199 Spanish; Puig 1987, p. 82; Rico 1863-1867, vol. 3, p. 205). The attribution to the Persians is also found in Ibn al-Raqqām's *Mustawfī Zīj* (p. 217). For a geographical latitude on the equator, all the non-ecliptic methods in this paper are equivalent to the Right Ascension method. It seems likely that this method was used by the pre-Islamic Iranian astronomers.

### 4.3 Oblique Ascension Method

See Figure 13. Points $L$ and $R$ are on the ecliptic such that

$$60^\circ = \alpha_\varphi(L) - \alpha_\varphi(P)\,,$$

$$60^\circ = \alpha_\varphi(P) - \alpha_\varphi(R)\,.$$

This method is described in an appendix to the treatise of al-Khwārizmī[61] (ca. 830) on the use of the astrolabe (Frank 1922, pp. 13, 17). This appendix was probably written by a later author (Frank 1922, p. 5). The method is also mentioned by al-Istijī, who attributes it to Ptolemy although it is not found in the *Tetrabiblos* (Samsó and Berrani 1999, pp. 303-304, and 2005, pp. 199-200, 234), and Ibn al-Raqqām (*Mustawfī Zīj*, p. 217-218; Abdurrahman 1996, p. 159).

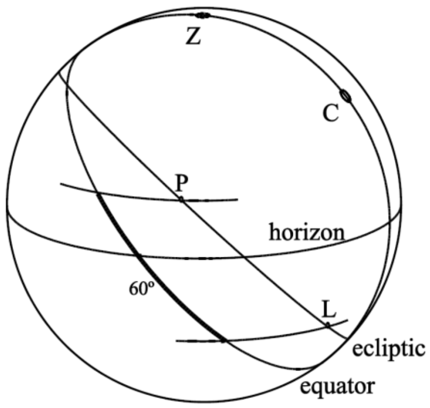


Figure 13

[61] On al-Khwārizmī see, for example, Brentjes 2007.

### *4.4 Single Position Semicircle Method*

Points $L$ and $R$ are on the ecliptic, and have to be determined according to the following condition:

If point $L$ ($R$) is rotated around the celestial axis over an angle of 60 degrees in the direction of the daily motion of the celestial sphere (or in the opposite direction), its image after rotation is on the position semicircle through $P$. We call this method the single position semicircle method, because it only involves the position semicircle through $P$.

Algebraically, let $\xi$ be the latitude of the position semicircle through $P$ in the Eastern hemisphere.

Then

$$60^\circ = \alpha_\xi(L) - \alpha_\xi(P)\,,$$

$$60^\circ = \alpha_\xi(P) - \alpha_\xi(R)\,.$$

For $P$ in the Western hemisphere, change $\xi$ to $-\xi$. See Figure 14, which is drawn for $P$ in the Eastern hemisphere and in which $\overline{\xi} = 90^\circ - \xi$ is the angle between the position circle and the celestial equator.

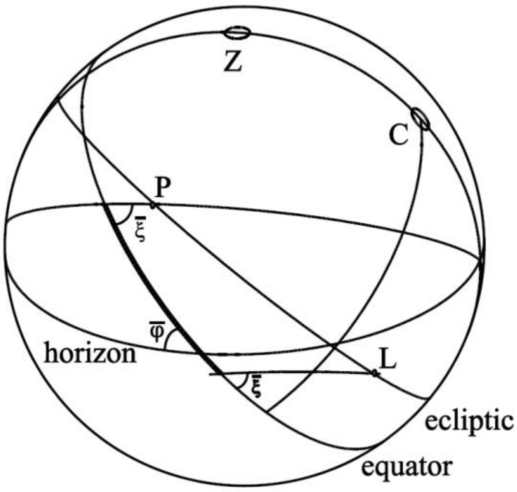


Figure 14

The method is the same as the Right Ascension Method for $P$ in the meridian plane and the Oblique Ascension Method for $P$ on the Eastern horizon.

The method is obviously related to the Position Semicircle Method for progressions, so the rays can also be determined by means of an astrolabe with the special plate described in Section 3.1.3 above. Put the spider in the position of the celestial bodies at birth, find $P$ on the spider, note (the projection of) the position semicircle $C$ through $P$ on the plate, and note the position of the pointer on the rim. Then turn the spider over 60 degrees to the right (or left). The position semicircle $C$ now intersects the ecliptic at the desired point $L$ (or $R$).

This method is mentioned in the treatise by al-Zarqālluh on the universal plate (Puig 1987, p. 85; Rico 1863-1867, vol. 3, p. 211), in the Alphonsine *Libro de las Armellas* (Rico 1863-1867, vol. 2, p. 65; Nolte 1922, pp. 45-46),[62] and in the treatise by Ḥusayn ibn Bāṣo on the universal astrolabe plate (Ibn Bāṣo 1993, Ch. 155, pp. 178-179 Arabic, 201-202 Spanish). These three sources attribute the method to Hermes, see Section 6.1 for a further discussion. Only Ulugh Beg attributes this method to Ptolemy (Sédillot 1853, p. 209).

### *4.5 Single Hour Line Method*

This method is attested in many medieval Islamic sources, and it can be considered the standard Islamic method for computing the projections of the rays. The method is explained by Kennedy and Krikorian (1972, par. 2. p. 5). It is based on the following geometrical principle:

Points $L$ and $R$ are on the ecliptic such that if $L$ (or $R$) is rotated around the celestial axis over an angle of 60 degrees in the direction of the daily motion of the celestial sphere (or in the opposite direction), its image after rotation is on the hour line through $P$ (see Figure 15). We call this method the Single Hour Line Method because only the hour line through $P$ is involved.

[62] The relevant text is as follows:
"Saca el cerco que es semeíante dell orizon sobre que está ell estrelle en aquella hora ... et cata quál grado dell yguador del dia cae sobre aquel cerco. et faz sobrél sennal. et annada sobrél .LX. grados al sestil siniestro ... et faz sennal do se allegar la cuenta. *et ponla sobrel cerco que es semeiante al cerco dell orizon* et cata quál grado del cerco de los signos cae sobre aquel cerco. et aquel grado es el logar dell echamiento del rayo siniestro que feziste". The italicized passage says that the second mark (on the equatorial ring) should be placed under the position circle. Thus it seems to us that the equatorial ring has to be rotated and the position circle is in a fixed position, hence this is a case of the single position semicircle method.

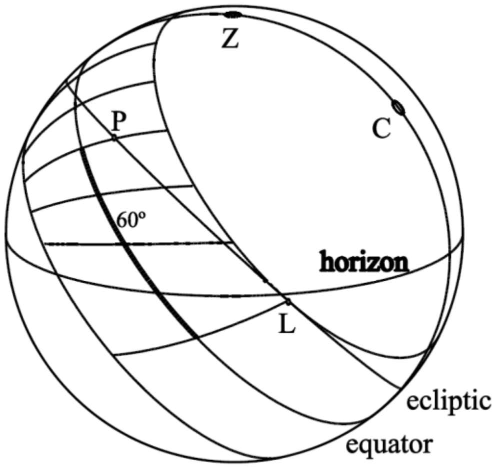


Figure 15

This method is related to the Hour Line Method for progressions and, therefore, the rays can be found with the astrolabe plates that we have mentioned in Section 3.1.4 above. In the field of analogical instruments, the Single Hour Line Method for the rays is also mentioned in treatises on the use of the astrolabe by, for example, al-Ṣūfī (1962, Chapters 154-155, pp. 128-132), and Ibn al-Samḥ (1986, pp. 68-70, 147-149). It is also found in the treatise on the universal plate by al-Zarqālluh (Puig 1987, p. 81; Rico 1863-1867, vol. 3, p. 205) with references to Wālyus the Egyptian and Ptolemy, and in the Alphonsine *Libro de las Armellas* (Rico 1863-1867, vol. 2, p. 62; Nolte 1922, p. 45), with an attibution to Vellix el egipciano and Ptolemy. This Vellix or Wālyus must be a pre-Islamic astrologer who may be identical to the Byzantine astrologer Vettius Valens.[63]

