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Tax Systems' Analysis

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PROSPECT THEORY AND TAX EVASION: A RECONSIDERATION OF THE YITZHAKI PUZZLE[*]

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ABSTRACT: The standard expected utility model of tax evasion predicts that evasion is decreasing in the marginal tax rate (the Yitzhaki puzzle). The existing literature disagrees on whether prospect theory overturns the puzzle. We disentangle four distinct elements of prospect theory and find loss aversion and probability weighting to be redundant in respect of the puzzle. Prospect theory fails to reverse the puzzle for various classes of endogenous specification of the reference level. These classes include, as special cases, the most common specifications in the literature. New specifications of the reference level are needed, we conclude.

MAIN RESULT: The standard model of tax evasion predicts that evasion decreases in the marginal tax rate. The literature on prospect theory illustrates cases in which the opposite is true. We provide a deeper analysis of the conditions under which evasion increases in the tax rate when agents behave according to prospect theory. We conclude that the reference income is crucial in determining the relationship between tax rate and evasion, while loss aversion and probability weighting play no role.

JEL Codes: H26, D81, K42

Keywords: Prospect theory, tax evasion, Yitzhaki puzzle, stigma, diminishing sensitivity, reference dependence, endogenous audit probability, endogenous reference level

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1 Introduction

If fines are imposed on the evaded tax, and if taxpayers' preferences satisfy the (theoretically and empirically plausible) assumption of decreasing absolute risk aversion (DARA), then the Expected Utility Theory (EUT) model of tax evasion predicts a negative relationship between tax rates and evasion (Yitzhaki, 1974).[1] Much empirical and experimental evidence, however, finds a positive relationship between evasion and the tax rate (see, e.g., Bernasconi *et al.*, in press, and the references therein).[2] Owing to its lack of empirical support, and its counter-intuitive nature, the negative relationship between tax rates and evasion predicted by the EUT model has sometimes been termed the "Yitzhaki paradox" or "Yitzhaki puzzle".

Prospect Theory (PT) has become a centrepiece of behavioural economics, for it is able to resolve many puzzles associated with EUT and provides a better fit to much empirical data (Bruhin *et al.*, 2010).[3] Our study seeks to (re)-examine whether the insights of PT reverse the Yitzhaki puzzle, as has been claimed in a number of recent papers (see, e.g., Bernasconi and Zanardi, 2004; Dhami and al-Nowaihi, 2007; Trotin, 2012; Yaniv, 1999). In a recent review article, Hashimzade *et al.* (2013: 16) consider some examples that cast doubt on this claim, and conclude that prospect theory does not provide a "compelling" resolution to the puzzle. We investigate this dichotomy: does PT reverse the Yitzhaki puzzle?

To investigate this question requires a general framework in which it is possible to vary (i) the specification of reference income – to understand the role of a dependency on the marginal tax rate and/or on the taxpayer's income declaration; (ii) the elements of PT that are assumed to hold – to separate out the distinct effects of reference-dependence, diminishing sensitivity, loss aversion, and probability weighting; and (iii) the properties of the probability of audit – when fixed exogenously and when a function of the taxpayer's declaration. The framework we use for this purpose – which includes PT as a special case – we term the Reference-Dependent (RD) framework.

When the audit probability is assumed exogenous to the model, our principal contribution is to show that the specifications of reference income in the existing literature belong to a set for which the RD framework (which includes PT) cannot reverse the Yitzhaki puzzle. PT,

[1] For expositions of the EUT model, see Allingham and Sandmo (1972) and Srinivasan (1973).

[2] The empirical evidence is not entirely consistent, however. See, e.g., Feinstein (1991) for a contrasting finding.

[3] PT was initially proposed by Kahneman and Tversky (1979), and subsequently extended to "cumulative" PT by Tversky and Kahneman (1992). In this study we use cumulative PT, but our qualitative conclusions apply equally to the original version of PT. See, e.g., Barberis (2013) and Camerer (2000) for reviews of further applications of PT beyond that to tax evasion.

as conventionally applied, does not resolve the Yitzhaki puzzle. When the audit probability is made endogenous to the model, we show that the RD framework with homogeneous utility – as assumed in Dhami and al-Nowaihi (2007) – cannot overturn the Yitzhaki puzzle either. We square this finding with that of Dhami and al-Nowaihi (2007) – who argue that PT unambiguously reverses the Yitzhaki puzzle – by noting that these authors augment the PT model with an assumed "stigma" cost associated with being caught cheating. When stigma is set to zero in their model, it no longer overturns the Yitzhaki puzzle.

A general feature of the results is sensitivity to the choice of reference level. Consistent with Hashimzade *et al.* (2013), we do find sets of specifications of reference income for which the RD framework does reverse the Yitzhaki puzzle. Interestingly, however, the literature has not so far advanced psychologically plausible specifications of reference income that belong to these sets. By untangling the separate elements of PT, we show the source of the sensitivity to the reference level lies in the multiplicative interaction between endogenous movements in the reference income and the taxpayer's risk preferences. We also uncover that loss aversion and probability weighting play no role in determining whether or not the RD framework overturns the puzzle.[4] Whenever the PT model does reverse the Yitzhaki puzzle, therefore, there always exists a more parsimonious model that does so too.

We do not claim that EUT is descriptively superior or inferior to PT over the full gamete of empirical regularities on tax related behaviour, and other evidence relating to behaviour in risky settings more generally. Our results do, however, lead us to claim that existing approaches to the application of PT to tax evasion fall short in respect of one of the most significant such empirical regularities: that tax evasion is increasing in the marginal tax rate. We conclude that, if the insights of PT can offer a plausible resolution to the Yitzhaki puzzle, then new approaches are needed, particularly regarding the specification of reference income.

