

Coincident brane nucleation and the neutralization of Λ

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Nucleation of branes by a four-form field has recently been considered in string motivated scenarios for the neutralization of the cosmological constant. An interesting question in this context is whether the nucleation of stacks of coincident branes is possible, and if so, at what rate does it proceed. Feng et al. have suggested that, at high ambient de Sitter temperature, the rate may be strongly enhanced, due to large degeneracy factors associated with the number of light species living on the worldsheet. This might facilitate the quick relaxation from a large effective cosmological constant down to the observed value. Here, we analyze this possibility in some detail. In four dimensions, and after the moduli are stabilized, branes interact via repulsive long range forces. Because of that, the Coleman–de Luccia (CdL) instanton for coincident brane nucleation may not exist, unless there is some short range interaction that keeps the branes together. If the CdL instanton exists, we find that the degeneracy factor depends only mildly on the ambient de Sitter temperature, and does not switch off even in the case of tunneling from flat space. This would result in catastrophic decay of the present vacuum. If, on the contrary, the CdL instanton does not exist, coincident brane nucleation may still proceed through a “static” instanton, representing pair creation of critical bubbles—a process somewhat analogous to thermal activation in flat space. In that case, the branes may stick together due to thermal symmetry restoration, and the pair creation rate depends exponentially on the ambient de Sitter temperature, switching off sharply as the temperature approaches zero. Such a static instanton may be well suited for the “saltatory” relaxation scenario proposed by Feng et al.

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I. INTRODUCTION

It has long been recognized that the effective cosmological constant Λ_{eff} may have contributions from a four-form field F , and that in such case

$$\Lambda_{eff} = (F^2/2) + \Lambda \quad (1)$$

may vary in space and time due to brane nucleation events. This has led to various proposals for solving the cosmological constant problem, starting with the pioneering work of Brown and Teitelboim [1]. These authors considered a cosmological scenario where Λ_{eff} is initially very large and positive, due to a large F^2 term. The additive constant Λ in (1) is assumed to be negative, but not fine-tuned in any way, so its absolute value is expected to be of the order of some cutoff scale to the fourth power. During the cosmological evolution, Λ_{eff} is “neutralized” through successive nucleation of closed 2-branes (charged with respect to the form field), which decrease the value of F , until eventually Λ_{eff} is relaxed down to the small observed value, Λ_{obs} .

One problem with the original scenario is that neutralization must proceed in very small steps, so that any initially large Λ_{eff} can be brought to Λ_{obs} without overshooting into negative values. For that, the charge of the branes should be tiny, ensuring that $\Delta\Lambda_{eff} \lesssim \Lambda_{obs}$ at each step. Also, the nucleation rate must be very small, or else the present vacuum

would quickly decay. These two constraints make the relaxation process extremely slow on a cosmological time scale. Meanwhile, ordinary matter in the universe is exponentially diluted by the quasi–de Sitter expansion, resulting in a disappointing empty universe.

Recently, Feng, March-Russell, Sethi, and Wilczek (FMSW) [2] have suggested that nucleation of coincident branes may offer a solution to the “empty universe” problem. Their proposal can be summarized as follows. In the context of M theory, a stack of k coincident D-branes supports a number of low energy degrees of freedom, corresponding to a $U(k)$ super Yang-Mills (SYM) theory living on the worldsheet. Consequently, the nucleation rate of coincident branes should be accompanied by large degeneracy factors, and could in principle be enhanced with respect to the nucleation of single branes. The charge of a stack of branes can be very large even if the individual charges are small, facilitating quick jumps from Λ_{eff} to Λ_{obs} . In this way, neutralization might proceed very rapidly, perhaps in just a few “multiple” steps of the right size. Finally, the stability of the present vacuum could be due to gravitational suppression of the nucleation rate [1,3].

FMSW argued, rather heuristically, that the nucleation rate of coincident branes should be enhanced by a factor of the form

$$D \sim e^S, \quad (2)$$

where S is the entropy of the worldsheet SYM fields. This entropy was estimated through simple thermodynamic arguments, as

$$S \sim g_* R^2 T^2, \quad (3)$$

where g_* is the effective number of worldsheet field degrees of freedom, and R is the size of the brane at the time of nucleation. However it remained unclear in [2] which temperature T should be used for the worldsheet degrees of freedom. Brane nucleation takes place in an ambient de Sitter (dS) space characterized by a Gibbons-Hawking temperature $T_o \propto \Lambda_{eff}^{1/2}$. The region inside the closed brane has a smaller value of the effective cosmological constant, and is therefore characterized by a smaller temperature T_i . Feng et al. considered two alternative possibilities for the temperature of the worldsheet degrees of freedom: $T_1 \sim T_o$ and $T_2 \sim (T_o T_i)^{1/2}$. The proposed enhancement of the nucleation rate and the resulting cosmological scenarios are quite different in both cases, and therefore it seems important to try and clarify the issue of which temperature is the relevant one.

The purpose of this paper is to present a more formal derivation of the nucleation rate corresponding to multiple brane nucleation. As we shall see, the temperature relevant for the worldsheet degrees of freedom is in fact determined by the internal geometry of the worldsheet [4]. For the Coleman–de Luccia (CdL) instanton, this worldsheet is a 2+1 dimensional de Sitter space of radius R , and the corresponding temperature is $T \sim R^{-1}$. When substituted into the naive expression (3), this leads to $S \sim g_*$, independent of R (and hence of the ambient dS temperatures). As we shall see, the actual result has a certain dependence on R due to the anomalous infrared behavior of light fields in the lower dimensional de Sitter space, but this results only in a rather mild dependence on the ambient dS temperatures.

We shall also see that de Sitter space allows for a “static” instanton that may be quite relevant to the nucleation of coincident branes. This is analogous to the instanton for thermal activation in flat space. It has a higher Euclidean action than the CdL solution, and hence (ignoring the degeneracy factor) it seems to represent a subdominant channel of decay. However, we shall argue that, depending on the short distance behavior of the interactions amongst the branes, the CdL instanton for coincident brane nucleation may simply not exist, and in this situation the static instanton may be the relevant one. In several respects, the static instanton appears to be better suited to the neutralization scenario proposed by Feng et al. than the CdL one.

The paper is organized as follows. In Sec. II we review different proposals for neutralization of Λ_{eff} via brane nucleation. Section III contains a discussion of coincident branes in 4 spacetime dimensions. These are obtained from dimensional reduction of type IIA supergravity in ten dimensions. In the 4 dimensional picture, the gravitational and four-form forces are both repulsive. However, the two are exactly balanced by the attractive force mediated by the scalar dilaton. In Secs. IV and V we discuss the stabilization of the dilaton, which is required in a more realistic scenario. After the dilaton acquires a mass, the remaining long range forces are

repulsive, rendering the stack of coincident branes unstable, or metastable at best. This has important implications, since the instanton for nucleation of coincident branes will only exist provided that some mechanism causes an attractive interbrane force at short distances.

Section IV contains a description of the CdL instanton for nucleation of coincident branes, highlighting a few limiting cases of interest. In Sec. VII we discuss the corresponding degeneracy factor in the nucleation rate, and we show that its dependence on the ambient de Sitter temperatures is rather mild. We also include a heuristic interpretation of this result based on the observation that the relevant temperature for the worldsheet degrees of freedom is determined by the inverse of the radius of the instanton. Implications for the scenario of [2] are briefly discussed.

Section VIII is devoted to a study of the “static” instanton, where the worldsheet has the topology $S^2 \times S^1$, and where the intrinsic temperature is comparable to T_o . In this case, the dependence of the nucleation rate on the ambient dS temperature is exponential. Coincident brane nucleation can be unsuppressed at large T_o but strongly suppressed at present. Our conclusions are summarized in Sec. IX. Some technical discussions are left to the Appendices.

To conclude this Introduction, a disclaimer may be useful. For most of the paper, we shall work directly in four dimensions, and our discussion will be certainly less than rigorous from the string theory point of view. In particular, we shall model the degrees of freedom of a stack of k coincident 2-branes by a weakly coupled $U(k)$ gauge theory on the worldsheet. This may or may not correspond to a true dimensional reduction from M theory, but it should at least represent some of the broad features of the degeneracy factors.

II. NEUTRALIZATION VERSUS RANDOMIZATION

In four dimensions, a four-form can be written as $\mathcal{F}_{\mu\nu\rho\sigma} = F \sqrt{-g} \epsilon_{\mu\nu\rho\sigma}$ where g is the determinant of the metric and ϵ is the Levi-Civita symbol. This field has no propagating degrees of freedom, since in the absence of sources the equation of motion $d^* \mathcal{F} = 0$ implies that F is a constant. This simply gives a contribution to the effective cosmological constant that gets added to the true cosmological constant, or vacuum energy density Λ ,

$$\Lambda_{eff} = \frac{F^2}{2} + \Lambda. \quad (4)$$

Four-forms may couple to “charged” 2-dimensional extended sources, or 2-branes, through a term of the form

$$q \int_{\Sigma} \mathcal{A}, \quad (5)$$

where the integral is over the worldsheet of the extended object and \mathcal{A} is the 3-form potential ($\mathcal{F} = d\mathcal{A}$). In this case, F changes by

$$\Delta F = q \quad (6)$$

across a brane of charge q . Consequently, F can decay through nucleation of closed spherical branes. The process is

analogous to pair creation in the presence of an electric field, and very similar to false vacuum decay in field theory [5]. The closed brane is the boundary of a newly formed “true” vacuum bubble, where the field strength differs from the original value by the amount (6). After nucleation, the radius of the bubble grows with constant proper acceleration, and the volume occupied by the new phase keeps increasing. Further nucleation events take place in the region with a low F , lowering Λ_{eff} even further. In the absence of gravity, this “neutralization” would proceed as long as $F > q/2$, wiping out any large initial value of F^2 and leaving us with a large negative cosmological constant $\Lambda_{eff}^{final} \sim \Lambda + O(q^2)$. Of course, this is not what we want.

Gravitational effects can improve the picture dramatically [1]. In particular, tunneling from flat or anti-de Sitter space (with a negative cosmological constant) is forbidden provided that the squared tension of the branes is sufficiently large compared with the jump $\Delta\Lambda_{eff} \sim qF$ in energy density across the brane,

$$\sigma^2 > (4/3)qFM_p^2. \quad (7)$$

Here σ is the brane tension and $M_p^2 = 1/(8\pi G)$ is the square of the reduced Planck mass. This gravitational “shutdown” of brane nucleation could be useful, since an initially large Λ_{eff} may eventually get “neutralized” to a value that is much smaller than Λ in absolute value [1]. Suppose that the true cosmological constant is negative $\Lambda < 0$. As long as $\Lambda_{eff} > 0$, branes keep nucleating. But once a vacuum with $\Lambda_{eff} \leq 0$ is reached, the process stops provided that Eq. (7) is satisfied. After that, the vacuum becomes absolutely stable. Brown and Teitelboim conjectured that we may live in one such vacuum, where the effective cosmological constant is expected to be of the order of the energy density gap between neighboring vacua

$$|\Lambda_{eff}^{final}| \sim F\Delta F \sim q|\Lambda|^{1/2}. \quad (8)$$

In this vacuum, $|F| \sim |\Lambda|^{1/2}$, and the huge negative bare cosmological constant is almost completely cancelled by the F^2 contribution in the final state.

For sufficiently small charge q , Eq. (8) leads to a suppression of $|\Lambda_{eff}^{final}|$ relative to the true vacuum energy $|\Lambda|$, which might be helpful in solving the “old” fine tuning problem of the cosmological constant. Particle physics models suggest $|\Lambda| \gtrsim (TeV)^4$. Hence, we need

$$q \lesssim 10^{-35}(eV)^2, \quad (9)$$

so that the final value $|\Lambda_{eff}^{final}|$ is consistent with the observed value $|\Lambda_{eff}^{obs}| \sim 10^{-11}(eV)^4$. The constraint (9) seems rather demanding, since in the context of supergravity we would expect q to be closer to the Planck scale. This is the so-called “gap problem.” FMSW argued that the smallness of the charge could be due to the wrapping of branes on degenerating cycles in the extra dimensions. A successful implementation of this idea has not yet been presented, but some plausibility arguments have been given in Ref. [2]. Alternatively, in a different context, it has been suggested that branes with

a very tiny charge q may arise due to symmetries of the theory. An explicit example was given in Ref. [6], where the branes are not fundamental objects but domain walls of a broken discrete symmetry. This same symmetry suppresses the coupling of the domain walls to the four-form F without any fine-tuning of parameters (see also Ref. [7] for a fuller discussion).

A more severe problem of the Brown-Teitelboim neutralization scenario is the “empty universe” problem, which we discussed in the Introduction. By combining the condition (9) with the stability condition (7), it can be shown that the time required to reach the value (8) is huge compared with the age of the universe [1]. By the time the effective cosmological constant would be wiped out, all other forms of matter would have also been diluted exponentially, in clear contradiction with observations. Furthermore, the endpoint of neutralization would be a state with vanishing or negative effective cosmological constant, whereas the observed value Λ_{eff}^{obs} is positive.

One way around the empty universe problem is to consider a slightly different scenario, where the effective cosmological constant is “randomized” (rather than neutralized) during inflation. Assume, for simplicity, that the energy scale of inflation is much larger than $\Delta\Lambda_{eff}$. In an inflationary phase, brane nucleation processes may increase as well as decrease the value of F^2 [4]. Thus, Λ_{eff} will randomly fluctuate up and down the ladder as a result of brane nucleation. Inflation is generically eternal to the future and there is an unlimited amount of time available for the randomization process to take place before thermalization [4]. Assume that the tunneling barriers are sufficiently high, so that no nucleation events happen in the last 60 e-foldings of inflation, or during the hot phase after thermalization up to the present time. In this scenario there is no empty universe problem: the local value of Λ_{eff} is decided many e-foldings before the end of inflation, and a wide range of values of Λ_{eff} will be found in distant regions of the universe, separated from each other by distances much larger than the present Hubble radius. Some of these regions will just happen to have a very tiny Λ_{eff} . In combination with anthropic selection effects, this approach may be used to explain the smallness of the observed effective cosmological constant [4,6,8,9]. This may also explain the so-called cosmic time coincidence, or why do we happen to live at the time when an effective cosmological constant starts dominating [4,9,10].