As for other texts not dealing with instruments, Kennedy and Krikorian (1972, pp. 3-4) found the method in nine Eastern sources, including al-Bīrūnī's *Qānūn* (1954-1956, pp. 1377-1385) and seven *zīj*es dated between the ninth and the fifteenth centuries. It is also found in the *Great Introduction to Astrology* by Abū Maʿshar (died A.D. 886) (Abū Maʿshar 1985, VII:7, pp. 408-410; 1995-1996, vol. 3, pp. 549-560), and in a medieval Latin trans-

[63] See Sezgin 1974-1984, vol. 7, pp. 38-41.

lation of a treatise on the astrolabe attributed to Māshā'allāh (ca. 780) but probably by the Andalusī astronomer Maslama al-Majrīṭī[64] (died A.D. 1007) (Frank 1922, pp. 24-25; Viladrich and Martí, p. 94; Nallino 1903, vol. 1, p. 311; Sezgin 1974-1984, vol. 6, p. 128). Another Andalusī astronomer, Ibn Muʿādh al-Jayyānī[65] (died A.D. 1093), wrote a treatise on the projection of rays[66] with the stated intention of warning about the errors of this method attributed to Ptolemy and "transmitted by Abū Maʿshar, among others". To this purpose, Ibn Muʿādh uses numerical demonstrations, including the following example for geographical latitude $\varphi = 49^{\circ}$: If $P$, the initial point of Capricorn on the ecliptic, is on the Eastern horizon, and $P'$ is the beginning of Cancer, that is, the point diametrically opposite $P$, we have $\alpha_\varphi(P') - \alpha_\varphi(P) = 120^{\circ}$, with the astrologically absurd consequence that the left trine ray and the opposite ray coincide (Casulleras 2007a, p. 40, and 2010, pp. 175, 203-210, 240-243: Spanish, 268-273: Arabic). The method is also found in later sources of the Islamic West, such as Ibn al-Kammād (*Muqtabis Zīj*, fols. 16*v*-17*r*: Chapter 28; Vernet 1949, pp. 74-78), Ibn al-Raqqām, who relates the method to Vettius Valens and Ptolemy (*Mustawfī Zīj*, p. 218), and Ibn ʿAzzūz, who attributes the method to Ptolemy and Hermes, and compiled on its basis the set of tables for the projection of rays that we have mentioned in 3.1.4 (Casulleras 2007b, pp. 63-64, 81, 89).

To find $L$ and $R$ the Islamic astrologers used an approximation as in Section 3.2, for $n = 60^{\circ}$.

Computation for $P$ in the Eastern hemisphere: First find points $L_0$ and $L_\varphi$ on the ecliptic such that

$$60^{\circ} = \alpha_0(L_0) - \alpha_0(P) \,,$$
$$60^{\circ} = \alpha_\varphi(L_\varphi) - \alpha_\varphi(P) \,.$$

Al-Bīrūnī calls $L_0$ the "first ray", $L_\varphi$ the "second ray" (1954-1956, pp. 1383-1384). The "first ray" is the result of the right ascension method, the "second ray" of the oblique ascension method.

If $P$ is on the Eastern horizon, $L = L_\varphi$. If $P$ is in the meridian plane, $L = L_0$.

If $P$ is between the Eastern horizon and the meridian, the left sextile ray is cast to point $L$ on the ecliptic defined by

[64] On Maslama al-Majrīṭī see, for example, Casulleras 2007d.
[65] On Ibn Muʿādh see Calvo 2007.
[66] For references to this work see Kennedy 1994; Casulleras 2004; Hogendijk 2005; Casulleras 2010 (edition and Spanish translation).

$$\lambda(L) = \lambda(L_0) + \frac{d(P)}{6} \times (\lambda(L_\varphi) - \lambda(L_0)) .$$

Here $d(P)$ is the distance in seasonal hours between $P$ and the meridian plane, and $\lambda$ denotes ecliptical longitude.

The right sextile ray is obtained in the same way by interpolating between the "first ray" $R_0$ and the "second ray" $R_\varphi$ obtained by

$$60^\circ = \alpha_0(P) - \alpha_0(R_0) ,$$

$$60^\circ = \alpha_\varphi(P) - \alpha_\varphi(R_\varphi) .$$

For $P$ in the Western hemisphere, change $\varphi$ to $-\varphi$.

Just as in Section 3.2, one may well ask whether the Islamic astrologers were aware of the relationship between computation and geometrical principle. The following evidence suggests that they were aware of the connection. Ḥusayn ibn Bāṣo mentions the geometrical principle and the corresponding determination of the rays on the special astrolabe plate with hour lines that we have mentioned in Section 3.1.4 (Ibn Bāṣo 1993, Ch. 151, pp. 163-165 Arabic, 193-194 Spanish), and he then explains how the rays can be found by means of his universal astrolabe plate, following the steps of the computation (1993, pp. 165-167 Arabic, 194-195 Spanish). Al-Bīrūnī says that the computation is incorrectly attributed to Ptolemy, but that the attribution is to be explained because the procedure is based on Ptolemy's method of progressions (al-Bīrūnī 1954-1956, pp. 1377, line 14, 1378, line 4), that is, the Hour Line method. Elsewhere (1954-1956, p. 1394), al-Bīrūnī says that this procedure for the rays was derived from the procedure for progressions, that is to say, the hour line method.

### *4.6 Four Position Circles Method*

Points $L$ and $R$ are determined as follows. Find the intersection of the celestial equator and the position semicircle through $P$. Let this intersection be an angular point of a regular hexagon and a square inscribed in the celestial equator. Draw the position semicircles through the other angular points. The planet $P$ casts its rays to the intersections of these position semicircles with the ecliptic. Thus points $L$ and $R$ are such that their position semicircles intersect the equator at points which are 60 degrees apart from the intersection of the equator and the position circle through $P$. See Figure 16, which shows the construction of $L$ only. This method involves a total number of four position circles.

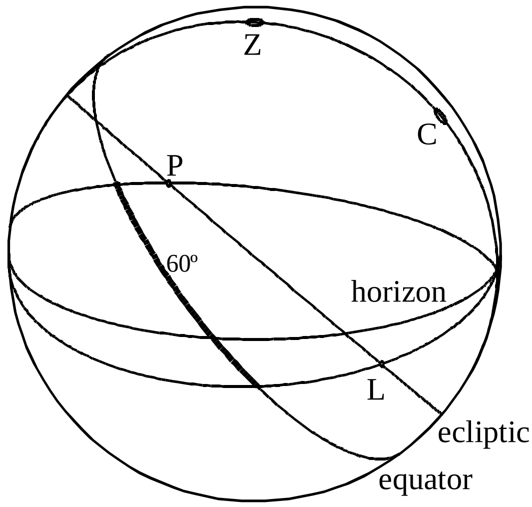


Figure 16

The rays can also be determined according to this method by means of the special astrolabe plate described in Section 3.1.3 above. Put the spider in the position of the celestial bodies at birth, find *P* on the spider, note the position circle *C* through *P* on the plate. Suppose the plate displays position circles for six-degree intervals of the equator. From the position circle *C*, count the position circles in counter-clockwise (or clockwise) direction, and let the tenth of these circles be $C_L$ (or $C_R$). Then the intersection of $C_L$ (or $C_R$) with the ecliptic will be the desired point *L* (or *R*). This procedure is illustrated in the Alphonsine *Libro dell Ataçir* written by Rabbī Isḥāq ibn Sīd (Viladrich and Martí 1983, pp. 92-93; Rico 1863-1867, vol. 2, p. 308).[67] The author uses an astrolabe with the special plate containing projections of the position circles.[68]

[67] Emilia Calvo's survey of methods for projection of rays (1998, Fig. 6) is consistent with ours if we interpret her Prime Vertical Method as our Single Position Semicircle Method and her Equatorial Method as our Four Position Circles method.

