Estimation of the degree of polarization in low-light 3D integral imaging

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ABSTRACT

The calculation of the Stokes Parameters and the Degree of Polarization in 3D integral images requires a careful manipulation of the polarimetric elemental images. This fact is particularly important if the scenes are taken in low light conditions. In this communication, we show that the Degree of Polarization can be effectively estimated when elemental images are recorded with few photons. First, we use the Maximum Likelihood Estimation approach for generating the 3D integral image. Nevertheless, this method produces very noisy images and thus, the degree of polarization cannot be calculated. We suggest using a Total Variation Denoising filter as a way to improve the quality of the generated 3D images. As a result, noise is suppressed but high frequency information is preserved. Finally, the degree of polarization is obtained successfully..

Keywords: 3D integral imaging, Photon counting, Polarization, Image reconstruction

1. INTRODUCTION

Polarimetric image techniques provide more information than conventional imaging systems. Polarization information is a powerful tool of analysis that provides information about the objects present in the scene and can be used in classification, segmentation, or pattern recognition among many others applications [1,2]. Integral image (InIm) [3-5] is an effective technique that extends 2D imaging concepts to the tridimensional world [6-8]. Recently, the use of polarimetric techniques has been proposed in 3D InIm [9]. This technique requires recording several polarimetric elemental images in order to calculate the Stokes parameters and the Degree of Polarization (DoP) of the 3D scene.

The problem becomes much more complex when the scene is illuminated in low light conditions. Several techniques have been proposed for producing an InIm from elemental images recorded with few photons [11-14]. In particular, the Maximum Likelihood Estimation (MLE) is a simple but effective method that produces reasonable results. Nevertheless, the estimation of the DoP using this technique provides wrong results since the evaluation of the degree of polarization involves several floating point calculations using noisy photon-counting elemental images. Recently, we proposed a successful alternative for estimating the DoP in low light conditions [14]. Polarimetric 3D InIm are obtained using the MLE and subsequently denoised using a Total Variance filter. DoP results were similar to those obtained in normal light conditions.

In the present paper we review basic concepts on the estimation of the DoP in photon-starved conditions. The original ideas were communicated in [A. Carnicer and B. Javidi, "Polarimetric 3D integral imaging in photon-starved conditions," Opt. Express **23**, 6408-6417 (2015)]. In section 2 we describe how to measure polarization in 3D InIm and some examples are provided. In section 3 we describe how results are improved by using MLE and total variation filters. Finally, conclusions are presented in section 4.

2. POLARIMETRIC 3D INTEGRAL IMAGING

Polarimetric imaging is a technique able to determine the polarization state in each pixel of the image. This methods are particularly useful in the analysis of complex images with multiple objects with diverse polarization behavior. The measure of the Stokes parameters and the DoP of the scene is straighforward by recording six polarimetric images with

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the help of a linear polarizer and a quarter wave plate placed in the front of the objetive of the camera [15]. The Stokes parameters (S_0, S_1, S_2, S_3) in each pixel of the image are obtained using the following formulae:

$$\begin{aligned} S_0 &= I^{0°,0} + I^{90°,0} \\ S_1 &= I^{0°,0} - I^{90°,0} \\ S_2 &= I^{45°,0} - I^{135°,0} \\ S_3 &= I^{45°,\pi/2} - I^{135°,\pi/2} \end{aligned} \tag{1}$$

where $I^{\alpha,\beta}$ is the recorded image when the axis of the polarizer is set at an angle α with respect to the x direction; β can take values 0 or $\pi/2$. In the first case, the image is recorded using a linear polarized but when $\beta=\pi/2$, a quarter wave plate is also required in addition to the linear polarizer. Note that S_0 is the energy of the scene (i.e. the conventional image) and S_1 compares the weight of the polarization in the x and y directions. S_2 provides the same information than S_1 but using two axes rotated 45° with respect to the x direction. S_3 indicates the circular content of the light in the corresponding pixel. Positive values mean the electric field rotates clockwise whereas negative values show a counterclockwise behavior. The four Stokes images can be combined in a single distribution: the DoP, defined as

