Hadronic spectra in holographic models

David Adame Carrillo
Facultat de Física, Universitat de Barcelona, Diagonal 645, 08028 Barcelona, Spain.[*]

Abstract: In this work we place ourselves in the framework of an holographic approach to QCD. We study the mass spectra of a family of dilaton backgrounds more general than the usual solvable one in a Soft-Wall model. Taking a particular limit of this family we recover the mass spectra of the Hard-Wall model.

I. INTRODUCTION

The theory we have for describing the strong interaction is Quantum Chromodynamics (QCD). This interaction is the responsible for quarks to stick together to form physical particles. As well, nucleons stay together in atomic nuclei thanks to the strong interaction. From this interaction arises what we call nuclear physics. The perfect understanding of this interaction may lead to a big number of advances. Unfortunately, QCD is still not perfectly understood.

Attempts to deal with this theory have been made following the path of holography. In holographic models, one tries to reproduce the quantum aspects of the theory by enlarging the number of dimensions. One of this models is based on what is called the Anti de Sitter/conformal field theory (AdS/CFT) correspondence, which states that the boundary of an AdS space can be regarded as the spacetime for a CFT [1].

The possibility of describing light mesons by means of a five-dimensional holographic model of QCD has attracted attention in recent years. Although these models still fail to give a general picture of the strong interaction, they reproduce interesting aspects of meson phenomenology. The Hard-Wall (HW) model has been proved to fit well some experimental data [2]. However, it does not yield a Regge-like trajectory of the mass spectrum (i.e., a linearly increasing mass spectrum). This has been accomplished (e.g., in Ref. [3]) by means of the Soft-Wall (SW) model. Whilst in the HW model one cuts off the fifth dimension, in the SW model one does so more smoothly by means of a dilaton background.

Inspired by those papers, in this work we explore the SW model by finding the spectra of a particular family of dilaton backgrounds. Even though changing the dilaton background from that used on Ref. [3] yields in a non-Regge-like trajectory, it is interesting to study its effect to the mass spectra.

This TFG paper is organized as follows. In Section II the holographic setup of the SW model is briefly reviewed, since its deep understanding is not the main purpose of this work. In Section III a family of dilaton background is proposed and its corresponding mass spectra are found and commented. A short summary of our results is contained in the concluding Section IV.

II. HOLOGRAPHIC SETUP

The purpose of this section is to comment on the holographic setup of the present work so that the experienced reader be able to understand where all the equations arise from. A more detailed explanation can be found in Ref. [3]. We consider a SW model in a 5D AdS space with the metric

$$ds^2 = \frac{R^2}{z^2}(dx_\mu dx^\mu - dz^2), \quad \mu = 0, 1, 2, 3, \tag{1}$$

where R is the radius of the AdS_5 space and $z > 0$ is the holographic coordinate. In the so called bottom-up approach, the global flavour symmetry group is $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. One postulates that the global symmetry becomes a gauge one. The simplest action (only the vector part of it) to include this symmetry is

$$S = -\frac{R^2}{g_5^2}\int d^4x dz \frac{e^{\varphi(z)}}{4z}\mathrm{Tr}\left\{V_{MN}^2\right\}, \tag{2}$$

where

$$V_{MN} = \partial_M V_N - \partial_N V_M - i\left[V_M, V_N\right], \tag{3}$$

$V_M = (L_M + R_M)/2$, L_M and R_M are 5D Abelian fields dual to the sources of the left and right 4D vector currents, g_5^2 is a constant fixed to match some asymptotic conditions [3], and the dilaton background $e^{\varphi(z)}$ is not yet fixed for generality.

We can still fix the gauge. For convenience, we choose to work in the axial gauge, i.e., $V_z = 0$. The equation of motion becomes

$$\partial_z\left(\frac{e^\varphi}{z}\partial_z V_\mu\right) - \frac{e^\varphi}{z}\partial_\mu^2 V_\mu = 0. \tag{4}$$

By making the 4D Fourier transform of Eq. (4) and assuming a standard plane wave ansatz $V_\mu(p, z) = \varepsilon_\mu v(z)$

[*]Electronic address: dadameca7@alumnes.ub.edu

for the 4D Fourier transform of $V_\mu(x,z)$, we obtain the equation

$$\partial_z\left(\frac{e^\varphi}{z}\partial_z v(z)\right)+\frac{e^\varphi}{z}p^2 v(z)=0. \quad (5)$$

The physical spectrum of mass ($p^2=m^2$) is given by the eigenvalues of this differential equation.

