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Cost-Based Models of Economic Growth*

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†This paper was begun before, stopped while, and finished after my father was in the hospital. In memoriam of José Manuel Sánchez-García.

Abstract: In this paper we highlight the importance of the operational costs in explaining economic growth and analyze how the industrial structure affects the growth rate of the economy. If there is monopolistic competition only in an intermediate goods sector, then production growth coincides with consumption growth. Moreover, the pattern of growth depends on the particular form of the operational cost. If the monopolistically competitive sector is the final goods sector, then per capita production is constant but per capita effective consumption or welfare grows. Finally, we modify again the industrial structure of the economy and show an economy with two different growth speeds, one for production and another for effective consumption. Thus, both the operational cost and the particular structure of the sector that produces the final goods determines ultimately the pattern of growth.

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Resum: En aquest article, es fa èmfasi en la importància dels costos operacionals en explicar el creixement econòmic i s'analitza com l'estructura industrial afecta la taxa de creixement de l'economia. Si només hi ha competència monopolística en sectors de béns intermedis, aleshores el creixement de la producció coincideix amb el creixement del consum. A més, l'esquema de creixement, endogen o semi-endogen, depèn de la forma particular del cost operacional. Si el sector on es produeix la competència monopolística és la de béns finals, aleshores la producció per càpita és constant però el consum per càpita efectiu o benestar creix. Finalment, si es modifica un altre cop l'estructura industrial de l'economia, es té una economia amb dos velocitats de creixement diferents, una per la producció i l'altre pel consum efectiu. Es conclou doncs que tant els costos operacionals com l'estructura particular del sector que produeix els béns finals determinen en últim terme l'esquema de creixement.

1 Introduction

In this paper we analyze growth in an economy with monopolistic competition characterized by the presence of operational costs. To this end, we first present a reduced model to show that the popular growth models based on R&D are specific cases of this reduced model. The growth source is simply the growth of the number of firms or goods in the economy, which is directly related to the opportunity of having profits. Second, we introduce microfoundations into the reduced model and analyze the relationship between the firms growth mechanism and the pattern of growth. In this part, and as the current models of economic growth, there is monopolistic competition only in an intermediate goods sector, and production growth coincides with consumption growth. Third, we modify the industrial structure of the economy by considering monopolistic competition in the final goods sector, and describe an economy where per capita production is constant but per capita effective consumption or welfare experiences growth. The economy seems to be at a standstill but individuals are better off. Finally, we show an economy with two different growth speeds, one for production and another for effective consumption.

The growth models based on R&D have a common property: there is an intermediate goods sector. A firm of this sector is the only producer of one intermediate good, what confers to it a certain degree of market power and, hence, positive profits. These profits are used to rent a patent, which in turn guarantees to the firm that it can produce and, moreover, it is the only producer of the good. Therefore, the existence of a sector with positive profits is necessary for R&D to arise. Otherwise, no economic agent would devote any amount of resources to engage in an R&D activity. This mechanism reveals that a requirement for growth to arise is not necessarily R&D, but profits. A patent is a type of operational cost, i.e., a cost that does not depend on the quantity produced but, at the same time, production cannot be engaged if this cost is not supported. Moreover, the operational cost is necessary to fix the number of intermediate firms. Therefore, any economy with a sector in which firms have both market power and an operational cost can experience growth. The pattern of growth will be determined by the specific type of operational cost. The rationale is that the existence of a free entry condition causes that the operational cost ultimately determines both the number of firms and the quantity of capital and labor per firm and, thus, growth.

In section 2, we show the influence of the operational cost and the number of firms on growth through a reduced model of monopolistic competition. The reduced model allows to compare growth rates by considering an ad-hoc rule that fixes the number of intermediate firms. In particular, by assuming a constant operational cost and fixing the evolution of firms as a linear function of both labor and the number of firms, we obtain the results of Romer (1990): endogenous growth with scale effects. Constant population is required for a balanced growth path to exist. Instead, by assuming an operational cost that is linear in total population and an evolution of firms that does depend non linearly on both labor and the number of firms, we obtain the results of Jones (1995): semiendogenous growth, i.e., there is growth if and only if there is population growth. The reduced model sheds light on the necessity in Jones (1995) of the operational cost to depend on total population instead of the workers employed by a firm, which denotes some type of scale effect.

We endogeneize the number of firms in order to deepen into the relationship between growth and operational costs in section 3. We depart from the model proposed by Coto-Martínez, Garriga and Sánchez-Losada (2007), where the number of firms or varieties is endogenous. The economy has two sectors: an intermediate goods sector with monopolistically competitive firms, and a competitive final goods sector where the firms combine the intermediate goods à la Dixit-Stiglitz. However, it is taken into consideration the formulation proposed by Ethier (1982) or Benassy (1996) that separates the returns to specialization from the monopolistic mark-up. Moreover, and in contrast with Romer (1990) and Jones (1995), the monopolistically competitive sector uses labor as a production input. In this type of economy, the entrance of a new firm in the market has two opposite effects on the incumbent firms: a complementary effect and a business-stealing effect. When a new firm enters the market, it does not take into account the positive effect on aggregate productivity, which increases the demand of the incumbent firms. This is the *complementary effect*. Moreover, at the same time the presence of a new intermediate firm means that the final good producers can choose among a greater variety of (partially substitutable) inputs, which decreases the demand of the incumbent firms. This is the *business-stealing effect*. The nature and evolution of the operational cost plays a crucial role in the determination of each effect and, thus, on the incentives for new firms to entry.

In this paper we consider an operational cost function that depends on the past history by assuming that these costs depend on how production has been organized in the past. This implies that since the production function does not vary, no firm will change its operational behavior if it implies a higher operational cost.[1] We assume an operational cost function such that only more capital intensive firms may improve the operational technology and induce growth. With this cost structure, we show that endogenous growth is characterized by some knife-edge condition:[2] the

[1]Future research should address the case where a higher operational cost is associated with an improvement of the total factor productivity of the production function.

[2]Growiec (2007) defines "a knife-edge condition as a condition imposed on parameter values

economy experiences endogenous growth only when the operational cost is asymptotically linear in the capital per firm. Otherwise, population growth is required to grow. We pay special attention to two particular cases that illustrate the importance of the operational costs. First, we assume that the operational cost is constant and independent of the quantity produced, as in Matsuyama (1995). Examples are fixed maintenance costs, managerial costs, or simply entry barriers as advertising. In this case, there is no mechanism to induce grow other than population growth. With constant population the complementary effect compensates the business-stealing effect and, therefore, there is no incentive for a new firm to enter the market. Instead, population growth translates into economic growth through the growth of the number of firms. When labor supply grows, the wage decreases in the short run, which alleviates the business-stealing effect and causes the complementary effect to reinforce entry, implying that in the long run labor demand increases. As a consequence, firms end up to become more capital intensive.

The second case assumes that the operational cost varies along the time. In particular, we assume that the operational cost is related to both the own capital and the (past) average level of capital per firm.[3] The idea is that the operational technology may be endogenously improved in economies where each firm accounts for and controls only a small amount of capital, and there is a process of diffusion of such operational technology improvements. Now, a positive population growth enhances economic growth but it is not necessary to experience growth. Positive economic growth only needs improvements in the technology associated to operational costs in order to compensate the negative effects of entry due to the business-stealing effect. The number of firms grows positively but each one becomes more capital intensive and at the same time smaller by hiring less capital and labor.[4] The firm has always the election between continuing with the old operational technology or adapting to the new one with a different operational cost. Since firms hire less capital, the operational cost technology is improved and, therefore, firms end up choosing the new technology. Assuming that the operational cost is not only positively related to the average level of capital per firm, but also to the mean of the labor growth used in a representative firm (what means that the operational cost depends on the number of workers) or to the growth of the number of firms (what allows to study either when the difficulty to exploit a new product increases with the number of products or the opposite, when technology spreads with the number of firms),

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such that the set of values satisfying this condition has an empty interior in the space of all possible values".