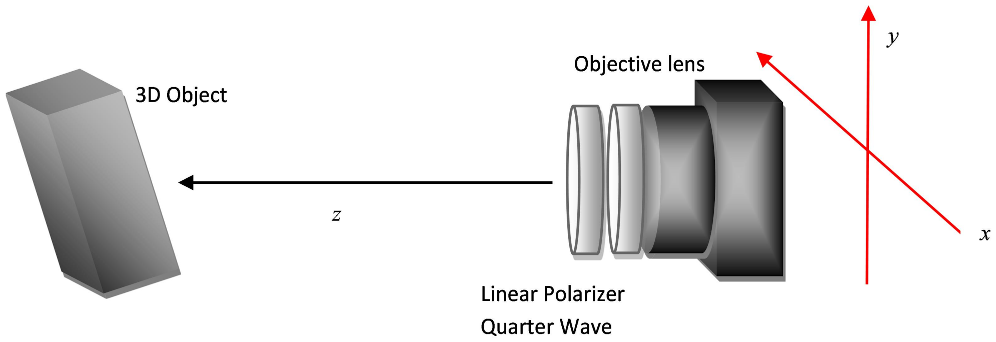

Figure 1. Sketch of the polarimetric 3D pick-up system.

$$\mathrm{DoP} = \frac{1}{S_0}\sqrt{S_1^{\,2} + S_2^{\,2} + S_3^{\,2}}. \tag{2}$$

This distribution specifies the portion of light which is polarize. Note that DoP ranges from 0 (unpolarized light) to 1 (totally polarized light).

A method for generating a 3D distribution of a scene $I(x,y,z)$ is based on combining elemental images $i(x,y)$ recorded from slightly different points of view. The information needed for producing 3D InIm is obtained by recording elemental images by moving a single camera to different locations while the scene is in the field of view. Then, the following formula is used to merge the elemental images:

$$I(x,y,z) = \sum_{k=0}^{N_x-1}\sum_{l=0}^{N_y-1} i\left(x - k\frac{N_x p f}{c_x z}, y - l\frac{N_y p f}{c_y z}\right). \tag{3}$$

N_x and N_y are the number of elemental images in the x- and y- directions, c_x x c_y is the size of the CCD pixel, p is the relative displacement of the camera for recording each elemental images, f is the focal length of the objective lens and z is the pick-up distance between the scene and the camera.

3D Polarimetric information is generated using the same approach. For every single (k,l) position of the camera, six elemental images $i_{k,l}^{\alpha,\beta}$ are recorded. Using Eq. (3), the six polarimetric distributions $I^{\alpha,\beta}(x,y,z)$ are calculated,

$$I^{\alpha,\beta}(x,y,z)=\sum_{k=0}^{N_x-1}\sum_{l=0}^{N_y-1} i_{k,l}^{\alpha,\beta}\left(x-k\frac{N_x p f}{c_x z},\, y-l\frac{N_y p f}{c_y z}\right), \tag{4}$$

and the four Stokes images are calculated using Eq. (1). If the system is illuminated in low light conditions, photon-counting elemental images $\hat{i}_{k,l}^{\alpha,\beta}(x,y)$ are modelled according to the Poisson distribution:

$$\hat{i}_{k,l}^{\alpha,\beta}(x,y)=\begin{cases}0, \text{ if } \mathrm{rand}(x,\ y)\le \exp\left(-n_{k,l}^{\alpha,\beta}(x,y)\right)\\ 1, \text{ otherwise}\end{cases} \tag{5}$$

where

$$n_{k,l}^{\alpha,\beta}(x,y)=\frac{N_p i_{k,l}^{\alpha,\beta}(x,y)}{\sum_{x,y} i_{k,l}^{\alpha,\beta}(x,y)} \tag{6}$$

and N_p is the number of photons in every elemental image. MLE provides a way of calculating the integral imaging in photon-starved conditions by means of averaging of the photon-counting irradiances $\hat{i}_{k,l}^{\alpha,\beta}$ of the polarimetric elemental images $i_{k,l}^{\alpha,\beta}$, i.e.

$$\hat{I}^{\alpha,\beta}(x,y,z)\propto\sum_{k=0}^{N_x-1}\sum_{l=0}^{N_y-1} \hat{i}_{k,l}^{\alpha,\beta}\left(x-k\frac{N_x p f}{c_x z},\, y-l\frac{N_y p f}{c_y z}\right). \tag{7}$$

A set of 6x6 elemental images has been recorded. The scene is composed of several objects located at different distances ranging from 420 to 720 mm. The focal length of the camera objective is 50 mm with a sensor size of 36 x 24 mm; images are 1000x1500 pixels and the displacement of the camera is 5mm in both horizontal and vertical directions. These figures are summarized in Table I. Figure 2 show the integral images calculated at z=450, 50 and 720 mm using Eq. (3).

