Effective description of the EW symmetry breaking

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The discovery of the Higgs boson in 2012 was a huge moment of achievement: the particle postulated more than 50 years ago was at last discovered. Even so, the particle acted as it was expected and, paradoxically, gave no new clues about to where to look next. In this report we briefly discuss what we call the Higgs mechanism, which gives mass to the W and Z when they interact with an invisible field, the Higgs field, that pervades the universe. Moreover, the Electroweak Chiral Lagrangian is presented, which allows us to consider an alternative way to perform an analysis of the model. A brief outline and some final reflections are exposed in the conclusion section.

I. INTRODUCTION

One of the greatest achievements in the history of physics is the development of a theory such as the Standard Model of fundamental interactions (SM). Three of the four known fundamental forces in the universe (the electromagnetic, weak and strong interactions) and how the basic building blocks of matter interact are very accurately explained by this well-tested physics theory.

Since the 20th century, principles of symmetry have been playing a critical role in fundamental physics, especially in quantum field theory. Moreover, an appealing connection is set between the gauge symmetries and the fundamental interactions. In a more formal manner, it is well established that the theoretical framework of the SM is based on the gauge symmetry group $\mathrm{SU}(3)_C\times \mathrm{SU}(2)_L\times \mathrm{U(Y)}_Y$. While the $\mathrm{SU}(3)_C$ gauge group describes the strong interaction, the $\mathrm{SU}(2)_L\times \mathrm{U(Y)}_Y$ is connected to the electroweak interaction.

Even though the SM seemed at first sight to be a complete description of the subatomic world, it was found experimentally that the mediators of the electroweak interaction, the W^+, the W^- and the Z, had masses different from zero. At that time, it was well understood that this fact was strictly forbidden by the gauge symmetry in the SM. Because of this, and because of others aspects, the so-called electroweak symmetry $\mathrm{SU}(2)_L\times \mathrm{U(Y)}_Y$ had to be necessarily broken.

On 4 July 2012, the ATLAS and CMS experiments at CERN's Large Hadron Collider (LHC) revealed one of the models proposed to explain the aforementioned symmetry breaking to be the correct one: the Higgs model. In fact, what the experiments at CERN's discovered was a new particle in the mass region around 126 GeV consistent with the Higgs Boson, the transparent manifestation of the Higgs mechanism. Consequently, this led to the award of the Nobel prize in physics to François Englert and Peter Higgs on 8 October 2013 [1].

The aim of the present work, therefore, is to analyse some general effective theory that enables us to comprehend the Higgs mechanism with a different approach. The report is organized as follows. Section II introduce the idea of Effective Field Theories and Effective Lagragians, a general tool to describe Field Theories only constructed by symmetry considerations.

Goldstone's Theorem is briefly mentioned in Section III to bring forward the Goldstone bosons, bosons that arise when a continuous symmetry is broken. Their introduction is determined by the critical role they will play in further development.

In Section IV we find a major section that explores the so-called Higgs mechanism in two different manners, the latter being of extreme relevance to reveal an important symmetry hidden in the relevant Lagrangian.

And finally, the paper ends in Section V by showing the construction of a general Lagrangian with all important symmetries that allows us to see beyond the Standard Model Lagrangian terms.

II. EFFECTIVE FIELD THEORIES

The theoretical framework known as the Standard Model (SM) is a quantum field theory that describes the electroweak and strong interactions of quark and leptons. Schematically, it consists of:

- A matter sector which is made of fermionic fields.
- Vector boson gauge fields.
- A symmetry breaking sector (SBS).

The last one is needed to provide masses for the fermions and EW bosons and it is not as well established as the other two.

In the aforementioned symmetry breaking sector we find the issue under discussion: the electroweak symmetry breaking (EWSB). A very appealing option to describe this symmetry breaking is to use effective theories. An Effective Field Theory is a quantum field theory that enables us to study the relevant physics of our system without the need of specifying a particular model at high energies. It can be constructed from the relevant symmetries of the light modes at low energies. Nonetheless,

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both light and heavy degrees of freedom are included, the latter having been integrated out.