[3]In Peretto (1999), firms only use labor and directly engage in R&D expenditures in order to improve the marginal productivity (or lower the marginal costs) of labor.

[4]A firm must be interpreted as a productive process.

gives the same qualitative conclusions than in the case of the operational cost only related to the average level of capital per firm. The results are consistent with the finding of Jones (2002) about the inexistence of a relationship between growth and the number of researchers. That the economy becomes more capital intensive in the growth process has been recently stressed, among others, by Givon (2006), Zuleta (2006) or List and Zhou (2007). However, and in contrast with these authors, our economy does not need any intended R&D expenditure made by firms to save labor in the production process. It is the evolution of the operational costs what makes the economy to become more capital intensive.

In the models of endogenous growth typically production growth coincides with consumption growth. In section 4 we show that a change in the sector that is monopolistically competitive changes the results about the growth rate. We consider an economy with differentiated consumption and investment goods. In particular, we assume that the individual buys several differentiated final goods and derive utility through a love of variety parameter from a mix of them, what we call effective consumption. Investment goods are produced by competitive firms through a mix of final goods. There is no aggregate returns to specialization for the investment. In this economy per capita production does not grow regardless of the assumed operational cost. However, having the same amount of final goods per capita does not mean that per capita effective consumption (or welfare) does not grow. In other words, real per capita production is the same but the subjective value the consumer gives to the production grows. In particular, with constant operational costs we have effective consumption semiendogenous growth whereas with operational costs positively related to the average capital per firm population growth is not necessary to have effective consumption growth.

Finally, we modify again in section 5 the industrial structure of the economy and show that assuming both love of variety and aggregate returns to specialization in the investment sector gives a different positive final goods growth rate than the effective consumption one. We analyze when population growth is needed to have both production and effective consumption growth.

The exercise made in this paper shows that both the operational cost and the particular industrial structure determines ultimately the pattern of growth. First, a technological change affecting the operational costs (ex. the promotion of technology diffusion through a Marshallian industrial district) can have dramatic consequences on economic growth by moving an economy from a semiendogenous to an endogenous growth pattern. Second, the industrial or sectorial structure can explain at least part of the economic growth[5] and, at the same time, it can hide part of this growth, i.e.,

[5] An example, from a different point of view, is Alonso-Carrera and Raurich (2006).

the effective consumption growth. Hence, accounting for growth should go beyond a Solow residual decomposition exercise.

2 Profits, number of firms and economic growth

In this section, we show that monopolistic competition can generate endogenous (semiendogenous) growth when the market structure evolves and becomes more (remains equally) competitive as a consequence of an increase in the number of firms. Assume there is a unique final good which is produced by competitive firms using a continuum of intermediate goods. Total population $\widehat{L}_t$ grows at a constant rate, so that $\widehat{L}_t/\widehat{L}_{t-1} = n$.

A final goods firm maximizes

$$Y_t - \int_0^{z_t} p_{it} x_{it} di, \tag{1}$$

where x_{it} and p_{it} are the quantity and price of the intermediate good i in period t, respectively, and z_t is the total number of intermediate goods in period t, which is taken as given by the (competitive) final goods sector. We have normalized the final goods price to one. The production function Y_t depends in a symmetric way on the available intermediate goods, which are not perfect substitutes. Therefore, from this profits maximization problem we can recover a demand function for each intermediate good.

In each period, new intermediate goods producers may enter and produce a new variety. Each firm produces at most one intermediate input x_{it}. In order to operate, firms have to pay an operational cost ψ_t. An intermediate goods firm i maximizes

$$\pi_{it} = p_{it} x_{it} - w_t L_{it} - (1 + r_t) K_{it} - \psi_t, \tag{2}$$

where π_{it} is the profits function, $x_{it} = K_{it}^{1-\alpha} L_{it}^{\alpha}$, K_{it} and L_{it} are the capital and labor used by firm i, respectively, w_t is the wage, and r_t is the interest rate, so that there is complete depreciation. We have assumed that the operational cost is measured in terms of the final good. Since intermediate goods are not perfect substitutes, firms in the intermediate goods sector face a downward slopping demand curve which confers them some degree of market power. Then, the profits function can be simplified to[6]

$$\pi_{it} = \eta p_{it} K_{it}^{1-\alpha} L_{it}^{\alpha} - \psi_t, \tag{3}$$

[6] We assume that the operational cost is completely out of the scope of the firm decisions. In the next section, we study a more general case where the operational cost can partially depend on the firm decisions.

where η is the inverse of the elasticity of the demand for each intermediate good and measures the degree of market power.

Since there is perfect competition in the final goods sector, in a symmetric equilibrium where all firms produce the same output level $x_{it} = x_t$, with the same quantity of inputs $K_{it} = K_t$ and $L_{it} = L_t$, set the same price $p_{it} = p_t$, and have the same profits $\pi_{it} = \pi_t$, we have that

$$Y_t = z_t p_t K_t^{1-\alpha} L_t^{\alpha}. \tag{4}$$

There is free entry in the intermediate goods sector. Thus, the total number of intermediate goods z_t is determined by the zero profit condition and, hence, Eq.(3) is equal to zero. Applying this condition to Eq.(4), the per capita final goods growth can be written as[7]

$$g_{y_{t+1}} = \frac{g_{z_{t+1}} g_{\psi_{t+1}}}{n}, \tag{5}$$

where $y_t = Y_t / \widehat{L}_t$ is per capita final goods production, and $g_{h_{t+1}} = h_{t+1}/h_t$ denotes the growth between $t+1$ and t of the variable h. From Eq.(5) it is clear that the particular pattern of growth depends on the assumed operational cost function. Moreover, we could identify different operational cost functions for either different historic times or different degrees of development. Therefore, if we identify the functional form of the cost function and the evolution of the number of firms, we would be able to also identify the growth rate of the economy. Obviously, the number of firms in the economy is the result of the free entry condition. Then, depending on the nature of the operational cost, associated either to the same intermediate sector or to another sector, we have different growth mechanisms. Next, we show two of the most popular cases of this mechanism when the operational cost is associated to a third sector.

In the endogenous growth model of Romer (1990), the operational cost is assumed to be constant and equal to the price of a patent, and the number of intermediate firms coincides with the number of patents, which are produced in another sector. Hence, by assuming that the number of patents evolves as

$$z_{t+1} - z_t = \delta L_{z,t} z_t, \tag{6}$$

where $L_{z,t}$ is labor used to produce (or search) a new patent at t, and δ is a positive constant, the per capita final goods growth becomes

$$g_{y_{t+1}} = \frac{1 + \delta L_{z,t}}{n}. \tag{7}$$

[7]In national accounting, final goods production corresponds to the Gross Domestic Product, i.e., $Y_t - z_t \psi_t$. Since in this paper the dynamics of Y_t is the same than the dynamics of $Y_t - z_t \psi_t$, hereinafter we concentrate on Y_t.

We need constant population for a balanced growth path to exist, i.e., $L_{z,t}$ must be fixed. Obviously, $L_{z,t}$ comes from the free entry condition. If there is population growth, then growth is not balanced, since $L_{z,t}$ in Eq.(7) is permanently growing whereas n is a constant. Moreover, without population growth, the country with the biggest population would have the biggest $L_{z,t}$ and, then, the biggest per capita final goods growth. This is the reason why this model is said to have scale effects.

