DETECTION OF ANOMALIES WITH THE POLARIZATION STATE OF THE MAGNETOTELURIC SIGNAL

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Abstract: It has been tested the usefulness of the technique developed by Escalas (2015) [1] for detecting and characterizing anomalies in the magnetoteluric signal. The system is based on the Morlet wavelet, wich allows making a polarization analysis in the time-frequency domain. For doing that, first, it has been made a synthetic signal with a magnetoteluric tensor and then, it has been analysed. Finally, it has been searched anomalies in real data.

I. INTRODUCTION

The magnetoteluric signal is the electromagnetic field created by natural sources from the Earth's subsurface. With this signal is possible to get the distribution of the electrical conductivity, wich leads to useful application in underground exploration. This analysis is a current field of research for deep mantle probing or earthquake prediction and many other applications. So the perfection of the evaluation technique is obviously motivated.

In this work, it will be analyzed different magnetoteluric signals (Mt for now on) with the developed technique by Escalas (2015)[1] that uses the wavelet transform. This way of evaluate the MT signal does not only give the frequencies (as a conventional Fourier Transform) it gives the time that a particular frequency occur and his polarization attributes. The polarization state of the signal is useful to see if there is any anomaly that comes from not geomagnetic or geoelectric sources. The natural magnetoteluric signal doesn't have a preferent polarization, so it is possible to find external signals (expanded in section VI). With the knowledge of these anomaly signals, it is possible to prospect some regions that are difficult to analyze eliminating these signals artificially or, study these anomalies.

For determining the effectiveness of these new technique it will be evaluated two types of signals. First one: simulating a synthetic signal with different polarization states in different frequencies changing in time using the magnetoteluric tensor. Second one: three real signals obtained in Samalus(Vallès), Jumilla(Murcia) and Vilada(Berguedà).

II. DEFINITION AND CHARACTERIZATION OF THE MT SIGNAL

The MT signal detected is:

$$\vec{E} = (E_{NS}, E_{EW}); \vec{B} = (B_{NS}, B_{EW}),$$

where $\vec{E}$ is the electric field (V/m), and $\vec{B}$ is the magnetic field (T); NS and EW indicate the orthogonal components. Clearly, this vector is different for each frequency and each time (As this is an experimental analysis, time is considered discrete). With this set up, the MT signal describes an ellipse in the surface plane. In this document, for each $\vec{E}$ and $\vec{B}$, the signal will be characterized with:

- **Ellipticity ϵ**: It is computed as: $\boldsymbol{\epsilon = r/R}$; where R is the semi-major axis and r is the semi-minor axis as shown in Fig.(1). This parameter goes from $\boldsymbol{\epsilon \in [-1, 1]}$. If $\boldsymbol{\epsilon > 0}$ the rotation will be clockwise. If $\boldsymbol{\epsilon < 0}$ the rotation will be anti-clockwise.

- **Polarization angle $\boldsymbol{\theta}$**:Which is the angle formed by the semi-major axis of the ellipse and the EW direction as shown in Fig.(1).$\boldsymbol{\Theta \in [0, 180]}$.

- **Phase difference**: $\boldsymbol{\Delta\phi}$: which is the phase difference between the two components of the signal. It will be computed as :$\boldsymbol{\Delta\phi = \phi_{NS} - \phi_{EW}}$; and it goes $\boldsymbol{\Delta\phi \in [-180, 180]}$.

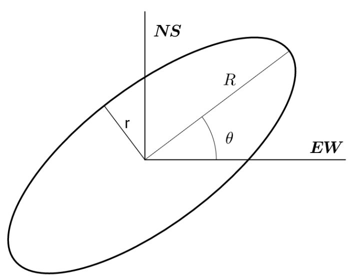

FIG. 1: Polarization ellipse of the MT signal for the electric and magnetic field.

