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The Structure of Justification

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The paper explores a structural account of propositional justification in terms of the notion of being in a position to know and negation. Combined with a non-normal logic for being in a position to know, the account allows for the derivation of plausible principles of justification. The account is neutral on whether justification is grounded in internally individuated mental states, and likewise on whether it is grounded in facts that are already accessible by introspection or reflection alone. To this extent, it is compatible both with internalism and with externalism about justification. Even so, the account allows for the proof of principles that are commonly conceived to depend on an internalist conception of justification. The account likewise coheres both with epistemic contextualism and with its rejection, and is compatible both with the knowledge-first approach and with its rejection. Despite its neutrality on these issues, the account makes propositional justification luminous and so is controversial. However, it proves quite resilient in the light of recent anti-luminosity arguments.

Introduction

This paper explores the prospects for a structural account of propositional justification. The account is structural in at least two respects. First, it helps to negotiate which formal principles govern this notion, using a limited set of axioms and rules. Secondly, it is largely silent on the issue of what grounds such justification, and so on what propositional justification materially consists in. In particular, the account is consistent both with internalism about justification and with its externalist opposition.

The account is furthermore neutral on the debate between epistemic contextualists and their invariantist opponents. It likewise coheres both with Williamson's so-called knowledge-first approach and with its rejection.

The account elucidates justification in terms of the notion of being in a position to know and negation. Being factive, the notion of one's being in a position to know is stronger than the notion of its merely being possible for one to know. The account thus differs from recent attempts to analyse justification in terms of the metaphysical possibility of knowing (Bird 2007; Ichikawa Jenkins 2014).

It validates many intuitively plausible principles of justification as well as certain controversial ones, without compromising its neutrality on the aforementioned issues. Its initial motivation does not depend on any assumptions extraneous to the account itself. However, it makes propositional justification luminous, which conflicts with certain externalist views. Surprisingly, though, the account proves quite resilient in the light of recent anti-luminosity arguments. Accordingly, the jury is still out on whether it ultimately founders just because of its commitment to luminosity.

1. Preliminaries

1.1 Propositional vs doxastic justification

The notion of justification to be elucidated in what follows is one of *propositional* rather than doxastic justification. There may be justification for φ in a given situation, even if no belief in φ is formed in that situation. By contrast, doxastic justification requires a justified belief. Uncontroversially, a justified belief in φ requires that φ itself be justified. Doxastic justification thus implies propositional justification, but not *vice versa*.

The relation between propositional and doxastic justification is often further characterized thus: *x* propositionally justifies φ in a given situation, only if any belief in φ, based on *x* in that situation, is a justified belief.[1] Plausibly, a belief is justified only if it is formed in non-question-begging ways. The alleged implication accordingly presupposes that one's belief in φ, formed on the basis of propositional justification for φ, cannot itself add to the propositional justification that there is for φ. For, one's belief

[1] See Turri (2010) for references.

in φ, based on itself, or on any of one's own prior beliefs in φ, would be formed in question-begging ways (unless φ is, say, the proposition that one believes something).

Plausibly, though, the fact that a competent subject comes to believe φ on the basis of propositional justification for φ in a given situation may add something to the propositional justification available for φ in that situation. After all, the fact that a competent subject deems the justification for φ sufficient for belief in φ is a further datum that counts in φ's favour; and such a datum may make a difference. The necessary amendment is this: *x* propositionally justifies φ in a given situation, only if any belief in φ, non-circularly based on *x* in that situation, is a justified belief. While others may non-circularly base their belief in φ on one's own belief in φ, one typically cannot do so oneself.

1.2 Non-factive vs factive justification

Propositional justification will be assumed to be *non-factive*, where this is to say that we cannot infer, from the fact that φ is justified in a given situation, that φ is true in that situation. In other words, where J is short for 'is justified',

$\mathbf{T_J}$ $\quad J\varphi \rightarrow \varphi$

will fail. Failure of $\mathbf{T_J}$ is consistent with the idea that justification for φ is sometimes grounded in factive states. Accordingly, for all that is here being assumed, φ may be justified in one's situation because one perceives φ in that situation, where 'one perceives φ' entails that φ is true.

Internalism about justification comes in two basic varieties: *mentalism* and *accessibilism* (cf. Conee and Feldman 2001). According to mentalism, justification is always fully grounded in, and hence supervenes upon, the subject's internally individuated mental states. According to accessibilism, justification is always fully grounded in, and so supervenes upon, facts that are introspectively or reflectively accessible to the subject. On the intended conception of the relevant mental states, mentalism is often taken to imply accessibilism. (See, however, Conee and Feldman 2001; Wedgwood 2002: 352.) The converse implication anyway fails, as basic logical truths are reflectively accessible; but arguably, very little besides internally individuated mental states is accessible by mere introspection. Internalism about knowledge then is the view that knowledge requires justification internalistically conceived.

Minimally characterized, externalism about justification is simply the denial of internalism about justification. After Gettier, even internalists concede that knowledge requires that certain external conditions be met. Externalism about knowledge thus is best construed as the denial of internalism about knowledge.

That propositional justification need not always be grounded in factive states is compatible with externalism as thus understood. Certain visible rock formations may justify the claim that there once was volcanic activity nearby, even if these formations did not result from volcanic eruptions but from some freak incident of an altogether different nature. Similarly, perceptible traces of slime on the pavement may justify the claim that the neighbourhood is infested with slugs, and yet on this occasion be the manifestations of an alien fungus. Neither rock formations nor slimy traces supervene upon internally individuated mental states, nor are they already accessible by reflection or introspection alone.

1.3 Justification as a prerequisite of being in a position to know

It will be assumed that one's being in a position to know φ in a given situation requires that φ be propositionally justified in that situation. If we let K be short for 'one is in a position to know' (rather than 'one knows'), we can accordingly lay down

K-J $K\varphi \rightarrow J\varphi$

Plausibly, to the extent that **K-J** holds, knowledge requires doxastic justification. One's knowing φ implies one's being in a position to know φ. It likewise implies one's believing φ. Doxastic justification implies propositional justification. It would be odd if one could not know φ if φ was not justified, and yet could know φ even if one's belief in φ was not based on the justification that there is for φ. If one could know φ, although one ignored the propositional justification for φ in forming one's belief in φ, why would the existence of propositional justification for φ be any precondition for one's knowledge of φ?

Proponents of Williamson's knowledge-first approach should not, *eo ipso,* have any misgivings about **K-J**, even if propositional justification is here understood to be non-factive. They are anyway happy to concede that one's knowledge of φ implies both that φ is true and that one believes φ (Williamson 2000). These concessions do not undermine their claim that, in the order of explanation, knowledge is both conceptually and metaphysically prior to any combinations of other epistemic notions that might be proffered in an attempt to analyse, or reduce, knowledge. To add **K-J** is not yet to add anything that would bring us any closer to such an analysis or reduction. Similarly, just as there is no pressure on the proponents of the knowledge-first approach to explain truth or belief in terms of knowledge, although knowledge implies truth and belief, there would seem to be no pressure on them to explain justification in terms of knowledge, even if **K-J** holds.[2]

Proponents of the knowledge-first approach may still harbour doubts about whether any such notion of justification – a notion that would, moreover, be apt for explanatory work – is forthcoming. These doubts will be allayed in what follows.