[68] Here is the relevant text:

"... sabe el grado dell ascendent. et pon aquel grado sobrell orizon oriental. et sabe quál de los cercos temporales passa por la estrella que tú quieres echar sus rayos. et faz sennal sobrella. et cata quál grado dell yguador del dia cae sobre aquel cerco et aquellos serán sos sobimientos

The Andalusī astronomer Maslama al-Majrīṭī computed tables for the (approximate) computation of the projections of rays. The lay-out and the mathematical structure of his tables show that he was a follower of the Four Position Circles Method. Al-Majrīṭī's tables were adaptations of earlier tables for the projection of rays by al-Khwārizmī but the geometrical motivation of the tables of al-Khwārizmī is not so clear. The tables of al-Majrīṭī were based on the concept of the "latitude" $\xi$ of a position circle *NPS* and the computation of the corresponding ascensions $\alpha_\xi$.[69] Al-Khwārizmī did not compute $\xi$. His computation is much simpler and based on the approximation of position circles by hour lines as explained in Section 3.1.4. Of course, it is an open question to what extent al-Khwārizmī was conscious of these mathematical subtleties.[70] Thus it is not clear whether we have to classify his tables as an instance of the Four Position Circles Method or rather of what we will call the Seven Hour Lines Method (see Section 4.8).

Without giving any details on the computation, al-Istijī attributes to the "geometers"[71] and to Hermes a procedure which Samsó and Berrani have conjecturally identified with the Four Position Circles Method (1999, pp. 304-305, and 2005, pp. 201-202, 234-235). In his work on the projection of rays, and in the *Tabulae Jahen*, Ibn Muʿādh al-Jayyānī argues that the Four Position Circles Method is the only correct method for projecting the rays.[72] He provides the only known algorithm for this method, and gives the Seven Hour Lines Method of Section 4.8 as an approximate alternative.

It is worth noting that all the sources mentioning this method come from the Western area (cf. Casulleras 2008/2009, 248-251).

sobre aquel cerco et guárdalos. Et si quisieres fazer rayos diestros (sinistros), mingua (annade) ... (60º, 90º, 120º) et lo que fuer de los sobimientos despues del annadimiento ó del minguamiento. faz sobrello sennal en el cerco dell yguador del dia. et cata quál cerco de los temporales (i.e. position circles) passa por y. et aquel será el cerco dell echamiento daquel rayo que quesiste saber."

69 For an explanation of these tables and their mathematical structure see Hogendijk 1989.

70 The tables of al-Khwārizmī possess a computational flaw, discussed in Hogendijk 1989, pp. 184-187, formula (3.10), with the consequence that only for planets in the first ten degrees of each zodiacal sign we have the following property: If $P$ casts its ray to $Q$, $Q$ also casts a ray to $P$. Note that formula (2.6) in Hogendijk 1989, p. 178 should be changed to $P^*Q^* = \alpha_\xi(\lambda_Q) - \alpha_\xi(\lambda_P)$.

71 *Muhandisūn*.

72 The mathematics is discussed in Hogendijk 2005.

### *4.7 Standard Houses Method*

This method was probably adapted from the Standard Method for defining the houses (see below, Section 5.4). The basic idea is as follows. Consider the position semicircle through *P* as an Eastern horizon for some latitude $\xi$ (with $-\varphi \leq \xi \leq \varphi$). For this horizon and point *P*, construct the division of the ecliptic into 12 houses according to the Standard Method. In the Standard Method for the houses, point *P* is always on the ecliptic but this need not be true in the analogous construction of the rays. The planet then casts its left sextile, left quartile, left trine, right trine, opposite, right quartile and right sextile rays to the cusps of the third, fourth, fifth, seventh, ninth, tenth and eleventh house respectively. Although *P* does not need to be on the ecliptic, its rays are always cast to points on the ecliptic.

The geometric construction is as follows (Figure 17): The circle through the celestial poles perpendicular to the position semicircle through *P* can be considered as a new "meridian plane". The planet *P* casts its quartile rays to the intersections between this plane and the ecliptic. Construct the circle through *P* parallel to the celestial equator, and trisect the two arcs of this circle between *P* and the new "meridian plane". The planet *P* casts its left and right sextile rays to the trisecting points *L* and *R* which are closest to the meridian plane. *P* casts its right and left trine ray to the points diametrically opposite *L* and *R* respectively.

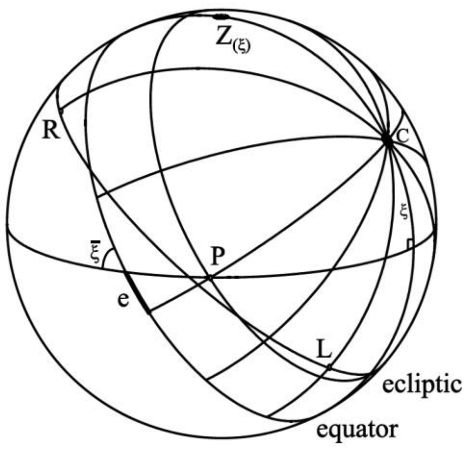


Figure 17

Algebraically, we first compute the "ascensional difference" $e = \alpha_0(P) - \alpha_\xi(P)$.

For the left quartile ray $Q$ we have

$$\alpha_0(Q) = \alpha_\xi(P) + 90^\circ = \alpha_0(P) + 90^\circ - e\,,$$

and for the sextile rays:

$$\alpha_0(L) = \alpha_0(P) + 60^\circ - \frac{2e}{3} = \alpha_\xi(P) + 60^\circ + \frac{e}{3}\,,$$

$$\alpha_0(R) = \alpha_0(P) - (60^\circ + \frac{2e}{3}) = \alpha_0(L) - 120^\circ\,.$$

This method is explained in the Zīj of Ulugh Beg, the king of Samarkand (ca. 1420) (Sédillot 1853, p. 210). Ulugh Beg says that there are many methods for the projections of rays, but that two methods are much more often used than others, namely (in our terminology) the Single Position Semicircle Method (which he attributes to Ptolemy) and the method of this section (which he attributed to unspecified authors other than Ptolemy) (Sédillot 1853, p. 209).

Samsó (1996, pp. 597-601) summarized one page from the manuscript of the Zīj al-Ṣafā'iḥ by Abū Jaʿfar al-Khāzin on the projection of rays. Samsó's reconstruction shows that al-Khāzin also used this method.[73]

## *4.8 Seven Hour Lines Method*[74]

In this method a planet $P$ casts its rays to the intersection points of the ecliptic with the seasonal hour lines indicating the seasonal hour of $P$ plus or minus 4 (sextiles), 6 (quatriles), 8 (trines) or 12 (opposition) seasonal hours. See Figure 18, in which the left sextile $L$ of $P$ is represented.

According to this definition, the rays can be found with the astrolabe

[73] In this connection, Samsó refers to a statement by al-Hāshimī (ca. 890) in his *Book on the Reasons behind Astronomical Tables* (1981, pp. 186, 323-324) on tables for the astrological rays. For the left sextile ray, al-Hāshimī introduces the quantity $\frac{x}{3}$ where $x = \alpha_\varphi(\lambda_P + 180^\circ) - \alpha_\varphi(\lambda_P)$. Since al-Hāshimī refers to tables, he may have thought of al-Khwārizmī, who uses $\frac{x}{3}$ in his tables for the astrological rays, see Section 4.6.

