DOCUMENTS DE TREBALL
DE LA FACULTAT DE CIÈNCIES
ECONÒMIQUES I EMPRESARIALS

Col·lecció d'Economia

E07/185

Incentives and Promotion
in Wage Hierarchies

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Abstract: Most of the large firms organization schemes consist in hierarchical structures of tiers with different wage levels. Traditionally the existence of this kind of organizations has been associated to the separation of productive and managerial or supervision tasks and to differences in the skills of the workers. However, many firms now employ workers with similar skills, and then the hierarchical structure can be related to an incentive scheme to ensure that workers supply effort. The model we present investigates how firm owners should determine the optimal wage distribution in order to maximize profits.
JEL Classification: J31, L23, M11,M51.

Keywords: Efficiency wages, Optimal hierarchies, firm structure, incentives scheme, moral hazard.

Resum: La majoria d'esquemes organitzatius en empreses consisteixen en estrucures jeràrquiques de diferents nivells amb differents nivells salarials. Tradicionalment l'existència d'aquest tipus d'esquemes organitzatius s'ha associat a la separció de tasques de gestió o supervisió i a diferències d'habilitats entre treballadors. Tot i això, actualment hi ha moltes empreses que contracten treballadors amb habilitats semblants, i aleshores l'estructura jeràrquica es pot relacionar a un esquema d'incentius per assegurar que els treballadors s'esforcen. El model que presentem invesiga com els propietaris de les empreses haurinen de determinar l'estructura de salaris òptima per tal de maximitzar els beneficis.
Paraules clau: Salaris d'eficiència, jerarquies òptimes, esquemes d'incentius, risc moral.

1 Introduction

The classic economic principles that date back to the period of Ricardo (1817) state that the wage earned by a worker in a free market economy should represent as closely as possible his or her marginal production. This statement is valid when it is easy to calculate the marginal product of a worker and there are few management tasks, which is the case in small firms, for example. Nowadays a large portion of the working population is employed by large firms, which often operate internationally and have thousands of employees around the world. In fact, many economic historians, such as Chandler (1962), have written that a significant proportion of the economic development over the last few centuries has been driven by large-scale organizations, some of them with hundreds of thousands of employees. In these large firms the wage is usually contracted before performance can be analyzed and all employees at the same level earn the same wage regardless of the effort supplied.

The production of these large firms depends on the effort supplied by their employees, which is often a hidden action that is not easily observed by the owners. This can encourage employees to lower their performance. Dickens, Katz, Lang and Summers (1989) present extensive evidence of the impact of worker shirking in American firms. Akerlof and Yellen (1990) show the correlation between worker effort and influences such as anger, jealousy and gratitude. In most of the moral hazard literature it is assumed that the cost of supervising the effort supplied by the employees is relatively high. This generates a situation of asymmetric information between the firm owner and the employee, in which only the employee knows the effort he or she supplies, the "hidden action". Effort, as a non-measurable variable, cannot usually be included in contracts between principal and agent. The creation of incentive mechanisms to increase the effort supplied by the agents in order to be competitive is an important issue for large firms.

One possible way of motivating the workforce to supply effort is through punishment schemes, which try to detect and penalize shirking, commonly referred to as "stick schemes". Models such as the one proposed by Shapiro and Stiglitz (1984) are used when firms have limited monitoring capabilities and incentive workers to maintain performance by applying a stochastic process that detects probability rates of shirking. Qian (1994), following the work of Calvo and Wellisz (1978, 1979), developed a model of optimal hierarchical organization, treating hierarchies as supervisory schemes in which the bottom tier of the firm consists of production workers and the remaining tiers are structured as a system in which every agent supervises the agents in the tier immediately below them. This vision is far from realistic in current firms, particularly those in the service sector, where the tasks performed by agents in the upper tiers of the hierarchy are diverse and often not specifically related to monitoring the effort supplied by agents in lower tiers. Moreover, it is usually difficult to prove irrefutably that a worker has been shirking continuously, and without such proof there are no legal grounds for dismissal.

The other possible way of ensuring worker effort is to introduce positive incentives, which are commonly known as "carrot schemes". Holmstrom (1982) formalized the model devised by Alchian and Demsetz (1972) for optimal contracts in cases of moral hazard in teams, which considers the firm to be a team for which only the aggregate output is observable. In addition, Tirole (1986) developed collusion-proof contracts, which are designed to prevent collusion among agents or between supervisors and workers. These models are interesting in contract theory, but they do not establish the hierarchical structure of firms and some types of contract are not permitted by law.

In large firms, which usually have highly hierarchical structures, the agents usually have a stronger incentive to work: promotion. In fact, the structure of many large firms is based on different tiers with increasing wages. The potential for higher future salaries provides workers with strong incentives to supply effort, particularly when there is considerable competition among workers to secure promotion. This type of incentive-based wage structure usually implies that agents will be payed below their marginal production in the lower tiers and above their marginal production in the higher tiers. In fact, there are many firms in which the large wage increases from one tier to the next make it barely conceivable that the productivity of an agent who gains promotion increases with wage, as Medoff and Abraham (1980) document. Moreover, it is often possible to find highly skilled workers in the job market or in lower tiers who are capable of supplying similar productivity to agents in high tiers for lower wages, but firms do not lower the wages of workers in higher tiers in order to maintain incentives for agents in lower tiers. Doeringer and Piore (1971) show that a high proportion of those in higher-paid jobs have been promoted from lower-paid jobs within the same organization.

In this paper I propose a model based on the observable behavior of firms and agents. I assume that when there is a vacancy in a certain tier on the salary ladder there is a lottery among the agents in the lower tier to occupy this vacancy. The individual probability of winning this lottery depends on the effort supplied. Then, the mechanism that incentive agents to supply effort is competition among agents to be promoted. The Nash equilibrium re

The paper is organized in four sections. After this introduction, the second section describes the general the model by analyzing the production, the probability of being promoted and the moral hazard involved. The third section presents the symmetric Nash equilibrium solution. The fourth section contains several comparisons of previous results, focusing particularly on two specific cases and the different scales of the problem. This section is also used to address inequality issues. Finally, we present the conclusions that can be obtained from the results presented.

