Pentaquarks with one heavy antiquark

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Abstract: The aim of this project is to construct a complete classification of all possible ground wave functions of a pentaquark consisting of four light quarks and a heavy antiquark. The existence of such a particle has not been established yet, but the theoretical interest in studying properties of pentaquarks has raised since the discovery, in July 2015, of an exotic baryon consisting of three light quarks (two *up* and one *down*) and a heavy pair *charm-anticharm*. We will study the symmetries of the internal degrees of freedom of flavour, colour and spin by computing the tensor product of irreducible representations of SU(3) and SU(2), and then identify which results correspond to particles that hold the quark model symmetry principles and thus could exist and might be discovered in the future.

I. INTRODUCTION

Since the LHCb collaboration reported the discovery of an exotic baryon consisting of four quarks and an antiquark in July 2015, several papers focusing on various aspects and predictions of pentaquarks have been published. [4]
In this project, we will study the specific case of the pentaquark consisting of four light quarks and an antiquark, $q^4\bar{Q}$, which has not been established yet.
We will focus on the internal properties corresponding to flavour, colour and spin and consider the spatial part to be totally symmetric, so that we make use as much as possible of symmetry principles and we do not introduce any dynamics.
In order to construct the states we will use the fact that, for each quark, an internal property corresponds to a given irreducible representation of a Lie Group ($SU(3)$ for flavour and colour and $SU(2)$ for spin). Then, each internal property of the multiquark system is obtained by computing the tensor product of the four representations, resulting in new irreducible representations which hold specific symmetric properties. When constructing the classification, we need to take into account the combinations of spin, flavour and colour in which the pentaquark wave function is a **colour singlet** and the system of four quarks obeys the **Pauli exclusion principle**, and thus it is totally antisymmetric.
We are interested in the wave functions because they predict the properties of the possible pentaquark states within the quark model.

II. MAIN PROPERTIES OF THE PENTAQUARK

We are going to study the pentaquarks consisting of a heavy antiquark ($\bar{Q}$) and four light quarks (q^4).
Each quark has a baryonic number of $+\frac{1}{3}$ and the antiquark corresponds to $-\frac{1}{3}$; the total baryonic number of the particle is $+1$. Its wave function contains contributions from each light quark connected to:

- Three flavour degrees of freedom *up*, *down* or *strange* (u, d, s), which transform under the fundamental representation of $SU(3)$.
- Three colour degrees of freedom r, g and b, which transform under the fundamental representation of $SU(3)$. The combination of the four light quarks and the antiquark color charge must be a colour singlet so that the state can exist as a free particle.
- Two spin degrees of freedom corresponding to a total spin of $\frac{1}{2}$, which transform under the fundamental representation of $SU(2)$.

The total parity of the pentaquark will be given by the equation:

$$P = P_{\bar{Q}} \cdot P_q \cdot P_q \cdot P_q \cdot P_q \cdot (-1)^L, \tag{1}$$

where $P_q(P_{\bar{Q}})$ is the parity of the quark(antiquark) wave function and L is the sum of relative angular momentum of the light quarks regarding the heavy antiquark. We will consider it to be 0 (ground state) so that the spatial wave function is totally symmetric. Then we need the ground state to have negative parity so that the Pauli exclusion principle holds.

III. SYSTEM OF FOUR LIGHT QUARKS

We associate each internal property of a quark with a tensor in $SU(3)$ for colour and flavour and $SU(2)$ for spin. Then, we combine the four quarks by computing the tensor product and obtain a direct sum of several irreducible representations.