III. MAGNETOTELURIC TENSOR $\bar{\bar{M}}$ (w)

$\vec{E}$ and $\vec{B}$ are related with the magnetoterulic tensor $\bar{\bar{M}}$ in the frequency domain:

$$\vec{E}(w) = \bar{\bar{M}}(w)\vec{B}(w),$$

$$\begin{pmatrix} E_i \\ E_j \end{pmatrix} = \begin{pmatrix} M_{ii} & M_{ij} \\ M_{ij} & M_{jj} \end{pmatrix} \begin{pmatrix} B_i \\ B_j \end{pmatrix}. \tag{1}$$

$\bar{\bar{M}}$ has velocity dimension. Note that we will have a tensor for each frequency w.

With this tensor it can be extract the useful following geoelectric parameters:

- **Apparent resistivity $\rho_{ij}(w)[\Omega m]$**: It gives the average resistivity per unit volume for each frequency. It is computed as:

$$\boldsymbol{\rho_{ij}(w) = \frac{\mu_o}{w}|M(w)_{ij}|^2,} \tag{2}$$

where $\boldsymbol{\mu_o}$ is the magnetic permeability in vacuum.

- **Phase $\varphi_{ij}(w)[rad]$**: which is the phase difference between **E** and **B**. Computed as:

$$\boldsymbol{\varphi_{ij}(w) = \arctan\left(\frac{\Im M_{ij}}{\Re M_{ij}}\right).} \tag{3}$$

IV. WAVELET TRANSFORM

Now, it will be summarized how wavelet transform works. *(It should be clear that the aim of this work is not going deeper in the procedure of the wavelet transform algorithm, neither mathematical details. This section only exist for the sake of completeness.)*

The WT of a continuous signal $S(t)$ is defined as:

$$W_\psi[s(\sigma,\tau)] = \sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} S(t) \cdot \psi^\star(\sigma,\tau,t)\, \mathrm{d}t, \tag{4}$$

where $\psi^\star$ is the conjugate of $\psi(\sigma,\tau,t)$, which is :

$$\psi(\sigma,\tau,t) = \frac{1}{\sigma} \cdot \psi_o\left(\frac{t-\tau}{\sigma}\right), \tag{5}$$

ψ_o is the mother wavelet which is going to be used for analyzing the $S(t)$ signal. σ is the scale parameter and τ is the translation parameter in time. The mother wavelet used is the **Morlet wavelet**:

$$\psi_o^M(t) = e^{i\beta t} \cdot e^{\frac{-t^2}{2}}, \tag{6}$$

β is the characteristic angular frequency, and it is related with the scale parameter $\sigma = \frac{\beta}{w}$

All of this mathematical structure is defined in continuous variable. The code used for doing the analysis discretizes this definition for experimental implementation. For further mathematical information consult [1]. For more information about the code consult [2].

V. CREATION AND ANALYSIS OF A SYNTHETIC SIGNAL

first it has been created the orthogonal components for the magnetic field $\vec{B}$. This signal has 3 different frequencies that change over time. All the components follow this form: $S(t) = \mathbf{A} \cdot \sin(2\pi \cdot \boldsymbol{f} \cdot t + \boldsymbol{\vartheta})$. Where **A** is the amplitude. $\boldsymbol{f}$ is the frequency in Hz. $\boldsymbol{\vartheta}$ is the phase shift in radians. The signal is defined in Tab.I. The polarization attributes previously defined in sec. II of the signal are shown in fig. 2 and 3 .

		$\Delta T(10^2 s)$									
		[0-3]	[3-8]		[8-10]	[10-13]		[13-15]		[15-20]	
B_{NS}	A	2	2	1	1	2	1	1	2	3	4
	f (Hz)	0.5	0.5	0.1	0.1	0.1	0.05	0.05	0.5	0.05	0.5
	$\vartheta(^\circ)$	0	0	0	0	45	45	45	0	90	0
B_{EW}	A	3	3	2	2	1	1	1	3	2	2
	f (Hz)	05	0.5	0.1	0.1	0.1	0.05	0.0	0.5	0.05	0.5
	$\vartheta(^\circ)$	45	45	0	0	45	45	0	45	0	30

TABLE I: Synthetic magnetic signal.