[74] The original Section 4.8 (Mathematical Properties of the Equatorial Methods) in Hogendijk 1998 corresponds to our current Section 4.10 below.

plates that we have mentioned in Section 3.1.4 above, but we have not found any evidence of the use of this method in the treatises on instruments. As we have seen in Section 4.6, Ibn Muʿādh presents this method as an approximate alternative for the Four Position Circles Method, an approach which is consistent with the idea of using approximations of position circles by hour lines (cf. Section 3.1.4), and which allows for the use of arithmetical rules instead of trigonometric functions (cf. Casulleras 2004, pp. 392-400 and 2010, p. 25).

We have mentioned in Section 4.6 the possibility that al-Khwārizmī's tables for the projection of rays were based on this method. Ibn al-Raqqām attributes to "the modern ones"[75] a procedure that is probably also the Seven Hour Lines Method (*Mustawfī Zīj*, pp. 218-220).

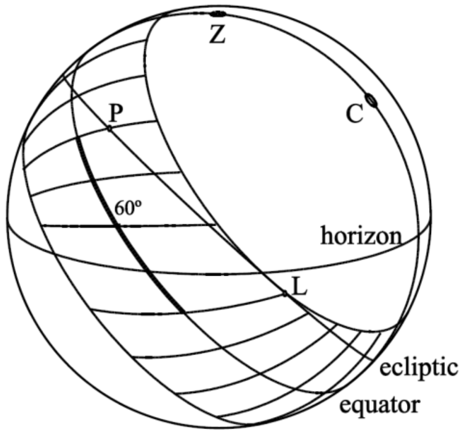


Figure 18

### *4.9 Use of the Prime Vertical*

In this section we deal with the existence of one or more methods for the projection of rays that use position circles arranged according to equal divisions of the prime vertical circle (defined in Section 3.1.6 above). A method for the division of houses based on divisions of this circle is well attested in

[75] *Al-muta'akhkhirūn.*

both Eastern and Western sources (cf. Section 5.2), but no evidence of the use of an analogous method for the projection of rays or the *tasyīr* has been found in the texts. However, we know of three astrolabes, all of them Andalusian, with plates that correspond to this geometric approach and can be used for the three astrological practices described here:[76]

- The oldest one is an astrolabe constructed in Toledo in A.D. 1029-1030. It includes two plates entitled "projection of rays for latitude 38º 30'" and "projection of rays for latitude 42º". The plate for 38º 30' was published (Woepke 1858, pp. 26-30, Fig. 12), and bears position circles which intersect the prime vertical at the endpoints of six-degree intervals around the horizon. The use of this plate for the division of houses is corroborated by the fact that the numbering of the position circles (6º, 12º, 18º, 24º, 30º) is repeated within the interval that corresponds to each astrological house (see also Hogendijk 2005, p. 99).

- In 1081-1082, two plates with a similar net of lines specifically devised for "the projection of rays in Valencia" and "Saragossa" were made by Muḥammad al-Ṣabbān. These plates have ordinal numbers for the astrological houses and position circles that correspond to divisions every ten degrees on the prime vertical, starting at the horizon (King 2005, 937 and 940).[77]

- Another plate which shows the same distribution and frequency of circles as al-Ṣabbān's plates belongs to the astrolabe made by Aḥmad b. Ḥusayn b. Bāṣo in 1304-1305, mentioned above (in Sections 3.1.3 and 3.1.4). The plate bears the inscription "Method of Hermes for the latitude 37º30'" (probably Granada), in this case without reference to the practices for which it was designed, but also showing specific ordinal numbers for

[76] We must recall that there is no verifiable evidence of the application of any of these plates to the *tasyīr* (cf. Section 3.1.6 above). To check that the position circles cross even divisions of the prime vertical circle in this type of plate, one can verify that the equatorial arcs between the East (West) point and the circles indicating the beginning of the 12th (8th) and 11th (9th) houses are close enough to, respectively, arctan (tan 30º / cos $\varphi$) and arctan (tan 60º / cos $\varphi$), where $\varphi$ is the geographical latitude for which the plate is designed. Cf. Hogendijk 2005, fn. 3 (on page 113).

[77] The plate for Valencia is at the bottom right on page 937. The current value for the latitude of Valencia, 39º 28', is consistent with the procedure proposed by Hogendijk (see the previous note) for checking this type of plate. The plate for Saragossa does not appear in the photographs.

the astrological houses (North 1986, p. 65).[78]

The method or methods to be applied for the "projection of rays" stated in two of these plates must be analogous to either the Single Position Semicircle Method or the Four Position Circles Method, using the prime vertical instead of the equator for measuring the arcs defining the aspects. Therefore, we can assume that at least the constructors of these plates had in mind this geometrical approach.

### *4.10 Mathematical Properties of the Methods for Casting the Rays*

In the simple ecliptic system, a planet $P$ casts its rays to points $P_1$, $P_2$, $P_3$, $P_4$, $P_5$, $P_6$ and $P_7$ which are defined as angular points of a regular hexagon $PP_1P_3P_4P_5P_7$ and a square $PP_2P_4P_6$ inscribed in the ecliptic (Figure 2). In this system, the rays have two properties, which we call the *symmetric property* and the *combination properties*, and which are consequences of the regularities of the figures.

By the symmetry property we mean the fact that if $P$ casts a ray to a point $Q$, a body at point $Q$ will also cast a ray to $P$. For example, if $P$ casts its left sextile ray to $Q$, we have $Q = P_1$, so $Q$ casts its right sextile ray to $P$. In terms of the ancient extramission theory of vision, if $P$ looks at $Q$, $Q$ will also look at $P$.

By combination properties we mean the following three properties for combinations of left rays, and three analogous properties for right rays: (1) If $P$ casts a left sextile ray to $P_1$, a body at $P_1$ casts its left sextile ray to the same point $P_3$ to which $P$ casts its trine ray. (2) If $P$ casts a left trine ray to $P_3$, a body at $P_3$ will cast its left sextile ray to the same point $P_4$ to which $P$ casts its opposite ray. (3) If $P$ casts a left quartile ray to $P_2$, a body at $P_2$ will cast its left quartile ray to the same point $P_4$ to which $P$ casts its opposite ray.

The following table shows whether these properties are maintained or lost in what we might call the "Equatorial Methods" (i.e. those systems in which the arcs defining the different aspects are placed along the celestial equator).

[78] If we make transparent copies of these last two plates with the same radius and place one over the other, we may easily verify that the lines on the two plates are practically identical, with very slight divergences due to the difference in latitude between the locations for which they were designed. Cf. Casulleras 2010, p. 100.

| Equatorial System | Symmetric Property | Combination Properties |
|---|---|---|
| Right ascension | yes | yes |
| Oblique ascension | yes | no |
| Single Position Semicircle | no | no |
| Single Hour Line | no | no |
| Four Position Circles | yes | yes |
| Standard Houses | no | no |
| Seven Hour Lines | yes | yes |

The two methods of Section 4.1 which take into account non-zero ecliptic latitude do not have the symmetric and combination properties.[79] Any methods based on divisions of the prime vertical circle (cf. Section 4.9) would have the same properties of either the Single Position Semicircle Method or the Four Position Circles Method, depending on their geometrical definition.

