Collective phenomena in social animal dynamics

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Abstract: Collective motion is a fascinating and well-proved behavior of social animals. Bird flocks, fish schools or sheep herds are common examples of that phenomenon. In the following paper, we analyze that kind of behavior of social animals, reproducing on that purpose the so called Vicsek model. In the model, animals copy their neighbors, averaging their direction of motion in each step with more or less precision depending on the intensity of a noise term. This gives raise to a phase transition between polarized and unpolarized states. We also analyze the effect of leadership and show that this new ingredient eliminates the phase transition.

I. INTRODUCTION

Fish, birds or sheep, but also bacteria and other subcellular entities tend to behave in a collective manner [1]. Therefore, collective motion is a phenomena that embraces a wide range of scales, and it has been waking the interest of different research fields.

Among those, of course, one can find several physics groups working actively on the subject. From our point of view, interest resides on the far from equilibrium range in which that phenomena takes place. After all, the individuals of any flock are active particles, and that organization seems to happen spontaneously, with no external influence that forces the collective behavior. The easiest model used to account for the behavior observed in animal groups is very similar to the XY model used in the study of magnetism, and is known by the name of Vicsek model, proposed by Tamás Vicsek *et al* ([2],[3]).

The observation and investigation of social animals behavior, showed that different groups behaved similarly. This fact led to the Vicsek model, which reproduced the experimental findings with certain precision, and had the advantage that it had not a great mathematical difficulty. This is why we decided to use the model in our work.

The basis of that model is the constant updating of the direction of motion of each particle [2]. After every time step, their direction of motion will be the mean of the directions of all its closest neighbors. Thus being the ideal case, one also introduces a noise term, disturbing the perfect averaging of the particles, which simulates the imperfect reasoning of actual animals. The interesting thing to observe is a phase transition between ordered and disordered states taking place at an intermediate noise value, which plays the role of an effective temperature in the classical XY model.

In section (II) we explain the way the model is supposed to work, how we did implement it, and we also reproduce the basic results, as well as extract some physical analogies with the XY model. Once that first part is proved, we will introduce new features in the social interactions among animals, collected in section (III), in the form of studying their behavior in the presence of one (III A) and two leaders with different preferences(III B). Studying their response in front of two opposed stimulations will be the final part of our work.

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II. THE VICSEK MODEL

The Vicsek model simplifies the interaction between animals in a system to a basic concept: imitation. The particles in our virtual box will be interacting among themselves by looking at their closest neighbors, and averaging their directions. Thus, we have got to define what closest means. In the Vicsek model, two particles will be able to interact depending on the euclidean distance that separates them. But in nature there is no perfect behavior, and to reflect that on our model, we introduce a noise term that will disturb the perfect averaging.

The model describes the motion of N self-propelled particles in an overdamped dynamics [2]. Each one of these particles is characterized by two variables: its position in the system, $\mathbf{r}_i(t)$, and its direction of motion, $\theta_i(t)$, both in a given time. Therefore, the change in the position after each time-step will be given by the velocity and direction of motion, following Eq. (1).

$$\mathbf{r}_i(t+\Delta t) = \mathbf{r}_i(t) + \Delta t \mathbf{v}_i(t), \quad (1)$$

where $\mathbf{v}_i(t)$ represents the velocity of each particle in a given time t, and Δt represents the time step. That velocity has, of course, two components in our two-dimensional model, depending on the time-changing angle $\theta_i(t)$, which we assume arbitrarily to be in the interval $[-\pi;\pi]$. We set a constant velocity modulus v_0, so that our components in each step are $\mathbf{v}_i(t) = v_0(\cos(\theta_i(t)); \sin(\theta_i(t)))$.