One of the simplest theory that allows us to describe the EWSB is the Electroweak Chiral Lagrangian (ECL). The choice of the name for the ECL is due to the similarity between the EWSB pattern and the one of the chiral symmetry in Quantum Chromodyamics.

III. THE GOLDSTONE THEOREM

Models exhibiting spontaneous breakdown of continuous symmetries lead to the appearance of massless particles. This is a general result known as *Goldstone theorem.* For each generator of the broken symmetry we have these massless particles known as Goldstone bosons (GB). In the case of Quantum Chromodynamics (QCD) we have an approximate symmetry in $\mathcal{L}_{QCD}$ known as the chiral symmetry. When it is spontaneously broken, the Goldstone bosons associated to his breaking are identified (at least approximately) with the three pions, $\pi^{\pm}$ and π^0. This example is one of the many light bosons seen in physics that may be interpreted as Goldstone bosons.

In the Electroweak Chiral Lagrangian the EWSB gives three Goldstone bosons, w^+, w^-, and w^0, corresponding to the longitudinal degrees of freedom of the electroweak mediators, W^+, W^- and Z, respectively. Moreover, the Higgs boson could also be interpreted as en extra Goldstone boson in some models such as the Minimal Composite Higgs Model [2], but this is not necessarily so.

A very transparent but general proof of Goldstone's theorem for classical scalar field theories is presented in [3].

IV. THE HIGGS MECHANISM

A. A first approach

The simplest way to give masses to the W's and Z is via the complex doublet of spin-zero Higgs field:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 - i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \tag{1}$$

where the superscripts are the Q electric charge assigments according to $Q = Y + T_3$. The conventional $\sqrt{2}$ ensures that the fields are normalized in the same way.

The lagrangian density that describes this scalar sector reads as:

$$\mathcal{L} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi^\dagger \Phi) \tag{2}$$

where the covariant derivate is defined as:

$$\begin{aligned} D_\mu \Phi &= (\partial_\mu + ig\vec{W}_\mu \vec{T} + ig' B_\mu Y)\Phi \\ &= (\partial_\mu + ig\vec{W}_\mu \frac{\vec{\tau}}{2} + i\frac{g'}{2} B_\mu I)\Phi \end{aligned} \tag{3}$$

The idea is to apply the concept of spontaneous symmetry breaking to the SU(2) × U(1) model of the electroweak interactions. The most general $SU(2)_L \times U(1)_Y$ invariant potential depends only on the combination $\Phi^\dagger\Phi \equiv \Phi^2$. Here we follow the particular form of the potential presented in [4]:

$$V(\Phi) = \lambda(\Phi^2 + \frac{\mu^2}{2\lambda})^2 \tag{4}$$

with λ and μ^2 arbitrary parameters. Furthermore, the literature commonly adopts an equivalent potential $V(\Phi) = -\mu^2 \Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$, where the constant factors are removed and the λ and μ^2 parameters are properly redefined [5][3].

Minimizing the potential V we find two solutions, the trivial solution $\langle\Phi\rangle_0 = 0$ and the nontrivial solution:

$$\langle\Phi^\dagger\Phi\rangle_0 = -\frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2} \tag{5}$$

Therefore, we have a minimum away from the origin provided that the quantity μ^2 in the potential is negative.

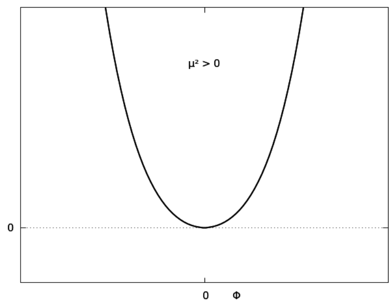

FIG. 1: Potential V(Φ) as a function of Φ for the positive sign of the μ^2 term. Trivial solution for the vacuum implies that the value of all the fields ϕ_i in the minimum energy state is zero.