The plan of the paper is the following. Section 2 introduces the baseline, EUT model, which (in section 3) we contrast with the RD framework under varying assumptions regarding the specification of reference income, audit probabilities, and the utility function. Section 4 concludes. All proofs are in the Appendix.

[4] Consistent with this finding, Eide (2001) shows that introducing (rank-dependent) probability weighting into the standard tax evasion model changes none of the qualitative comparative statics results.

2 The EUT model

As a springboard for our later analysis, we begin with a development of the standard EUT model. Consider a taxpayer with an exogenous taxable income Y (which is known by the taxpayer but not by the tax authority). The government levies a proportional income tax at marginal rate t on declared income X. The probability of audit is given by $p \in (0,1)$. Following Yitzhaki (1974), audited taxpayers face a fine at rate $f > 1$ on all undeclared tax. The taxpayer's expected utility may be written as

$$V = pv\left(Y^c\right) + [1-p]\, v\left(Y^n\right), \tag{1}$$

where $Y^n = Y - tX$ is the taxpayer's income when not caught, $Y^c = Y^n - tf\,[Y - X]$ is the taxpayer's income when caught (audited), and v is an increasing and strictly concave utility function. The first and second order conditions for a maximum are given by

$$\frac{\partial V}{\partial X} = t\left[p\,[f-1]\, v'\left(Y^c\right) - [1-p]\, v'\left(Y^n\right)\right] = 0; \tag{2}$$

$$\frac{\partial^2 V}{\partial X^2} = D = t^2\left[p\,[f-1]^2\, v''\left(Y^c\right) + [1-p]\, v''\left(Y^n\right)\right] < 0; \tag{3}$$

where the latter is satisfied by the strict concavity of v. The derivative $\partial X/\partial t$, found implicitly from (2), is

$$\frac{\partial X}{\partial t} = t\frac{[1-p]\, Xv''\left(Y^n\right) - p\,[f-1]\,[X + [Y-X]\, f]\, v''\left(Y^c\right)}{-D}. \tag{4}$$

A mode of derivation that shall prove insightful once we move to analysing variants of the RD framework is to add and subtract $t^{-1}D\,[Y-X]$ in the numerator of (4), in which case we obtain

$$\frac{\partial X}{\partial t} = \frac{1}{t}\,[Y-X] - \frac{tY\left[p\,[f-1]\, v''\left(Y^c\right) - [1-p]\, v''\left(Y^n\right)\right]}{-D}, \tag{5}$$

to which application of (2) yields

$$\frac{\partial X}{\partial t} = \frac{1}{t}\left[[Y-X] - \frac{Y\left[A\left(Y^n\right) - A\left(Y^c\right)\right]}{[f-1]\, A\left(Y^c\right) + A\left(Y^n\right)}\right], \tag{6}$$

where $A(x) = -v''(x)/v'(x)$ is the Arrow-Pratt coefficient of absolute risk aversion. If, as is conventional, we assume decreasing absolute risk aversion (DARA), i.e., $A'(x) < 0$ – which implies that $A\left(Y^c\right) > A\left(Y^n\right)$ – equation (6) yields Yitzhaki's (1974) puzzle: *under EUT and DARA, $\partial X/\partial t > 0$ at an interior maximum.* The Yitzhaki puzzle should be understood as a pure income effect. An increase in the tax rate lowers expected income, which, under DARA, makes taxpayers more risk averse. Hence taxpayers find it optimal to evade less.

3 Departures from the EUT model

We now depart from the EUT model. As we shall often wish to employ a stripped-down version of PT – so as to disentangle competing effects – we first introduce variants of a "reference-dependent" framework that each share reference-dependence as a common assumption, but that allow additionally for further elements of PT.

Reference Dependence

Reference dependence can be introduced into the EUT model independently of the remaining elements of PT. This is performed by writing the taxpayers' objective function in (1) as:

$$V_R = pv\left(Y^c - R\right) + \left[1-p\right]v\left(Y^n - R\right), \tag{7}$$

where R is the reference level of income.[5] In order to analyse various different approaches to the setting of reference income in the literature, we will allow R to be a function of the marginal tax only, or to be a function of both the marginal tax rate and the taxpayer's declaration.

Diminishing sensitivity

Diminishing sensitivity cannot meaningfully be introduced into the EUT model independently of reference dependence. In equation (7) it requires utility to be convex when its argument is negative. For $x < 0$, we therefore replace $v\left(x\right)$ with $\underline{v}\left(x\right)$, where $\underline{v}'' > 0$ such that the coefficient of absolute risk aversion is $\underline{A}\left(x\right) < 0$. As is widely noted in the literature, under diminishing sensitivity an interior maximum must satisfy $Y^n - R > 0$, for otherwise the taxpayer's objective function is globally convex.[6] Moreover, if $Y^c - R > 0$, then the results with or without diminishing sensitivity are unchanged. Hence, when examining the RD framework with diminishing sensitivity, we focus on the only interesting case, in which $Y^n > R > Y^c$. In this case we can write the taxpayers' objective function as

$$V_{DS} = p\underline{v}\left(Y^c - R\right) + \left[1-p\right]v\left(Y^n - R\right). \tag{8}$$

The first and second derivatives of (8) with respect to X are given by

[5] For axiomatisations of frameworks that allow for reference dependence separately of the remaining elements of PT see, e.g., Sugden (2003) and Apesteguia and Ballester (2009).