In the endogenous growth model of Jones (1995), the operational cost is also assumed to be equal to the price of a patent, but not necessarily constant, and the number of intermediate firms coincides with the number of patents, which are produced in another sector. In particular, it is assumed $\psi_t = \psi \widehat{L}_t$, where ψ is a positive constant. This in turn means that the price of a patent increases if and only if population grows.[8] The number of patents is assumed to evolve as

$$z_{t+1} - z_t = \delta L_{z,t} z_t^{\phi} l_{z,t}^{\lambda-1}, \tag{8}$$

where $l_{z,t}$ is the mean of the labor used in a representative firm that produces patents, i.e., an externality accruing from the duplication of R&D, and ϕ and λ are constants. The parameter ϕ captures the fact that the stock of current patents can affect either positively or negatively the production of new patents. In a symmetric equilibrium, where $L_{z,t} = l_{z,t}$, the growth of the number of firms is

$$g_{z_{t+1}} = \left(1 + \delta L_{z,t}^{\lambda} z_t^{\phi-1}\right). \tag{9}$$

Population cannot be constant in a balanced growth path, since therefore the growth of the number of firms z_t does not allow the factor $1+\delta L_{z,t}^{\lambda} z_t^{\phi-1}$ to be constant. Such factor remains constant if $g_z = n^{\lambda/(1-\phi)}$. Rewriting Eq.(5), we have

$$g_y = n^{\lambda/(1-\phi)}. \tag{10}$$

Hence, we need population growth for a balanced growth path to exist. This is the reason why this model is said to be a semiendogenous growth model. When population does not grow, the operational cost for the intermediate goods firms collapses into a constant and, then, growth is not possible.

In order to realize the role of the operational cost in the determination of growth, assume instead that $\psi_t = \psi l_{z,t}$. Then, in a symmetric equilibrium and a balanced growth path, as both $z_t L_{z,t}$ and $z_t L_t$ are constant proportions of the total population, we have from Eq.(5) that $g_y = 1$ regardless of the number of patents accumulation law given by Eq.(8). In this sense, we can say that the necessity in Jones (1995) of the operational cost to depend on total population instead of the labor mean employed by a firm denotes some type of scale effect.

[8]It can be inferred from equations (A2) and (A15) in Jones (1995).

Obviously, g_{z_t} depends on the assumed ψ_t. We detail this relationship in the next section.

3 A cost-based model of endogenous growth

We construct an economy with aggregate returns to scale to analyze how the free entry condition together with the particular evolution of the operational costs determine the growth rate of the economy. In standard models (see Mankiw and Whinston, 1986), free entry reduces welfare in the sense that the increase in the aggregate operational costs is not compensated by the positive benefits arising from increasing competition or business-stealing effect: new firms entering the market have a negative impact on the incumbent firms' demand. Hence, the market equilibrium can generate excessive entry. However, in the presence of increasing returns to specialization the entrance of a new intermediate firm has a complementary effect: it increases the aggregate productivity and, thus, the incumbents' demands. Therefore, when considering the complementary effect the free entry may yield an inefficiently low number of firms. The evolution of the operational costs will determine the new entry in the market and, thus, the aggregate productivity growth, and at the same time the growth of the aggregate losses due to the presence of the operational costs.

Final goods production: There is a unique final good which is produced by competitive firms through a continuum of intermediate goods, with the following technology (as in Benassy, 1996):[9]

$$Y_t = \left(z_t^{v(1-\eta)-\eta} \int_0^{z_t} x_{it}^{1-\eta} di \right)^{\frac{1}{1-\eta}}, \quad \eta \in (0,1), \ v \in (0,1). \tag{11}$$

In a symmetric equilibrium, all the firms in the intermediate goods sector produce the same output level x_t and, thus, aggregate output is $Y_t = z_t^{v+1} x_t$. Then, the elasticity of output with respect to the number of firms z_t is given by the "degree of returns to specialization" v, as in Ethier (1982). This parameter measures the degree to which society benefits from specializing production between a large number of intermediate goods z_t. As a result, an increase in the number of intermediate goods improves the total factor productivity of the final goods technology. This formulation allows to separate the effect of the mark-up from the economies of scale.[10] Since

[9] We maintain the same definition of the variables as in the previous section.

[10] The conventional formulation established by Dixit and Stiglitz (1977) corresponds to the case $v = \eta / (1 - \eta) < 1$, where there exists a one-to-one relationship between the market power and the degree of returns to specialization.

there is free entry in the intermediate goods sector, at the aggregate level the number of intermediate goods z_t is determined by the zero profits condition. However, the representative firm in the final goods sector takes this value as given. From the profits maximization problem, given by

$$\max_{\{x_{it}\}} z_t^{\frac{v(1-\eta)-\eta}{1-\eta}} \left(\int_0^{z_t} x_{it}^{1-\eta} di \right)^{\frac{1}{1-\eta}} - \int_0^{z_t} p_{it} x_{it} di, \tag{12}$$

we obtain the inverse demand function for each intermediate input,

$$x_{it} = (p_{it})^{-\frac{1}{\eta}} z_t^{v\frac{(1-\eta)}{\eta}-1} Y_t. \tag{13}$$

Intermediate goods production: Each intermediate goods firm solves

$$\max_{\{p_{it}, K_{it}, L_{it}\}} \pi_{it} = p_{it} x_{it} - (1 + r_t) K_{it} - w_t L_{it} - \psi_t, \tag{14}$$

subject to the final goods sector demand, Eq.(13), and where $x_{it} = K_{it}^{1-\alpha} L_{it}^{\alpha}$. The operational cost can depend partially on the firm decisions, but not completely. Note that an operational cost means that there are goods neither consumed directly by individuals nor invested in capital. We have assumed there is complete capital depreciation.[11] The associated first-order conditions of the firm problem yield

$$1 + r_t = p_{it} (1 - \eta) (1 - \alpha) K_{it}^{-\alpha} L_{it}^{\alpha} - \psi_{K_{it}}, \tag{15}$$

$$w_t = p_{it} (1 - \eta) \alpha K_{it}^{1-\alpha} L_{it}^{\alpha-1} - \psi_{L_{it}}, \tag{16}$$

where ψ_h is the partial derivative of ψ_t with respect to h.

In a symmetric equilibrium final output is equal to

$$Y_t = z_t^{v+1} K_t^{1-\alpha} L_t^{\alpha}, \tag{17}$$

and the price, by substituting Eq.(17) into Eq.(13), is

$$p_t = z_t^v. \tag{18}$$

The free entry condition (each intermediate firm makes zero profits, i.e., $\pi_t = 0$) determines the number of firms. Formally, and using Eq.(18), we have

$$\eta z_t^v K_t^{1-\alpha} L_t^{\alpha} = \psi_t - \psi_{K_t} K_t - \psi_{L_t} L_t. \tag{19}$$

Since the final cost is defined in terms of the final output, the entry of any firm reduces the relative price between final output and intermediate goods $1/p_t = z_t^{-v}$ and,

[11] Capital depreciation does not vary the qualitative results.

thus, it makes entry more profitable. However, individual firms do not internalize this (complementary) effect. Note that in our model we obtain the standard formulation where $p_t = 1$ when $v = 0$. In this case, aggregate returns to specialization are absent and, hence, there would be only one (normalized) firm.

Consumers: We assume Solow individuals: at any period t, each individual j saves a constant fraction of her income R_t^j and is endowed with one unit of labor that she supplies inelastically. Therefore, savings for the individual j are

$$S_t^j = sR_t^j, \tag{20}$$

where $s \in (0,1)$ is the constant propensity to save.[12]

Labor market clearing condition: In equilibrium, labor demand and supply coincides, i.e.,

$$z_t L_t = \widehat{L}_t. \tag{21}$$

As population grows at a constant rate, we have

$$g_{z_{t+1}} g_{L_{t+1}} = g_{\widehat{L}_{t+1}} = n. \tag{22}$$

Note that assuming another sector employing labor would modify the labor market clearing condition and, thus, the particular industrial structure of the economy would become crucial in determining the growth rate of the economy.