III. HADRONIC SPECTRA

The starting point of this work is Eq. (5). It can be brought into a Schrdinger-like equation by performing the substitution

$$v(z)=\sqrt{z/e^\varphi}\,\Psi(z), \quad (6)$$

which leads to

$$-\frac{d^2\Psi}{dz^2}+\left[\frac{\varphi''}{2}-\frac{\varphi'}{2z}+\frac{\varphi'^2}{4}+\frac{3}{4}\frac{1}{z^2}\right]\Psi_{\pm,n}=p^2\Psi. \quad (7)$$

We shall impose the boundary condition $\Psi(z=0)=0$ [3]. In a SW model, the boundary condition at $z=\infty$ is requiring the action to be finite.

This way, we have transformed the problem of finding the mass spectrum of a meson into finding the energy spectrum of a non-relativistic particle moving in a central potential with angular momentum $l(l+1)=3/4$. The central potential is a function of the first an second derivatives of the background $\varphi(z)$.

With the aim of finding solvable model, the background is set to $\varphi=-\lambda^2z^2$ in Ref. [3]. This background yields an harmonic potential and, therefore, a Regge-like trajectory,

$$\frac{m_-^2}{\lambda^2}=4n, \quad n=1,2,... \quad (8)$$

Yet, it is worthwhile to explore some other behaviours so as to acquire a wider picture of the theory. We let the background be

$$\varphi(z)=\pm(\lambda z)^\alpha \quad \alpha>0. \quad (9)$$

Even though only the (-) case leads to a meaningful dilaton background, studying the (+) case is also of interest.

Substituting Eq. (8) in Eq. (7), and introducing the dimensionless variable $y=\lambda z$, one finds for the discrete mass spectrum $m_n^2=p_n^2$,

$$-\frac{d^2\Psi_{\pm,n}}{dy^2}+\left[V_\alpha^\pm(y)+\frac{3}{4}\frac{1}{y^2}\right]\Psi_{\pm,n}=\frac{m_{\pm,n}^2}{\lambda^2}\Psi_{\pm,n}. \quad (10)$$

where

$$V_\alpha^\pm(y)=\frac{\alpha y^{\alpha-2}}{2}\left(\frac{\alpha y^\alpha}{2}\pm(\alpha-2)\right). \quad (11)$$

In Figures 1 to 4, $V_\alpha^\pm$ is plotted for different ranges of α and both $\pm$ sign.

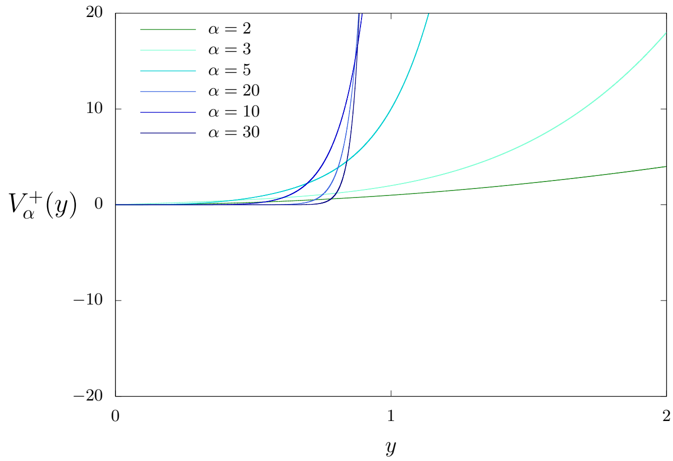

Figure 1: V_α^+ for some values of $\alpha>2$. The shape of the potential gets closer to an infinte well as α increases. For $\alpha=2$ we recover the harmonic potential.

