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**Worksharing and access discounts in the
postal sector with asymmetric information**

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Abstract

This paper analyses the optimal worksharing discount granted to mailers and entrants in a liberalized postal sector when there is asymmetric information about the Post Office's costs. When the regulator is unable to ascertain which part of the total cost of sorting has to be attributed to each sorting facility, the optimal 'access discount' given to the entrants is set in a procompetitive way, thus facilitating the entry of firms that are less efficient than the Post Office. However, with the same asymmetry of information, the optimal 'worksharing discount' given to the mailers is set to favor the Post Office, even when it is less efficient than the mailers in providing the sorting.

JEL Classification Numbers: L51

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Resum

Este artículo analiza los descuentos óptimos concedidos a los grandes usuarios y a los entrantes en el sector postal que realizan una parte de la clasificación de su correspondencia antes de entregarla en la red de distribución del operador postal. Cuando el regulador es incapaz de determinar que parte de los costes totales de clasificación del operador postal tiene que ser asignado a cada una de las fases de clasificación, el descuento óptimo ofrecido a los entrantes se establece de forma pro-competitiva, facilitando la entrada de empresas que son menos eficientes que el operador postal. Sin embargo, con el mismo tipo de asimetría de información, el descuento óptimo ofrecido a los grandes consumidores se establece para evitar el *bypass* del operador postal, incluso cuando este es menos eficiente que los consumidores en el proceso de clasificación.

1 Introduction

The reform in the postal sector initiated in the last decade has been based, at least in the European Union, on the division between the regulation and the management of the Post Office, the liberalization of the commercial mail and the maintenance of the monopoly over the single-piece mail in order to secure the provision of the universal service. In this situation, one of the main challenges for the regulatory policy is to ensure that new operators can access the facilities of the incumbent Post Office to distribute the non local letters. The regulator must establish an adequate access discount for the letters that are collected and pre-sorted by the entrants, but that are introduced in the Post Office's network for its delivery. On the other hand, some mailers are more efficient than the Post Office in pre-sorting their correspondence and would find attractive to bypass the Post Office's facilities. As in the previous case, the challenge of the regulatory policy is to establish an adequate worksharing discount over the price of the letters that promotes the bypass of the mailers that are more efficient than the Post Office. The objective of this paper is to analyze the determination of the worksharing and access discounts when there is asymmetric information about the Post Office's sub-cost for the sorting activity, a problem that has not been studied by the literature.

In order to deliver a letter it is necessary to undertake different activities, such as the collection of bulk mail, different stages of sorting, transport and distribution. In this paper we consider that the transport and the distribution of non-local letters are provided exclusively by the Postal Operator. However, we consider that the Post Office, the entrants and the mailers contend for the recollection and the pre-sorting of the letters. Moreover, we consider that there is a regulator how can grant a discount to the entrants or the mailers that pre-sort their mail. On the one hand, she can grant a discount over the final price of the letters to the mailers that pre-sort their mail.¹ On the other hand, she can concede a discount to the entrants that provide a part of the sorting, but that use the Post Office network to distribute their letters.² As we see, both the mailers and the entrants can obtain a discount over the price of the letter when they pre-sort their mail. However, we show that with asymmetric information about the costs the optimal policy for each discount is different. When the regulator is unable to ascertain which part of the Post Office's total cost of sorting has to be attributed to each stage of the activity, the optimal policy consists of conceding a pro-competitive access discount to the entrants. This implies that some entrants that are less efficient than the Post

¹Sherman (2001) states that "it in the Postal market, instead of purchasing an input, the mailer supplies one. The input is not an 'essential facility', it is a common input that the mailer may supply more efficiently than the postal administration".

²Another way to interpret this situation is to suppose that the entrants pay an access price that consists of the final price of the letters minus the discount.

Office may enter the market.³ Interestingly, however, we also found that with the same type of asymmetric information, the optimal worksharing discount is more restrictive. Even when the Post Office is less efficient than the mailers, the optimal policy may consist of fixing a low discount to avoid the bypass.

As it is standard in the literature, in this paper we assume that asymmetric information enters our model in two ways.⁴ First of all, the regulator is unable to know the productivity of the Post Office. And secondly, the regulator can not control the effort that the company's manager put into reducing the costs. When a part of the classification is produced outside the Post Office, the regulator needs to concede the Post Office less informational rents to encourage him to increase the effort. This occurs when the mailers or the entrants pre-sort their letters. However, the discounts conceded to the mailers and to the entrants affect the prices of the letters in different ways. When an entrant pre-sorts the letters of all the mailers, the regulator have a better knowledge of the costs of the other stages of classification that the Post Office produces. As a consequence, it does not need to distort the prices to make the firm increase its effort. The entry reduces the informational rents given to the Post Office and does not distort the prices. Therefore, the optimal policy consist of having a pro-competitive attitude with respect to entry.

This situation does not occur when there is no entry but there is a group of efficient mailers that pre-sort their letters. In this case, the regulator still needs to identify which part of the total costs of classification of the Post Office is due to the pre-sorting. This requires to distort prices with respect to marginal cost pricing. As a consequence, the optimal policy may mean to avoid the pre-sorting of the efficient mailers if the difference of costs between the mailers and the Postal Office is small.

Finally, there are some cases where the optimal policy for the two regimes are reconciled. This occurs, for example, when all mailers have the same sorting costs. In this case, the worksharing discount for the mailers is also set in a pro-competitive way, and it may be optimal to allow the bypass of mailers that are less efficient than the Post Office. This result is interesting, because it gives a hint of what occurs in the presence of an entrant that is able to pre-sort the mail of all mailers.

The difference between worksharing and access discounts has not been analyzed by the literature, but it deserves special attention because its consequences can be rather different. On the other hand, our conclusion may support the current practice of some regulatory agencies with respect to the discounts, which consists of regulating the access charges and offering some commercial flexibility to the Post Office in order to fix the worksharing discount for the mailers.

The idea behind this paper is related to the literature on worksharing discounts

³De Fraja (1999) obtains the same result in a model for the telecommunications sector. Indeed, our model closely follows his paper.

⁴See Laffont and Tirole (1990 and 1993) and De Fraja (1999).

in the postal sector. Crew and Kleindorfer (1991) introduced the problem of the worksharing discounts in the postal sector, worried about an excessive discount that could encourage inefficient entry into the pre-sorting business. In particular, these authors consider that the letters may be distributed during the peak or the off-peak periods. The urgent letters can be sorted in any of these periods, but during the peak period the sorting costs are higher. They show that when marginal costs for each class of service are constant, the welfare optimal pre-sort discount (when the break-even constraint of the incumbent is not an issue) is set at exactly the unit cost of pre-sorting. When marginal costs are increasing the pre-sort discount depends on the magnitude of the peak problem, with larger discounts given to ameliorate peak loads.

Sherman (2001) uses a variant of Armstrong, Doyle and Vickers (1996), who analyze the optimal access charge in the telecommunications sector. He shows that with perfect information about the Post Office costs, the optimal worksharing discount depends on whether mailers workshare all their mail or not. If they workshare all their mail, their marginal decisions affect the usage of the mail, and it is optimal to apply the normal Ramsey inverse-elasticity rule, that is, it is optimal to establish separate Ramsey prices for single-piece and workshared letters. If the mailers have an increasing marginal worksharing cost, they workshare until these costs equal the discount. In this case, the discount affects the fraction of mail that is workshared by the mailer. Moreover, the worksharing discount should be less than the cost savings for the Postal Service, which implies that it is optimal to establish a greater margin on the workshared letters than for the single-piece letters. In a different model, Billete de Villemeur *et al.* (2002) determine the optimal Ramsey prices of a regulated postal operator, when there is a group of clients that never bypasses the Post Office, and another group of heterogeneous mailers that may or may not pre-sort their mail. They show that only under special circumstances the worksharing discount is equal to the marginal (avoided) costs. Moreover, under plausible conditions, the worksharing discount exceeds the Efficient Component Pricing Rule (ECPR) level.⁵ This depends on the demand superelasticities of the single-piece and workshared letters.

There are few papers that analyze the entry in the postal sector. Crew and Kleindorfer (2001) analyze the effects of different kinds of entry on the viability of the Universal Service Obligation (USO) provided by the Post Office. They consider a model where entrants provide end-to-end services for selected customers, and can hand in some selected mail to the Post Office for the delivering of some routes. Their approach relies on the idea that incumbents lose customers and not routes or

⁵The ECPR was originally proposed by Willig (1979). In particular, the ECPR establishes that the access charge should be equal to the direct cost of the access plus the incumbent's opportunity costs when it supplies one access unit to the rivals. See Baumol (1983) or Laffont y Tirole (2000) for a complete analysis of the properties of the ECPR.

products to rivals. They argue that entrants would only provide end-to-end service on the lowest cost, and highest profit routes. Crew and Kleindorfer (2002 and 2003) analyze the effects of access charges in the USO. They extend the previous paper to consider the possibility that the regulators can establish different access charges as a function of the last destination of their mail. They name this access system Delivery-Zone Access Pricing (DAP). "This makes entrants access the Post Office network at the two levels, either for delivery or at the single piece rate". They show that when an uniform price for the letters is fixed, the traditional access policy does not guarantee the viability of the USO. On the other hand, they emphasize that the DAP system is more likely to enable the preservation of the USO. Indeed, the DAP partially eliminates subsidies that would otherwise promote inefficient entry and use of the Post Office facilities for downstream access at rates that do not cover marginal cost of such access. Finally, Billete de Villemeur *et al.* (2003a and 2003b) analyze the optimal access pricing rules in the postal sector when there is perfect and imperfect competition, respectively. In particular, in the second paper they consider a model of differentiated oligopoly where operators interact strategically, and analyze the impact of the uniform pricing on access pricing. Their finding suggest that, many general principles governing access pricing in competitive fringe setting remain applicable under imperfect competition.

In contrast to this literature, we are interested in analyzing the worksharing discount when there is asymmetric information about the Post Office costs.⁶ For this purpose, our model closely follows De Fraja's (1999), which analyses the access charge regulation in the telecommunications sector. This author studies the optimal price policy when there is an incumbent firm which has the monopoly on local calls, but faces a competitor for the long distance calls. In this model, the regulator does not know which part of the incumbent's total calls has to be attributed to each part of the network. The regulator can neither infer the firm's cost parameter, nor his effort in reducing costs. Moreover, the competitor may be less, equally or more efficient than the incumbent. In this situation, De Fraja (1999) shows that the regulator will take a pro-competitive attitude, allowing entry even when the incumbent is more efficient than the entrant. This occurs because entry prevents the regulated firm from exploiting the regulator's inability to observe sub-costs.⁷

⁶Some papers analyze the bypass problem when there is asymmetric information about the type of consumers. This is the case, for instance, of Einhorn (1987), Laffont and Tirole (1990) and Curien, Jullien and Rey (1998). In these works it is assumed that the regulated firm can price-discriminate consumers. On the contrary, in this paper I assume that the regulator observes the type of consumers, but the firm can not price-discriminate. This may represent a strong assumption for the discount offered to consumers, but allows us to compare the worksharing discount with the access discount given to entrants.

⁷Demski, Sappington and Spiller (1987) and Caillaud (1990) take a different approach, considering that the costs of the regulated firm and the entrants are correlated. Therefore, entry of inefficient firms can be efficient, because it provides information about the regulated firm's costs.

We encounter the same mechanism in our model for the optimal access charge in the postal sector. But when we consider the worksharing discount given to the mailers we find that the optimal policy consist of forbidding the bypass, because pre-sort by the mailers distort the prices with respect to marginal cost pricing.

The rest of the paper continues as follows. Section 2 briefly reviews the recent liberalization of the postal sector. Section 3 analyses the optimal policy for the worksharing discount offered to the mailers. Section 4 analyses the optimal policy for the access discount given to one entrant that uses the Post Office's network. Finally, Section 5 concludes.

2 Competition in the postal market?

In the postal sector, the delivery of a letter implies different activities: collection, sorting, transport and distribution. Leaving aside distribution, which is labor intensive, the rest of the activities are contestable. New technologies make self-supply of sorting efficient and economically feasible. On the other hand, developments in electronic data interchange and computerized systems are lowering the cost of transport with private companies. Therefore, some mailers or the new operators can profitably produce these activities with specialized machinery.