Bousso and Polchinski [11] have proposed a somewhat related “randomization” scenario, which, moreover, does not rely on branes with the exceedingly small charge q satisfying (9). In the context of string theory one may expect not just one but many different four-form fluxes F_i coupled to branes with different charges q_i . Each one of these fluxes is quantized in units of the charge, so that $F_i = n_i q_i$. In this case, the condition for a generic negative cosmological constant to be compensated for by the fluxes is $|\Lambda_{eff}| = |\sum_{i=1}^J (q_i^2 n_i^2)/2 + \Lambda| \leq \Lambda_{obs}$. The larger the number of different fluxes, the denser is the discretuum of possible values of Λ_{eff} , and the easier it is to find a set of values of n_i such that the above

inequality is satisfied. A sufficiently dense discretuum is typically obtained provided that the number of fluxes is sufficiently large $J \sim 100$, even if the individual charges q_i are Planckian (a smaller number of fluxes $J \geq 6$ may be enough in a scenario with large extra dimensions, where the charges q_i are suppressed with respect to the Planck scale by a large internal volume effect). In the Bousso-Polchinski model, Λ_{eff} is typically very large, and drives an exponential expansion at a very high energy scale. Suppose we start with a single exponentially expanding domain characterized by a set of integers $\{n_i\}$. Whenever a brane of type j nucleates in this region, the integer n_j will change by one unit inside of the brane. The newly formed region will itself expand exponentially, creating a huge new domain. The nucleation of further branes within this region will cause an endless random walk of the values of $\{n_i\}$, which will sample the whole discretuum of values of Λ_{eff} . Eventually, a bubble will nucleate where Λ_{eff} is comparable to the observed value. The nucleation of this last bubble is still a high energy process, which kicks some inflaton field off its minimum, and starts a short period of inflation within this last bubble. This period of ‘‘ordinary’’ inflation is necessary in order to produce the entropy we observe, thereby avoiding the empty universe problem. Of course, some anthropic input is still necessary in this approach in order to explain why, out of the discretuum of possibilities, we live in a vacuum with small Λ_{eff} .

Nucleation of coincident branes would drastically modify the neutralization scenario of Brown and Teitelboim, as well as the randomization scenarios sketched above. As proposed in [2], in the case of neutralization, this modification may lead to a solution of the empty universe problem. In the randomization scenarios there is no such problem, and it is unclear whether an enhancement of the multiple brane nucleation rate is desirable at all. This enhancement would trigger large jumps in the effective cosmological constant, making the calculation of its spatial distribution more complicated than that for single brane nucleation. Thus, it is of interest to understand the conditions under which multiple brane nucleation is allowed, and what are the degeneracy factors that might enhance their nucleation rate relative to the nucleation of single branes. Before addressing this issue, it will be convenient to present a short discussion of coincident branes.

III. COINCIDENT BRANES IN 4D

In the context of string theory, one may consider stacks of k coincident D- p -branes. Each brane in the stack has charge q with respect to the form field \mathcal{A} . Thus, in four noncompact dimensions, a pair of parallel 2-branes repel each other with a constant force per unit area given by

$$f_q = q^2/2, \tag{10}$$

due to the four-form field interaction. In the ten dimensional theory, the repulsive force due to \mathcal{A} is balanced with other contributions from the closed string sector, such as the graviton and dilaton. As a result, there are no net forces amongst the different branes on the stack.

It is in principle possible to maintain this delicate balance by suitable compactification from 10 to 4 dimensions. In 4D, the branes look like domain walls, and their interaction through the ordinary graviton leads to a mutual repulsive force given by [12]

$$f_\sigma = 3\sigma^2/4M_p^2. \tag{11}$$

Hence, both forces given by (10) and (11) tend to push the branes apart from each other. On the other hand, some of the higher dimensional gravitational degrees of freedom are represented by scalars in 4D, and these, together with the dilaton, lead to attractive forces.

A. Dimensional reduction

Let us first consider the case of D-2-branes in 10 dimensional type IIA supergravity. The relevant part of the action is given by (see e.g. [13])

$$S_{10} = \frac{M_{10}^8}{2} \int d^{10}x \sqrt{G} \left[e^{-2\phi} [R + 4(\nabla\phi)^2] - \frac{1}{2 \times 4!} \hat{F}^2 \right] - T_2 \int_{\Sigma} d^3\xi \sqrt{G_{\Sigma}} e^{-\phi + \hat{q}_2} \int_{\Sigma} d^3\xi \hat{\mathcal{A}}. \tag{12}$$

Here \sqrt{G} and $\sqrt{G_{\Sigma}}$ are the determinants of the 10 dimensional metric G_{AB} and of the metric induced on the worldsheet Σ , respectively, whereas R is the Ricci scalar corresponding to G_{AB} . The carets on the four-form, the gauge potential, and the corresponding charge are introduced in order to distinguish them from the four dimensional ones that will be used below, and which differ from those by constant normalization factors. Compactifying on a Calabi-Yau manifold K through the ansatz

$$ds_{10}^2 = e^{2\phi - 6\psi} g_{\mu\nu} dx^\mu dx^\nu + e^{2\psi} dK_6^2,$$

where the Greek indices run from 0 to 3, we readily obtain the following four-dimensional action:

$$S_4 = \frac{M_p^2}{2} \int d^4x \sqrt{g} \left\{ \mathcal{R} - \frac{2}{7} (\partial\hat{\phi})^2 - \frac{6}{7} (\partial\hat{\sigma})^2 - \frac{1}{2 \times 4!} e^{-2\hat{\phi}} \hat{F}^2 \right\} - T_2 \int_{\Sigma} d^3\xi \sqrt{\gamma} e^{\hat{\phi} + \hat{q}_2} \int_{\Sigma} \hat{\mathcal{A}}. \tag{13}$$

Here $M_p^2 = M_{10}^8 V_6$, where V_6 is the coordinate volume of the manifold K , \mathcal{R} is the Ricci scalar for the metric g , and we have introduced two linear combinations of the internal volume modulus ψ and the dilaton ϕ

$$\hat{\phi} = 2\phi - 9\psi, \quad \hat{\sigma} = \phi - \psi.$$

The field $\hat{\sigma}$ decouples from the branes, and shall be ignored in what follows. In Ref. [2], a different expression was given for the dimensionally reduced action, because no modulus was introduced for the size of the internal space. However, as we shall see, such modulus is necessary for the cancellation of forces among the branes.

A more direct route to the 4-dimensional theory (13), which will be slightly more convenient for our discussion, is to start directly from 11-dimensional supergravity and compactify on a 7 dimensional internal space. The action in eleven dimensions is given by

$$S_{11} = \frac{M_{11}^9}{2} \int d^{11}x \sqrt{G} \left[R - \frac{1}{2 \times 4!} \hat{\mathcal{F}}^2 \right] - T_2 \int_{\Sigma} d^3 \xi \sqrt{G_{\Sigma}} + \hat{q}_2 \int_{\Sigma} \hat{\mathcal{A}} \quad (14)$$

where $T_2 = \hat{q}_2 = 2\pi M_{11}^3$. Introducing the ansatz

$$ds_{11}^2 = e^{-7\hat{\psi}} g_{\mu\nu} dx^{\mu} dx^{\nu} + e^{2\hat{\psi}} d\Omega_7^2, \quad (15)$$

with a Ricci flat internal manifold, we find

$$S_4 = \frac{M_p^2}{2} \int d^4x \sqrt{g} \left\{ \mathcal{R} - \frac{2}{7} (\partial\hat{\varphi})^2 - \frac{1}{2 \times 4!} e^{-2\hat{\varphi}} \hat{\mathcal{F}}^2 \right\} + \dots, \quad (16)$$

where $M_p^2 = M_{11}^9 V_7$ and

$$\hat{\varphi} = -\frac{21}{2} \hat{\psi}.$$

Not surprisingly, this has the same form as (13), since after all the 10-dimensional Lagrangian can be obtained from the 11-dimensional one by compactifying on a circle.

Equation (16) is a particular case of the slightly more general action in four dimensions:

$$S_4 = \frac{1}{2} \int d^4x \sqrt{g} \left\{ M_p^2 [\mathcal{R} - (\partial\varphi)^2] - \frac{1}{4!} e^{-2\alpha\varphi} \mathcal{F}^2 \right\} - \sigma \int_{\Sigma} d^3 \xi \sqrt{\gamma} e^{\alpha\varphi} + q \int_{\Sigma} \mathcal{A}. \quad (17)$$

The parameter α characterizes the scalar charge of the brane. As we shall see below, linearizing in φ around $\varphi=0$, it can be easily shown that the scalar force is given by

$$f_e = -e^2/2, \quad (18)$$

where we have introduced the scalar charge

$$e \equiv \alpha\sigma/M_p. \quad (19)$$

Thus, from (10), (11), and (18), the branes will be in indifferent equilibrium provided that the following relation holds:

$$Q^2 \equiv \left[\frac{e^2 - q^2}{2} - \frac{3\sigma^2}{4M_p^2} \right] = 0. \quad (20)$$

From (19), this condition can be rewritten as

$$q = \left(\alpha^2 - \frac{3}{2} \right)^{1/2} \frac{\sigma}{M_p}. \quad (21)$$

Note that the case of type IIA supergravity discussed above, corresponds to $\alpha = \sqrt{7/2}$, $q = (\sqrt{2}/M_p) \hat{q}_2$, and $\sigma = T_2$ [note that $\hat{\mathcal{A}} = (\sqrt{2}/M_p) \mathcal{A}$]. Therefore (21) is satisfied provided that $T_2 = \hat{q}_2$, the usual Bogomol'nyi-Prasad-Sommerfield (BPS) condition.

B. Multiple brane solutions

Multiple brane solutions to (17) can easily be constructed provided that (21) is satisfied. In the bulk, the equation of motion for \mathcal{A} leads to

$$\mathcal{F}_{\alpha\beta\gamma\delta} = F e^{2\alpha\varphi} \sqrt{g} \epsilon_{\alpha\beta\gamma\delta},$$

where F is a constant and ϵ is the Levi-Civita symbol. At the branes, this constant jumps by the amount

$$\Delta F = q.$$

It can be checked that the remaining equations of motion for the scalar and the gravitational field follow from the action

$$S_4 = \sum_i \int_i d^4x \sqrt{g} \left\{ \frac{M_p^2}{2} [\mathcal{R} - (\partial\varphi)^2] - V_i(\varphi) \right\} - \sigma \int_{\Sigma} d^3 \xi \sqrt{\gamma} e^{\alpha\varphi}, \quad (22)$$

where

$$V_i(\varphi) = \frac{F_i^2}{2} e^{2\alpha\varphi}, \quad (23)$$

and the sum is over the regions with different values of F . With the metric ansatz

$$ds^2 = w^2(z) \eta_{ab} dx^a dx^b + dz^2,$$

where Latin indices run from 0 to 2, the solution is given by

$$e^{-\alpha\varphi} = c_i \pm \frac{\alpha^2}{(\alpha^2 - 3/2)^{1/2}} \frac{|F_i|}{M_p} z, \quad (24)$$

and

$$w(z) = e^{-\varphi/2\alpha}. \quad (25)$$

Here c_i are integration constants. These, and the sign option in (24), must be chosen so that the junction conditions for the gravitational field and for the scalar field are satisfied at the branes. For the gravity part, the condition is [14]

$$[K_{ab}] = -4\pi G \sigma e^{\alpha\varphi} \gamma_{ab}, \quad (26)$$

where $[K_{ab}]$ is the difference of extrinsic curvature on the two sides of the brane and γ_{ab} is the worldsheet metric. In the present case, this reduces to

$$\left[\frac{w'}{w} \right] = -\frac{\sigma}{2M_p^2} e^{\alpha\varphi}. \quad (27)$$

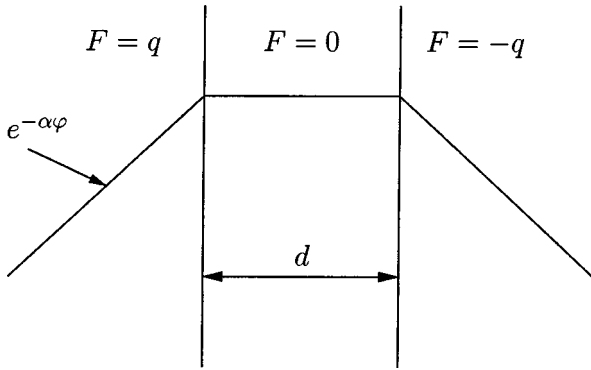


FIG. 1. Configuration of two parallel branes.

For the scalar field, the junction condition that follows from the equation of motion at the brane reduces to

$$[\varphi'] = \alpha \frac{\sigma}{M_p^2} e^{\alpha\varphi}, \tag{28}$$

which is consistent with (27) and (25). For instance, a solution with two branes separated by a distance d is given by flat space ($w = 1, \varphi = 0$) in the region between the branes, and by

$$e^{-\alpha\varphi} = 1 - \alpha^2 \frac{\sigma}{M_p^2} (|z| - d/2) \quad (|z| > d/2) \tag{29}$$

in the exterior region (see Fig. 1). The solution corresponds to two branes of charge q interpolating between regions with $F = q$ and $F = -q$, separated by a region with $F = 0$.

Note that the solution (29) contains a singularity at a distance $z \sim M_p^2/\alpha^2\sigma$ (where the warp factor w and the volume of the internal space vanish). In what follows, however, we shall not be interested in flat, infinite branes, such as the ones discussed above. Rather, we shall be interested in compact instanton solutions of finite size, in a theory with a stabilized dilaton. In this case, the singularities due to linear potentials of the form (29) should not arise, but the solutions discussed above should remain a good description in the vicinity of the branes. Another consequence of stabilizing the dilaton is that the perfect balance of forces amongst the branes will be spoiled at distances larger than the inverse mass of the moduli, as we now discuss.