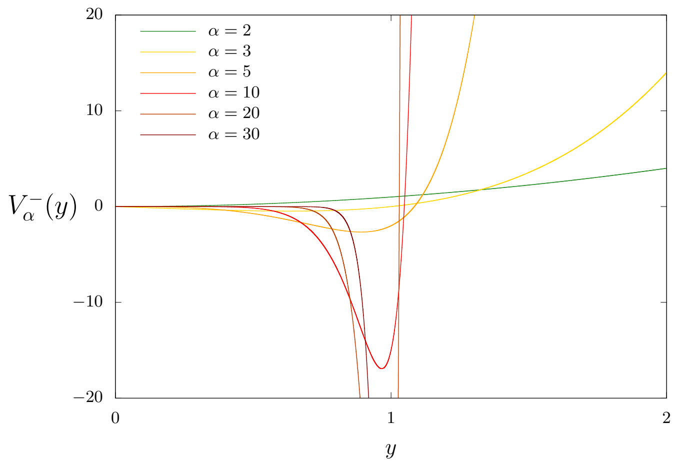

Figure 2: V_α^- for some values of $\alpha>2$. As α increases an infite wall appears at $y=1$. This potential features a valley that gets close to $y=1$, narrows and gets deeper as α increases. For $\alpha=2$ we recover the harmonic potential.

One can solve equation Eq. (10) numerically to find the mass spectra for different values of α, which are shown in Figures 5 to 9. For $\alpha<1$ the potential does not present bounded states.

Given that V_α^+ converges towards an infinite potential well as $\alpha\to\infty$ (see Figure 1), one should expect the eigenvalues m_+^2/λ^2 to converge towards those of that potential for large values of α. This actually happens, as shown in Figure 5.

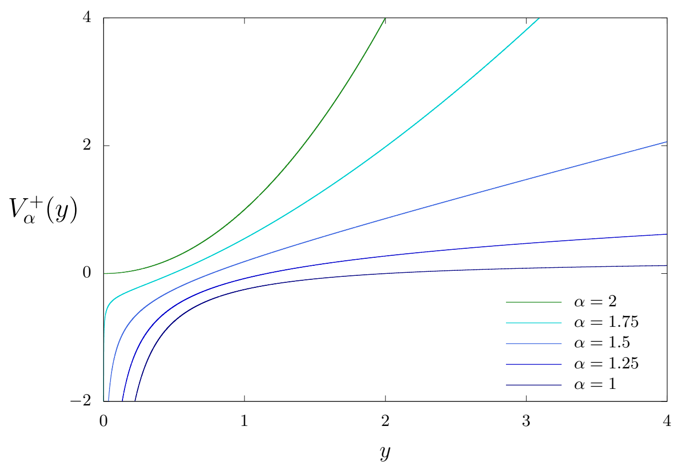

Figure 3: V_α^+ for some values of $1 < \alpha < 2$. The potential diverges as $y^{2(\alpha-1)}$ for large y and $-1/y^{2-\alpha}$ for $y \to 0$. For $\alpha = 1$ we recover an atom-like potential ($V^+ \propto 1/y$).

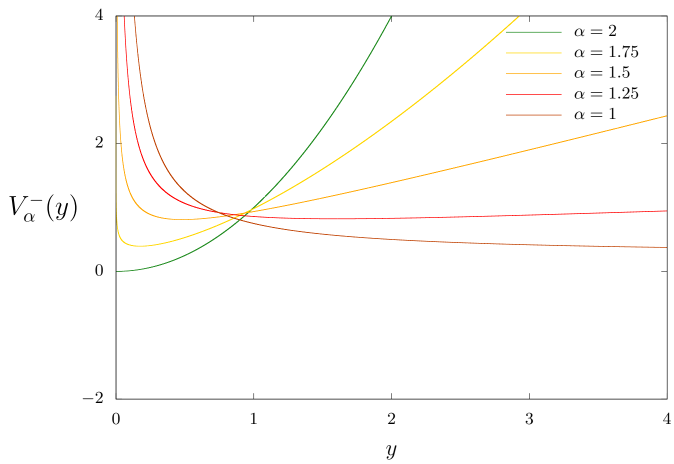

Figure 4: V_α^- for some values of $1 < \alpha < 2$. The potential diverges as $y^{2(\alpha-1)}$ for large y and $1/y^{2-\alpha}$ for $y \to 0$. These potentials do not present any further feature.