The symmetry and combination properties are natural properties for a system for projecting the rays, because these properties reflect the properties of the regular polygons which are used in the definition of the rays. Similar properties hold in the theory of aspects between zodiacal signs, and the analogies with these aspects were often used in the astrological interpretation of the rays. The Simple Ecliptic Method, the Right Ascension Method, the Four Position Circles Method and the Seven Hour Lines Method are, in this sense, natural methods for projecting the rays. Thus we can assume that the Oblique Ascension, Single Position Semicircle and Single Hour Line methods were derived from the corresponding methods for progressions, and that the Standard Houses method was derived from the standard methods for dividing the ecliptic into houses. As for methods based on the use of the prime vertical for the rays, we can presuppose that they would be related to the Prime Vertical Method for the houses, but we can only conjecture about whether these methods were originally designed for the division of houses, the projection of rays, or the progressions (*tasyīr*). These relations will be further discussed in Section 6.

[79] In the method of al-Battānī, a point *P* outside the ecliptic casts its rays to points in the ecliptic. This method has some (but not all) of the combination properties and it does not have the symmetric property. In the method of al-Bīrūnī, the rays are not cast to points on the ecliptic, except the quartile rays. This method has neither the symmetric property nor any of the combination properties.

## 5. Houses

> Die Angriffe unsachverständiger Astrologie-Gegner richten sich mit Vorliebe gegen das Häusersystem, welches als willkürliches, törichtes Gemengsel einfältiger Kombinationen hergestellt wird.
> (Koch 1959, p. 95)

Although the astrological houses are not the main subject of this paper, we will list the systems which are related to the above-mentioned systems for progressions and rays. We use the same terminology as Kennedy (1996, pp. 538-545) and North (1986, pp. 46-47), and we only add references to ancient and medieval sources if they are relevant for our discussion of rays and progressions.

### *5.1 Equatorial (Fixed Boundaries) Method*

This is Kennedy-North, no. 4. Modern astrologers attribute this system to Regiomontanus (1436-1476), an author who owned a Latin translation of Ibn Muʿādh al-Jayyānī's *Tabulae Jahen*.[80] The cusps of the houses are the intersections of the ecliptic with the position circles which divide the celestial equator into equal intervals of 30 degrees (Figure 19).

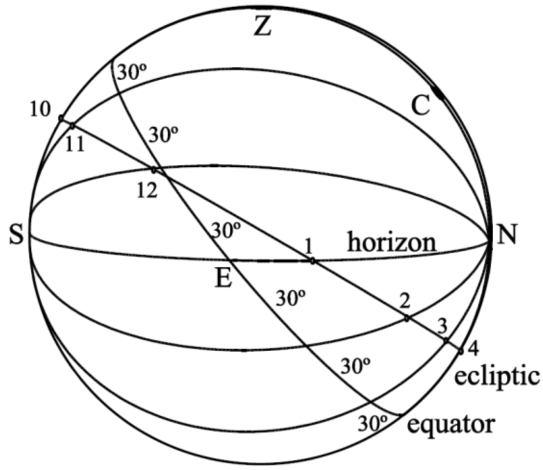


**Figure 19**

[80] North 1987, p. 49.

Ibn Muʿādh mentions this method in both his *Tabulae Jahen* and his treatise on the projection of rays, and he says that it is the correct method because it is analogous to the Four Position Circle method, which he considers as the correct method for the projection of rays (Hogendijk 2005). Thus many later authors attributed this method for the houses to Ibn Muʿādh (or: Abenmoat).[81]

All the astrolabe plates mentioned in Section 3.1.3 above allow for the application of this method for the houses, but the evidence of markings or numbers for the limits of the houses lacks on the oldest one, made in 984-985. Therefore, we can not assume that this plate, which explicitly mentions the projection of rays and the *tasyīr*, was also designed for the division of houses. On the plate of the astrolabe of Aḥmad ibn Ḥusayn ibn Bāṣo, the method is attributed to one al-Ghāfiqī. According to North (1986, p. 65), this individual was Abū'l-Qāsim Aḥmad ibn ʿAbdallāh ibn ʿUmar al-Ghāfiqī ibn al-Ṣaffār (died 1035). Following this identification, it is possible to consider that the method was already known to Ibn al-Ṣaffār, if not before (cf. Hogendijk 2005, p. 98). Perhaps Ibn al-Ṣaffār invented the mathematical method and Ibn Muʿādh gave the philosophical motivation and some mathematical algorithms for the computation. Nevertheless, al-Ghāfiqī is a fairly common *nisba* in al-Andalus (cf. for instance Lirola and Puerta 2004, p. 754: index of *nisba*s) and it seems somewhat risky to take North's identification for granted. Among others, the same *nisba* is attached to the Andalusian mathematician and astronomer Ibn al-Hā'im (Sevilla?, second half of the 12$^{th}$ c – Marrakesh?, first half of the 13$^{th}$ c).[82] Moreover, we can not know whether the "method of al-Ghāfiqī" refers to this solution for the houses or to a more general approach that uses the same geometrical pattern for the projection of rays (or the progressions, cf. Sections 3.1.6 and 4.9 above). Another astrolabe of Aḥmad ibn Ḥusayn ibn Bāṣo, dated in A.D. 1294-1295 (694 of the Hegira) and published in García Franco 1955, pp. 297-309,[83] seems to have four plates with position circles for the division of houses according to the

[81] See North 1986, pp. 35-38.
[82] On this author see Puig 2007, pp. 555-556 and the references there given.
[83] García Franco read the Hegira date on the plate, written in *abjad* numerals, as 664 (A.D. 1265-1266), but Calvo (in Ibn Bāṣo 1993, p. 31: fn. 86) showed that it seems more reasonable to read 694 (see also Vernet and Samsó 1992, p. 225; Eiroa Rodríguez 2006, pp. 69-70, no. 70).

Equatorial Method.[84]

In the thirteenth-century *Libro dell astrolabio redondo*, a work written for king Alphonso X by Rabbī Isḥāq ibn Sīd, the method is described and attributed to Hermes and al-Zarqālluh (died 1100).[85] Traces of this method have yet to been found in the Eastern Islamic world (see Section 6).

### *5.2 Prime Vertical Method*

This is Kennedy-North no. 3. Modern astrologers attribute this system to Campanus of Novara (ca. 1210-1296). The cusps of the houses are the intersections of the ecliptic with the position circles which divide the prime vertical (defined in Section 3.1.6 above) into equal intervals of 30 degrees (Figure 20).

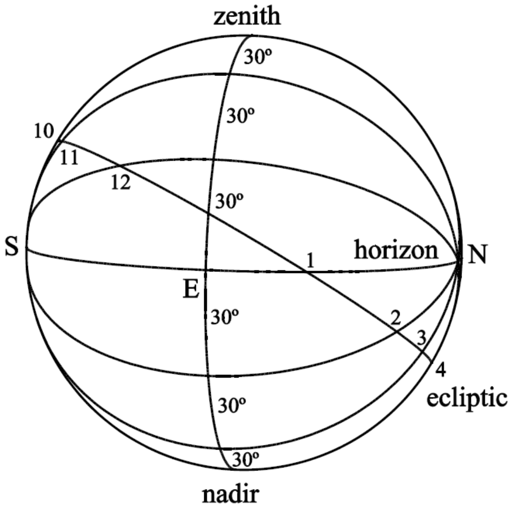


Figure 20

[84] The published photographs are not clear enough to identify all the markings. The published description implies that circles crossing twelve equal divisions of the equator and passing through the North point of the horizon were drawn on these plates. A new inspection of this device, preserved in Madrid, Real Academia de la Historia, would clarify its details.

[85] As pointed out by Calvo (1998, p. 36), the *Libro dell astrolabio redondo* does not give the Prime Vertical Method of Section 5.2 and thus North 1986, p. 34, has to be corrected. The description in the *Libro dell astrolabio redondo* is also explained in Seemann 1925, pp. 15, 28.