Capital market clearing condition: The amount saved by individuals at t equals the stock of physical capital at $t+1$; i.e.,

$$\int_0^{\widehat{L}_t} S_t^j dj = s \int_0^{\widehat{L}_t} R_t^j dj = z_{t+1} K_{t+1}. \tag{23}$$

Noting that $\int_0^{\widehat{L}_t} R_t^j dj = w_t \widehat{L}_t + (1+r_t) z_t K_t = z_t \left[w_t L_t + (1+r_t) K_t \right] =$ $(1-\eta) z_t^{v+1} K_t^{1-\alpha} L_t^{\alpha} - z_t \left(\psi_{K_t} K_t + \psi_{L_t} L_t \right)$, where we have used the definition of the individuals income and Eqs (21), (15), (16) and (18), the previous equation becomes

$$s \left[(1-\eta) z_t^{v+1} K_t^{1-\alpha} L_t^{\alpha} - z_t \left(\psi_{K_t} K_t + \psi_{L_t} L_t \right) \right] = z_{t+1} K_{t+1}, \tag{24}$$

or

$$s \left(Y_t - z_t \psi_t \right) = z_{t+1} K_{t+1}. \tag{25}$$

Balanced growth path: The dynamics of the model can be reduced to the capital accumulation Eq.(24) and the free entry condition Eq.(19), which using Eqs (21) and (22) can be written as

$$\eta n \frac{g_{K_{t+1}}}{g_{L_{t+1}}} = s \left[(1-\eta) \frac{\psi_t}{K_t} - \psi_{K_t} - \psi_{L_t} \frac{L_t}{K_t} \right], \tag{26}$$

[12] An infinite horizon consumer with CES preferences gives the same qualitative results.

and

$$\eta \widehat{L}_t^v K_t^{-\alpha} L_t^{\alpha-v} = \frac{\psi_t}{K_t} - \psi_{K_t} - \psi_{L_t} \frac{L_t}{K_t}. \tag{27}$$

Thus, for a balanced growth path to exist, from Eq.(26) we have that in equilibrium the operational cost function must be asymptotically of the form

$$\psi_t = \frac{1}{(1-\eta)} \left[HK_t + \psi_{K_t} K_t + \psi_{L_t} L_t \right] \tag{28}$$

where H denotes a positive constant.[13] In that case, from Eq.(27) we have

$$g_L = g_K^{\frac{\alpha}{\alpha-v}} n^{\frac{-v}{\alpha-v}}, \tag{29}$$

and combining it with Eq.(26) gives

$$\eta n^{\frac{\alpha}{\alpha-v}} g_K^{\frac{-v}{\alpha-v}} = sH. \tag{30}$$

From Eqs (17), (21) and (29), we obtain

$$g_y = \left(\frac{n}{g_K} \right)^{\frac{v}{\alpha-v}}. \tag{31}$$

For $\alpha > v$, this equation clearly informs us about the negative (positive) effect of the capital per firm growth (population growth) on the growth of per capita output; i.e., small firms (or productive processes) are the source of growth. Thus, any mechanism whose effects are translated into $g_K < 0$ will induce positive growth.[14,15]

Next, in order to complete the analysis, we assume a particular functional form for the operational cost.

3.1 Homogeneous operational costs

Operational costs consist of resources that the firm consumes, i.e., they constitute final goods that are neither consumed directly by individuals nor invested in capital. We assume the following specification of the operational cost:

$$\psi_t = \psi K_t^{\gamma\xi} L_t^{\gamma(1-\xi)} \Phi \left(\{k_{t-\tau}\}_{\tau=1}^{\infty} \right), \tag{32}$$

[13]Note that Eq.(27) informs that ψ_t cannot be homogeneous of degree one in K_t and L_t, since then there would be no equilibrium.

[14]Note that $g_K < 0$ is compatible with more capital intensive firms. Also note that this result depends crucially on the fact that the production function does not vary.

[15]Although capital per firm growth has the opposite sign than the output growth, we have a balanced growth path because aggregate capital growth and output growth coincide.

where $k_{t-\tau}$ is the average level of capital per firm at period $t-\tau$, $\psi > 0$, $\gamma \in [0,1)$ and $\xi \in [0,1]$. The function Φ is defined as $\Phi\left(\{k_{t-\tau}\}_{\tau=1}^{\infty}\right) = \min_{\tau=1,\ldots,\infty}\left\{k_{t-\tau}^{\beta}\right\}$, with $\beta > 0$. This means that the operational cost depends on how production has been organized in the past. In particular, large productive structures are less able to improve the operational technology than small ones. Moreover, at any time the firm has always the election between continuing with the old technology or adapting to the new one with a different operational cost. However, in order to simplify notation and since the economy will evolve such that k_{t-1} is decreasing, we directly write $\Phi\left(\{k_{t-\tau}\}_{\tau=1}^{\infty}\right) = k_{t-1}^{\beta}$. Note that this operational cost function is homogenous of degree $\gamma < 1$ in K_t and L_t, but in aggregate it exhibits homogeneity of degree $\gamma + \beta$. This formulation guarantees the existence of a balanced growth path as it satisfies Eq.(28). Note that the cases in which $\gamma \neq 0$ mean that the firm realizes that the operational cost partially depends on production.

In the Appendix we show both the existence and the stability properties of the balanced growth path for any parameter configuration. Endogenous growth is obtained if and only if $\gamma + \beta = 1$ and $\xi = 1$; i.e., the operational cost function is homogeneous of degree one in aggregate with respect to k_{t-1} and K_t. Thus, the usual knife-edge condition for endogenous growth to arise is reduced to the property that operational costs must be "linear" on the average capital per firm. We next illustrate the importance of the operational costs on economic growth through two special cases: $\psi_t = \psi$ and $\psi_t = \psi K_t^{\gamma} k_{t-1}^{1-\gamma}$.[16]

3.1.1 $\gamma = \beta = 0$

This parameter configuration gives a constant operational cost, i.e., $\psi_t = \psi$ for all t, as in Matsuyama (1995). This means that the operational cost is not only independent of the produced quantity, but also on how the technology has been used in the past. We need to assume that $v < \alpha$ in order to have convergence. This means that if the aggregate returns to specialization are too high, the complementary effect can never be compensated by the business-stealing effect and, then, multiplying the number of firms simply by reducing the size of each firm makes growth to be explosive. In this case, from Eq.(28) we have $H = (1-\eta)\,\psi/K_t$ and, as it must be asymptotically constant, the unique possible balanced growth path must satisfy that K_t is asymptotically constant, so that $g_K = 1$. From Eqs (29), (22) and (31),

[16] Other different operational cost functions, as $\psi K_t^{\gamma} k_{t-1}^{\beta}\left(L_t/l_{t-1}\right)^{\varepsilon}$, where l_{t-1} is the average level of labor per firm at period $t-1$, $\psi K_t^{\gamma} k_{t-1}^{\beta}\left(l_t/l_{t-1}\right)^{\varepsilon}$ or $\psi K_t^{\gamma} k_{t-1}^{\beta}\left(L_t/l_{t-1}\right)^{\varepsilon}\left(z_t/z_{t-1}\right)^{\varphi}$ give the same qualitative conclusions.

we obtain

$$g_K = 1, \quad g_z = n^{\frac{\alpha}{\alpha - v}},$$
$$g_L = n^{\frac{-v}{\alpha - v}}, \quad g_y = n^{\frac{v}{\alpha - v}}. \tag{33}$$

Finally, Eq.(30) yields $K = s(1-\eta)\psi/\eta n^{\frac{\alpha}{\alpha-v}}$. The economy experiences semiendogenous growth. When population grows, the number of firms grows at a positive rate and labor demand per firm grows at a negative rate. Thus, firms become smaller and more capital-intensive, implying a higher labor productivity, which in turn makes per capita growth to increase. Note that not only the productivity of labor increases, but also the productivity of capital can increase because of the complementary effect. If there is no population growth, then any variable grows regardless of the returns to specialization.