1.4 All-things-considered vs pro tanto justification

Justification will be assumed to be justification *all things considered*. While there may be both *pro tanto* justification for φ and *pro tanto* justification for ¬φ, it is ruled out that there is all-things-considered justification for both φ and ¬φ. If the *pro tanto* reasons for φ (¬φ) prevail, ¬φ (φ) is not justified overall. If the *pro tanto* reasons for φ and ¬φ, respectively, are on a par, neither is justified overall. If there are no *pro tanto* reasons at all for either of φ and ¬φ, again, neither is justified overall. In other words, it will be assumed that the following principle holds:

D$_J$ $J\neg\varphi \rightarrow \neg J\varphi$

D$_J$ is not all that the notion of all-things-considered justification encodes. Reasons for ¬φ are not the only factors that, on balance, can outweigh or weaken the *pro tanto* reasons for φ, so as to preclude that φ is justified overall. There may also be reasons for thinking that one's *pro tanto* reasons for φ are misbegotten or too weak (cf. Pollock and Cruz 1999; Bergmann 2005). Insofar as justification is here understood to be epistemic, all-things-considered justification for the thought that one is in no position to know φ on the basis of one's *pro tanto* reasons for φ, if any, should correspondingly be understood

[2] Given that the relevant notion of justification is *epistemic*, proponents of the knowledge-first approach may nonetheless be prone to looking for a reductive account of justification in terms of knowledge. Note, though, that the epistemic nature of justification may already be guaranteed by principles like **K-J** and others to be discussed below, where these principles need not conspire to yield a reductive account.

to undercut their power to bestow all-things-considered justification on φ. Accordingly, we lay down:

(1) $J\neg K\varphi \rightarrow \neg J\varphi$

It will be assumed that any structural account of non-factive propositional all-things-considered justification must have the resources to vindicate (1), $\mathbf{D_J}$ and **K-J**.

1.5 Desiderata for a structural account of justification

The goal accordingly is to devise an account of justification that is propositional, non-factive and all-things-considered and underwrites **K-J**, $\mathbf{D_J}$ and (1). These general features still allow for different precisifications of the notion, and the account to be devised offers only one such precisification.

Being structural, the account aims to be as neutral as possible on the question of which kinds of fact ground justification, and so on what it is that justification materially consists in. Given that **K-J** holds, it is therefore natural to lay down the following desideratum:

[D1] The account should be compatible both with internalism and with externalism about justification and knowledge.

Recall that externalism was minimally characterised as the denial of internalism. As such it allows for different concretizations. [D1] should not be understood to demand that the account be consistent with all externalist views. To meet [D1], it will be enough if the account neither implies nor precludes that justification is fully grounded in internally individuated mental states or in facts that are already introspectively or reflectively accessible.

This observation matters, because the account will remain as faithful as possible to the job description that internalists commonly assign to justification, *viz.* that it make beliefs, appropriately formed on its basis, epistemically responsible. Not all externalist views will be concerned with this job description, which finding is consistent with the idea that there is such a notion available even to externalists.

Ever since its first detailed defence by Williamson (2000), the knowledge-first approach has gained a considerable number of supporters. Accordingly, in order to ensure that the account is of broad enough interest to epistemologists, we should accept the following:

[D2] The account should be compatible both with the knowledge-first approach and with rejection of that approach.

These are not the only desiderata. The debate between contextualists and invariantists about knowledge is still unresolved.[3] A structural account of justification should take no sides, but allow for implementation of the idea that the standards for knowledge vary from context to context; thus:

[D3] The account should be compatible both with invariantism and with contextualism about knowledge.

As indicated, an account of justification, as this notion is here being understood, should have the resources to vindicate **K-J**, $\mathbf{D_J}$ and (1). But a structural account of justification should do more than this; thus:

[D4] The account should license independently plausible principles of justification, including **K-J**, $\mathbf{D_J}$ and (1), and help to decide controversies about others.

[3] Cf. e.g. Kornblith (2000); Hawthorne (2004); Wright (2005); DeRose (2009).

To fulfil [D4], the account of justification must be supplemented by a suitable logic. It may initially seem that, for this reason, we need to negotiate which principles of justification hold before the account is given, in which case the account itself does nothing to license them. However, as we shall see, the logic that needs to be in place is not a logic of justification, but a logic governing the notion of being in a position to know. Such a logic may be motivated on independent grounds. This is not to say that the account cannot help to motivate systematic extensions of this logic, for example because **K-J**, $\mathbf{D_J}$ and (1) must be validated in order not to miss the target. In the end, fulfilment of [D4] will have to be assessed on the basis of considerations of reflective equilibrium.

An account of justification requires initial motivation. It is not enough to show that it has nice consequences, as long as it is wildly implausible to start with. Such an initial motivation, or rationale, may have to appeal to certain principles of justification, however implicitly so. Since an initial rationale does not pretend to already be a justification, this need not render fulfilment of [D4] useless as a criterion of success. However, once backed by a suitable epistemic logic, the account should have the resources to sanction the principles, if any, on which its initial motivation depends; thus:

[D5] The account should not rely, for its initial rationale, on any principles of justification that it cannot be shown to imply.

This completes our list of desiderata against which to assess the structural account of justification to be proposed. If the account can be shown to meet all of them, this is already a considerable feat. There may be further desiderata which the account proves to have trouble fulfilling. Whether there are any such further desiderata is left for future work to ascertain.

1.6 The notion of being in a position to know, and its logic

Principle **K-J** already links justification to the notion of being in a position to know. As adumbrated, the background logic presupposed in what follows is a logic governing this notion. We therefore must clarify how this notion is to be understood and outline which axioms and rules it is, minimally, assumed to underwrite. Later, we will add to this axiomatic base in systematic ways.

Roughly, being in a position to know φ is like knowing φ, but with the condition that φ be believed systematically being replaced by the condition that one be physically and psychologically capable of believing φ. It already follows from this rough characterization that knowing implies being in a position to know, but not *vice versa*. Standard epistemic logics are logics governing the notion of knowledge. Since knowledge requires belief, these logics are typically bound to make strong assumptions about the psychology of subjects. Such assumptions are largely irrelevant to epistemology. A logic of being in a position to know, by contrast, abstracts from the doxastic states of subjects.

Given the aforementioned characterization, it likewise follows that one is only ever in a position to know what is the case. Thus, like knowledge but unlike justification, being in a position to know is *factive*. Again using K as short for 'one is in a position to know' (rather than 'one knows'), we thus lay down:

$\mathbf{T_K}$ $\quad K\varphi \rightarrow \varphi$

Accordingly, one's being in a position to know φ differs from its being possible for one to know φ. But it also differs from its being possible for one to know φ where φ is guaranteed to be an actual truth. For, unlike the latter, being in a position to know φ implies that φ's being the case is 'open to one's view, unhidden, even if one does not yet see it' (Williamson 2000: 95).[4]

Standard epistemic logics presuppose the rule of necessitation according to which all logical theorems are known (Hintikka 1962; cf. also Lenzen 1978). These logics likewise presuppose that if $\varphi \rightarrow \psi$ is a logical theorem, then if φ is known, ψ is known. The idealisations inherent in these rules are two-fold: not only are subjects said to have the resources to know what follows from what, they are furthermore said to be aware of what follows from what, using these resources. The logic for being in a position to know only makes the first kind of idealisation, leaving what is believed to one side. To this extent at least, it is less demanding. We thus lay down:

RN$_\mathbf{K}$ If $\vdash \varphi$, then $\vdash K\varphi$

RM$_\mathbf{K}$ If $\vdash \varphi \rightarrow \psi$, then $\vdash K\varphi \rightarrow K\psi$

These rules still require extremely strong idealisations – an issue to which we will return in due course.

