Ferromagnetic instabilities in neutron matter at finite temperature with the Gogny interaction

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The properties of spin-polarized neutron matter are studied both at zero and finite temperature using the D1 and the D1P parametrizations of the Gogny interaction. The results show two different behaviors: whereas the D1P force exhibits a ferromagnetic transition at a density of $\rho_c \sim 1.31$ fm^{-3} whose onset increases with temperature, no sign of such a transition is found for D1 at any density and temperature, in agreement with recent microscopic calculations.

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The possible existence of a phase transition of neutron matter to a ferromagnetic state has motivated many investigations of the equation of state (EOS) of spin-polarized neutron matter. In addition to the interest that such a transition could have in the context of neutron stars [1], this problem has gained interest in itself and has been addressed in the framework of very different theoretical approaches [2–11]. Whereas some of these calculations, like for instance those based on Skyrme-like interactions, predict a transition at densities in the range $(1-4)\rho_0$ (with $\rho_0 = 0.16$ fm^{-3} the saturation density of symmetric nuclear matter), others, like recent Monte Carlo [7] or Brueckner–Hartree–Fock (BHF) calculations [9,10] using modern two- and three-body realistic interactions, exclude such a transition at least up to densities around five times ρ_0. In spite of this discrepancy, it is interesting to study how temperature influences the ferromagnetic transition [12,13]. In Ref. [12] an analysis of the temperature effects in the framework of a Hartree-Fock calculation with Skyrme interactions was performed. In the present Brief Report, we study the influence of the finite-range terms of the interaction on the ferromagnetic transition. To this end, we consider the Gogny interaction. This is an effective nucleon-nucleon force with both zero- and finite-range terms and a simple spin-isospin structure. In addition to the original D1 parametrization [14], several other parametrizations of this force are available. The D1S force, for instance, was introduced to improve the pairing properties and surface effects of finite nuclei [15], whereas more recently the D1P [16] interaction has been introduced with the aim of reproducing the EOS of pure neutron matter given by a variational microscopic calculation with realistic interactions [17].

The properties of nuclear matter deduced from Gogny interactions have already been treated in the literature [18]. Indeed, several instabilities produced by these forces at zero temperature have been studied in previous works [19]. The isospin instability, for instance, is a common feature to all the existing Gogny parametrizations as well as of most Skyrme forces. This instability is signaled by the fact that, above a certain critical density, the energy per particle of nuclear matter becomes more repulsive than that of neutron matter. For the D1P force, this instability takes place at $\rho_I \sim 7\rho_0$, whereas for D1 it occurs at $\rho_I \sim 3\rho_0$. Furthermore, D1 and D1S also exhibit a spinodal instability in neutron matter, i.e., the energy per particle does not increase monotonically with density. Instead, it reaches a maximum value at a critical density (which for D1 and D1S is around $\rho_S \sim 4\rho_0$) and then decreases from that density onward. For the discussion of our results, we will choose the D1P and D1 parametrizations of the Gogny *NN* force, because they represent two qualitatively different behaviors regarding ferromagnetic instabilities.

In general, any Gogny interaction can be casted in the following form:

$$V_{NN}(\vec{r}) = \sum_{i=1}^{2} e^{-\frac{r^2}{\mu_i}} (W_i + B_i P_\sigma - H_i P_\tau - M_i P_\sigma P_\tau) + t_0 (1 + x_0 P_\sigma) \rho_N^\alpha \, \delta(\vec{r}), \qquad (1)$$

with $\vec{r}$ the distance between two nucleons. The spin-isospin structure of the force is given by the spin (isospin) exchange operators P_σ (P_τ). Notice that the usual spin-orbit term of the Gogny interaction has been omitted, because it does not give any contribution in infinite matter. The Gogny force includes a sum of two Gaussian-shaped terms that mimic the finite-range effects of a realistic interaction in the medium. Usually, it also contains one density-dependent zero-range term, even though in the case of the D1P parametrization two of these terms were used to make the fitting procedure more flexible.