3.1.2 $\xi = 1$ and $\beta = 1 - \gamma$

This parameter configuration means that the operational cost is only related to the capital per firm, i.e., $\psi_t = \psi K_t^{\gamma} k_{t-1}^{1-\gamma}$. In equilibrium, where $k_{t-1} = K_{t-1}$,[17] we have $H = \psi g_K^{\gamma-1}(1-\eta-\gamma)$, which in turn does impose the restriction $\gamma + \eta < 1$ to have a positive balanced growth path. We need to assume that $v < \alpha(1-\gamma)/(2-\gamma)$ in order to have convergence. Eqs (30), (29), (22) and (31) yield

$$g_K = D^{-(\alpha-v)} n^{\frac{-\alpha}{\alpha(1-\gamma)-v(2-\gamma)}}, \quad g_z = D^{\alpha} n^{\frac{\alpha(2-\gamma)}{\alpha(1-\gamma)-v(2-\gamma)}},$$
$$g_L = D^{-\alpha} n^{\frac{-[v(2-\gamma)+\alpha]}{\alpha(1-\gamma)-v(2-\gamma)}}, \quad g_y = D^{v} n^{\frac{v(2-\gamma)}{\alpha(1-\gamma)-v(2-\gamma)}}, \tag{34}$$

where $D = [\eta/s\psi(1-\gamma-\eta)]^{\frac{1}{\alpha(1-\gamma)-v(2-\gamma)}}$. The economy experiences a combination of endogenous growth and semiendogenous growth. In case of zero population growth, we have positive growth if $\psi < \eta/s(1-\gamma-\eta)$. We need a sufficiently low unit operational cost in order that firms enter the market. Otherwise, the operational cost cannot be compensated by profits. With positive growth, the number of firms increases but each firm hires less capital and labor, such that they become more capital intensive. The complementary effect due to the growth in the number of firms dominates the business-stealing effect (less production per firm), which in turn makes production per capita to grow. Population growth reinforces the magnitude of each growth rate. The fact that labor and capital per firm grows at a negative rate regardless of the population growth means that the operational cost decreases and, therefore, firms choose the new technology.[18] However, note that

[17] If instead of k_{t-1} we have k_t by assuming $\Phi(\{k_{t-\tau}\}_{\tau=0}^{\infty})$ then there would be no dynamics.

[18] Alternatively, we could think that technology becomes more standard or, what is the same, that knowledge diffusion is faster and, then, the operational cost associated to create a new firm decreases.

$g_z g_{\psi_t} > 1$, which means that the aggregate operational cost increases.

4 Monopolistic competition in the final goods sector

> *By the problem of economic development I mean simply the problem of accounting for the observed pattern, across countries and across time, in levels and rates of growth of per capita income. This may seem too narrow a definition, and perhaps it is, but thinking about income patterns will necessarily involve us in thinking about many other aspects of societies too, so I would suggest that we withhold judgment on the scope of this definition until we have a clearer idea of where it leads us.* (R.E. Lucas, Jr., 1988, p. 3)

In this section, we show that a different industrial structure dramatically changes the interpretation of the results obtained regarding growth. In particular, we show that accounting for the observed pattern in levels and rates of growth of per capita income does not necessarily explain economic development. We consider a formulation with differentiated consumption and investment goods. In particular, we assume that the individual j buys several differentiated final goods and derives utility from the following mix, that we call effective consumption,

$$c_t^j = \left(z_t^{v(1-\eta)-\eta} \int_0^{z_t} \left(x_{c_{it}}^j \right)^{1-\eta} di \right)^{\frac{1}{1-\eta}}, \quad \eta \in (0,1)\,,\ v \in (0,1)\,, \tag{35}$$

where $x_{c_{it}}^j$ is the final good produced by firm i and consumed by individual j, and now v is a love of variety parameter. Investment goods, I_t, are produced by competitive firms through the following technology:

$$I_t = \left(z_t^{-\eta} \int_0^{z_t} x_{I_{it}}^{1-\eta} di \right)^{\frac{1}{1-\eta}}, \quad \eta \in (0,1)\,, \tag{36}$$

where $x_{I_{it}}$ is the final good produced by firm i used to produce investment goods. Note that in order to concentrate on the love of variety we have assumed that there is no aggregate returns to specialization for the investment. Also note that the same varieties are used to consume and to produce investment goods and that both share the inverse of the elasticity of the demand for each intermediate good η.

Consumers: Since individuals save a constant fraction of their income, each individual solves

$$\max_{\left\{\left\{x_{c_{it}}\right\}_{i=0}^{z_t}\right\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t \left(z_t^{v(1-\eta)-\eta} \int_0^{z_t} \left(x_{c_{it}}^j\right)^{1-\eta} di \right)^{\frac{1}{1-\eta}} \tag{37}$$

$$s.to \quad \int_0^{z_t} p_{it} x_{c_{it}}^j di = (1-s) R_t^j. \tag{38}$$

The first order condition is

$$\beta^t \left(c_t^j\right)^{\eta} z_t^{v(1-\eta)-\eta} \left(x_{c_{it}}^j\right)^{-\eta} = \lambda_t p_{it}, \quad \forall i, \tag{39}$$

where λ_t is the Lagrange multiplier associated to Eq.(38). From Eq.(39) we have

$$x_{c_{mt}}^j = \left(\frac{p_{it}}{p_{mt}}\right)^{\frac{1}{\eta}} x_{c_{it}}^j, \quad \forall i, m. \tag{40}$$

In order to recover each individual good demand, combine Eqs (38) and (40) to get

$$\int_0^{z_t} p_{mt} x_{c_{mt}}^j dm = \int_0^{z_t} p_{mt}^{1-\frac{1}{\eta}} p_{it}^{\frac{1}{\eta}} x_{c_{it}}^j dm = p_{it}^{\frac{1}{\eta}} x_{c_{it}}^j \int_0^{z_t} p_{mt}^{1-\frac{1}{\eta}} dm = (1-s) R_t^j, \tag{41}$$

where the second equality comes from the fact that good i is infinitesimal. From this equation we have the good i demand as

$$\int_0^{\widehat{L}_t} x_{c_{it}}^j dj = \left(\frac{(1-s)\int_0^{\widehat{L}_t} R_t^j dj}{\int_0^{z_t} p_{mt}^{1-\frac{1}{\eta}} dm} \right) p_{it}^{-\frac{1}{\eta}}. \tag{42}$$

Savings are used by the individuals to buy investment goods that are rented to the final goods production firms.

Investment goods production: Each firm solves

$$\max_{\left\{x_{I_{it}}\right\}_{i=0}^{z_t}} \quad P_{I_t} I_t - \int_0^{z_t} p_{it} x_{I_{it}} di \tag{43}$$

subject to Eq.(36), and where P_{I_t} is the price of the investment good. The inverse demand function for each intermediate good is

$$x_{I_{it}} = \left(\frac{p_{it}}{P_{I_t}}\right)^{-\frac{1}{\eta}} z_t^{-1} I_t. \tag{44}$$

Final goods production: Each firm solves

$$\max_{\left\{p_{it}, K_{it}, L_{it}\right\}} \quad p_{it} \left(\int_0^{\widehat{L}_t} x_{c_{it}}^j dj + x_{I_{it}} \right) - P_{I_t} (1+r_t) K_{it} - w_t L_{it} - P_{I_t} \psi_t, \tag{45}$$

$$s.to \quad \int_0^{\widehat{L}_t} x_{c_{it}}^j dj + x_{I_{it}} = K_{it}^{1-\alpha} L_{it}^{\alpha}, \tag{46}$$

and Eqs (42) and (44), where K_{it} denotes the investment goods rented to the firm by the individuals.[19] The associated first-order conditions of the firm problem yield

$$P_{I_t} (1 + r_t) = p_{it} (1 - \eta) (1 - \alpha) K_{it}^{-\alpha} L_{it}^{\alpha} - P_{I_t} \psi_{K_{it}}, \tag{47}$$

$$w_t = p_{it} (1 - \eta) \alpha K_{it}^{1-\alpha} L_{it}^{\alpha-1} - P_{I_t} \psi_{L_{it}}. \tag{48}$$

