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Excitation energy dependence of the symmetry energy of finite nuclei

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A finite-range density and momentum-dependent effective interaction is used to calculate the density and temperature dependence of the symmetry energy coefficient $C_{\rm sym}(\rho, T)$ of infinite nuclear matter. This symmetry energy is then used in the local density approximation to evaluate the excitation energy dependence of the symmetry energy coefficient of finite nuclei in a microcanonical formulation that accounts for thermal and expansion effects. The results are in good harmony with the recently reported experimental data from energetic nucleus-nucleus collisions.

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The symmetry energy $E_{\rm sym}(\rho, T)$ represents with a very good accuracy the energy cost per nucleon to convert all the protons to neutrons in symmetric nuclear matter at the density ρ and temperature T. Study of its density and energy dependence is of utmost contemporary importance. It is essential not only for understanding many aspects of exotic nuclear physics induced by collisions of radioactive nuclei but also a number of important issues in the astrophysical scenario like supernovae explosions [1], explosive nucleosynthesis, cooling of protoneutron stars [2], and abundances of relatively heavier elements. Even on a more mundane level, the neutron skin thickness of heavier nuclei is intimately correlated to the density derivative of the symmetry energy [3,4] as it reflects the pressure difference on the neutrons and protons.

In addition to a kinetic contribution, the symmetry energy has a contribution arising from the difference between the neutron-proton (n-p) interaction and that between like pairs (n-n, p-p). Given an interaction, it is straightforward to calculate the symmetry energy at different densities and temperatures for infinite matter. There have been several attempts in this direction. Calculations of the nuclear equation of state (EOS) in the microscopic framework using both bare [4,5] and effective interactions [6,7] have been done. The outcome of these calculations for the symmetry energy is similar ($\sim$ 30–35 MeV) at saturation density but is considerably different at subnormal as well as at supranormal densities where the available data from experiment to confront with theory are more scarce.

Laboratory information on the density dependence of the symmetry energy can be obtained from energetic nucleus-nucleus collision experiments. At densities above the normal, it can be inferred from the comparison of theoretical predictions with experimental data on the differential flow of neutrons and protons, from the π^-/π^+, K^0/K^+ ratios, etc. [8]. At subnormal densities, disassembly of a hot expanded nucleus offers the best tool to study the characteristics of the symmetry energy [9,10]. Experimental data related to isotopic distributions, isospin diffusion, and isoscaling try to constrain the density dependence in the subnormal region, but there is still considerable uncertainty.

A nucleus expands with excitation with increasing temperature in general. This implies an excitation energy dependence of the symmetry energy because of the density change. Experimentally, this information is generally extracted [10] from the fit of the experimental isotopic distributions at different excitation energies to those obtained from a model for multifragmentation like the statistical multifragmentation model (SMM) [11] or from isoscaling [12]. Currently, calculations for the energy dependence of the symmetry energy are available for infinite matter, but no microscopic calculation has yet been performed for the energy dependence of finite nuclei. The main purpose of this communication is to report such a calculation.

For an expanding system pursuing the equilibrium configuration, the surface diffuseness is likely to play an important role [13], thus a zero-range interaction like the Skyrme force widely used to explore the nuclear ground-state properties may not be the most adequate for generating such a density profile. It is further noted that a constrained expanded system in a Thomas-Fermi approach may lead to numerical instabilities [14] and the gradient (surface) terms in the energy density functional were replaced with a suitable Yukawa interaction [15]. We have therefore chosen the modified Seyler-Blanchard (SBM) effective interaction [16] for our microscopic calculation in the finite temperature Thomas-Fermi formulation. This interaction is of finite range and momentum and density dependent. The interaction reproduces quite satisfactorily the ground-state bulk properties of nuclei over the whole periodic table for $A > 16$. The EOS calculated [17] with this interaction agrees very favorably with those obtained microscopically with a realistic interaction in a variational approach [18,19]. The SBM interaction is given by

$$v(\mathbf{r}_1, \mathbf{r}_2, p, \rho) = -C_{l,u}\left\{1 - \frac{p^2}{b^2} - d^2\left[\rho(\mathbf{r}_1) + \rho(\mathbf{r}_2)\right]^n\right\} \times \frac{\exp(-r/a)}{(r/a)}. \quad (1)$$