2 The model

2.1 Production

Let's first establish the production side of the firm. We assume that the firm, through the effort of the agents, produces a homogeneous product, that is sold at a nominal price $p > 0$ taken as given by the firm. We also assume that there are no other production costs than the wages paid to the agents. This assumption is equivalent to assume that the costs other than the wages are proportional to the production, and the net revenue of every unit of good is its price. The firm owner, then, have as nominal revenue the overall production of the firm multiplied by the price of this product, and the wages paid to the workers as costs.

We assume that the firm organization consists in $Q \in \mathbb{N}$ jobs grouped in $N \leq Q$ subsets or tiers. We also assume that every tier has a large number of jobs given by Q_n, and has an associated wage, w_n, with $n = 1, ..., N$. We assume that the wages are arranged in rising order, $w_1 \leq ... \leq w_N$. At every instant of time Q individuals, called agents, belong to the firm, and occupy one and only one job. Every job belongs to one tier, and then $\sum_{n=1}^{N} Q_n = Q$. We assume that the wages that the firm owner can establish have a lower limit, given by a minimum wage w_b. This minimum wage can be established by the government, for example through minimum wage law, or by the market, for example as an opportunity cost of the agents.

Every agent can supply an effort given by[1] $\lambda^{\mathrm{i}} \in \mathbb{R}^+$. We assume that the production depends linearly on the effort and the productivity of the jobs of the tier where the agent works. The real productivity per unity of effort worked by an agent of the tier n will be noted by A_n, thus the production of an agent in the tier n supplying an effort λ^{i} is $A_n.\lambda^{\mathrm{i}}$. The nominal productivity is given by the product of the real productivity times the price of the good. The overall nominal production Y^{n} will be given by the following function

$$Y^{\mathrm{n}} = \sum_{n=1}^{N} p.A_n.\bar{\lambda}_n.Q_n \ ,$$

[1]From now we note $\mathbb{R}^+$ as the non negative reals, including the 0. For the moment we assume en effort without upper bound, that can seen non realistic, but as we will see it will be naturally bounded.

where $\bar{\lambda}_n$ is the average effort supplied by the agents in the tier n. Because the firm owner can not observe the effort supplied by the agents, he/she pays the same wage to all the agents in a single tier regardless the effort they supply. Then, the nominal production costs C^{n} are given by

$$C^{\mathrm{n}} = \sum_{n=1}^{N} w_n.Q_n \ .$$

The nominal profits of the firm are given by the nominal production minus the nominal costs. Then, the profits function P^{n} is given by the following formula:

$$P^{\mathrm{n}} = Y^{\mathrm{n}} - C^{\mathrm{n}} = \sum_{n=1}^{N} (p.A_n.\bar{\lambda}_n - w_n).Q_n \ .$$

The firm owner faces the problem of maximize this nominal profits function. In general he/she will set the wages of all existing tiers, the constraint being that they mist be higher or equal than the minimum wage w_b. In addition, we will consider the case where the firm owner can also choose the firm organization, that is, the number of agents in each tier $\{Q_n\}$, and maximize the profits optimizing also these numbers. In general, the firm owner cannot decide the average effort of each tier, because every agent decides the effort he/she supplies and it can not be observed. However, it can influence their decisions through changes in the incentives of being promoted, changing the wages and modifying the firm organization.

All of the nominal variables presented here can be translated to the real analogous variables simply by dividing them by the general price level of the economy. We will consider that the production of the firm represents a small proportion of the overall production of the specific type of goods, which in turn represents only a small portion of the consumer basket. Thus, changes in the price of the product without changes in nominal wages will not affect the real wages of the firmĠs workers. Nevertheless, when we consider the overall economy and aggregate production later in the paper, the price of the aggregate production will represent the general price level, in which case the real variables will be strongly dependent on the price.

2.2 Probability of promotion

We assume that the agents employed by the firm can leave it, a process that will be called "death". An agent can "die" in the literal sense, but also by reaching retirement age or deciding to move to another firm. We use a simplifying assumption that the death rate is the same for all agents working in the firm, regardless of age or the tier to which they belong. Then, the process of dying can be treated as a stochastic exponential process with a constant rate, which is represented by d. At any point of time, the probability that an agent will ÒsurviveÓ for a period of τ from the present is $e^{-d.\tau}$. When $\tau = \Delta t t$ is small, the probability of death can be approximated by $d.\Delta t$. The life expectancy of every agent in the firm is $1/d$.

When an agent employed by the firm dies, he/she leaves a vacancy in the respective tier. We assume that this vacancy can only be occupied by an agent from the tier immediately below, through a selection process that will be discussed later in the paper. We also assume that the agents can not be demoted or fired.[2] Then, vacancies in any tier can only arise due to the promotion of an agent from to the tier immediately above or due to the death of an agent in the tier. This implies that the death of an agent belonging to a certain tier produces a cascade effect in which there is a chain of promotion from all lower tiers. When there is a vacancy in the lowest tier, tier 1, the firm automatically hires another worker.

We assume that the process of filling the vacancies is instantaneous. This assumption ensures that the number of agents in each level is always constant. The automatic hiring of a new agent can be understood as the existence of a minimum wage set by the government and a pool of unemployed agents willing to work for the wage $w_1 \geq w_b$ In this case, we can assume that their reserve utility is suffciently low that they will immediately accept a job in the firm. Another possibility is to assume a reserve utility established by the opportunity available to agents in the labor market. We can then assume that there is no minimum wage and that the only restriction on wages is that they should be non-negative. The results in this case are similar to a government-established minimum wage, and we will obviate this case.

We will now consider the probability of promotion in a given tier. One agent will be promoted when a vacancy arises in the next tier. In general, the probability rate[3] of one vacancy in a certain tier will be constant. In fact, it is easy to show that when the death rate is constant, the number of deaths of agents in tiers above a specific tier follows a Poisson process, with constant probability rates. We use qn to denote the probability rate of a vacancy arising due to a promotion in tier n. The condition of replacing all of the agents that leave tier n is given by:

$$0 = \dot{Q}_n = -\overbrace{d.Q_n}^{\text{die}_n} - \overbrace{q_n.Q_n}^{\text{upg}_n} + \overbrace{q_{n-1}.Q_{n-1}}^{\text{upg}_{n-1}} .$$

This condition states that in order to maintain the number of agents in each tier n, the number of agents promoted from tier n to tier $n+1$ plus the number of agents in

[2]Examples of the impossibility of fire workers are the functionaries, that can not be fired except in extreme cases, and labor laws that do not allow to fire workers except when there are irrefutable proofs of their shirking, that are difficult to obtain.