Table 1. Integral imaging variables

# Elemental images	$N_x = 6$, $N_y = 6$
Scene depth range	420 – 720 mm
Focal length	f= 50 mm
Sensor size	c_x = 36 mm, c_y = 24 mm
Resolution (# of pixels)	r = 1000, c = 1500
Camera relative displacement	p = 5 mm

Figure 2. Reconstructed conventional InIm at z=450, 530 and 720 mm.

Figure 3 shows the InIm Stokes parameters calculated at z=720 mm. Note that the toy cars partially polarize light, whereas the rest of the light in the scene remains unpolarized.

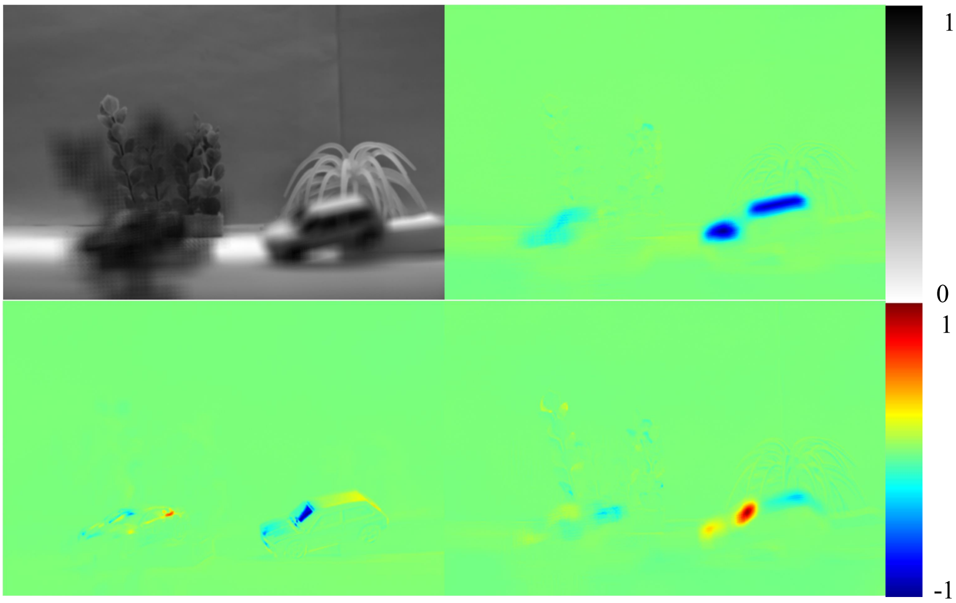

Figure 3. InIm Stokes parameters (S_0, S_1, S_2, S_3) calculated at z=720 mm. S_0 is shown in gray level whereas S_1, S_2 and S_3 are displayed using the jet colomap.

Figure 4 shows the InIm Stokes parameters calculated at z=720 mm in photon starved conditions. The number of photons is N_P = 75000, i.e. a 5% of the number of pixels in the image. The MLE [Eq. (7)] is used to generate the Stokes images. Note that in this case all pixels in images S1, S2, and S3 are close to zero in even in those areas where the effect of polarization is not negligible. When the number of photons is low, MLE underestimates polarization.