Standard epistemic logics are normal modal logics: they not only include **RN**$_\mathbf{K}$ and **RM**$_\mathbf{K}$, but also

K$_\mathbf{K}$ $K(\varphi \rightarrow \psi) \rightarrow (K\varphi \rightarrow K\psi)$

Accordingly, in standard epistemic logics, **RM**$_\mathbf{K}$ is a derived rule. The logic for being in a position to know cannot be normal. To see this, reflect first that if we also had **K**$_\mathbf{K}$ in addition to **RN**$_\mathbf{K}$, we could derive the principle according to which K agglomerates over conjunction, i.e.

[4] Accordingly, if there is beer in the fridge while you do not yet know this, then even if you could easily open the door and look inside, this alone does not yet put you in a position to know that there is beer in the fridge, for in your current situation this fact remains hidden.

$$K\varphi \,\&\, K\psi \rightarrow K(\varphi \,\&\, \psi)$$

Next note that where ψ is 'No one ever knows φ', the conjunction φ & ψ cannot possibly be known, given only that knowledge is factive and distributes over conjunction (Fitch 1963). Plausibly, if it is impossible for one to know a given proposition, one is not *in a position to know* that proposition either. But for all that, one may both be in a position to know φ and be in a position to know ψ. Just let φ be a fleeting truth of little interest, e.g. that there are exactly seven blossoms on the bougainvillea, in a context where 'the fact is open to one's view, unhidden, even if one does not yet see it' (Williamson 2000: 95), and one knows that a storm is about to hit, that one is presently the only one around, and also, by introspection, that one is far too unconcerned ever to form any belief about the matter (Heylen 2016; Rosenkranz 2016a).

Failure of $\mathbf{K_K}$ implies that the logic for K is non-normal. This *inter alia* means that we cannot give a standard Kripkean semantics for it. Something like a neighbourhood semantics is needed instead (Chellas 1980). This is not the place to devise such a semantics, though.

There are other principles, familiar from modal logic, that are bound to fail in the case of K. Among these are

$\mathbf{B_K}$ $\quad \varphi \rightarrow K\neg K\neg\varphi$

$\mathbf{5_K}$ $\quad \neg K\varphi \rightarrow K\neg K\varphi$

$\mathbf{B_K}$ wrongly predicts that the truth of a proposition is always sufficient in order for one to be in a position to discard any claim to knowledge of its negation – which it evidently is not, since one's evidence may be misleading. $\mathbf{5_K}$ wrongly predicts that whenever one's evidence is misleading, one is in a position to know that it is.

On any mildly externalist construal of being in a position to know, whether one is in that position may depend on environmental conditions, and as long as φ does not entail that such conditions prevail, one may be in a position to know φ without being in a position to know that one is in that position. The account to be proposed is meant to be neutral on the issue of whether externalism is true. Hence, the following principle should be resisted:

$\mathbf{4_K}$ $\quad K\varphi \rightarrow KK\varphi$

For the time being, we will only assume $\mathbf{T_K}$, $\mathbf{RN_K}$ and $\mathbf{RM_K}$ as part of the logic for K. Later we will consider an extension of this minimal base. It is understood to be a constraint on this extension that none of $\mathbf{K_K}$, $\mathbf{B_K}$, $\mathbf{5_K}$ and $\mathbf{4_K}$ becomes derivable in the process. However, since no formal semantics will here be provided, demonstration that the account meets this constraint will have to await another occasion.[5]

1.7 Idealisations

As indicated, $\mathbf{RN_K}$ and $\mathbf{RM_K}$ require very strong idealisations. Subjects are assumed to be in a position to know each and every logical theorem, and likewise each and every logical consequence of what they are in a position to know. This *inter alia* implies that subjects are presumed to have all the conceptual resources necessary to grasp each and every proposition expressible in the language. To these idealisations we add yet another.

Thus, it will henceforth be assumed that subjects never fail to be in a position to know a given proposition already because they suffer from what would count as physical or psychological deficiencies, e.g. impairments of their sensory apparatus, lack

[5] As one of the referees for this journal has pointed out, there are fairly simple models of neighbourhood semantics in which all of the desired axioms and rules are valid, including one we will consider in §3.1, while each of $\mathbf{K_K}$, $\mathbf{B_K}$, $\mathbf{5_K}$ and $\mathbf{4_K}$ fails. Limitations of space preclude detailed discussion.

of intellectual sophistication, inattentiveness, drunkenness, madness, death, *etc.* They may still fail to be in that position for other reasons, e.g. because they lack relevant information, or because the prevailing circumstances are not knowledge-conducive, or because being in a position to know requires a safety margin, or because the proposition in question is false or structurally unknowable.

Idealisations do no harm, as long as they are flagged as idealisations. However, any such idealisation makes it harder to explain what relevance the resultant logic has for ordinary subjects (cf. Williamson 1993; Stalnaker 2006: 172). This is the kind of question we cannot address here. For present purposes, and with hindsight, it may suffice to point out that we are here ultimately interested in an account of propositional justification in terms of K. As long as we do not assume that whenever a given proposition is justified in one's situation, one can easily avail oneself of this justification, irrespective of any further assumptions about one's epistemic capacities, the idealisations inherent in the characterisation of K need not automatically render the account of propositional justification irrelevant for the case of less than ideal subjects.

2. A structural account of justification

*2.1 Equivalence **E***

The account of justification to be proposed is a combination of two theses. The first is:

$\mathbf{E_1}$ $\quad J\varphi \to \neg K \neg K \varphi$

Here is an initial rationale for $\mathbf{E_1}$. If there is all-things-considered justification for φ in one's situation, then one cannot, in that situation, be in a position to know that this justification fails to have the right kind of pedigree or is obtained under unfavourable conditions or is otherwise insufficient to underwrite one's aspirations to know. For, in order for one to be in that position, there would have to be some information or some discernible feature of one's epistemic situation that outbalanced or undermined the justification for φ, in which case the justification for φ would not be justification all things considered (cf. Williamson 2000: 255-56; Smithies 2012c: 738).

This informal reasoning would seem to presuppose both **K-J** and (1). It remains to be seen whether these presuppositions can be redeemed, just as both [D4] and [D5] demand.

The second thesis is the converse of the first, i.e.

E₂ $\neg K \neg K\varphi \rightarrow J\varphi$

We have already idealised away physical and psychological deficiencies, conceptual limitations and limitations on reasoning power. So $\mathbf{E_2}$ is not already refuted by trees, dogs, small infants, or madmen.[6] To initially motivate $\mathbf{E_2}$, let us first ask what an epistemic situation must be like in which subjects, so idealised, are in no position to know that they are in no position to know φ. It cannot be a situation in which, if only one surveyed all the available information and was responsive to all the discernible features of one's situation, one would come to know that they failed to provide φ with sufficient support – which arguably would already be the case if all the available information and discernible features failed to bear on the matter at all or supported ¬φ at

[6] The present account is not the first that seeks to characterize an epistemic phenomenon in purely negative terms. For example, Martin explores a disjunctivist view according to which 'there is no more to the phenomenal character' of visual hallucinations 'than that of being indiscriminable from corresponding visual perceptions' (Martin 2006: 369). For a discussion of the corresponding problem posed by beings with degraded epistemic powers, and of the role of idealizations in solving it, see Martin (2006: 373-87), Hawthorne and Kovakovich (2006: 164-74) and Sturgeon (2006: 195-98). See also the *ex negativo* account of belief discussed in Lenzen (1979), Stalnaker (2006) and Halpern *et al.* (2009), which is structurally just like **E** and which we will briefly comment on in §4.1 below.

least as strongly as φ. What else, then, might block one's path to knowing that one is in no position to know φ?