Now let us focus on the angular variation with time, which includes the interactions of the particles with their neighbors, as well as noise irregularities. After each time-step, the i-th particle will change its direction of motion, as the result of the averaging direction from all its neighbors found in a circle with a radius R_0 centered in that particle i. Therefore, one would obtain the new angle of motion as

$$\theta_i(t + \Delta t) = \arg(\sum_{k\in S} \mathbf{v}_i(t)) + \eta\xi_i(t), \quad (2)$$

where the sum extends to all particles found inside the surface S, defined by the radius R_0, as $S = \pi R_0^2$. Note that, as the function arg is defined as the arc tangent of the two projections of the velocity in y and x respectively, $\arg(z) \equiv \arctan(\frac{y}{x})$, it is not necessary to normalize the components of each direction dividing by the number of particles inside the surface S. It is important to notice that in Eq. (2) we consider the particle i has no privilege over its neighbors, as it has the same weight in the averaging.

One can observe that we have introduced a second term in Eq. (2), $\eta\xi_i(t)$, which we call *noise* term. $\xi_i(t)$ is a uniformly distributed random number defined within the interval $[-\pi; \pi]$. The multiplicative factor η will be our way to control the noise intensity. We will be able to change it at our pleasure, in the range $[0; 1]$. $\xi_i(t)$ is a delta-correlated noise with zero mean.

Taking a step further in our analogy with the XY model, one introduces an order parameter. We introduce it in order to analyze the collective behavior, and to take a close look into the transition between ordered and disordered states. The parameter is defined as shown in Eq. (3).

$$|\phi(t)| = \frac{1}{Nv_0} |\sum_{i=1}^{N} \mathbf{v}_i(t)|. \quad (3)$$

One can see that the order parameter only indicates us how ordered the direction of motion of the particles is, as it only takes the mean of the velocities of all particles, normalized with the constant modulus. The order parameter takes a value in the $[0; 1]$ range, and achieves its maximum value when the system is completely ordered. Introducing a noise term will make the system less and less ordered as its intensity grows. The order parameter will show this fact, decreasing to 0 when $\eta \geq \eta_0$. Furthermore, the noise term will introduce fluctuations in the order parameter, as in two different times, particles will have different directions and may be more or less ordered. Anyhow, the zero value for the order parameter will never be a reality in our model, as N randomly moving vectors make $\phi \sim \frac{1}{N}$.

In our experiments we will consider a box in a two-dimensional system of N particles, initially randomly distributed, and with a density, $\rho = \frac{N}{L^2}$, where L will be the length of the box's side. Our particles will move with a constant speed-modulus of $v_0 = 0.03$, as we want to study the process in the low velocity regime, where the particles are able to interact with themselves during a sufficiently large amount of time. Were the velocity higher, our particle would be interacting with new neighbors at every time-step. This one we will assume to be $\Delta t = 1$. Our radius of influence, which sets the surface in which we will take into account particles on determining the mean direction, will be always set as $R_0 = 1$. We also will consider that the density does not change, fixing it in $\rho = 1$. N and η will be our control parameters, that we will change during our experiments.

As we will be working in a finite box, the finite size effects will have to be reduced to the minimum. We therefore force the system to be working on periodic boundary conditions, where a particle exiting the box by one side will enter it by the opposed one. Those conditions will have to be applied not only in the variation of the position after each time-step, but also in the conditions set for particles to interact. We said above that the new direction of one particle would be the averaged one of all its neighbors, including itself. Therefore, a particle placed in the limits of the box will have to interact, not only with their immediate neighbors, but also with those occupying places near the box-limit on the opposite site, that would fall inside the area S it they where moved a distance L, equal to the box's side.

The first step in our work will be to reproduce some basic results of the Vicsek model [1], so that we can make our analogies with magnetism. The top image of Fig. (1) shows us the randomly-distributed particles in the beginning of each simulation. Not only their initial positions have been set randomly, but also their directions of movement. We can see that we find ourselves in a completely disordered system. On the other hand, the bottom image, shows the same system once a long-enough time has passed, and one can see that almost all the randomness has disappeared, giving raise to a well-ordered, polarized, collective motion, at low noise intensities.