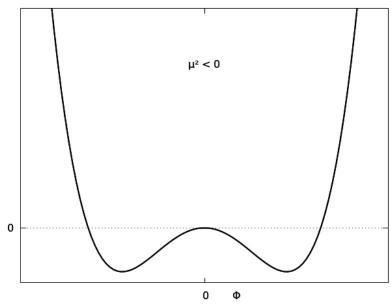

FIG. 2: Potential V(Φ) as a function of Φ for the negative sign of the μ^2 term. Nontrivial solution for the vacuum implies that at least one of the four fields (ϕ_i) must be non-zero. Hence, in general, we cannot treat all four components of Φ in a symmetric manner.

A nontrivial vaccum Higgs configuration which obeys the constraint Eq.5 and preserve the quantum properties and quantum numbers of the vacuum is:

$$\langle\Phi\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \tag{6}$$

for real v.

Of course, this is purely conventional: one can make an SU(2) × U(1) transformation to make any vaccum expectation value (VEV) of $\Phi^\dagger$ and Φ^0 have the aforementioned form. See a demonstration in [4].

B. Spontaneous symmetry breaking

It is convenient now to introduce the following conjugate doublet:

$$\tilde{\Phi} \equiv i\tau_2\Phi^* \tag{7}$$

And introduce the matrix [6]

$$M(x) = \sqrt{2}\left(\tilde{\Phi}\Phi\right) = \sqrt{2}\begin{pmatrix} {\phi^0}^\dagger & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix} \tag{8}$$

which allows us to recast the Standard Model Lagrangian involving the Higgs doublet in a completely equivalent form as:

$$\begin{aligned}\mathcal{L}_{SBS} = &\frac{1}{4}Tr[(D_\mu M)^\dagger(D^\mu M)] \\ &-\frac{1}{4}\lambda[\frac{1}{2}Tr(M^\dagger M) + \frac{\mu^2}{\lambda}]^2\end{aligned} \tag{9}$$

with

$$D_\mu M = \partial_\mu M + L_\mu M - MR_\mu \tag{10}$$

and

$$L_\mu M = ig\frac{\vec{\tau}}{2}\vec{W}_\mu \quad R_\mu = ig'\frac{\tau_3}{2}B_\mu \tag{11}$$

It is easy to show now that the Lagrangian $\mathcal{L}_{SBS}$ is invariant under the so-called electroweak chiral symmetry $SU(2)_L \times SU(2)_R$. One can check it by performing two global and independent chiral transformations on M

$$M \to LMR^\dagger \quad L, R \in SU(2)_{L,R} \tag{12}$$

and check that the $\mathcal{L}_{SBS}$ remains the same. That symmetry was not so obvious before introducing the equivalent formalism with matrix M. However, this hidden symmetry now appears as evident.

The interesting issue is that this symmetry is spontaneously broken into the custodial symmetry group

$$SU(2)_L \times SU(2)_R \to SU(2)_C = SU(2)_{L+R} \tag{13}$$

To see that, it is useful for our purpose to notice that the matrix M can be written as $M(x) = \sigma(x)U(x)$, where $\sigma(x)$ is a real scalar field and $U(x)$ is an $SU(2)$ field. Computing $M^\dagger M$ one can get convinced that $\sigma^2 = 2(|\phi^0|^2 + \phi^+\phi^-) \geq 0$. Thus, defining the quantity $\frac{\mu^2}{\lambda}$ to be positive the Lagrangian reads:

$$\begin{aligned}\mathcal{L} = &\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{\sigma^2}{4}Tr[(D_\mu U)^\dagger(D^\mu U)] \\ &-\frac{\lambda}{4}(\sigma^2 - \frac{\mu^2}{\lambda})^2\end{aligned} \tag{14}$$

If we minimize the potential as we did before, we get, apart from the trivial solution $\sigma = 0$ (which corresponds to unestable maximum), the nontrivial vaccum expectation value (VEV):

$$\sigma = \pm\sqrt{\frac{\mu^2}{\lambda}} \equiv \pm v \tag{15}$$

which implies:

$$M_0 = \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix} = vI \tag{16}$$

Therefore, the unitary matrix U in the vacuum turns out to be $U_0 = I$ and now the vacuum is left invariant only if L= R. Hence, the global symmetry group $SU(2)_L \times SU(2)_R$ has been broken to $SU(2)_{L+R}$ and the three Goldstone bosons associated to symmetry breaking are contained in the matrix U.