A. Young Tableaux and Clebsch-Gordan decomposition

The irreducible representations of $SU(n)$ with m indices are associated to the irreducible representations of

the permutation group S_m because of their connection with symmetries. **Young Tableaux** are a useful tool to identify the dimension and the symmetry of the representations of the permutation group and thus we can use them to label and compute the tensor product between $SU(n)$ irreducible representations.
In a Young Tableaux, a row represents a multiplet of symmetric combinations and a column an antisymmetric multiplet. All other configurations are of mixed symmetry.
When working with Young Tableaux, we need to take the following rules into account:

- As in $SU(n)$, a Tableaux with more than n boxes in any column will vanish.
- Tableaux which are the same except for a column with n boxes correspond to the same irreducible representation. Such a column corresponds to the totally antisymmetric tensor ϵ.
- (**Clebsch-Gordan decomposition**) When computing the tensor product of two Tableaux we will label the rows of the second one by ordered numbers. Then, we will add the boxes from the second Tableaux to the first one in all possible ways which hold that, when reading the obtained Tableaux along the rows from right to left from top row down to the bottom row, the number of $1's$ must be equal or greater than the number of $2's$, which must be equal or greater than the number of $3's$ and so on. The result of the tensor product will be the direct sum of all tensors corresponding to the Young Tableaux obtained by this method.
- To calculate the dimension of an irreducible representation from its Young Tableaux, we need to introduce the *factors over hooks* rule, which is a special case of Weyl's character formula. For $SU(n)$, put an n on the upper left hand box of the Tableaux, add one when moving right and subtract one when moving down. The product of all these factors is F. We associate a hook to each box of a Young Tableaux i. A **hook** contains the given box together with all those boxes that are to the right in the same row and lower in the same column. The number of boxes is called the **hook length** l. The dimension of the representation is:

$$h_\nu = \frac{F}{\prod_i l_i} \tag{2}$$

For **SU(3)**,

$$\boxed{1}\otimes\boxed{2}\otimes\boxed{3}\otimes\boxed{4}$$

we obtain the direct sum of the Young Tableaux from TABLE I.

Computing the dimensions, we get:

$$3\otimes3\otimes3\otimes3 = 15_{ts}\oplus15_a\oplus15_a\oplus15_a\oplus\bar{6}_{sa}\oplus\bar{6}_{sa}\oplus3_s\oplus3_s\oplus3_s \tag{3}$$

[4], **15**	$\begin{ytableau} 1 & 2 & 3 & 4 \end{ytableau}$ (ts)		
[31], **15**	$\begin{ytableau} 1 & 2 & 3 \\ 4 \end{ytableau}$ (a4)	$\begin{ytableau} 1 & 2 & 4 \\ 3 \end{ytableau}$ (a3)	$\begin{ytableau} 1 & 3 & 4 \\ 2 \end{ytableau}$ (a2)
[22], **6**	$\begin{ytableau} 1 & 2 \\ 3 & 4 \end{ytableau}$ (sa12)	$\begin{ytableau} 1 & 3 \\ 2 & 4 \end{ytableau}$ (sa13)	
[211], **3**	$\begin{ytableau} 1 & 4 \\ 2 \\ 3 \end{ytableau}$ (s4)	$\begin{ytableau} 1 & 3 \\ 2 \\ 4 \end{ytableau}$ (s3)	$\begin{ytableau} 1 & 2 \\ 3 \\ 4 \end{ytableau}$ (s2)

TABLE I: Young Tableaux label and dimension for the Yamanouchi basis vectors for $SU(3)$.

This is the general result for SU(3), which is valid to describe the **flavour states**. As for the **colour states**, we must impose that, when adding the antiquark to the resulting representation, we obtain the totally antisymmetric tensor. The antiquark corresponds to the Young Tableaux $[1^{n-1}]$:

$$\begin{ytableau} \; \\ \; \end{ytableau}$$

which only combined with the [211] Young Tableaux leads to the colour singlet. Therefore, we will only consider representations of dimension 3_s for the four light quarks as colour states.

[4], **5**	$\begin{ytableau} 1 & 2 & 3 & 4 \end{ytableau}$ (ts)		
[31], **3**	$\begin{ytableau} 1 & 2 & 3 \\ 4 \end{ytableau}$ (a4)	$\begin{ytableau} 1 & 2 & 4 \\ 3 \end{ytableau}$ (a3)	$\begin{ytableau} 1 & 3 & 4 \\ 2 \end{ytableau}$ (a2)
[22], **1**	$\begin{ytableau} 1 & 2 \\ 3 & 4 \end{ytableau}$ (sa12)	$\begin{ytableau} 1 & 3 \\ 2 & 4 \end{ytableau}$ (sa13)	

TABLE II: Young Tableaux label and dimension for the Yamanouchi basis vectors for $SU(2)$.

