Exercises on Adverse Selection

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1. A Principal has the following objective function

$$P(e, w) = 16e - w.$$

She may hire an agent that can be of type G or B. Let $\frac{1}{2}$ be the probability that the agent is G (or B). The elementary utility function of a type-G agent is

$$U^G(w, e) = w - e^2,$$

and a type-B agent has utility

$$U^B(w, e) = w - 2e^2.$$

The reservation utility of the agents is zero.

(a) Find the contract or contracts that will be offered by the principal when there is symmetric information.

(b) Find the contract or contracts in case of asymmetric information (adverse selection).

(c) Compute the information rents paid to different types of agents.

(d) Can the shut policy be profitable for the principal?

2. Consider a principal who wants to hire a risk neutral agent for a job. In exchange for the effort e made by the agent, which is observed by the principal, the agent would receive a wage w. Effort level e generates a revenue $x(e) = \sqrt{e}$ to the principal. This agent can be of two types: either A with probability q, or B with probability $1-q$. Agents' utilities are given:

$$U^A(w, e) = w - e \quad \text{and} \quad U^B(w, e) = w - 2e$$

(a) Find the contract(s) offered by P if she can distinguish the type of the agent.

(b) Find the contract(s) offered by P in the case of adverse selection and provide an explanation. How do the contracts vary as a function of q?

3. Monopolistic Pricing with Hidden Consumers' Types. A monopolist can produce a good in different qualities. The cost of producing a unit of quality s is s^2. Consumers buy at most one unit of the good and have utility function:

$$u(q, \theta) = \begin{cases} \theta s & \text{if they consume one unit of quality } s \\ 0 & \text{if they do not consume} \end{cases}$$

where θ is the valuation of the good.

The monopolist decides on the qualities it is going to produce and the prices p. Consumers observe qualities and prices and decide which quality to buy if at all.

(a) Characterize the first-best solution.

(b) Suppose that the monopolist cannot observe θ and suppose that:

$$\theta = \begin{cases} \theta_H & \text{with probability } 1-\beta \\ \theta_L & \text{with probability } \beta \end{cases}$$

with $\theta_H > \theta_L > 0$. What happens if the monopolist continues to use the first-best price-quality schedule?

(c) Characterize the second-best solution and the consumers' information rents.