Here $r = |\mathbf{r}_1 - \mathbf{r}_2|$ and $p = |\mathbf{p}_1 - \mathbf{p}_2|$ are the relative separation of the interacting nucleons in coordinate and momentum space, $\rho(\mathbf{r}_1)$ and $\rho(\mathbf{r}_2)$ are the densities at the sites of the two interacting nucleons, and C_l and C_u are the strengths for like pair and unlike pair nucleon-nucleon interaction. The density exponent n controls the stiffness of the nuclear EOS. The values of the parameters C_l, C_u, b, d, a, and n are given in Refs. [16,20]. The energy per nucleon $e(\rho, T)$ calculated with this interaction for infinite nuclear matter is given by

$$e(\rho, T) = \frac{1}{\rho}\sum_\tau \rho_\tau \left[T J_{3/2}(\eta_\tau)/J_{1/2}(\eta_\tau)\left(1 - m_\tau^k V_\tau^1\right) + \frac{1}{2}V_\tau^0 \right], \tag{2}$$

where τ refers to the isospin index (n, p). Here $J_q(\eta)$ are the Fermi integrals, m_τ^k the effective k mass of the nucleon, and η_τ the fugacity given by

$$\eta_\tau = \left(\mu_\tau - V_\tau^0 - V_\tau^2\right)/T, \tag{3}$$

with μ_τ as the nucleon chemical potential. In Eqs. (2) and (3), the V_τ's are the different components of the single-particle potential whose expressions can be found in Ref. [16].

The symmetry energy per nucleon of asymmetric nuclear matter with asymmetry $X = (\rho_n - \rho_p)/\rho$ is

$$e_{\text{sym}}(\rho, T, X) = e(\rho, T, X) - e(\rho, T, X = 0). \tag{4}$$

It can be written as

$$e_{\text{sym}}(\rho, T, X) = C_{\text{sym}}(\rho, T)X^2 + \mathcal{O}(X^4). \tag{5}$$

The terms beyond X^2 are negligible over a considerable range of X (as involved in finite nuclei). The symmetry energy coefficient C_{sym} is obtained from

$$C_{\text{sym}}(\rho, T) = \frac{1}{2}\frac{\partial^2}{\partial X^2} e_{\text{sym}}(\rho, T, X)|_{X=0}. \tag{6}$$

In the top panel of Fig. 1, the density dependence of C_{sym} at $T = 0$ is displayed in the density region $0.1 \leqslant \rho/\rho_0 \leqslant 1.0$

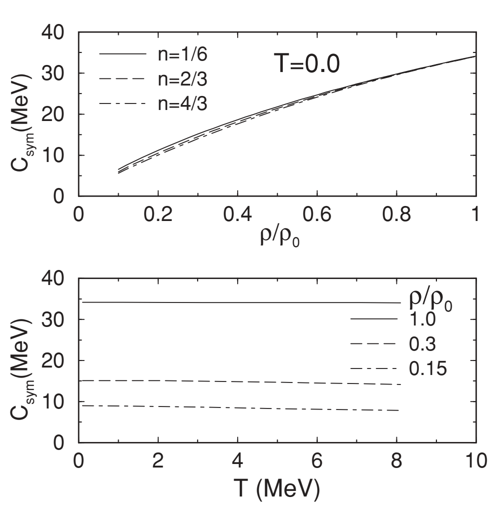

FIG. 1. Symmetry energy coefficient of infinite nuclear matter as a function of density for different variants of the SBM interaction (see text) at $T = 0.0$ MeV (top panel). In the bottom panel the temperature dependence of C_{sym} at several fixed densities is shown for $n = 1/6$.

for three variants of the SBM interaction with density exponents $n = 1/6, 2/3$, and $4/3$, in increasing order of the stiffness of the nuclear EOS. The values of the nuclear incompressibility with these three interactions are $K_\infty = 238, 300$, and 380 MeV. The symmetry energy coefficient calculated with them can be very nicely represented by $C_{\text{sym}}(\rho) \sim C_{\text{sym}}(\rho_0)(\rho/\rho_0)^\gamma$ with $C_{\text{sym}}(\rho_0) = 34.0$ MeV and $\gamma = 0.65, 0.68$, and 0.70, respectively. The value of the exponent γ appears not very sensitive to the nuclear EOS. The agreement of the functional form of the symmetry energy coefficient and the value of γ with those obtained recently [10] from experimental data ($\gamma \simeq 0.69$) is excellent. The nuclear incompressibility with $n = 1/6$ compares very well with the presently accepted value of $K_\infty \sim 230$ MeV; all the subsequent calculations are therefore reported for the SBM interaction with $n = 1/6$. In the bottom panel of Fig. 1, the temperature dependence of the symmetry energy coefficient at different densities ($\rho/\rho_0 = 0.3, 0.6$, and 1.0) is shown. At a fixed density, dependence on temperature is not much evident.