In the following, we consider spin-polarized neutron matter, characterized by the spin-polarization parameter $\Delta = (\rho_\uparrow - \rho_\downarrow)/(\rho_\uparrow + \rho_\downarrow)$, where $\rho_{\uparrow(\downarrow)}$ is the density corresponding to neutrons having spin up (down) respect to a given direction, and by the total density, $\rho = \rho_\uparrow + \rho_\downarrow$. The energy per particle in the Hartree–Fock approximation is given by

$$e(\rho_\uparrow, \rho_\downarrow, T) = \frac{1}{\rho} \sum_{\sigma,k} \frac{\hbar^2 k^2}{2m} n_\sigma(k, T) + \frac{1}{2\rho} \times \sum_{\sigma_1,k_1;\sigma_2,k_2} \langle \vec{k}_1\sigma_1, \vec{k}_2\sigma_2 | V_{NN} | \vec{k}_1\sigma_1, \vec{k}_2\sigma_2 \rangle_A \times n_{\sigma_1}(k_1, T) n_{\sigma_2}(k_2, T), \qquad (2)$$

where $n_\sigma(k, T)$ are the momentum distributions:

$$n_\sigma(k, T) = \frac{1}{1 + e^{\beta[\epsilon_\sigma(k) - \mu_\sigma]}} \qquad (3)$$

of each spin component σ. At each temperature T and density ρ_σ, a self-consistent procedure has to be performed to compute the chemical potential of each species, μ_σ, from the normalization condition:

$$\rho_\sigma = \sum_k n_\sigma(k, T). \tag{4}$$

Once the momentum distribution is determined, the entropy per particle of the system is given by:

$$s(\rho_\uparrow, \rho_\downarrow, T) = \frac{1}{\rho} \sum_{\sigma,k} \{n_\sigma(k, T) \ln n_\sigma(k, T) + [1 - n_\sigma(k, T)] \ln[1 - n_\sigma(k, T)]\}. \tag{5}$$

From the internal energy and the entropy one can readily compute the free energy per particle, $f(\rho_\uparrow, \rho_\downarrow, T) = e(\rho_\uparrow, \rho_\downarrow, T) - Ts(\rho_\uparrow, \rho_\downarrow, T)$, and, from this, the inverse magnetic susceptibility is given by:

$$\frac{1}{\chi} = \frac{1}{\mu^2 \rho} \left(\frac{\partial^2 f}{\partial \Delta^2}\right)_{\Delta=0}, \tag{6}$$

with μ, the magnetic moment of the neutron.

An important quantity for our analysis is the single-particle (sp) energy:

$$\varepsilon_\sigma(k) = \frac{\delta e}{\delta n_\sigma(k)} = \frac{\hbar^2 k^2}{2m} + U_\sigma(k), \tag{7}$$

where $U_\sigma(k)$ is the sp potential. Note that in contrast to the sp spectrum $\epsilon_\sigma(k)$ appearing in Eq. (3), this sp energy contains the rearrangement effects. The momentum dependence of the sp potentials can be characterized by the effective mass $m^*_\sigma(k)$:

$$\frac{m^*_\sigma(k)}{m} = \left[\frac{m}{\hbar^2 k'} \frac{d\varepsilon_\sigma}{dk'}\right]^{-1}\Bigg|_{k'=k} = \left[1 + \frac{m}{\hbar^2 k'} \frac{dU_\sigma}{dk'}\right]^{-1}\Bigg|_{k'=k}. \tag{8}$$

Let us start our analysis with the properties of the sp potential, $U(k)$. Figure 1 reports the sp potential obtained from D1P for neutrons with spin up at saturation density in a nonpolarized system ($\Delta = 0$) and in a fully polarized one ($\Delta = 1$) for $T = 0$ (left panels) and 40 MeV (right panels). The splitting due to the different spin polarizations can be understood in terms of both the dependence of the sp potentials in phase space and the dependence on spin of the effective *NN* interaction [13]. Moreover, the increase of temperature makes the sp potentials slightly less attractive in both cases due to the dependence on the transferred momentum of the two-body matrix element which, at finite temperature, is explored in a wider range of momenta. In spite of the fact that the effective mass depends on momentum, for low enough values of k the sp potential can be casted in a quadratic form:

$$U^{(2)}_\sigma(k) = U_\sigma(0) + \left[\frac{1}{2k} \frac{dU_\sigma(k)}{dk}\right]\Bigg|_{k_F^\sigma} k^2, \tag{9}$$

in which the derivative $dU_\sigma(k)/dk$ and the k-independent term $U_\sigma(0)$ are determined from the full sp spectrum at the corresponding temperature. Figure 1 indeed illustrates that, for the density and polarization under study, this quadratic approximation (dotted lines) is very close to the full k-dependent $U_\sigma(k)$ sp potential (full lines), at least for momenta up to k_F^σ. The approximation of Eq. (9) is devised to reproduce the slope of the spectrum at k_F together with its value at $k = 0$. The choice of the Fermi momentum k_F^σ is related to the fact that the most relevant changes in the occupation of the sp states, when changing ρ and T, essentially involve the neighboring region of the Fermi surface. The quadratic approximation of Eq. (9) leads to an approximated form for the sp energies in terms of the effective mass at the Fermi surface and a k-independent term of $U^{(2)}_\sigma(k)$:

$$\epsilon_\sigma(k, T) \approx \frac{\hbar^2 k^2}{2m^*_\sigma(k_F^\sigma, T)} + U_\sigma(k = 0, T). \tag{10}$$

Finally, let us note that a similar picture is obtained for the other Gogny parametrizations.

In Fig. 2 we report the ratio $\chi_{\rm Free}/\chi$ (where $\chi_{\rm Free}$ is the magnetic susceptibility of the free Fermi sea at the corresponding temperature) as a function of density. Results for the D1P (D1) forces are shown on the left (right) panels

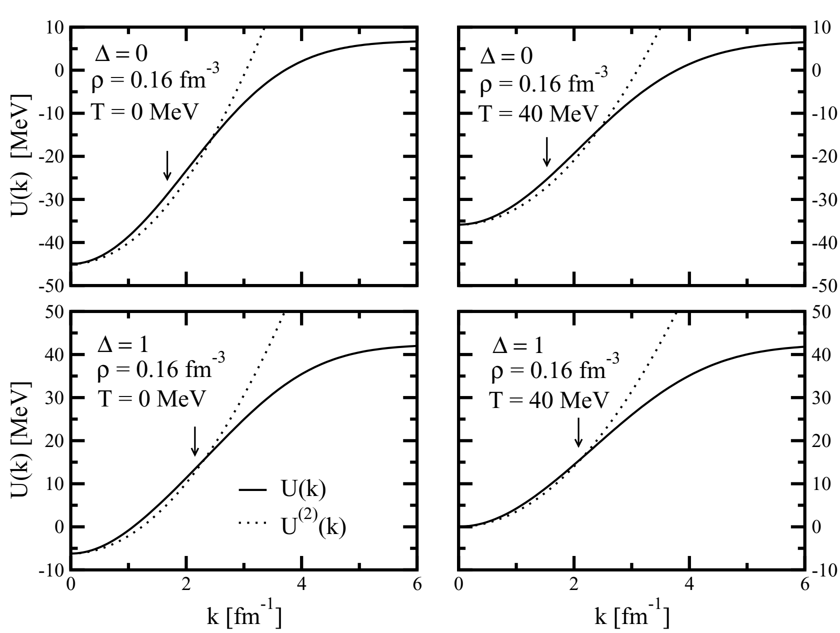

FIG. 1. Single-particle potential $U(k)$ for up neutrons in nonpolarized (top panels) and totally polarized (bottom panels) neutron matter at $\rho = 0.16$ fm^{-3} for $T = 0$ (left panels) and $T = 40$ MeV (right panels) within the D1P parametrization. $U(k)$ and its quadratic approximation $U^{(2)}$ are displayed in full and dashed lines, respectively. The arrows denote the value of $k_F^\uparrow$.