For **SU(2)**,

$$\boxed{1}\otimes\boxed{2}\otimes\boxed{3}\otimes\boxed{4}$$

we obtain the direct sum of the Young Tableaux from TABLE II.

Computing the dimensions, we get:

$$2\otimes2\otimes2\otimes2 = 5_{ts}\oplus3_a\oplus3_a\oplus3_a\oplus1_{sa}\oplus1_{sa} \tag{4}$$

Note that if we label each representation by the the **total spin** of the system we get:

$$\frac{1}{2}\otimes\frac{1}{2}\otimes\frac{1}{2}\otimes\frac{1}{2} = 2\oplus1\oplus1\oplus1\oplus0\oplus0 \tag{5}$$

IV. THE PENTAQUARK WAVE FUNCTIONS

The next step is to identify the combinations of the flavour, spin and colour states obtained in Section III A so that the system of the four light quarks is described by a totally antisymmetric wave function.

A. Representation Theory of S_n: the Yamanouchi Basis

In order to check that the wave function is antisymmetric under any exchange of the four quarks, we will need some results of Representation Theory.
As mentioned above, the inequivalent irreducible representations of S_n, or the labelled Young tableaux, match with a given symmetry between quarks. We will name the labelled Young tableaux **Yamanouchi basis vectors**. A **partition** will denote a Young Tableaux with no label, which corresponds to a given state.

For example, the Yamanouchi basis vector $Y_m^{[\nu]}$ denotes the partition $[\nu]$, labelled in a way m, where there is $\frac{n!}{\prod_i l_i}$ possibilities for m.

We know that the permutation group, S_4, is generated by the adjacent transpositions $(12), (23), (34)$, so we just need to prove that the total wave function is antisymmetric under the effect of any of the generators of the group.

A Yamanouchi basis vector has no symmetry between non-adjacent indices that are at the same row or column or between adjacent indices from different row and column. That is why we need to define the **Yamanouchi matrix elements**, which will allow us to study all mixed symmetries under the effect of the generators of the permutation group S_n. We will define the Yamanouchi matrix elements as follows:

1. $$(i-1, i) \mid Y_m^{[\nu]}\rangle = \pm \mid Y_m^{[\nu]}\rangle \tag{6}$$

   when $i-1$ and i are in the same row (+) or same column (-).

2. $$\langle Y_{m'}^{[\nu]} \mid (i-1, i) \mid Y_m^{[\nu]}\rangle = \begin{cases} \frac{1}{\sigma} & \text{if } m' = m \\ \sqrt{\frac{\sigma^2-1}{|\sigma|}} & \text{if } Y_{m'}^{[\nu]} = (i-1, i) Y_m^{[\nu]} \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

   when $i-1$ and i are not in the same row or column. σ is the **axial distance** from $i-1$ to i in $Y_m^{[\nu]}$ and it is defined to be:

   $$\sigma = c_i - c_{i-1} - (r_i - r_{i-1}) \tag{8}$$

   where $r_i, r_{i-1}, c_i, c_{i-1}$ are the row and columns numbers of the letters i and $i-1$ respectively in the Young Tableaux $Y_m^{[\nu]}$.

In (A.I) we can find the Yamanouchi matrices for the partitions we are interested in.