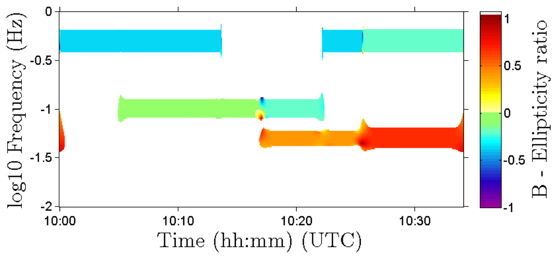

FIG. 2: Ellipticity of the synthetic signal in the time-frequency domain.

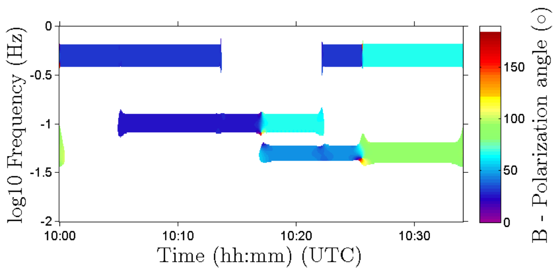

FIG. 3: Polarization of the synthetic signal in the time-frequency domain.

With this results we can determine easily the state of polarization for each frequency and time, keeping in mind: $\epsilon = 0$ (linear) $\epsilon = 1$ (circular). The state of polarization is shown in table II.

	f(Hz)	$\Delta T(10^2 s)$		
		[0-3]	[3-8]	[8-10]
Polaritation state	0.5	elliptical anticlockwise	elliptical anticlockwise	
	0.1		linear	linear
	0.05			

		[10-13]	[13-15]	[15-20]
Polarization state	0.5		elliptical anticlockwise	elliptical anticlockwise
	0.1	elliptical clockwise		
	0.05	elliptical clockwise	elliptical clockwise	elliptical clockwise

TABLE II: Polarization state of the synthetic magnetic signal.

Now that the synthetic magnetic signal is well characterized, it has been obtained the synthetic electric signal with the magnetoteluric tensor Eq.1. For doing that, it has been proposed three tensors, shown in Eq.7, for each frequency: 0.5 hz, 0.1hz and 0.05 hz respectively. To be consistent with the dimensions and magnitude of $\bar{\bar{M}}(w)$; these tensor must be multiplied by a constant proportional to $\frac{\mu_o}{w}$, but for simplicity, this factor is not considered.

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix};\begin{pmatrix} e^{\frac{i\pi}{6}} & -e^{\frac{i\pi}{3}} \\ e^{\frac{i\pi}{3}} & e^{\frac{i\pi}{6}} \end{pmatrix};\begin{pmatrix} 2.25e^{\frac{i\pi}{6}} & 0.5e^{\frac{i\pi}{3}} \\ 1.25e^{\frac{i\pi}{3}} & -1.5e^{\frac{i\pi}{4}} \end{pmatrix}; \quad (7)$$

For computing the electric signal, I write a new Matlab script. The procedure is the following: the magnetic signal has been transformed to the frequency domain and multiplied each frequency by Eq.7 . This multiplication is not trivial, because the fourier transform is not six Dirac deltas (One for every frequency including negatives frequencies). The multiplication has been made dividing the fourier transform in six different sections. First, the frequencies has been localized. Then, it has been defined the limits of each section as half the distance between the surrounding frequencies. After that is has been multiplied the six sections with his particular tensor. Finally, an antifourier transform was applied to recover E in the time domain. We can see the transformation in Fig. 4: It is shown the temporal series of the magnetic field and the spectrum of that signal, immediately after, in red, it is shown the spectrum of the electric signal after applying the magnetoteluric tensor on the signal. The last plot shows the electric signal in the time domain.