Taking into account the costs' reductions in the postal sector, in the last decade, several countries have reformed the industry and have introduced competition for the provision of some activities. However, very few of them have completely liberalized the sector. This is the case of Argentina, Finland, New Zealand and Sweden.⁸ In the rest of the countries, the borderline between the monopoly and the competitive sector takes different forms. The Post Office generally retains part of the monopoly over the standard correspondence in order to protect the overall service. The reserved area enables enough revenues as to cross-subsidize the non-commercial services, specially with regards to the distribution of the letters in areas with high costs.⁹ This is the case of the United States and the European Union. In the European Union, the Post Offices retain the national monopoly over the letters that weigh less than 100 grams (50 grams from 2006), but this has not deterred some private operators from entering in the most profitable cities. In the particular case of Spain, the delivery of urban letters was completely liberalized in 1960, but despite this, private operators rely on the Post Office's network for distributing non-local letters.

⁸Campbell (2001) analyses the reform in Sweden, Germany, New Zealand, Australia, the United Kingdom and Netherlands. For a study of the reform in the United States see Pickett, Treworgy and Conrad (2000).

⁹See Cremer *et al.* (2000), Crew and Kleindorfer (2000) and Panzar (2000) for an analysis of the universal service in the postal sector.

Not surprisingly, the major defenders of the deregulation of the postal market are the big mailers, dissatisfied with the high cost of the service and its low quality. This situation has motivated many countries to establish a system of worksharing discounts that allows the efficient mailers to bypass some of the Post Office facilities. Bypassing confers services as well as cost savings to the mailers. Indeed, by controlling when and where their mail enters the postal network, mailers may increase the quality of their letters. On the other hand, upstream entry may reduce the number of days for delivery.

In some countries the worksharing discounts are determined by the number of sorts and facilities that the consumer bypasses before the Post Office accepts their mail. Generally, the discounts are based on the avoided cost of the facilities that are bypassed. This methodology encourages bypassing when mailers can save costs because they are more efficient than the Post Office, or when they can provide a higher quality service.

In the United States, Germany and the Netherlands, the Post Office provides substantial discounts to large mailers that barcode and pre-sort their own material. Such discounts encourage the use of the mail by businesses, who frequently issue mass mailings for advertising purposes. This has promoted competition in the upstream activities, as mailers and third party consolidators exploit the difference between their own cost and the discounts. Cohen *et al.* (2000) explain that "over 60 percent of the United States postal volume receives some type of worksharing discount. Yet, less than a 16 per cent of total volume is pre-sorted all the way to the carrier route level and dropped at the local or area offices by the private sector". Indeed, small mailings may not contain enough volume to be sorted and transported efficiently to delivery areas. In this case, consolidators or new operators may find it profitable to supply this service to small mailers.¹⁰

3 The bypass problem

In this section, we follow the work of De Fraja (1999) to consider a simplified market of non local mail service, bearing in mind that this service is monopolized by the Post Office. We do not consider local delivery, which is supposed to be a competitive activity.¹¹ Mail supply requires two components. The first corresponds to a composite activity, including collection and several stages of sorting. The second component is transportation that connects different areas or cities and delivery to the final users.

¹⁰See also Crew and Kleindorfer (2002).

¹¹In the countries where the postal sector is liberalized, the new operators frequently enter the big cities, where they can take advantage of the density economies and where the most profitable consumers are concentrated.

The Post Office provides both types of activities. The collection and the first stage of the sorting is supplied with a constant average and marginal costs of β . This is distributed in $[\underline{\beta}, \bar{\beta}]$ according to a distribution $F(\beta)$, with density $f(\beta) > 0$.

The consumers bypass the first stage of the sorting when they find profitable to do this activity by themselves. Alternatively, they can send a single-piece letter through the Post Office. We consider an heterogeneous group of mailers. The mailers provide the sorting with a constant average and marginal cost of k , where k is distributed over $[\underline{k}, \bar{k}]$ according to a cumulative distribution $G(k)$, with density $g(k)$. Initially, we make a simplifying assumption about $G(k)$. Afterwards, we analyze the effects of relaxing it.

Assumption 1. $G(k)$ is uniformly distributed between $[\underline{k}, \bar{k}]$.

When there are no fixed costs, the efficiency in the performance of the sorting is measured directly by β and k . Mailers can be less, equally or more efficient than the Post Office. However, in what follows we consider the simplification that $\underline{k}, \bar{k} \in [\underline{\beta}, \bar{\beta}]$. This facilitates the display of the results, but does not affect our conclusions.

The marginal cost of the additional stages of sorting and transport is given by $\alpha\beta$, where $\alpha > 0$ measures the weight of the first stage of the sorting itself relative to the rest of the sorting activities. Moreover, we consider that in this stage of the sorting the Post Office can exert an effort, e , to reduce his operating cost. But this effort implies a cost $\psi(e)$ for the firm, with $\psi', \psi'' > 0, \psi''' \geq 0$.¹²

Assumption 2. The unit cost of handing in a letter to the addressee is $(1+\alpha)\beta - e$ when the incumbent supplies the complete postal service, and it is $k + \alpha\beta - e$ when the mailers bypass the first sorting facility.

Finally, we consider that it is prohibitively costly for mailers to duplicate the transport and distribution network. Therefore, they are provided exclusively by the incumbent Post Office. In order to simplify the exposition of the model, we consider that there is not a distribution cost.¹³

Next we consider Billete de Villemeur *et al.* (2002) to derive the consumer's indirect utility function. Let S denote the surplus of sending non-local letters. This is a function of the consumption level. We can obtain the net surplus by subtracting the payment to the Post Office, plus the sorting cost, if applicable. Taking this into account we define the demand function for the non-local letters as

¹²See Laffont and Tirole (1990 and 1993).

¹³This may seem too simplistic, but if we consider that the regulator has perfect information about the distribution costs it does not modify the main insights of the model. In our model, in order to account for these costs it would be sufficient to add the distribution cost to the final equation of the price of the letters.

$$q(p) = \operatorname{argmax}\{S(q) - pq\}. \quad (1)$$

Substituting the demand function in the net surplus yields the indirect utility function, V :

$$CS(p, p_0, k) = \begin{cases} V(p) = S(q(p)) - pq(p) & \text{if } p_0 + k > p \\ V(p_0 + k) = S(q(p_0 + k)) - (p_0 + k)q(p_0 + k) & \text{if } p_0 + k \leq p \end{cases}$$

where p is the price of the letters when the Post Office provides the collection and all sorting activities, and p_0 is the price of the workshared letters. That is, $p - p_0$ is the worksharing discount conceded to the mailers that bypass the sorting facilities of the Post Office. Taking this into account, $q(p_0 + k)$ and $q(p)$ are the demands for each type of service. We consider that V satisfies the following properties.

Assumption 3. $V'(p) \leq 0, V''(p) < 0$.

We consider that there exist a regulator that determines the Post Office's prices, p and p_0 , as well as the value of a transfer, t , conceded to the firm.¹⁴ The prices and the transfer are regulated so as to maximise a social welfare function, W .¹⁵

Assumption 4. *The regulator's welfare function is given by the weighted sum of the consumer's surplus, and the Post Office's profit, U , reduced by the transfers of public funds, t , conceded to the Post Office.*

$$W = \int_{p-p_0}^{\bar{k}} V(p)g(k)dk + \int_{\underline{k}}^{p-p_0} V(p_0 + k)g(k)dk + \lambda U - t, \quad (2)$$

where $\lambda \in (0, 1)$ is the weight of profits relative to consumer's surplus.

¹⁴The interpretation of t as a direct transfer to the Post Office could be somehow appealing to the postal sector. Alternatively, t could be considered as a reward to managers that depend on their performance.

¹⁵As stated by De Fraja (1999, footnote 4), this objective function is the same as the adopted by Baron and Myerson (1982). On the other hand, the analysis of Laffont and Tirole (1986) and Laffont and Tirole (1993) is conducted on the assumption of equal weights for consumer's and producers' surplus, and a shadow cost of public funding. We consider the same setting than De Fraja (1999) in order to obtain marginal cost pricing in conditions of symmetric information, instead of Ramsey pricing. By doing so, we can more adequately compare the optimal worksharing discount of this section and the optimal access discount of section 4.

The profit of the Post Office in equation (2) differs when the firm provides the sorting to all mailers, U^N , from when a part or all mailers provide the first stage of the sorting by themselves, U^E . If the Post Office supplies to all mailers its profits are

$$U^N = (p - (1 + \alpha)\beta + e) \int_{\underline{k}}^{\bar{k}} q(p)g(k)dk + t - \psi(e). \quad (3)$$

When a part of the mailers workshare their letters, the profit is given by

$$U^E = (p - (1 + \alpha)\beta + e) \int_{p-p_0}^{\bar{k}} q(p)g(k)dk \\ + (p_0 - \alpha\beta + e) \int_{\underline{k}}^{p-p_0} q(p_0 + k)g(k)dk + t - \psi(e). \quad (4)$$

In contrast to the previous case, here the firm has two sources of revenues. The first one comes from delivering single-piece letters of those mailers with a high sorting cost. The second one arrives from delivering workshared letters. In this case, the service is provided with a worksharing discount. On the other hand, we consider that the firm can refuse to produce if the regulatory contract does not guarantee a minimum level of expected profit. This forces the regulator to respect a "participation" constraint for the firm.

Finally, we consider that the regulator has limited capacity to audit the regulated firm's sub-cost. However, it knows the mailers' sorting cost. This is summarized in the following assumption.

Assumption 5. *The regulator fixes p , p_0 , and t . She knows, k , α and the distribution of β . But she can not know the true value of β and the effort undertaken for reducing the cost, e . The regulator observes the ex post total cost of the Post Office and conditions the values of p , p_0 , and t to the observed cost levels.*

We establish therefore, that the regulator is unable to know which proportion of the firm's total cost of sorting is to be attributed to the first stage and which to the rest. However, the realized total cost, the quantities produced and the prices are all verifiable.

3.1 The optimal worksharing discount

Following De Fraja (1999), in the next proposition we determine the optimal p , p_0 , e and t , when the regulator can observe β . We calculate the values of these variables when bypassing does not occur, and when some mailers classify their own mail. Further on, in the next proposition we will determine the optimal 'cut-off'

point, that is, the value of β under which entry will not be allowed. In order to ensure the sufficiency of the first order conditions, we first introduce the following assumption.

Assumption 6. For all admissible p, p_0 and e : $\psi''(e) > -\tilde{Q}'_1(p(\beta))$ and $\psi''(e) > -Q'_1(p(\beta), p_0(\beta)) - Q'_0(p(\beta), p_0(\beta))$.

where $\tilde{Q}_1(p(\beta))$ is the aggregate supply when bypassing is not allowed, and where $Q_1(p(\beta), p_0(\beta))$ and $Q_0(p(\beta), p_0(\beta))$ represent the aggregate supply of the single-piece letters and workshared letters when bypassing occurs.

The optimal policies when bypassing is not allowed and when it is permitted are defined in the following proposition. See the proofs in the Appendix.

Proposition 1. Considering assumptions 1-6, when the regulator know β and the cut-off point is β^* , the optimal policy is

$$p = (1 + \alpha)\beta - e, \quad (5)$$

$$p_0 = \begin{cases} > \alpha\beta - e & \text{if } \beta \leq \beta^*, \\ \alpha\beta - e & \text{if } \beta > \beta^*, \end{cases} \quad (6)$$

$$\psi'(e) = \begin{cases} \tilde{Q}_1(p(\beta)) & \text{if } \beta \leq \beta^*, \\ Q_1(p(\beta), p_0(\beta)) + Q_0(p(\beta), p_0(\beta)) & \text{if } \beta > \beta^*, \end{cases} \quad (7)$$

$$t = \psi(e). \quad (8)$$

Note that when the regulator know β the prices of the simple letters, p , and of the workshared letters, p_0 , are equal to the marginal costs of each service, and the effort is efficiently set. Therefore, when bypass occurs the discounts conceded to the mailers, $d = p - p_0$, is equal to the avoided cost. On the other hand, in each regime no rent is left to the firm.

The next proposition determines the optimal cut-off point, β^* . For all values of β below this level bypassing is not allowed. Taking into account the two equations in (6) we define $e^N(\beta)$ and $e^E(\beta)$ as the functions for the effort level, when bypassing is not allowed and when it is.

Proposition 2. For every $\underline{k}, \bar{k} \in [\underline{\beta}, \bar{\beta})$, the optimal cut-off point is $\beta^* = \underline{k}$, which satisfies

$$\begin{aligned} & \int_{\underline{k}}^{\bar{k}} V((1 + \alpha)\beta^* - e^N(\beta^*))g(k)dk - \psi(e^N(\beta^*)) \\ &= \int_{p^E - p_0^E}^{\bar{k}} V((1 + \alpha)\beta^* - e^E(\beta^*))g(k)dk + \int_{\underline{k}}^{p^E - p_0^E} V(\alpha\beta^* - e^E(\beta^*) + k)g(k)dk - \psi(e^E(\beta^*)). \end{aligned}$$

The left-hand side of the equation represents the social surplus when the incumbent provides the sorting to all mailers and the right hand side is the social surplus when a group of mailers bypass the Post Office. The equality is satisfied for $\beta^* = \underline{k}$. This implies that bypassing is allowed only to the mailers that are more efficient than the Post Office. As a consequence, the higher the difference between β and \underline{k} , the greater the proportion of mailers that bypass the sorting.