Plausibly, some information must be available, or some feature of one's situation must be discernible, that, whether it be knowledge-conferring or not, favours φ all things considered – and does so to such a degree that knowledge that it is too weak a justification for φ, or lacks the right pedigree required for knowledge of φ, or is acquired under conditions that are not knowledge-conducive, *etc.* is foreclosed. Such information or feature constitutes all-things-considered justification for φ.

This informal reasoning would seem to presuppose the following principle:

(2) $\neg J\varphi \rightarrow K\neg J\varphi$

It remains to be seen whether the account ultimately sanctions (2), just as [D5] demands. It also remains to be seen whether its reliance on (2) makes the account vulnerable to principled objections.

Note that with $\mathbf{E_2}$ in place, **K-J** can immediately be established. By the factivity of K, i.e. $\mathbf{T_K}$, and contraposition, we have

(3) $K\varphi \rightarrow \neg K\neg K\varphi$

which latter, together with $\mathbf{E_2}$, yields **K-J**. With both (1) and $\mathbf{E_2}$ in place, we could also straightforwardly derive

(4) $J\neg K\varphi \rightarrow K\neg K\varphi$

(4) has a certain plausibility. Even if $J\neg K\varphi$ is grounded in misleading information to the effect that the prevailing circumstances are not conducive to knowledge of φ, the mere availability of this misleading information is enough to put one in a position to know

that one is in no position to know φ. For the *de facto* misleading nature of the information is immaterial. One is anyway in a position to know, in its light, that one is not in a position to know φ, since its availability is already enough to put one in a position to know that there is no all-things-considered justification for φ. This line of reasoning, if cogent, also sucessfully undermines Smithies' alleged counterexample to $\mathbf{E_2}$ (Smithies 2012b: 270). However, the reasoning still depends on the assumption of (2); and it remains to be seen whether this assumption can in the end be vindicated.

So far on the intuitive rationale for $\mathbf{E_1}$ and $\mathbf{E_2}$. They jointly yield the equivalence

E $\quad J\varphi \leftrightarrow \neg K \neg K\varphi$

Ultimately, the justification for **E** will depend on its power to fulfill the aforementioned desiderata, and in particular on its inferential potential, given a suitable logic for K.

2.2 Fulfilling desiderata [D1] to [D3]

E is silent on whether propositional justification supervenes upon the subject's internally individuated mental states *(mentalism)*. It is likewise silent on whether all factors relevant for propositional justification are already introspectively or reflectively accessible *(accessibilism)*. **E** would thus seem neutral with respect to the controversy between internalists and externalists about knowledge and justification, thereby fulfilling desideratum [D1].

Thus, according to **E**, subjects who are connected to the Matrix but are otherwise fully functional have justification for the claim that they have hands, since they are in no position to know that they are in no position to know that they have hands. The only facts accessible to them are facts that are accessible to them through introspection or reflection. Plausibly, therefore, the justification such subjects have available for the claim that they have hands is grounded in such facts. Indeed, it is the induced inner appearance as of having hands that, in the absence of defeaters, stops them from being in a position to know that they are in no position to know that they have hands.

However, there is no pressure to conclude that the justification *we* have for the claim that we have hands is likewise grounded in introspectively or reflectively accessible facts, such as easily available inner appearances as of having hands. For, what stops us from being in a position to know that we are in no position to know that we have hands is that we are in a position to *see* that we have hands. Arguably, being in a position to see is a case of being in a position to know (cf. Williamson 2000: 34). Accordingly, given **K-J**, whenever one is in a position to see that one has hands, one has propositional justification for the claim that one has hands. Such a justification, grounded in seeing, is unavailable to subjects connected to the Matrix, but it is nonetheless available to us.

E is controversial. However, to the extent that propositional justification is what makes beliefs, appropriately formed on its basis, epistemically responsible (see §1.5), propositional justification, and its absence, should be something believers can in principle be responsive to and appreciate on occasion of forming their beliefs. Thus understood, **E** should seem less contentious, even to externalists. This is not to say that **E** coheres with all externalist views, e.g. ones according to which justification depends on objective features whose absence in a given situation cannot even empirically be ascertained in that situation. However, in the light of the minimal characterisation of externalism given in §1.2, compliance with [D1] does not require coherence with such views.

Although **E** is assumed to be a necessary equivalence, there is no presumption that it yields a conceptual or any other kind of reduction of justification. But **E** surely leaves room for such a reduction. If **E** is consistent with the reducibility of justification in terms of K, it is *a fortiori* consistent with the knowledge-first approach. **E** is therefore compatible both with the knowledge-first approach and with its rejection. **E** thus fulfills desideratum [D2].

E allows for, but does not entail, contextual variation in the standards for knowledge. It thus fulfills desideratum [D3]. In fact, **E** provides an explanation of why, if the standards for knowledge vary, so do the standards for justification. If it becomes easier to be in a position to know φ, because the standards for Kφ are lowered, it becomes comparatively harder to be in a position to know that one is in no position to know φ, and so comparatively easier not to be in that position. Hence, given **E**, the standards for Jφ are correspondingly lowered. The same holds, *mutatis mutandis*, for a raising of standards.

2.3 Towards fulfilling desiderata [D4] and [D5]

We have already seen how **K-J** can be derived with the help of **E** and

T_K $K\varphi \rightarrow \varphi$

Besides **T_K**, the minimal logic for K also comprises the two rules

RN_K If |- φ, then |- Kφ
RM_K If |- $\varphi \rightarrow \psi$, then |- $K\varphi \rightarrow K\psi$

Once **E** is added, the following two rules can accordingly be established:

RN_J If |- φ, then |- Jφ

which straightforwardly follows from **K-J** and **RN_K**, and

RM_J If |- $\varphi \rightarrow \psi$, then |- $J\varphi \rightarrow J\psi$

Proof: Let $\varphi \rightarrow \psi$ be a theorem. Then by $\mathbf{RM_K}$, so is $K\varphi \rightarrow K\psi$. By contraposition and another application of $\mathbf{RM_K}$, this yields $K\neg K\psi \rightarrow K\neg K\varphi$. By contraposition and **E**, $J\varphi \rightarrow J\psi$ follows.

With $\mathbf{RM_J}$ in place, we can now easily prove the following two highly plausible principles of justification:

(5) $J(\varphi \,\&\, \psi) \rightarrow J\varphi \,\&\, J\psi$

(6) $J\varphi \rightarrow J(\varphi \vee \psi)$

The addition of **E** likewise allows us to prove all of the following:

(7) $\neg J(\varphi \,\&\, \neg K\varphi)$

Proof: Given $\mathbf{RM_K}$, K distributes over conjunction. This together with $\mathbf{T_K}$ entails $\neg K(\varphi \,\&\, \neg K\varphi)$, whence by $\mathbf{RN_K}$ we derive $K\neg K(\varphi \,\&\, \neg K\varphi)$. Given **E**, (7) follows.