If the noise intensity were set to be bigger, the randomness in the direction of the particles would be greater, up to the point that the system would not move as one entity, but as N independent individuals. As in the bottom image in Fig. (1) the noise intensity is small, $\eta = 0.1$, the orientation of each individual is slightly different, but the whole system moves in a preferred direction. Now let us study a bit further the dynamical evolution of our system in terms of the noise intensity, analyzing the behavior of the order parameter introduced before. In an ideal system with the noise intensity set to zero, the particles would move averaging its direction with all its neighbors, including themselves, in each time-step. Therefore, one would expect on the order parameter of the system to start in a value near to $\phi_0 = 0$, and to grow with time up to a perfect order, namely to $\phi(t) = 1$.

The effect of an increasing noise intensity, would make its appearance as shown in Fig. (2). The noise term would break the perfect order of our system, as it will introduce in each time-step a modification of the direction of each particle, that will be smaller or greater depending on the fixed intensity, η. As a result, the order parameter will never reach the ideal case of $\phi = 1$, but it will still make it to a steady state, around which there will be fluctuations that will get more and more important as the noise term gains power.

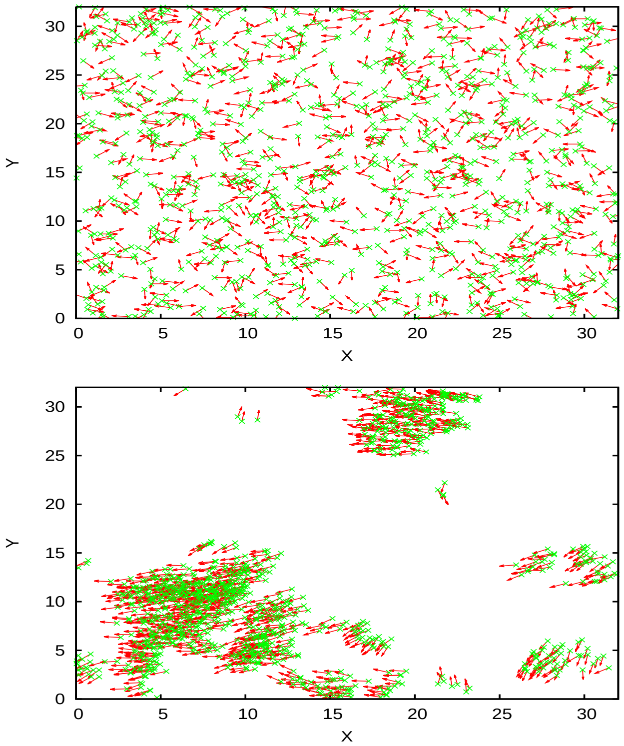

FIG. 1: (Top) Initial random distribution of the particles. (Bottom) Final ordered state in a system with $N = 1024$, $\rho = 1$ and $\eta = 0.1$.

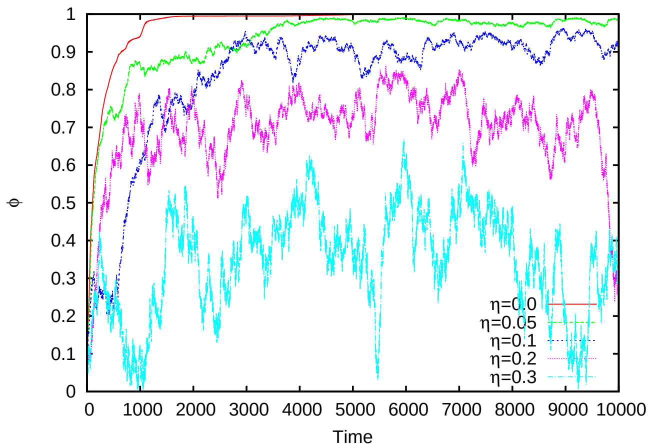

FIG. 2: Evolution of the order parameter with time, for different noise intensities.