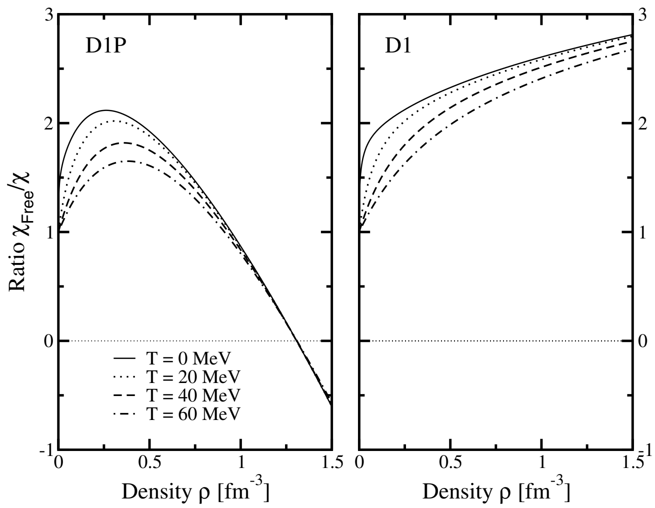

FIG. 2. Ratio between the inverse magnetic susceptibility of interacting neutron matter and that of the corresponding free Fermi sea as a function of density for several temperatures.

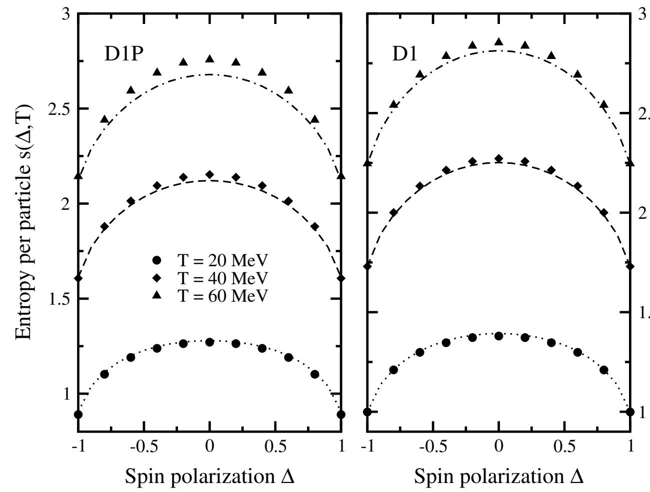

FIG. 3. Entropy per particle as a function of the spin polarization at $\rho = 0.16$ fm^{-3} and several temperatures. The exact Hartree-Fock values are depicted with circles, diamonds, and triangles, whereas the lines illustrate the results obtained from a quadratic approximation of the single-particle spectrum [Eq. (10)].

of the figure at several temperatures. A different qualitative behavior for the two parametrizations is clearly observed. On the one hand, the D1P parametrization leads, similarly to what was found in the case of the Skyrme interaction [12], to a ferromagnetic phase transition, signaled by a vanishing ratio $\chi_{\rm Free}/\chi$. The critical density of this transition at $T = 0$ MeV is $\rho_c = 1.31$ fm^{-3}, a much larger value than the ones obtained with Skyrme interactions, all of which were systematically below 1 fm^{-3} (see Table II of Ref. [12]). Moreover, and even though it is difficult to distinguish this in the figure, the critical density slightly increases with temperature. This is in agreement with the intuitive idea that thermal disorder increases the onset density of ferromagnetism, but it is opposite to what was found with Skyrme forces. On the other hand, no trace of a ferromagnetic transition is seen for D1 at any density or temperature. This behavior is very similar to what was found in a BHF calculation with realistic interactions [13]. In addition, the temperature dependence (the higher the temperature, the lower the ratio becomes) is also in agreement with such microscopic calculations.