IV. STABILIZING THE MODULI

As it stands, the dimensionally reduced supergravity Lagrangian (13) [or its generalization (17)] is not useful for discussing the neutralization of the cosmological constant. The Lagrangian does not include the bare cosmological term Λ , which is precisely the subject of our interest, and the term proportional to F^2 does not behave as an effective cosmological constant, but rather as an exponential potential for a modulus φ (which is not flat enough to mimick the vacuum

energy¹). A more realistic model is obtained by introducing a stabilization mechanism that fixes the expectation value of φ , and gives it some mass m . Once φ is stabilized the F^2 term does behave as a contribution to Λ_{eff} . Stabilization is also desirable because the dilaton and the radion moduli (corresponding to the size of extra dimensions) mediate scalar interactions of gravitational strength, which are severely constrained by observations. The study of mechanisms for stabilization is currently an active topic of research (see e.g. Refs. [16,17] and references therein). Although the details of stabilization will not be too important in our subsequent discussion, it may be nevertheless illustrative to have in mind a specific toy mechanism (for which we do not claim any rigorous justification in the context of string theory).

The general problem is that the potential (23) has no minimum and leads to a runaway dilaton. In order to create a minimum, let us consider two contributions that may be added to (23). First of all, instead of using a Ricci flat internal manifold in (15) we may compactify on an Einstein manifold, with

$$R_{ab}^\Omega = 6K g_{ab}^\Omega. \tag{30}$$

Upon dimensional reduction, the curvature K contributes an exponential term to the effective potential for φ . However, this will still not be sufficient for stabilizing the internal volume in an interesting way. In fact, as shown by Maldacena and Nunez [18], there are no static compactifications of the classical supergravity Lagrangian with a positive effective four-dimensional cosmological constant $\Lambda_{eff} > 0$, and so far we have added nothing to the classical Lagrangian. Thus, in order to implement the four dimensional situation of our interest, a third term related to quantum corrections has to be considered. Following Candelas and Weinberg [19], we may consider the Casimir energy of bulk fields (we are of course assuming that supersymmetry is broken, so that the Casimir contributions of bosons and fermions do not cancel each other exactly).

In the example considered by Candelas and Weinberg [19], besides the Casimir energy term, a higher dimensional cosmological term Λ_{4+n} was used, and the internal manifold was taken to be a space of constant positive curvature ($K = 1$). In 11D supergravity a cosmological constant is not allowed, so here we use the F^2 flux instead. Also, we will

¹Exponential potentials such as the one appearing in (22) have been thoroughly studied in the literature, and it is known that they can drive cosmological solutions with a power-law scale factor [15]. Such attractor solutions are approached for a wide range of initial conditions, and the resulting expansion can be accelerating or decelerating, depending on whether $\alpha^2 < 1/2$ or $\alpha^2 > 1/2$. For $\alpha^2 < 3/2$ the cosmological scale factor approaches $a(t) \sim t^{1/2\alpha^2}$, where t is cosmological time. This solution corresponds to an effective equation of state $p = [(4\alpha^2/3) - 1]\rho$, where the ratio of kinetic and potential energies of the scalar field φ remains constant. In our case, from (21), we need $\alpha^2 > 3/2$ and therefore the kinetic term becomes completely dominant in the long run, which leads to $p = +\rho$. Hence, by itself, the F^2 term does not behave like an effective cosmological constant.

need to compactify on a negatively curved internal manifold ($K = -1$), or else the dilaton would be stabilized at negative Λ_{eff} .

The versatility of negatively curved compactifications has been stressed in Ref. [20]. In particular, they have the interesting property of rigidity, which means they do not lead to other moduli besides the size of the internal space. Compact hyperbolic manifolds (CHM) can be obtained from the maximally symmetric negatively curved space H_7 through identifications by a discrete isometry group Γ . The volume of H_7/Γ is given by

$$V_7 = r_c^7 e^\gamma, \quad (31)$$

where r_c is the curvature radius of the manifold, related to the curvature parameter in (30) by $K = -1/r_c^2$. The factor e^γ depends on the topology, and it is bounded below but not above. If L is the largest distance around the manifold, then for $L \gg r_c/2$ we have $e^\gamma \sim e^{6L/r_c}$. The Kaluza-Klein (KK) spectrum in this manifold is believed to have a mass gap, bounded below by $m_{KK} \sim e^{-\hat{\psi}} r_c^{-1}$. From the 11 dimensional point of view the Casimir energy density scales like

$$\rho_C^{(11)} = C m_{KK}^{11} = C e^{-11\hat{\psi}} r_c^{-11}. \quad (32)$$

The factor C can be estimated by naive dimensional analysis as $C \sim \beta \nu$, where $\nu \sim 10^3$ is the number of physical polarizations of bulk fields, and β is some small one-loop factor. This factor will depend on the precise topology of the compact hyperbolic manifold, but it could plausibly be in the range $\beta \sim 10^{-2} - 10^{-4}$. Hence, the parameter C could be of order one. Explicit calculations for different choices of the manifold H_7/Γ have not been performed, and are well beyond the scope of the present paper (see e.g. Ref. [21] and references therein). In what follows we shall leave C unspecified, assuming that a compactification exists where $|C| \gtrsim 1$. Multiplying the higher dimensional energy density $\rho_C^{(11)}$ given in (32) by the size of the internal manifold, $V_7 e^{7\hat{\psi}}$, and by a factor $e^{-14\hat{\psi}}$ that arises from the four-dimensional volume element in going to the Einstein frame, the effective potential that appears in Eq. (22) gets replaced by

$$V_i(\varphi) = -21M_{11}^9 V_7 K e^{-9\hat{\psi}} + C V_7 r_c^{-11} e^{-18\hat{\psi}} + \frac{F_i^2}{2} e^{-21\hat{\psi}}, \quad (33)$$

where we have also added the curvature contribution. Here $\hat{\psi} = -(2/21)\alpha\varphi$, where $\alpha = \sqrt{7/2}$.

We can always adopt the convention that $r_c = 1/M_{11}$, since a change of $r_c \rightarrow e^{-\lambda} r_c$ can be reabsorbed by a shift in $\hat{\psi} \rightarrow \hat{\psi} + \lambda$ and a constant re-scaling of the four-dimensional metric $g_{\mu\nu} \rightarrow e^{7\lambda} g_{\mu\nu}$ (in this frame, the curvature of the manifold is of the order of the higher dimensional Planck scale for $|\hat{\psi}| \lesssim 1$). Since $F_i = n_i q$, where n_i is an integer and $q = (\sqrt{2}/M_p)\hat{q}_2 = 2\sqrt{2}\pi M_{11}^3/M_p$, we have

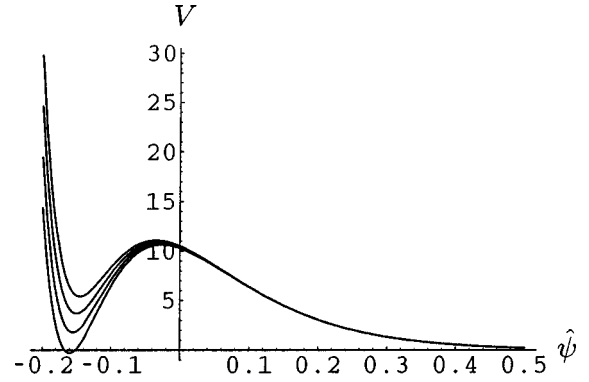


FIG. 2. The effective potential (34), for different values of the integer n_i which characterizes the quantized flux of the four-form F .

$$V_i(\varphi) = M_p^2 M_{11}^2 \left[21e^{-9\hat{\psi}} + C e^{-18\hat{\psi}} + 4\pi^2 n_i^2 \left(\frac{M_{11}}{M_p} \right)^4 e^{-21\hat{\psi}} \right]. \quad (34)$$

As illustrated by Eq. (31) a large value of V_7 in units of $r_c = 1/M_{11}$ can be obtained by using a manifold with sufficiently complicated topology. Since $M_p^2 = M_{11}^9 V_7 = M_{11}^2 e^\gamma$, the factor $(M_{11}/M_p)^4 = e^{-2\gamma}$ in the last term of (34) can be rather small. The scale M_{11} could be as low as the TeV , in which case that factor can be as low as 10^{-64} . Moreover, as emphasized in Ref. [20], this can be achieved for CHM even if the linear size L of the internal manifold is not very much larger than r_c . In Fig. 2 we plot the effective potential for $C = -20$, $e^{-\gamma} = 10^{-3}$ and values of $n_i = 485, 487, 489$ and 491 .

When a single brane nucleates, it changes n_i by one unit, and hence changes the value of the effective potential at the minimum (changing therefore the effective cosmological constant). If the discretuum of values of F_i were sufficiently dense, then there would always be one of the minima of the effective potential where the vacuum energy is sufficiently small to match observations. In the case we have considered here, the discretuum is not dense enough. The cancellation between the last term in (33) and the other two requires $n_i \sim (M_p/M_{11})^2$, and so the gap between levels near $V_i = 0$ can be estimated as $\Delta V_i \sim M_{11}^4$, which is far too large. This situation can be remedied by considering 5M branes wrapped around 3-cycles in the internal space. As emphasized by Bousso and Polchinski [11], these are coupled (with different charges) to additional fluxes, and a large number of fluxes will result in a much denser spectrum. Alternatively, FMSW have suggested that the branes may wrap a degenerating cycle [2], in which case the individual charges might themselves be exponentially smaller than the fundamental scale, resulting also in a sufficiently dense discretuum.

Before closing this section, we should note that the location where the modulus sits is basically determined by the competition between the two first terms in (33), corresponding to curvature and Casimir energy. The physical curvature radius of the internal space is therefore stabilized at $r_{phys} = r_c e^{\hat{\psi}} \sim M_{11}^{-1} C^{1/(D-2)}$, where $D = 11$ is the spacetime dimension. Thus, unless the constant C in Eq. (32) is exceed-

ingly large, the compactification scale is comparable to the inverse of the higher dimensional cut-off scale M_{11} . In such a case the semiclassical analysis that we have entertained above is not justified, since higher order corrections will be just as important as the one loop effect that we have included. This appears to be a generic problem when we try to stabilize by making the curvature and the Casimir terms comparable, as in the Candelas and Weinberg example [19]. There, the problem was not quite as poignant, since the constant C could be made very large by adding a sufficient number of fields (also, as it is clear from the above estimate of r_{phys} , the problem is somewhat milder if the number of extra dimensions is smaller). Here we shall not dwell on this problem, since the main purpose of the above discussion is just to illustrate the role of the four-form in obtaining a discretuum of states. For more fundamental approaches to moduli stabilization in the present context, the reader is referred to Refs. [16] and references therein. Our ensuing discussion will be largely independent of the details of the stabilization mechanism.

V. INTERBRANE FORCES

Inevitably, the stabilization of φ spoils the perfect balance of forces. At distances larger than the inverse of the mass of the modulus m^{-1} , the remaining gravitational and four-form interactions are both repulsive, and lead to a linear potential per unit surface of the form

$$\mathcal{V}(d) \approx - \left[\frac{q^2}{2} + \frac{3\sigma^2}{4M_p^2} \right] d \quad (d \gg m^{-1}) \tag{35}$$

where d is the distance between the branes.

To investigate the behavior of the interaction potential at distances shorter than m^{-1} , let us first consider the situation where gravity and the 3-form gauge potential \mathcal{A} are ignored, and the branes interact only through a scalar field φ of mass m . The action is given by

$$S = - \frac{M_p^2}{2} \int d^4x [(\partial\varphi)^2 + m^2\varphi^2] - \sigma \int_{\Sigma} d^3\xi \sqrt{\gamma} e^{\alpha\varphi}. \tag{36}$$

The solution with a single brane on the plane $z=0$ has the cusp profile

$$\varphi = \varphi_0 e^{-m|z|}, \tag{37}$$

where φ_0 is a solution of

$$\varphi_0 e^{-\alpha\varphi_0} = - \frac{\alpha\sigma}{2mM_p^2} \equiv - \frac{e}{2mM_p}. \tag{38}$$

The energy per unit area of this configuration is given by

$$\sigma_1 = \sigma e^{\alpha\varphi_0} + \frac{M_p^2}{2} \int dz (\varphi'^2 + m^2\varphi^2) = \sigma e^{\alpha\varphi_0} + mM_p^2\varphi_0^2. \tag{39}$$

For small charge and tension, we have

$$\sigma_1 \approx \sigma - \frac{e^2}{4m} \quad (e^2 \ll 2m\sigma).$$

Due to the scalar field dressing, the effective tension of the brane, denoted by σ_1 , is smaller than the parameter σ that appears in the action. This effect becomes more dramatic if we place a large number k of branes on top of each other. Since both σ and e scale like k , the effective tension of the stack is given by

$$\sigma_k \approx k \left[\sigma - k \frac{e^2}{4m} \right] \quad (k e^2 \ll 2m\sigma), \tag{40}$$

which grows with k but less than linearly. In the limit of very large k , using (38) with σ replaced by $k\sigma$ we have

$$\sigma_k \approx \frac{m\sigma^2}{e^2} \ln^2 \left(\frac{ke^2}{2m\sigma} \right) \quad (k e^2 \gg 2m\sigma) \tag{41}$$

and so the tension almost saturates, growing only logarithmically with k . This last expression should not be taken too literally, however, since in (36) we are neglecting gravitational effects. As shown above, nonlinear effects of gravity become important at a distance given by $M_p^2/\alpha^2\sigma_k$, which in the limit given by (41) is smaller than m^{-1} . Hence the cusp solution (37), which has typical width $\sim m^{-1}$, will receive sizable gravitational corrections in the limit of large k . Nevertheless, it still seems likely that the effective tension of the stack of branes will grow with k much slower than linearly. Physically, the reason is that the scalar charge of the stack increases with k . According to (28), this means that the cusp in the field φ on the branes grows stronger, which in turn means that the value of the field φ_0 on the branes has to be further displaced into negative values. Hence, the brane contribution to the effective tension $k\sigma e^{\alpha\varphi_0}$ shows only a very modest growth with k , and the tension for large k is in fact dominated by the potential and gradient energy of the scalar near the brane.