B. Calculation of the wave functions

The way to proceed is:

1. Assume the wave function ψ^A contains all possible combinations of the Yamanouchi basis vectors of given partitions (states) of spin, flavour and colour. For example:

   $$\psi^A = \sum_{\substack{i \in a4,a3,a2 \\ j \in a4,a3,a2 \\ k \in s2,s3,s4}} \alpha_{i,j,k} \psi_i^F \psi_j^S \psi_k^C \tag{9}$$

2. Apply the permutation (12) to ψ^A by using the matrices from (A.I) corresponding to each partition of flavour, spin and color. Get rid of the elements of ψ^A which are not antisymmetric. Continuing with the previous example, we are left with:

$$\begin{aligned} \psi^A =+ \quad & \alpha_{a2,a4,s2}\psi_{a2}^F\psi_{a4}^S\psi_{s2}^C + \quad \alpha_{a4,a4,s3}\psi_{a4}^F\psi_{a4}^S\psi_{s3}^C \\ + \quad & \alpha_{a4,a4,s4}\psi_{a2}^F\psi_{a4}^S\psi_{s4}^C + \quad \alpha_{a3,a3,s3}\psi_{a3}^F\psi_{a3}^S\psi_{s3}^C \\ + \quad & \alpha_{a3,a3,s4}\psi_{a3}^F\psi_{a3}^S\psi_{s4}^C + \quad \alpha_{a2,a2,s3}\psi_{a2}^F\psi_{a2}^S\psi_{s3}^C \\ + \quad & \alpha_{a2,a2,s4}\psi_{a2}^F\psi_{a2}^S\psi_{s4}^C + \quad \alpha_{a4,a3,s3}\psi_{a4}^F\psi_{a3}^S\psi_{s3}^C \\ + \quad & \alpha_{a4,a3,s4}\psi_{a4}^F\psi_{a3}^S\psi_{s4}^C + \quad \alpha_{a3,a2,s2}\psi_{a3}^F\psi_{a2}^S\psi_{s2}^C \\ + \quad & \alpha_{a2,a3,s2}\psi_{a2}^F\psi_{a3}^S\psi_{s2}^C + \quad \alpha_{a4,a2,s2}\psi_{a4}^F\psi_{a2}^S\psi_{s2}^C \\ + \quad & \alpha_{a3,a4,s3}\psi_{a3}^F\psi_{a4}^S\psi_{s3}^C + \quad \alpha_{a3,a4,s4}\psi_{a3}^F\psi_{a4}^S\psi_{s4}^C \end{aligned} \tag{10}$$

3. Apply the permutation (23) and find the combinations of the coefficients $\alpha_{i,j,k}$ such that the result is antisymmetric under the exchange $2 \leftrightarrow 3$ for all elements of ψ^A. In the previous example, we are left with:

$$\begin{aligned} \psi^A =+ \quad & \alpha\psi_{a2}^F\psi_{a4}^S\psi_{s2}^C - \quad \alpha\psi_{a3}^F\psi_{a4}^S\psi_{s3}^C + \quad \beta\psi_{a2}^F\psi_{a2}^S\psi_{s4}^C \\ + \quad & \beta\psi_{a3}^F\psi_{a3}^S\psi_{s4}^C + \quad \gamma\psi_{a4}^F\psi_{a3}^S\psi_{s3}^C - \quad \gamma\psi_{a4}^F\psi_{a2}^S\psi_{s2}^C \\ + \quad & \lambda\psi_{a3}^F\psi_{a3}^S\psi_{s3}^C - \quad \lambda\psi_{a2}^F\psi_{a2}^S\psi_{s3}^C + \quad \lambda\psi_{a2}^F\psi_{a3}^S\psi_{s2}^C \\ + \quad & \lambda\psi_{a3}^F\psi_{a2}^S\psi_{s2}^C + \quad \delta\psi_{a4}^F\psi_{a4}^S\psi_{s4}^C \end{aligned} \tag{11}$$

4. Finally, apply the permutation (34) and find the value of the coefficients such that the wave function is antisymmetric under this transposition too. In the previous example, we are left with the following

normalized wave function:

$$\begin{aligned}\psi^A =& + \frac{1}{3}\big(\frac{1}{\sqrt{2}}\psi^F_{a2}\psi^S_{a4}\psi^C_{s2} - \frac{1}{\sqrt{2}}\psi^F_{a3}\psi^S_{a4}\psi^C_{s3} \\ &+ \frac{1}{\sqrt{2}}\psi^F_{a2}\psi^S_{a2}\psi^C_{s4} + \frac{1}{\sqrt{2}}\psi^F_{a3}\psi^S_{a3}\psi^C_{s4} - \frac{1}{\sqrt{2}}\psi^F_{a4}\psi^S_{a3}\psi^C_{s3} \\ &+ \frac{1}{\sqrt{2}}\psi^F_{a4}\psi^S_{a2}\psi^C_{s2} + \psi^F_{a3}\psi^S_{a3}\psi^C_{s3} - \psi^F_{a2}\psi^S_{a2}\psi^C_{s3} \\ &+ \psi^F_{a2}\psi^S_{a3}\psi^C_{s2} + \psi^F_{a3}\psi^S_{a2}\psi^C_{s2} - \sqrt{2}\psi^F_{a4}\psi^S_{a4}\psi^C_{s4}\big)\end{aligned} \tag{12}$$

C. Results

By the procedure described in Section IV B we have identified seven possible ground state wave functions with their corresponding spin and flavour states for the four quark system:

1. **Spin** 1**, Flavour** 15_{ts}

$$\psi^A = + \frac{1}{\sqrt{3}}\left(\psi^F_{ts}\psi^S_{a4}\psi^C_{s4} + \psi^F_{ts}\psi^S_{a3}\psi^C_{s3} + \psi^F_{ts}\psi^S_{a2}\psi^C_{s2}\right) \tag{13}$$

2. **Spin** 2**, Flavour** 15_a

$$\psi^A = + \frac{1}{\sqrt{3}}\left(\psi^S_{ts}\psi^F_{a4}\psi^C_{s4} + \psi^S_{ts}\psi^F_{a3}\psi^C_{s3} + \psi^S_{ts}\psi^F_{a2}\psi^C_{s2}\right) \tag{14}$$

3. **Spin** 1**, Flavour** 15_a

$$\begin{aligned}\psi^A =& + \frac{1}{3}\big(\frac{1}{\sqrt{2}}\psi^F_{a2}\psi^S_{a4}\psi^C_{s2} - \frac{1}{\sqrt{2}}\psi^F_{a3}\psi^S_{a4}\psi^C_{s3} \\ &+ \frac{1}{\sqrt{2}}\psi^F_{a2}\psi^S_{a2}\psi^C_{s4} + \frac{1}{\sqrt{2}}\psi^F_{a3}\psi^S_{a3}\psi^C_{s4} - \frac{1}{\sqrt{2}}\psi^F_{a4}\psi^S_{a3}\psi^C_{s3} \\ &+ \frac{1}{\sqrt{2}}\psi^F_{a4}\psi^S_{a2}\psi^C_{s2} + \psi^F_{a3}\psi^S_{a3}\psi^C_{s3} - \psi^F_{a2}\psi^S_{a2}\psi^C_{s3} \\ &+ \psi^F_{a2}\psi^S_{a3}\psi^C_{s2} + \psi^F_{a3}\psi^S_{a2}\psi^C_{s2} - \sqrt{2}\psi^F_{a4}\psi^S_{a4}\psi^C_{s4}\big)\end{aligned} \tag{15}$$

4. **Spin** 0**, Flavour** 15_a

$$\begin{aligned}\psi^A =& + \frac{1}{\sqrt{12}}\big(\psi^S_{sa12}\psi^F_{a2}\psi^C_{s2} + \sqrt{2}\psi^S_{sa12}\psi^F_{a4}\psi^C_{s3} \\ &+ \psi^S_{sa12}\psi^F_{a3}\psi^C_{s3} - \sqrt{2}\psi^S_{sa12}\psi^F_{a3}\psi^C_{s4} - \sqrt{2}\psi^S_{sa13}\psi^F_{a4}\psi^C_{s2} \\ &+ \psi^S_{sa13}\psi^F_{a3}\psi^C_{s2} - \psi^S_{sa13}\psi^F_{a2}\psi^C_{s3} - \sqrt{2}\psi^S_{sa13}\psi^F_{a2}\psi^C_{s4}\big)\end{aligned} \tag{16}$$