In calculating the excitation energy or density dependence of the symmetry energy coefficient of a finite nucleus with N neutrons and Z protons ($A = N + Z$) with excitation energy E^*, we remind ourselves that the hot nucleus prepared in the laboratory in energetic nuclear collisions is an isolated system with a fixed total excitation E^* and thus should be described by microcanonical thermodynamics [21]. Left to itself, the system expands due to unbalanced thermal pressure in search of maximal entropy where the total pressure vanishes and the system is in equilibrium in a bloated mononuclear configuration with the same excitation energy. The expansion is simulated through a self-similar scaling approximation for the density:

$$\rho_\lambda(r) = \lambda^3\rho(\lambda r), \tag{7}$$

where λ is the scaling parameter ($0 < \lambda \leqslant 1$) and $\rho(r)$ is the base density profile. The base density, employing the SBM interaction, is generated in the self-consistent Thomas-Fermi framework. The subtraction scheme [22,23] is used to render the density profile independent of the box size with an effective temperature T chosen so as to give the maximum entropy for the given excitation E^*. The excitation energy is calculated as

$$E^* = E(\lambda_{\text{eq}}, T) - E(\lambda = 1, T = 0), \tag{8}$$

where λ_{eq} is the scaling parameter for the equilibrium density profile corresponding to this excitation.

The SBM interaction, being momentum dependent, renormalizes the bare nucleon mass m to an effective k mass. A frequency dependent mass factor m_ω/m is further phenomenologically incorporated [24,25] in the calculation. It is very relevant in the present context; the ω mass m_ω/m is generally larger than unity, it has the effect of bringing down the excited states from higher to lower energy near the Fermi surface, thus increasing the many-body density of states at low excitations that allow comparatively more accommodation of entropy at a given excitation energy. Details on the generation of the equilibrium density profile, effective temperature, frequency dependent mass, etc., as employed in this calculation, are given in Refs. [26,27].

Once the equilibrium density $\rho(r)$ of a nucleus at excitation E^* is known, the symmetry energy is calculated in the local density approximation as

$$C_{\text{sym}}(E^*)\left(\frac{N-Z}{A}\right)^2 = \frac{1}{A}\int \rho(r)C^{l}_{\text{sym}}[\rho(r),T] \times \left[\frac{\rho_n(r)-\rho_p(r)}{\rho(r)}\right]^2 d\mathbf{r}. \quad (9)$$

Here $C^{l}_{\text{sym}}[\rho(r),T]$ is the symmetry energy coefficient at temperature T of infinite nuclear matter at a value of the local density $\rho(r)$. The local isospin density is given by $\rho_n(r)-\rho_p(r)$. It may be mentioned that both the volume and the surface terms in the liquid drop type mass formula are asymmetry dependent [28]. The symmetry energy coefficient $C_{\text{sym}}(E^*)$ defined through Eq. (9) may therefore be taken as an effective parameter incorporating both the volume and surface contributions from asymmetry and may be written as

$$C_{\text{sym}} = C^{\text{vol}}_{\text{sym}} - C^{\text{surf}}_{\text{sym}}/A^{1/3}. \quad (10)$$

In a microcanonical formulation, it has been found that the equilibrium density at a given excitation depends on the mass and asymmetry of the nucleus concerned [26]. In investigating the excitation energy dependence of the symmetry energy coefficient, it would then be worthwhile to investigate its system dependence. We have therefore chosen three systems, two isobars of $A = 150$, namely Cs and Sm, and a lighter system ^{40}S. In the bottom panel of Fig. 2, the coefficient C_{sym} as calculated using Eq. (9) is displayed as a function of E^*/A for the isobars of $A = 150$. It is found that at a fixed excitation, including the ground state, C_{sym} is somewhat sensitive to the asymmetry of the nucleus; it increases with increasing proton fraction of the system. This is at variance with the expectation from the liquid-drop formula where the effective symmetry energy coefficient given by Eq. (10) is independent of charge for a given mass. This may be understood from the fact that the parameters of the liquid-drop formula are based on a global fit to the binding energies of nuclei over the entire periodic table around the stability line, excluding the very light ones. Here, the Coulomb energy $(= a_c Z^2/A^{1/3})$ coefficient a_c is taken as a constant for the whole mass range. In our calculations for isobars, it is seen that with increasing charge as the proton distribution is pushed outward, a_c decreases and C_{sym} increases. This effectively explains the isobaric variation of binding energies.

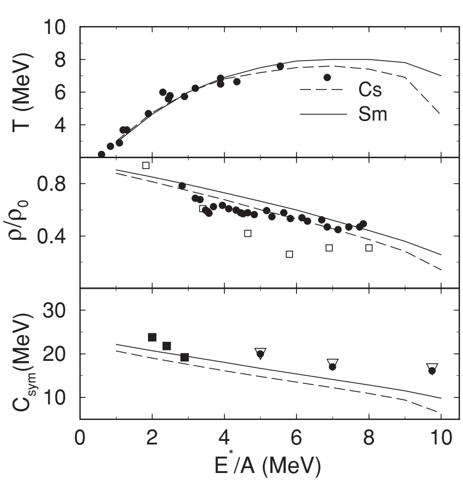

FIG. 2. The equilibrium temperature (top panel), equilibrium central density (middle panel) and the symmetry energy coefficient (bottom panel) as a function of excitation energy for the $A = 150$ isobars (Cs and Sm). The experimental data for T are from Ref. [33], those for ρ/ρ_0 are from Refs. [30] (circles) and [31,32] (squares), and those for C_{sym} are from Refs. [10] (inverted triangles and circles) and [29] (squares).