This kind of position circles are plotted on the astrolabe plates mentioned in Section 4.9, which have also numbers indicating that are to be used for the division of houses. Another plate related to this method was published on the cover of Comes, Puig and Samsó, 1987. It belongs to an astrolabe that, according to Samsó, can be attributed to one of the Banū Bāṣo: Ḥasan b. Muḥammad b. Bāṣo[86]. This plate is only designed for the division of houses at a latitude of 37º30'. It has the inscription "for the Method of Hermes" and shows the projection of twelve position circles corresponding to the limits of the houses as defined in the Prime Vertical Method.

Ḥusayn Ibn Bāṣo presents a trial-and-error solution according to this method by means of his universal astrolabe plate (1993, Ch. 154, pp. 175-177 Arabic, 199-201 Spanish). He attributes the method to Hermes, and he says that Ibn al-Samḥ also employed it in his *Zīj* using an incorrect computation. This incorrect computation was criticized and corrected by Ibn Muʿādh in his treatise on the projection of rays (Kennedy 1994).

### *5.3 Hour Lines Method*

This is Kennedy-North no. 0, compare Sections 3.1.4, 3.2 and 4.5. Modern astrologers attribute this system to Placidus, that is, Placido de Titi (1590-1668) (North 1986, p. 21). The cusps of the houses are the intersections of the ecliptic with the lines of the even seasonal hours. Thus the cusps of the eleventh and twelfth house lie on the lines of 2 and 4 seasonal hours before noon, as in Figure 21.

This procedure can be performed with any standard astrolabe plate having the lines for the seasonal hours. Nevertheless, some preserved plates bear explicit numbers of the houses, which were engraved in counter-clockwise order along the lines for the even-numbered seasonal hours. Thus these instruments were also constructed for determining the houses by means of the Hour Line Method. This is the case of the two plates of Aḥmad b. Ḥusayn b. Bāṣo's astrolabe mentioned in Section 3.1.4.

The application of an astrolabe to this function is found in several treatises on this instrument (cf. Calvo 1998, p. 36). Ibn al-Samḥ says that Ḥabash [al-Ḥāsib][87] (ninth century) attributed the procedure to Ptolemy (Ibn al-Samḥ

[86] The first note on this astrolabe appeared in Firneis 1987, pp. 229-230. On the identification of the members of the Banū Bāṣo family, see Calvo 2002; Ibn Bāṣo 1993, pp. 23-25.
[87] On this author see Charette 2007.

1986, pp. 66, 124). No trace of any method for determining the houses appears in the extant works of Ptolemy, but Viladrich argues that the reference can be explained by the fact that in the *Tetrabiblos*, Ptolemy mentions hour lines in connection with progressions (Ibn al-Samḥ 1986, p. 67, fn. 140). Ibn al-Ṣaffār[88] (died 1035) describes the technique without any attribution (Millás 1955, pp. 44, 75; Catalan translation in Millàs 1931, pp. 70-71). Abraham b. ʿEzra gives the procedure in three works: his treatise on the astrolabe (Millás 1940, p. 22), *Sefer ha-moledot* (North 1986, pp. 20-27) and *De rationibus* (Millás 1947, pp. 159-161), where he explains that its application is very easy with an astrolabe[89] (cf. Samsó 2012, p. 190). In the framework of the Alfonsine Books, the method is attributed to Ptolemy and Vetius Valens ("Veles") in the treatise on the spherical astrolabe (Seemann 1925, p. 28), and it appears without attribution in the treatise on the flat astrolabe.

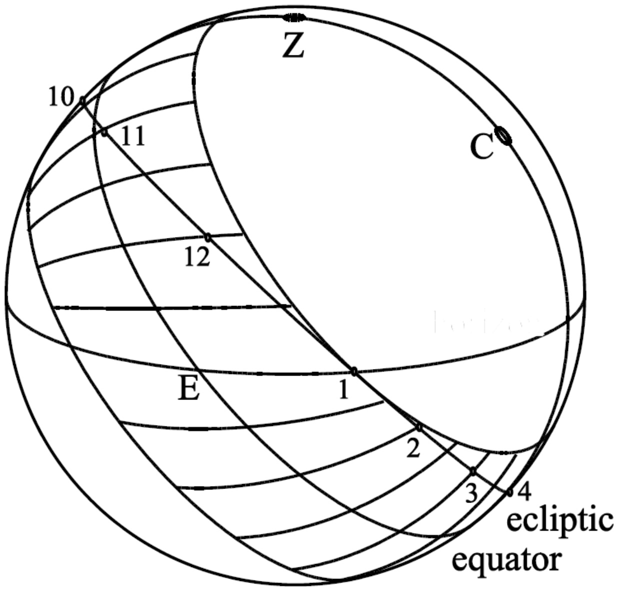


Figure 21

[88] On this author see Rius 2207b.
[89] "Nos vero in astrolabio docuimus facile distinguere domus" (Millás 1947, p. 160; quoted in Samsó 2012, p. 190).

In the end of Section 3.1.4 above reference has been made to the quantities which the astrologer Māshā'allāh (who died ca. A.D. 815) called "ascensions in the houses". For example, the "ascension" of an ecliptical arc $\lambda$ in the eleventh house was defined as $\alpha_0(\lambda) + \frac{1}{3}\ (\alpha_\varphi(\lambda) - \alpha_0(\lambda))$. Now the question arises what system of houses Māshā'allāh used, if this computation was based on any geometric motivation. His "ascensions" in the tenth and the first house are the right and oblique ascensions. These depend on the meridian and horizon plane, so they are derived from the *cusps* of the tenth and first house. If one assumes that this is also true for the other houses, there are good grounds for believing that Māshā'allāh defined the houses by means of the hour lines method.[90]

### *5.3.1 Split Differences Method*

This is Kennedy no. 8. Kennedy found this method in the *Zīj* of Ibn al-Raqqām, who worked in Tunis and Granada around 1280. It is as follows:

"Trisect the quadrant of the equator between upper midheaven and the east point of the horizon, this determining two equatorial points. For each of them find two ecliptic points which are its inverse right and oblique ascensions respectively. Trisect both of the ecliptic segments thus determined. Then the initial point of the eleventh house is in the upper segment, a third of the distance from the inverse right ascension to the inverse oblique ascension. The initial point of the twelfth house is in the lower segment, but it is a third of the distance from the inverse oblique ascension to the inverse right ascension" (Kennedy 1996, pp. 544-545, see also p. 568).

We now reconstruct the geometric rationale of this method, using Sections 3.2 and 4.6. In Figure 22, the ecliptic and the celestial equator intersect the meridian above the horizon at $P$ and $Q$ respectively, and the cusp of the eleventh house is $F$. Let $\varphi$ be the geographical latitude of the locality. Since $Q$ is

[90] Let $V$ be the vernal point on the ecliptic and denote the "ascension in the eleventh house" of any ecliptical arc $VL$ as the equatorial arc $VE$. Assume that the boundary between the tenth and eleventh house is defined by some curve $C$. We now give the notion "ascension in the eleventh house" the following geometrical interpretation: if $L$ is on $C$, $E$ must also be on $C$ (note that this interpretation is also valid for Māshā'allāh's ascensions in the tenth and the first house). Using the definition of the "ascension in the eleventh house" one can now prove that $C$ is either the hour line 2 seasonal hours before noon, or a line obtained by rotating this hour line on the sphere around the celestial axis. The second possibility can be excluded on historical grounds.

on the equator, we have $\alpha_\varphi(P) = \alpha_0(P) = \alpha_0(Q)$. The construction of the cusp of the eleventh house can now be rendered thus:

Find points $F_0$ and $F_\varphi$ on the ecliptic such that

$$30^\circ = \alpha_0(F_0) - \alpha_0(P)\,,$$
$$30^\circ = \alpha_\varphi(F_\varphi) - \alpha_\varphi(P)\,.$$

Let $\lambda$ denote ecliptical longitude. Then $F$ is defined by

$$\lambda(F) = \lambda(F_0) + \frac{1}{3}(\lambda(F_\varphi) - \lambda(F_0))\,.$$

These formulas resemble formula (4) in Section 3.2 for $n = 30^\circ$, but instead of an interpolation factor $\frac{d(P)}{6}$ in (4), which belongs to the hour line through $P$, we now have the interpolation factor $\frac{1}{3}$. The following two remarks can be made:

- The interpolation factor $\frac{1}{3}$ corresponds to the hour line two seasonal hours before noon.
- Point $Q$ is on the celestial equator, so if this point is made to "progress" (in the way of Section 3, but in the opposite direction to the daily motion of the universe) to the hour line two seasonal hours before noon, the progression arc $n$ is 30 degrees.