[6] We do not investigate the properties of corner solutions, for the descriptive validity of tax evasion as an all-or-nothing activity appears weak. Note, in particular, that the focus on specifications of reference income that are consistent with interior maxima rules out the choice of reference income as $R = Y$. This specification is, however, allowable once we endogenise p in section 3.3.

$$\frac{\partial V_{DS}}{\partial X} = t\left[p\left[f-1\right]\underline{v}'\left(Y^c-R\right)-\left[1-p\right]v'\left(Y^n-R\right)\right]; \tag{9}$$

$$\frac{\partial^2 V_{DS}}{\partial X^2} = t^2\left[p\left[f-1\right]^2\underline{v}''\left(Y^c\right)+\left[1-p\right]v''\left(Y^n\right)\right]. \tag{10}$$

The second derivative in (10) is ambiguous in sign. The condition for it to be negative cannot be guaranteed by any easily interpretable restriction on the parameters. The second order condition for a maximum may, therefore, not be satisfied. Moreover, under diminishing sensitivity it is possible – because of the possibility of corner solutions – that the first and second order conditions do not describe the solution of the maximisation problem. Local maxima may also arise, so the first order condition may not possess a unique solution.[7]

As these difficulties of the PT model are well understood, we choose to set them aside here. Henceforth, when analysing the RD framework with diminishing sensitivity, we proceed under the maintained assumption that indeed the first order condition describes a unique, and genuinely optimal, interior choice for the taxpayer.

Loss aversion

Loss aversion with respect to a utility function v requires that $-v(-x) > v(x)$ for $x > 0$. Note that this condition necessarily holds if v is strictly concave, hence loss aversion is already implied by the EUT model and by the RD framework with globally concave utility.[8] Loss aversion is no longer guaranteed, however, once reference dependence and diminishing sensitivity are assumed. Under these assumptions, loss aversion holds if $\underline{v}(\cdot)$ is assumed to satisfy $-\underline{v}(-x) > v(x)$ for $x > 0$.

Probability weighting

Probability weighting can be introduced in the EUT model on its own, or in combination with any of the remaining elements of PT. It may be introduced into either of equations (7) or (8) by replacing p with $w(p)$, where $w(0) = 0$, $w(1) = 1$ and $w' > 0$.[9]

[7]See Hashimzade *et al.* (2013) for a detailed discussion of these difficulties.

[8]We use the original definition of loss aversion in Kahneman and Tversky (1979). Unlike this “global” condition, Köbberling and Wakker (2005) propose an alternative “local” definition of loss aversion – which is not satisfied by the EUT model or the RD framework with concave utility – according to which v displays loss aversion if and only if $\lim_{x\uparrow 0}\partial v(x)/\partial x > \lim_{x\downarrow 0}\partial v(x)/\partial x$.

[9]Hence, the objective probability distribution is $(p, 1-p)$ and the transformed probability distribution is $(w(p), 1-w(p))$. PT allows for different weighting functions to apply to outcomes that fall above or below the reference level. As pointed out by Dhami and al-Nowaihi (2007) and Prelec (1998), however, empirically the same weighting function is found to apply above and below the reference level, so we assume there to be a single weighting function w.

3.1 Reference as a function of t

We begin by considering the case in which reference income is a decreasing function of t: $R_t \equiv \partial R/\partial t \leq 0$. Among the specifications of R that satisfy this assumption is the taxpayer's post-tax income if they do not evade (the legal post-tax income): $R = Y[1-t]$. This specification for reference income was first proposed by Dhami and al-Nowaihi (2007) and has subsequently been employed in Trotin (2012). Assuming reference dependence only, the derivative $\partial X/\partial t$ (equation 6) becomes

$$\frac{\partial X}{\partial t} = \frac{1}{t}\left[[Y-X] - \frac{[Y+R_t]\left[A\left(Y^n - R\right) - A\left(Y^c - R\right)\right]}{[f-1]\,A\left(Y^c - R\right) + A\left(Y^n - R\right)}\right]. \tag{11}$$

Reference dependence combined with diminishing sensitivity yields

$$\frac{\partial X}{\partial t} = \frac{1}{t}\left[[Y-X] - \frac{[Y+R_t]\left[A\left(Y^n - R\right) - \underline{A}\left(Y^c - R\right)\right]}{[f-1]\,\underline{A}\left(Y^c - R\right) + A\left(Y^n - R\right)}\right], \tag{12}$$

where the denominator is signed positive by the second-order condition at an interior maximum.[10]

Proposition 1 *Assume $R_t \leq 0$ and $R_X = 0$. Then:*
(i) assuming DARA, there exists a threshold level $\widetilde{R}_t < -Y$ such that, at an interior maximum, $\partial X/\partial t < 0$ for $R_t < \widetilde{R}_t$ and $\partial X/\partial t \geq 0$ for $R_t \geq \widetilde{R}_t$.
(ii) assuming diminishing sensitivity, there exists a threshold level $\widetilde{R}_{t,DS} \in (-Y, 0)$ such that, at an interior maximum, $\partial X/\partial t < 0$ for $R_t > \widetilde{R}_{t,DS}$ and $\partial X/\partial t \geq 0$ for $R_t \leq \widetilde{R}_{t,DS}$.
(iii) parts (i) and (ii) hold if loss aversion and/or probability weighting are additionally assumed.