Let us give a last turn in our analogy with the magnetic system, by studying the phase transition between ordered and disordered states. With that purpose in mind we analyze both, the order parameter mean value and its fluctuations as a function of the noise intensity, as shown in Fig. (3). That last variable, called variance, has been implemented following Eq. (4),

$$\chi = < \phi^2 > - < \phi >^2 \, . \tag{4}$$

The two plots in Fig. (3) give us, as said, information about the phase transition in the Vicsek model. One can easily see that the peak of the variance, χ is given approximately in the change of curvature of the order parameter, ϕ. In a clear analogy with ferromagnetism, the peak observed corresponds to the position where the critical point is to be found. So, the variance gives us the critical noise intensity, $\eta = \eta_c$ at which the system will undergo a phase transition [5].

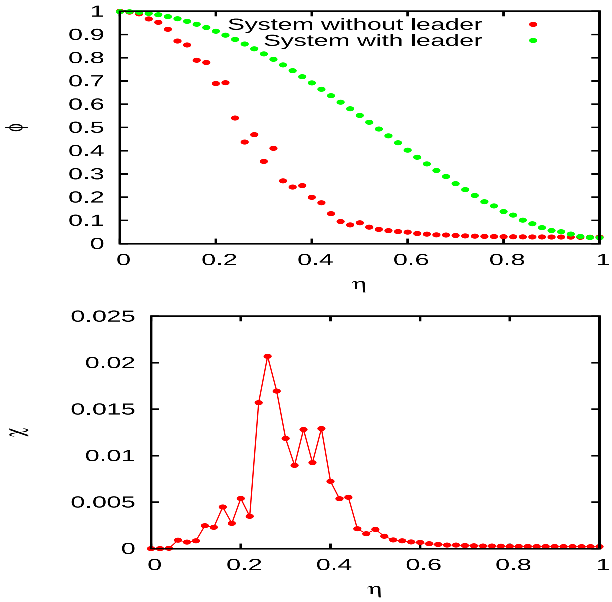

FIG. 3: (Top)Variation of the order parameter of two kind of systems. One representing the traditional Vicsek model, and the other showing the effects of leadership. (Bottom) Evolution of the variance in the traditional model. We always consider $\rho = 1$ and $N = 1024$.

III. THE EFFECT OF LEADERSHIP

A. Single leader

We now consider a new ingredient in the Vicsek model, that is leadership. In many animal groups that we can find in nature, one or a few individual play that role. The leader takes the decisions and the group follows the leader, so that one could expect that the time the system takes to reach an ordered phase will be reduced. In our case, we will consider a leader that exerts a kind of dictatorial authority. Thus meaning that the individual selected with that responsibility will not attend the other individuals opinion, nor will see himself influenced by the noise that affects the rest of the group. We will then consider a leader that moves perpetually along the same direction. Furthermore, we will consider the power of the leader being so, that every individual in our system can see him, so that all of them will interact with him at each time-step, no matter their relative position. Eq. (5) shows the new dynamic model followed by all the particles inside the box, except for the leader himself, who will maintain his direction unaltered through time.

$$\theta_i(t+\Delta t) = \arg(\sum_{k\in S} \mathbf{v}_i(t) + \mathbf{v}_{lead}) + \eta\xi_i(t). \quad (5)$$

with $\mathbf{v}_{lead}$ being the velocity of the leader.

The results of that kind of leadership interaction are shown in Fig. (4). One can see that the dynamics observed is quite similar to that shown in Fig. (2), in a system without a leader. The main difference between both evolutions is the time needed to reach a stationary state, as well as the fluctuations around that state. The effect of leadership reduces drastically the convergence time. One can justify these results arguing that a leader would play a similar role to an external magnetic field in a magnetic system. An external field eliminates the phase transition at a finite T_c, and our leader produces the same qualitative effect at η_c [5]. A disordered phase is only observed at the maximum value of disorder, $\eta = 1$.