5. **Spin** 1**, Flavour** $\bar{6}_{sa}$

$$\begin{aligned}\psi^A =& + \frac{1}{\sqrt{12}}\big(\psi^F_{sa12}\psi^S_{a2}\psi^C_{s2} + \sqrt{2}\psi^F_{sa12}\psi^S_{a4}\psi^C_{s3} \\ &+ \psi^F_{sa12}\psi^S_{a3}\psi^C_{s3} - \sqrt{2}\psi^F_{sa12}\psi^S_{a3}\psi^C_{s4} - \sqrt{2}\psi^F_{sa13}\psi^S_{a4}\psi^C_{s2} \\ &+ \psi^F_{sa13}\psi^S_{a3}\psi^C_{s2} - \psi^F_{sa13}\psi^S_{a2}\psi^C_{s3} - \sqrt{2}\psi^F_{sa13}\psi^S_{a2}\psi^C_{s4}\big)\end{aligned} \tag{17}$$

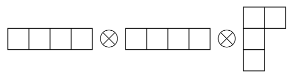
6. **Spin** 1**, Flavour** 3_s

$$\begin{aligned}\psi^A =& + \frac{1}{\sqrt{6}}\big(\psi^F_{s2}\psi^S_{a4}\psi^C_{s3} - \psi^F_{s3}\psi^S_{a4}\psi^C_{s2} \\ &+ \psi^F_{s2}\psi^S_{a3}\psi^C_{s4} + \psi^F_{s3}\psi^S_{a2}\psi^C_{s4} - \psi^F_{s4}\psi^S_{a3}\psi^C_{s2} \\ &- \psi^F_{s4}\psi^S_{a2}\psi^C_{s3}\big)\end{aligned} \tag{18}$$

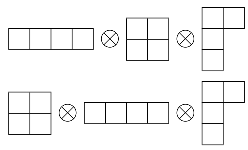
7. **Spin** 0**, Flavour** 3_s

$$\begin{aligned}\psi^A =& + \frac{1}{\sqrt{12}}\big(\sqrt{2}\psi^F_{s4}\psi^S_{sa12}\psi^C_{s2} + \sqrt{2}\psi^F_{s4}\psi^S_{sa13}\psi^C_{s3} \\ &+ \sqrt{2}\psi^F_{s2}\psi^S_{sa12}\psi^C_{s4} - \sqrt{2}\psi^F_{s3}\psi^S_{sa13}\psi^C_{s4} + \psi^F_{s2}\psi^S_{sa12}\psi^C_{s3} \\ &+ \psi^F_{s2}\psi^S_{sa13}\psi^C_{s2} + \psi^F_{s3}\psi^S_{sa12}\psi^C_{s2} - \psi^F_{s3}\psi^S_{sa13}\psi^C_{s3}\big)\end{aligned} \tag{19}$$

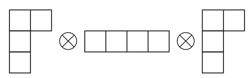
In order to prove that this classification is complete, we need to prove that the ground wave functions described are the only possible pentaquark states, i.e. any other combination violates the Pauli exclusion principle. The other possible combinations are the following combinations of flavour, spin and color representations respectively:

- 

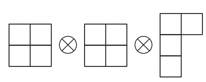
  The combination of flavour [4] and spin [4] is completely symmetric and thus when we combine the three representations we get the same symmetry as the colour [211] representation, which is not completely antisymmetric.

- 

  If we disregard the [4] completely symmetric representation we are left with the combination of two representations: a [211], which has completely antisymmetric properties only when combined with its conjugate [31], and a [22]. Thus, the result is not completely antisymmetric.

- 

  By the same argument as the combination above the result violates the Pauli exclusion principle.

- 

  When we apply the procedure described in section IV B to this combination of representations, we are left with the following wave function:

$$\begin{aligned}\psi^A =& - \sqrt{2}\psi^F_{sa12}\psi^S_{sa12}\psi^C_{s3} + \psi^F_{sa12}\psi^S_{sa12}\psi^C_{s4} \\ &- \sqrt{2}\psi^F_{s13}\psi^S_{sa13}\psi^C_{s3} + \psi^F_{sa13}\psi^S_{sa13}\psi^C_{s4}\end{aligned} \tag{20}$$

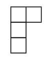
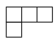
which is not antisymmetric under the permutation (23). Therefore, this state violates the Pauli exclusion principle.