[3]As in the process of dying, the relevant concept is the probability rate and not the probability itself. It is due we are considering the probability of being update per unity of time.

this tier who die must be the same as the number of agents promoted from tier $n-1$ to tier n. Note that only the agents in the tier immediately below can occupy the vacancy, and not agents in lower tiers or higher tiers. From the previous equation we can express the promotion rate of a given tier with respect the promotion rate of the following tier. We obtain the following expression:

$$q_{n-1} = \frac{(d + q_n).Q_n}{Q_{n-1}} \ . \tag{2.1}$$

When the structure of the firm is finite with N tiers, the probability rate of promotion for the top tier, q_N, must be 0. For a given organizational structure, i.e. for given values of Q_n, and a probability rate of death d, the relation (2.1) establishes a recurrence for the promotion rates that can be solved analytically. Then, for every n ¡ N, we can obtain the following expression for q_n:

$$q_n = \frac{d. \sum_{i=n+1}^{N} Q_i}{Q_n} \ . \tag{2.2}$$

Let us interpret (2.2). If we rearrange this equation, multiplying both sides by a small period of time Δt, we obtain the following equation

$$\Delta t.Q_n.q_n = \Delta t.d. \sum_{i=n+1}^{N} Q_i \ . \tag{2.3}$$

The left side of this equation is the number of people promoted from tier n in a short period of time Δt. On the right side we have the people in higher tiers that die during this period of time Δt. In order to supply sufficient employees to preserve the number of agents in each tier, the number of promotions from a given tier must be equal to the number of deaths in all higher tiers.

Although agents are identical they can supply different levels of effort. We will treat the promotion process when there is a vacancy in a certain tier as a lottery between all of the agents in the tier immediately below. We assume that the probability of winning this lottery and being promoted may be different for the various agents in the tier, depending on the individual effort supplied and the distribution of effort supplied by the rest of the agents in the tier, which every agent takes as given. Consequently, the agents know that supplying effort does not guarantee promotion when there is a vacancy, but that is alters the probability of promotion. This implies that the process is not a game in a strict sense, because the strategies of the individual players are not mutually influenced. Nevertheless, the distribution of strategies among the other agents in the same tier will affect the strategy of a particular agent, in which case some concepts of Game Theory may be considered.

The literature takes different approaches to this type of lottery. The optimal contract theory usually considers that the results of the work performed by agents are stochastically dependent on the effort supplied. When the level of success of an agentĠs work is included in the promotion criteria, the process is clearly dependent on the individual effort. Moreover, when the promotion criteria include worker experience, the business network and other aspects that depend stochastically on effort, the selection of promotion candidates depends stochastically on the individual effort supplied. The literature on ranking tournaments, pioneered by Lazear and Rosen (1981) and Green and Stokey (1983), is another important example of the use of effort-dependent lotteries in order to incentize worker effort.

The probability rate of promotion of every individual agent in tier n depends on his/her own effort, but also on the effort supplied by the other agents in the same tier. Then, given a distribution of effort supplied by the other agents in the same tier $\tilde{\lambda}_n$, we note the individual probability rate of promotion of a given agent supplying an effort λ^{i} as $q_n^{\mathrm{i}}(\lambda^{\mathrm{i}}, \tilde{\lambda}_n)$. The mean probability of promotion of all agents must be equal to the probability of promotion of any individual agent, so we have the following condition:

$$\overline{q_n^{\mathrm{i}}(\lambda^{\mathrm{i}}, \tilde{\lambda}_n)} = q_n \ , \tag{2.4}$$

where the upper bar is average among the agents in the tier n.

In some parts of this paper we will consider the simplifying assumption that the individual probability rate of promotion depends on the distribution of effort of the other agents in the same tier only through its mean, $\bar{\lambda}_n$. It is easy to show that the only function that depends on individual effort and the average effort that verifies the property (2.4) for all possible values of $\tilde{\lambda}_n$ is the linear probability rate given by:

$$q_n^{\mathrm{i}}(\lambda^{\mathrm{i}}, \bar{\lambda}_n) = \frac{a(\bar{\lambda}_n) + b(\bar{\lambda}_n).\lambda^{\mathrm{i}}}{a(\bar{\lambda}_n) + b(\bar{\lambda}_n).\bar{\lambda}_n} \cdot q_n \ ,$$

with $a(\bar{\lambda}_n) \geq 0$. The particular value $b(\cdot) = 0$ is the case in which the probability of promotion is independent of the effort supplied by the agents. Another possible requirement is that $\frac{\partial}{\partial \lambda^{\mathrm{i}}} q_n^{\mathrm{i}}(\lambda^{\mathrm{i}}, \bar{\lambda}_n) > 0$, which is equivalent to the condition that the probability of promotion must increase with effort, which implies $b(\cdot) > 0$. In this case the probability rate of promotion becomes:

$$q_n^{\mathrm{i}}(\lambda^{\mathrm{i}}, \bar{\lambda}_n) = \frac{c(\bar{\lambda}_n) + \lambda^{\mathrm{i}}}{c(\bar{\lambda}_n) + \bar{\lambda}_n} \cdot q_n, \quad \text{with} \ \ c(\bar{\lambda}_n) = \frac{a(\bar{\lambda}_n)}{b(\bar{\lambda}_n)} \geq 0 \ . \tag{2.5}$$

In general, the probability rate of promotion under these assumptions when the effort is 0 is given by $\frac{c(\bar{\lambda}_n)}{\bar{\lambda}_n + c(\bar{\lambda}_n)} \cdot q_n$.