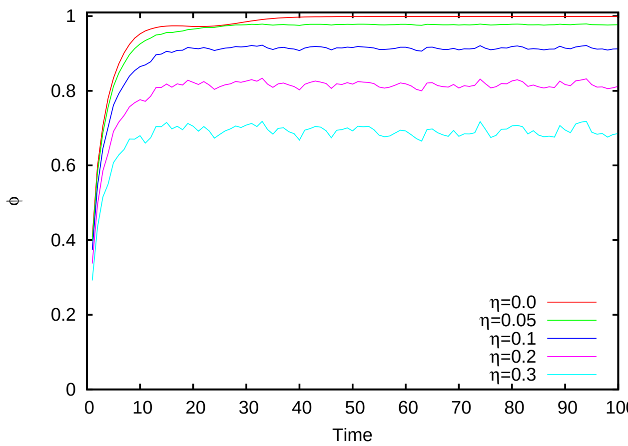

FIG. 4: Dynamic evolution of the order parameter in a system with an absolute leadership, and with $N = 1024$ and $\rho = 1$.

Furthermore, something that we can see in Fig. (4), is the difference in the fluctuations once the stationary state is reached. We saw in section (II) that at intermediate noise intensities the fluctuations where quite big. Now, one can see that the steady state is much smoother, and that the variations around the stationary value are not very significative.

One can also observe from Fig. (3) that the steady behavior of the order parameter does present notable changes. The order parameter curve is better aligned in the leader case, as a consequence of the small fluctuations. Thus, the implementation of a leader makes the order prevail even though high levels of noise are present in the system.

B. Two leaders

Let us end our work with the introduction of a second leader to our system that is in competition with the first one. Clearly, if each leader exerts its influence over the rest of the system, without being itself influenced by the rest of the particles, we would simply have an effective leader imposing a preferred direction that would be the average of the two leaders direction. If that was our case, the system would be exactly the same as the one studied above. Therefore, we are going to study the dynamics of the system with a new variable.

We are going to consider that each one of the particles in our box follows one of the two leaders present in the system. Thus, we will be varying the proportion of particles following one or the other leader, being our control parameter the number of followers the second leader has, w. In that situation, the first leader will have $N(1-w)$ followers, and the second, Nw. We are going to study this case in a system without noise.

An example of a case as the explained above can be found in Fig. (5). There we consider the case where a 70% of particles follow one leader, and the remaining 30% follow the other one. In the top image of that figure we can see the distribution of particles in our box, once a long enough time has passed, so that the system has reached a steady state.