Note that the dimension of the flavour symmetry state indicates the number of different pentaquarks that have the corresponding wave function. We also need to consider the spin of the heavy antiquark to obtain the total spin of the particle. When adding the spin $\pm\frac{1}{2}$ we can obtain two different states which will have approximately the same mass, as the interaction between the light quarks and the heavy antiquark does not depend on the heavy antiquark spin.

V. CONCLUSIONS

In this project, we have developed a mathematical method to identify the ground wave functions of the pentaquarks containing one heavy antiquark and four light quarks only in terms of the permutation symmetry properties regarding colour, spin and flavour.
The classification is complete and relates each ground wave function to the internal properties of the particle.
The results obtained are useful to identify detected particles. From the flavour symmetry we can obtain the relation between the masses of the different multiplets of isospin of the four quark system, i.e, the relation between the masses of the pentaquarks corresponding to a given wave function. They differ because of the heavier mass of the *strange* quark and that is computed through the *Gell-Mann-Okubo* formula. It is also useful to know the combinations of flavour and spin because then we can infer the spin, which can be difficult to measure, by knowing that the flavour must be conserved.
Another interesting question would be if any of these states is the result of the combination of a baryon (qqq) and a meson ($q\bar{q}$). Any baryon will have a completely antisymmetric wave function obtained from the combination of partitions from (A.II). When a given baryon is combined with a meson, the symmetry of the irreducible representations we obtain is restricted even though we get states which match the spin and flavour properties of our results. We can not construct the ground wave functions that we obtained in this project and therefore they are genuinely exotic states.

VI. APPENDIX

	(12)	(23)	(34)
	$\begin{pmatrix}1&0&0\\0&-1&0\\0&0&-1\end{pmatrix}$	$\begin{pmatrix}-\frac{1}{2}&\frac{\sqrt{3}}{2}&0\\\frac{\sqrt{3}}{2}&\frac{1}{2}&0\\0&0&-1\end{pmatrix}$	$\begin{pmatrix}-1&0&0\\0&-\frac{1}{3}&\frac{\sqrt{8}}{3}\\0&\frac{\sqrt{8}}{3}&\frac{1}{3}\end{pmatrix}$
	$\begin{pmatrix}1&0&0\\0&1&0\\0&0&-1\end{pmatrix}$	$\begin{pmatrix}1&0&0\\0&-\frac{1}{2}&\frac{\sqrt{3}}{2}\\0&\frac{\sqrt{3}}{2}&\frac{1}{2}\end{pmatrix}$	$\begin{pmatrix}-\frac{1}{3}&\frac{\sqrt{8}}{3}&0\\\frac{\sqrt{8}}{3}&\frac{1}{3}&0\\0&0&1\end{pmatrix}$
	$\begin{pmatrix}1&0\\0&-1\end{pmatrix}$	$\begin{pmatrix}-\frac{1}{2}&\frac{\sqrt{3}}{2}\\\frac{\sqrt{3}}{2}&\frac{1}{2}\end{pmatrix}$	$\begin{pmatrix}1&0\\0&-1\end{pmatrix}$

(A.I) Yamanouchi matrices in the basis of the Yamanouchi basis vectors.

[3], **10**, **4**	$\boxed{1}\boxed{2}\boxed{3}$ (ts)	
[21], **8**, **2**	$\begin{array}{ll}\boxed{1}&\boxed{2}\\\boxed{3}&\end{array}$ (s2)	$\begin{array}{ll}\boxed{1}&\boxed{3}\\\boxed{2}&\end{array}$ (s3)
[111], **1**, /	$\begin{array}{l}\boxed{1}\\\boxed{2}\\\boxed{3}\end{array}$ (ta)	

(A.II) Young Tableaux label and dimension for $SU(3)$ and $SU(2)$ for the Yamanouchi basis vectors for a baryon qqq.

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