Ignoring the fluctuations around the vacuum v, we replace M $\to v$U to get

$$\mathcal{L} = \frac{v^2}{4}Tr(D_\mu U D^\mu U^\dagger) \tag{17}$$

To interpret this theory, suppose that the system is near one of the minima (say the positive one). Then it is convenient to define the shift field

$$\bar{\sigma} = \sigma - v \tag{18}$$

and rewrite $\mathcal{L}$ in terms of $\bar{\sigma}$. Dropping the constant term, equation 14 becomes:

$$\begin{aligned}\mathcal{L} = &\frac{1}{2}\partial_\mu\bar{\sigma}\partial^\mu\bar{\sigma} + \frac{(\bar{\sigma}+v)^2}{4}Tr[(D_\mu U)^\dagger(D^\mu U)] \\ &-\frac{\lambda}{4}((\bar{\sigma}+v)^2 - \frac{\mu^2}{\lambda})^2 \\ = &\frac{1}{2}\partial_\mu\bar{\sigma}\partial^\mu\bar{\sigma} + \frac{(\bar{\sigma}+v)^2}{4}Tr[(D_\mu U)^\dagger(D^\mu U)] \\ &-\frac{1}{2}(\sqrt{2}\mu)^2\bar{\sigma}^2 - \lambda v\bar{\sigma}^3 - \frac{\lambda}{4}\bar{\sigma}^4\end{aligned} \tag{19}$$

This Lagrangian describes a simple scalar field of mass $M_H = \sqrt{2}\mu = v\sqrt{2\lambda}$ with $\bar{\sigma}^3$ and $\bar{\sigma}^4$ interactions. This is the so-called Higgs field, whose mass is usually represented by M_H. Predicted more than 50 years ago, the Higgs boson was at last discovered with a mass nowadays of $M_H = 125.09 \pm 0.21$ GeV [7].

V. ELECTROWEAK CHIRAL LAGRANGIAN

In the Electroweak Chiral Lagrangian (ECL), the Electroweak Goldstone bosons (EW GB) are introduced in a non-linear exponential representation

$$U = exp\left(i\frac{\sigma^a\omega^a}{v}\right), \tag{20}$$

where ω^a are the EW GB fields, σ^a are the Pauli matrices, both for a=1,2,3 and $v = 246$GeV is the VEV of the SM scalar doublet.

Our aim is to construct the most general ECL of QED, with interactions terms invariant under chiral SU(2) transformations. The simplest chiral and Lorentz invariant term involving the U field is the derivative-free term [5]:

$$Tr(UU^\dagger) = 2 \tag{21}$$

Since this is a constant it can always be removed from the Lagrangian. Thus, the general Lagrangian will contain terms with arbitrary number of derivatives classifiying the operators according to their energy dimension:

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + ... \tag{22}$$

where $\mathcal{L}_n$ denotes generally the term with n derivatives.