An interesting, specific form of the function $c(\cdot)$ is the linear form $c(s) = c.s$, with c a non -negative constant. The derivative of the probability of promotion with respect the effort supplied is a decreasing function of c, so we interpret c as a measure of the insensitivity of the prize of being awarded promotion. In fact, the derivative of the individual probability of promotion with respect the inidividual effort supplied is $\frac{\bar{\lambda}_n^{-1}}{1+c} \cdot q_n$. The larger the value of c, the less sensible the probability rate of promotion $q_n^{\mathrm{i}}(\lambda^{\mathrm{i}}, \bar{\lambda}_n)$ to changes in the individual effort supplied, λ^{i}. Moreover, when $c = 0$ the probability of promotion of an agent supplying nil effort is 0 meanwhile for a general c is $\frac{c}{1+c} \cdot q_n$, which is a decreasing function of c.

2.3 Utility and moral hazard

As we have seen, the firm owner pays the same wage to all of the agents in a given tier. On the one hand, since the effort supplied by the agents cannot be observed and they do not like to supply effort, there are incentives not to do so. On the other hand, since the probability of promotion increases with the effort supplied, if the expected utility of the agents in the next tier is higher than the expected utility of their own tier, the agents have incentives to supply more effort in order to gain promotion and improve their level of utility. Consequently, the agents solve the maximization problem of deciding the amount of effort to supply by taking as given the wages of all tiers, the structure of the firm, the utility of the next tier and the effort supplied by the other agents in the tier.

The agents being hired have preferences regarding the effort they supply and the wage they earn. In the Ramsey model (1928), the agents want to maximize intertemporal utility throughout their working lives, perhaps sacrificing present utility in order to improve future utility. Since agents do not know the exact point at which they will ÒdieÓ, there is a discount term $e^{-d.t}$, which takes into account the probability of being alive at time t. Then, when agents make decisions they first make expectations about their expected utility and then reach a decision. An agent with an expected wage[4] $w(t)$ and effort $\lambda^{\mathrm{i}}(t)$, for each $t \in \mathbb{R}^+$, has an expected utility given by:

$$U(w, \lambda^{\mathrm{i}}) = \int_0^\infty e^{-d.t}.u\big(w(t), \lambda^{\mathrm{i}}(t)\big).\mathrm{d}t \ ,$$

where $u(w, \lambda^{\mathrm{i}})$ is the instantaneous utility when the wage is w and the effort is λ^{i}. We will assume that $u_w > 0$, $u_\lambda < 0$, $u_{ww} < 0$ and $u_{\lambda\lambda} < 0$.

[4]Expected in the sense that depends stochastically on the effort supplied but also in the sense that the agents do not know when they will die, and then, although the expected wage is defined for all times $t \in \mathbb{R}^+$, the wage will not be earned and the effort will not supplied after their death.

The agents will determine their effort throughout their working life, $\lambda^{\mathrm{i}}(t)$, and this decision will imply an expected wage at each point of time t, given by $w(t)$. The agents take the effort of the other agents as given. In general, the wage throughout the working life of an agent, or at least his/her expected wage at each point of time, will be a function of his/her effort. We can then suppose that agents have individual expectations about their wage at each point, $w(t, \lambda^{\mathrm{i}})$. The task faced by the agents is to solve the following problem:

$$U = \max_{\lambda^{\mathrm{i}}} \int_0^{\infty} e^{-d.t}.u\big(w(t, \lambda^{\mathrm{i}}), \lambda^{\mathrm{i}}(t)\big).\mathrm{d}t \ ,$$

with a given initial wage $w(0, \lambda^{\mathrm{i}})$. We define U_n as the maximum utility of the people who at time $t = 0$ are in the tier n, and then have as initial wage $w(0, \lambda^{\mathrm{i}}) = w_n$. The expected utility presented above depends only on the initial tier of the agent, which determines the initial value of the wage. The maximizer agent solves the same problem of determining the effort he/she supplies depending on the tier to which he/she belongs at any given point. We can then assume that each agent supplies a constant effort when he /she is in a given tier, which may be different of the effort of the other agents in the same tier. Moreover, agents in the same tier are indistinguishable, in the sense that the decisions are taken on the basis of future utility, and the same decisions affect all agents at any given point in time. Since the agents maximize their expected utilities, they will choose the maximum expected utility, so the expected utility of every agent in a given tier will be the same, given by U_n.

Now suppose that the expected utility of a certain tier $n+1$, U_{n+1}, is given and known. In this case, agents who are in the lower tier face the following maximization problem, in the form of a Bellman equation:

$$\begin{aligned} U_n &= \max_{\lambda^{\mathrm{i}}} \left[\int_0^{\infty} q_n^{\mathrm{i}}(\lambda^{\mathrm{i}}, \tilde{\lambda}_n).e^{-q_n^{\mathrm{i}}(\lambda^{\mathrm{i}}, \tilde{\lambda}_n).t}. \left(\int_0^t e^{-d.s}.u(w_n, \lambda^{\mathrm{i}}).\mathrm{d}s + e^{-d.t}.U_{n+1} \right).\mathrm{d}t \right] \\ &= \max_{\lambda^{\mathrm{i}}} \left[\frac{u(w_n, \lambda^{\mathrm{i}}) + q_n^{\mathrm{i}}(\lambda^{\mathrm{i}}, \tilde{\lambda}_n).U_{n+1}}{d + q_n^{\mathrm{i}}(\lambda^{\mathrm{i}}, \tilde{\lambda}_n)} \right] \ , \end{aligned} \tag{2.6}$$

where $\tilde{\lambda}_n$ is the distribution of the efforts of the agents in the tier n, which each agent takes as given, and $q_n^{\mathrm{i}}(\lambda^{\mathrm{i}}, \tilde{\lambda}_n)$ is the probability rate of promotion of an agent in the tier n who supplies an effort λ^{i} when the distribution of efforts in this tier is $\tilde{\lambda}_n$. This formula provides us with a relationship between the utilities of different tiers, and will be useful in the future in providing a recurrent solution to the above problem.

The agents in the highest tier, tier N, do not have any incentive to work because they cannot be promoted further. These agents do not supply any effort and only leave this tier when they die. The utility of the agents in tier N is given by:

$$U_N = \frac{u(w_n, 0)}{d} \ . \tag{2.7}$$

This formula can be used in equation (2.6), to obtain U_{N-1}, and then U_{N-2},... We can therefore obtain all of the utilities from the top to the bottom tier.