Let us now look at the interaction potential between two branes separated from each other. For simplicity, we shall restrict our attention to the case of small scalar charge, $e^2 \ll 2m\sigma$. Then, the last term in (36) is well approximated by its linearized expression:

$$S = - \frac{M_p^2}{2} \int d^4x [(\partial\varphi)^2 + m^2\varphi^2] - \int_{\Sigma} d^3\xi \sqrt{\gamma} (\sigma + eM_p\varphi). \tag{42}$$

Placing the two branes at $z = \pm d/2$, the solution for the scalar field has the ‘‘Golden Gate’’ profile shown in Fig. 3:

$$\begin{aligned} \varphi &= - \frac{e}{mM_p} e^{-md/2} \cosh mz, & |z| < d/2 \\ \varphi &= - \frac{e}{mM_p} \cosh(md/2) e^{-m|z|}, & |z| > d/2. \end{aligned} \tag{43}$$

The energy per unit area of this configuration, as a function of the interbrane distance, is given by

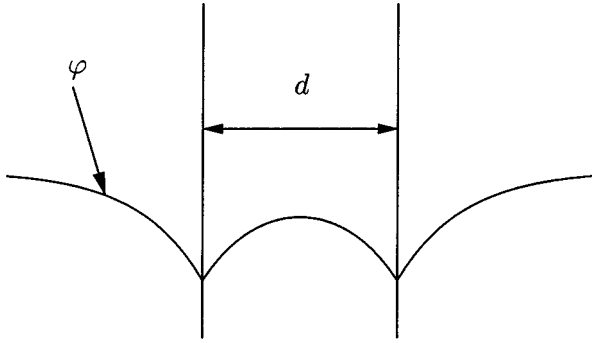


FIG. 3. Profile of a massive dilaton in the presence of two branes.

$$\sigma_2(d) = 2\sigma_1 - \frac{e^2}{2m} e^{-md}. \quad (44)$$

From this expression, and adding the long range contributions from gravity and the four-form, the interaction potential per unit surface is given by

$$\mathcal{V}(d) = - \left[\frac{q^2}{2} + \frac{3\sigma^2}{4M_p^2} \right] d - \frac{e^2}{2m} e^{-md}.$$

At short distances, this takes the form

$$\mathcal{V}(d) = - \frac{e^2}{2m} + Q^2 d - \frac{me^2}{4} d^2 + \dots \quad (45)$$

where the parameter

$$Q^2 \equiv \left[\frac{e^2 - q^2}{2} - \frac{3\sigma^2}{4M_p^2} \right], \quad (46)$$

was introduced in (20). As shown in Sec. III, dimensional reduction from the supergravity Lagrangian with BPS charges gives $Q^2 = 0$. In this case, the linear term in (45) disappears. The quadratic term is negative, which means that the stack of coincident branes is unstable and tends to dissolve. Thus the stabilization of the modulus φ seems to make the superposition of branes an unstable configuration.

As we shall see in more detail in Sec. VII, stacks of branes in marginal or unstable equilibrium will not be appropriate for constructing instantons, since in particular these would have too many negative modes. The instanton is only meaningful if the branes attract each other. The above analysis shows that at the classical level, the branes with BPS values of the charges would not attract each other, and consequently the nucleation of coincident branes is not allowed at least in the semiclassical description.

There may be several escape routes to this conclusion. For instance, after supersymmetry breaking, the charges of the branes get renormalized, and it is possible that the corrected charges satisfy $Q^2 > 0$. In this case the branes in the stack would attract each other with a linear potential. Other mechanisms by which nearby branes attract each other are conceivable, but here we shall not try to pursue their study. It should be emphasized, however, that this remains an important open

question that needs to be addressed in order to justify the semiclassical description of coincident brane nucleation. In Sec. VII, where we discuss the degeneracy factor, we shall simply postulate that an attractive interaction exists at short distances.

VI. COLEMAN-DE LUCCIA (CDL) INSTANTONS

The brane nucleation rate per unit volume is given by an expression of the form [5]

$$\Gamma = D e^{-B}, \quad (47)$$

where $B = S_E(I) - S_E(B)$. Here $S_E(I)$ is the Euclidean action of the instanton corresponding to the decay of the four-form field, and $S_E(B)$ is the action for the background solution before nucleation. The prefactor D will be discussed in the next section. The formal expression (47) can be used both in flat space and in curved space. Also, it can be used at zero or at finite temperature. The difference is in the type of instanton and background solutions to be used in each case.

In this section, we shall concentrate in the maximally symmetric instanton. In flat space, this represents the decay of a metastable vacuum at zero temperature. The Euclidean solution can be described as follows [5]. At infinity, the field strength takes the value F_o , which plays the role of a false vacuum. Near the origin, we have $F = F_i = F_o - q$. This plays the role of a true vacuum phase. Both phases are separated by the Euclidean worldsheet of the brane, which is a three-sphere of radius

$$R = \frac{3\sigma}{\epsilon}. \quad (48)$$

Here σ is the tension of the brane and

$$\epsilon = \frac{1}{2} (F_o^2 - F_i^2) = q[F_o - (q/2)] \quad (49)$$

is the jump in the energy density across the brane. The difference in Euclidean actions between the instanton and the background solution is given in this case by [5]

$$B^{(flat)} \approx \frac{27\pi^2}{2} \frac{\sigma^4}{\epsilon^3}. \quad (50)$$

If instead of considering a single brane, we are looking at the nucleation of k coincident branes, an analogous solution of the Euclidean equations of motion should be considered. The only difference is that the effective charge is a factor of k higher, and the effective tension is also higher by approximately the same factor. Using (40), we should replace

$$\begin{aligned} \epsilon &\rightarrow \epsilon_k = kq[F_o - (kq/2)], \\ \sigma &\rightarrow \sigma_k \approx k[\sigma - (ke^2/4m)], \end{aligned} \quad (51)$$

in Eq. (50) for the ‘‘bounce’’ action. As shown in Sec. V, the approximate form of σ_k is valid when the second term in the r.h.s. of (51) is small compared with the first. In the model considered in Sec. IV, the modulus is stabilized with a mass

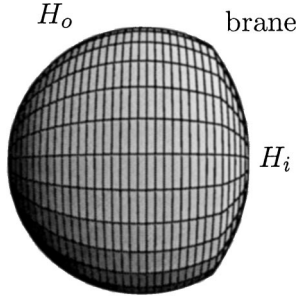


FIG. 4. Coleman–de Luccia instanton, which is obtained by gluing together two different four-spheres of radii H_o^{-1} and H_i^{-1} at the worldsheet of the brane, itself a three-sphere.

of order $m \sim M_{11}$. This may be regarded as a conservative upper bound for the general case. Since $e = \alpha\sigma/M_p$, the condition $(ke^2/4m) \ll \sigma$ requires

$$k \ll \frac{1}{\alpha^2} \left(\frac{M_{11}^3}{\sigma} \right) \left(\frac{M_p}{M_{11}} \right)^2. \quad (52)$$

For larger k , gravitational corrections to the brane profile become important on a length scale comparable to the inverse mass of the dilaton, as we discussed in the preceding section. In this case the instanton solution will depend on the detailed dynamics of the dilaton near the brane. The investigation of these dilatonic instantons is per se an interesting problem, which we leave for further research. Here, for simplicity, we shall assume a scenario where either $\sigma \ll M_{11}^3$, due perhaps to the wrapping of branes on a degenerating cycle, or where the extra dimensions are relatively large, so that $M_p \gg M_{11}$, and we shall restrict attention to instantons where the number of branes is bounded by (52). This gives

$$B_k^{(flat)} \approx \frac{27\pi^2}{2} \frac{k\sigma^4}{q^3 [F_o - (kq/2)]^3}. \quad (53)$$

Nucleation in flat space is impossible when the number of branes is too large, since otherwise we would be jumping to vacua with a higher energy density. Therefore, we must restrict ourselves to $k < 2F_o/q$. In fact, the minimum value of F_i is achieved for the largest k satisfying $k < (F_o/q) + (1/2)$. Note that the action increases faster than linearly as we increase the number of branes k , but we still have

$$B_k^{(flat)} \sim kB^{(flat)}$$

throughout this range.

When gravity is taken into account, the maximally symmetric instanton was given by Coleman and De Luccia [3]. It is constructed by gluing two different de Sitter solutions (that is, two four-spheres) at the worldsheet of the brane, which is still a three-sphere. The instanton is sketched in Fig. 4. The four-spheres have the radii H_o^{-1} and H_i^{-1} , where

$$H_o^2 = \frac{\Lambda_{eff}}{3M_p^2}, \quad H_i^2 = \frac{\Lambda_{eff} - \epsilon}{3M_p^2}, \quad (54)$$

TABLE I. Values of the radius R of the Coleman–de Luccia instanton and the corresponding bounce action B in different limits.

	$\sigma_k \ll M_p^2 H_o$	$\sigma_k \gg M_p^2 H_o$
$\sigma_k H_o / \epsilon_k \ll 1$	$R \approx 3\sigma_k / \epsilon_k$ $B \approx B_k^{(flat)}$	$R \approx 1/2\pi G\sigma_k$
$\sigma_k H_o / \epsilon_k \gg 1$	$R \approx H_o^{-1}$ $B \approx 2\pi^2\sigma_k H_o^{-3}$	$B \approx \pi/GH_o^2$

are the Hubble rates of the de Sitter phases before and after the nucleation event, respectively. Here, and in what follows, we are assuming that the effective cosmological constant is still positive after nucleation, since these are the final states we are interested in. The bubble radius at the time of nucleation (which coincides with the radius of the three-sphere) is bounded by $0 < R < H_o^{-1}$. Analytic expressions for R are given in Refs. [1,3]. The general expression is cumbersome and not particularly illuminating, so we shall concentrate on a few limiting cases of interest.

As discussed in Sec. III, the gravitational field of a brane is repulsive, and is characterized by an “acceleration” of order σ/M_p^2 . This gravitational field will be negligible provided that the corresponding Rindler radius (or inverse of the acceleration) is much larger than the radius R of the Euclidean worldsheet, which in turn is smaller than H_o^{-1} :

$$\sigma \ll M_p^2 H_o. \quad (55)$$

In this regime, we can distinguish two cases. For $\sigma H_o / \epsilon \ll 1$, the radius of the Euclidean worldsheet is much smaller than the de Sitter radius, and the flat space expression (50) holds. In the opposite limit, $\sigma H_o / \epsilon \gg 1$, we have $(H_o^{-1} - R) \ll H_o^{-1}$ and

$$B^{(wall)} \approx 2\pi^2\sigma H_o^{-3}. \quad (56)$$

The vacuum energy difference ϵ is unimportant in this case, and the action coincides with that for domain wall nucleation [22].

Finally, the gravitational field of the brane is important when

$$\sigma \gg M_p^2 H_o. \quad (57)$$

In this case, the radius of the worldsheet is given by $R \approx (1/2\pi G\sigma)$, and

$$B^{(wall)} \approx \frac{\pi}{GH_o^2}. \quad (58)$$

In this limit, the action of the instanton is much smaller than the action of the background, and this is the reason why (58) is independent of the tension. The same arguments apply of course to the instantons with coincident branes, and the corresponding expressions for the action and radii in the different regimes are summarized in Table I.

VII. THE PREFACTOR FOR CDL INSTANTONS

The prefactor D in Eq. (47) is given by [5]

$$D = \frac{Z'}{Z_B}. \quad (59)$$

Here Z and Z_B are the Gaussian integrals of small fluctuations around the instanton and the background solutions respectively. Expanding all brane and bulk fields (which we generically denote by ϕ) around the instanton configuration ϕ_I as $\phi = \phi_I + \sum_j \delta\phi_j$, we have $S_E[\phi] = S_E[I] + S^{(2)}[I, \delta\phi_j] + \dots$, where $S^{(2)}$ includes the terms quadratic in $\delta\phi_j$. At the one loop order we have

$$Z' = \int \prod_j \mathcal{D}'(\delta\phi_j) e^{-S^{(2)}}.$$

In the functional integral, there are some directions that correspond to spacetime translations of the instanton. The primes in the numerator of (59) and in the preceding equation indicate that the translational zero modes are excluded from the integration, and replaced by the corresponding spacetime volume. The latter is subsequently factored out in order to obtain a nucleation rate per unit time and volume.

The degrees of freedom that live on the brane will make a contribution to the numerator but not to the denominator. Consider, for instance, a free bosonic field Φ of mass m_Φ living on the worldsheet. Its contribution to the prefactor is

$$D_\Phi = Z_\Phi = e^{-W_\Phi} = \int \mathcal{D}\Phi e^{\int \gamma^{1/2} \Phi (\Delta^2 - m_\Phi^2) \Phi d^3\xi}.$$

Here the integral in the exponent is over the Euclidean worldsheet of the brane. If we have k^2 of such fields, their effect on the nucleation rate is to replace

$$B_k \rightarrow B_k + k^2 W_\Phi, \quad (60)$$

in the naive expression for the nucleation rate, $\Gamma \sim e^{-B_k}$, a replacement which can become very important as we increase the number of fields. This is, in essence, the observation made by Feng et al. that the large number of worldsheet fields might strongly affect the nucleation rate.

A. Scalars

The Euclidean worldsheet in the Coleman–de Luccia instanton is a 3-sphere. Determinantal prefactors due to scalar fields were considered in some detail in Ref. [23]. They are given by

$$W_\Phi = -\zeta'_R(-2) + (y^2/2) \ln(\sin \pi y) - \frac{1}{\pi^2} \int_0^\pi x \ln(\sin x) dx, \quad (61)$$

where $y^2 = 1 - m_\Phi^2 R^2$ and ζ_R is the usual Riemann zeta function. For instance, the contribution of a conformally coupled scalar field can be obtained by taking $m_\Phi^2 = (3/4)R^{-2}$, which gives

$$W_c = -\zeta'_R(-2) + \frac{1}{8} \ln 2 - \frac{7}{16\pi^2} \zeta_R(3) \approx 0.0638.$$

Hence, the effective degeneracy factor contributed by a conformal scalar field is given by

$$D_c \approx e^{-W_c} \approx 0.94 < 1. \quad (62)$$

The first thing to note is that this factor is not an enhancement, but a suppression. Hence, the prefactor cannot simply be thought of as the exponential of an entropy. More generally, from Eq. (61), the prefactor is independent of the expansion rate of the ambient de Sitter space. This has implications for the scenario proposed by FMSW, as we shall discuss in the next section.