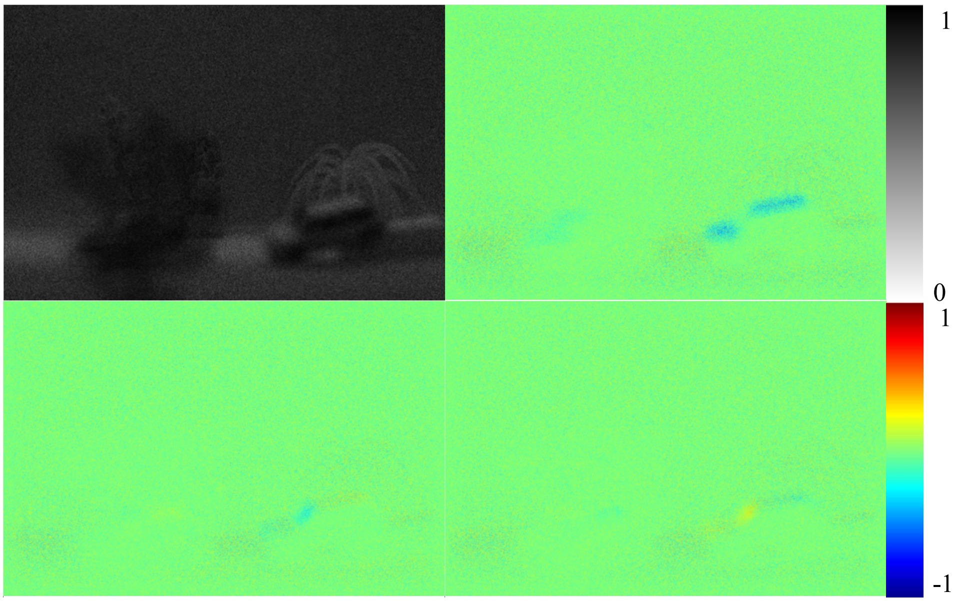

Figure 4. InIm Stokes parameters (S_0, S_1, S_2, S_3) in photon counting conditions (N_P = 75000), using the Maximum Likelihood Estimation at z=720 mm

Figure 5 shows the DoP when the scene is conventionally illuminated and in photon starved conditions. Note that MLE provides a wrong measurement of the DoP.

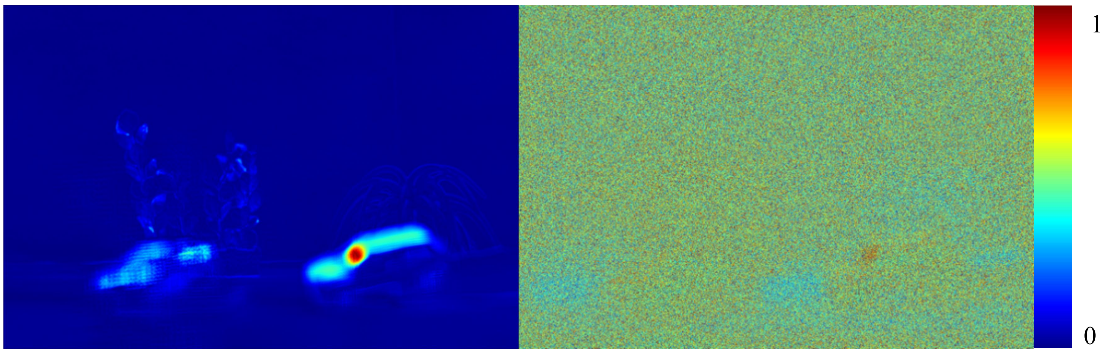

Figure 5. DoP when the scene is conventionally illuminated and in photon starved conditions

3. TOTAL VARIATION DENOISING POLARIMETRIC 3D INTEGRAL IMAGING

Total variance denoising filters are designed to minimize the effect of noise while preserving the details of the image [16]. This technique performs better when compared with other denoising techniques based on smoothing signals, because high frequency information is kept. In this paper we use MLE 3D InIm reconstruction in combination with the Chambolle implementation of total variation filters [17]. In particular calculations are carried out using the scikit-image library [18].

Figures 6 and 7 show the same Stokes images and the DoP presented in Fig. 4 and 5(b) but after applying the Chambolle total variance denoising filter. A visual comparison of the present images with the results obtained using conventional illumination [Figs. 3 and 5(a)] show a better agreement. In particular, the DoP calculated using MLE + a total variance filter with *Np*=75000 photons displays very similar values to the distributions obtained using conventional illumination.