(8) $\neg J(J\varphi \,\&\, \neg\varphi)$

Proof: Assume for *reductio* $K(J\varphi \,\&\, \neg\varphi)$. By $\mathbf{T_K}$ and **E**, $\neg K\neg K\varphi$. By $\mathbf{T_K}$, $\neg\varphi \rightarrow \neg K\varphi$. By $\mathbf{RM_K}$, $K\neg\varphi \rightarrow K\neg K\varphi$. Hence by *modus tollens*, $\neg K\neg\varphi$. From our assumption, by $\mathbf{RM_K}$, $K\neg\varphi$ follows. Hence, $\neg K(J\varphi \,\&\, \neg\varphi)$ holds. By $\mathbf{RN_K}$, $K\neg K(J\varphi \,\&\, \neg\varphi)$ follows. So by **E**, we obtain (8).

(9) $\neg J(\varphi \,\&\, \neg J\varphi)$

Proof: Assume for *reductio* $K(\varphi \,\&\, \neg J\varphi)$. By $\mathbf{T_K}$, $\neg J\varphi$, and so by **K-J**, $\neg K\varphi$. By $\mathbf{RM_K}$ from our assumption, $K\varphi$. Hence, $\neg K(\varphi \,\&\, \neg J\varphi)$ holds. By $\mathbf{RN_K}$, $K\neg K(\varphi \,\&\, \neg J\varphi)$ follows. So by **E**, we obtain (9).

This is good news, since (7) to (9) are eminently plausible (Smithies 2012a: 283-84).[7]

E explains why $\mathbf{T_J}$ should fail: J is not factive, because $\neg K\neg K$ is not factive. The axiomatic base for K includes none of $\mathbf{K_K}$, $\mathbf{B_K}$, $\mathbf{5_K}$ and $\mathbf{4_K}$. **E** shows why $\mathbf{B_K}$ and $\mathbf{5_K}$ are both bad. Given **E**, each of $\mathbf{B_K}$ and $\mathbf{5_K}$ would make justification factive. **E** likewise shows why $\mathbf{4_K}$ is problematical, for given **E**, $\mathbf{4_K}$ would entail $J\varphi \rightarrow JK\varphi$. Yet, the

[7] Smithies (2012a) suggests that we need to assume the characteristic **S4** and **S5** principles for J, in order to derive (8) and (9). The foregoing shows that, to this end, those principles are not needed.

justification for φ may be altogether silent on whether the environmental conditions are conducive to knowing φ on its basis.[8]

For all that, the account would so far seem to be incomplete, as we still need to derive all of the following

D$_J$ $J\neg\varphi \rightarrow \neg J\varphi$

(1) $J\neg K\varphi \rightarrow \neg J\varphi$

(2) $\neg J\varphi \rightarrow K\neg J\varphi$

In the next section, an extension of the minimal logic for K is considered and defended against objections. This extension allows for the derivation of (1), (2) and **D$_J$**. As we shall see, it also allows for the derivation of potentially more controversial principles of justification that are often taken to presuppose internalism.

3. Justification as luminous

3.1 The luminosity principle

It was part of the initial characterisation of non-factive propositional all-things-considered justification, given in §1, that

(1) $J\neg K\varphi \rightarrow \neg J\varphi$

holds. By **E** and contraposition, (1) is equivalent to

Lum $\neg K\neg K\varphi \rightarrow K\neg K\neg K\varphi$

In the context of a normal modal logic for K, **Lum** would define a class of Kripke frames with the following property: $\forall x\forall y\forall z\ (xRy\ \&\ xRz \rightarrow \exists u(zRu\ \&\ \forall w(uRw \rightarrow$

[8] On the status of **K$_K$**, **B$_K$**, **5$_K$** and **4$_K$**, see footnote 5.

yRw))).[9] Although this is ultimately of limited interest, since the logic for K is not normal, this observation might still help to clarify the import of **Lum**. *Modulo* **E**, **Lum** is in turn equivalent to

(10) $J\varphi \rightarrow KJ\varphi$

Let us call a condition *luminous* just in case its obtaining, in one's situation, guarantees that, in that situation, one is in a position to know that it obtains (cf. Williamson 2000). Accordingly, given **E**, **Lum** amounts to the claim that justification is luminous. In what follows, we investigate the option of adding **Lum** to the minimal logic for K.

3.2 Fulfilment of desiderata [D1] to [D3]

Addition of **Lum** does not affect the account's neutrality with respect to the debate between internalists and externalists that [D1] demands. **Lum** is consistent with the idea that one's propositional justification for the claim that one has hands is grounded in one's seeing that one has hands, while one may nonetheless not be in a position to know that the environmental conditions are conducive to seeing. Even if justification is luminous, the grounds of justification may not be luminous. Just like subjects connected to the Matrix, one may be in a position to know ¬K¬K(One has hands), without being in a position to know that one sees that one has hands.

Again, this is not to say that addition of **Lum** can be made to cohere with *any* externalist view, e.g. one according to which justification depends on features whose presence in a given situation cannot even empirically be ascertained in that situation. However, in the light of the minimal characterisation of externalism given in §1.2, fulfilment of [D1] does not require coherence with such views.

[9] Here I am indebted to José Martínez and Fabrice Correia.

Addition of **Lum** is likewise compatible both with the knowledge-first approach and with its rejection, just as [D2] demands. It is also compatible both with contextualism and with invariantism about knowledge, just as [D3] demands.

3.3 Towards fulfilling desideratum [D4]: further proofs

Adding **Lum** to $\mathbf{T_K}$, $\mathbf{RN_K}$, $\mathbf{RM_K}$ and **E** allows us to derive $\mathbf{D_J}$ and a host of other principles of justification:

$\mathbf{D_J}$ $\quad J\neg\varphi \rightarrow \neg J\varphi$

Proof: By $\mathbf{T_K}$, contraposition and $\mathbf{RM_K}$, we obtain $K\varphi \rightarrow K\neg K\neg\varphi$, whence by contraposition and another application of $\mathbf{RM_K}$, we derive $K\neg K\neg K\neg\varphi \rightarrow K\neg K\varphi$. By **Lum**, we have $\neg K\neg K\neg\varphi \rightarrow K\neg K\neg K\neg\varphi$. So by the transitivity of $\rightarrow$, $\neg K\neg K\neg\varphi \rightarrow K\neg K\varphi$ follows. Given **E**, this is equivalent to $\mathbf{D_J}$.

(11) $\quad JJ\varphi \rightarrow J\varphi$

Proof: By $\mathbf{T_K}$, we get $K\neg K\varphi \rightarrow \neg K\neg K\neg K\varphi$. By **Lum**, we obtain $\neg K\neg K\neg K\varphi \rightarrow K\neg K\neg K\neg K\varphi$. By the transitivity of $\rightarrow$, $K\neg K\varphi \rightarrow K\neg K\neg K\neg K\varphi$ follows, whence by contraposition, we obtain $\neg K\neg K\neg K\neg K\varphi \rightarrow \neg K\neg K\varphi$. Given **E**, this is equivalent to (11).

$\mathbf{4_J}$ $\quad J\varphi \rightarrow JJ\varphi$

Proof: Assume $J\varphi$, i.e. given **E**, $\neg K\neg K\varphi$, whence by **Lum**, $K\neg K\neg K\varphi$ follows. Now assume for *reductio* $\neg JJ\varphi$, i.e. given **E**, $K\neg K\neg K\neg K\varphi$, whence by $\mathbf{T_K}$, $\neg K\neg K\neg K\varphi$ follows. Contradiction! Thus, $\mathbf{4_J}$ holds.