On the other hand, the shape of V_α^- is less clear in the limit $\alpha \to \infty$, as a result of the valley that appears near $y = 1$ (see Figure 2). One could expect the eigenvalues m_-^2/λ^2 to converge towards those of a 3D infinite well, if the negative area of the valley shrinked as α increased, to the point that in the limit $\alpha \to \infty$ the area was null. The value of this area is clearly

$$A(\alpha) = \int_0^{\tilde{y}(\alpha)} V_\alpha^-(y)dy, \qquad (12)$$

where the upper limit satisfies $V_\alpha^-(\tilde{y}) = 0$. One can easily check $\lim_{\alpha\to\infty} A(\alpha) \to \infty$. Therefore, V_α^- does not converge towards an infinite well for large α whatsoever. However, in this situation, the dilaton background e^φ converges to a step function $\theta(\lambda z - 1)$. Therefore, Eq. (5) becomes that of a HW model. We expect in this limit ($\alpha \to \infty$) the spectra to converge to that of the HW model, i.e.,

$$J_0\left(m_- z_m\right) = 0, \qquad (13)$$

according to Ref. [2]. z_m in Eq. (13) is the cutoff radius of the holographic dimension. It is easy to check that our background cuts the dimension at $z = 1/\lambda$, so $z_m = 1/\lambda$. The spectum actually converges as expected as shown in Figure 6.

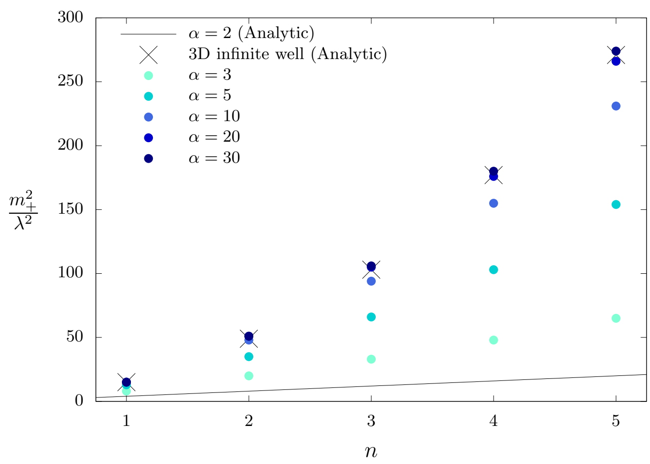

Figure 5: Eigenvalue spectra of V_α^+ for some values of $\alpha > 2$. The spectra rise up from the Regge-like trajectory for $\alpha = 2$. As α increases, the spectrum converges towards that of an 3D infinite well.

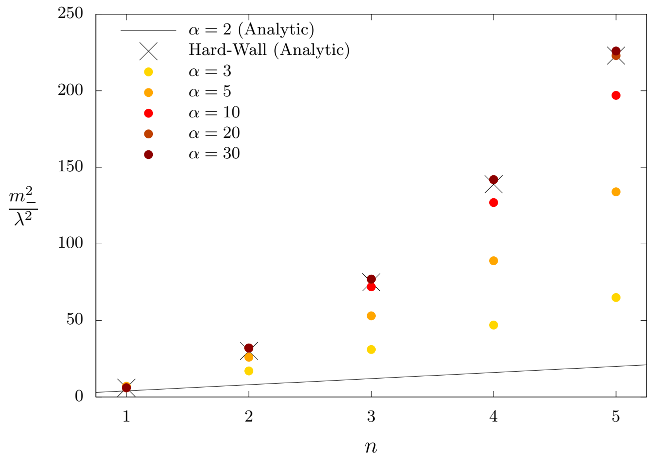

Figure 6: Eigenvalue spectra of V_α^- for some values of $\alpha > 2$. The spectra rise up from the Regge-like trajectory for $\alpha = 2$. As α increases, the spectrum converges towards that of the HW model.