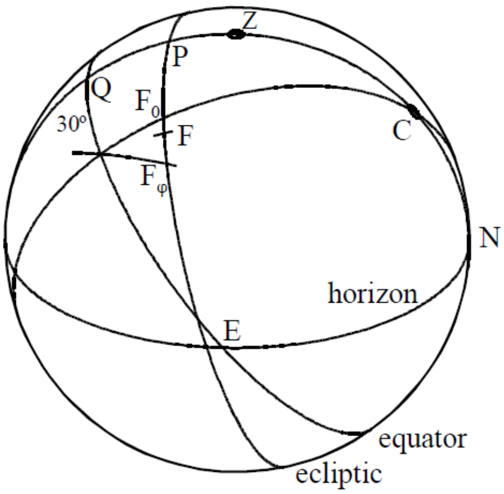


Figure 22

We conclude that Ibn al-Raqqām computed the cusp of the eleventh house as the intersection between the ecliptic and the hour line two seasonal hours before noon. Thus the Split Differences Method is an approximate computation of the houses according to the hour lines method.[91] This confirms North's conjecture in (North 1996, p. 580): "This new method (i.e. the *Split Differences* method of Ibn al-Raqqām) I find interesting because, to my eyes, it looks like an approximative method, rather than a new method".

### *5.4 Standard Method*

Compare Section 4.7 and Figure 17. This system was the most popular one among medieval Islamic authors of astrological texts and tables, and it is therefore called the Standard Method by Kennedy and North (no. 1), and "the well-known method" by al-Bīrūnī (1985, vol. 3, pp. 1357-1359). Through the ascendent $P$ draw a semicircle parallel to the celestial equator with endpoints on the meridian plane. Thus one obtains the half day arc and the half night arc of the ascendent. Trisect these arcs and draw great circles through the division points and the celestial pole $C$. These great circles define the boundaries of the houses. This system is attested in a Greek horoscope by Rhetorius who lived in the fifth century A.D. (North 1986, p. 6), so it is pre-Islamic.

## 6. Final remarks

### *6.1 Ptolemy and Hermes*

In the preceding sections we have seen that many methods for progressions, rays and houses are attributed to either Ptolemy or Hermes. The question arises what these attributions really mean. It is clear that the Hour Line method for progressions was correctly attributed to Ptolemy because it is found in the *Tetrabiblos*. No trace of the Single Hour Line method for rays and the Hour Lines Method for the houses are found in the extant works of Ptolemy.

We think that these attributions are best explained following the suggestion by al-Bīrūnī in Section 4.5 above. In Section 4.10 we have argued that the Hour Line method for rays is unnatural because it does not have the ma-

[91] An exact computation was not feasible because it would have involved the numerical solution of a cubic equation, see North 1986, pp. 22-23.

thematical properties one expects for a sensible theory of rays. The method had probably been adapted from the Hour Line method for progressions. Since that method was the work of Ptolemy, the method for rays was also called "method of Ptolemy". In the same way one can explain why the hour lines method for houses was attributed to Ptolemy. Thus the term "method of Ptolemy" means the same as "using hour lines" and it does not mean "invented by Ptolemy".

The case of the attributions to Hermes seems to be similar. The tenth-century astrologer Ibn Hibintā says: "Hermes said in his book related to 'The Latitude' that the trine, and sextile, and quartile (rays) are made in equal degrees (i.e. on the equator)"[92] (Ibn Hibintā 1987, vol. 1, p. 293, vol. 2, p. 66; Kennedy and Krikorian 1972, p. 13). Hermes is a mythical figure and his astrological "Book of Latitude" is now lost, but the term "method of Hermes" must somehow have become synonymous to "by means of position semicircles". Perhaps the lost book by Hermes on progressions contained the position semicircle method as well.[93] This explains why almost all methods involving position (semi)circles were attributed by at least one medieval Islamic author to Hermes.[94] Thus it is not necessary to assume a pre-Islamic origin of either the Prime Vertical Method or the Equatorial (fixed boundaries) Method of houses.

One may compare the modern usage of "algorithm" for method of computation. This term does not imply that the method in question was invented by al-Khwārizmī.[95]

[92] This is Kennedy's translation. Ibn Hibinta continues that Dorotheus did "the same" (*kadhalika*) and that this means the following: the ecliptic degrees have to be transformed to ascensions, which are used to find the exact position of the ray (presumably on the equator).

[93] On Hermes and his astrological works see Sezgin 1974-1984, vol. 7, pp. 50-58. The work on progressions is p. 58 no. 1.

[94] Note that the position semicircle method for progressions is found in Ptolemy's *Tetrabiblos*, so it should have been attibuted to Ptolemy.

[95] Our interpretation of the attributions to Ptolemy and Hermes is confirmed in Chapter 52 of the Alfonsine *Libro Segundo de las Armellas*. This text establishes, on the one hand, an identity between the 'opinions' of Ibn Muʿādh and Hermes in connection with the methods using position circles or semicircles for the projection of rays and the *tasyīr* and, on the other hand, a relationship between Ptolemy and the methods using hour lines. Moreover, the author also expresses his doubts on the attributions to Ptolemy, stating that they can be derived from mistakes in the translation (or the transmission: the text refers to "yerro en el trasladar"), and that a correct understanding of the *Tetratiblos* shows that the 'opinion' of Ptolemy is most near to that of Ibn Muʿādh: "Et quando entendieres bien el *quarto partido* de Ptolomeo. entendrás que mas tira su opinion á lo que dixo Aben-Mohat. que non á otro." (Rico 1863-1867, vol. 2, p. 66;

### *6.2 Chronological survey of systems and their relations*

We will conclude this paper by a chronological list of some of the systems and their relationships. References to the sources can be found in the sections in which the methods are described.

#### *6.2.1 A.D. 0 - 200*

Most of the systems for progressions date back to the time of Ptolemy (around A.D. 150) or before. In the *Tetrabiblos*, Ptolemy mentioned the Right Ascension system for planets on the meridian, the Oblique Ascension system as the method usual in his time, the Position Semicircle system as the true system, and the Hour Line system as the system to be used in computations. Ptolemy mentions the astrological rays but the *Tetrabiblos* does not indicate how they have to be computed, so he may have used the simple system.[96]

Ibn Hibintā preserved a confused quotation from an astrological work by Dorotheus, who lived in the first century after Christ.[97] We do not understand the details of the quotation but what is clear is that, if the quotation is authentic, Dorotheus used some kind of equatorial theory for the projection of rays, perhaps the Oblique Ascension Method.