In this context, it is interesting to study the behavior of the entropy. To this end, plots of the entropy per particle as a function of spin polarization at a fixed value of density ($\rho = \rho_0$) and several temperatures are shown in Fig. 3. We consider the entropy given by Eq. (5) computed with both (i) the full momentum-dependent spectrum $\epsilon_\sigma(k)$ (symbols) and (ii) the quadratic approximation to $\epsilon_\sigma(k)$ of Eq. (9) (lines). In the two cases the exact chemical potential determined from the normalization of the momentum distribution, Eq. (4), are used. The agreement between the values of the entropy coming from the exact and the approximated sp energies is rather satisfactory, the most significant discrepancies not being larger than a 3%. Both the D1 and the D1P parametrizations give rise to similar results: the entropy per particle is symmetric with respect to the nonpolarized state and shows a maximum at zero polarization. In addition, the values of the entropy increase with temperature, which is again an indication that the entropy per particle behaves as naively expected. This so-considered "natural" behavior was also found in the BHF analysis of Ref. [13]. In contrast, for Skyrme forces the entropy per particle of the polarized phase is seen to be higher than the nonpolarized one above a certain density [12]. This defines a kind of "critical" density, which is smaller than ρ_0 for most Skyrme parametrizations. Such a nonintuitive behavior of the entropy as a function of the polarization can be related to the dependence of the entropy on the effective mass, and a condition for the effective masses can then be derived:

$$\frac{m^*(\rho, \Delta = 1)}{m^*(\rho, \Delta = 0)} < 2^{2/3}, \qquad (11)$$

if the quantity $s(\rho, \Delta = 1) - s(\rho, \Delta = 0)$ has to be always negative. Most of the Skyrme forces analyzed in Ref. [12] violate this criterion and thus lead to an "anomalous" temperature dependence for the onset density of ferromagnetism.

Notice, however, that for Skyrme forces the effective mass is momentum and temperature independent. This is actually not the case of neutron matter described by means of Gogny forces: the effective masses do depend on both momentum and temperature. Nevertheless, as it has been previously discussed, the sp spectrum and the entropy per particle are correctly described by a quadratical momentum dependence, with an effective mass calculated at $k = k_F^\sigma$ for each temperature. Within this approximation, one can prove that the criterion of Eq. (11) is still valid at each temperature. We have checked that both the D1 and the D1P forces fulfill the criterion of Eq. (11) in a vast region of the (T, ρ) parameter space, which includes both the classical and degenerate limits. Thus, as it has been previously observed, we do not expect that the ferromagnetic transition (only present for D1P) has an anomalous thermal behavior.

One of the questions that needs for a closer insight is why neutron matter described with Gogny forces does not present a ferromagnetic instability or why it is present only at very high densities. Intuitively, one expects that the instability will be related to the zero-range term. This can be understood just by taking into account that the pure zero-range term involves only S-wave contributions and, therefore, due to Pauli principle, it is not possible to have a couple of neutrons with the same spin interacting through the contact term. Therefore there is no contribution to the total energy per particle of fully polarized matter from a zero-range term, whereas for nonpolarized neutron matter this term is both strongly density dependent and repulsive, thus contributing to the spin instability. In contrast, the behavior of finite-range terms, both direct and exchange, has the opposite dependence with polarization: for a given density, the higher the polarization, the higher these contributions become. In other words, if only finite-range terms were considered, the nonpolarized system would be energetically favored. In addition, the density dependence of these contributions is softer than that of the zero-range term. Therefore, the competition between zero and finite-range effects is resolved in favor of the first one at high values of the density. From this, it is also obvious to understand why no transition is found for the D1 and the D1S parametrizations. In these two cases, the fitting parameters are such that the zero-range term does not contribute for any polarization, because $x_0 = 1$. The zero-range term is therefore not active in neutron matter, whereas the direct and exchange parts of the finite-range terms behave as mentioned, energetically favoring the nonpolarized case at all densities. In contrast, for the D1P force $x_0 \neq 1$, and the zero-range term is active. As a consequence, the instability arises once we come across a high enough critical density.

In this work, we have studied the properties of polarized neutron matter with neutrons interacting through Gogny forces, both at zero and finite temperature. The results show two different qualitative behaviors for the two parametrizations under study. On the one hand, the D1P exhibits a ferromagnetic transition at what can be considered a very high density, $\rho_c \sim 1.31$ fm^{-3}. On the other hand, no sign of a ferromagnetic transition is found for the D1 parametrization at any density or temperature. The cause of these two different behaviors is related to the zero-range term, which contributes for D1P but not for D1. Finally, we have checked that the temperature dependence of the ferromagnetic transition with D1P is the one expected by intuition, in agreement with the fact that the criterion of Eq. (11) for the effective masses is respected in a wide range of densities and temperatures.

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