The results of the analysis of the electric signal are shown in Fig. 6 and 5. θ_{EB} is the angle between the angle of polarization between the two signals: $\theta_{EB} = |\theta_E - \theta_B|$. ϵ_{EB} is the difference between the angle of polarization between the two signals: $\epsilon_{EB} = |\epsilon_E| - |\epsilon_B|$.

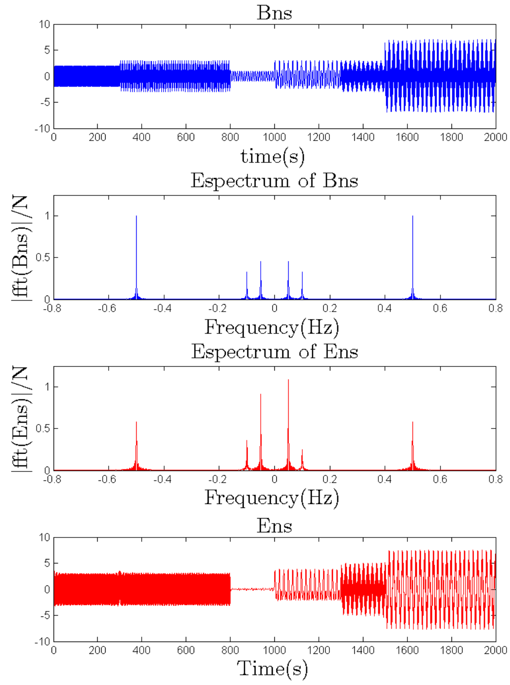

FIG. 4: Temporal series and espectrums of B_{NS} and E_{NS}.

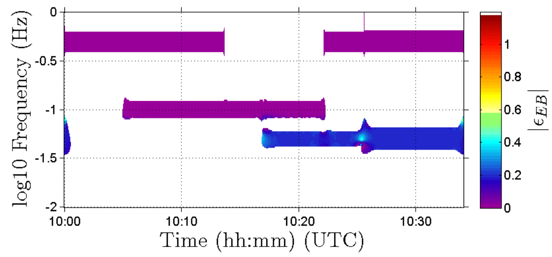

FIG. 5: Comparison between the ellipticity ratio of the electric and magnetic signal.

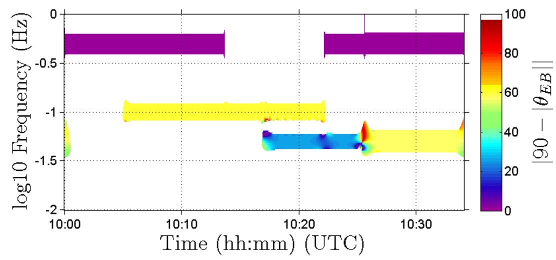

FIG. 6: Comparison between the polarization angle of the electric and magnetic signal.

The frequency 0.5Hz does not change his attributes. The signals are perpendicular due to Eq.7. At 0.1Hz, the signal has changed his polarization angle, therefore, the comparison differs from the 90 degrees. The ellipticity does not change. At 0.05 Hz, where the tensor is totally 3D, the ellipticity vary because the tensor change the amplitude of the signal. The polarization angle change over time.

This analysis show that the properties of the medium can change drastically the polarization attributes of the signal. It must be clear that the magnetoteluric tensor applied is not a realistic one (It can be for a not geophysic purpose).

VI. ANALYSIS OF A REAL SIGNAL

Before the analysis, it is useful to know the most common anomalies:

- **Sun-Earth interaction**: $f < 1Hz$ This fluctuaction is caused by the solar wind emitted by the sun interacting with the magnetosphere of the Earth. This signal depends on the solar activity, so it is natural to expect a regular variation and punctual magnetic storms. One interesting phenomenon are the geomagnetic pulsations: The more common ones are called Pc2 and Pc3 with $f \in [(0.1; 0.2)Hz]$ and $f \in [(0.022; 0.1)Hz]$, respectively.