In general, the degeneracy factor will depend on R and on the mass of the particle. For light minimally coupled scalars, Eq. (61) gives

$$D_s \approx \frac{e^{\zeta'_R(-2)}}{\pi^{1/2} m_\Phi R} \quad (m_\Phi R \ll 1). \quad (63)$$

There can be a strong enhancement in the nucleation rate if there are very light scalar fields. In the limit $m_\Phi \rightarrow 0$ the factor goes to infinity. This is because a massless scalar has a normalizable zero mode on the sphere, corresponding to the symmetry $\Phi \rightarrow \Phi + \text{const}$. In this case, the zero mode must be treated as a collective coordinate. The nucleation rate is proportional to the range $\delta\Phi$ of the field Φ , because the bubbles can nucleate with any average value of the scalar field with equal probability [23]

$$\begin{aligned} D_s(m_\Phi^2 = 0) &= \lim_{m_\Phi^2 \rightarrow 0} [m_\Phi D_s(m_\Phi)] (\pi R^3)^{1/2} \delta\Phi \\ &= e^{\zeta'_R(-2)} R^{1/2} \delta\Phi. \end{aligned} \quad (64)$$

Finally, for large mass the expression (61) leads to

$$D_s \approx \exp(\pi m_\Phi^3 R^3/6) \quad (m_\Phi R \gg 1). \quad (65)$$

The exponent of this expression can be interpreted as a renormalization of the tension of the stack of branes, due to the heavy scalars living on it [24]. Indeed, the effective potential for a scalar field in 2+1 dimensions in the flat space limit is proportional to m_Φ^3 , and the factor of R^3 is just due to the volume of the worldsheet. The factor D_s is plotted in Fig. 5 for different values of $m_\Phi R$.

Note that there is an enhancement both at small and at large mass, but the two have very different origin. The large value of (63) for light fields can be interpreted as a phase space enhancement. As we shall discuss in Sec. VII D, quantum fluctuations of fields living on the worldsheet of the brane are characterized by a temperature $T = 1/2\pi R$. The corresponding fluctuations in the potential term are of order $m_\Phi^2 \Phi^2 \sim T^3$, which corresponds to a root mean squared expectation value for Φ of the order

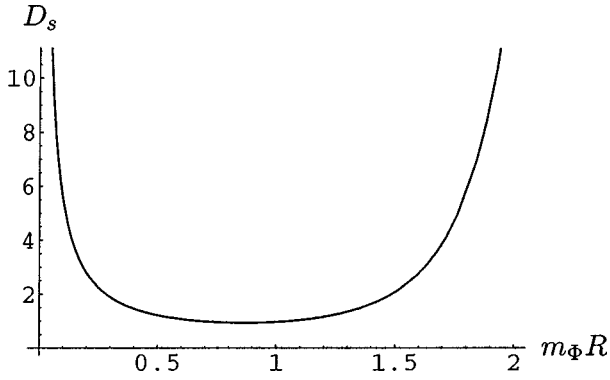


FIG. 5. Contribution of a scalar field to the prefactor D in the nucleation rate (47), as a function of its mass m_ϕ (measured in units of the inverse radius of the instanton). At low mass, the enhancement is due to large phase space: bubbles can nucleate with values of the field in the range (66), which becomes larger for smaller masses. The enhancement at large mass can be understood as a finite renormalization of the brane tension.

$$\delta\Phi \sim \frac{1}{mR^{3/2}}. \tag{66}$$

Equation (63) is recovered if we insert the range (66) in Eq. (64). The lighter the field, the larger is the phase space factor $\delta\Phi$, and the larger is the nucleation rate. For $m_\phi R \gg 1$ this argument cannot be used, since the field does not behave as effectively massless. In that limit the field decouples, as it should, and its effect is felt as a renormalization of the parameters in the Lagrangian. For a scalar field, the leading effect is to renormalize the tension of the brane, making it lower (see Appendix A). This causes an exponential enhancement of the nucleation rate.

B. $U(k)$ fields

Besides scalar fields, gauge bosons and fermions may live on a brane. At low energies, the field content on a stack of coincident branes will be model dependent. The idea is that it will correspond to a gauge theory whose symmetry is enhanced when branes are coincident, giving rise to a large number of light species on the worldsheet. The details of the theory, however, will depend on whether we start from 2D branes that descend directly from the ten dimensions, or from higher dimensional p -branes wrapped on $(p-2)$ -cycles. They will also depend on the details of compactification. Rather than building a particular scenario from first principles, here we shall try to gain some intuition by considering a toy model directly in four dimensions. The degrees of freedom on the stack of k coincident branes will be bluntly modeled by a weakly coupled SUSY $U(k)$ gauge theory on the 2+1 dimensional worldsheet. This contains a $U(k)$ gauge field, (k^2-1) scalar degrees of freedom in the adjoint representation of $SU(k)$, and a scalar singlet; plus the corresponding fermionic degrees of freedom. The action for the worldsheet fields is given by

$$S_{SYM} = - \int \sqrt{\gamma} d^3 \xi \left[\frac{1}{2} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) + \text{Tr}(D_\mu \phi D^\mu \phi) + \mathcal{V}(\phi) + \dots \right], \tag{67}$$

where the ellipsis indicate the terms containing fermions. Here $F_{\mu\nu}$ is the field strength of the gauge field $A^\mu = A^\mu_a \lambda_a$, where λ_a are the generators of $U(k)$, normalized by $\text{Tr}(\lambda_a \lambda_b) = \delta_{ab}/2$, and $\phi = \phi_a \lambda_a$, where ϕ_a are real scalar fields ($a, b = 1, \dots, k^2$). By analogy with the well known case of D-branes in 10 dimensions, we shall assume that if the coincident branes are flat (as in the case when there is no external four-form field), then the theory is supersymmetric and all degrees of freedom are massless. The scalar field ϕ is a Hermitian matrix and can always be diagonalized by a suitable $U(k)$ gauge transformation, $\phi = \text{diag}(\varphi_1, \dots, \varphi_k)$. The eigenvalues φ_i , $i = 1, \dots, k$ are then interpreted as the positions of the different parallel branes along an axis perpendicular to them (if the codimension of the branes were higher, there would be additional scalar matrices representing the positions of the branes along the additional orthogonal directions, but here we are interested in the case of codimension one). In the supersymmetric case, the potential for the scalar field vanishes, $\mathcal{V}(\phi) = 0$. However, for nonflat branes, the displacement of the stack of branes no longer behaves as an exactly massless field [23,25], but one which couples to a combination of the worldsheet and extrinsic curvatures, as well as to the background four-form field. Also, after moduli stabilization, the forces amongst branes are nonvanishing, and this also contributes to the potential for the relative displacements of the different branes. Thus, as we shall see, there will be a nonvanishing potential $\mathcal{V}(\phi) \neq 0$ in the physical situation of our interest.

When the positions are not coincident, the $U(k)$ symmetry breaks to a smaller group because some of the gauge bosons acquire masses $m_A^2 = (g^2/2)(\varphi_i - \varphi_j)^2$. There are always at least k massless vectors (corresponding to Maxwell theory on the individual branes on the stack) and the remaining $k^2 - k$ have double degeneracy. For example, in the case of a single brane, the scalar field will represent the Goldstone mode of the broken translational symmetry, associated to transverse displacement of the brane. For the case of two branes, there are two such scalars. One of them, $\varphi_+ = (\varphi_1 + \varphi_2)/\sqrt{2}$, will corresponds to simultaneous motion of both branes, and is a singlet under $SU(2)$. The other one, $\varphi_- = (\varphi_1 - \varphi_2)/\sqrt{2}$, will correspond to relative motion of the branes and it transforms under $SU(2)$. When the two branes move apart, φ_- acquires an expectation value and two of the gauge bosons get a mass, breaking the symmetry $U(2) \rightarrow U(1) \times U(1)$.

The case of interest to us is not a flat brane, but the world-volume of the 2-brane in the CdL instanton, which forms a 3-sphere of radius R . In this situation, we do not expect the theory to be supersymmetric (in particular, corrections to the effective action will appear at one loop, which will be related in fact to the determinantal prefactor in the nucleation rate). The case of a single brane is very similar to the case of a

vacuum bubble, and in that case we know that the transverse displacements correspond to a scalar field of negative mass squared [23,25]

$$m_+^2 = -3R^{-2}. \quad (68)$$

The origin and precise value of this mass term can be understood geometrically, since it leads to four normalizable zero modes which are the spherical harmonics with $l=1$. These correspond to the four space-time translational modes of the instanton, which have to be treated as collective coordinates. This scalar field has also a single negative mode, which is the constant $l=0$ mode. A negative mode is precisely what is needed for an instanton to contribute to the imaginary part of the vacuum energy, and hence to contribute to false vacuum decay [5]. Integrating out the transverse displacement of the brane gives a determinantal prefactor of the form [23]

$$D_+ = \frac{\sigma^2 R^2}{4} e^{\zeta'_R(-2)} \Omega, \quad (69)$$

where $\Omega = VT$ is the spacetime volume. The prefactor in the nucleation rate (47) per unit time and volume is obtained after dividing by Ω . The above argument neglects gravity, and it is a good approximation when $\sigma_0 \ll M_p^2 H_o$. The case of strong gravity $\sigma_0 \gg M_p^2 H_o$ is far more complicated, since one has to integrate over fluctuations of the gravitational field in the bulk, and is left for further research.

If there are 2 coincident branes, then there are two independent transverse displacements corresponding to the eigenvalues φ_1 and φ_2 . The ‘‘center of mass’’ displacement φ_+ behaves just like in the case of a single brane. The orthogonal combination φ_- represents the brane separation, and the two-brane instanton will only be relevant if this second combination acquires a positive mass through some mechanism, so that there is a single negative mode, not two, and four normalizable zero modes in total. In other words, the branes must attract each other. As shown in Sec. V, if the branes have BPS charges, they in fact tend to repel each other once the dilaton is stabilized, and the configuration with coincident branes is unstable. In this case, we do not expect that there will be any instanton representing the nucleation of multiple branes.

One possible way out of this conclusion is to assume that the charges are different from their BPS values, due to supersymmetry breaking effects, so that the sum Q^2 defined in Eq. (46) is positive. In that case, the two branes attract each other with a linear potential. The canonical field φ_- is related to the interbrane distance d by $|\varphi_-| = \sigma^{1/2} d$. Hence, the interbrane potential (45) takes the form

$$\mathcal{V}(\varphi_-) = Q^2 \sigma^{-1/2} |\varphi_-| - \frac{m e^2}{4\sigma} \varphi_-^2 + \dots \quad (70)$$

This potential is attractive at small distances, and has a maximum at $\varphi_- = \varphi_m \sim Q^2 \sigma^{1/2} / m e^2$. Classically, the branes will attract at short distances. However, there is a danger that they will be separated by quantum fluctuations. As we discussed in the preceding subsection (and we will argue more at

length in Sec. VII D) quantum fluctuations of fields living on the worldsheet of the brane are characterized by a temperature $T = 1/2\pi R$. The fluctuations in the potential are of order $\mathcal{V} \sim T^3$, and these correspond to a root mean squared expectation value for φ_- of the order

$$\delta\varphi_- \sim \sigma^{1/2} / R^3 Q^2. \quad (71)$$

The stability of the two-brane instanton requires that $\delta\varphi_- \ll \varphi_m$. Otherwise, unsuppressed quantum fluctuations take the field over the barrier and the distance between the branes starts growing without bound. This requires

$$Q^4 \gg \frac{m e^2}{R^3}. \quad (72)$$

If this condition is satisfied, then φ_- is trapped near the origin, and the branes stay together. It can be shown that for $Q^4 \gg \sigma R^{-5}$ the field behaves as approximately massless in the range given by (71), so from (64) its contribution to the prefactor can be estimated as

$$D_- \sim R^{1/2} \delta\varphi_- \sim \frac{\sigma^{1/2}}{R^{5/2} Q^2}. \quad (73)$$

As in the case of the massive field discussed in the preceding section, this expression is only justified when $D_- \gg 1$. If Q^2 is too large, then the field will not behave as massless in the range (71), and we expect that for $Q^4 \gg \sigma R^{-5}$ the sole effect of the field will be to renormalize the coefficients of operators such as the brane tension in the classical Lagrangian. An inconvenient feature of the linear potential (70) is that it is nonanalytic at the origin, and hence an explicit calculation in the limit of large slope is not straightforward. Moreover, we cannot write down an expression for it in terms of the matrix operator ϕ , but just in terms of its eigenvalues φ_i .

Another possibility, which is more tractable from the formal point of view, is to assume that there is an attractive interbrane potential which is quadratic at short distances. That is, as in (70) with $Q^2 = 0$ but with a positive coefficient in front of the second term. In terms of the eigenvalues φ_i , which represent the displacements of the branes, we assume the following expression for the potential

$$\mathcal{V}(\varphi_j) = m_+^2 \varphi_+^2 + \frac{1}{2k} m_-^2 \sum_{ij} (\varphi_i - \varphi_j)^2 + \dots, \quad (74)$$

where $\varphi_+ = k^{-1/2} \sum_{i=1}^k \varphi_i = k^{-1/2} \text{Tr}(\phi)$ and $m_+^2 = -3R^{-2}$ [as given in Eq. (68)], while $m_-^2 > 0$ is a new parameter which characterizes the attractive interaction at short distances. In terms of the field ϕ , we can write the potential as

$$\begin{aligned} \mathcal{V}(\phi) &= m_+^2 k^{-1} (\text{Tr } \phi)^2 + m_-^2 [\text{Tr } \phi^2 - k^{-1} (\text{Tr } \phi)^2] \\ &= \frac{1}{2} m_+^2 \phi_1^2 + \frac{1}{2} m_-^2 \sum_{b=2}^k \phi_b^2, \end{aligned} \quad (75)$$

where in the last equality we have expanded $\phi = \phi_a \lambda_a$ in the basis of generators λ_a , and we have used $\lambda_1 = (2k)^{-1/2} \mathbf{1}$ and

$\text{Tr} \lambda_b = 0$ for $b = 2, \dots, k^2$. In the symmetric phase (and assuming $m_- R \ll 1$) each one of the adjoint fields ϕ_b will contribute a determinantal prefactor of the form

$$D_- \approx \frac{e^{\zeta_R'(-2)}}{\pi^{1/2} m_- R}, \quad (m_- R \ll 1) \quad (76)$$

where we have used (63). This shows a somewhat milder dependence in R than in the case of a linear interaction between branes, given in Eq. (73), but still of power law form. In the limit of large mass $D_- \approx \exp(\pi m_-^3 R^3/6)$, which as discussed before amounts to a finite renormalization of the brane tension.