In order to provide more insight, the correlation coefficient $\rho(i_1,i_2)$ between images i_1 and i_2 is defined as:

$$\rho(i_1,i_2) = \frac{\sum_{m,n}\left[i_1(m,n)-\langle i_1\rangle\right]\left[i_2(m,n)-\langle i_2\rangle\right]}{\sqrt{\sum_{m,n}\left[i_1(m,n)-\langle i_1\rangle\right]^2\sum_{m,n}\left[i_2(m,n)-\langle i_2\rangle\right]^2}} \tag{8}$$

Note that this parameter is normalized, i.e. $0 \le \rho(i_1,i_2) \le 1$; $\rho(i_1,i_2)$ is calculated using S_0 and DoP in conventional illumination conditions as reference. Table 2 shows correlation values for the MLE or MLE + filter cases.

Table 2. Correlation coefficient

	MLE	MLE + TV filter
S_0	0.935	0.980
DoP	0.683	0.999

It is apparent that when the TV filter is used in addition to MLE, correlation values are close to 1.

4. CONCLUDING REMARKS

In this communication, we showed that the DoP can be effectively estimated in 3D InIm when elemental images are recorded with few photons. The Maximum Likelihood Estimation approach for generating the 3D integral image in combination with a Total Variation Denoising filter suppresses noise while high frequency information is preserved. Results obtained in photo-starved conditions are similar with those obtained using conventional illumination.

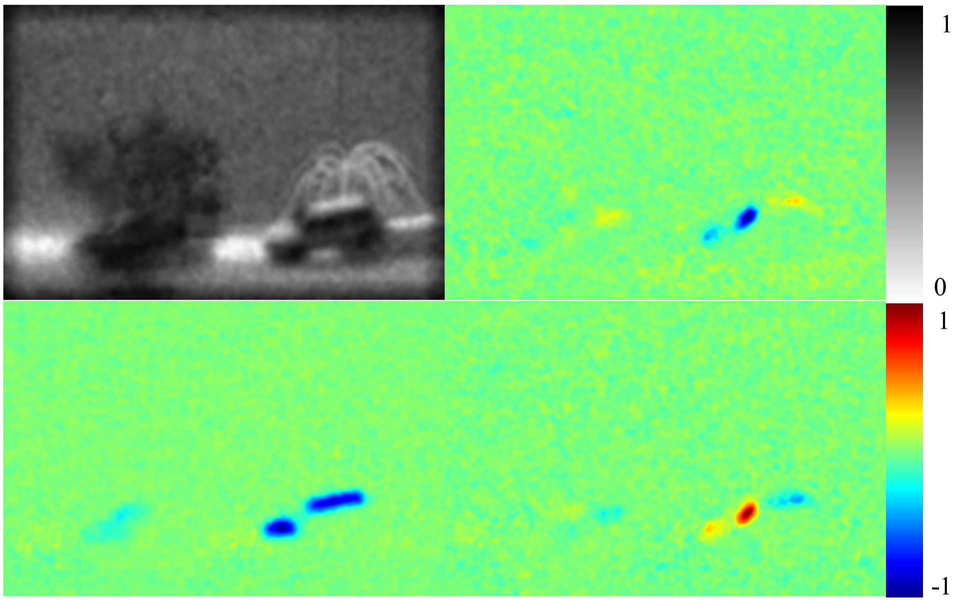

Figure 6. InIm Stokes parameters $\left(S_0, S_1, S_2, S_3\right)$ in photon counting conditions (N_P = 75000), using the Maximum Likelihood Estimation and the Chambolle Total Variance denoising filter at z=720 mm

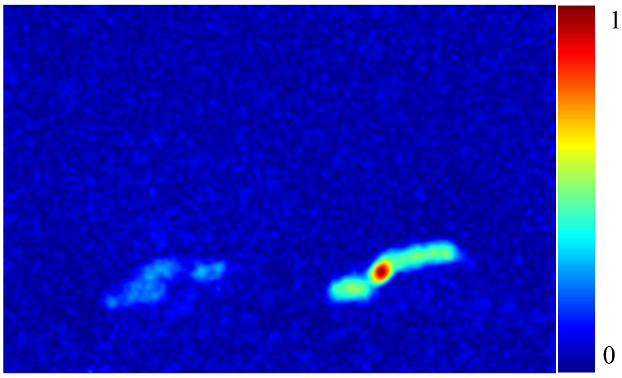

Figure 7. DoP using MLE + Chambolle total variation denoising filter

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