- **Electric storms (Shumman resonances)** $f > 1Hz$: When an electrical discharge occurs in a storm, the ionosphere and the surface of the Earth act as a resonant cavity and it occurs resonance. The frequencies are 8 Hz, 14Hz, 20Hz, 26Hz, and so on.

- **Cultural noise** : This is the anthropogenic generated signal. The main two signals are the electric lines and the railroad tracks, which are not perfect and always there is some losses. This signal is easy to find because it has a fundamental frequency of 50 Hz (in Europe). This cultural noise is easily detected because it has a linear polarization.

The natural magnetoteluric signal does not have a preferent polarization, so an abrupt polarization tendency is the evidence of an anomaly. It has been analysed 3 real data. The sites are Samalus, Vilada and Jumilla. Next, there are the descriptions of the data with the results of the analysis made with the code described in [2]. Not all the results are shown, only the more illustrative and clear are displayed:

A. Samalus data and analysis

The signal has a length of 2 hours. The sampling frequency is 4Hz, wich means long wavelength. So the maximum frequency to compute is 2Hz. This type of data is ideal to see possible Pc3 and Pc2. Interesting results are shown in Fig. 7 , 8 and 9.

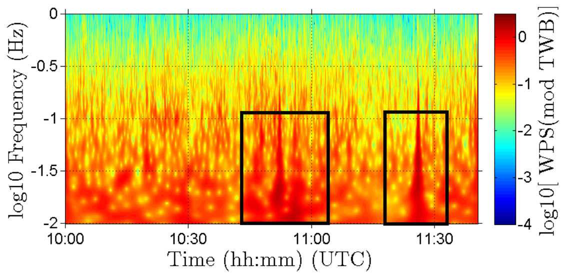

FIG. 7: Scalogram of the magnetic Samalus data.

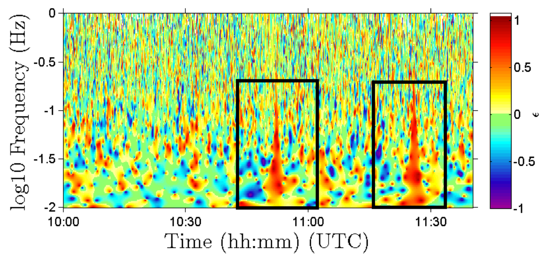

FIG. 8: Ellipticity ratios of the magnetic Samalus data.

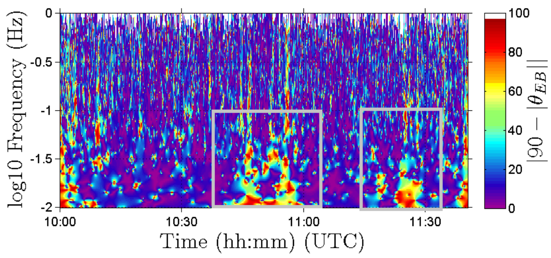

FIG. 9: Comparition of the angle of polarization between $\vec{E}$ and $\vec{B}$ of the Samalus data.

There are two anomalies found in the rage of 0.1 and 0.01 Hz with the polarization attributes: for $\vec{B}$: ϵ=[0.3 - 0.7], θ=[30° - 65°] and $\Delta\phi$=[30° - 100°]. In the magnetic field (Fig.8) it is clear that there is a polarization tendency. In the Electric field it is not that clear (not shown). The ultimate evidence is when the two fields are compared (Fig. 9) : There is a clear change of state in both anomalies. Due to the range of frequencies, it is probably a Pc3.

B. Vilada data and analysis

The sampling frequency is the same as the Samalus data but it has a length of two weeks. It has been analyzed the data and nothing clear was found. Maybe there is two Pc3 in different days, but is very unclear and not worth to mention it.

C. Jumilla data and analysis

The sampling frequency of the data is 512Hz which means that the maximum frequency to compute is 256Hz.

This type of acquisition allows to visualize possible cultural noise and Schumann resonances.