Therefore, we are able to construct the operators of the ECL order by order compatible with the relevant symmetries. In a general sense, we consider an analytic function of σ that is locally given by:

$$f(\sigma) = f(0) + f'(0)\sigma + \frac{1}{2}f''(0)\sigma^2 + ... \tag{23}$$

This way, at the lowest order of the Higgless EW Chiral Lagrangian we will have:

$$\begin{aligned}\mathcal{L}_2 &= \frac{1}{4}[f_2(\sigma)]^2 Tr(D_\mu U D^\mu U^\dagger) \\ &= \tfrac{1}{4}[v^2 + 2av\sigma + (vb + a^2)\sigma^2 + ...]Tr(D_\mu U D^\mu U^\dagger)\end{aligned} \tag{24}$$

where we have defined $v \equiv f_2(0)$, $a \equiv f_2'(0)$, $b \equiv f_2''(0)$ and so on. For the next-to-leading order Lagrangian

$$\begin{aligned}\mathcal{L}_4 &= (a_4 + ...)[Tr((D_\mu U)U^\dagger(D_\nu U)U^\dagger)]^2 \\ &+(a_5 + ...)[Tr((D_\mu U)U^\dagger(D^\mu U)U^\dagger)]^2\end{aligned} \tag{25}$$

with $a_4, a_5 \equiv f_4(0)$ and always understanding the covariant derivative of the matrix U as:

$$D_\mu U = \partial_\mu U + i\frac{g}{2}W_\mu^a\sigma^a U - i\frac{g'}{2}UB_\mu\sigma^3 \tag{26}$$

with the Pauli matrices σ^a and ω^a the EW GB fields (a=1,2,3).

Hence, we have constructed a more general Lagrangian than the one given by the Standard Model. Indeed, replacing $a = b = 0$ and $a_4 = a_5 = 0$ we recover the Lagrangian given by equation 17.

The Lagrangian involving terms with four derivatives depends on v but also on other effective coefficients usually known as chiral parameters or chiral coefficents. They are of great importance since they encode the information on the heavy modes that have been integrated out. As they parametrize the interaction between EW gauge bosons and the Higgs, the determination of their numerical value will provide us critical information about the mechanism under the EWSB. But, moreover, an appeling issue is that different sets of values for chiral parameters correspond to different theories, including the SM. That is, potential desviations from the SM predictions, that could be detected experimentally, depend on the value of theses couplings or parameters.

A. Tree Level

We are able now to extract the Tree Level interactions of these two contributions to the Lagrangian, $\mathcal{L}_2$ and $\mathcal{L}_4$. This can be done by expanding the exponential as

$$U = 1 + \frac{i\sigma^a\omega^a}{v} - \frac{\sigma^a\sigma^b\omega^a\omega^b}{2v^2} - \frac{i\sigma^a\sigma^b\sigma^c\omega^a\omega^b\omega^c}{6v^3} + ... \tag{27}$$

For the sake of simplicity, we will omit the B_μ field in the calculation due to no conceptual change is made in this. For our purpose, we will only keep those terms involving up to four ω fields or two ω fields with one W_μ^a.

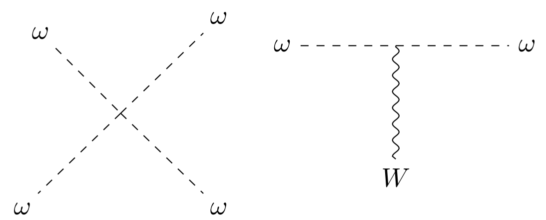

FIG. 3: Diagrams contributing at tree level

The terms that appear in the Higgless EW Chiral Lagrangian are presented below:

$$\begin{aligned}\mathcal{L}_2 &= \frac{1}{2}\partial_\mu\omega^a\partial^\mu\omega^a + \frac{1}{6v^2}[(\omega^a\partial_\mu\omega^a)(\omega^b\partial^\mu\omega^b) \\ &-(\omega^b\partial_\mu\omega^a)(\omega^b\partial^\mu\omega^a)] - \frac{g\epsilon^{abc}}{2}\partial_\mu\omega^a W^{b\mu}\omega^c\end{aligned} \tag{28}$$

$$\begin{aligned}\mathcal{L}_4 &= 4\frac{a_4}{v^4}\partial_\mu\omega^a\partial_\nu\omega^a\partial^\mu\omega^b\partial^\nu\omega^b \\ &+4\frac{a_5}{v^4}\partial_\mu\omega^a\partial^\mu\omega^a\partial_\nu\omega^b\partial^\nu\omega^b\end{aligned} \tag{29}$$