(12) $\quad J\neg J\varphi \rightarrow \neg J\varphi$

Proof: By **K-J**, contraposition and $\mathbf{RM_J}$, we have $J\neg J\varphi \rightarrow J\neg K\varphi$. Given **E**, **Lum** is equivalent to (1), i.e. $J\neg K\varphi \rightarrow \neg J\varphi$. By the transitivity of $\rightarrow$, (12) follows.

$\mathbf{5_J}$ $\quad \neg J\varphi \rightarrow J\neg J\varphi$

Proof: By **Lum**, $\neg K\neg K\varphi \rightarrow K\neg K\neg K\varphi$, whence by contraposition, $\neg K\neg K\neg K\varphi \rightarrow K\neg K\varphi$. From this, by $\mathbf{RM_K}$, we obtain $K\neg K\neg K\neg K\varphi \rightarrow KK\neg K\varphi$, which by contraposition yields $\neg KK\neg K\varphi \rightarrow \neg K\neg K\neg K\neg K\varphi$. From the latter, by **E** and (11), $\neg KK\neg K\varphi \rightarrow \neg K\neg K\varphi$ follows. Now assume $\neg J\varphi$, i.e. given **E**, $K\neg K\varphi$. Assume for *reductio*, $\neg J\neg J\varphi$, i.e. given **E**, $K\neg KK\neg K\varphi$, whence by $\mathbf{T_K}$, $\neg KK\neg K\varphi$ follows. By the foregoing, the latter yields $\neg K\neg K\varphi$. Contradiction! Thus, $\mathbf{5_J}$ holds.

Smithies (2012a: 296-99) argues that (11), (12), $\mathbf{4_J}$ and $\mathbf{5_J}$ are best explained by mentalist internalism. Their derivability from externalist-friendly premises shows that this contention is mistaken. Smithies would seem to rely on the questionable assumption, rejected above, that if facts about justification are luminous, so must be their grounds, or on the equally questionable assumption that the luminosity of such facts requires these facts themselves to be already accessible by introspection or reflection alone.

There is no reason to expect that, after addition of **Lum**, the unwanted principles $\mathbf{T_J}$, $\mathbf{K_K}$, $\mathbf{B_K}$, $\mathbf{5_K}$ and $\mathbf{4_K}$ suddenly become derivable. However, without a suitable formal semantics, we admittedly remain short of a demonstration that none of these principles can be derived.[10]

3.4 Fulfilling desideratum [D5]

With **Lum** in place, we can finally redeem the remaining presupposition underlying the initial rationale for **E**, i.e.

(2) $\neg J\varphi \rightarrow K\neg J\varphi$

Proof: We have already proved (12), i.e. $J\neg J\varphi \rightarrow \neg J\varphi$. By $\mathbf{RM_K}$, we accordingly have $KJ\neg J\varphi \rightarrow K\neg J\varphi$. By **Lum** and **E**, $J\neg J\varphi \rightarrow KJ\neg J\varphi$. By the transitivity of $\rightarrow$, this yields $J\neg J\varphi \rightarrow K\neg J\varphi$. Given $\mathbf{5_J}$, (2) follows.

Given (2), we accordingly now also have:

(4) $J\neg K\varphi \rightarrow K\neg K\varphi$

Thus, the account fulfils desideratum [D5] and hence does not, by way of motivation, presuppose anything that it cannot redeem.

[10] See footnote 5.

3.5 Margin-for-error principles

However, **Lum** is controversial, as a cursory look into the recent epistemological literature confirms. Thus, consider what happens once we conjoin

(10) $J\varphi \rightarrow KJ\varphi$

with the margin-for-error principle

$\mathbf{M_K}$ $KJ\varphi_\alpha \rightarrow J\varphi_\beta$

where α and β are *relevantly close* cases that differ only marginally in whatever factors conspire to determine whether or not φ is justified. It takes little to see that we obtain the soritical

$J\varphi_\alpha \rightarrow J\varphi_\beta$

Hence **$\mathbf{M_K}$** puts pressure on **Lum**. Williamson (2000) derives margin-for-error principles for K from corresponding margin-for-error principles for knowledge. Let *k* be short for 'one knows'; then the principle corresponding to **$\mathbf{M_K}$** is

$\mathbf{M}_k$ $kJ\varphi_\alpha \rightarrow J\varphi_\beta$

where here, as before, α and β are relevantly close in that they differ only marginally in whatever factors conspire to determine whether or not φ is justified. Principles like **$\mathbf{M}_k$** are motivated by the idea that one's knowledgeable belief must be *safe*, in the sense that there must be no easy possibility of one's believing falsely.

However, unlike $\kappa\varphi$, $K\varphi$ does not require that φ be believed. In order to derive principles like **$\mathbf{M_K}$** from principles like **$\mathbf{M}_k$**, Williamson (2000: 128) appeals to principles of the kind exemplified by

If one is in a position to know Jφ in case α, there is a case α' such that, in α', one *does* know Jφ and, in α', φ is justified on the basis of the very same factors as in α – so that whatever cases are relevantly close to α are relevantly close to α' and *vice versa*.

Friends of **Lum** accordingly have two main lines of response available: either they reject the margin-for-error principles for knowledge, i.e. $\mathbf{M}_{k}$, or they reject **#**. Important work has already been done in pursuit of the first strategy (Berker 2008; Zardini 2012). Here, I intend to pursue the second and cast doubts on **#**.

Being in a position to know requires propositional justification but no justified belief. By contrast, knowledge requires doxastic justification, and hence a justified belief. J encodes propositional rather than doxastic justification. As argued in §1.1, a competent subject's justified belief in ψ may add to the strength of the propositional justification that there is for ψ in a given situation. Let ψ be Jφ. Given its commitment to (11), i.e. $JJ\varphi \rightarrow J\varphi$, the account under scrutiny allows that a strengthening of the justification for Jφ provides a strengthening of the justification for φ, as long as the latter is not already as strong as it can get (which close to the borderline it is not). Objections to a given account should not underestimate its resources. On the account under scrutiny, what is relevantly close to cases in which *no* justified belief in Jφ is formed may accordingly not be relevantly close to cases in which such a justified belief *is* formed. Thus, unless $\mathbf{M}_{\mathbf{K}}$ can independently be motivated, we may reject it while still accepting $\mathbf{M}_{k}$. Without $\mathbf{M}_{\mathbf{K}}$, there is so far no argument against **Lum**.

Thus, for all that has been said, one may well be in a position to know the last case of Jφ in a Sorites series for Jφ, even if actually knowing what one thus is in a position to know would require that the last case of Jφ was further along the series. 'But if in the next case $\neg$Jφ holds, doesn't this block one's path to knowing that Jφ holds?' Not if not knowing what one is in a position to know blocks that path. One could push the last case of Jφ further ahead by knowing, even if in fact one never does.[11]

3.6 The unmarked clock

Even if **M_K** should fail for such reasons, there may still be margin-for-error principles governing Kφ for φ that do not attribute propositional justification. It remains to be seen whether this concession already puts pressure on **Lum**, quite independently from considerations of vagueness. To this end, consider Williamson's case of an extremely accurate and precise clock with an unmarked dial and a single hour hand (Williamson 2014). Let us assume with Williamson[12] that for all relevant specifications φ of what time it is, at time n, one is in a position to know φ by looking at the unmarked clock, only if φ holds at all times within the interval $n \pm$ 5min, i.e.