From the last proposition we also obtain the following result.

Corollary 1. *If $\beta > \underline{k}$, bypass occurs, and (1.1) $\frac{de^E}{dk} < 0$, $\frac{de^E}{d\beta} < 0$. (1.2) $\frac{de^N}{d\beta} > \frac{de^E}{d\beta}$.*

The first part of the corollary implies that, given that bypassing occurs, a lower cost of either firm requires more effort from the Post Office. The second part states that a higher cost for the Post Office implies less effort when bypassing occurs than when it is prohibited. Therefore, as the amount of effort required depends on the quantity produced, the Post Office will need to put in less effort when bypassing is not allowed. This occurs for two reasons. First of all, because an increase in β reduces even more the demand when bypassing is prohibited. And second, because when β increases, more mailers find the bypass option profitable. This implies that the quantity of single-piece letters is smaller and the quantity of workshared letters is greater. Again, as the effort required depends on the quantity produced of each service, the first effect is reinforced.

3.2 Bypass with asymmetric information

We next analyse the more complex case where the regulator can not observe the sub-cost of the firm. Thus, we consider the situation where there exists a cut-off point $\beta^* \in [\underline{\beta}, \bar{\beta}]$ such that if $\beta \leq \beta^*$ the incumbent supplies the single-piece letters to all mailers, and if $\beta > \beta^*$ the efficient mailers provide the sorting by themselves.

The constraints imposed on the regulator's maximization program are of two kinds, and are shown in the following proposition.

Proposition 3. *A policy $p(\beta)$, $p_0(\beta)$, $e(\beta)$, $t(\beta)$, β^* , is "incentive compatible" if and only if it satisfies*

$$\dot{U}(\beta) = -(\alpha + 1)\psi'(e(\beta)) \quad \text{if } \beta \leq \beta^*, \quad (9)$$

$$\dot{U}(\beta) = -\left(\alpha + \frac{Q_1(p(\beta), p_0(\beta))}{Q_0(p(\beta), p_0(\beta)) + Q_1(p(\beta), p_0(\beta))}\right)\psi'(e(\beta)) \quad \text{if } \beta > \beta^*, \quad (10)$$

$$\lim_{\beta \rightarrow \beta_-^*} U(\beta) = \lim_{\beta \rightarrow \beta_+^*} U(\beta) : U(\beta) \quad \text{is continuous at } \beta^*, \quad (11)$$

and the policy satisfies the Post Office's "individual rationality constraint" if and only if it satisfies $U(\bar{\beta}) \geq 0$.

Note that when $Q_0(p(\beta), p_0(\beta))$ is not zero, $\frac{Q_1(p(\beta), p_0(\beta))}{Q_0(p(\beta), p_0(\beta)) + Q_1(p(\beta), p_0(\beta))} < 1$. That is, the informational rent necessary to make the Post Office reveal the true value of β is higher when $\beta \leq \beta^*$ (no bypass) than when $\beta > \beta^*$ (bypass).

In view of Proposition 3, we can now write the program for the overall maximization of expected welfare. Take an arbitrary cut-off point β^* . Using $U(\beta)$ to simplify $t(\beta)$ we obtain two separate problems. When $\beta \leq \beta^*$ the regulator considers

$$\begin{aligned} & \max_{\substack{U(\beta); p(\beta), \\ p_0(\beta), e(\beta)}} \int_{\underline{\beta}}^{\beta^*} \left\{ \int_{\underline{k}}^{\bar{k}} V(p)g(k)dk - (1 - \lambda)U^N - \psi(e^N) \right. \\ & \left. + (p - (1 + \alpha)\beta + e) \int_{\underline{k}}^{\bar{k}} q(p)g(k)dk \right\} f(\beta)d\beta, \\ & \text{s.t. } \dot{U}(\beta) = -(\alpha + 1)\psi'(e(\beta)), \quad U(\beta^*) = U^*, \quad p(\beta) - p_0(\beta) - \underline{k} < 0, \end{aligned}$$

where U^* is the incumbent's reservation utility in the problem $\beta > \beta^*$, when the cost is β^* . When $\beta > \beta^*$, the regulator considers the following problem

$$\begin{aligned} & \max_{\substack{U(\beta); p(\beta), \\ p_0(\beta), e(\beta)}} \int_{\beta^*}^{\bar{\beta}} \left\{ \int_{p-p_0}^{\bar{k}} V(p)g(k)dk + \int_{\underline{k}}^{p-p_0} V(p_0 + k)g(k)dk \right. \\ & \left. - (1 - \lambda)U^E - \psi(e^E) + (p - (1 + \alpha)\beta + e) \int_{p-p_0}^{\bar{k}} q(p)g(k)dk \right. \\ & \left. + (p_0 - \alpha\beta + e) \int_{\underline{k}}^{p-p_0} q(p_0 + k)g(k)dk \right\} f(\beta)d\beta + \Phi(\beta^*, U^*), \\ & \text{s.t. } \dot{U}(\beta) = -\left(\alpha + \frac{Q_1(p(\beta), p_0(\beta))}{Q_0(p(\beta), p_0(\beta)) + Q_1(p(\beta), p_0(\beta))}\right)\psi'(e(\beta)), \\ & U(\bar{\beta}) = 0, \quad p(\beta) - p_0(\beta) - \underline{k} \geq 0. \end{aligned}$$

where $\Phi(\beta^*, U^*)$ is the value of the first problem. As in the complete information case, we solve the problem in two stages. We first solve the optimal policy for each control problem. Afterwards, we find the optimal cut off point, β^* .

Proposition 4. *The optimal policy for $p(\beta), p_0(\beta), e(\beta)$ and $U(\beta)$ when $\beta \leq \beta^*$ is*

$$p(\beta) = (1 + \alpha)\beta - e(\beta), \quad (12)$$

$$p_0(\beta) > (1 + \alpha)\beta - e(\beta) - \underline{k}, \quad (13)$$

$$\psi'(e(\beta)) = \tilde{Q}_1(p(\beta)) - (1 - \lambda) \frac{F(\beta)}{f(\beta)} (\alpha + 1) \psi''(e(\beta)), \quad (14)$$

$$U(\beta) = (\alpha + 1) \int_{\beta}^{\beta^*} \psi'(e(\tilde{\beta})) d\tilde{\beta}, \quad (15)$$

and, when $\beta > \beta^*$, is

$$p(\beta) = (1 + \alpha)\beta - e(\beta) + (1 - \lambda) \frac{F(\beta)}{f(\beta)} \frac{Q_0}{(Q_0 + Q_1)^2 \tilde{\eta}_0} \psi'(e(\beta)), \quad (16)$$

$$p_0(\beta) = \alpha\beta - e(\beta) - (1 - \lambda) \frac{F(\beta)}{f(\beta)} \frac{Q_1}{(Q_0 + Q_1)^2 \tilde{\eta}_1} \psi'(e(\beta)), \quad (17)$$

$$\psi'(e(\beta)) = Q_1(p(\beta), p_0(\beta)) + Q_0(p(\beta), p_0(\beta)) \quad (18)$$

$$-(1 - \lambda) \frac{F(\beta)}{f(\beta)} \left(\alpha + \frac{Q_1}{Q_0 + Q_1} \right) \psi''(e(\beta)), \quad (19)$$

$$U(\beta) = \int_{\beta^*}^{\bar{\beta}} \left(\alpha + \frac{Q_1}{Q_0 + Q_1} \right) \psi'(e(\tilde{\beta})) d\tilde{\beta}. \quad (20)$$

where we have used $Q_1 = Q_1(p(\beta), p_0(\beta))$ and $Q_0 = Q_0(p(\beta), p_0(\beta))$. On the other hand, $\tilde{\eta}_0$ and $\tilde{\eta}_1$, which are defined in the appendix, are the price superelasticities for the complete and the workshared services. Superelasticities are modified elasticities of demand which account for possible substitution or complementarity between services.¹⁶

Comparing the two regimes, it follows that asymmetric information does not create a distortion in the prices in the first regime, but it does so in the second. Indeed, when $\beta > \beta^*$ the price of the single-piece letters is raised above the marginal cost, and the price of the workshared letters is reduced below the marginal cost in such a way as to keep total net revenues at zero. Since the regulator is unable to

¹⁶Notice that these superelasticities are not exactly defined as in Laffont and Tirole (1994). Indeed, in our model when the aggregate demand functions are independent, superelasticities are equal to 1. This does not occur when there are cross price effects.

observe the firm's sub-cost, she needs to give some informational advantage to the firm. As this rent is more important for the complete service than for the sorted, the regulator increases the price of the first service and reduces the price of the second. With this policy, total revenue is kept at zero, and welfare is maximized. In contrast, when $\beta \leq \beta^*$ the regulator does not require to know the cost of the first stage of the sorting. As a consequence, it is not necessary to distort the prices. Finally, observe that the effort is reduced for $\beta > \underline{\beta}$ in order to reduce the rent. This occurs in the two regimes, when worksharing is allowed and when it is not.

Finally, let us notice that the worksharing discount conceded to the consumer that bypasses the Post Office is given by $d = p^E - p_0^E$. The next corollary states when bypassing occurs.

Corollary 2. *A consumer with type k bypasses the Post Office when*

$$d = \beta + (1 - \lambda) \frac{F(\beta)}{f(\beta)} \frac{Q_0 \tilde{\eta}_1 + Q_1 \tilde{\eta}_0}{(Q_0 + Q_1)^2 \tilde{\eta}_1 \tilde{\eta}_0} \psi'(e(\beta)) > k.$$

Therefore, when bypassing is allowed, even mailers that are less efficient than the Post Office find it profitable to bypass. This is a direct consequence of the distortion in prices.

The previous Corollary shows which mailers will pre-sort their letters when bypass is allowed. The next proposition determines the cut-off point above which bypass will be allowed. This further characterizes the optimal policy.

Proposition 5. *For $\underline{k} \in [\underline{\beta}, \bar{\beta})$, the optimal cut-off point, β^* , for $\beta^* \in (\underline{k}, \bar{k})$, satisfies*

$$\begin{aligned} & \int_{\underline{k}}^{\bar{k}} V((1 + \alpha)\beta^* - e^N(\beta^*))g(k)dk - \int_{p^E - p_0^E}^{\bar{k}} V((p^E(\beta^*))g(k)dk \\ & - \int_{\underline{k}}^{p^E - p_0^E} V(p_0^E(\beta^*) + k)g(k)dk = \psi(e^N(\beta^*)) - \psi(e^E(\beta)) \\ & + (1 - \lambda) \frac{F(\beta^*)}{f(\beta^*)} \left\{ (\alpha + 1)\psi'(e^N(\beta^*)) - \left(\alpha + \frac{Q_1}{Q_0 + Q_1} \right) \psi'(e^E(\beta^*)) \right\}. \end{aligned}$$

The left-hand side of the equation represents the difference in social surplus when the Post Office supplies the complete service to all mailers and when a group of mailers bypass the sorting facilities. The right-hand side represents the difference in rents that are given to the Post Office in each case. For the optimal cut-off point, β^* , both sides are negative and equal.

The next proposition clarifies which mailers will bypass the Post Office. In particular, it studies who will pre-sort the letters when the Post Office and the mailers have the same level of efficiency.

Proposition 6. *Given $\underline{k} \in [\underline{\beta}, \bar{\beta}]$, $\beta^*(\underline{k})$ satisfies: (6.1) $\beta^*(\underline{k}) > \underline{k}$ for every $\underline{k} \in [\underline{\beta}, \bar{\beta}]$; (6.2) $\beta^*(\underline{k})$ is continuous; and (6.3) $\beta^*(\underline{k})$ is concave.*

The proposition shows that the prohibition of bypassing occurs more often with asymmetric information than when the regulator knows the firm's sub-costs. Condition (6.1) states that the Post Office produces the sorting to all mailers even when it is less efficient than the most efficient type, \underline{k} . This is an important conclusion. Asymmetric information implies that the Post Office is favored. On the other hand, the concavity of $\beta^*(\underline{k})$ implies that the bias in favor of the Post Office is less strong when the mailers are less efficient.