Aside from scalars, we should also consider the contributions from gauge bosons and fermions. For the case of a 3-sphere, these have been studied in Ref. [24]. For vectors of mass m_A , the result is

$$W_A = -\frac{1}{2} \log \left(\frac{g^2 R}{4 \pi^2} \right) - \log \left(\frac{\sinh(\pi m_A R)}{\pi m_A R} \right) - \int_0^{m_A R} y^2 \frac{d}{dy} \times \log(\sinh \pi y) dy - \zeta_R'(-2) + 2 \zeta_R'(0), \quad (77)$$

where g is the gauge coupling, which is dimensionful in three dimensions. When the branes are coincident, the theory is in the symmetric phase and the gauge bosons are massless. A massless gauge boson gives a contribution of the form

$$D_A = e^{-W_A} = g R^{1/2} e^{\zeta_R'(-2)} \quad (m_A = 0), \quad (78)$$

which again behaves as a power of R . A Dirac fermion of mass m_Ψ yields the contribution [24]

$$W_\Psi = \frac{1}{4} \log \cosh(\pi m R) + \pi \int_0^{mR} u^2 \tanh(\pi u) du + 2 \zeta_R'(-2, 1/2) - \frac{1}{2} \zeta_R'(0, 1/2). \quad (79)$$

For the massless case, the R dependent terms vanish and we have

$$D_\Psi = 2^{-1/4} e^{(3/2)\zeta_R'(-2)} \quad (m_\Psi = 0), \quad (80)$$

which is a constant independent of the radius R .

C. Nucleation rate

Collecting all one-loop contributions, the prefactor in Eq. (47) due to the weakly coupled $U(k)$ gauge theory in the unbroken phase is given by

$$D = \frac{D_+}{\Omega} (D_-)^{k^2-1} (D_A)^{k^2} (D_\Psi)^{k^2}. \quad (81)$$

Using (69), (76), (78), and (80) we are led to a nucleation rate per unit volume of the form

$$\Gamma_k \approx \frac{\pi^{1/2} m_- \sigma_k^2}{4} R^3 \left(\frac{e^{7\zeta_R'(-2)} g^2}{\sqrt{2} \pi m_-^2 R} \right)^{k^2/2} e^{-B_k} \quad (m_- R \ll 1), \quad (82)$$

where B_k is the corresponding bounce action for the nucleation of k coincident branes. Note that the R dependence of the prefactor is simply as a power law. Here we have used the form (76) for the scalar contribution D_- , corresponding to an attractive interaction amongst branes which is quadratic at short distances, with a curvature of the potential characterized by some mass parameter m_- .² The prefactor in Eq. (82) has the exponential dependence on k^2 which counts the number of worldsheet field degrees of freedom, while the Euclidean action B_k behaves approximately linearly with k . Hence, as suggested in Ref. [2], the prefactor can be quite important in determining the nucleation rate.

In the scenario proposed in Ref. [2] it was also desirable that the enhancement in the nucleation rate would switch off at the present time, in order to prevent the vacuum from decaying further. Unfortunately, the expression (82) does not seem to have this property. The prefactor depends only on the radius R , which is itself a function of various parameters, such as the brane tension, the charge and the ambient expansion rate, as summarized in Table I. According to (82), an enhancement of the nucleation rate of coincident branes will occur for

$$R \sim \text{Min} \left\{ \frac{3 \sigma_k}{\epsilon_k}, H_o^{-1}, \frac{4 M_p^2}{\sigma_k} \right\} \ll (g/m_-)^2, \quad (83)$$

where we have used the results of Table I in the first step. Consider first the situation where $\sigma_k M_p^{-2} \ll H_o$. Note that even in the regime when $R \approx H_o^{-1}$, the dependence of the degeneracy factor on the corresponding dS temperature is only power law, and not exponential as suggested in Ref. [2]. Also, it is clear the stability of our vacuum is not guaranteed by the smallness of the present expansion rate. The enhancement will persist provided that $\sigma_k / \epsilon_k \ll (g/m_-)^2$, even if the ambient de Sitter temperature vanishes. More worrisome is the fact that for sufficiently large k we enter the regime where $\sigma_k M_p^{-2} \gg H_o$. In that case, we have

$$R \sim M_p^2 / \sigma_k,$$

which can get smaller and smaller as we increase the number of coincident branes, eventually leading to a catastrophic decay rate, regardless of the value of H_o .

The expression (82) for the nucleation rate is valid for $m_- R \ll 1$. In the opposite limit, $m_- R \gg 1$, the scalars decouple, contributing a finite renormalization of the parameters in the action (such as the brane tension and induced Newton's constant). For completeness, this is discussed in

²If we assume instead a linear interaction at short distances, we should use (73) and the behavior changes to $D \propto R^{(9/2)-2k^2}$, but in any case the dependence is still a power of the radius R .

Appendix A, where it is shown that the renormalization of parameters can have a very significant impact on the nucleation rate.

D. Discussion: Temperature of a vacuum bubble

In Ref. [2], the prefactor in the nucleation rate for the nucleation of coincident branes was estimated as an entropy enhancement

$$D \sim e^S,$$

where, from dimensional analysis, the entropy was estimated as $S \propto T^2 R^2$ per field degree of freedom. The factor R^2 is due to the area of the bubble, and T is some effective temperature. Although the interpretation of the prefactor as the exponential of an entropy should not be taken too literally, let us try and phrase the results of the preceding subsection in this intuitive language.

A particle detector following a geodesic in a de Sitter space responds as if it was at rest in a thermal bath in flat space, at the temperature $T_o = H_o/2\pi$. It should be kept in mind, however, that the dS invariant quantum state is in fact a pure state, and hence rather different from a true thermal state. For instance, any two detectors in geodesic relative motion observe the same temperature, with a perfectly isotropic distribution. This is a consequence of de Sitter invariance, and is in contrast with the situation in a thermal bath in flat space, where moving observers detect a temperature blue-shift in the direction of their motion relative to the bath.

The fields living on a nucleated brane will experience some thermal effects too. The bubble is embedded in a dS space characterized by a temperature T_o . The interior of the bubble is also a dS space characterized by a different expansion rate, with corresponding temperature T_i . The existence of two different de Sitter spaces in contact with the brane led the authors of Ref. [2] to consider two different possibilities for the effective temperature of the fields on the brane: $T = T_o$ and $T = (T_o T_i)^{1/2}$. However, there is in fact no ambiguity in the temperature of such fields [4], which is determined as follows.

The worldsheet of the brane is an S^3 of radius R , the Euclidean de Sitter space in 2+1 dimensions. If interactions with bulk fields are neglected, brane fields are only sensitive to the geometry of the worldsheet, and do not know about the properties of the ambient space. In this approximation, the relevant temperature is clearly the intrinsic temperature of the lower dimensional de Sitter space,

$$T_R = 1/2\pi R. \quad (84)$$

This conclusion remains unchanged when we include interactions with bulk fields. The simplest way to see this is to consider the limiting case where gravity can be ignored and the nucleation takes place in a flat space. There, the ambient temperature vanishes $T_o = 0$, but the fields on the brane will feel the temperature T_R , because the nucleated brane expands with constant acceleration $a = 1/R$. An accelerating observer in the Minkowski vacuum will detect a Rindler temperature $T_R = a/2\pi$, which happens to coincide with the

intrinsic worldsheet temperature. Hence, the de Sitter vacuum in the 2+1 dimensional worldsheet is in equilibrium with the Minkowski vacuum in the bulk. This conclusion is quite general, and applies also to bubbles nucleating in de Sitter. The CdL instanton has an $O(4)$ symmetry under Euclidean rotations. This becomes an $O(3,1)$ symmetry after analytic continuation into Lorentzian time. The quantum state after bubble nucleation is expected to inherit this symmetry [5,26,27], and the only way to achieve it is if the fields on the brane are in their intrinsic de Sitter vacuum, which is characterized by temperature T_R .

Note that T_R is a relatively high temperature. The radius of the instanton is always smaller than H_o^{-1} (see Table I), and therefore T_R is strictly larger than T_o and T_i . Nevertheless, the product $k^2 T_R^2 R^2 \sim k^2$, and hence the ‘‘entropy enhancement,’’ is independent of the ambient de Sitter temperature. As shown in the previous subsections, the independence of the prefactor D on the ambient expansion rate is only approximate, due to the anomalous behavior of light fields in de Sitter space. This introduces a dependence of the effective action $W_{eff} = -\log D$ on the radius R of the instanton (which in turn may depend on H_o in certain regimes). This dependence, however, is quite different from the one proposed in Ref. [2], where it was suggested that the nucleation rate of coincident branes would be enhanced at high H_o , and would switch off at low H_o due to the drop in ambient temperature. What we find instead is that, if the CdL instanton for nucleation of coincident branes really exists, then the corresponding degeneracy factor does not necessarily switch off.

We shall return to a discussion of this point in the concluding section. Before that, let us turn our attention to a different instanton, which may be relevant to the FMSW scenario.

VIII. PAIR CREATION OF CRITICAL BUBBLES

Euclidean de Sitter space is compact in all spacetime directions, and (as we just discussed) it behaves in some respects as a system at finite temperature. One may then ask whether there are instantons similar to the thermal ones in flat space. These correspond to static bubbles, in unstable equilibrium between expansion and collapse.

Static instantons with $O(3)$ symmetry have previously been considered in a variety of contexts, notably for the description of false vacuum decay in the presence of a black hole (see e.g. Refs. [28,29] and references therein). The particular instanton we shall consider corresponds to pair creation of critical bubbles in de Sitter, and to our knowledge it does not seem to have received much attention in the past. This is perhaps not surprising, since its action is higher than that of the action for the maximally symmetric CdL instanton. However, if the CdL instanton does not exist for coincident branes, the static one may turn out to be relevant once the degeneracy factors are taken into account.

A. The instanton solution

The energy of a critical bubble is different from zero, and consequently, the metric outside of the bubble is no longer

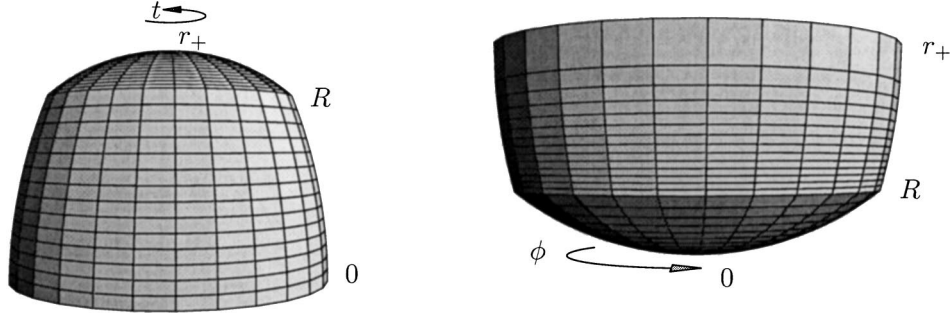


FIG. 6. Static instanton in de Sitter space. The left figure shows the geometry induced on the plane r, t , while keeping angular coordinates fixed, whereas the right figure shows the geometry induced on the plane r, ϕ , keeping θ and t fixed. The vertical direction corresponds to the coordinate r , common to both pictures. The cosmological horizon is at $r=r_+$, the brane is at $r=R$, and $r=0$ is the center of the static bubble of the new phase.

pure de Sitter, but Schwarzschild–de Sitter (SdS). The instanton is a solution of the Euclidean equations of motion, with two metrics glued together at the locus of the wall, which is a surface of constant r in the static chart of SdS (see Fig. 6). The metric outside is given by

$$ds^2 = f_o(r)dt^2 + f_o^{-1}(r)dr^2 + r^2d\Omega^2, \quad (85)$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$, and

$$f_o(r) = \left(1 - \frac{2GM}{r} - H_o^2 r^2\right). \quad (86)$$

The metric inside is given by

$$ds^2 = C^2 f_i(r)dt^2 + f_i^{-1}(r)dr^2 + r^2d\Omega^2, \quad (87)$$

where

$$f_i(r) = (1 - H_i^2 r^2), \quad (88)$$

corresponding to a de Sitter solution. The constant C is determined by the condition that on the brane (i.e., at $r=R$) the two metrics must agree, which leads to $C = [f_o(R)/f_i(R)]^{1/2}$.

The parameters M and R depend on σ_k , H_o and H_i . Their values are determined by the junction conditions at the brane [14],

$$[K_{ab}] = -4\pi G\sigma_k\gamma_{ab}, \quad (89)$$

where $[K_{ab}]$ is the difference in the extrinsic curvature $K_{ab} = (1/2)f^{1/2}\partial_r g_{ab}$ on the two sides and γ_{ab} is the worldsheet metric. Equation (89) gives rise to the junction conditions,

$$[g] = -4\pi G\sigma_k, \quad [g'] = 0, \quad (90)$$

where we have introduced the new function $g(r) = f^{1/2}(r)/r$. Using Eqs. (86) and (88), we have

$$g_o g'_o = -\frac{1}{r^3} + \frac{3GM}{r^4}, \quad g_i g'_i = -\frac{1}{r^3}. \quad (91)$$

Hence, using (90), $g'_o(R) = g'_i(R) = -3M/4\pi\sigma_k R^4$, and then $g_i(R)$ and $g_o(R)$ are easily obtained from Eqs. (91):

$$g_i(R) = \frac{4\pi\sigma_k R}{3M}, \quad g_o(R) = g_i(R) \left(1 - \frac{3GM}{R}\right). \quad (92)$$

From (86) and (88) we have

$$g_o^2(R) = \frac{1}{R^2} - \frac{2GM}{R^3} - H_o^2, \quad g_i^2(R) = \frac{1}{R^2} - H_i^2. \quad (93)$$

Inserting (92) in (93) we finally obtain a quadratic equation for $g_i(R) \equiv x$. The solution is

$$x = \frac{\epsilon}{4\sigma_k} + \frac{3\sigma_k}{16M_p^2} + \left[\left(\frac{\epsilon}{4\sigma_k} + \frac{3\sigma_k}{16M_p^2} \right)^2 + \frac{H_i^2}{2} \right]^{1/2}, \quad (94)$$

where we have used $H_o^2 - H_i^2 = 8\pi G\epsilon/3 = \epsilon/3M_p^2$. Then the parameters M and R are given in terms of x by

$$R^{-2} = x^2 + H_i^2, \quad M = 4\pi\sigma_k R/3x. \quad (95)$$

This concludes the construction of the instanton solution for given values of the physical parameters σ_k , H_o and H_i .