In the symmetric equilibrium, $x_{c_{it}}^j = x_{c_t}^j$, $x_{I_{it}} = x_{I_t}$ and $p_{it} = p_t$ for all i. Note that this implies that $x_{c_{it}} = x_{c_t}$, i.e., all the aggregate demands are equal. Substituting Eq.(36) into Eq.(44) gives the relative prices

$$P_{I_t} = p_t. \tag{49}$$

As in the previous section, we consider one final good p_{it} as the numéraire and normalize its price to one.[20] Hence, we have that $p_t = P_{I_t} = 1$, and final output is equal to

$$Y_t = z_t K_t^{1-\alpha} L_t^{\alpha}. \tag{50}$$

The free entry condition yields

$$\eta K_t^{1-\alpha} L_t^{\alpha} = \psi_t - \psi_{K_t} K_t - \psi_{L_t} L_t. \tag{51}$$

Capital market clearing condition: Since now the investment goods have a price, the condition becomes

$$\int_0^{\widehat{L}_t} S_t^j dj = s \int_0^{\widehat{L}_t} R_t^j dj = P_{I_t} z_{t+1} K_{t+1}. \tag{52}$$

Noting that $\int_0^{\widehat{L}_t} R_t^j dj = w_t \widehat{L}_t + P_{I_t} (1 + r_t) z_t K_t = z_t [w_t L_t + P_{I_t} (1 + r_t) K_t] =$ $(1 - \eta) z_t K_t^{1-\alpha} L_t^{\alpha} - z_t \left(\psi_{K_t} K_t + \psi_{L_t} L_t\right)$, where we have substituted for the prices, the condition becomes

$$s \left[(1 - \eta) z_t K_t^{1-\alpha} L_t^{\alpha} - z_t \left(\psi_{K_t} K_t + \psi_{L_t} L_t\right)\right] = z_{t+1} K_{t+1}. \tag{53}$$

Consumption: From Eqs (35) and (38) we have

$$c_t^j = z_t^{v+1} x_{c_t}^j = z_t^v (1 - s) R_t^j. \tag{54}$$

[19] We have measured the operational cost in terms of the investment good in order to analyze the cases where the operational cost is completely out of the scope of the firm.

[20] Alternatively, we could assume that the numéraire is the investment good price, $P_{I_t} = 1$. Therefore, in a symmetric equilibrium monopolistic firms choose their prices such that they are also one.

Aggregating, and using Eq.(53), we have

$$\widehat{L}_t c_t = \int_0^{\widehat{L}_t} c_t^j dj = z_t^v (1-s) \int_0^{\widehat{L}_t} R_t^j dj = \frac{(1-s)\, z_t^v z_{t+1} K_{t+1}}{s}, \tag{55}$$

where c_t is the effective consumption per capita.

Balanced growth path: As in the previous section, both the free entry and the capital accumulation conditions determine the dynamics of the economy. Combining Eqs (22), (53) and (51), these conditions can be written as

$$\eta n \frac{g_{K_{t+1}}}{g_{L_{t+1}}} = s\left[(1-\eta)\frac{\psi_t}{K_t} - \psi_{K_t} - \psi_{L_t}\frac{L_t}{K_t}\right], \tag{56}$$

and

$$\eta K_t^{1-\alpha} L_t^{\alpha} = \psi_t - \psi_{K_t} K_t - \psi_{L_t} L_t. \tag{57}$$

These conditions are the same as Eqs (26) and (27), evaluated with a different price for the intermediate goods. Hence, the properties regarding existence and stability are the same than those of the previous section. Therefore, combining Eqs (51) and (22) give

$$g_K = g_L, \tag{58}$$

$$g_z g_K = n. \tag{59}$$

From Eqs (55) and (59) we obtain

$$g_c = \left(\frac{n}{g_K}\right)^v. \tag{60}$$

And Eqs (50), (51) and (59) yield

$$g_y = 1. \tag{61}$$

Hence, the economy does not grow regardless of the assumed operational cost. However, having the same amount of final goods per capita does not mean that per capita effective consumption does not grow. We illustrate this assertion through the same two operational costs of the previous section.

$\boldsymbol{\gamma = \beta = 0}$: If $\psi_t = \psi$ for all t, then we have

$$g_K = g_L = 1, \quad g_z = n, \quad g_c = n^v. \tag{62}$$

Eq.(56) determines the level of capital per firm. The economy experiences semiendogenous effective consumption growth. Once capital per worker has been fixed, the size of the firms does not vary with the population growth, but it does the number of firms. As a result, although the amount of consumption of each good decreases, the total amount of goods consumed remains unchanged and the effective consumption increases due to the love of variety parameter.

$\boldsymbol{\xi = 1}$ **and** $\boldsymbol{\beta = 1 - \gamma}$: If $\psi_t = \psi K_t^{\gamma} k_{t-1}^{1-\gamma}$, then we have

$$g_K = g_L = G^{-1} n^{\frac{-1}{1-\gamma}}, \quad g_z = G n^{\frac{2-\gamma}{1-\gamma}},$$

$$g_c = G^{v} n^{\frac{v(2-\gamma)}{1-\gamma}}, \tag{63}$$

where $G = [\eta/s\psi\,(1-\eta-\gamma)]^{1/(1-\gamma)}$. The economy experiences endogenous and semiendogenous effective consumption growth.

Discussing prices and output growth: In this economy, growth of final goods (zero) does not coincide with growth of the individual effective consumption (positive). In other words, real production is the same but the subjective value the consumer gives to the production grows. In the market, per capita expenditure in consumption $\int_0^{z_t} p_{it} x_{c_{it}}^{j} di$ remains unchanged. The reason is that in adding different goods up, we need to use relative prices and, then, to measure output in units of one specific good. In the literature, it is typically assumed an effective consumption price index P_{c_t}.[21,22] In our case, we would have $P_{c_t} = p_t z_t^{-v}$, so that if we measure in units of effective consumption then $g_y = g_c$. The main problem with this unit of measure is that it does not exist as a good in the economy and, then, there is no economic reason why goods should be measured in terms of a good that does not exist in the economy. Moreover, if we consider a different love of variety parameter for each individual, then each individual has her own price index, and it would be impossible to find out an aggregate price index. On the other hand, since the prices of all the goods of this economy are the same, a direct estimation of the national product would give no growth at all. Nevertheless, this particular economic structure shows that the way production is measured by the current governments underestimates real (effective consumption) growth. Thus, accounting for the observed pattern in levels and rates of growth of per capita income does not necessaily explain economic development.

5 Love of variety and aggregate returns to scale

Next, we show that assuming both love of variety and aggregate returns to scale in the investment sector gives a different positive final goods growth rate than the

[21]In particular, in our economy it would be $P_{c_t} = z_t^{-v+\eta/(1-\eta)} \left(\int_0^{z_t} p_{it}^{(\eta-1)/\eta} di\right)^{\eta/(\eta-1)}$.

[22]An example of effective consumption without love of variety is Peretto (1998).

effective consumption one. In particular, we assume that individuals effective consumption is

$$c_t^j = \left(z_t^{v_c(1-\eta)-\eta} \int_0^{z_t} \left(x_{c_{it}}^j \right)^{1-\eta} di \right)^{\frac{1}{1-\eta}}, \quad \eta \in (0,1), \ v_c \in (0,1), \tag{64}$$

and that investment goods are produced through the following technology:

$$I_t = \left(z_t^{v_I(1-\eta)-\eta} \int_0^{z_t} x_{I_{it}}^{1-\eta} di \right)^{\frac{1}{1-\eta}}, \quad v_I \in (0,1), \tag{65}$$

where v_c and v_I are the love of variety and the aggregate returns to specialization parameters, respectively. This model nests the economy of the previous sections. For this reason we skip all the steps.