The intuition behind the Proposition is that, when the regulator allows the bypass, the prices are distorted away from Ramsey prices, so as to give an informational rent to the firm. As the distortion of prices reduces the welfare, if the cost advantage of the mailers is not important the optimal policy consist of forbidding the bypass. The regulator must spend on informational rents in order to force the Post Office to report the true value of β . This occurs, in both cases, when the Post Office supplies the complete service to all mailers, and when a group of mailers provide the sorting themselves. Moreover, the informational rent that must be given to the firm decreases as more mailers bypass the sorting. However, when $\beta(\underline{k}) = \underline{k}$ the informational rents are equal in the two regimes. Given that bypassing distorts the prices away from Ramsey prices it must be that $\beta^* > \underline{k}$. To sum up, there is a trade-off. On the one hand, bypassing distorts the prices. On the other hand, it reduces the firm's informational rent. For $\beta^*(\underline{k}) = \underline{k}$ bypassing is prohibited, because although the informational rents given to the Post Office are the same in the two regimes, the distortion in prices reduces social welfare.

The next lemma characterizes further the regulator's solution.

Lemma 3. *If $\underline{k} > \underline{\beta}$, then (1.1) $p_0^E(\beta^*) + k < p^N(\beta^*)$, and (1.2) $e^E(\beta^*) > e^N(\beta^*)$.*

When β moves from β^* to $\beta^* + \epsilon$, the cost of sorting suddenly decreases from β^* to \underline{k} . On the other hand, effort suddenly increases, which also decreases the cost. Therefore, when bypassing is allowed, the two effects decrease the price. In spite of this, the consumer's surplus is a continuous function in β .

Corollary 4. *The ex-post regulator's payoff is a continuous function of β .*

If t is interpreted as a transfer paid by consumers, it follows from this corollary that the net consumer's surplus, $V(p(\beta)) - t(\beta)$, is also a continuous function in

β , and therefore, the lower price paid when bypassing occurs is compensated for exactly a higher fixed charge.

Finally, notice that Proposition 6 relies on the uniform distribution of k considered in Assumption 1. The next proposition shows that this result can be reversed. In particular, we consider the case where the unit cost of sorting for all mailers is \underline{k} .

Proposition 7. *Given $\underline{k} \in [\underline{\beta}, \overline{\beta}]$, when all mailers have the type \underline{k} , then $\beta^*(\underline{k}) < \underline{k}$.*

The intuition behind this result is that when all mailers bypass the Post Office, the distortion of prices disappears because the regulator has perfect information about the total cost of the workshared service, and because there is only one service sold. On the other hand, even when all mailers bypass the Post Office the regulator does not know which is the true value of β and the effort. Therefore, she must give some rent to encourage effort. However, the rent that she must give to the Post Office is lower when all mailers bypass the firm.

This result is important because it gives an inclination into what will occur in the presence of a competitor of the Post Office that provides the sorting to all mailers. Indeed, if we consider the situation where all mailers have type \underline{k} and delegate the sorting to one of them, the mailer that provides the sorting must access the Post Office network to distribute all the correspondence. In this case, the result that $\beta^*(\underline{k}) < \underline{k}$ implies that there is a pro-competitive bias in favor of the competitor. Hence, in this situation even if the competitor is less efficient than the Post Office the optimal policy consists of allowing him to provide the sorting. In the next section we study the problem in more detail.

4 The access charge problem

In this section, we extend the previous framework to consider that the postal operator has one competitor in the sorting activity. For this analysis we closely follow the work of De Fraja (1999) about the access charge in the telecommunication industry. Our analysis adapts his model to the postal sector, and we obtain a similar conclusion than him. Moreover, it is very interesting to compare the worksharing discount that we have obtained in the previous section with the optimal access discount.

It is now common in many countries that new postal firms operate a local network. However, they generally rely on the Post Office network for delivering non-local letters. The competitors collect the bulk correspondence of some mailers and separate the local and non-local letters. Finally, they distribute their local mail through their network, and pay an access charge to the Post Office for the delivery of the rest of the letters.

As in the previous section, we restrict our analysis to the non-local letters. Furthermore, to facilitate the comparison with the optimal policy for the worksharing discount, we make the simplifying assumption that the competitor only provides the first stage of the sorting. The access charge that the competitor pays to the Post Office consists of the price of the letters, p , minus a worksharing discount, a . That is, $p_0 = p - a$.

We consider that the potential competitor has a constant average and marginal cost of ξ for supplying the sorting, where $\xi \in (\underline{\beta}, \bar{\beta}]$. The mailers and the Post Office has the same cost structure than before. β is distributed in $[\underline{\beta}, \bar{\beta}]$. But, in order to simplify the model, we consider that $\underline{\beta} = \beta^*$, where β^* is the optimal cutt-off point of the previous section, when there is perfect information and when there exists asymmetric information. With this simplification we guarantee that at least a group of mailers will find it profitable to bypass the Post Office.

As in the previous section, the regulator observes ex post the total cost of the two firms, but she can not infer the Post Office's sub-costs. On the other hand, we consider that the regulator maximizes a welfare function, that in this case incorporates the competitor's profit, π . This modifies assumption 4 as follows.

Assumption 7. *The regulator's welfare function is given by the weighted sum of the consumer's surplus, and the firms' profit, U and π , reduced by any direct transfer of public funds to the Post Office, t :*

$$W = \int_{p-p_0}^{\bar{k}} V(p)g(k)dk + \int_{\underline{k}}^{p-p_0} V(p_0+k)g(k)dk + \lambda(U + \pi) - t, \quad (21)$$

The firms' profits can take two different forms. If the Post Office supplies the sorting to the inefficient mailers we have

$$U^N = (p - (1 + \alpha)\beta + e) \int_{p-p_0}^{\bar{k}} q(p)g(k)dk \quad (22)$$

$$+(p_0 - \alpha\beta + e) \int_{\underline{k}}^{p-p_0} q(p_0+k)g(k)dk + t - \psi(e), \quad (23)$$

$$\pi^N = 0. \quad (24)$$

In this case the competitor's profit is zero, because it stays out of the market. When the competitor supplies the first stage of the sorting to the inefficient mailers, the firms' profits are

$$U^E = (p_0 - \alpha\beta + e) \int_{\underline{k}}^{\bar{k}} q(p)g(k)dk + t - \psi(e), \quad (25)$$

$$\pi^E = (p - p_0 - \xi) \int_{p-p_0}^{\bar{k}} q(p)g(k)dk. \quad (26)$$

Our hypothesis on the observability of costs implies that the regulator is able to prevent the entrant from obtaining any profit from his activities.

4.1 The optimal access discount

As in the bypass problem, we first study the model given that the regulator knows β . In order to determine the optimal access policy, we consider the case where there is no entry separately from the case where a competitor provides the sorting. We then determine the optimal cut-off point in a second stage. This point represents the value of the Post Office's sorting cost under which entry is not allowed.

To ensure the sufficiency of the first order conditions, we introduce the following assumption.

Assumption 8. For all admissible p , p_0 and e : $\psi''(e) > -Q'_1(p(\beta), p_0(\beta)) - Q'_0(p(\beta), p_0(\beta))$.

Notice that in this assumption, $Q_1(p(\beta), p_0(\beta))$ and $Q_0(p(\beta), p_0(\beta))$ represent the aggregate supply of the complete and the sorted mail when bypassing is possible.

The optimal policy is defined in the following proposition. The proofs can be found in the Appendix.

Proposition 8. When the regulator know β , and the cut-off point is ξ , the optimal policy is

$$p = \begin{cases} (1 + \alpha)\beta - e & \text{if } \beta \leq \xi, \\ \xi + \alpha\beta - e & \text{if } \beta > \xi, \end{cases} \quad (27)$$

$$p_0 = \alpha\beta - e, \quad (28)$$

$$\psi'(e) = Q_1(p(\beta), p_0(\beta)) + Q_0(p(\beta), p_0(\beta)) \quad (29)$$

$$a = \begin{cases} > p - p_0 & \text{if } \beta \leq \xi, \\ = p - p_0 & \text{if } \beta > \xi, \end{cases} \quad (30)$$

$$t = \psi(e). \quad (31)$$

Note that the prices of the single-piece letters and of the pre-sort letters are equal to the marginal costs, and effort is efficiently set. On the other hand, in each regime no rent is left for the firms. Finally, let us notice that $a = \xi$ when $\beta > \xi$. That is, when the regulator knows the entrant's costs, the optimal policy does not consist of fixing a discount equal to the cost saved by the Post Office. On the contrary, the optimal policy maintains the entrant's profit equal to zero.

4.2 Access with asymmetric information

In this section we analyze the case where the regulator cannot observe the firm's sub-costs. As in the perfect information case, we consider the situation where there is a cut-off point $\beta^* \in [\underline{\beta}, \bar{\beta}]$ such that if $\beta \leq \beta^*$ the Post Office supplies the first stage of the sorting to the inefficient mailers, and if $\beta > \beta^*$ the competitor supplies the sorting to the inefficient mailers.

The following proposition shows two kinds of restrictions that must satisfy the regulator's maximization problem.

Proposition 9. *A policy $p(\beta)$, $p_0(\beta)$, $e(\beta)$, $t(\beta)$, β^* , is "incentive compatible" if and only if it satisfies*

$$\dot{U}(\beta) = -\left(\alpha + \frac{Q_1(p(\beta), p_0(\beta))}{Q_0(p(\beta), p_0(\beta)) + Q_1(p(\beta), p_0(\beta))}\right)\psi'(e(\beta)) \quad \text{if } \beta \leq \beta^*, \quad (32)$$

$$\dot{U}(\beta) = -\alpha\psi'(e(\beta)) \quad \text{if } \beta > \beta^*, \quad (33)$$

$$\lim_{\beta \rightarrow \beta_-^*} U(\beta) = \lim_{\beta \rightarrow \beta_+^*} U(\beta) : U(\beta) \quad \text{is continuous at } \beta^*,. \quad (34)$$

and the policy satisfies the Post Office's "individual rationality constraint" if and only if it satisfies $U(\bar{\beta}) \geq 0$.

Note that the informational rents necessary to make the Post Office reveal the true value of β are higher when $\beta \leq \beta^*$ (no entry) than when $\beta > \beta^*$ (entry). This occurs because when the competitor enters, the regulator is able to know the Post Office's sub-cost at the second stage of the sorting. Indeed, when the competitor enters the Post Office only provides this activity. In spite of this, the regulator cannot observe the incumbent's productivity and effort. Therefore, it is still necessary to give some incentives to the incumbent.

Taking into account the previous result, we can now write the program for the overall maximization of the expected welfare. Take an arbitrary cut-off point β^* . When $\beta \leq \beta^*$ the regulator considers

$$\begin{aligned}
& \underset{\substack{U(\beta); p(\beta), p_0(\beta) \\ a(\beta), e(\beta)}}{\max} \int_{\underline{\beta}}^{\beta^*} \left\{ \int_{p-p_0}^{\bar{k}} V(p)g(k)dk + \int_{\underline{k}}^{p-p_0} V(p_0+k)g(k)dk \right. \\
& - (1-\lambda)U^N - \psi(e^N) + (p - (1+\alpha)\beta + e) \int_{p-p_0}^{\bar{k}} q(p)g(k)dk \\
& \left. + (p_0 - \alpha\beta + e) \int_{\underline{k}}^{p-p_0} q(p_0+k)g(k)dk \right\} f(\beta) d\beta \\
& \text{s.t. } \dot{U}(\beta) = -\left(\alpha + \frac{Q_1(p(\beta), p_0(\beta))}{Q_0(p(\beta), p_0(\beta)) + Q_1(p(\beta), p_0(\beta))}\right)\psi'(e(\beta)), \\
& U(\beta^*) = U^*, \quad p(\beta) - p_0(\beta) - a(\beta) < 0.
\end{aligned}$$

where U^* is the incumbent's reservation utility in the problem $\beta > \beta^*$, when the cost is β^* . On the other hand, when $\beta > \beta^*$, the regulator considers the problem

$$\begin{aligned}
& \underset{\substack{U(\beta); p(\beta), p_0(\beta) \\ a(\beta), e(\beta)}}{\max} \int_{\beta^*}^{\bar{\beta}} \left\{ \int_{p-p_0}^{\bar{k}} V(p)g(k)dk + \int_{\underline{k}}^{p-p_0} V(p_0+k)g(k)dk \right. \\
& - (1-\lambda)U^E - \psi(e^E) + (p_0 - \alpha\beta + e) \int_{\underline{k}}^{\bar{k}} q(p)g(k)dk \\
& \left. + \lambda(p - p_0 - \xi) \int_{p-p_0}^{\bar{k}} q(p)g(k)dk \right\} f(\beta) d\beta + \Phi(\beta^*, U^*), \\
& \text{s.t. } \dot{U}(\beta) = -\alpha\psi'(e(\beta)), \quad U(\bar{\beta}) = 0, \quad p(\beta) - p_0(\beta) - a(\beta) \geq 0.
\end{aligned}$$

As in the complete information case, in order to find the optimal policy we solve the regulator's problem in two stages. Firstly, we solve the optimal policy for each control problem. Then, we will find the optimal cut off point, β^* .