By using the expression for the expected utility (2.6) we determine that the incentive compatibility condition for an agent belonging to the tier n who supplies an effort λ^{i} and for an effort distribution on the tier $\tilde{\lambda}_n$, can be expressed in the following form:

$$\frac{u(w_n, \lambda^{\mathrm{i}}) + q_n^{\mathrm{i}}(\lambda^{\mathrm{i}}, \tilde{\lambda}_n).U_{n+1}}{d + q_n^{\mathrm{i}}(\lambda^{\mathrm{i}}, \tilde{\lambda}_n)} \begin{cases} = \frac{u(w_n, \lambda'^{\mathrm{i}}) + q_n^{\mathrm{i}}(\lambda'^{\mathrm{i}}, \tilde{\lambda}_n).U_{n+1}}{d + q_n^{\mathrm{i}}(\lambda'^{\mathrm{i}}, \tilde{\lambda}_n)} & \text{if } \lambda'^{\mathrm{i}} \in \tilde{\lambda}_n \ , \\ \geq \frac{u(w_n, \lambda'^{\mathrm{i}}) + q_n^{\mathrm{i}}(\lambda'^{\mathrm{i}}, \tilde{\lambda}_n).U_{n+1}}{d + q_n^{\mathrm{i}}(\lambda'^{\mathrm{i}}, \tilde{\lambda}_n)} & \text{if } \lambda'^{\mathrm{i}} \notin \tilde{\lambda}_n \ . \end{cases} \tag{2.8}$$

This expression establishes that if one an agent that chooses λ^{i} and another agent chooses λ'^{i}, their expected utility must be the same.

In this problem a stable state will be given by a set $\{w_n, \tilde{\lambda}_n, Q_n\}$ for $n = 1, ..., N$ that verifies equations (2.6) and (2.8). In this case, by using the incentive compatibility condition, we see that the agents have no incentive to change the individual effort supplied in any tier, taking as given the distribution of effort of the other agents in the same tier, $\tilde{\lambda}_n$. Therefore, every agent faces the following problem:

$$0 = \frac{\partial U(w_n, \lambda^{\mathrm{i}})}{\partial \lambda^{\mathrm{i}}} = \frac{\partial}{\partial \lambda^{\mathrm{i}}} \left(\frac{u(w_n, \lambda^{\mathrm{i}}) + q_n^{\mathrm{i}}(\lambda^{\mathrm{i}}, \tilde{\lambda}_n).U_{n+1}}{d + q_n^{\mathrm{i}}(\lambda^{\mathrm{i}}, \tilde{\lambda}_n)} \right)$$

Finally, this relationship can be arranged in order to obtain an interpretable expression for the incentive compatibility condition:

$$-u_\lambda(w_n, \lambda^{\mathrm{i}}) = \frac{(d.U_{n+1} - u(w_n, \lambda^{\mathrm{i}})) \cdot \frac{\partial}{\partial \lambda^{\mathrm{i}}} q_n^{\mathrm{i}}(\lambda^{\mathrm{i}}, \tilde{\lambda}_n)}{d + q_n^{\mathrm{i}}(\lambda^{\mathrm{i}}, \tilde{\lambda}_n)} \ . \tag{2.9}$$

This equation means that the marginal increment of expectancy gain of utility (right side) must be equal to the marginal disutility of increasing this amount of effort (left side).

3 Symmetric Nash equilibrium

Now we assume that every agent can choose the effort he/she supplies among from all the non-negative reals. The agents will only choose to increase the effort supplied if the decision also increases their expected utility. The problem can therefore be considered a game in players are the agents trying to choose the best strategy, the effort they supply, in order to maximize the outcome of the game, the expected utility.

We consider the symmetric Nash equilibrium (SNE) of our problem, that is, a situation in which all agents have individual incentives to supply the same effort. The SNE is a fundamental concept in In many problems with identical players that are analyzed using Game Theory. SNE is used to obtain results in many different problems, that range from firms competition to consumer's behavior. In this case, we can consider the competition between the agents in each tier as a game with a large number of identical players, where the strategy of each agent is the decision of the individual effort he/she supplies. The final level of effort that is common between the agents in a certain tier is not determined by either the firm owner or the individual agents, but through a process of competition between the agents that produces a state in which all agents individually decide to supply the same level of effort as the others.

The fundamental equations that determine the decisions of the agents are (2.6) and (2.9). These equations show that, when the agents choose the effort they supply among all real values, they choose a local maximum. In our case, in a symmetric solution, all agents supply the average effort of the tier considered, that is $\lambda^{\mathrm{i}} = \bar{\lambda}_n$ and $q_n^{\mathrm{i}} = q_n$. We assume a linear individual probability rate of promotion, as the shown in (2.5). Moreover we choose a function $c(\cdot)$ with a linear form $c(s) = c.s$, with c a non-negative constant.

We add to our model some simplifying assumptions that will allow us to obtain analytical results. These assumptions are not too restrictive and the the results obtained will be general and useful to interpret the firm structure in depth. We first assume that the utility can be decomposed in two additive parts, one depending (positively) on the wage and the other (negatively) on the effort. Moreover, in most of the calculations, we will assume that these two parts take the form of a constant relative risk aversion function. Then, the utility is given by

$$u(w, \lambda) = u^w(w) - u^\lambda(\lambda) = w^\alpha - B.\lambda^\beta \ , \tag{3.1}$$

that verify $(u^w)' > 0$, $(u^\lambda)' > 0$, $(u^w)'' < 0$ and $(u^\lambda)'' > 0$. In the particular form we have $0 < \alpha < 1$, $B > 0$ and $\beta > 1$.

From (2.6) and (2.9) the wage of the tier n can be expressed as a function of the expected utility U_n and the effort λ_n in the following way

$$w_n = \left(d.U_n - B.((1 + c).\beta - 1).\lambda_n^\beta\right)^{\frac{1}{\alpha}} \ . \tag{3.2}$$

This result allows us to implement the previously mentioned process to obtain recurrently first the expression for the expected utility U_N, then the expression for

U_{N-1},... These utilities are obtained in terms of the efforts $\lambda_1, ..., \lambda_{N-1}$ and the highest tier expected utility U_N. The result for the expected utility of the tier n is

$$U_n = U_1 + \beta.B.(1+c).\sum_{i=1}^{n-1} \frac{\lambda_i^\beta}{q_i} \tag{3.3}$$

It is important to note that there is a bijection between these N variables and the N wages that the firm owner tries to decide. Then, in general, this system provides the solution of our problem.