In the variants of the RD framework considered in parts (i) and (ii) of Proposition 1, the sign of $\partial X/\partial t$ is seen to switch around a threshold value. A key observation that we shall use in our next result, however, is that in part (i) the Yitzhaki puzzle is reversed if reference income is sufficiently *sensitive* to the tax rate (R_t sufficiently negative), whereas in part (ii) the Yitzhaki puzzle is reversed if reference income is sufficiently *insensitive* to the tax rate (R_t sufficiently close to zero).

The intuition for the pattern of switching observed for $\partial X/\partial t$ is straightforward once it is noted that, when reference income is a decreasing function of the tax rate, taxpayers

[10] This point may be seen by substituting the first order condition in equation (9) into equation (10).

need not feel poorer after an increase in the tax rate, for the fall in Y^c and Y^n is offset in the utility function by a fall in R. In part (i), if R is less sensitive to the tax rate than is the expected value of the tax gamble, taxpayers feel poorer (and so more risk averse) and $\partial X/\partial t > 0$. If, however, R responds more to the tax rate than does the expected value of the tax gamble, taxpayers feel richer (relative to the reference income) and $\partial X/\partial t > 0$. In part (ii) the presence of diminishing sensitivity implies that, as taxpayers feel poorer, they become *less* (not more) risk averse. The chain of intuition therefore goes as follows: if R is less sensitive to the tax rate than is the expected value of the tax gamble, the taxpayer feels poorer (and so less risk averse) and $\partial X/\partial t > 0$. If, however, R responds more to the tax rate than does the expected value of the tax gamble, taxpayers feel richer relative (and so more risk averse) and $\partial X/\partial t < 0$. The final part of Proposition 1 clarifies that allowing for loss aversion and probability weighting does not alter these intuitions.

We may now state a straightforward corollary of Proposition 1:

Corollary 1 *(i) Assume $R_X = 0$, and $R_t \in \left(\widetilde{R}_t, \widetilde{R}_{t,DS}\right)$. Then, at an interior maximum, $\partial X/\partial t > 0$ whether or not diminishing sensitivity is assumed.*
(ii) Assume $R = Y[1-t]$, which implies $R_t = -Y$. Then, from equations (11) and (12), $\partial X/\partial t = t^{-1}[Y - X] > 0$ whether or not diminishing sensitivity is assumed.

According to part (i) of Corollary 1, there is an interval of R_t such that reference income is insufficiently sensitive to the tax rate for the RD framework without diminishing sensitivity to resolve the Yitzhaki puzzle, but too sensitive for the RD framework with diminishing sensitivity to resolve the Yitzhaki puzzle. Hence, in this range, whatever the form of v, the RD framework cannot resolve the Yitzhaki puzzle. Part (ii) clarifies, for emphasis, that the result in part (i) applies to the specification of R as the legal post-tax income – as adopted in much of the literature. Note that, in this case, the ability of the RD framework to reverse the Yitzhaki puzzle is strictly weaker than that of the EUT model. The latter can always reverse the puzzle, albeit by invoking the unsatisfactory assumption of increasing absolute risk aversion (and this must be sufficiently strong), whereas the RD framework cannot reverse the puzzle for any choice of preferences consistent with an interior maximum.

The preceding findings have implications for some of the existing literature. First, Bernasconi and Zanardi (2004) reverse the Yitzhaki puzzle in a PT model. Different from the remaining literature, these authors do not specify the reference income, but examine taxpayer behaviour for all possible values of R. By not specifying the reference income, these authors, in effect, assume that R is independent of the marginal tax rate t (and the taxpayer's declaration X), which can be understood in the context of our analysis as corresponding to the case in which $R_t = 0$. In this case we have $R_t = 0 > \widetilde{R}_{t,DS}$, which, by part (ii) of Proposition 1, implies $\partial X/\partial t < 0$. Hence we recover Bernasconi and Zanardi's (2004) finding (although they use the full apparatus of PT, including loss aversion and probability weighting). In light of our analysis, however, we note the role in this (positive) result of not specifying R. As the authors discuss in their conclusion, the predictions of their model vary greatly with the assumed level of reference income, so determining this parameter is unavoidable if the PT model is to yield clear and testable predictions. We show, however, that once a particular specification for R is adopted (even one which is psychologically plausible), the PT model may no longer reverse the Yitzhaki puzzle.

Second, Yaniv (1999) examines a PT model with reference income specified as $R = Y - D$, where D is the amount of an advance tax payment. The advance payment D is specified (up to a constant) as $D = \alpha t b$, implying $R_t = -\alpha b$, where b is the tax authority's estimate of the taxpayer's income (which could under- or over-estimate the true Y), and $\alpha \in [0, 1]$. By Proposition 1, a necessary (and still not sufficient) condition for $\partial X/\partial t < 0$ at an interior maximum is that $R_t > -Y$, which implies $\alpha b < Y$. For this condition to hold for any $\alpha \in [0, 1]$ it must be that $b < Y$. Hence, the Yitzhaki puzzle is resolved in Yaniv's model when the tax authority *under-estimates* a taxpayer's income. A difficulty with this finding is that the empirical evidence on advanced tax payments finds that taxpayers who are under-withheld at filing exhibit lower rates of compliance than those who are over-withheld (e.g., Cox and Plumley, 1988; Chang and Schultz, 1987; Robben *et al.*, 1990), which suggests that a revenue-maximising tax authority always *over-estimates* a taxpayer's income (see, e.g., Elffers and Hessing, 1997). Last, in an unpublished working paper, Trotin (2012) claims (her Proposition 8) to resolve the Yitzhaki puzzle in a PT model with reference income as the taxpayer's legal post-tax income. Corollary 1 shows this claim to be false.[11]