Proposition 10. *The optimal policy for $p(\beta), p_0(\beta), a(\beta), e(\beta)$ and $U(\beta)$ when $\beta \leq \beta^*$ is*

$$p(\beta) = (1 + \alpha)\beta - e(\beta) + (1 - \lambda) \frac{F(\beta)}{f(\beta)} \frac{Q_0}{(Q_0 + Q_1)^2 \tilde{\eta}_0} \psi'(e(\beta)), \quad (35)$$

$$p_0(\beta) = \alpha\beta - e(\beta) - (1 - \lambda) \frac{F(\beta)}{f(\beta)} \frac{Q_1}{(Q_0 + Q_1)^2 \tilde{\eta}_1} \psi'(e(\beta)), \quad (36)$$

$$\begin{aligned} \psi'(e(\beta)) &= Q_1(p(\beta), p_0(\beta)) + Q_0(p(\beta), p_0(\beta)) \\ &\quad - (1 - \lambda) \frac{F(\beta)}{f(\beta)} \left(\alpha + \frac{Q_1}{Q_0 + Q_1} \right) \psi''(e(\beta)), \end{aligned} \quad (37)$$

$$a(\beta) > p(\beta) - p_0(\beta), \quad (38)$$

$$U(\beta) = \int_{\beta}^{\beta^*} \left(\alpha + \frac{Q_1}{Q_0 + Q_1} \right) \psi'(e(\tilde{\beta})) d\tilde{\beta}, \quad (39)$$

and, when $\beta > \beta^*$, is

$$p(\beta) = \alpha\beta + \xi - e(\beta), \quad (40)$$

$$p_0(\beta) = \alpha\beta - e(\beta), \quad (41)$$

$$\begin{aligned} \psi'(e(\beta)) &= Q_1(p(\beta), p_0(\beta)) + Q_0(p(\beta), p_0(\beta)) \\ &\quad - (1 - \lambda) \frac{F(\beta)}{f(\beta)} \alpha \psi''(e(\beta)), \end{aligned} \quad (42)$$

$$a(\beta) = p(\beta) - p_0(\beta), \quad (43)$$

$$U(\beta) = \alpha \int_{\beta^*}^{\bar{\beta}} \psi'(e(\tilde{\beta})) d\tilde{\beta}, \quad (44)$$

where we have used $Q_1 = Q_1(p(\beta), p_0(\beta))$ and $Q_0 = Q_0(p(\beta), p_0(\beta))$. On the other hand, $\tilde{\eta}_0$ and $\tilde{\eta}_1$, are price supereslasticities, which are defined as in the previous section.

Comparing the two regimes, it follows that asymmetric information creates a distortion in the prices of the first regime, but not in the second. As we have seen in section 3, when $\beta < \beta^*$, the regulator raises the price of the single-piece letters

over the marginal cost and reduces the price of the workshared letters in such a way as to keep total net revenues at zero. This allows the regulator to give less informational rents to the Post Office. In contrast, when $\beta \geq \beta^*$ it is not necessary to distort the prices because the regulator knows which is the cost of the second stage of the sorting. On the other hand, observe that effort is reduced for $\beta > \underline{\beta}$ when entry is allowed and when it is not.

We now determine the optimal cut-off point, above which entry is allowed.

Proposition 11. *For $\xi \in (\underline{\beta}, \overline{\beta}]$, the optimal cut-off point, β^* , for $\beta^* \in (\underline{\beta}, \overline{\beta})$, implies*

$$\begin{aligned} & \int_{p^N - p_0^N}^{\overline{k}} V(p^N(\beta^*))g(k)dk + \int_{\underline{k}}^{p^N - p_0^N} V(p_0^N(\beta^*) + k)g(k)dk \\ & - \int_{p^E - p_0^E}^{\overline{k}} V(\xi + \alpha\beta^* - e^E(\beta^*))g(k)dk - \int_{\underline{k}}^{p^E - p_0^E} V(\alpha\beta^* - e^E(\beta^*) + k)g(k)dk \\ & = \psi(e^N(\beta^*)) - \psi(e^E(\beta^*)) + (1 - \lambda) \frac{F(\beta^*)}{f(\beta^*)} \left\{ \left(\alpha + \frac{Q_1}{Q_0 + Q_1} \right) \psi'(e^N(\tilde{\beta}^*)) - \alpha \psi'(e^E(\tilde{\beta}^*)) \right\}. \end{aligned}$$

The left hand side of the equation represents the difference in social surplus when the Post Office provides the sorting to the less efficient mailers and when it is the entrant who provides them this service. The right-hand side of the equation represents the difference in rents that are given to the Post Office in each case. For the cut-off point β^* both sides are equal.

Finally, we study in which situation the Post Office will be allowed to provide the sorting to the mailers. In particular, we analyze who will be in charge of providing the sorting when the Post Office and its competitor are equally efficient.

Proposition 12. *Given $\xi \in [\underline{\beta}, \overline{\beta}]$, let $\beta^*(\xi)$ be the optimal cut-off point. Then $\beta^*(\xi)$ satisfies: (11.1) $\beta^*(\underline{\beta}) = \underline{\beta}$; (11.2) $\beta^*(\xi) < \xi$ for every $\xi \in (\underline{\beta}, \overline{\beta}]$; (11.3) $\beta^*(\xi)$ is continuous.*

As in De Fraja (1999), the proposition shows that with asymmetric information there is a bias against the Post Office.¹⁷ Despite being equally efficient than the entrant, the Post Office is not allowed to supply the first stage of the sorting.

The intuition behind this conclusion is that the entry of a competitor reduces the informational rents that must be given to the Post Office, and does not distort

¹⁷In fact, our result can be seen as an extension of De Fraja (1999), when there are heterogeneous consumers. It is important to emphasize that our representation of the consumers does not modify its interesting conclusion, when there is a uniform distribution of k .

the prices from marginal cost pricing. Therefore, entry of an inefficient competitor may be allowed, because although it reduces the productive efficiency, it increases the allocative efficiency and improves the ability of the regulator to observe the Post Office's cost. This result contrasts with the optimal worksharing discount that we have seen in Section 3.2. In that case, the Post Office is protected because bypassing implies the distortion from marginal cost pricing.

Finally, it is important to remember again that the result of the previous proposition is obtained with a uniform distribution of the mailers. With a different distribution this result can be modified. The next proposition shows an example.

Proposition 13. *Given $\underline{k} \in [\underline{\beta}, \overline{\beta}]$, when all mailers have the type \underline{k} , then $\beta^*(\xi) = \xi$.*

Intuitively, if all the mailers have the type \underline{k} , all of them will bypass the Post Office and will not use the competitor. On the other hand, observe that when it increases the number of mailers close to \underline{k} the pro-competitive bias against the Post Office is reduced.

5 Conclusions and policy implications

In this paper we have analyzed the optimal access policy in a liberalized postal market. In particular, we have studied how must be established the discounts over the price of the letters that receive those agents that undertake some sorting activities that were traditionally provided by the Post Office. We distinguish two types of worksharing discounts. The discounts conceded to the mailers that bypass some sorting facilities of the Post Office's network. And the discounts granted to the firms that carry out some stages of the sorting, but that have to relay on the Post Office's network to deliver their non-local correspondence. Apparently, the two types of discounts should be determined in the same way, because in both cases the discount has to take into account the cost avoided to the Post Office. The literature about worksharing discounts shows that this is indeed the case when the regulator has perfect information about the Post Office sub-costs. However, we show that this result can no longer be held in our model with asymmetric information.

When there is asymmetric information between the regulator and the Post Office, the regulator has to concede informational rents to the firm to encourage it to make some effort. When there are some agents (mailers or firms) that pre-sort their letters, the informational rents that must be given to the firm are reduced, but the prices of the letters maybe distorted with respect to marginal cost pricing. When the agent that pre-sort their letters is a group of mailers that are as efficient as the Post Office, the informational rents that must be given to the Post Office is equal when the regulator allows the mailers to pre-sort their mail and when it forbids it.

However, if only a group of mailers (not all of them) pre-sort their correspondence, the informational rents conceded to the Post Office distort the tariffs away from marginal cost pricing. As a result, the optimal policy is to forbid the bypass of the mailers. That is, only when the difference between the mailers costs and the Post Office's is sufficiently large, should the regulator allow the bypass. A different case is when all the mailers have more or less the same sorting cost. In this situation, the regulator may allow bypassing by the inefficient mailers when the difference with the Postal Office's cost is sufficiently small.

In the access discount problem the optimal policy is different and may consist of allowing the entry of firms that are less efficient than the Post Office. This occurs because entry reduces the informational rents that must be given to the Post Office. On the other hand, entry permits to observe the Post Office's cost in the second stage of the sorting. As a consequence, it is possible to establish a marginal cost pricing policy. As we have shown, however, this pro-competitive bias in favor of the entrant is reduced when the main part of the mailers is more efficient than the Post Office.

The main contribution of this paper is to show that with asymmetric information the optimal worksharing discount conceded to the mailers and the optimal access discount conceded to the entrants in the postal market have opposite characteristics. While in the first case the regulator should restrict the bypass, in the second the regulator should take a pro-competitive attitude. This may support the view of the postal authorities of some countries that consists of fixing the access discounts slightly lower than the Post Office's avoided cost, and delegating to the regulated Post Office the responsibility of fixing the worksharing discounts.

Appendix

Proof of Proposition 1. The welfare function for any given cut-off point can be written as two separate problems. When we eliminate t using the definition of U , we can write the function as follows:

$$\max_{\substack{U(\beta); p(\beta), \\ p_0(\beta), e(\beta)}} \int_{\underline{k}}^{\bar{k}} V(p)g(k)dk - (1 - \lambda)U^N - \psi(e^N) + (p - (1 + \alpha)\beta + e) \int_{\underline{k}}^{\bar{k}} q(p)g(k)dk$$

when $\beta \leq \beta^*$, and

$$\begin{aligned} & \max_{\substack{U(\beta); p(\beta), \\ p_0(\beta), e(\beta)}} \int_{p-p_0}^{\bar{k}} V(p)g(k)dk + \int_{\underline{k}}^{p-p_0} V(p_0 + k)g(k)dk - (1 - \lambda)U^E - \psi(e^E) \\ & + (p - (1 + \alpha)\beta + e) \int_{p-p_0}^{\bar{k}} q(p)g(k)dk + (p_0 - \alpha\beta + e) \int_{\underline{k}}^{p-p_0} q(p_0 + k)g(k)dk, \end{aligned}$$

when $\beta > \beta^*$. Consider the following definitions:

$$\tilde{Q}_1(p(\beta)) = \int_{\underline{k}}^{\bar{k}} q(p)g(k)dk; \quad Q_1(p(\beta), p_0(\beta)) = q(p)[1 - G(p(\beta) - p_0(\beta))];$$

$$Q_0(p(\beta), p_0(\beta)) = \int_{\underline{k}}^{p-p_0} q(p_0(\beta) + k)g(k)dk.$$

Taking this into account, the two problems can be solved in the standard manner, giving the policy of the statement. ■

Proof of Proposition 2. Given $\underline{k}, \bar{k} \in [\underline{\beta}, \bar{\beta}]$, define $G(\tilde{\beta})$ as the social welfare given an arbitrary $\tilde{\beta}$ as the cut-off point, and the optimal policies above and below it, given by Proposition 1.

$$\begin{aligned} G(\tilde{\beta}) &= \int_{\underline{k}}^{\tilde{\beta}} [\int_{\underline{k}}^{\bar{k}} V((1 + \alpha)\beta - e^N(\beta))g(k)dk - \psi(e^N(\beta))] f(\beta) d\beta \\ &+ \int_{\tilde{\beta}}^{\bar{\beta}} [\int_{p^E - p_0^E}^{\bar{k}} V((1 + \alpha)\beta - e^E(\beta))g(k)dk + \int_{\underline{k}}^{p^E - p_0^E} V(\alpha\beta - e^E(\beta) + k)g(k)dk - \psi(e^E(\beta))] f(\beta) d\beta \end{aligned}$$

Differentiation yields

$$\begin{aligned} \frac{G'(\tilde{\beta})}{f(\tilde{\beta})} &= [\int_{\underline{k}}^{\bar{k}} V((1+\alpha)\beta - e^N(\beta))g(k)dk - \psi(e^N(\beta))] \\ &- [\int_{p^E - p_0^E}^{\bar{k}} V((1+\alpha)\beta - e^E(\beta))g(k)dk - \int_{\underline{k}}^{p^E - p_0^E} V(\alpha\beta - e^E(\beta) + k)g(k)dk - \psi(e^E(\beta))]. \end{aligned}$$