where we have used the well-known trace properties of Pauli matrices

$$\begin{aligned}tr(\sigma_a\sigma_b) &= 2\delta_{ab} \\ tr(\sigma_a\sigma_b\sigma_c) &= 2i\epsilon_{abc} \\ tr(\sigma_a\sigma_b\sigma_c\sigma_d) &= 2(\delta_{ab}\delta_{cd} - \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc})\end{aligned} \tag{30}$$

Notice there is no mass term for the above Lagrangians since they describe Goldstone bosons.

B. Feynman Rules

In pursuit of a more detailed analysis, we are able to compute the Feynman rules from the interaction terms of $\mathcal{L}_2$ and $\mathcal{L}_4$ Lagrangians.

For the first interaction term in $\mathcal{L}_2$ Lagrangian we find:

$$\begin{aligned}&\frac{i}{3v^2}[(\delta^{ab}\delta^{cd}+\delta^{ad}\delta^{bc}-2\delta^{ac}\delta^{bd})t\\&+(\delta^{ab}\delta^{cd}+\delta^{ac}\delta^{bd}-2\delta^{ad}\delta^{bc})u\\&-(\delta^{ad}\delta^{bc}+\delta^{ac}\delta^{bd}+2\delta^{ab}\delta^{cd})s]\end{aligned} \tag{31}$$

where we have introduced the Mandelstam variables:

$$\begin{aligned}s&=(p_1+p_2)^2=(p_3+p_4)^2\\t&=(p_1-p_3)^2=(p_2-p_4)^2\\u&=(p_1-p_4)^2=(p_2-p_3)^2\end{aligned} \tag{32}$$

Likewise, the second term gives the following contribution

$$\frac{g\epsilon^{abc}}{2}(\delta^{ad}\delta^{ce}p_{1\mu}+\delta^{ae}\delta^{cd}p_{2\mu})W^{b\mu} \tag{33}$$

For the $\mathcal{L}_4$ contribution we put Feynman rules for the first term

$$\begin{aligned}&\frac{ia_4}{v^4}[(3\delta^{ab}\delta^{cd}+4\delta^{ad}\delta^{bc})t^2\\&+(4\delta^{ab}\delta^{cd}+4\delta^{ac}\delta^{bd})u^2\\&+(\delta^{ab}\delta^{cd}+4\delta^{ac}\delta^{bd}+4\delta^{ad}\delta^{bc})s^2]\end{aligned} \tag{34}$$

as an example.

VI. OUTLOOK AND CONCLUSIONS

Throughout this work we have dealt with electroweak chiral symmetry $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$, a global symmetry included in the electroweak Standard Model and with the required global symmetry breaking pattern $SU(2)_L\times SU(2)_R\to SU(2)_{L+R}$. After presenting a first approach of the Higgs mechanism, an equivalent formalism assisted by the abovementioned matrix M is developed. It is worth highlighting that the Higgs field is introduced *ad hoc* in the theory. That is, relevant quantities of the Higgs such as the Higgs self coupling cannot be predicted. Therefore, there are some issue left that SM cannot explain yet.

The fact that matrix M could always be written as $M=\sigma U$ was of great help for a better understanding. One can appreciate that the potential in equation 14 is only a function of σ, V=V(σ). Thus, the components of M that are not σ (i.e. U) lead us to a different but equivalent vacua. Indeed, moving along these directions cost no energy at all.

In the final section we focus in a very appeling way of approaching to the matter under discussion. Effective Lagrangians brings us the opportunity to consider a more general view of this kind of theoretical descriptions. Based on the relevant symmetries, we are able to treat with more interactions terms beyond those ones predicted by the SM. These terms could change the strengh of the interactions or bring some new effects that could, eventually, be detected experimentally.

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