3.2 Reference as a function of t and X

We now turn to the case in which reference income is a function of both the marginal tax rate and the taxpayer's declaration. Although we know of no published application to tax evasion that employs this specification for reference income, it is of interest for at least two reasons. First, Kőszegi and Rabin (2006) make a general argument, designed to be portable

[11]The difference in findings appears due to a non sequitur in the proof of Trotin's Proposition 8. In particular, we are unable to replicate the expression for $\partial \Phi_{\overline{R}}(x^*, t)/\partial t$ in the first line of the proof.

across contexts, that the reference level should reflect the expected outcome of the lottery. If, accordingly, reference income is set as the expected value of the tax gamble,

$$R = pY^c + [1-p]\,Y^n = Y - tX - pft\,[Y-X]\,, \tag{13}$$

then it is a function of both X and t.[12] Second, Hashimzade *et al.* (2013) briefly consider an example in which $R = [1-t]\,X$, but do not draw out the more general implications of allowing for dependency upon X. Note that both specifications of R discussed above satisfy the following properties: (i) at an interior maximum, $R_t < 0$, $R_X < 0$, $R_{XX} = 0$; and (ii) R_X is homogeneous of degree one in t, such that $t^{-1}R_X$ is independent of t.[13] If, accordingly, we endow R with these two properties then an increase in the tax rate affects the declared income in the following way:[14]

$$\frac{\partial X}{\partial t} = \frac{1}{t}\left[[Y-X] - \frac{\phi\,[A\,(Y^n-R) - A\,(Y^c-R)]}{[ft-t-R_X]\,A\,(Y^c-R) + [t+R_X]\,A\,(Y^n-R)}\right], \tag{14}$$

or, for the case of diminishing sensitivity,

$$\frac{\partial X}{\partial t} = \frac{1}{t}\left[[Y-X] - \frac{\phi\,[A\,(Y^n-R) - \underline{A}\,(Y^c-R)]}{[ft-t-R_X]\,\underline{A}\,(Y^c-R) + [t+R_X]\,A\,(Y^n-R)}\right], \tag{15}$$

where $\phi = t\,[Y+R_t] + R_X\,[Y-X]$.

Proposition 2 *Assume $R_t < 0$, $R_X < 0$, $R_{XX} = 0$ and R_X homogeneous of degree one in t. Then parts (i)-(iii) of Proposition 1 hold unchanged, and so does its Corollary 1.*

Proposition 2 is a strong result: it states that additionally allowing reference income to depend upon X (as well as t) in the manner so far considered in the literature leaves the predictive power of the RD framework in respect of the Yitzhaki puzzle entirely unaltered. The proof proceeds by establishing that equations (14) and (15) have identical roots to (11) and (12). Hence, it remains the case that, for any reference level such that $R_t \in \left(\widetilde{R}_t, \widetilde{R}_{t,DS}\right)$, the

[12] This specification for R guarantees that, for any $X \in [0, Y]$, the taxpayer's income is (weakly) below the reference level if caught ($-ft\,[1-p]\,[Y-X] \geq 0$) and (weakly) above the reference level if caught ($pft\,[Y-X] \geq 0$) for any $X \in [0, Y]$. The set of specifications of reference income that possess this property is, therefore, somewhat larger than has hitherto been recognised in the literature (see, e.g., Proposition 3 in Dhami and al-Nowaihi, 2007).

[13] To sign R_X in the case in which R is the expected value of the tax gamble, we make use of the fact that $pf < 1$ at an interior maximum. This is the standard condition that the tax gamble must be better than fair.

[14] We write the FOC as $\partial V/\partial X = t\left[p\left[[f-1] - t^{-1}R_X\right]v'\,(Y^c) - [1-p]\left[1 + t^{-1}R_X\right]v'\,(Y^n)\right] = 0$. As $t^{-1}R_X$ is independent of t, we may then simply apply the steps of Section 2.

RD framework is unable to reverse Yitzhaki's puzzle whether or not diminishing sensitivity is assumed. A straightforward Corollary of Proposition 2 is as follows:

Corollary 2 *If R is the expected value of the gamble, or if $R = [1-t]\,X$ as in Hashimzade et al. (2013), then $\partial X/\partial t > 0$ whether or not diminishing sensitivity is assumed.*

According to Corollary 2 neither of these specifications of reference income can overturn the Yitzhaki puzzle in any variant of the RD framework. Together, the results of sections 3.1 and 3.2 imply that the RD framework does not reverse the Yitzhaki puzzle for any of the specifications of reference income we observe in the literature.