Balanced growth path: If one final good is the numéraire, then $p_t = 1$ and $P_{I_t} = z_t^{-v_I}$. In the balanced growth path, we have the following relationships:

$$\eta n^{\frac{\alpha}{\alpha - v_I}} g_K^{\frac{-v_I}{\alpha - v_I}} = s \left[(1-\eta) \frac{\psi_t}{K_t} - \psi_{K_t} + \psi_{L_t} \frac{L_t}{K_t} \right], \tag{66}$$

$$g_z = g_K^{-\frac{\alpha}{\alpha - v_I}} n^{\frac{\alpha}{\alpha - v_I}}, \tag{67}$$

$$g_c = g_K^{\frac{-\alpha v_c - v_I(1-\alpha)}{\alpha - v_I}} n^{\frac{v_I(1-\alpha)+\alpha v_c}{\alpha - v_I}}, \tag{68}$$

$$g_y = g_K^{-\frac{v_I(1-\alpha)}{\alpha - v_I}} n^{\frac{v_I(1-\alpha)}{\alpha - v_I}}. \tag{69}$$

$\boldsymbol{\gamma = \beta = 0}$: If $\psi_t = \psi$ for all t, then we have $g_K = 1$, and

$$g_z = n^{\frac{\alpha}{\alpha - v_I}}, \quad g_L = n^{\frac{-v_I}{\alpha - v_I}},$$
$$g_y = n^{\frac{v_I(1-\alpha)}{\alpha - v_I}}, \quad g_c = n^{\frac{v_I(1-\alpha)+v_c\alpha}{\alpha - v_I}}, \tag{70}$$

where $v_I < \alpha$ in order to have convergence. The economy experiences semiendogenous final goods growth and semiendogenous effective consumption growth, but the last is greater than the final goods growth.

$\boldsymbol{\xi = 1}$ **and** $\boldsymbol{\beta = 1 - \gamma}$: If $\psi_t = \psi K_t^{\gamma} k_{t-1}^{1-\gamma}$, then we have

$$g_K = M^{v_I - \alpha} n^{\alpha}, \quad g_z = M^{\alpha} n^{\frac{\alpha(2-\gamma)}{\alpha(1-\gamma)-v_I(2-\gamma)}}, \quad g_L = M^{-\alpha} n^{\frac{-\alpha - v_I(2-\gamma)}{\alpha(1-\gamma)-v_I(2-\gamma)}},$$
$$g_y = M^{v_I(1-\alpha)} n^{\frac{v_I(1-\alpha)^2}{\alpha - v_I}}, \quad g_c = M^{v_I(1-\alpha)+v_c\alpha} n^{\frac{v_I(1-\alpha)^2+\alpha(1-\alpha)v_c}{\alpha - v_I}}, \tag{71}$$

where $M = \left[\eta / s\psi \left(1 - \eta - \gamma\right)\right]^{\frac{1}{\alpha(1-\gamma)-v_I(2-\gamma)}}$ and $v_I < \alpha(1-\gamma)/(2-\gamma)$ in order to have convergence. The economy experiences endogenous and semiendogenous growth for both final goods and effective consumption, but again effective consumption growth is greater than final goods growth.

6 Concluding remarks

The main contribution of this paper is to stress the importance of the evolution of operational costs, as they alter the market structure of the economy, which in turn determines the growth pattern. Moreover, also the industrial structure has been found to be crucial when interpreting growth. In particular, we show that a real per capita production growth can be different than the per capita effective consumption growth or the subjective value the consumer gives to production.

We still know not much about growth. We need to investigate more on the relationship between technology, market structure and preferences. Also, factors as human capital or public infrastructures could accelerate an industrial structure change. Obviously, the endogeneization of the mark-up, different mark-ups for different sectors, or different economic structures, would shed more light about this relationship.

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Appendix

Existence and stability of the balanced growth path

Applying the operational cost Eq.(32) to the free entry condition Eq.(19) we have

$$\eta z_t^v K_t^{1-\alpha} L_t^{\alpha} = (1-\gamma)\,\psi_t. \tag{A.1}$$

Combining Eqs (24), (32) and (A.1) gives

$$z_t s \left(\frac{1-\gamma-\eta}{\eta}\right) \psi_t = z_{t+1} K_{t+1}. \tag{A.2}$$

Using Eqs (32) and (21), Eqs (A.1) and (A.2) transform in equilibrium into

$$\widehat{L}_t^v K_t^{1-\alpha-\gamma\xi} L_t^{\alpha-v-\gamma(1-\xi)} K_{t-1}^{-\beta} = \frac{(1-\gamma)\,\psi}{\eta}, \tag{A.3}$$

$$s\psi \left(\frac{1-\gamma-\eta}{\eta}\right) \widehat{L}_t L_t^{\gamma(1-\xi)-1} K_t^{\gamma\xi} K_{t-1}^{\beta} = \widehat{L}_{t+1} L_{t+1}^{-1} K_{t+1}. \tag{A.4}$$

In view of Eq.(A.3), the following result is immediate.

Proposition 1 *When $\alpha - v - \gamma(1-\xi) = 0$ and $v \neq 1-\gamma-\beta$, then*

1. $g_K = n^{-v/(1-v-\gamma-\beta)}$.
2. *The balanced growth path is stable if $\beta/(1-v-\gamma) \in (-1,1)$.*

In case that $\alpha - v - \gamma(1-\xi) \neq 0$, substituting L_t from Eq.(A.3) into Eq.(A.4), and after applying growth rates, yields

$$F\left(g_{K_{t+1}}, g_{K_t}, g_{K_{t-1}}\right) = n^{-E_1} g_{K_{t-1}}^{-\beta E_2} g_{K_t}^{E_3+\beta} g_{K_{t+1}}^{-E_4} = 1, \tag{A.5}$$

where $E_1 = v\gamma(1-\xi)$, $E_2 = 1-\alpha+v$, $E_3 = (1-\gamma)(1-\alpha) - v\gamma\xi$, and $E_4 = 1-v-\gamma$. We concentrate on g_{K_t} since if it exists, Eq.(31) informs that g_{y_t} exists, too. Defining $E_5 = -\beta E_2 + E_3 + \beta - E_4$, note that Eq.(A.5) allows to calculate the balanced growth path only when $E_5 \neq 0$. In case that $E_5 = 0$, the balanced growth path may either not exist or it cannot be inferred from Eq.(A.5).

Proposition 2 *If $E_5 = 0$ and $E_1 \neq 0$ and $n \neq 1$, then no balanced growth path exists.*

Proposition 3 *If $E_5 \neq 0$ then $g_K = n^{E_1/E_5}$.*

Note that the case $\gamma = \beta = 0$, implying that $E_1 = 0$, belongs to the last Proposition. Next, we show sufficient conditions for the balanced growth path to be stable when $E_5 \neq 0$. The first order Taylor's expansion around the balanced growth path is

$$\begin{aligned} P_1(g,g,g) &= F(g,g,g) + F_{g_{K_{t+1}}}(g,g,g)\left(g_{K_{t+1}} - g\right) + \\ &+ F_{g_{K_t}}(g,g,g)\left(g_{K_t} - g\right) + F_{g_{K_{t-1}}}(g,g,g)\left(g_{K_{t-1}} - g\right), \end{aligned} \tag{A.6}$$

where $F_{g_{K_{t+1}}}(g,g,g) = -E_4 F(g,g,g)\, g^{-1}$; $F_{g_{K_t}}(g,g,g) = [E_3 + \beta]\, F(g,g,g)\, g^{-1}$; and $F_{g_{K_{t-1}}}(g,g,g) = -\beta E_2 F(g,g,g)\, g^{-1}$. Thus, the linearized differential equations can be written as

$$P_1(g,g,g) - F(g,g,g) = 0 = -E_4 g_{K_{t+1}} + [E_3 + \beta]\, g_{K_t} - \beta E_2 g_{K_{t-1}} - gE_5. \tag{A.7}$$

For the case $E_4 \neq 0$, defining $N_t = K_{t-1}$ yields

$$\begin{bmatrix} g_{K_{t+1}} \\ g_{N_{t+1}} \end{bmatrix} = \begin{bmatrix} -\frac{gE_5}{E_4} \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{E_3+\beta}{E_4} & \frac{-\beta E_2}{E_4} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} g_{K_t} \\ g_{N_t} \end{bmatrix}, \tag{A.8}$$

which eigenvalues satisfy

$$p(\lambda) = \lambda^2 + b\lambda + c = 0, \tag{A.9}$$

where $b = -(E_3 + \beta)/E_4$ and $c = \beta E_2/E_4$. Since $E_2 > 0$, it is immediate that $E_4 < 0$ implies that $p(\lambda)$ has real solutions. In this case, stability is assured as far as $p(-1) > 0$ and $p(1) > 0$, since $p(0) = c < 0$; i.e., $1 - b + c > 0$ and $1 + b + c > 0$. As $E_4 < 0$, the previous inequalities can be written as $E_4 + E_3 + \beta(1 + E_2) < 0$ and $E_4 - E_3 - \beta(1 - E_2) < 0$. Thus, the following proposition holds.