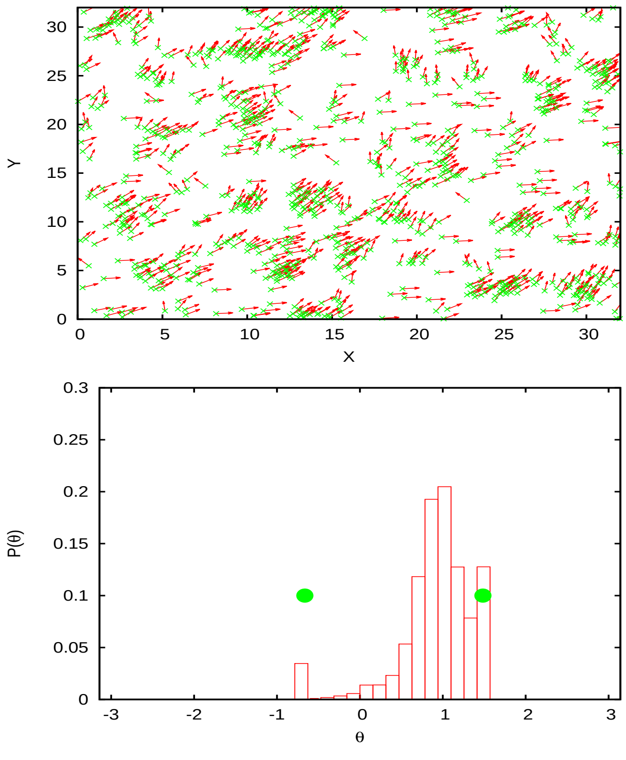

FIG. 5: (Top) Steady state of a system with two leaders, with $w = 30\%$ of the population following the second leader. (Bottom) Angular histogram distribution of the particles in the system, with the leaders orientations marked in green.

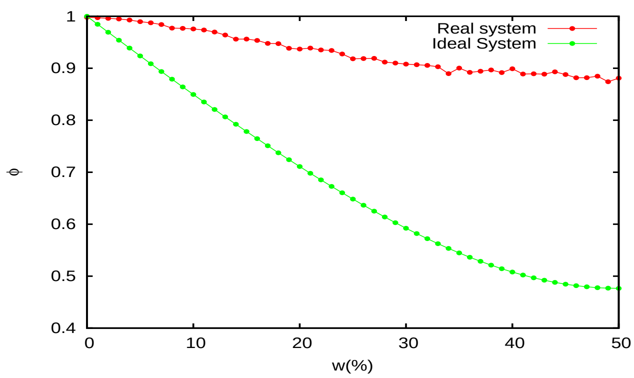

FIG. 6: Evolution of the order parameter with the proportion of followers between both leaders, compared with the ideal case. System with $N = 1024$, $\rho = 1$ and $\eta = 0$.

In the bottom image in Fig. (5), we represented the angular histogram, which shows the angular distribution once the steady state has been achieved. One can see that the boxes in which the leaders are found are also marked. If we had an ideal gas system, in which particles did not interact between them and each one would only follow its leader, we would see that distribution as two delta functions centered in the leaders position.

But the effect of the interaction between particles causes that the system aligns itself in a preferred direction, much closer to the strongest leader, while the followers of the weaker one are clearly reduced. Thus, interactions at first neighbors actually modify the dynamics of a system.

With those results in mind, it would be interesting to see how the order parameter evolves as the proportion of particles changes. With the data that we saw in Fig. (5), we could see that the interaction between particles seemed to order the system to higher levels than an ideal system where the particles followed exclusively their leaders. Therefore, we compare in Fig. (6) how the order parameter changes in an ideal system and in a real one.

The results observed show that the interaction between individuals actually favor order. The order of the system gets reduced as the differences of opinion grow in the group, up to when half of the system follows one leader, and the other 50% follows the other. But we see that the particles always seem to have a preferred direction of motion, as the order parameter takes values around 0.9 in the most extreme case. On the other hand, in an ideal gas system, the order would be very poor.

IV. CONCLUSIONS

We began our work with the intention of studying the collective motion of animal groups through the Vicsek model. In the first place, we reproduced the basic model, and proved the expected results found on the bibliography by studying the two-dimensional case, finding the phase transition between polarized and unpolarized states at intermediate noise values. Once that part was done, we focused our work on the effects of leadership. We observed that by making a particle an absolute leader, the system reached the order in a very short time, and that the phase transition observed in the traditional Vicsek model disappeared.

Finally we put two leaders with different preferences in our system, and studied how the system evolved by changing the ratio of followers each leader had. Thus, we found the most interesting result, that the interaction between particles defined by the Vicsek model, actually favored the order of a system compared to an ideal case, where particles would only follow their leaders.

Although the work is not shown in this paper because we could not finish it on time, we began to study the effect that a group of uninformed particles would have in our system. Following the studies of I. Couzin [4], we would expect those particles to play a central role in the ordering of the system. We will keep on studying that phenomenon in the near future, as to keep on working in that world that still has a lot of questions to be answered.

Acknowledgments

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