Some representative experimental results [10] for C_{sym} obtained from the analysis of isoscaling data for lighter fragments are shown in the bottom panel of Fig. 2 as inverted open triangles and solid circles. At relatively lower excitations ($E^*/A \sim$ 2–3 MeV), experimental data from isoscaling for heavier fragments [29] are also shown as solid squares. Our calculated energy dependence of the symmetry energy coefficient agrees favorably with these experimental findings, the calculated values are somewhat lower. The variation of C_{sym} with excitation stems basically from the changing equilibrium density with excitation energy, which is shown in the middle panel of the figure for the two systems. Because the density has a profile, the choice of a single value of the density leaves room for ambiguity; we have taken $\rho_c/\rho_{c,0}$ as the measure of the density where ρ_c is the central equilibrium density at the relevant excitation and $\rho_{c,0}$ is the ground-state central density. The theoretically calculated results are in nice agreement with the experimental data from Ref. [30] (solid circles of the middle panel) derived from the analysis of caloric curve measurements. The data obtained from Coulomb barrier systematics [31,32] are shown with open squares. For completeness, in the top panel, the caloric curves along with the experimental data compiled by Cibor *et al*. [33] are also displayed. It is seen that the plateau of the caloric curve shows little sensitivity to the asymmetry of the nucleus; this is consistent with the recent calculations of Hoel *et al*. [34].

The mass dependence of the effective symmetry energy $C_{\text{sym}}(E^*/A)$ is displayed in the bottom panel of Fig. 3. The calculations are done for ^{150}Sm and ^{40}S, both having nearly

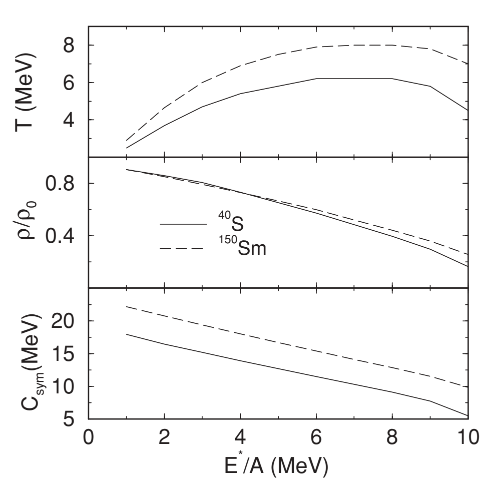

FIG. 3. Same as in Fig. 2 for the systems ^{40}S and ^{150}Sm to show the mass dependence.

the same asymmetry. The reduction in C_{sym} for ^{40}S can be understood from the role played by the surface asymmetry as given by Eq. (10). For completeness, the equilibrium central densities and temperatures as a function of E^*/A are also shown in the middle and top panels of the figure, respectively.

To conclude, calculations on the density and excitation energy dependence of the symmetry energy of finite nuclei have been reported in this communication in a microscopic formulation within the microcanonical framework. It has been stressed in a recent calculation [13] that the surface diffuseness of the expanded mononuclear system plays a key role in making the system softer toward instability, limiting the maximum excitation energy a mononucleus can hold to $\sim$5 MeV/nucleon with free variation of the surface diffuseness. Our model calculation does not leave any room for free variation of the surface diffuseness. It is determined in two stages: the increased diffuseness of the base density profile of the hot nucleus over that of the ground state and then its subsequent stretching from the self-similar expansion. The surface diffuseness so obtained is found to be somewhat less than that reported in Refs. [13,34]. Exploring the weakening of the symmetry energy with excitation using free variation of the surface diffuseness would be interesting to look into. In the present calculation, the density dependence of $C_{\mathrm{sym}}(\rho)$ of infinite nuclear matter is found out to be $\sim(\rho/\rho_0)^\gamma$ with $\gamma = 0.65$, very close to the recently extracted experimental value of $\gamma \simeq 0.69$ [10]. At constant density, $C_{\mathrm{sym}}(E^*)$ of infinite nuclear matter is practically constant. For finite nuclei, however, density changes with excitation; their excitation energy dependence can be well represented by $C_{\mathrm{sym}}(E^*) \simeq C_{\mathrm{sym}}(E^* = 0)(1 - \alpha E^*)$ with $\alpha \simeq 0.06$. These are in good consonance with the experimental data obtained from nuclear multifragmentation.

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