3.1 At a single firm level

We first consider the single firm problem. A single firm takes the price of the good produced and the nominal minimum wage of the economy (both indexed in monetary units and not in product units) as given. Because the price level of the economy is independent of the price of the good produced, the nominal and real wages are equivalent in the decisions of the agents.

Assume that the firm maximizes the profits with a wage distribution such that $w_1 > w_b$. If the firm owner decreases the lowest tier wage, this change does not affect the expected utilities of the higher tiers. This implies that agents in the higher levels supply exactly the same effort than before the changes. Otherwise, equations (3.2) and (3.3) show that when the wage w_1 decreases the average effort of the tier 1 agents increases. Thus, a decrease in the wage of the level 1 increases the production and decreases the costs. The conclusion is then that in the optimal wage distribution the wage that the firm owner choose for the agents of the tier 1 is the minimum possible wage, i.e., the minimum wage, $w_1 = w_b$. This argument cannot be used in the other tiers higher than 1, because changes the wage of the tier $n > 1$ affect the expected utility of the agents in this tier, and then also affect the effort supplied by the agents in all the lower tiers.

From (3.2) and (3.3) it is easy to observe that the wage of a certain tier does not depend on the efforts of the lower levels. In particular, w_N and w_{N-1} are the only wages that depend on λ_{N-1}. From these statements we can easily obtain the FOC for this effort turns, that turns to be specially simple. The FOC for λ_{N-1} states that:

$$w_{N-1}^\alpha = w_N^\alpha - \left(\frac{d^\beta.\alpha^\beta.p^\beta.A^\beta}{q_{N-1}^{\beta-1}.\big(\beta.(1+c).(d+q_{N-1}) - q_{N-1}\big).\beta^\beta.B.w_N^{\beta.(1-\alpha)}} \right)^{\frac{1}{\beta-1}} . \tag{3.4}$$

To obtain this result we used $q_{N-1}.Q_{N-1} = d.Q_N$, that can be easily obtained from (2.1). This process can be iterated to obtain the expression for the wages of all the tiers, but unfortunately no concise analytical results can be obtained.

In order to obtain analytical results we now consider a firm with only two tiers, $N = 2$. Although the simplifying assumption of a firm (or the whole economy) organized only two tiers may seem too simplified, it will allow us to obtain many interesting analytical results. In fact, most of the models that consider more than one role usually consider only two roles. Models with workers and firms, with highly skilled and low skilled workers, models with producers and predators, models with landowners and peasants,..., are models that help us to explain some observed facts and that allow us to understand them better. Albeit the reality is far of being as simple as only having two roles, because there are a lot of different roles and people assuming many of them at the same time, simple models allow us to understand better some aspects of the reality.

Now the effort supplied by the second tier agents is nil, $\lambda_2 = 0$, because the second is the highest tier and its agents have no incentives to work. Since no agent of the tier 2 can be promoted, we have $q_2 = 0$. Then, we can use the equation (2.2) to obtain that the number of agents in the tiers 1 and 2 with respect q_1, that are respectively $Q_1 = \frac{d}{d+q_1} \cdot Q$ and $Q_2 = \frac{q_1}{d+q_1} \cdot Q$. Because in this case the parameter q_1 determines the organization of the firm we will call it the structure parameter. The equation (3.4), for $N = 2$, establishes the relation between w_b and w_2. This relation can not be inverted, that is, we can not write analytically $w_2(w_b)$, but this relation is implicitly established. From this result we can finally write the effort λ_1 as a function of the wage w_2:

$$\lambda_1 = \left(\frac{d.\alpha.p.A}{\big(\beta.(1+c).(d+q_1) - q_1\big).\beta.B.w_2^{1-\alpha}} \right)^{\frac{1}{\beta-1}} . \tag{3.5}$$

This equation, considering the relation $w_2(w_b)$ that can be obtained from (3.4), implicitly defines the relation $\lambda(w_b)$. Note that λ_1 and w_2 are negatively correlated.

If the firms in one sector face extraordinary profits, new firms are attracted to this sector, and then the price decreases, until the moment that profits are 0. The 0 profits price takes in general the following form:

$$p = \frac{w_b.d + w_2.q_1}{\lambda_1.A.d} = \frac{w_b}{\lambda_1.A} + \frac{w_2.q_1}{\lambda_1.A.d} .$$

Once we take in consideration that $\lambda_1 = \lambda_1(q_1)$ and $w_2 = w_2(q_1)$, it is easy to show that this price function $p(q_1)$ is a U-shaped function, considering, and then it has an interior minimum.

A natural parameter[5] to represent the different variables is the number of agents in the tier 2, given by Q_2. This parameter ranges from 0 (all agents in the tier 1)

[5] Q_2 is a parameter when the firm can not choose the structure of the firm. Nevertheless if we consider that the firm owner can choose the number of agents in every tier, it turns to be a variable.

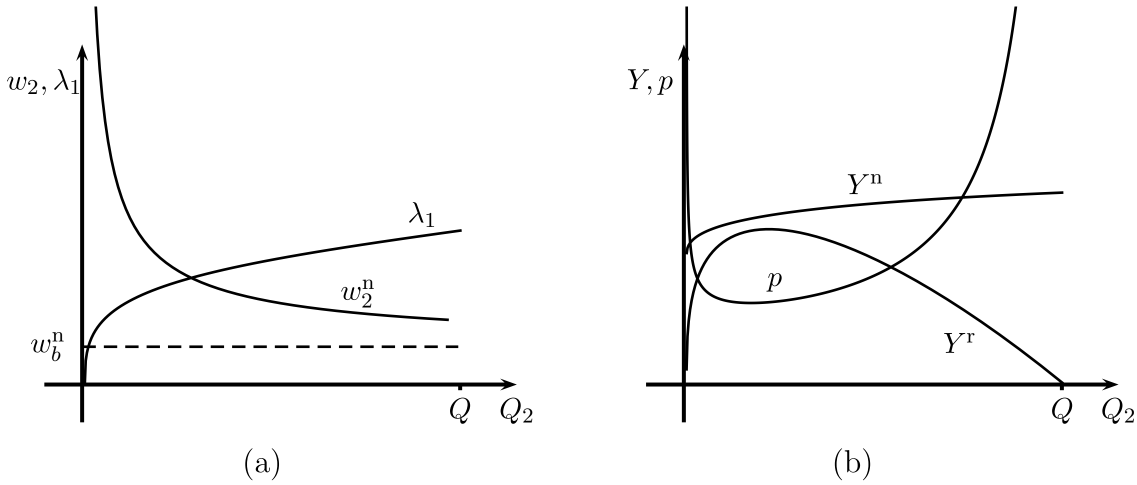

Figure 1: In (a) we represent the nominal wages and the efforts in a single firm. In (b) are depicted the nominal and real productions of a single firm and the price when profits are 0.