Note that as $G'(\beta) \geq 0$ and $G'(\bar{\beta}) < 0$, in view of the continuity of G' , the point $\beta = \underline{k}$ satisfies $G'(\beta^*) = 0$, and $G''(\beta^*) \geq 0$. This establishes the Proposition. ■

Proof of Proposition 3. The regulator can only observe total cost. Therefore, if the incumbent wants to misreport $\tilde{\beta}$ when the value of the cost is β , it will need to exert an effort's level such that the actual cost is the same as would be observed if $\tilde{\beta}$ where the true cost parameter. If no mailer workshare their letters this is satisfied for

$$\begin{aligned} (\alpha + 1)\tilde{\beta} - e(\tilde{\beta}) &= (\alpha + 1)\beta - x, \\ x &= e(\tilde{\beta}) + (\beta - \tilde{\beta})(\alpha + 1). \end{aligned}$$

When a part of the mailers make the sorting, the effort's level have to satisfy

$$\begin{aligned} Q_0(p(\beta), p_0(\tilde{\beta}))(\alpha\tilde{\beta} - e(\tilde{\beta})) + Q_1(p(\tilde{\beta}), p_0(\tilde{\beta}))((1+\alpha)\tilde{\beta} - e(\tilde{\beta})) \\ = Q_0(p(\tilde{\beta}), p_0(\tilde{\beta}))(\alpha\beta - x) + Q_1(p(\tilde{\beta}), p_0(\tilde{\beta}))((1+\alpha)\beta - x) \end{aligned}$$

which, after rearranging gives us

$$x = e(\tilde{\beta}) + (\beta - \tilde{\beta})(\alpha + \frac{Q_1(p(\tilde{\beta}), p_0(\tilde{\beta}))}{Q_0(p(\tilde{\beta}), p_0(\tilde{\beta})) + Q_1(p(\tilde{\beta}), p_0(\tilde{\beta}))}).$$

■

Proof of Proposition 4. Consider the following derivatives for the demands functions:

$$\begin{aligned} \frac{\partial Q_1(p, p_0)}{\partial p} &= [1 - G(p - p_0)] \frac{\partial q(p)}{\partial p} - q(p)g(p - p_0), \\ \frac{\partial Q_0(p, p_0)}{\partial p_0} &= \int_{\underline{k}}^{\beta^*} \frac{\partial q(p_0 + k)}{\partial p_0} g(k)dk - q(p_0 + k)g(p - p_0), \\ \frac{\partial Q_1(p, p_0)}{\partial p_0} &= q(p)g(p - p_0), \quad \frac{\partial Q_0(p, p_0)}{\partial p} = q(p_0 + k)g(p - p_0). \end{aligned}$$

Next, define the following superelasticities

$$\tilde{\eta}_1 = \frac{\eta_1\eta_0 - \eta_{01}\eta_{10}}{\eta_1\eta_0 + \eta_{10}\eta_{01}}, \quad \tilde{\eta}_0 = \frac{\eta_0\eta_1 - \eta_{10}\eta_{01}}{\eta_0\eta_1 + \eta_{01}\eta_{10}}.$$

Taking into account these definitions, and given U^* , the regulator's problem for $\beta \leq \beta^*$ can be solved in the standard manner. The problem for $\beta > \beta^*$ is an optimal control problem with an initial value, which is represented by $\Phi(\beta^*, U^*)$. Taking into account the transversability conditions of this problem we solve it, and the proposition is established. ■

Proof of Corollary 2. The proof follows directly from condition (16) and (17). ■

Proof of Proposition 5. In order to proof this proposition we follow the proof of Proposition 6 in De Fraja (1999). Consider that $G(\tilde{\beta})$ is the social welfare when the cut-off point is arbitrary fixed at $\tilde{\beta}$, and Proposition 4 determines the optimal policies for $\beta \leq \beta^*$ and $\beta > \beta^*$.

$$\begin{aligned} G(\tilde{\beta}) &= \int_{\underline{\beta}}^{\tilde{\beta}} [\int_{\underline{k}}^{\bar{k}} V(p^N(\beta))g(k)dk - (1 - \lambda)\tilde{U}^N(\beta, \tilde{\beta}) - \psi(e^N(\beta))]f(\beta)d\beta \\ &+ \int_{\tilde{\beta}}^{\bar{\beta}} [\int_{p^E - p_0^E}^{\bar{k}} V(p^E(\beta))g(k)dk + \int_{\underline{k}}^{p^E - p_0^E} V(p_0^E(\beta) + k)g(k)dk - (1 - \lambda)\tilde{U}^E(\beta, \tilde{\beta}) \\ &- \psi(e^E(\beta))]f(\beta)d\beta. \end{aligned}$$

where $\tilde{U}^j(\beta, \tilde{\beta})$, $j = N, E$ is the profit of the Post Office, when it has cost β and the cut-off point is $\tilde{\beta}$. After differentiating this equation we obtain

$$\begin{aligned} G'(\tilde{\beta}) &= \left\{ \int_{\underline{k}}^{\bar{k}} V(p^N(\beta))g(k)dk - \int_{p^E - p_0^E}^{\bar{k}} V(p^E(\beta))g(k)dk \right. \\ &- \left. \int_{\underline{k}}^{p^E - p_0^E} V(p_0^E(\beta) + k)g(k)dk - \psi(e^N(\beta)) + \psi(e^E(\beta)) \right\} f(\tilde{\beta}) \\ &- (1 - \lambda) \left\{ \int_{\underline{k}}^{\tilde{\beta}} \frac{\partial U^N}{\partial \beta} f(\beta)d\beta - \int_{\tilde{\beta}}^{\bar{k}} \frac{\partial U^E}{\partial \beta} f(\beta)d\beta \right\}. \end{aligned}$$

Using the fact that $\frac{\partial U^N}{\partial \beta} = (\alpha + 1)\psi'(e^N(\tilde{\beta}))$ and $\frac{\partial U^E}{\partial \beta} = (\alpha + \frac{Q_1}{Q_0 + Q_1})\psi'(e^E(\tilde{\beta}))$ we obtain

$$\begin{aligned}
G'(\tilde{\beta}) &= f(\tilde{\beta}) \left\{ \int_{\underline{k}}^{\bar{k}} V(p^N(\beta))g(k)dk - \int_{p^E - p_0^E}^{\bar{k}} V(p^E(\beta))g(k)dk \right. \\
&\quad \left. - \int_{\underline{k}}^{p^E - p_0^E} V(p_0^E(\beta) + k)g(k)dk - \psi(e^N(\beta)) + \psi(e^E(\beta)) \right\} \\
&\quad - (1 - \lambda)F(\tilde{\beta}) \left\{ (\alpha + 1)\psi'(e^N(\tilde{\beta})) - \left(\alpha + \frac{Q_1}{Q_0 + Q_1} \right) \psi'(e^E(\tilde{\beta})) \right\}.
\end{aligned}$$

Next we write the following two functions:

$$\begin{aligned}
\tilde{G}(\tilde{\beta}) &= \int_{\underline{k}}^{p^E - p_0^E} V((1 + \alpha)\tilde{\beta} - e^N(\tilde{\beta}))g(k)dk - \int_{\underline{k}}^{p^E - p_0^E} V(\alpha\tilde{\beta} - e^E(\tilde{\beta}) + k)g(k)dk \\
&\quad - \psi(e^N(\beta)) + \psi(e^E(\beta)) - (1 - \lambda)F(\tilde{\beta}) \left\{ (\alpha + 1)\psi'(e^N(\tilde{\beta})) - \left(\alpha + \frac{Q_1}{Q_0 + Q_1} \right) \psi'(e^E(\tilde{\beta})) \right\}.
\end{aligned}$$

$$\begin{aligned}
R(\tilde{\beta}, e^N, e^E) &= \int_{\underline{k}}^{p^E - p_0^E} V((1 + \alpha)\tilde{\beta} - e^N)g(k)dk - \int_{\underline{k}}^{p^E - p_0^E} V(\alpha\tilde{\beta} - e^E + k)g(k)dk \\
&\quad - \psi(e^N) + \psi(e^E) - (1 - \lambda)F(\tilde{\beta}) \left\{ (\alpha + 1)\psi'(e^N) - \left(\alpha + \frac{Q_1}{Q_1 + Q_0} \right) \psi'(e^E) \right\}.
\end{aligned}$$

From the effort optimality condition (14), $e^N(\underline{\beta})$ maximizes $R(\underline{\beta}, e^N, e^E(\underline{\beta}))$. Consequently $R(\underline{\beta}, e^N(\underline{\beta}), e^E(\underline{\beta})) > R(\underline{\beta}, e^E(\underline{\beta}), e^E(\underline{\beta}))$. Taking this into account we obtain:

$$\begin{aligned}
\frac{G'(\underline{\beta})}{f(\underline{\beta})} &= R(\underline{\beta}, e^N(\underline{\beta}), e^E(\underline{\beta})) > R(\underline{\beta}, e^E(\underline{\beta}), e^E(\underline{\beta})) \\
&= \int_{\underline{k}}^{p^E - p_0^E} V((1 + \alpha)\underline{\beta} - e^E(\underline{\beta}))g(k)dk - \int_{\underline{k}}^{p^E - p_0^E} V(\alpha\underline{\beta} - e^E(\underline{\beta}) + k)g(k)dk \geq 0.
\end{aligned}$$

This is a strict inequality if $\underline{\beta} < \underline{k}$. On the other hand, $R(\bar{\beta}, e^E(\bar{\beta}), e^E(\bar{\beta})) < 0$. Indeed,

$$\begin{aligned}
R(\bar{\beta}, e^E(\bar{\beta}), e^E(\bar{\beta})) &= \int_{\underline{k}}^{p^E - p_0^E} V((1 + \alpha)\bar{\beta} - e^E(\bar{\beta}))g(k)dk \\
&\quad - \int_{\underline{k}}^{p^E - p_0^E} V(\alpha\bar{\beta} - e^E(\bar{\beta}) + k)g(k)dk - (1 - \lambda) \frac{F(\bar{\beta})}{f(\bar{\beta})} \left(1 - \frac{Q_1}{Q_0 + Q_1} \right) \psi'(e^E(\bar{\beta})) < 0.
\end{aligned}$$

Notice that when $\bar{\beta} > \underline{k}$ the first two terms are negative. Therefore, if $R(\underline{\beta}, e^E(\underline{\beta}), e^E(\underline{\beta})) \geq 0$ and $R(\bar{\beta}, e^E(\bar{\beta}), e^E(\bar{\beta})) < 0$, in view of the continuity of R , there is a β such that $R(\beta, e^E(\beta), e^E(\beta)) = 0$.

On the other hand, we have seen that $G'(\underline{\beta}) > 0$. If $G'(\bar{\beta}) > 0$ we have nothing to prove. Bypass will never occur, and there is not a cut-off point. Consider that $G'(\bar{\beta}) < 0$. We have

$$\frac{G'(\bar{\beta})}{f(\bar{\beta})} > \tilde{G}(\bar{\beta}) = R(\bar{\beta}, e^N(\bar{\beta}), e^E(\bar{\beta})) > R(\bar{\beta}, e^E(\bar{\beta}), e^E(\bar{\beta})).$$

Consider the following result. Lemma: $\frac{G'(\beta)}{f(\beta)} > \tilde{G}(\beta)$. Proof of the Lemma:

$$\begin{aligned} \frac{G'(\beta)}{f(\beta)} - \tilde{G}(\beta) &= \int_{p^E - p_0^E}^{\bar{k}} V((1 + \alpha)\beta - e^N(\beta))g(k)dk - \int_{p^E(\beta) - p_0^E}^{\bar{k}} V(p^E)g(k)dk \\ &+ \int_{\underline{k}}^{p^E - p_0^E} V(\alpha\beta - e^E(\beta) + k)g(k)dk - \int_{\underline{k}}^{p^E - p_0^E} V(p_0^E(\beta) + k)g(k)dk \geq 0. \end{aligned}$$

Indeed, p^E and p_0^E give zero total revenue. Marginal cost prices also give zero total revenue. However, consumer's surplus is maximized, for given revenue requirement, by Ramsey prices, in this case marginal cost pricing. Considering assumption 1 and 3, give the statement of the lemma. ■