Casulleras 2010, pp. 112-115). Consequently, it seems that the Alfonsine author assumes that Ptolemy also intended to use position (semi)circles. Another case of possible identification of the procedures attributed to "Ptolemy and Hermes" is found in Ibn ʿAzzūz, who puts both names together as the source of "the best method and the most correct of what is said" on the projection of rays (cf. Casulleras 2007b, pp. 63-64, 81-82: English and 86-88: Arabic).

[96] The Standard and the Hour Line methods for the houses are also attributed to Ptolemy, and al-Istijī says that Ptolemy made their projections using oblique ascensions, but we should take this kind of attributions cautiously, as they cannot be substantiated from the extant works.

[97] Here is the quotation, with our remarks in square brackets [ ]. "Dorotheus said the same thing, namely: you look at the degree of the planet, then you make the projection of any ray you want in (a degree) equal to the degree of the sign in which (the planet) is [ignore latitude?], then you transform it to the ascension of its sign, [take some kind of ascension] [Here Dorotheus probably stated that 60, 90, etc. degrees should be added or subtracted in a passage which is missing in the text we have] and there will come out for you the exact place of its ray, and who tried it has stated that it is correct, then you transform the degree of the planet itself to the ascensions of its sign, and the place of its body will come out for you. [unclear; does he mean that a point on the celestial equator has to be transformed to a point on the ecliptic by taking some kind of inverse ascension?]" (Ibn Hibintā 1987, vol. 1, p. 293, vol. 2, p. 66; translated in Kennedy and Krikporian 1972, p. 13).

### *6.2.2 Late antiquity*

The works attributed to Hermes and Wālis are of uncertain authorship, but they must have been written in late antiquity in Greek.[98] The quotation from the "Book of Latitude" in Section 6.1 above suggests that "Hermes" also used some equatorial theory for the projection of rays. Therefore it is likely that the position semicircle method had been adapted from progressions to rays at this time. The fact that the Single Hour Line Method for rays and the Hour Lines Method for houses were attributed to "Wālis" shows that these methods were probably known in late antiquity as well. The Standard method for houses is found in a horoscope from late antiquity.

### *6.2.3 Sassanid Iran*

The Right Ascension Method, which is one of the natural methods for casting the rays, was used in pre-Islamic Iran. A reference to the Persians using right ascensions also for the *tasyīr* is found in al-Istijī. Some of the methods listed under "Ninth-century Baghdād" may also be of Iranian origin.

### *6.2.4 Ninth-century Baghdād*

In the eleventh century Ibn Muʿādh presents the Seven Hour Lines Method for the rays as an approximation to the Four Position Circles Method, but we do not know whether these two methods had been created from the beginning as two different methods or as two different solutions for the same geometrical approach. In any case, we find the first trace of this approach for the rays in the tables of al-Khwārizmī, and must have been conceived by one of his contemporaries or predecessors, perhaps under influence of the Iranian right ascension method and the single position semicircle method of late antiquity.

Further research may well lead to a number of surprises. Al-Kindī stated in a lost astrological work that if a planet is at the ascendent, it casts its sextile rays to the cusps of the eleventh and third house, its quartile rays to the intersections of ecliptic and meridian, and its trine rays to the cusps of the fifth and ninth houses.[99] One wonders what systems of rays and houses al-

[98] See Sezgin 1974-1984, vol. 7, pp. 38-41, 50-58.
[99] The source of this information is the treatise by Ibn Muʿādh al-Jayyānī on the projection of rays (Hogendijk 2005, pp. 98-99, 100: Arabic, 102-103: English). Ibn Hibintā presents a simi-

Kindī used. There are four possibilities: (1) the Standard Method for the houses, combined with the Standard Houses Method for the rays; (2) the Hour Lines Method for the houses, combined with the Seven Hour Lines Method for the rays; (3) the Equatorial (fixed boundaries) Method for the houses, combined with the Four Position Circles Method for the rays; and (4) the use of the prime vertical with four position circles, combined with the Prime Vertical Method for the houses. Since Ibn Muʿādh's treatise seems very comprehensive about the different systems for the houses, and he does not mention the use of the standard houses for the rays, the first option may be discarded. It is also unlikely that he was referring to operations on the prime vertical, because the use of this circle for the division of houses is harshly criticized in other passages. The solution of the Seven Hour Lines Method for the rays in the second option corresponds to the method presented by Ibn Muʿādh as an approximation to the Four Position Circles. The remaining possibility is the combination that corresponds to the only methods for the rays and the houses approved by Ibn Muʿādh in his works. In this last case, the Equatorial (fixed boundaries) Method for the houses has a history in the Eastern Islamic world and perhaps the ancient world as well. This is not impossible, for if hour lines are considered as approximations of position circles as in the *Tetrabiblos* of Ptolemy, then the Hour Lines method for the houses of Ḥabash and "Veles" is an approximation of the Equatorial Method.

Ḥabash al-Ḥāsib used the Hour Lines method for the houses. It is likely that he used the same computation as Ibn al-Raqqām.

### *6.2.5 The tenth and eleventh centuries*

Around 950, Abū Jaʿfar al-Khāzin used the standard houses method for the projection of rays.

The end of the tenth or the early eleventh century witnessed the invention of the Prime Vertical Method for the houses by al-Bīrūnī in Iran or Afghanistan and (perhaps independently) by Ibn al-Samḥ in al-Andalus. We also have the first evidence of a design using the prime vertical for the projection of rays in the astrolabe of Toledo A.D. 1029-1030.

If the Equatorial (fixed boundaries) Method was not transmitted from the

lar theory (1987, vol. 1, pp. 291-292 = vol. 2, pp. 64-65). Ibn ʿEzra uses this principle to find the rays by interpolation if the houses are known (Viladrich and Martí 1983, pp. 95-96).

East, it was invented in this period in al-Andalus, where Ibn Muʿādh formulates the first correct computation known for this method.

The computations of the *tasyīr* and the projection of rays along the ecliptic using the simplest methods are strongly defended by al-Istijī.

### *6.2.6 From the eleventh century on*

We do not find any new methods for the astrological practices in our area of study from the eleventh century on, but some authors make interesting compendia of the existing alternatives, proposing new computational approaches (*Libros del Saber*, Ibn al-Raqqām), compiling tables (Ibn ʿAzzūz), or making instruments with plates for a variety of methods (Aḥmad ibn Ḥusayn ibn Bāṣo).

Between the 13th and 17th centuries, Islamic methods for finding the houses became known in Europe under the names of Campanus of Novara, Regiomontanus and Placido de Titi. Thus all traces of the Islamic background of these methods disappeared.

After al-Istijī's defence of the simple ecliptic methods for the progressions and the aspects, their number of supporters increases in the Maghrib (Ibn ʿAzzūz, Ibn Qunfudh, al-Baqqār). The abandon of the astrolabe, an instrument based on the projection of the celestial sphere on the equatorial plane, may have contributed to the modern use by the astrologers of the same simple methods.

### *6.3 Open questions*

In this paper we have been concerned with texts and instruments, and most of the sources which have been discussed throw light on the history of *theoretical* mathematical astrology. It is obvious that the picture which we have sketched will be modified in the light of future research. The study of extant but unpublished or unexplored sources will provide much information on the history of houses, progressions and rays in the ninth to eleventh centuries. The history of these concepts in pre-Islamic Iran is a missing link which will be much more difficult to fill. It is an open question which methods were used by *practicing* astrologers in Islamic civilization. It would be interesting to locate unpublished horoscopes with astrological interpretations in archives (an example is Elwell-Sutton 1977) and then investigate this question in the light of the classifications of methods for the different practices, as has been done by Samsó (1999, 2004 and 2009). Thus one could try to find out which parts of the history described in this paper went beyond the textbooks on mathematical astrology.

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