to Q (all agents in the tier 2). Moreover, Q_2 is closely related with q_1, what ensures that determine the other variables uniquely. The number of agents in the tier 2 does not allow us to write analytical expressions, but allows us to compare graphically the different relevant variables. Moreover, the proportion of agents among the two tiers many times may be given by specificities of the productive process of every concrete good, what makes this variable more suitable when we analyze a concrete good.

The figure 1 (a) corresponds to the wages and efforts in a single firm for every given quantity of agents in the tier 2. In this figure the price used in every point verifies the free entry condition, implying nil profits, price that depends on the structure of the firm. Note that higher is the quantity of the workers in the tier 2, higher is the effort of the agents in the tier 1, but lower is 2nd tier wage. Although for high Q_2 the premium is lower ($U_2 = w_2^2/d$ is lower) the increasing on the probability rate of promotion q_1 increases the competition among the agents, implying a high effort.

In the figure 1 (b) we represent the nominal production, the real production (measured in number of units produced) and the price of the product that implies nil profits. We observe that the real production is a hill-shaped function of Q_2. In fact, larger is Q_2, high is the effort the agents of the tier 1 supply, although the difference of wages between the tiers decreases. This is due to the increase on the probability of being promoted for high Q_2, that enhances the incentives to work. Nevertheless, for Q_2 sufficiently high, the effect of the increase of the effort of the tier

1 agents is counterbalanced by the effect of the decrease on the number of agents in the tier 1. Then, the real production achieves a maximum and since then decreases. There is a trade off between increase the effort of the agents of the tier 1, and then have more productive working agents, and decrease the number of agents in the tier 1, decreasing the number of working agents. If the firm has an excessively large number of agents in the tier 1 the small probability of being promoted discourages the workers and they do not effort. If the firm has an excessively large number of agents in the tier 2 then there is a large number of non-producing agents, making the firm inefficient.

We observe that the nil profits price is different for different firm organizations. Our model predicts differences of prices among different goods due the differences in the organizations of the firm that produce them. Nevertheless, when firms can choose their structure, firms will tend to choose Q_2 near the structure that lowers the price in order to be more competitive and then obtain additional profits. Then, the competitive market makes the firms choose a structure that verifies the condition $\partial p/\partial Q_2 = 0$, that corresponds to the minimum observed in the figure 1 (b). It is important to note that the maximum of the real production does not coincide in general with the minimum price, and then the market does not maximize the real production.

3.2 At the whole economy level

We now analyze the whole economy, that is, the effects observed when all firms of all sectors are considered. When treat the whole economy, the price level is obtained by weighting the consumption of every good by its price. Once the nominal minimum wage is established by the government, the price of every good is set up by the competition among firms. We assume that this process leads the economy to an equilibrium, where perfectly competitive sectors supply goods at the price of nil profits. This quantities and prices of equilibrium generate a price level, that itself jointly with the nominal minimum wage establishes a real minimum wage. Intuitively, when we aggregate the production the process is similar to consider only one good produced, and then the agents are paid in real terms with respect to this good.

In order to study the aggregate production we model the whole economy as a single firm behaving facing nil profits. This firm produces a good that is sold at price p, that coincides with the price index. Real wages are obtained by dividing nominal wages by the price level. The agents will make the decisions about choosing the effort supplied taking in account the real wages of every tier. In fact, at firm level these two wages where exogenously proportional, because every single can not

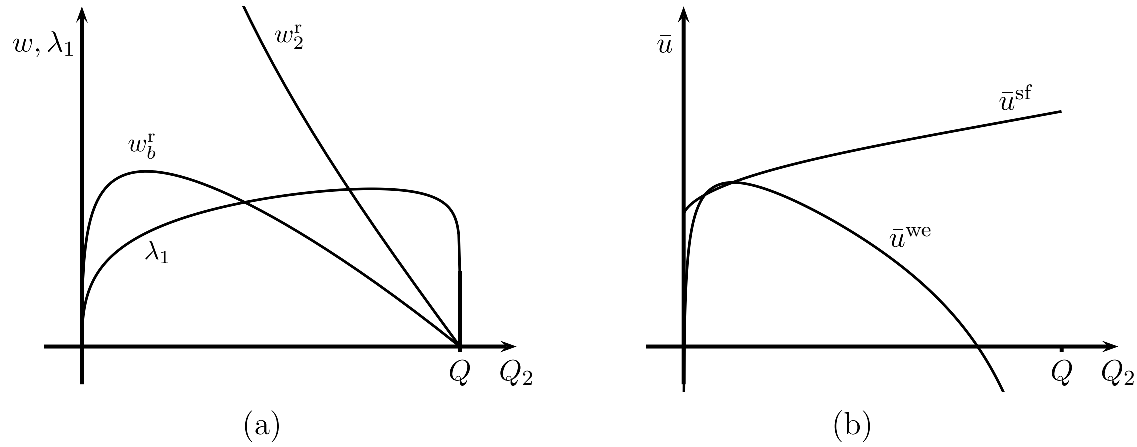

Figure 2: In (a) are depicted real wages and the efforts when the whole economy is considered. In (b) are depicted the average instantaneous utility in a single firm case ($\bar{u}^{\mathrm{sf}}$) and when the whole economy is considered ($\bar{u}^{\mathrm{we}}$).

influence the price level of the economy as given. Now the price of the aggregated good does influence the real wages. It is easy to show that the formulas presented for the wages and the effort, (3.5) and (3.4), are now valid by replacing the nominal wages by real wages.