Proposition 4 *If $E \neq 0$ and $E_4 < 0$, a balanced growth path is stable whenever $\beta < -(E_4 + E_3)/(1 + E_2)$ and either*

1. *$E_2 < 1$ and $\beta > (E_4 - E_3)/(1 - E_2)$, or*
2. *$E_2 > 1$ and $\beta < (E_4 - E_3)/(1 - E_2)$, or*
3. *$E_2 = 1$ and $E_4 - E_3 < 0$.*

When $E_5 \neq 0$ and $E_4 > 0$, as $c > 0$, $p(\lambda)$ may have either real or complex solutions. If solutions are complex, they are $\lambda = (-b/2) \pm i\,(c - b^2/4)^{1/2}$ and, therefore, the square of the absolute value of the module is c. Hence, stability occurs whenever $c < 1$. For the case of real solutions, since $p(0) = c > 0$, the requirement is either[23] $0 < -b/2 < 1$ and $1 + b + c > 0$, or $-1 < -b/2 < 0$ and $1 - b + c > 0$.

Proposition 5 *If $E \neq 0$ and $E_4 > 0$, a balanced growth path is stable whenever*

1. *The solutions of $p(\lambda)$ are complex, i.e., $(E_3 + \beta)^2 < 4\beta E_2 E_4$, and $\beta < E_4/E_2$.*
2. *The solutions of $p(\lambda)$ are real, i.e., $(E_3 + \beta)^2 > 4\beta E_2 E_4$, and*
   (a) *$\beta > -E_3$ and $\beta < 2E_4 - E_3$ and either*
      i. *$E_2 < 1$ and $\beta < (E_4 - E_3)/(1 - E_2)$.*
      ii. *$E_2 > 1$ and $\beta > (E_4 - E_3)/(1 - E_2)$.*
      iii. *$E_2 = 1$ and $E_4 - E_3 > 0$.*
   (b) *$\beta < -E_3$ and $\beta > -2E_4 - E_3$ and $\beta > -(E_4 + E_3)/(1 + E_2)$.*

Proposition 6 *If $E_5 \neq 0$ and $E_4 = 0$ we distinguish two situations:*

1. *$E_3 + \beta = 0$ or $\beta E_2 = 0$, in which cases we have stability (no transition).*
2. *$E_3 + \beta \neq 0$ and $\beta E_2 \neq 0$ and thus the balanced growth path is stable if $\beta E_2/(E_3 + \beta) \in (-1, 1)$.*

When $E_5 = 0$ and either $E_1 = 0$ or $n = 1$, we have to check each possible case. We concentrate on $E_5 = 0$ and $E_1 = 0$. This happens when $(1 - \beta)(v - \alpha) + \gamma(\alpha - v\xi) = 0$ and either $\xi = 1$ or $\gamma = 0$, which gives six possible cases. However, the cases $\xi = 1$, $v = \alpha$ and $\beta \neq 1 - \gamma$; $\xi = 1$, $v = \alpha$ and $\beta = 1 - \gamma$; $\gamma = 0$, $v = \alpha$ and $\beta \neq 1$; and $\gamma = 0$, $v = \alpha$ and $\beta = 1$ belong to Proposition 1. Thus, we analyze the other two cases.

If $\xi = 1$, $\beta = 1 - \gamma$ and $v \neq \alpha$, combining Eqs (A.2) and (32), evaluating in equilibrium, and applying growth rates, gives

$$s\psi \left(\frac{1 - \gamma - \eta}{\eta} \right) g_{K_t}^{\gamma - 1} = g_{z_{t+1}} g_{K_{t+1}}. \tag{A.10}$$

[23] These conditions guarantee that $p(\lambda)$ attains a minimim in $(0, 1)$ and $p(1) > 0$, respectively, or the minimum is in $(-1, 0)$ and $p(-1) > 0$.

And from Eqs (A.1), (21) and (32), evaluating in equilibrium, and applying growth rates, we obtain

$$g_{z_{t+1}}^{v-\alpha} n^{\alpha} = g_{K_{t+1}}^{\gamma+\alpha-1} g_{K_t}^{1-\gamma}. \tag{A.11}$$

From these two equations we have

$$g_{K_{t+1}} = \left[s\psi \left(\frac{1-\gamma-\eta}{\eta} \right) \right]^{\frac{\alpha-v}{1-\gamma-v}} n^{\frac{-\alpha}{1-\gamma-v}} g_{K_t}^{\frac{(1-\gamma)(1+v-\alpha)}{1-\gamma-v}}, \tag{A.12}$$

from where

$$g_K = \left[\frac{\eta}{s\psi\left(1-\gamma-\eta\right)} \right]^{\frac{-(\alpha-v)}{\alpha(1-\gamma)-v(2-\gamma)}} n^{\frac{-\alpha}{\alpha(1-\gamma)-v(2-\gamma)}}, \tag{A.13}$$

and the economy converges to a unique stable balanced growth path when

$$0 < \frac{(1-\gamma)(1+v-\alpha)}{1-\gamma-v} < 1, \tag{A.14}$$

i.e.,

$$v(2-\gamma) < \alpha(1-\gamma). \tag{A.15}$$

If $\gamma = 0$, $\beta = 1$ and $v \neq \alpha$, combining Eqs (A.10) and (A.11) with $\gamma = 0$ gives

$$g_{K_{t+1}} = \left[\frac{s\psi\left(1-\eta\right)}{\eta} \right]^{\frac{\alpha-v}{1-v}} n^{\frac{-\alpha}{1-v}} g_{K_t}^{\frac{1+v-\alpha}{1-v}}, \tag{A.16}$$

from where

$$g_K = \left[\frac{s\psi\left(1-\eta\right)}{\eta} \right]^{\frac{\alpha-v}{\alpha-2v}} n^{\frac{-\alpha}{\alpha-2v}}, \tag{A.17}$$

which is stable if

$$-1 < \frac{1+v-\alpha}{1-v} < 1. \tag{A.18}$$

Proposition 7 *When $\xi = 1$, $\beta = 1-\gamma$ and $v \neq \alpha$, then*

1. $g_K = \left[\eta/s\psi\left(1-\gamma-\eta\right)\right]^{-(\alpha-v)/[\alpha(1-\gamma)-v(2-\gamma)]} n^{-\alpha/[\alpha(1-\gamma)-v(2-\gamma)]}$.
2. *The balanced growth path is stable if $v(2-\gamma) < \alpha(1-\gamma)$.*

Proposition 8 *When $\gamma = 0$, $\beta = 1$ and $v \neq \alpha$, then*

1. $g_K = \left[s\psi\left(1-\eta\right)/\eta\right]^{(\alpha-v)/(\alpha-2v)} n^{-\alpha/(\alpha-2v)}$.

2. *The balanced growth path is stable if* $(1 + \upsilon - \alpha) / (1 - \upsilon) \in (-1, 1)$.

The particular parameter combinations of the last two propositions are called knife-edge conditions by Christiaans (2004). He shows that the knife-edge conditions are consequence of the need to have a system of linear equations in order to have endogenous growth. But he does not wonder where they come from. In our case, we are simply assuming that the operational cost is a Cobb-Douglas function.