3.3 Endogenous Audit Probability

Suppose now that the probability of audit is not exogenous, but instead depends on declared income.[15] Consistent with the literature on optimal auditing (e.g., Reinganum and Wilde, 1986) we assume that higher income declarations are less likely to be audited ($\partial p/\partial X \leq 0$). The models discussed so far are for the special case of this assumption in which $\partial p/\partial X = 0$. Under this new assumption the analysis becomes more complex and few, if any, general results are possible. We therefore follow Dhami and al-Nowaihi (2007) who analyse a model in which v and $\underline{v}$ are homogeneous of degree $\beta \in [0,1]$ – as in Tversky and Kahneman (1992) – and in which $R = [1-t]\,Y$.[16] Applying this framework in equation (8), but now assuming $p = p\,(X)$, homogeneity implies that

$$V_p = t^{\beta}[Y-X]^{\beta}\left[1-p-p\,[f-1]^{\beta}\right]v(1). \tag{16}$$

The first order condition corresponding to (16) is

$$-t^{\beta}[Y-X]^{\beta-1}\left[[Y-X]\left[1+[f-1]^{\beta}\right]p' + \beta\left[1-p-p\,[f-1]^{\beta}\right]\right]v(1) = 0. \tag{17}$$

From (17) we obtain:

Proposition 3 *Assume endogenous reference dependence, with $R = Y\,[1-t]$, v homogeneous of degree β, and $p' \leq 0$. Then, at an interior maximum, $\partial X/\partial t = 0$.*

[15]Hashimzade *et al.* (2013) discuss this version of the RD model only cursorily in their footnote 5.

[16]The homogeneous form is standard in applications of PT, and is axiomatised under PT by al-Nowaihi *et al.* (2008).

Proposition 3 clarifies that, when reference income is the legal post-tax level of income, the RD framework with homogenous preferences makes the same prediction for the relationship between the tax rate and evasion at an interior maximum, irrespective of whether $p'(X) = 0$ or $p'(X) < 0$ is assumed. In either case, Yitzhaki's puzzle remains. Moreover, Proposition 3 holds irrespective of whether utility is assumed globally concave, or to display diminishing sensitivity. The only distinction of note between the two cases is that, as described by Dhami and al-Nowaihi (2007), for $p'(X) = 0$ the dynamics of the optimum are bang-bang. Hence, except in the special case in which an interior solution is weakly optimal, the RD framework is simply incapable of delivering an interior solution for X. This difficulty is, however, mitigated when $p'(X) < 0$, for the function $p(X)$ can be chosen to make the taxpayer's objective function strictly concave.

How can Proposition 3 be squared with Proposition 4 of Dhami and al-Nowaihi (2007), which these authors interpret as showing that PT resolves the Yitzhaki puzzle? The answer is that these authors allow for a feature additional to those of PT: a "stigma" cost $s > 0$, such that income when caught becomes $Y_s^c = Y^c - s[Y - X]$.[17] Rewriting in our notation the expression for $\partial X/\partial t$ in their equation (8.26), we obtain

$$\frac{\partial X}{\partial t} = -s\frac{\beta\theta[s + [f-1]\,t]^{\beta-1}}{t[Y-X]^{1-\beta}}\left[\beta w(p) - [Y-X]\,w'(p)\,p'\right], \tag{18}$$

where θ is a parameter such that $\theta > 1$ implies loss aversion in their formulation. For $s > 0$, and assuming $p' < 0$, equation (18) indeed yields $\partial X/\partial t < 0$, and this result continues to hold without the assumptions of loss aversion and probability weighting ($w(p) = p$, $\theta = 1$). When stigma is removed from the model ($s = 0$), however, equation (18) yields $\partial X/\partial t = 0$, which accords with our Proposition 3: the Yitzhaki puzzle remains.

PT is not a sufficient condition to resolve the Yitzhaki puzzle in Dhami and al-Nowaihi's model, but is it a necessary condition? That is, can stigma be combined with a theory of decision-making other than PT and still reverse the Yitzhaki puzzle?

Proposition 4 *Assume EUT, stigma, $p' < 0$, and risk neutrality. Then, at an interior maximum, $\partial X/\partial t < 0$.*

Proposition 4 clarifies that, once stigma is introduced into the EUT model, it too may

[17] See Dhami and al-Nowaihi (2007) for a discussion on the empirical relevance of stigma and its use in the tax evasion literature.

readily reverse the Yitzhaki puzzle.[18] Proposition 4 appears of roughly equal generality to Dhami and al-Nowaihi's Proposition 4: the latter may not hold for sufficient deviations from the assumption of homogeneity, while the former may not hold for sufficient deviations from risk neutrality. Overall, therefore, we find no evidence to suggest that the RD framework systematically improves upon the predictions of the EUT model in respect of the Yitzhaki puzzle in this case. As such, Dhami and al-Nowaihi's (2007) result should, in our view, be interpreted as illuminating the role of stigma in reversing the Yitzhaki puzzle – a contribution of potential import in itself – but not as demonstrating a descriptive advantage of PT over EUT in respect of the Yitzhaki puzzle.

Although any positive level of stigma is sufficient to overturn the Yitzhaki puzzle in the EUT model of Proposition 4, much larger levels of stigma must be assumed to resolve a further difficulty with the EUT model: it predicts far more tax evasion than is empirically observed.[19] By contrast – as loss aversion and probability weighting help reduce predicted evasion levels – PT is shown by Dhami and al-Nowaihi (2007) to be able to match empirically observed levels of evasion for much more moderate levels of the parameter s. Thus, it can be argued, the PT model should be preferred to the EUT model on these grounds.[20] We recognise this argument, but note two points. First, the validity or otherwise of this argument is orthogonal to our analysis, which is concerned solely with the ability of models to resolve the Yitzhaki puzzle. Second, it is equally possible to resolve the levels puzzle without resort to either PT or stigma costs. For instance, PT assumes that taxpayers observe the true audit probability p, which is then psychologically exaggerated (for small p) in the decision-making process. An alternative view is that taxpayers face ambiguity over p, the value of which they do not know for sure. Snow and Warren (2005) show that introducing ambiguity over p into the tax evasion model decreases predicted evasion if taxpayers are ambiguity averse. Also, Kleven *et al.* (2011) show that when the EUT model is extended to allow for plausible levels of third-party reporting, the predicted level of compliance falls to levels in line with those observed empirically. The latter explanation can be straightforwardly integrated into the EUT and RD models we consider here, so as to make them consistent with level data, without altering the predictions of these models concerning the Yitzhaki puzzle.