$M_{K\varphi}$ $\quad K\varphi_n \rightarrow \varphi_m$, for all $m \in \{n \pm 5\text{min}\}$

Let 't_0' abbreviate 'the present time' so that $(t_0 = n)_n$, for all n. Then **$M_{K\varphi}$** entails

(13) $\quad K(t_0 \in \{m \pm 5\text{min}\})_n \rightarrow n = m$

(14) $\quad \neg K(t_0 \neq m)_n$, for any $m \in \{n \pm 5\text{min}\}$

For, if $n \neq m$, the proposition that the present time lies within the interval $m \pm$ 5min will not hold at all times within the interval $n \pm$ 5min; and trivially, the proposition that the present time is distinct from m does not hold at m.

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[11] It would follow that one can never know all one is in a position to know. Let the superscript in 'necessary^i' encode i-1 iterations of 'necessary'. It would follow that **Lum** won't be necessary^n for all n, if some ¬Jφ-case in the series is not even possibly^m a Jφ-case, for some m. For then it won't hold that for every case x, every n and every possible^n circumstance in which one is in a position to know x to be a Jφ-case, where x is the last such case in the series, there is a possible^{n+1} circumstance in which one knows x to be a Jφ-case and some case later than x is the last Jφ-case. For all that, **Lum** might still be necessary.

[12] Although he puts these points in terms of knowledge, certain passages suggest that Williamson intends them to likewise apply to K (e.g. Williamson 2014: 972).

Again following Williamson, assume in addition that at time n, one is indeed in a position to know by looking that the present time lies within the interval $n \pm 5\text{min}$, i.e.

(15) $K(t_0 \in \{n \pm 5\text{min}\})_n$

Let p be the proposition that the present time lies within the interval 4 o'clock ± 5min. From (13) and (15), we derive

(16) $Kp \leftrightarrow$ It's 4 o'clock

Since the clock is extremely precise and accurate, there will be a huge number of possible pointer positions corresponding to times within the interval 4 o'clock ± 5min which, given (14), one is in no position to rule out as candidates for being present. Given suitable background assumptions, it follows that at 4 o'clock, it is almost certain on one's evidence that it isn't 4 o'clock. To the extent that (16) is known, it would seem that, at 4 o'clock, it is almost certain on one's evidence that K*p* is false. Since, plausibly, one cannot be in a position to know something whose falsity is almost certain on one's evidence, at 4 o'clock, KK*p* fails, whence it would follow that K*p* is not luminous (Williamson 2014: 979-81).

Even so, at 4 o'clock, it does not equally seem to be almost certain on one's evidence that J*p* is false. For given **E**, this would require that it then be almost certain on one's evidence that *one is in a position to know* that one is in no position to know *p* – and so, given (16), that one is in a position to know conditions to prevail that prevail iff it isn't 4 o'clock. Such a diagnosis would seem entirely unwarranted.

This rejoinder might be thought too quick as an attempt to salvage **Lum**. To begin with, note that from the foregoing we can derive

(17) $\neg K\neg$(It's 4 o'clock) $\leftrightarrow p$

For, at any time m within the interval 4 o'clock ± 5min, 4 o'clock lies within the interval m ± 5min, and so by (14) is, for all one is in a position to know, present. Likewise, at any time m outside the interval 4 o'clock ± 5min, 4 o'clock lies outside the interval m ± 5min, and so by (15) and $\mathbf{RM_K}$, one is in a position to know it not to be present.

Let us now assume that one is in a position to know (17), and also in a position to know that one is in a position to know (16), i.e. K(17) & KK(16). Focusing on k rather than K, Williamson (2014: 973-79) assumes an epistemic logic that is normal. If we similarly had

$\mathbf{K_K}$ $\quad K(\varphi \rightarrow \psi) \rightarrow (K\varphi \rightarrow K\psi)$

we could, by contraposition, infer from K(16) – and hence by $\mathbf{T_K}$, also from KK(16) – that the following holds:

(18) $\quad \neg K \neg K p \leftrightarrow \neg K \neg$(It's 4 o'clock)

Ex hypothesi, KK(16) holds. From this, by $\mathbf{K_K}$ and $\mathbf{RN_K}$, we could now derive K(18). Together with (17), (18) yields

(19) $\quad \neg K \neg K p \leftrightarrow p$

Ex hypothesi, K(17) holds. Accordingly, by $\mathbf{RM_K}$, K(19) would follow. By another application of $\mathbf{K_K}$, we would obtain

(20) $\quad K \neg K \neg K p \leftrightarrow K p$

From (16), (19) and (20), by the transitivity of the conditional, **Lum** would then allow us to infer the absurd

(21) $p \rightarrow$ It's 4 o'clock.

This argument against **Lum** not only assumes that one is in a position to know (17) and that one is in a position to know that one is in a position to know (16) – which may be contested – it also makes heavy-duty use of $\mathbf{K_K}$. We already know that $\mathbf{K_K}$ is bound to fail (see §1.6) and so need not be overly impressed by this argument.

It might at first seem that this defence of **Lum** is *ad hoc*, because even if $\mathbf{K_K}$ does not in general hold, certain of its instances may after all do. Yet, arguably, failure of $\mathbf{K_K}$ in the present context has nothing in particular to do with **Lum**. Suppose that one is in a position to know (17). Given (16), $\mathbf{K_K}$ would then imply

(22) $K\neg K\neg$(It's 4 o'clock) $\rightarrow$ It's 4 o'clock

But even at one minute past 4 o'clock, it may persistently seem to one that, for all one is in a position to tell by looking at the unmarked clock, it is 4 o'clock. Plausibly, it can be taken as known that as long as one enjoys such an appearance, with nothing to discount it, it blocks one's path to knowing that it isn't 4 o'clock. But then, by another application of $\mathbf{K_K}$, being in a position to know, at one minute past 4 o'clock, that one enjoys such an appearance, with nothing to discount it, should put one in a position to know that one is in no position to know that it isn't 4 o'clock – which would contradict (22) (cf. Rosenkranz 2016b for further discussion).

4. Why the logic for J cannot be normal

4.1 Normality and the agglomeration of J over conjunction

Before closing, let us consider whether the resultant logic for J can be a normal modal logic. This question is of more than merely technical interest, since the logic for J is normal just in case J agglomerates over conjunction; and the question whether J so agglomerates is undoubtedly of philosophical interest. Given $\mathbf{RN_J}$, if we also had

$\mathbf{K_J}$ $\quad J(\varphi \rightarrow \psi) \rightarrow (J\varphi \rightarrow J\psi)$

the resultant logic for J would be normal. We next prove that $\mathbf{K_J}$ is equivalent to

(23) $\quad J\varphi \,\&\, J\psi \rightarrow J(\varphi \,\&\, \psi)$

> *Proof:* (i) Assume (23). Next assume $J(\varphi \rightarrow \psi)$ and $J\varphi$. By (23), the latter yield $J(\varphi \,\&\, (\varphi \rightarrow \psi))$. By $\mathbf{RM_J}$, we have $J(\varphi \,\&\, (\varphi \rightarrow \psi)) \rightarrow J\psi$. Accordingly, $\mathbf{K_J}$ holds. (ii) Assume $\mathbf{K_J}$. Next assume $J\varphi$ and $J\psi$. By $\mathbf{RN_J}$, we have $J(\varphi \rightarrow (\psi \rightarrow \varphi \,\&\, \psi))$. By two applications of $\mathbf{K_J}$, $J(\varphi \,\&\, \psi)$ follows. Accordingly, (23) holds.