In the figure 2 (a) we observe the different real wages and the consequent efforts reached when the whole economy is considered, as functions of the number of agents in the tier 2. It is easy to show that does not depends on the nominal value established by the government. In fact, when the government changes the minimum wage, competition among firms adjust the price level in a way that w_b^{n}/p remains constant. If the nominal minimum wage w_b^{n}/p is held exogenously, the real minimum wage is simply $w_b^{\mathrm{r}} = w_b^{\mathrm{n}}/p$. Because, as we can see in the figure 1 the competitive price diverges both when $Q_2 \to 0$ and when $Q_2 \to Q$, the real minimum wage in these limits is 0. Moreover, in this limit the real production goes to 0, and then the real wage of the tier 2 agents also approaches to 0. This convergence among the two real wages implies that the effort supplied by the tier 1 agents also decrease to 0 when $Q_2 \to 1$, differently from the single firm effort. When firm owners can choose the structure of the firms, competition moves the economy to the minimum of the price function. In this case, this point coincides with the maximum of the real minimum wage. In this case, the free market maximizes the purchasing power of the less well paid workers.

When Q_2 is such that the real minimum wage is maximal it is easy to show that the effort supplied by the agents of the tier 1 an increasing function of Q_2. Then, the instantaneous utility of the tier 1 agents is also a decreasing function of Q_2.

The instantaneous utility of the agents in the tier 2 is also decreasing, because w_2^r is strictly decreasing on Q_2. Then, this point may seem nonPareto efficient, because we can decrease Q_2 an this increases the instantaneous utility of both the tier 1 agents and the tier 2 agents, and then we make everyone better off. Nevertheless, the utility that the agents consider when they make decisions is not the instantaneous utility, but the expected utility. The expected utility of the agents in the tier 1 can increase with Q_2 because the decrease of the instantaneous utilities can be compensated by the increase of the probability of being promoted, caused by the increasing of agents in the tier 2.

From (2.6) we can obtain the form of the expected utilities of the first tier workers:

$$\begin{aligned} U_1 &= \frac{\frac{u(w_1,\lambda_1)}{d} + q_1.U_2}{d+q_1} = \frac{u(w_1,\lambda_1)}{d} \cdot \frac{d}{d+q_1} + \frac{u(w_2,0)}{d} \cdot \frac{q_1}{d+q_1} \\ &= \frac{u(w_1,\lambda_1)}{d} \cdot \frac{Q_1}{Q} + \frac{u(w_2,0)}{d} \cdot \frac{Q_2}{Q} = \frac{\bar{u}}{d} , \end{aligned}$$

where we have used the fact that the all the agents in the tier 1 supply effort. The expected utility of the workers that belong to the firm in the tier 1, U_1, coincides with the average instantaneous utility of the economy, $\bar{u}$, divided by the death rate d. Note that even when the instantaneous utilities of all the agents in the economy decrease with Q_2, an increase of the number of agents in the tier 2 (with high instantaneous utility) can compensate the decrease of the instantaneous utilities, and make the average utility rise.

The figure 2 (b) represents the different average utilities in the single firm and at the whole economy level depending on Q_2. Like nominal and real productions, the average utilities behave very different in a single firm and at the whole economy level. In a single firm the average utility is a increasing function of Q_2, like the nominal production, while the average utility when the whole economy is considered is a hill-shaped function, like the real production. An important consequence of the form of these curves is that single firms tend to prefer to have a regulated structure defined by the government, with Q_2 as higher as possible. Note that when a sector is not regulated and firms can choose the structure, Q_2 tends to the point where minimizes the competitive price, and then lowers the average utility of the firm. As we have seen in the figure 1 (b), when Q_2 is fixed high the price and the nominal production are high and, what is more important, the average utility of the members of this sector is high. But if it is applied to the whole economy, the price level increases and the nominal wages decrease, making the whole average utility decrease. It is an example of the fact that what is better for every part of the economy (a regulation with high Q_2 for a sector) can be worse for the whole economy.

Conclusions

The model we have presented helps us to understand organization of firms and the wage distribution between the different jobs tiers.

When variables that are important to determine the level of production cannot be observed by the firm owner, he/she must establish a system of incentives to ensure that the agents provide a certain level of effort. We modelled this process a a promotion “lottery” among agents, where the probability of promotion depends stochastically on the effort individually supplied. This is a realistic assumption because the agents know that their success in the firm depends on the effort they supply, but not in a deterministic way. Agents in the same tier compete for available vacancies in the next tier. This element of competition ensures that agents supply effort, even though it is not directly observable.

Competition between firms implies that prices decrease until the moment that profits are 0. Because this competitive price is strongly related to the organizational structure, firms that produce similar goods will have similar structures and wages. Furthermore, technological changes in the productive process that change the organization towards the price minimizer scheme will be favored by the market, because firms that initially adopt of this kind of technologies will have positive profits temporally.

We have seen that when firms can be restructured, that is, if the productive process allows different organization schemes, the structure is changed in order to decrease the prices and then be more competitive. This process makes firms more similar, because there is an unique organization scheme that minimizes the price. When it is applied at the whole economy level the election of the structure coincides with the maximization of the real wage of the agents in the tier 1. When the firm owner chooses the structure of the firm there is a tradeoff, Increasing the number of agents in higher tiers produces a decrease in the number of working agents, but also an increase in the effort supplied by them, The firm owner should compensate the agency problem of the higher tiers with high incentives in the lower tiers.

Firms in a sector are better off when there are regulations that increase the expected quantity of high tier workers. This can be done by regulating the structure or the relative wage among tiers, because although they make 0 profits we have seen that the average utility is increased. Although this kind of regulations generates productive inefficiencies, they increase the nominal output and the welfare of the regulated sector, by rising the price of the good. Nevertheless, as we have seen, when these regulations applied to the whole economy the result is a decrease in the overall welfare, and the economy becomes inefficient.

Although the model presented is simple, specially with the assumption of firms with only two tiers, it is helpful to understand a wide range of observed effects. We modelled the conflict of interests between workers and firms, the tradeoffs in the hierarchical organization, the behavior of a single firm and the whole economy,... This first step opens the door to future generalizations of the model, to be able test the predictions that it implies.

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