Proof of Proposition 6. (6.1) Let $\underline{k} > \underline{\beta}$, and let β^* be any zero of $G'(\beta)$. Lemma 5 implies $\tilde{G}(\tilde{\beta}) < \frac{G'(\tilde{\beta})}{f(\tilde{\beta})} = 0$, and, since $\tilde{G}(\underline{\beta}) = \frac{G'(\underline{\beta})}{f(\underline{\beta})}$, the zero of $\tilde{G}(\underline{\beta})$, call it $\tilde{\beta}^*$, satisfies $\tilde{\beta}^* < \beta^*$. Now consider $0 = \tilde{G}(\tilde{\beta}^*) = R(\tilde{\beta}^*, e^N(\tilde{\beta}^*), e^E(\tilde{\beta}^*)) > R(\tilde{\beta}^*, e^E(\tilde{\beta}^*), e^E(\tilde{\beta}^*))$. Again, this follows from the fact that $e^N(\tilde{\beta}^*)$ maximises $R(\tilde{\beta}, e^N, e^E(\tilde{\beta}^*))$. Therefore the zero of $R(\tilde{\beta}^*, e^E(\tilde{\beta}), e^E(\tilde{\beta}^*))$, $\tilde{\tilde{\beta}}$, satisfies $\tilde{\tilde{\beta}} < \tilde{\beta}^*$. Finally, $R(\tilde{\tilde{\beta}}^*, e^E(\tilde{\tilde{\beta}}^*), e^E(\tilde{\tilde{\beta}}^*)) = 0$ implies

$$\begin{aligned} &\int_{\underline{k}}^{p^E - p_0^E} V((1 + \alpha)\tilde{\tilde{\beta}}^* - e^E(\tilde{\tilde{\beta}}^*))g(k)dk - \int_{\underline{k}}^{p^E - p_0^E} V(\alpha\tilde{\tilde{\beta}}^* - e^E(\tilde{\tilde{\beta}}^*) + \underline{k})g(k)dk \\ &= (1 - \lambda) \left[\frac{F(\tilde{\tilde{\beta}}^*)}{f(\tilde{\tilde{\beta}}^*)} \right] \left(1 - \frac{Q_1}{Q_0 + Q_1} \right) \psi(e^E(\tilde{\tilde{\beta}}^*)) = 0, \end{aligned}$$

and hence $\tilde{\tilde{\beta}}^* = \underline{k}$. Indeed, when $R(\underline{k}, e^E(\underline{k}), e^E(\underline{k})) = 0$, $Q_0(p(\beta), p_0(\beta)) = 0$. Therefore, $\beta^* > \underline{k}$. ■

(6.2) Consider the second derivative of G . Write $G'(\beta) = f(\beta)[\frac{G'(\beta)}{f(\beta)}]$, and $G''(\beta) = f'(\beta)[\frac{G'(\beta)}{f(\beta)}] + f(\beta)(\frac{d}{d\beta})(\frac{G'(\beta)}{f(\beta)})$. The first term is zero at a stationary point. Now consider

$$(\frac{d}{d\beta})(\frac{G'(\beta)}{f(\beta)}) = -(1 - \lambda)(\frac{d}{d\beta})(\frac{F(\beta)}{f(\beta)})[(\alpha + 1)\psi'(e^N(\beta)) - (\alpha + \frac{Q_1}{Q_1+Q_0})\psi'(e^E(\beta))] < 0,$$

at every stationary point of G . As a consequence, there can be at most one such point, establishing the continuity of $\beta^*(\underline{k})$. ■

(6.3) Total differentiation of G' gives $G''(\beta^*)d\beta^* + (\frac{\partial G'(\beta^*)}{\partial \underline{k}}) > 0$.

Now $(\frac{\partial G'(\beta^*)}{\partial \underline{k}}) = q(p_0^E(\beta^*) + k)$, again the effort optimality condition implies that the terms in $(\frac{\partial e^E(\beta^*)}{\partial \underline{k}})$ vanish. If we differentiate $\frac{d\beta^*}{d\underline{k}}$ with respect to \underline{k} we obtain,

$$\frac{d^2\beta^*}{d\underline{k}^2} = -\frac{1}{G''(\beta^*)^2}[\frac{\partial^2 G'(\beta^*)}{\partial \underline{k}^2}G''(\beta^*) - \frac{\partial G'(\beta^*)}{\partial \underline{k}}\frac{\partial G''(\beta^*)}{\partial \underline{k}}] > 0.$$

Indeed,

$$\frac{d^2\beta^*}{d\underline{k}^2} = q'(p_0^E(\beta^*) + k) < 0,$$

$$\frac{\partial G''(\beta^*)}{\partial \underline{k}} = f(\beta^*)(\alpha + 1)(1 - \lambda)\frac{d}{d\beta}(\frac{F(\beta^*)}{f(\beta^*)})\psi''(e^E(\beta^*))\frac{de^E(\beta^*)}{d\underline{k}} < 0,$$

$$\frac{de^E(\beta^*)}{d\underline{k}} = \frac{Q_1'(p^E(\beta^*))}{\psi''(e^E(\beta^*)) + (\alpha + \frac{Q_1}{Q_0+Q_1})(1 - \lambda)(\frac{F(\beta^*)}{f(\beta^*)})\psi'''(e^E(\beta^*)) + Q_1'(p^E(\beta^*))} < 0.$$

■

Proof of Lemma 3. (1.1) Clearly $p^N(\beta) \leq p_0^E(\beta) + k$. If $p^N(\beta) < p_0^E(\beta) + k$, there is nothing to prove. Let β_0 be such that $p^N(\beta_0) = p_0^E(\beta_0) + k$. At this point we have that $p^N(\beta_0) < p^E(\beta_0)$, and from the effort optimality condition $e^N(\beta_0) < e^E(\beta_0)$. We also have that $\frac{de^N(\beta_0)}{d\beta} < \frac{de^E(\beta_0)}{d\beta}$. Therefore, $\frac{dp^N(\beta_0)}{d\beta} = (1 + \alpha) - [\frac{de^E(\beta_0)}{d\beta}] > \alpha - [\frac{de^E(\beta_0)}{d\beta}] > \frac{dp_0^E(\beta_0)}{d\beta}$, which implies that $\beta > \beta_0$ implies $p^N(\beta) > p_0^E(\beta) + k$.

Next, notice that for $\beta = \underline{k}$, $e^N = e^E$ implies that $p^N > p_0^E + k$. Therefore, for $\beta^* > \underline{k}$ we have that $p^N > p_0^E + k$. This implies that, $\beta_0 < \beta^*$.

(1.2) Notice that $e^N(\beta_0) = e^E(\beta_0) = e$ for $\beta_0 = \underline{k}$. Therefore, $\beta_0 < \beta^*$. ■

Proof of Corollary 4. Define $t^N(\beta^*)$, $U^N(\beta^*)$, and $e^N(\beta^*)$. The corollary establishes that

$$S(p^N(\beta^*)) + \lambda U^N(\beta^*) - t^N(\beta^*) = S(p^E(\beta^*)) + \lambda U^E(\beta^*) - t^E(\beta^*).$$

Using the definition of t , we obtain

$$S(p^N(\beta^*)) - (1 - \lambda)U^N(\beta^*) - \psi(e^N(\beta^*)) = S(p^E(\beta^*)) - (1 - \lambda)U^E(\beta^*) - \psi(e^E(\beta^*)).$$

Taking into account Proposition 3, and substituting from Proposition 5 yields

$$(1 - \lambda) \frac{F(\beta^*)}{f(\beta^*)} [(\alpha + 1)\psi'(e^N(\beta^*)) - (\alpha + \frac{Q_1}{Q_0 + Q_1})\psi'(e^E(\beta^*))] = 0.$$

This is satisfied because U is continuous at β^* . ■

Proof of Proposition 7. If all mailers have the type \underline{k} , then from the proof of proposition 5 we have that $\frac{G'(\beta)}{f(\beta)} < \tilde{G}(\beta)$. In this case, since $e^E(\tilde{\beta})$ minimises $R(\beta, e^N(\beta), e^E)$ for every $\beta \in [\underline{\beta}, \bar{\beta}]$, it follows that

$$\frac{G'(\bar{\beta})}{f(\bar{\beta})} < \tilde{G}(\bar{\beta}) = R(\bar{\beta}, e^N(\bar{\beta}), e^E(\bar{\beta})) < R(\bar{\beta}, e^N(\bar{\beta}), e^N(\bar{\beta})).$$

Using the same approach than in proof of proposition (6.1), we have that $\beta^* < \underline{k}$. ■

Proof of Proposition 8. The welfare function for an arbitrary cut-off point $\tilde{\beta}$ can be split into two separate problems. When $\beta \leq \tilde{\beta}$,

$$\begin{aligned} & \max_{\substack{U(\beta); p(\beta), \\ p_0(\beta), e(\beta)}} \int_{p-p_0}^{\bar{k}} V(p)g(k)dk + \int_{\underline{k}}^{p-p_0} V(p_0+k)g(k)dk - (1 - \lambda)U^N - \psi(e^N) \\ & + (p - (1 + \alpha)\beta + e) \int_{p-p_0}^{\bar{k}} q(p)g(k)dk + (p_0 - \alpha\beta + e) \int_{\underline{k}}^{p-p_0} q(p_0+k)g(k)dk, \end{aligned}$$

and when $\beta > \tilde{\beta}$

$$\begin{aligned} & \max_{\substack{U(\beta); p(\beta), \\ p_0(\beta), e(\beta)}} \int_{p-p_0}^{\bar{k}} V(p)g(k)dk + \int_{\underline{k}}^{p-p_0} V(p_0+k)g(k)dk - (1 - \lambda)U^E - \psi(e^E) \\ & + \lambda(p - a - \xi) \int_{p-p_0}^{\bar{k}} q(p)g(k)dk + (p_0 - \alpha\beta + e) \int_{\underline{k}}^{p-p_0} q(p_0+k)g(k)dk \\ & + (a - \alpha\beta + e) \int_{p-p_0}^{\bar{k}} q(p)g(k)dk, \end{aligned}$$

Consider the following definitions:

$$Q_1(p(\beta), p_0(\beta)) = q(p)[1 - G(p(\beta) - p_0(\beta))]$$

$$Q_0(p(\beta), p_0(\beta)) = \int_{\underline{k}}^{p-p_0} q(p_0(\beta) + k)g(k)dk.$$

Taking this into account, the two problems can be solved in the standard manner. Afterwards, we find that the cut off is $\tilde{\beta} = \xi$. ■

Proof of Proposition 9. Following the same analysis than in Proposition 3, if the competitor stays out of the market the Post Office effort's level satisfies,

$$x = e(\tilde{\beta}) + (\beta - \tilde{\beta})\left(\alpha + \frac{Q_1(p(\tilde{\beta}), p_0(\tilde{\beta}))}{Q_0(p(\tilde{\beta}), p_0(\tilde{\beta})) + Q_1(p(\tilde{\beta}), p_0(\tilde{\beta}))}\right).$$

Indeed, this is the same condition that we had in Section 3, when a part of the mailers make the sorting. On the other hand, when the competitor provides the first stage of the sorting to the inefficient mailers we have

$$\alpha\tilde{\beta} - e(\tilde{\beta}) = \alpha\beta - x,$$

$$x = e(\tilde{\beta}) + (\beta - \tilde{\beta})\alpha.$$

■

Proof of Proposition 10. We solve the two problems as in Proposition 4. ■

Proof of Proposition 11. Fix ξ . Consider that $G(\tilde{\beta})$ is the social welfare when the cut-off point is arbitrary fixed at $\tilde{\beta}$, and Proposition 10 determines the optimal policies for $\beta \leq \beta^*$ and $\beta > \beta^*$.

$$\begin{aligned} G(\tilde{\beta}) &= \int_{\underline{\beta}}^{\tilde{\beta}} \left[\int_{p^N - p_0^N}^{\bar{k}} V(p^N(\beta))g(k)dk + \int_{\underline{k}}^{p^N - p_0^N} V(p_0^N(\beta) + k)g(k)dk \right. \\ &\quad \left. - (1 - \lambda)\tilde{U}^N(\beta, \tilde{\beta}) - \psi(e^N(\beta)) \right] f(\beta)d\beta + \int_{\tilde{\beta}}^{\bar{\beta}} \left[\int_{p^E - p_0^E}^{\bar{k}} V(p^E(\beta))g(k)dk \right. \\ &\quad \left. + \int_{\underline{k}}^{p^E - p_0^E} V(p_0^E(\beta) + k)g(k)dk - (1 - \lambda)\tilde{U}^E(\beta, \tilde{\beta}) - \psi(e^E(\beta)) \right] f(\beta)d\beta, \end{aligned}$$

where $\tilde{U}^j(\beta, \tilde{\beta})$, $j = N, E$ is the profit of the Post Office when it has cost β and the cut-off point is $\tilde{\beta}$. After differentiating we obtain

$$\begin{aligned}
G'(\tilde{\beta}) &= \left\{ \int_{p^N - p_0^N}^{\bar{k}} V(p^N(\beta))g(k)dk + \int_{\underline{k}}^{p^N - p_0^N} V(p_0^N(\beta) + k)g(k)dk \right. \\
&- \int_{p^E - p_0^E}^{\bar{k}} V(p^E(\beta))g(k)dk - \int_{\underline{k}}^{p^E - p_0^E} V(p_0^E(\beta) + k)g(k)dk \\
&\left. - \psi(e^N(\beta)) + \psi(e^E(\beta)) \right\} f(\tilde{\beta}) - (1 - \lambda) \left\{ \int_{\underline{k}}^{\tilde{\beta}} \frac{\partial U^N}{\partial \beta} f(\beta) d\beta - \int_{\tilde{\beta}}^{\bar{k}} \frac{\partial U^E}{\partial \beta} f(\beta) d\beta \right\}.
\end{aligned}$$