The equation $f_o(r) = 0$ has three real solutions for $27H_o^2 M^2 G^2 < 1$. One of them, say r_- , is negative and the other two are positive. The two positive roots correspond to the black hole and cosmological horizons. We call them respectively r_s and r_+ . Therefore we can write

$$f_o(r) = -\frac{H_o^2}{r}(r-r_-)(r-r_s)(r-r_+). \quad (96)$$

Of course, in our instanton the horizon at r_s is not present, since the exterior metric is matched to an interior metric at some $r=R > r_s$ (see Fig. 6). For $r < R$ the metric is just a ball of de Sitter in the static chart, and it is regular down to the center of symmetry at $r=0$. In general, the size of the cosmological horizon is given by

$$H_o r_+ = \frac{2}{\sqrt{3}} \cos\left(\frac{\varphi + \pi}{3}\right), \quad (97)$$

where we have introduced the angle

$$\varphi = -\arctan \sqrt{\frac{1}{27H_o^2 M^2 G^2} - 1}. \quad (98)$$

In the limit $M \rightarrow 0$ the angle $\varphi \rightarrow -\pi/2$, and $H_o r_+ \rightarrow 1$.

According to Eq. (92), at the brane we have $f_o(R) = x^2(R - 3GM)^2$, so the equation $f_o(R) = 0$ has a double zero instead of two different roots. This means that the radius of the instanton will coincide with the radius of one of the horizons only in the special case where both horizons have the same size, $r_s = r_+ = R = 3GM$. In the limit $r_s = r_+$ the exterior metric becomes the Nariai solution [30,31], which has $r = (\sqrt{3}H_o)^{-1}$. Note that in the limit $3GM \rightarrow (\sqrt{3}H_o)^{-1}$, $\varphi \rightarrow 0$ and $H_o r_+ \rightarrow 1/\sqrt{3}$, as expected.

Like in the case of instantons describing the production of black holes [30] or monopoles [22] in de Sitter metric, the instanton presented here describes the creation of *pairs* of bubbles. As we shall see, the Euclidean solution is periodic in the time direction, so that time runs on a circle S^1 (see Fig. 6). The geometry at the time of nucleation is obtained by slicing the compact instanton through a smooth spacelike surface that cuts the S^1 factor at two places, say, $t=0$ and $t=\pi$. The resulting geometry contains two different bubbles separated by a distance comparable to the inverse expansion rate.

B. Temperature and action

In order to calculate the temperature of the worldsheet in the static instanton we must first find the time periodicity β . This is determined by the regularity of the Euclidean metric at the cosmological horizon. For $r \rightarrow r_+$, we have

$$f_o(r) \approx A^2 \left(1 - \frac{r}{r_+}\right), \quad (99)$$

where

$$A^2 = H_o^2 (r_+ - r_-)(r_+ - r_s) = 3H_o^2 r_+^2 - 1. \quad (100)$$

In terms of the new coordinates

$$\rho = \frac{2r_+}{A} \sqrt{1 - \frac{r}{r_+}}, \quad \phi = \frac{A^2}{2r_+} t, \quad (101)$$

the metric (85) for $r \rightarrow r_+$ reads

$$ds^2 = \rho^2 d\phi^2 + d\rho^2 + r_+^2 d\Omega^2, \quad (102)$$

so it is clear that ϕ is an angle, $0 \leq \phi \leq 2\pi$, and t varies in the range $0 \leq t \leq 4\pi r_+ / A^2$. Therefore the value of β is

$$\beta = \frac{4\pi r_+}{3H_o^2 r_+^2 - 1} = \frac{2\pi r_+^2}{r_+ - 3GM}. \quad (103)$$

The temperature of the worldsheet instanton is given by the proper time periodicity $\beta_R \equiv \int_0^\beta f_o^{1/2}(R) dt = f_o^{1/2}(R) \beta = C f_i^{1/2}(R) \beta$. Hence, the inverse temperature is given by

$$\beta_R = 2\pi x r_+^2 \frac{R - 3GM}{r_+ - 3GM}. \quad (104)$$

We shall also be interested in the Euclidean action, which turns out to have a rather simple expression in terms of r_+ . This is derived in Appendix B, where it is shown that the difference in Euclidean actions between the instanton and the background solutions is given by

$$B = \frac{\pi}{GH_o^2} (1 - r_+^2 H_o^2). \quad (105)$$

C. Some limiting cases

Let us start with the case of low tension branes, $\sigma_k / M_p^2 \ll H_o, H_o - H_i$. In this case the parameter x is large compared with H_o , $R \approx x^{-1}$ is small, and $GMH_o \ll 1$. In this limit the angle φ in Eq. (98) is close to $-\pi/2$ and $H_o r_+ \approx 1$. We have

$$B \approx \frac{2\pi}{H_o} \frac{16\pi\sigma_k^3}{3\epsilon^2}, \quad \beta_R \approx \frac{2\pi}{H_o}. \quad (106)$$

This is just the flat space expression for the energy of a critical bubble, multiplied by Euclidean time periodicity of the low curvature de Sitter space in which this bubble is embedded.

Next, we may consider the case of intermediate tension $H_o - H_i \ll \sigma_k / M_p^2 \ll H_o, H_i$. In this case, $x \approx H_i / \sqrt{2}$, $R \approx (\sqrt{3}x)^{-1}$, $H_o r_+ \approx 1 - GMH_o$, with $GMH_o \ll 1$, and we have

$$B \approx \frac{16\pi^2}{3\sqrt{3}} \frac{\sigma_k}{H_o^3}, \quad \beta_R \approx \frac{2\pi}{\sqrt{3}H_o}. \quad (107)$$

In this case, the difference in pressure between inside and outside of the brane is insignificant compared with the brane tension term, which is balanced against collapse by the cosmological expansion.

Finally, in the limit of very large σ_k we find that $3GM$ becomes larger than R , namely, $3GM \rightarrow 4R/3$. This means that $f_o(R)$ vanishes for some value $\sigma_k = \sigma_{\max}$, given below in Eq. (118), so it is not sensible to consider the limit of very large σ_k but just the limit $\sigma_k \rightarrow \sigma_{\max}$. As we have mentioned [see the discussion below Eq. (96)], the exterior metric in this limit corresponds to the Nariai solution, with $r_s = r_+ = (\sqrt{3}H_o)^{-1}$. Replacing this value in (105) we find readily

$$B = \frac{2\pi}{3GH_o^2}. \quad (108)$$

It is interesting to compare this value of B with the corresponding one for the nucleation of black holes in the same de Sitter universe. This is described by the Nariai instanton [30], which has the bounce action

$$B_N = \frac{\pi}{3GH_o^2}. \quad (109)$$

Note that the difference $B - B_N = \pi/3GH_o^2 = A_{bh}/4G$, where $A_{bh} = 4\pi r_s^2$ is the area of the black hole horizon in the Nariai solution. Hence, the probability of nucleating black holes divided by the probability of nucleating brane configurations characterized by the same mass parameter is just the exponential of the black hole entropy, as expected from general considerations (in this argument, we are of course neglecting the entropy stored in the field degrees of freedom living on the branes, which will only show up when the determinantal prefactor in the nucleation rate is evaluated).

Let us consider the value of β_R in the limit $\sigma_k \rightarrow \sigma_{max}$. This is a singular limit in Eq. (104) due to the simultaneous vanishing of numerator and denominator. Thus we will need to change to more appropriate coordinates. The fact that $r_s = r_+$ does not mean, though, that both horizons coincide, since the coordinates r, t become inadequate in this case. Near this limit the metric outside takes the form (85), with

$$f_o(r) \approx A^2 \left(1 - \frac{r}{r_+} \right) - \left(1 - \frac{r}{r_+} \right)^2, \quad (110)$$

and $r \approx r_+$, plus higher orders in A . The constant A is the same parameter defined in (100), but in this limit tends to zero, $A^2 = \sqrt{3}H_o(r_+ - r_s)$. Now we define new coordinates ψ and λ by

$$\cos \psi = 1 - \frac{2}{A^2} \left(1 - \frac{r}{r_+} \right), \quad \lambda = \frac{A^2}{2} t, \quad (111)$$

so that the metric becomes

$$ds^2 = \sin^2 \psi d\lambda^2 + r_+^2 d\psi^2 + r_+^2 d\Omega^2. \quad (112)$$

In these coordinates the cosmological horizon is at $\psi = 0$ and the black hole horizon is at $\psi = \pi$. Now in the limit $A \rightarrow 0$ we just replace $r_+ = (\sqrt{3}H_o)^{-1}$.

We must determine the position ψ_R of the brane, which is given as before by the matching conditions (89), where now the metric outside is (112). So, on the brane, we have

$$ds_{\sigma_k}^2 = \sin^2 \psi_R d\lambda^2 + r_+^2 d\Omega^2 \quad (113)$$

$$= f_i(R) dt'^2 + R^2 d\Omega^2 \quad (114)$$

and the extrinsic curvature on the outside of the brane is $-(1/2)\partial_\psi g_{ab}$, with $g_{00} = \sin^2 \psi$ and $g_{\Omega\Omega} = r_+^2$, i.e., $K_{00} = -(1/r_+)g_{00} \cot \psi$, $K_{\Omega\Omega} = 0$. The curvature inside is as before $K_{00} = g_{00}\partial_r f_i^{1/2}$ and $K_{\Omega\Omega} = g_{\Omega\Omega}f_i^{1/2}/r$, with $f_i(r) = (1 - H_i^2 r^2)$, so the Israel conditions give

$$-\frac{1}{r_+} \cot \psi_R - (f_i^{1/2})'|_R = -4\pi G \sigma_k, \quad (115)$$

$$f_i^{1/2}(R)/R = 4\pi G \sigma_k. \quad (116)$$

These equations are easily solved and give

$$\sin \psi_R = \left(\frac{3H_o^2 - H_i^2}{6H_o^2 - H_i^2} \right)^{1/2}, \quad (117)$$

$$\sigma_k = \sigma_{max} = 2M_p^2 \sqrt{3H_o^2 - H_i^2}, \quad (118)$$

so H_i must be less than $\sqrt{3}H_o$. Now regularity at the cosmological horizon $\psi = 0$ implies that $0 \leq \lambda/r_+ \leq 2\pi$, so $\beta_R = \sin(\psi_R)2\pi r_+$. Hence,

$$\beta_R = \frac{2\pi}{\sqrt{3}H_o} \left(\frac{3H_o^2 - H_i^2}{6H_o^2 - H_i^2} \right)^{1/2}. \quad (119)$$

Thus, also in this case, the effective temperature of the field degrees of freedom living on the worldsheet will be of order H_o or higher.

D. The prefactor for static instantons

In flat space, and at finite temperature $T \gg \epsilon_k/\sigma_k$, the instanton which is relevant for vacuum decay is static and spherically symmetric in the spatial directions. The fluctuations are periodic in Euclidean time, with periodicity $\beta = 1/T$. The worldsheet of the brane has the topology $S^1 \times S^2$, where the S^1 is the direction of imaginary time, and the S^2 is the boundary of the ‘‘critical’’ bubble, a closed brane in unstable equilibrium between expansion and collapse (this is in contrast with the zero temperature instanton, where the worldsheet is a 3-sphere). The radius of the critical bubble is given by

$$R_\beta = \frac{2\sigma_k}{\epsilon_k},$$

and the difference of the instanton action and the action for the background is given by

$$B_\beta = \beta E^{(0)},$$

where $E^{(0)} = (4\pi/3)\sigma_k R_\beta^2$ is the classical energy of the critical bubble. The one loop quantum correction can be written as (see e.g. Ref. [32])

$$W_\Phi = \beta F_\Phi = \beta(E_\Phi - TS_\Phi). \quad (120)$$

Here F_Φ denotes the free energy and E_Φ is the correction to the energy of the critical bubble due to the field Φ . This includes the zero point energy of Φ in the presence of the bubble, as well as the thermal contributions. Finally, S_Φ is the entropy. Thus, the nucleation rate takes the form

$$\Gamma_\beta = D e^{-B_\beta} \sim e^{-(B_\beta + k^2 W_\Phi)} \sim e^{-\beta F} \sim e^{-E/T} e^S,$$

where $E = E^{(0)} + k^2 E_\Phi$ is the total energy, $F = E^{(0)} + k^2 F_\Phi$ is the total free energy, and $S = k^2 S_\Phi$ is the total entropy. Thus, for thermal instantons the determinant prefactor does indeed include the exponential of the entropy. This is, however, not the only role of the prefactor, since there is also some correction to the energy of the bubble.

Consider, for simplicity a massless field Φ . At sufficiently high temperature $T \gg R_\beta^{-1}$, the entropy behaves as $S_\Phi \propto T^2 R_\beta^2$. From $S_\Phi = -\partial F_\Phi / \partial T$, it follows that $F_\Phi \approx -TS_\Phi/3$. Hence

$$\Gamma_\beta = D e^{-B_\beta} \sim e^{-B_\beta e^{+k^2 S_\Phi/3}}.$$

In this case, the prefactor clearly represents an ‘‘entropy enhancement.’’ On the other hand, at lower temperatures, it is not clear whether the prefactor represents an enhancement or a suppression. At temperatures comparable to R_β^{-1} , the vacuum energy term can be as important as the thermal contributions, and the logarithm of the prefactor D can have either sign.