As for $\alpha < 2$, the difference between the shape of V_α^+ and V_α^- leads to different spectra. Yet, they, the spectra, are not significantly different; except for the fact that for V_1^+ one finds an atom-like spectrum (infinite and bounded) that can be found analytically,

$$\frac{m^2_{+,n}}{\lambda^2} = \frac{1}{4}\left(1 - \frac{1}{2n^2}\right) \qquad n = 1, 2, ..., \tag{14}$$

but for V_1^- there are not bounded solutions. These spectra are shown in Figures 7 and 8.

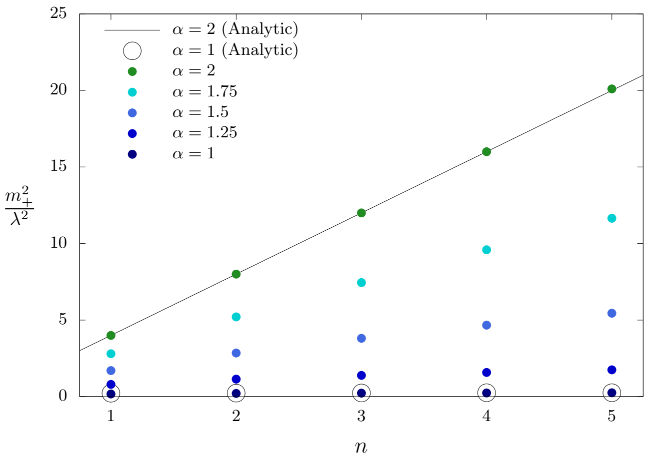

Figure 7: Eigenvalue spectra of V_α^+ for some values of $\alpha < 2$. The spectra come down from the Regge-like trajectory for $\alpha = 2$. For $\alpha = 1$ we recover the spectrum of an Hydrogen-like atom shifted.

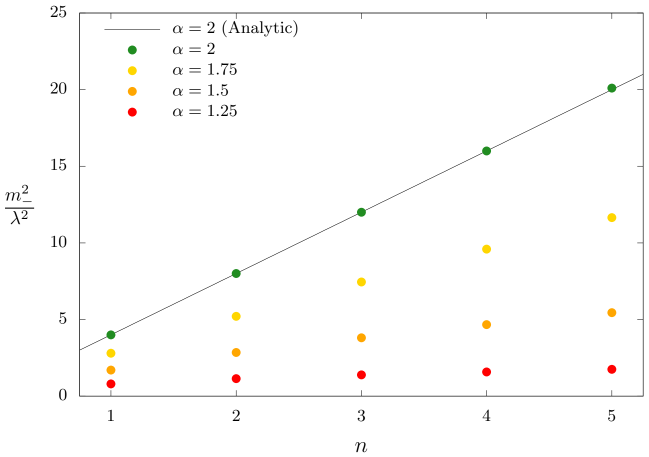

Figure 8: Eigenvalue spectra of V_α^- for some values of $\alpha < 2$. The spectra come down from the Regge-like trajectory for $\alpha = 2$. These spectra do not present any further features.

IV. CONCLUSIONS

We have considered the family of dilaton backgrounds given by Eq. (9) in the Soft-Wall model. We have simplified the problem into that of an Schrödinger-like equation following Ref. [3]. Once the problem was simpler, we have studied its features in all the possible ranges. The most interesting ones are the following.

On the one hand, in the $(+)$ situation, although the resulting dilaton backgroun is not plausible, we have seen the convergence of the spectrum towards that of a particle in a spheric box as $\alpha \to \infty$. Likewise, for $\alpha = 1$, we find an atom-like bounded spectrum that can be found analyticaly.

On the other hand, and more remarkably, in the $(-)$ situation, we have seen the convergence of the Soft-Wall spectrum towards that of the Hard-Wall as $\alpha \to \infty$. In our model, λ plays the role of $1/z_m$ in the Hard-Wall model when it comes to the mass spectrum.

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