[18] Although we believe Proposition 4 to be new as stated, the idea that stigma can overturn the Yitzhaki puzzle in the EUT model is not. Variations of this idea, but under different assumptions over how stigma enters the taxpayer's objective function, are found in, e.g., al-Nowaihi and Pyle (2000), Dell'Anno (2009), Gordon (1989) and Kim (2003).

[19] See, e.g., Alm *et al.* (1992: footnote 3) for a detailed discussion of the levels puzzle, and al-Nowaihi and Pyle (2000) for the levels of stigma needed to resolve it.

[20] We are grateful to Sanjit Dhami for this point.

4 Conclusion

Albeit with limitations, (see, e.g., Levy and Levy, 2002; List, 2003), PT is widely viewed as the best available description of how people behave in risky settings. Barberis (2013: 73) notes, however, that there are "few well-known and broadly accepted applications of prospect theory in economics." The reason, Barberis argues, is that PT is not straightforward to apply: in particular, the most appropriate choice of the reference level is often unclear.[21]

In this paper we focused on tax evasion and in particular on the Yitzhaki puzzle: the EUT model of tax evasion predicts a decrease in tax evasion when the tax rate increases. Does PT resolve the puzzle? We address this question in a general formulation that encompasses different specifications of reference income and which disentangles the different elements of PT. We find that existing applications of PT to tax compliance do not convincingly resolve the Yitzhaki puzzle. In particular, we show that, in the widely considered version of the tax evasion model in which audit probability is exogenous, popular specifications of reference income imply that PT, and all variants of it encompassed within the RD framework, are incapable of reversing the Yitzhaki puzzle. We demonstrate a similar impossibility result for the case in which audit probability is endogenous.

Barberis's generic point over the difficulty of proper identification of the reference level shines through in the tax evasion context. In particular, when reference income is not specified – as in Bernasconi and Zanardi (2004) – the PT assumption of diminishing sensitivity enables it to reverse the Yitzhaki puzzle. When, however, reference income is a decreasing function of the tax rate – as is the case for the psychologically plausible specifications of reference income advanced so far in the literature – PT readily fails to reverse the Yitzhaki puzzle. In this sense, different views over the interpretation of reference income can yield (very) different outcomes.

What do our findings suggest for the importance of the individual elements of PT? We show in Proposition 1 that diminishing sensitivity is neither necessary nor sufficient for the RD framework to reverse the Yitzhaki puzzle. It is not necessary as Yitzhaki's puzzle can be reversed by endogeneity of reference income alone, and it is not sufficient, as it does not always reverse the puzzle. Curiously, diminishing sensitivity lies behind both the ability of PT to reverse the Yitzhaki puzzle when reference level is not specified, and for its inability to do so for conventional specifications of reference income.

[21] Existing specifications of the value and weighting functions of PT are also problematic (see, e.g., Neilson and Stowe, 2002).

We find that loss aversion and probability weighting are irrelevant in respect of the predictions of the RD framework for the sign of $\partial X/\partial t$. Invoking Occam's razor, we believe that results relating to the Yitzhaki puzzle that have been attributed to "prospect theory" may more properly be interpreted as being attributable to simpler reference-dependent models that contain only a subset of the elements of PT.

We do not take our findings to imply that PT is necessarily unimportant for the tax evasion decision. Indeed, given the range of systematic deviations from EUT that PT can explain, it might be surprising if this were the case. The PT model does reverse the puzzle in some well-defined situations, but, interestingly, these situations are not those that existing applications of PT have argued to be psychologically plausible. If PT is to offer a convincing explanation of the puzzle, new approaches will need to be considered. We see two strands of research that might further illuminate the role of PT. The first is the further investigation of the specification of the reference level: are there psychologically plausible specifications of reference income that satisfy the conditions required for PT to resolve the Yitzhaki puzzle? As reference income must be sufficiently insensitive to the tax rate for PT to reverse the puzzle (Proposition 1) one possibility is to assume an adaptive process for R in an explicitly dynamic framework. In this vein, Bernasconi *et al.* (in press) allow for reference income to adapt over time to changes in the tax rate and show that, under these conditions, PT can predict an upward drift in tax evasion (after an initial fall), following an increase in the tax rate.

Alternatively, it has long been known that taxpayers do not, in the most part, treat the evasion decision as a simple gamble (e.g., Baldry, 1986). Researchers might, therefore, investigate whether PT adds value in combination with other plausible developments of the standard model. For instance, Rablen (2010) introduces PT into a version of the tax evasion model that allows for taxes to fund the provision of a public good. The author shows that reference dependence and diminishing sensitivity are sufficient to overcome a puzzling result that arises under expected utility: that taxpayer evasion is decreasing (increasing) in the tax rate when the public good is over-provided (under-provided). For now, however, with respect to applications of PT, Yitzhaki's puzzle remains a puzzle.

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Appendix

Proof of Proposition 1.