The case of the static instanton in de Sitter space is somewhat close to this low temperature situation, and without an explicit calculation it is not clear whether the prefactor represents an enhancement or a suppression of the nucleation rate. An interesting possibility would be that at sufficiently high Gibbons-Hawking temperature $T_o \sim H_o$, the thermal contribution may be sufficient to restore the symmetry, creating the desired attractive force amongst the branes. This is currently under research.

The static instanton presented in this section may perhaps be better suited to the scenario proposed by Feng et al. [2], than the Coleman–de Luccia instanton. Ignoring the degeneracy factors, the action of the static instanton is always larger than the action of the CdL instanton. In this sense, it seems to correspond to a subdominant decay channel. However, as we have discussed in previous sections, it might well be that the CdL instanton for multiple brane nucleation simply does not exist because of the repulsive force amongst the branes. This does not exclude the possibility that in the case of the static thermal instantons the symmetry is restored at high ambient (and worldsheet) dS temperature $\sim H_o$. In this situation, the decay through nucleation of coincident branes would only be possible through the static instanton.³ At low H_o , the thermal contribution might not be sufficient to restore the symmetry and stacks of branes may simply not hold together, destroying the possibility of further decay by coincident brane nucleation. Also, the prefactor and the Euclidean action have an exponential dependence on the ambient temperature $\sim H_o$, and can be much suppressed at the present epoch, contributing to the stability of the present vacuum (in contrast with the Coleman–de Luccia case).

E. An entropy bound

A potentially worrying aspect of coincident brane nucleation in the CdL case is whether the degeneracy factor may grow without bound as we increase the number of branes [2]. As we have seen, this will not happen for the case of the static instanton discussed in this section, since nucleation of coincident branes cannot involve arbitrarily large k . Indeed,

there is a maximum value of the combined tension of the branes $\sigma_k \ll \sigma_{max} \sim M_p^2 H_o$, given in Eq. (118), beyond which the instanton simply does not exist. In this limit, the metric outside of the branes approaches the Nariai solution.

From this observation, we can easily derive a bound on the maximum entropy which can be stored in the stack of branes. Indeed, the static instanton represents a spherical bubble in unstable equilibrium between undeterred expansion or collapse into a black hole. The entropy can only increase when the stack of branes collapses, and hence the entropy of the coincident branes cannot exceed the entropy of the Nariai black hole. The stack of branes with tension $\sigma_k \rightarrow \sigma_{max}$ has the same radius as the horizon of the Nariai black hole, and so, the entropy of the stack of branes is bounded by one fourth of its own area, in natural units. A corollary is that the nucleation rate of coincident branes in the limit $\sigma_k \rightarrow \sigma_{max}$ is bounded above by the nucleation rate of Nariai black holes of the same mass.

IX. CONCLUSIONS AND DISCUSSION

In this paper we have investigated the possibility of coincident brane nucleation by a four-form field, in connection with string motivated scenarios for the neutralization of the effective cosmological constant.

In four dimensions, and after the moduli are stabilized, the branes repel each other at distances larger than the inverse mass of the moduli. At shorter distances, their interactions will be model dependent, but in the simplest models the branes do not attract at the classical level. In this situation, it is unclear whether the Coleman–de Luccia (CdL) instanton for nucleation of coincident branes really contributes to the semiclassical decay rate, since it would have too many zero modes and negative modes.

Assuming that the CdL instanton exists for the nucleation of coincident branes (that is, assuming an attractive short range interaction amongst the branes in the stack), we have investigated the degeneracy factor accompanying the formula for the nucleation rate, due to the large number of worldsheet degrees of freedom. We have modeled such degrees of freedom by a weakly coupled SYM $U(k)$ gauge theory, which is unbroken when the branes are coincident. We find that the degeneracy factor does not depend very strongly on the ambient de Sitter (dS) temperatures before or after the nucleation event. Rather, it depends only on the radius of the instanton. Hence the degeneracy factors can be quite important even when the ambient dS temperature is as low as it is today. This may indicate that nucleation of coincident branes via the CdL instanton is in fact impossible, otherwise the present vacuum would immediately decay.

If the CdL instanton for coincident branes does not exist, stacks of branes may still nucleate through a ‘‘static’’ instanton which represents pair creation of critical bubbles, in unstable equilibrium between expansion and collapse. This is the analog of the instanton for thermal activation in flat space. Despite the absence of a classical attractive force, the branes could be held together by thermal corrections to the interbrane potential, which tend to favor the symmetric phase (where branes are on top of each other). The calculation of

³In the weak coupling limit, we have checked that indeed the $U(k)$ symmetry is not restored in the case of the CdL instanton [24].

this thermal effective potential for the static instanton is currently under research. One may ask whether a similar symmetry restoration may not happen for the CdL instanton. In this case the calculation has been done in Ref. [24], where it is shown that the one loop potential does not help restoring the symmetry. So it is conceivable that the branes may stick together for the static instanton but not for the CdL instanton, in which case the former would be the relevant decay channel.

To conclude, we have presented some evidence that the “saltatory” relaxation scenario of Ref. [2] may be difficult to implement via the CdL instanton, since saltation would be hard to stop at present. Rather, we have speculated that it may be easier to implement through the static instanton. In the scenarios proposed in Ref. [2] for the saltatory relaxation of the cosmological constant, two different possibilities were suggested for the effective temperature of the worldsheet degrees of freedom, namely $T_1 \sim H_o$ and $T_2 \sim (H_o H_i)^{1/2}$, where H_i and H_o are the expansion rates before and after nucleation. We have shown that for the static instanton, the relevant temperature is comparable to the ambient de Sitter temperature $\sim H_o$ before the tunneling. Hence, the nucleation rate of coincident branes would be unsuppressed at large ambient de Sitter temperature, but exponentially suppressed at present, which is of course desirable.

Clearly, many issues need to be addressed before a scenario based on coincident brane nucleation can be used to successfully explain the smallness of the observed cosmological constant. A considerable advance would be to understand why the large Λ_{eff} relaxes to the small Λ_{obs} instead of plunging directly into deep AdS space (the latter jump involves a larger number of coincident branes and would be rewarded by a larger degeneracy factor). In Ref. [2] an explanation was offered, based on a “uniquely weak” form of the anthropic principle. As explained in Sec. II, any relaxation mechanism requires the gap $\Delta\Lambda$ in the discretuum of Λ_{eff} not to be much larger than Λ_{obs} (otherwise it becomes a problem to understand why, accidentally, there happens to be an allowed vacuum so close to zero, at $\Lambda_{eff} = \Lambda_{obs} \ll \Delta\Lambda$). In Ref. [2] it was proposed that $\Delta\Lambda = a\Lambda_{obs}$ with $a \sim 1$, saturating the above requirement. Then the allowed Λ_{eff} would take values in the sequence $\dots, (1-a)\Lambda_{obs}, \Lambda_{obs}, (1+a)\Lambda_{obs}, (1+2a)\Lambda_{obs}, \dots$. If we start from a large Λ_{eff} , then the enhancement of brane nucleation for large k favors a jump to the lowest value in the above list which is still compatible with the existence of observers. FMSW suggested that the value $(1-a)\Lambda_{obs}$ may already be too small for observers to emerge, making the vacuum with the value Λ_{obs} the favorite destination.

Finally, one should try to embed this scenario in a cosmological context, taking into account the restrictions imposed by homogeneity and isotropy. If unsuppressed saltation happened after inflation, then we would have seen signals of it in the microwave background. Indeed, bubbles that nucleate after thermalization are still rather small at the time of decoupling, and we would see different domains with different values of Λ_{eff} separated by fast moving stacks of branes, which would presumably cause large perturbations in the gravitational potential. Hence, saltation should occur during

inflation, and switch off somewhat before the end of it. This may impose certain constraints on the space of parameters such as the tension and charges of the branes, or alternatively, on the ambient temperature below which the instanton with coincident branes simply does not exist (e.g. because thermal symmetry restoration is no longer effective). Also, it should be clarified what might be the advantages of a saltatory “neutralization” scenario over the “randomization” scenarios discussed in Sec. II. A possible advantage is that saltatory relaxation operates very quickly, and hence it does not require eternal inflation to take place (as required in the randomization scenarios). A fuller discussion of these issues is left for further research.

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APPENDIX A: HEAVY FIELDS ON THE BRANES

The expression (82) for the nucleation rate is only valid for $m_{-}R \ll 1$. For completeness, here we shall discuss the limit $m_{-}R \gg 1$. In this case the heavy degrees of freedom decouple, and they simply contribute a finite renormalization of the parameters in front of different operators in the classical Lagrangian.

For scalar fields we have

$$W_{-} = -\frac{\pi}{6} m_{-}^3 R^3 + \frac{\pi}{4} m_{-} R + \dots \quad (A1)$$

As discussed in Sec. VII, the first term in this expansion corresponds to a finite renormalization of the brane tension, multiplied by the worldvolume of the stack of branes. The second term correspond to a finite renormalization of the coefficient in front of the worldsheet Ricci scalar. This term was not present in the classical action we started with, but evidently it can be generated by quantum corrections. The scalar contribution (A1) tends to decrease the tension of the stack of branes. This tends to favor the nucleation rate at large $m_{-}R$, as represented in Fig. 5. However, whether an actual enhancement really occurs will be model dependent, since all massive species, and not just the scalars, contribute finite renormalizations of the parameters in the action. In Sec. VII we have assumed that there is an attractive short range force amongst the branes, which we have modeled as a mass term for the scalars representing the relative positions of the branes. One may expect that the same mechanism which generates a potential for the scalars may generate masses also for their fermionic partners. From (79), heavy fermions give a contribution to the effective potential of the form

$$\frac{1}{2} W_{\Psi} = +\frac{\pi}{6} m_{\Psi}^3 R^3 + \frac{\pi}{8} m_{\Psi} R + \dots \quad (m_{\Psi} R \gg 1) \quad (A2)$$

per degree of freedom. This gives a positive renormalization of the brane tension (which tends to suppress the nucleation rate). In the special case where $m_- = m_\Psi$, the brane tension does not renormalize, but each pair of fields will still contribute a finite renormalization of the subleading term $\Delta W = (3\pi/8)m_-R$ which would suppress the nucleation rate. Generically, however, scalars and fermions may wind up with different masses (since supersymmetry is broken at some level), and the tension will be renormalized. Dividing the leading term in W by the worldvolume $\text{Vol}[S^3] = 2\pi^2 R^3$, each scalar and fermionic degree of freedom contributes a brane tension renormalization of the form

$$\Delta\sigma \approx \frac{m_\Psi^3 - m_-^3}{12\pi}. \quad (\text{A3})$$

Similarly, there will be a renormalization of the induced Newton's constant G_N on the worldsheet, of the order $\Delta G_N^{-1} \sim (m_\Psi - m_-)$ for each pair of heavy field species. These changes will modify the instanton solution. For k coincident branes, the number of such fields grows as k^2 , and the effect of these finite renormalizations can be quite dramatic. The nucleation rate will take the form

$$\Gamma_k \sim \sigma^2 R^2 (A g^2 R)^{k^2/2} e^{-B_k^{\text{ren}}} \quad (m_- R \gg 1) \quad (\text{A4})$$

where $A \sim 1$ is a constant that depends on whether some (or all) of the fermionic species have decoupled or not. The bounce action B_k^{ren} in the exponent is calculated by using the renormalized values of the parameters. If $\Delta\sigma < 0$, then the renormalized tension $\sigma_k \sim k\sigma + (k^2 - 1)\Delta\sigma$ sharply decreases for large k , leading to unsuppressed tunneling rate. On the contrary, for $\Delta\sigma > 0$ the nucleation of coincident branes is strongly suppressed.

APPENDIX B: EUCLIDEAN ACTION FOR THE STATIC INSTANTON

The action is given by

$$S_E(I) = \sigma \int d^3\xi \sqrt{\gamma} + \int d^4x \sqrt{g} \left(\rho_V - \frac{\mathcal{R}}{16\pi G} \right). \quad (\text{B1})$$

On shell, the scalar curvature is given by

$$\mathcal{R}\sqrt{g} = 32\pi G \rho_V \sqrt{g} + 24\pi G \sigma \int d^3\xi \sqrt{\gamma} \delta^{(4)}(x - x(\xi)), \quad (\text{B2})$$

and hence the instanton action is given by

$$S_E(I) = -\frac{\sigma}{2} \int d^3\xi \sqrt{\gamma} - \int d^4x \rho_V \sqrt{g}. \quad (\text{B3})$$

The first integral in (B3) is just the volume of a two-sphere of radius R times β_R . The second integral in (B3) splits into two contributions from the two regions,

$$\begin{aligned} \rho_i \int_0^R C dt dr 4\pi r^2 + \rho_o \int_R^{r_+} dt dr 4\pi r^2 \\ = \rho_i C \beta \frac{4}{3} \pi R^3 + \rho_o \beta \frac{4}{3} \pi (r_+^3 - R^3) \end{aligned} \quad (\text{B4})$$

$$= \rho_i C \beta \frac{4}{3} \pi R^3 + \rho_o \beta \frac{4}{3} \pi (r_+^3 - R^3) \quad (\text{B5})$$

So the instanton action is

$$\begin{aligned} S_E(I) = -2\pi R^2 \sigma f_o^{1/2}(R) \beta - R^3 \frac{H_i^2 f_o^{1/2}(R)}{2G f_i^{1/2}(R)} \beta \\ - (r_+^3 - R^3) \frac{H_o^2}{2G} \beta. \end{aligned} \quad (\text{B6})$$

After some algebra $S_E(I)$ can be written in the simple form

$$S_E(I) = -\frac{\pi r_+^2}{G}. \quad (\text{B7})$$

The exponent B which gives the probability for brane nucleation is the difference in Euclidean actions between instanton and background. The action of the background is just $S_E = -\pi/GH_o^2$, so the difference in Euclidean actions leads to Eq. (105).

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