Notice that $\frac{\partial U^N}{\partial \beta} = (\alpha + \frac{Q_1}{Q_0 + Q_1})\psi'(e^N(\tilde{\beta}))$ and $\frac{\partial U^E}{\partial \beta} = \alpha\psi'(e^E(\tilde{\beta}))$. As a consequence,

$$\begin{aligned}
G'(\tilde{\beta}) &= f(\tilde{\beta}) \left\{ \int_{p^N - p_0^N}^{\bar{k}} V(p^N(\beta))g(k)dk + \int_{\underline{k}}^{p^N - p_0^N} V(p_0^N + k)g(k)dk \right. \\
&- \int_{p^E - p_0^E}^{\bar{k}} V(p^E(\beta))g(k)dk - \int_{\underline{k}}^{p^E - p_0^E} V(p_0^E(\beta) + k)g(k)dk \\
&\left. - \psi(e^N(\beta)) + \psi(e^E(\beta)) \right\} - (1 - \lambda)F(\beta) \left\{ (\alpha + \frac{Q_1}{Q_0 + Q_1})\psi'(e^N(\tilde{\beta})) - \alpha\psi'(e^E(\tilde{\beta})) \right\}.
\end{aligned}$$

Taking this into account, consider the following functions:

$$\begin{aligned}
\tilde{G}(\tilde{\beta}) &= \int_{p^N - p_0^N}^{\bar{k}} V((1 + \alpha)\beta - e^N(\beta))g(k)dk + \int_{\underline{k}}^{p^N - p_0^N} V(\alpha\beta - e^N(\beta) + k)g(k)dk \\
&- \int_{p^E - p_0^E}^{\bar{k}} V(\xi + \alpha\beta - e^E(\beta))g(k)dk - \int_{\underline{k}}^{p^E - p_0^E} V(\alpha\beta - e^E(\beta) + k)g(k)dk \\
&- \psi(e^N(\beta)) + \psi(e^E(\beta)) - (1 - \lambda)\frac{F(\beta)}{f(\beta)} \left\{ (\alpha + \frac{Q_1}{Q_0 + Q_1})\psi'(e^N(\tilde{\beta})) - \alpha\psi'(e^E(\tilde{\beta})) \right\}.
\end{aligned}$$

$$\begin{aligned}
R(\tilde{\beta}, e^N, e^E) &= \int_{p^N - p_0^N}^{\bar{k}} V((1 + \alpha)\beta - e^N)g(k)dk + \int_{\underline{k}}^{p^N - p_0^N} V(\alpha\beta - e^N + k)g(k)dk \\
&- \int_{p^E - p_0^E}^{\bar{k}} V(\xi + \alpha\beta - e^E)g(k)dk - \int_{\underline{k}}^{p^E - p_0^E} V(\alpha\beta - e^E + k)g(k)dk \\
&- \psi(e^N) + \psi(e^E) - (1 - \lambda)\frac{F(\beta)}{f(\beta)} \left\{ (\alpha + \frac{Q_1}{Q_0 + Q_1})\psi'(e^N) - \alpha\psi'(e^E) \right\}.
\end{aligned}$$

Note that, from the effort optimality condition, $e^N(\underline{\beta})$ maximises $R(\underline{\beta}, e^N, e^E(\underline{\beta}))$. Therefore, $R(\underline{\beta}, e^N(\underline{\beta}), e^E(\underline{\beta})) > R(\underline{\beta}, e^E(\underline{\beta}), e^E(\underline{\beta}))$ and we can write

$$\begin{aligned}
\frac{G'(\underline{\beta})}{f(\underline{\beta})} &= R(\underline{\beta}, e^N(\underline{\beta}), e^E(\underline{\beta})) > R(\underline{\beta}, e^E(\underline{\beta}), e^E(\underline{\beta})) \\
&= \int_{p^N - p_0^N}^{\bar{k}} V((1 + \alpha)\underline{\beta} - e^N(\underline{\beta}))g(k)dk - \int_{p^E - p_0^E}^{\bar{k}} V(\xi + \alpha\underline{\beta} - e^N(\underline{\beta}))g(k)dk \\
&\quad - \int_{\underline{k}}^{p^N - p_0^N} V(\alpha\underline{\beta} - e^N(\underline{\beta}))g(k)dk - \int_{\underline{k}}^{p^E - p_0^E} V(\alpha\underline{\beta} - e^N(\underline{\beta}) + k)g(k)dk \geq 0.
\end{aligned}$$

when $\beta = \underline{\beta}$, $p^N - p_0^N = p^E - p_0^E$. Therefore, if $\underline{\beta} < \underline{k}$ this is a strict inequality. On the other hand, $R(\bar{\beta}, e^N(\bar{\beta}), e^N(\bar{\beta})) \leq 0$. Indeed,

$$\begin{aligned}
&R(\bar{\beta}, e^N(\bar{\beta}), e^N(\bar{\beta})) \\
&= \int_{p^N - p_0^N}^{\bar{k}} V((1 + \alpha)\bar{\beta} - e^N(\bar{\beta}))g(k)dk - \int_{p^E - p_0^E}^{\bar{k}} V(\xi + \alpha\bar{\beta} - e^N(\bar{\beta}))g(k)dk \\
&\quad \int_{\underline{k}}^{p^N - p_0^N} V(\alpha\bar{\beta} - e^N(\bar{\beta}))g(k)dk - \int_{\underline{k}}^{p^E - p_0^E} V(\alpha\bar{\beta} - e^N(\bar{\beta}) + k)g(k)dk \\
&\quad - (1 - \lambda) \frac{F(\bar{\beta})}{f(\bar{\beta})} \left\{ \frac{Q_1}{Q_0 + Q_1} \psi'(e^N(\bar{\beta})) \right\} < 0.
\end{aligned}$$

Taking into account assumptions 1 and 3 this is negative. Now consider that

$$\frac{G'(\bar{\beta})}{f(\bar{\beta})} < \tilde{G}(\bar{\beta}) = R(\bar{\beta}, e^N(\bar{\beta}), e^E(\bar{\beta})) < R(\bar{\beta}, e^N(\bar{\beta}), e^N(\bar{\beta})) \leq 0.$$

The first inequality is shown in the following lemma. Lemma: $\frac{G'(\beta)}{f(\beta)} < \tilde{G}(\beta)$
Proof of the Lemma.

$$\begin{aligned}
\frac{G'(\beta)}{f(\beta)} - \tilde{G}(\beta) &= \int_{p^N - p_0^N}^{\bar{k}} V(p^N(\beta))g(k)dk - \int_{p^N - p_0^N}^{\bar{k}} V((1 + \alpha)\beta - e^N(\beta))g(k)dk \\
&\quad \int_{\underline{k}}^{p^N - p_0^N} V(p_0^N(\beta) + k)g(k)dk - \int_{\underline{k}}^{p^N - p_0^N} V(\alpha\beta - e^N(\beta) + k) < 0.
\end{aligned}$$

If $G'(\beta) \geq 0$ and $G'(\bar{\beta}) < 0$, in view of the continuity of G' , there exists at least a point β^* where $G(\beta^*) = 0$, $G''(\beta^*) \leq 0$. ■

Proof of Proposition 12. (12.1) Let $\xi > \underline{\beta}$, and let β^* be any zero of $G'(\tilde{\beta})$. The lemma in the proof of the previous proposition implies $\tilde{G}(\tilde{\beta}) > \frac{G'(\tilde{\beta})}{f(\tilde{\beta})} = 0$, and,

since $\tilde{G}(\underline{\beta}) = \frac{G'(\underline{\beta})}{f(\underline{\beta})}$, the zero of $\tilde{G}(\underline{\beta})$, call it $\tilde{\beta}^*$, satisfies $\tilde{\beta}^* > \beta^*$. Now consider $0 = \tilde{G}(\tilde{\beta}^*) = R(\tilde{\beta}^*, e^N(\tilde{\beta}^*), e^E(\tilde{\beta}^*)) < R(\tilde{\beta}^*, e^N(\tilde{\beta}^*), e^N(\tilde{\beta}^*))$. Again, this follows from the fact that $e^E(\tilde{\beta}^*)$ minimizes $R(\tilde{\beta}, e^N(\tilde{\beta}^*), e^E)$. Therefore the zero of $R(\tilde{\beta}^*, e^N(\tilde{\beta}), e^N(\tilde{\beta}^*))$, $\tilde{\tilde{\beta}}$, satisfies $\tilde{\tilde{\beta}} > \tilde{\beta}^*$. Finally, $R(\tilde{\tilde{\beta}}, e^N(\tilde{\beta}^*), e^N(\tilde{\tilde{\beta}})) = 0$ gives

$$\begin{aligned} & \int_{p^N - p_0^N}^{\bar{k}} V((1 + \alpha)\tilde{\tilde{\beta}} - e^N(\tilde{\beta}^*))g(k)dk - \int_{p^E - p_0^E}^{\bar{k}} V(\xi + \alpha\tilde{\tilde{\beta}} - e^N(\tilde{\beta}^*))g(k)dk \\ & + \int_{\underline{k}}^{p^N - p_0^N} V(\alpha\tilde{\tilde{\beta}} - e^N(\tilde{\beta}^*) + k)g(k)dk - \int_{\underline{k}}^{p^E - p_0^E} V(\alpha\tilde{\tilde{\beta}} - e^N(\tilde{\beta}^*) + k)g(k)dk \\ & = (1 - \lambda)\frac{F(\tilde{\tilde{\beta}})}{f(\tilde{\tilde{\beta}})}\left(\frac{Q_1}{Q_0 + Q_1}\right)\psi(e^N(\tilde{\beta}^*)) > 0. \end{aligned}$$

Notice that when $\tilde{\tilde{\beta}} = \xi$ we have

$$- \int_{p^E - p_0^E}^{p^N - p_0^N} V((1 + \alpha)\tilde{\tilde{\beta}} - e^N(\tilde{\beta}^*))g(k)dk - \int_{p^E - p_0^E}^{p^N - p_0^N} V(\alpha\tilde{\tilde{\beta}} - e^N(\tilde{\beta}^*) + k)g(k)dk < 0,$$

which contradicts the previous result. Therefore, it is necessary that $\tilde{\tilde{\beta}} < \xi$. ■

(12.2) Take the second derivative of G . Write $G'(\beta) = f(\beta)\left[\frac{G'(\beta)}{f(\beta)}\right]$, and $G''(\beta) = f'(\beta)\left[\frac{G'(\beta)}{f(\beta)}\right] + f(\beta)\left(\frac{d}{d\beta}\right)\left(\frac{G'(\beta)}{f(\beta)}\right)$. The first term is zero at a stationary point. Now consider

$$\left(\frac{d}{d\beta}\right)\left(\frac{G'(\beta)}{f(\beta)}\right) = -(1 - \lambda)\left(\frac{d}{d\beta}\right)\left(\frac{F(\beta)}{f(\beta)}\right)\left[\left(\alpha + \frac{Q_1}{Q_0 + Q_1}\right)\psi'(e^N(\beta)) - \alpha\psi'(e^E(\beta))\right] < 0.$$

at every stationary point of G . As a consequence, there can be at most one such point, establishing the continuity of $\beta^*(\xi)$. ■

Proof of Proposition 13. From the proof of Proposition 10, if all mailers have the type \underline{k} , we have that $p_0^N(\beta) = \alpha\beta - e^N(\beta)$. From Proposition 11 we have that, $\frac{G'(\beta)}{f(\beta)} = \tilde{G}(\beta)$. Finally, from Proposition 12 we have that for $R(\tilde{\beta}^*, e^N(\tilde{\beta}^*), e^N(\tilde{\beta}^*)) = 0$ to be satisfied it is necessary that $\tilde{\tilde{\beta}} = \xi$. Moreover, it is satisfied that $\tilde{\tilde{\beta}} = \tilde{\beta}^* = \beta^* = \xi$. ■

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