(i) We begin by establishing the existence of a point such that $\partial X/\partial t = 0$. To do this, we first show that $\partial X/\partial t > 0$ when $R_t = 0$: in this case equation (11) becomes

$$\frac{\partial X}{\partial t} = \frac{1}{t}\left[[Y - X] - \frac{Y\left[A\left(Y^n - R\right) - A\left(Y^c - R\right)\right]}{[f-1]\,A\left(Y^c - R\right) + A\left(Y^n - R\right)}\right] > 0. \tag{A.1}$$

We now show that $\partial X/\partial t < 0$ as $R_t \downarrow -\infty$: the first order condition for X is written as

$$\frac{\partial V_{RD}}{\partial X} = t\left[p\,[f-1]\,v'\left(Y^c - R\right) - [1-p]\,v'\left(Y^n - R\right)\right],$$

which does not depend on R_t. Hence we may vary R_t in the right-side of equation (11), holding X (and R) fixed, in which case this expression is seen to be monotonically decreasing and unbounded below as $R_t \downarrow -\infty$. By continuity, there must therefore exist a value $\widetilde{R}_t < 0$ such that $\partial X/\partial t = 0$. It follows, by monotonicity, that $\partial X/\partial t < 0$ for $R_t < \widetilde{R}_t$ and $\partial X/\partial t \geq 0$ for $R_t \geq \widetilde{R}_t$. From equation (11) we may also infer the inequality $\widetilde{R}_t = -Y + [Y - X]\,\frac{[f-1]A(Y^c - R) + A(Y^n - R)}{A(Y^n - R) - A(Y^c - R)} < -Y$.

(ii) Proceeding as in part (i), we now show that $\partial X/\partial t < 0$ when $R_t = 0$: in this case equation (11) becomes

$$\frac{\partial X}{\partial t} = \frac{1}{t}\left[[Y - X] - \frac{Y\left[A\left(Y^n - R\right) - \underline{A}\left(Y^c - R\right)\right]}{[f-1]\,\underline{A}\left(Y^c - R\right) + A\left(Y^n - R\right)}\right], \tag{A.2}$$

which can be rearranged as

$$\frac{\partial X}{\partial t} = \frac{1}{t}\left[\frac{[Y - X]\,f\underline{A}\left(Y^c - R\right) - X\left[A\left(Y^n - R\right) - \underline{A}\left(Y^c - R\right)\right]}{[f-1]\,\underline{A}\left(Y^c - R\right) + A\left(Y^n - R\right)}\right] < 0. \tag{A.3}$$

Similarly to part (i), it can be shown that – as $\partial^2 X/\left[\partial t \partial R_t\right] < 0$ – it holds that $\partial X/\partial t > 0$ as $R_t \downarrow -\infty$. There must, therefore, exist a value $\widetilde{R}_{t,DS} < 0$ such that $\partial X/\partial t = 0$. By monotonicity, it follows that $\partial X/\partial t < 0$ for all $R_t > \widetilde{R}_{t,DS}$, and $\partial X/\partial t \geq 0$ for all $R_t \leq \widetilde{R}_{t,DS}$. Finally, from equation (12) we may also infer the inequality $\widetilde{R}_{t,DS} = -Y + [Y - X]\,\frac{[f-1]\underline{A}(Y^c - R) + A(Y^n - R)}{A(Y^n - R) - \underline{A}(Y^c - R)} > -Y$.

(iii) Introducing loss aversion and/or probability weighting leaves (11) and (12) symbolically unchanged.

■

Proof of Proposition 2. (i) Setting equation (14) to zero and re-arranging for R_t we obtain $\widetilde{R}_t = -Y + [Y - X] \frac{[f-1]A(Y^c-R)+A(Y^n-R)}{A(Y^n-R)-A(Y^c-R)}$.

(ii) Similarly, but using equation (15), we obtain $\widetilde{R}_{t,DS} = -Y + [Y - X] \frac{[f-1]\underline{A}(Y^c-R)+A(Y^n-R)}{A(Y^n-R)-\underline{A}(Y^c-R)}$.

(iii) Introducing loss aversion and/or probability weighting leaves (14) and (15) symbolically unchanged. ■

Proof of Corollary 2. If R is the expected value of the gamble then, from (13), we have $R_t = -pfY + X[pf-1]$ and $R_X = -t[1-pf]$. Hence $\phi = 0$, so $\partial X/\partial t = t^{-1}[Y - X] > 0$. If, alternatively, $R = [1-t]X$ then $R_t = -X$. We know from Proposition 2 that $\partial X/\partial t > 0$ for any $R_t > \widetilde{R}_t$, with $\widetilde{R}_t < -Y$. We have then $R_t = -X > -Y > \widetilde{R}_t$. ■

Proof of Proposition 3. Equation (17), which implicitly defines X, can be rewritten as $[Y - X]\left[1 + [f-1]^\beta\right] p' = -\beta \left[1 - p - p[f-1]^\beta\right]$. As this equality does not depend on t, it holds that $\partial X/\partial t = 0$. ■

Proof of Proposition 4. The objective function under risk neutrality ($v(X) = X$) is given by $V = p[Y^c - s[Y - X]] + [1-p][Y^n]$, from which we obtain the first order condition

$$-p'f[Y - X] + pf - 1 = \frac{[p'[Y - X] - p]s}{t}. \tag{A.4}$$

The derivative of t with respect to X is

$$\frac{\partial X}{\partial t} = -\frac{-p'f[Y - X] + pf - 1}{D}, \tag{A.5}$$

where $D = \partial^2 V/\partial X^2 < 0$. Using (A.4) into (A.5), we obtain

$$\frac{\partial X}{\partial t} = -\frac{[p'[Y - X] - p]s}{tD} = \frac{s}{[s + tf]D} < 0. \tag{A.6}$$

■

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