In figures 10 , 12 and 11 it is shown two anomalies found. The first one has a frequency of 50Hz and the following polarization attributes: for $\vec{E}$: ϵ=0, θ=94° and $\Delta\phi$=[160° - 165°]; for $\vec{B}$: ϵ=0.36, θ=32° and $\Delta\phi$=43°. It is undoubtably cultural noise due to the ellipticity of the electric field and his frequency. It is possible to see the next harmonics of that signal (100Hz and 150Hz). This cultural noise is visible in all the data collected. It is known that the acquisition was made with an electric line at 146°E.The polarization angle in the magnetic signal is 32 degrees, so it can be compatible and coherent if the electric polarization angle is arround 146 degrees. This is not the case. It is very possible that the medium between the site and the electrical line modifies the signal, as it is shown in Sec.V. The two vertical lines are not identified as a common anomaly, they can be losses related to the harmonics (they have the same polarization). It will be researched in more depth in next acquisitions on the same site.

In Fig.12, it is easy to recognize three anomalies at about 8Hz, 15Hz and 25Hz. The polarization attributes are not clear. It is very likely that this anomalies are Shumman resonances.

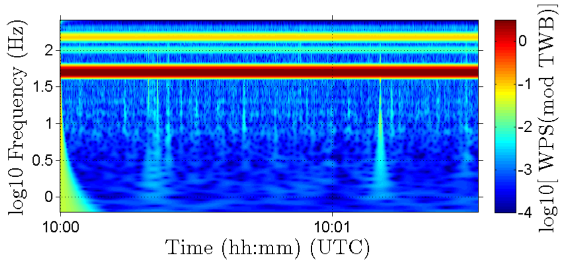

FIG. 10: Scalogram of the magnetic Jumilla data.

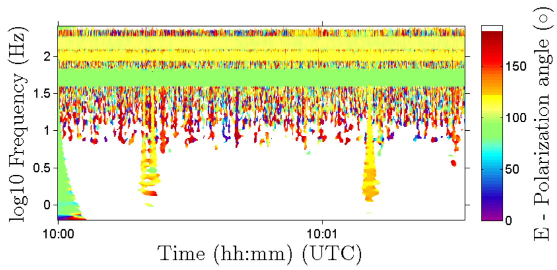

FIG. 11: Polarization angle of the electric field.

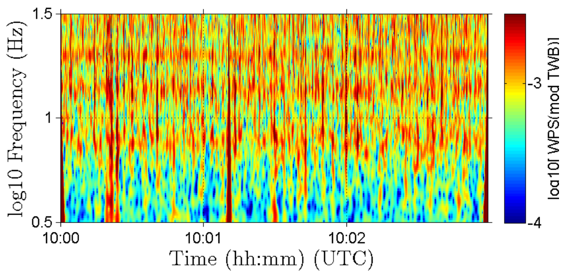

FIG. 12: Scalogram of the magnetic field in shumman's frequency range.

VII. SUMMARY AND CONCLUSIONS

It has been tested the usefulness of the method made by Escalas (2015) analyzing synthetic and real data with different characteristics. We can conclude:

- The use of different kinds of real data(range of frequency) allow to find different types of anomalies.
- This technique is very useful for finding and characterizing external anomalies of a magnetoteluric signal. It can enormously help to study all this phenomenon in more depth. For example; it can help to understand better the Sun-Earth interaction or the electromagnetic properties of the atmosphere.
- To get the magnetoteluric tensor it is necessary to expand the code (filtering the signal of anomalies and compute the attributes of the tensor), this update of the code is difficult but not unreachable.
- It can be applied in a lot of other areas such as telecommunications, medical signals , photonics or any other field that requires analysis of the polarization state in time-frequency domain.

Acknowledgments

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[1] Magdalena Escalas Oliver (2015), *Polarisation analysis of the magnetotelluric signal in the time-frequency domain*, (University of Barcelona. 279p).

[2] Escalas (2015), *MTWAVELETS Tutorial-v1* , 25p.