By **E** and contraposition, (23) is in turn equivalent to

Agg $\quad K\neg K(\varphi \,\&\, \psi) \rightarrow (\neg K\neg K\varphi \rightarrow K\neg K\psi)$

Thus, the logic for J will be normal iff J agglomerates over conjunction iff **Agg** holds. In the context of a normal modal logic for K, **Agg** would define a class of Kripke frames with the following property: $\forall x\forall y_1\forall y_2(xRy_1 \,\&\, xRy_2 \rightarrow \exists y_3(xRy_3 \,\&\, \forall z(y_3Rz \rightarrow y_1Rz \,\&\, y_2Rz)))$.[13] Although this is ultimately of limited interest, since the logic for K is agreed not to be normal, this observation might still help to clarify the import of **Agg**.

Agg implies **Lum**, and hence its addition would make separate addition of **Lum** superfluous:

[13] Here I am indebted to Ramon Jansana.

Proof: Assume Jφ and assume for *reductio* ¬KJφ. From the latter, by **E**, J¬Kφ follows. By **Agg**, this together with the first assumption yields J(φ & ¬Kφ) – which contradicts (7), i.e. ¬J(φ & ¬Kφ). **Lum** follows.

Accordingly, by combining **E** with **Agg**, we could still derive $\mathbf{D_J}$, $\mathbf{4_J}$ and $\mathbf{5_J}$, as we did in §3.3, but would now have $\mathbf{K_J}$ in addition. Lenzen (1979), Stalnaker (2006) and Halpern *et al.* (2009) consider the principle Bφ ↔ ¬*k*¬ *k*φ, where B is short for 'one believes', and show that, once supplemented by this principle, normal epistemic logics for *k* stronger than **S4** but weaker than **S5** allow us to derive **KD45** for B.[14] The principle in question is structurally just like **E**. The foregoing shows that the same result can be obtained using a *non-normal epistemic logic* distinct from any logic that is at least as strong as **S4**.

The addition of **Agg** is, however, highly problematical; and ultimately, the logic for J cannot be normal, and J does not agglomerate over conjunction. This will be argued in the remainder.

4.2 Lotteries and the preface

Given **E**, **Agg** is equivalent to (23). Many critics point to the preface and lottery paradoxes in order to discredit (23). But given **E**, these criticisms would seem to misfire. First consider lotteries (Kyburg 1961: 197). Arguably, one *is* in a position to know that one is in no position to know that one's ticket will lose. So given **E**, lotteries present no clear counterexamples to (23).

[14] In order to prove this result, these authors assume the epistemic logic **S4.4**, where $\mathbf{4.4}_k$ is φ → (¬*k*¬*k*φ → *k*φ). Jointly with Bφ ↔ ¬*k*¬*k*φ, $\mathbf{4.4}_k$ entails *k*φ ↔ (φ & Bφ), which is highly implausible given Gettier cases. However, in this context, Lenzen (1979: 42) and Stalnaker (2006: 179-80) also consider the epistemic logic **S4.2**, where **S4.2** results from **S4** by adding Geach's axiom. In the end, this is no better: as argued, given only the mildest forms of externalism about knowledge, **S4** is too strong to serve as an epistemic logic.

Next consider the preface paradox (Makinson 1965). Given **E**, for an author *not* to have justification for the conjunction of all the propositions composing her treatise, she must be in a position to *know* that she is in no position to know this conjunction. But how can past records of her fallibility put her in a position to *know* that she fails to know on this occasion? She would have to have an independent grip on how the relative frequency of falsehoods among the presently believed propositions compares to the relative frequency of falsehoods among the propositions believed in the past. Yet, she has no such independent grip. For all she is in a position to know, on this occasion she gets everything right (cf. Rosenkranz 2015: 637-38).

It thus is unclear whether, with **E** in place, the preface and lottery paradoxes provide counterexamples to (23). It is thus equally unclear whether they can serve to discredit **Agg**.

4.3 Why J does not agglomerate over conjunction

But **Agg** is bad for reasons that are independent from its equivalence to (23). Here is one such reason. Plausibly, $K\neg K(\varphi$ & No one ever knows $\varphi)$ holds. Yet, once both **Agg** and $\mathbf{T_K}$ are assumed, it follows that $K\varphi$ and K(No one ever knows φ) cannot simultaneously hold – which, as already argued in §1.6, is highly implausible on the intended interpretation of K (Heylen 2016; Rosenkranz 2016a).

Here is another reason, related to vagueness.[15] Plausibly, where α and β are adjacent cases in a Sorites series for $J\varphi$,

$$K\neg K(J\varphi_{\alpha} \ \& \ \neg J\varphi_{\beta})$$

holds, whence by **E** and **Agg** we obtain

[15] The argument is due to Patrick Greenough.

$$JJ\varphi_\alpha \to \neg J\neg J\varphi_\beta$$

However, **Agg** implies **Lum**; and on the basis of **Lum**, we have already proved both $\mathbf{4}_J$, i.e. $J\varphi \to JJ\varphi$, and $\mathbf{5}_J$, i.e. $\neg J\varphi \to J\neg J\varphi$. Hence, we can now once again derive the soritical

$$J\varphi_\alpha \to J\varphi_\beta$$

Accordingly, **Agg** ultimately fails and so does $\mathbf{K}_J$. Hence J does not agglomerate over conjunction, and just like the logic for K, the logic for J cannot be normal.

Conclusion

The proposed structural account of non-factive propositional all-things-considered justification comprises the following five principles governing the notion of being in a position to know:

E $J\varphi \leftrightarrow \neg K\neg K\varphi$
$\mathbf{T}_K$ $K\varphi \to \varphi$
$\mathbf{RN}_K$ If $\vdash \varphi$, then $\vdash K\varphi$
$\mathbf{RM}_K$ If $\vdash \varphi \to \psi$, then $\vdash K\varphi \to K\psi$
Lum $\neg K\neg K\varphi \to K\neg K\neg K\varphi$

Given its commitment to $\mathbf{RN}_K$ and $\mathbf{RM}_K$, the account involves hefty idealisations – idealisations which are, however, familiar from more standard epistemic logics. Embodying a non-normal logic for K, it yields a logic with **D45** as the non-normal logic for J. The account is neutral on whether justification is fully grounded in internally individuated mental states or in facts that are already introspectively or reflectively accessible. Thus, it allows one's justification to be grounded in externally individuated mental states or features of one's environment that are accessible only through outer sense. To this extent, it is open to both internalists and externalists. Even so, it provides a host of interesting principles of justification, of which some are often taken to presuppose internalist conceptions of what justification consists in. The account is likewise neutral on the debate between contextualists and invariantists about knowledge. It furthermore coheres both with the knowledge-first approach and with its rejection. As such, it should be of interest to epistemologists of different persuasion.

The account does, however, imply that justification is luminous, and so something that believers can be responsive to and appreciate when forming their beliefs. Its commitment to luminosity makes the account controversial. Yet, pending further argument, it has resources to resist recent anti-luminosity arguments, which makes it interesting albeit controversial.[16]

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